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Simplified Mechanism for Anisotropic Failure in Modular Material Models

Al 6061-T6 Puncture Model Progression

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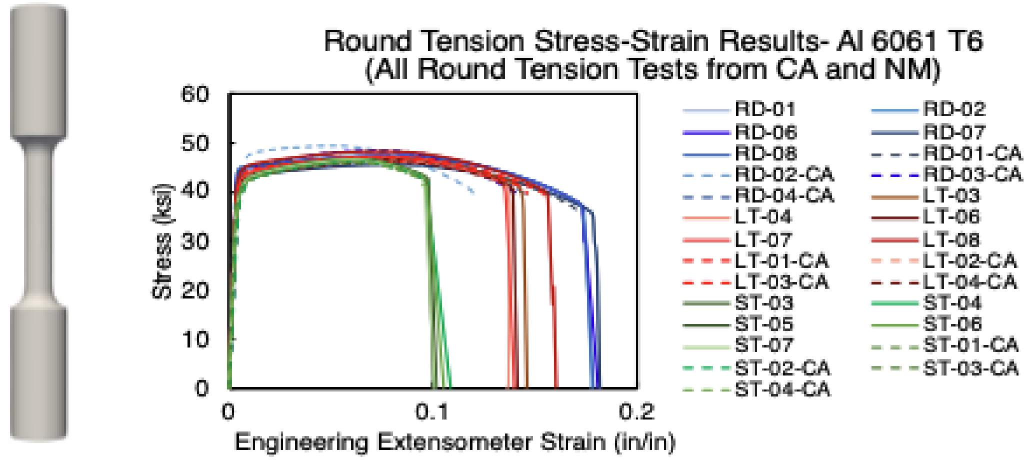
Anisotropic Failure Method

ASTME8 smooth tension



Anisotropic Failure!

Problem

Desire: anisotropic damage/failure model

Opportunity: many anisotropic damage models in literature

Challenge: “too many parameters”,
“difficult to calibrate”,
“difficult to implement”,
”expensive to run / fine mesh”,
“not robust”

Proposed Solution

Concept: turn isotropic model into anisotropic model; establish a directionally-weighted equivalent plastic strain rate;

Caveats: not derived from physics / microstructure, limited representation from weighting order

Upside: easy, low-cost, familiar

Anisotropic Failure Method

Current Practice in Sierra/LAMÉ: modular material modeling

Modular Plasticity Model

Yield Surface
+
Hardening Model
+
Rate-Dependence

Modular Failure Model

Johnson-Cook,
Tearing Parameter,
Wilkins,
Bammann-Chiesa-Johnson,
etc.

Complete simulation with failure

Softening,
Element Death,
Nonlocal Regularization

$\dot{\varepsilon}^p$
Plastic Strain Rate

d
Damage/Failure Variable

Anisotropic Failure Method

Concept: establish a directionally-weighted equivalent plastic strain rate

Modular Plasticity Model

Yield Surface
+
Hardening Model
+
Rate-Dependence

Anisotropic Weighting

2nd-order weighting,
4th-order weighting

Modular Failure Model

Johnson-Cook,
Tearing Parameter,
Wilkins,
Bammann-Chiesa-Johnson,
etc.

Complete simulation with failure

Softening,
Element Death,
Nonlocal Regularization

$\dot{\varepsilon}^p, N^p$
Plastic Strain Rate,
Flow Direction

$\hat{\dot{\varepsilon}}^p$
Weighted Plastic Strain

d
Damage/Failure Variable

Anisotropic Failure Method

Concept: establish a directionally-weighted equivalent plastic strain rate

Plastic Strain: $\dot{\varepsilon}^p_{ij} = \dot{\gamma}\frac{\partial \sigma_*}{\partial \sigma_{ij}}$ $\dot{\gamma}$: scalar plasticity rate $\frac{\partial \sigma_*}{\partial \sigma_{ij}}$: flow rule

Equivalent Plastic Strain Rate: $\dot{\varepsilon}^p_{ij} = \dot{\bar{\varepsilon}}^p N_{ij}$ $\dot{\bar{\varepsilon}}^p = \dot{\gamma}, \quad N_{ij} = \frac{\partial \sigma_*}{\partial \sigma_{ij}}$

solve: $\dot{\bar{\varepsilon}}^p = \frac{\sqrt{\dot{\varepsilon}^p_{ij}\dot{\varepsilon}^p_{ij}}}{\sqrt{N_{ij}N_{ij}}} = \frac{\sqrt{\dot{\varepsilon}^p : \dot{\varepsilon}^p}}{\sqrt{N : N}}$

Weighted Plastic Strain Rate:

2nd-order formulation

$$\hat{\dot{\bar{\varepsilon}}}^p = \sqrt{\frac{\dot{\varepsilon}^p_{ij} A_{jk} \dot{\varepsilon}^p_{kl} \delta_{kl}}{N_{ij}N_{ij}}} = \sqrt{\frac{\mathrm{tr}(\dot{\varepsilon}^p \cdot A \cdot \dot{\varepsilon}^p)}{N : N}} = \dot{\bar{\varepsilon}}^p \sqrt{\frac{N_{ij} A_{jk} N_{kl} \delta_{kl}}{N_{ij}N_{ij}}} = \dot{\bar{\varepsilon}}^p \sqrt{\frac{\mathrm{tr}(N \cdot A \cdot N)}{N : N}}$$

4th-order formulation

$$\hat{\dot{\bar{\varepsilon}}}^p = \sqrt{\frac{\dot{\varepsilon}^p_{ij} B_{ijkl} \dot{\varepsilon}^p_{kl}}{N_{ij}N_{ij}}} = \sqrt{\frac{\dot{\varepsilon}^p : B : \dot{\varepsilon}^p}{N : N}} = \dot{\bar{\varepsilon}}^p \sqrt{\frac{N_{ij} B_{ijkl} N_{kl}}{N_{ij}N_{ij}}} = \dot{\bar{\varepsilon}}^p \sqrt{\frac{N : B : N}{N : N}}$$

Developing Flow Stresses from Plasticity Models

Yield Surface:

- J2 / Von Mises

$$\sigma_\star = \sqrt{\frac{3}{2}\sigma^{dev} : \sigma^{dev}} \qquad N = \frac{\partial \sigma_\star}{\partial \sigma_{ij}} = \frac{3}{2}\frac{\sigma^{dev}}{\sigma_\star} = \sqrt{\frac{3}{2}}\frac{\sigma^{dev}}{\sqrt{\sigma^{dev} : \sigma^{dev}}}$$

- Hill

$$\sigma_\star = \sqrt{\frac{3}{2}\sigma^{dev} : H : \sigma^{dev}} \qquad N = \frac{\partial \sigma_\star}{\partial \sigma_{ij}} = \frac{3}{2}\frac{H : \sigma^{dev}}{\sigma_\star} = \sqrt{\frac{3}{2}}\frac{H : \sigma^{dev}}{\sqrt{\sigma^{dev} : H : \sigma^{dev}}}$$

- Hosford

$$\sigma_\star = \left(\frac{|\sigma_I - \sigma_{II}|^a + |\sigma_{II} - \sigma_{III}|^a + |\sigma_{III} - \sigma_I|^a}{2}\right)^{\frac{1}{a}} \qquad N = \frac{\partial \sigma_\star}{\partial \sigma_{ij}} = V(\sigma)^T A V(\sigma),$$

$$A = \operatorname{diag}\begin{pmatrix} \frac{|\sigma_I-\sigma_{II}|^{a-2}(\sigma_I-\sigma_{II})-|\sigma_{III}-\sigma_I|^{a-2}(\sigma_{III}-\sigma_I)}{2} \\ \frac{|\sigma_{II}-\sigma_{III}|^{a-2}(\sigma_{II}-\sigma_{III})-|\sigma_I-\sigma_{II}|^{a-2}(\sigma_I-\sigma_{II})}{2} \\ \frac{|\sigma_{III}-\sigma_I|^{a-2}(\sigma_{III}-\sigma_I)-|\sigma_{II}-\sigma_{III}|^{a-2}(\sigma_{II}-\sigma_{III})}{2} \end{pmatrix}$$

(etc)

Anisotropic Failure Method

Application to Failure Models:

- integrate against weighted equivalent plastic strain rate/increment $\{d\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p\} \rightarrow \{d\hat{\bar{\varepsilon}}^p, \dot{\hat{\bar{\varepsilon}}}^p\}$

- Johnson-Cook:

$$D = \int \frac{d\bar{\varepsilon}^p}{\left(d_1 + d_2 \exp(d_3 T)\right)\left(1 + d_4 \frac{\dot{\bar{\varepsilon}}^p}{\dot{\bar{\varepsilon}}^p_0}\right)\left(1 + d_5 \hat{\theta}\right)} \rightarrow \int \frac{d\hat{\bar{\varepsilon}}^p}{\left(d_1 + d_2 \exp(d_3 T)\right)\left(1 + d_4 \frac{\dot{\bar{\varepsilon}}^p}{\dot{\bar{\varepsilon}}^p_0}\right)\left(1 + d_5 \hat{\theta}\right)}$$

- Tearing Parameter:

$$D = \frac{1}{\psi_{crit}} \int \left\langle \frac{2\sigma_1}{3\left(\sigma_1 - \sigma_m\right)} \right\rangle^{\xi} d\bar{\varepsilon}^p \rightarrow \frac{1}{\psi_{crit}} \int \left\langle \frac{2\sigma_1}{3\left(\sigma_1 - \sigma_m\right)} \right\rangle^{\xi} d\hat{\bar{\varepsilon}}^p$$

- Bammann-Chiesa-Johnson damage:

$$\dot{\phi} = \sqrt{\frac{2}{3}} \dot{\bar{\varepsilon}}^p \frac{1 - (1-\phi)^{m+1}}{(1-\phi)^m} \sinh\left[\frac{2(2m-1)}{2m+1} \frac{\langle p \rangle}{\sigma_f}\right] + (1-\phi)^2 \dot{\eta} v_0$$

$$\rightarrow \sqrt{\frac{2}{3}} \dot{\hat{\bar{\varepsilon}}}^p \frac{1 - (1-\phi)^{m+1}}{(1-\phi)^m} \sinh\left[\frac{2(2m-1)}{2m+1} \frac{\langle p \rangle}{\sigma_f}\right] + (1-\phi)^2 \dot{\eta} v_0$$

$$\dot{\eta} = \eta \dot{\bar{\varepsilon}}^p N_4 \left(N_1 \left[\frac{4}{27} - \frac{J_3^2}{J_2^3}\right] + N_2 \frac{J_3}{J_2^{3/2}} + N_3 \left[\frac{|p|}{\sigma_f}\right] \right)$$

$$\rightarrow \eta \dot{\hat{\bar{\varepsilon}}}^p N_4 \left(N_1 \left[\frac{4}{27} - \frac{J_3^2}{J_2^3}\right] + N_2 \frac{J_3}{J_2^{3/2}} + N_3 \left[\frac{|p|}{\sigma_f}\right] \right)$$

- Simple Direct Integration:

$$D = \frac{1}{\psi_{crit}} \int_0^t \frac{d\hat{\bar{\varepsilon}}^p}{dt'} dt' \rightarrow \frac{1}{\psi_{crit}} \int_0^t \frac{d\hat{\bar{\varepsilon}}^p}{dt'} dt'$$

Anisotropic Failure Method

Weighting Notes:

- Symmetric operation requires major symmetry: $A_{jk} = A_{kj}, \quad B_{ijkl} = B_{klij}$
- Symmetric stress tensor, flow-direction tensor → requires minor symmetry:

$$B_{ijkl} = B_{ijlk} = B_{jikl} = B_{jilk}$$

- → Voigt notation can be used for 4th-order weighting: $B_{ijkl} \Rightarrow B^v_{IJ}, \quad N_{ij} \Rightarrow N^v_I$

$$\hat{\dot{\varepsilon}}^p = \dot{\varepsilon}^p \sqrt{\frac{N^v_I B^v_{IJ} N^v_J}{N^v_K N^v_K}} = \dot{\varepsilon}^p \sqrt{\frac{N^v \cdot B^v \cdot N^v}{N^v \cdot N^v}}$$

- Reproducing lower-order weights…
  - Isotropic:

$$A_{ij} = \delta_{ij} = I_{(3x3)}, \quad B_{ijkl} = \frac{1}{2}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right), \quad B^v = I_{(6x6)}$$

  - 2nd-order (diagonal):

$$B^v = \mathrm{diag}\left[A^v_1, A^v_2, A^v_3, \frac{1}{2}(A^v_1 + A^v_2), \frac{1}{2}(A^v_2 + A^v_3), \frac{1}{2}(A^v_3 + A^v_1)\right]$$

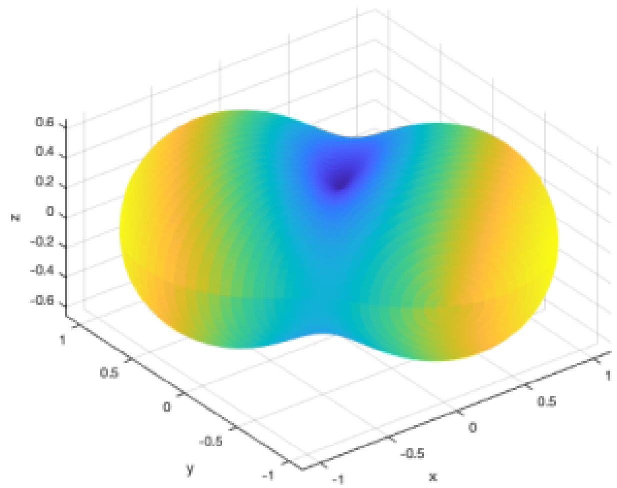
Anisotropic Failure Method

Weighting Notes:

- From eigenvalue decomposition…
  - 2^{nd}-order tensor (6 entries) contains: 3 unique weights + rotations
  - 4^{th}-order tensor (21 entries) contains: 6 unique weights + “rotations"

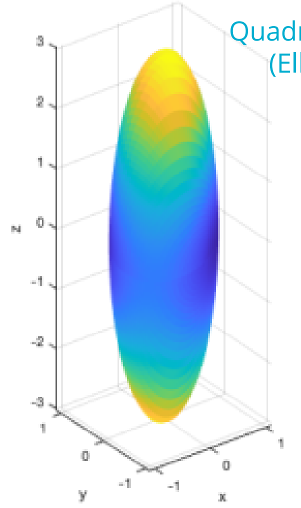
$\{\rho, \theta, \phi\}; \{\theta, \phi\} = S^2$

$$A = \begin{bmatrix} 1 & -0.5 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$



Weighting surface:

$\rho = \sqrt{N(\theta,\phi) \cdot A \cdot N(\theta,\phi)}$

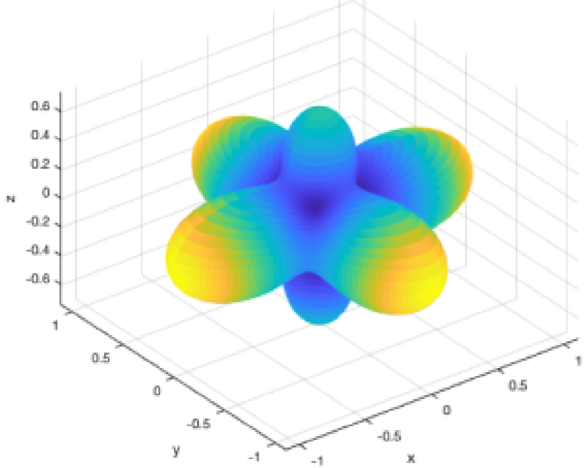


Quadric Surface (Ellipsoid)

Failure surface:

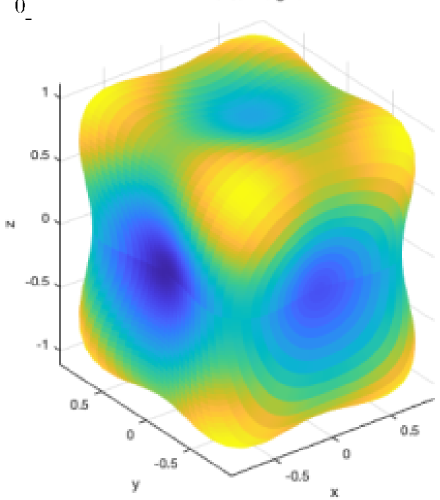
$\rho = 1/\sqrt{N(\theta,\phi) \cdot A \cdot N(\theta,\phi)}$

$$B'' = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



Weighting surface:

$\rho = \sqrt{N(\theta,\phi) : B : N(\theta,\phi)}$



Failure Surface:

$\rho = 1/\sqrt{N(\theta,\phi) : B : N(\theta,\phi)}$

Anisotropic Failure Method

Weighting Notes:

- sqrt term requires some constraints…
- For general (real-valued) matrix N, we should require that $\{A, B\}$ are positive semi-definite…

$$\mathrm{tr}(N \cdot A \cdot N) \geq 0, \quad N : B : N \geq 0$$

  - Positive Semi-Definite equivalent to…
    - Non-negative eigenvalues: $\min(eig(A)) \geq 0$
    - All three invariants non-negative $I_1, I_2, I_3 \geq 0$
- BUT, for many common plasticity models, N is known to be traceless
  - This weakens the PSD conditions on $\{A, B\}$. We now *only* require:
    - $I_1 = \lambda_1 + \lambda_2 + \lambda_3 \geq 0$
    - $I_2 = \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 \geq 0$
    - $I_3 = \lambda_1\lambda_2\lambda_3$ may be negative

$$\text{where } \lambda_i = eig(A)$$

$$\text{example: } A = \begin{bmatrix} 1.0 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & -0.1 \end{bmatrix}$$

  - Unknown impact on B…
- Practical implementation:
  - PSD wrt. traceless N is ideal, but not required. If $N \cdot A \cdot N < 0$, set to zero in weighting calc

Anisotropic Failure Method

Weighting Notes:

- Could be parametrized from analytical forms and/or calibration

- Analytical forms (speculative)…
  - Borrow from literature of anisotropic yield surfaces:

$$F(\bar{\varepsilon}^p, f) = [\sigma : B : \sigma] - 1 \leq 0$$

$$F(\bar{\varepsilon}^p, \bar{\varepsilon}^p_{crit}) = [\bar{\varepsilon}^p : B : \bar{\varepsilon}^p] - 1 \leq 0 \ ?$$

J2 / Von Mises

$$B^v = \frac{1}{f^2}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

Hill

$$B^v = \begin{bmatrix} G+H & -H & -G & 0 & 0 & 0 \\ -H & F+H & -F & 0 & 0 & 0 \\ -H & -F & F+G & 0 & 0 & 0 \\ 0 & 0 & 0 & 2N & 0 & 0 \\ 0 & 0 & 0 & 0 & 2L & 0 \\ 0 & 0 & 0 & 0 & 0 & 2M \end{bmatrix}$$

$$\begin{cases} 2F = \frac{1}{f_y^2} + \frac{1}{f_z^2} - \frac{1}{f_x^2} & 2L = \frac{1}{f_{yz}^2} \\ 2G = \frac{1}{f_z^2} + \frac{1}{f_x^2} - \frac{1}{f_y^2} & 2M = \frac{1}{f_{xz}^2} \\ 2H = \frac{1}{f_x^2} + \frac{1}{f_y^2} - \frac{1}{f_z^2} & 2N = \frac{1}{f_{xy}^2} \end{cases}$$

Potential Extension (asymmetry):

$$F(\sigma, f) = [\sigma : B : \sigma + A : \sigma] - 1 \leq 0$$

$$F(\bar{\varepsilon}^p, \psi_{crit}) = [\bar{\varepsilon}^p : B : \bar{\varepsilon}^p + A : \bar{\varepsilon}^p] - 1 \leq 0 \ ?$$

- Mises-Schleicher
- Drucker-Prager
- Tsai-Wu
- Hoffman
- Etc.

S. Oller, E. Car, J. Lubliner. Definition of a general implicit orthotropic yield criterion. *CMAME*. 2003.

Calibration Examples

2nd-order weighting:

- 3 uniaxial tension tests:

$$\{\bar{\varepsilon}_1^p, \bar{\varepsilon}_2^p, \bar{\varepsilon}_3^p\} = \{8\%, 14\%, 18\%\} \qquad N_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \quad N_2 = \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \quad N_3 = \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Tearing Parameter + uniaxial loading…

$$D = \frac{1}{\psi_{crit}} \int d\hat{\bar{\varepsilon}}^p = \frac{d\hat{\bar{\varepsilon}}}{\psi_{crit}}$$

- Assuming coordinate-system aligned…

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

- Solve as linear system:

$$\begin{bmatrix} \psi_{crit} \\ \psi_{crit} \\ \psi_{crit} \end{bmatrix} = \begin{bmatrix} \hat{\bar{\varepsilon}}^p \\ \hat{\bar{\varepsilon}}^p \\ \hat{\bar{\varepsilon}}^p \end{bmatrix} = \begin{bmatrix} \hat{\varepsilon}_1^p \sqrt{\frac{\mathrm{tr}(N_1 \cdot A \cdot N_1)}{N_1 : N_1}} \\ \hat{\varepsilon}_2^p \sqrt{\frac{\mathrm{tr}(N_2 \cdot A \cdot N_2)}{N_2 : N_2}} \\ \hat{\varepsilon}_3^p \sqrt{\frac{\mathrm{tr}(N_3 \cdot A \cdot N_3)}{N_3 : N_3}} \end{bmatrix} = \begin{bmatrix} \hat{\varepsilon}_1^p \sqrt{\frac{2}{3}a + \frac{1}{6}b + \frac{1}{6}c} \\ \hat{\varepsilon}_2^p \sqrt{\frac{1}{6}a + \frac{2}{3}b + \frac{1}{6}c} \\ \hat{\varepsilon}_3^p \sqrt{\frac{1}{6}a + \frac{1}{6}b + \frac{2}{3}c} \end{bmatrix}$$

$$\begin{bmatrix} (\bar{\varepsilon}_1^p)^{-2}\psi_{crit}^2 \\ (\bar{\varepsilon}_2^p)^{-2}\psi_{crit}^2 \\ (\bar{\varepsilon}_3^p)^{-2}\psi_{crit}^2 \end{bmatrix} = \begin{bmatrix} 2/3 & 1/6 & 1/6 \\ 1/6 & 2/3 & 1/6 \\ 1/6 & 1/6 & 2/3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5/3 & -1/3 & -1/3 \\ -1/3 & 5/3 & -1/3 \\ -1/3 & -1/3 & 5/3 \end{bmatrix} \begin{bmatrix} (0.08)^{-2}\psi_{crit}^2 \\ (0.14)^{-2}\psi_{crit}^2 \\ (0.18)^{-2}\psi_{crit}^2 \end{bmatrix} = \begin{bmatrix} 233.12\psi_{crit}^2 \\ 22.66\psi_{crit}^2 \\ -17.65\psi_{crit}^2 \end{bmatrix}$$

Calibration Examples

2nd-order weighting (continued):

- Solution, normalized:

$$\left\{\psi_{crit} = 1, A = \begin{bmatrix} 233.12 & 0 & 0 \\ 0 & 22.66 & 0 \\ 0 & 0 & -17.65 \end{bmatrix}\right\};$$

$$\left\{\psi_{crit} = 0.06550, A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.0972 & 0 \\ 0 & 0 & -0.0757 \end{bmatrix}\right\} \quad \text{(one value set to 1);}$$

$$\left\{\psi_{crit} = 0.1122, A = \begin{bmatrix} 2.9368 & 0 & 0 \\ 0 & 0.2855 & 0 \\ 0 & 0 & -0.2224 \end{bmatrix}\right\} \quad (\mathrm{tr}(A) = 3)$$

- Add additional uniaxial test in X-Y direction, add shear term:

$$A = \begin{bmatrix} a & d & 0 \\ d & b & 0 \\ 0 & 0 & c \end{bmatrix} \qquad \{\bar{\varepsilon}_4^p\} = \{16\%\} \qquad N_4 = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{3}{4} & \frac{1}{4} & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} (\bar{\varepsilon}_1^p)^{-2}\psi_{crit}^2 \\ (\bar{\varepsilon}_2^p)^{-2}\psi_{crit}^2 \\ (\bar{\varepsilon}_3^p)^{-2}\psi_{crit}^2 \\ (\bar{\varepsilon}_4^p)^{-2}\psi_{crit}^2 \end{bmatrix} = \begin{bmatrix} 2/3 & 1/6 & 1/6 & 0 \\ 1/6 & 2/3 & 1/6 & 0 \\ 1/6 & 1/6 & 2/3 & 0 \\ 5/12 & 5/12 & 1/6 & 1/2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \rightarrow \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 5/3 & -1/3 & -1/3 & 0 \\ -1/3 & 5/3 & -1/3 & 0 \\ -1/3 & -1/3 & 5/3 & 0 \\ -1 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} (0.08)^{-2}\psi_{crit}^2 \\ (0.14)^{-2}\psi_{crit}^2 \\ (0.18)^{-2}\psi_{crit}^2 \\ (0.16)^{-2}\psi_{crit}^2 \end{bmatrix}$$

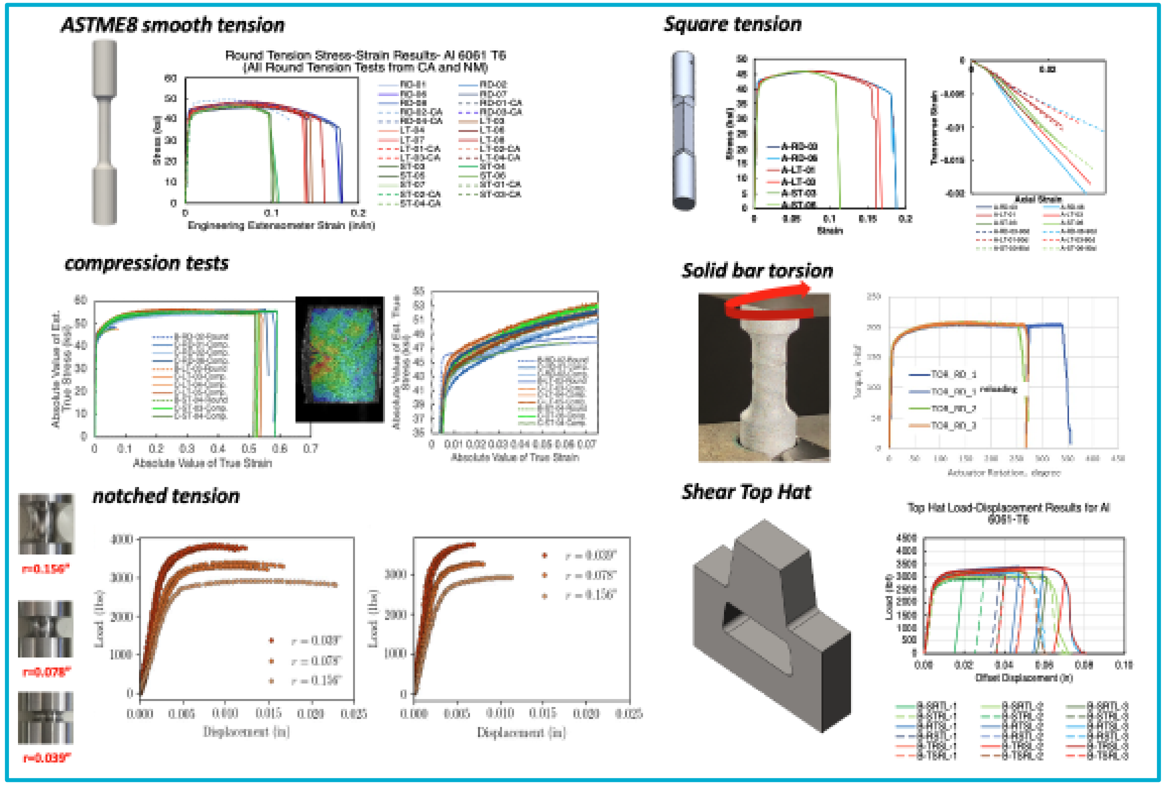
$$\psi_{crit} = 0.1122, A = \begin{bmatrix} 2.9368 & -1.6270 & 0 \\ -1.6270 & 0.2855 & 0 \\ 0 & 0 & -0.2224 \end{bmatrix}$$

*$\bar{\varepsilon}_4^p$ = **9.823%** would result in no change to calibrated parameters, $\boldsymbol{d} = \mathbf{0}$!

Calibration Examples

2nd-order weighting (continued):

$$A = \begin{bmatrix} 2.9368 & 0 & 0 \\ 0 & 0.2855 & 0 \\ 0 & 0 & -0.2224 \end{bmatrix}$$



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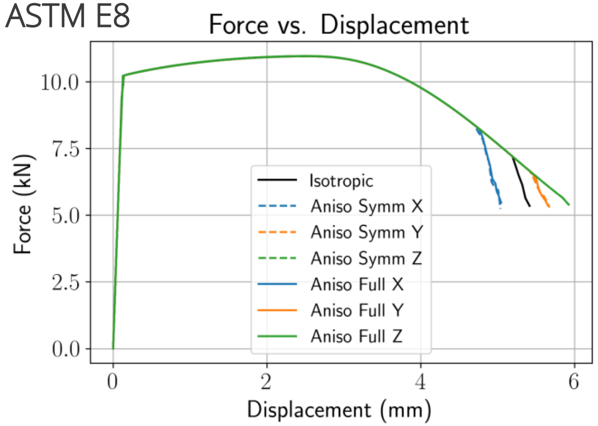
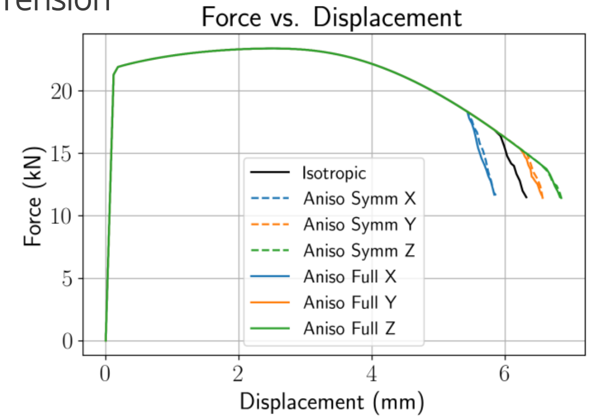
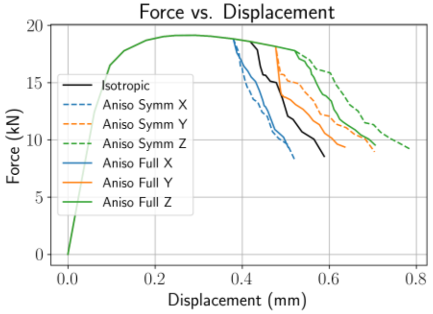


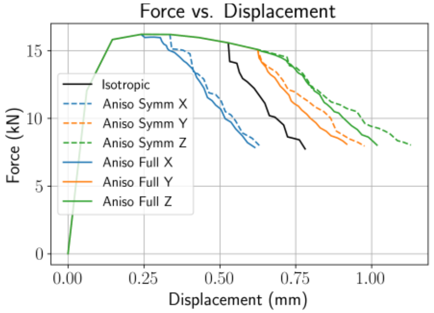
Square Tension



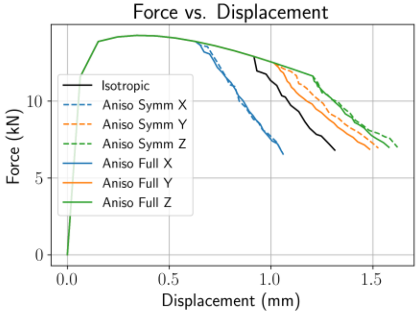
Notched Tension r=0.039"



Notched Tension r=0.078"



Notched Tension r=0.156"



Hill Plasticity Model Review (from the Lame 5.4 manual)

Kinematics

Deviatoric Plastic Flow

$$D_{ij} = D^{\mathrm{e}}_{ij} + D^{\mathrm{p}}_{ij}$$

Objective Stress Rate

Isotropic Elastic Tensor

$$\overset{\circ}{\sigma}_{ij} = \mathbb{C}_{ijkl} D^{\mathrm{e}}_{kl}$$

Yield Surface

$$f\left(\sigma_{ij}, \bar{\varepsilon}^p\right) = \phi\left(\sigma_{ij}\right) - \bar{\sigma}\left(\bar{\varepsilon}^p\right) = 0$$

Plastic Flow Direction

$$D^{\mathrm{p}}_{ij} = \dot{\gamma}\frac{\partial \phi}{\partial \sigma_{ij}}.$$

Square of the Effective Stress

$$\phi^2\left(\sigma_{ij}\right) = F\left(\hat{\sigma}_{22} - \hat{\sigma}_{33}\right)^2 + G\left(\hat{\sigma}_{33} - \hat{\sigma}_{11}\right)^2 + H\left(\hat{\sigma}_{11} - \hat{\sigma}_{22}\right)^2 + 2L\hat{\sigma}_{23}^2 + 2M\hat{\sigma}_{31}^2 + 2N\hat{\sigma}_{12}^2.$$

Note that pressure will drop out
Hence, with Associative Plastic Flow,
There flow direction is deviatoric

Note, ij are *Material Directions*

Mapping of Yield Parameters Onto Material Strengths

$$F = \frac{(\bar{\sigma})^2}{2}\left[\frac{1}{\left(\sigma^y_{22}\right)^2} + \frac{1}{\left(\sigma^y_{33}\right)^2} - \frac{1}{\left(\sigma^y_{11}\right)^2}\right] \quad ; \quad L = \frac{(\bar{\sigma})^2}{2}\left[\frac{1}{\left(\tau^y_{23}\right)^2}\right]$$

$$G = \frac{(\bar{\sigma})^2}{2}\left[\frac{1}{\left(\sigma^y_{33}\right)^2} + \frac{1}{\left(\sigma^y_{11}\right)^2} - \frac{1}{\left(\sigma^y_{22}\right)^2}\right] \quad ; \quad M = \frac{(\bar{\sigma})^2}{2}\left[\frac{1}{\left(\tau^y_{31}\right)^2}\right]$$

$$H = \frac{(\bar{\sigma})^2}{2}\left[\frac{1}{\left(\sigma^y_{11}\right)^2} + \frac{1}{\left(\sigma^y_{22}\right)^2} - \frac{1}{\left(\sigma^y_{33}\right)^2}\right] \quad ; \quad N = \frac{(\bar{\sigma})^2}{2}\left[\frac{1}{\left(\tau^y_{12}\right)^2}\right].$$

Define Anisotropy Ratios Used in Sierra

$$R_{11} = \frac{\sigma^y_{11}}{\bar{\sigma}} \quad ; \quad R_{12} = \sqrt{3}\frac{\tau^y_{12}}{\bar{\sigma}}$$

$$R_{22} = \frac{\sigma^y_{22}}{\bar{\sigma}} \quad ; \quad R_{23} = \sqrt{3}\frac{\tau^y_{23}}{\bar{\sigma}}$$

$$R_{33} = \frac{\sigma^y_{33}}{\bar{\sigma}} \quad ; \quad R_{31} = \sqrt{3}\frac{\tau^y_{31}}{\bar{\sigma}}.$$

Anisotropic Failure Weighting Calibration for CFRP

Recall The Plastic Flow Direction

$$D^{\mathrm{p}}_{ij} = \dot{\gamma}\frac{\partial \phi}{\partial \sigma_{ij}}.$$

Take Stress States From Experiments (just at failure),

Rotate into the material frame,

Determine the Plastic Flow Direction

Uniaxial Stress 11, 22, 33 Directions

$$\boldsymbol{N}^1 = \frac{1}{\sqrt{G+H}}\left((H+G)\boldsymbol{e}_1\boldsymbol{e}_1 - H\boldsymbol{e}_2\boldsymbol{e}_2 - G\boldsymbol{e}_3\boldsymbol{e}_3\right)$$

$$\boldsymbol{N}^2 = \frac{1}{\sqrt{F+H}}\left(-H\boldsymbol{e}_1\boldsymbol{e}_1 + (F+H)\boldsymbol{e}_2\boldsymbol{e}_2 - F\boldsymbol{e}_3\boldsymbol{e}_3\right)$$

$$\boldsymbol{N}^3 = \frac{1}{\sqrt{F+G}}\left(-G\boldsymbol{e}_1\boldsymbol{e}_1 - F\boldsymbol{e}_2\boldsymbol{e}_2 + (F+G)\boldsymbol{e}_3\boldsymbol{e}_3\right)$$

Flow Directions in the out-of-plane shears (1-3, 2-3)

$$N^5 = \sqrt{\frac{L}{2}}\,(e_2e_3 + e_3e_2)$$

$$N^6 = \sqrt{\frac{M}{2}}\,(e_1e_3 + e_3e_1)$$

45 Degrees from the material 1 direction in the 1-2 plane

$$N^4 = \sqrt{\frac{1}{F+G+2N}}\,(Ge_1e_1 + Fe_2e_2 - (F+G)e_3e_3 + N(e_1e_2 + e_2e_1))$$

The equivalent plastic strain at failure will be needed in each of these directions. We will approximate the total strain at failure in these directions as the associated equivalent plastic strain at failure in that direction

Anisotropic Failure Weighting Calibration for CFRP

Assume the 4th Order Plastic Weighting Tensor Is Diagonal in the Material Frame

$$B^v = \begin{bmatrix} a & 0 & 0 & 0 & 0 & 0 \\ 0 & b & 0 & 0 & 0 & 0 \\ 0 & 0 & c & 0 & 0 & 0 \\ 0 & 0 & 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 & e & 0 \\ 0 & 0 & 0 & 0 & 0 & f \end{bmatrix}$$

Voigt Form (11:1, 22:2, 33:3, 12,21:4, 23,32:5, 13,31:6)

Define the failure variable, D. Failure occurs when D = 1

$$D = \frac{1}{\psi_{crit}} \int d\hat{\varepsilon}^p = \frac{d\hat{\varepsilon}}{\psi_{crit}}$$

Identify the Critical Failure Equivalent Plastic Strain, ψ_{crit}, using the Most Ductile Direction ~0.12 for CFRP

Identify the plastic failure strains (here the total strains) at failure in the 6 calibration Experiments. Solve the 6x6 system of Equations defined by:

Here, I goes 1 to 6

$$\left(\frac{\psi_{crit}}{\hat{\varepsilon}_I^p}\right)^2 = \frac{N^I \cdot B \cdot N^I}{N^I \cdot N^I}$$

Anisotropic Failure Weighting Calibration for CFRP

- 4 uniaxial tension tests (11,22,33,13) + 2 pure shear tests:

$$\{\bar{\varepsilon}_1^p, \bar{\varepsilon}_2^p, \bar{\varepsilon}_3^p, \bar{\varepsilon}_4^p, \bar{\varepsilon}_5^p, \bar{\varepsilon}_6^p\} = \{0.2\%, 0.3\%, 0.4\%, 11\%, 12\%, 13\%\}$$

$$N_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \quad N_2 = \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \quad N_3 = \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad N_4 = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{3}{4} & \frac{1}{4} & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \quad N_5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad N_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

- Critical-EQPS failure (TP, $\xi = 0$)...

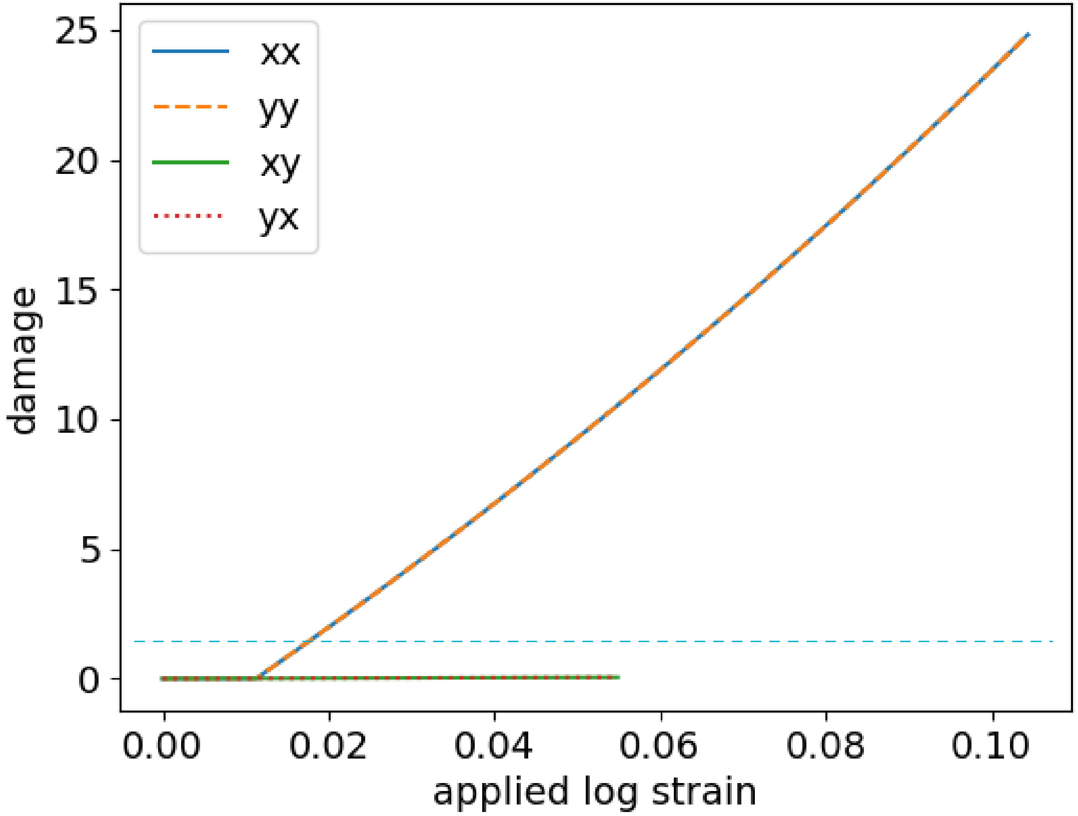
$$D = \frac{1}{\psi_{crit}} \int d\hat{\bar{\varepsilon}}^p = \frac{d\hat{\bar{\varepsilon}}}{\psi_{crit}}$$

- Assuming coordinate-system aligned, simplest possible...
- Solve as linear system:

$$B^v = \begin{bmatrix} a & 0 & 0 & 0 & 0 & 0 \\ 0 & b & 0 & 0 & 0 & 0 \\ 0 & 0 & c & 0 & 0 & 0 \\ 0 & 0 & 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 & e & 0 \\ 0 & 0 & 0 & 0 & 0 & f \end{bmatrix}$$

$$\begin{bmatrix} (\bar{\varepsilon}_1^p)^{-2}\psi_{crit}^2 \\ (\bar{\varepsilon}_2^p)^{-2}\psi_{crit}^2 \\ (\bar{\varepsilon}_3^p)^{-2}\psi_{crit}^2 \\ (\bar{\varepsilon}_4^p)^{-2}\psi_{crit}^2 \\ (\bar{\varepsilon}_5^p)^{-2}\psi_{crit}^2 \\ (\bar{\varepsilon}^6)^{-2}\psi_{crit}^2 \end{bmatrix} = \begin{bmatrix} 2/3 & 1/6 & 1/6 & 0 & 0 & 0 \\ 1/6 & 2/3 & 1/6 & 0 & 0 & 0 \\ 1/6 & 1/6 & 2/3 & 0 & 0 & 0 \\ 1/24 & 1/24 & 1/6 & 3/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} \rightarrow \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} 5/3 & -1/3 & -1/3 & 0 & 0 & 0 \\ -1/3 & 5/3 & -1/3 & 0 & 0 & 0 \\ -1/3 & -1/3 & 5/3 & 0 & 0 & 0 \\ 0 & 0 & -1/3 & 4/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.002^{-2} \\ 0.003^{-2} \\ 0.004^{-2} \\ 0.11^{-2} \\ 0.12^{-2} \\ 0.13^{-2} \end{bmatrix} = \begin{bmatrix} 3587.96 \\ 810.19 \\ -162.04 \\ -414.46 \\ 1.39 \\ 1.18 \end{bmatrix}$$

Comparison of “Damage” (Failure Parameter) Material Axis Loading in the 1-2 plane



Failure is accumulating in the pure shear directions But VERY slowly. At 10% strain, it is only about 0.06

Failure occurs in less than 0.5% strain after yield is reached in the 11, 22 directions

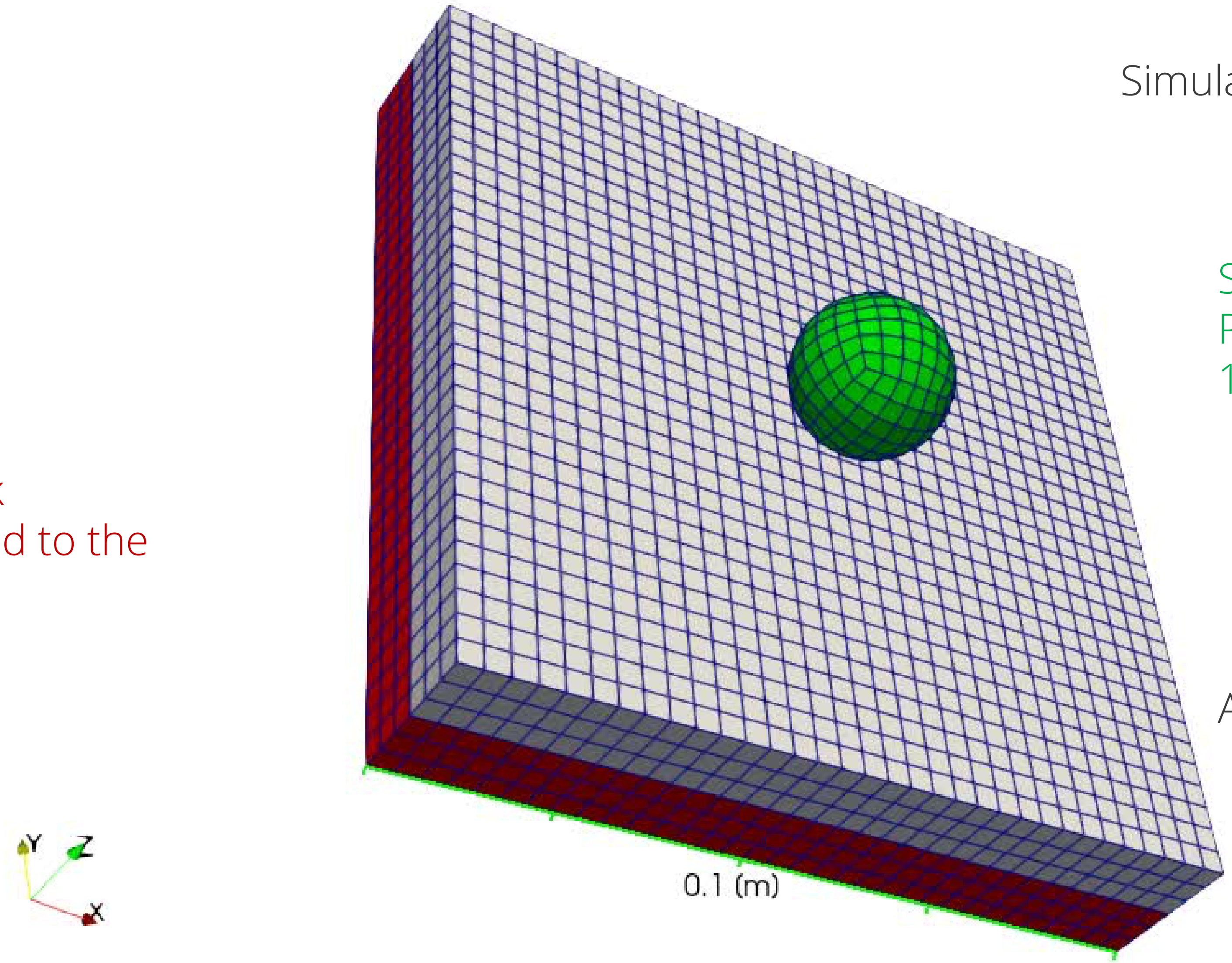
The tension 45 direction needs to be examined.

Demonstration in a plate puncture problem

Simulation Terminated at 500 ms

Steel Ball (elastic-plastic)
Prescribed Impact Velocity
1000 m/s

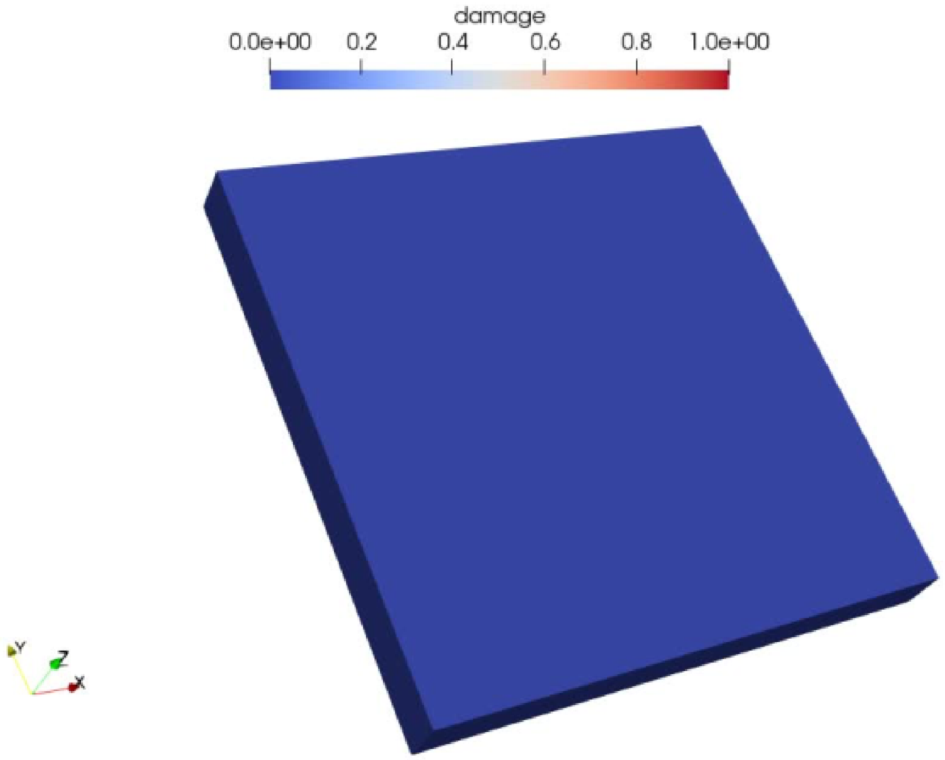
CFRP 1 cm thick
Perfectly Bonded to the
Aluminum Plate



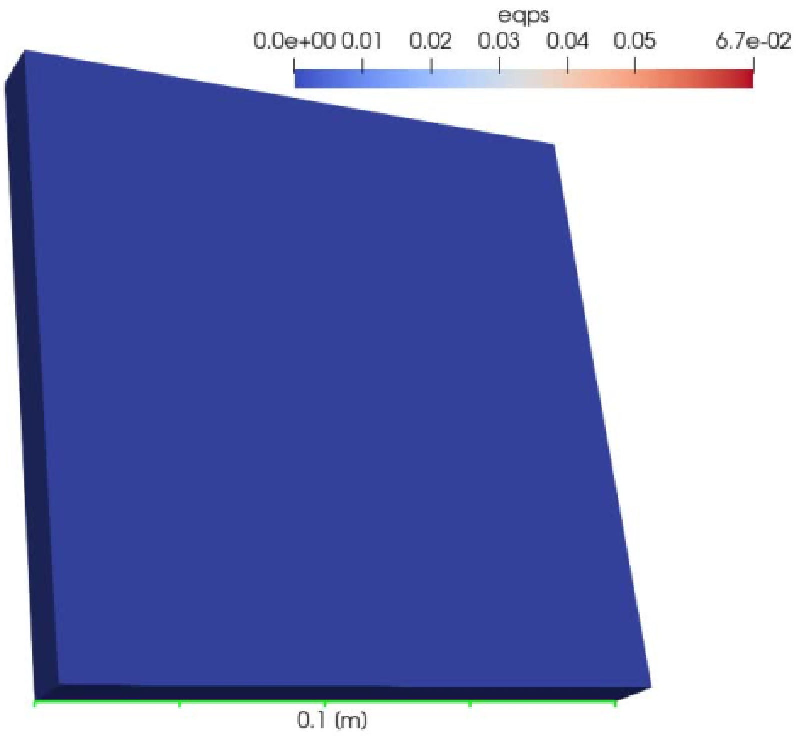
Aluminum Plate (1 cm thick)

Demonstration in a plate puncture problem
Isotropic Vs. Anisotropic Failure (with a critical failure eqps of 0.12)

Isotropic

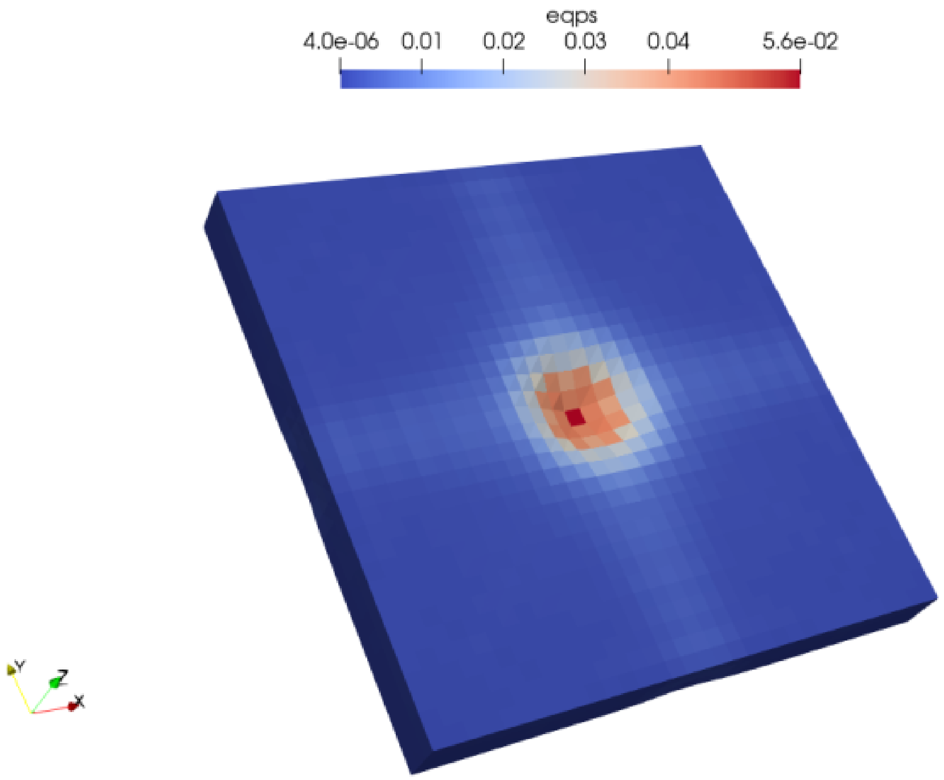


Anisotropic

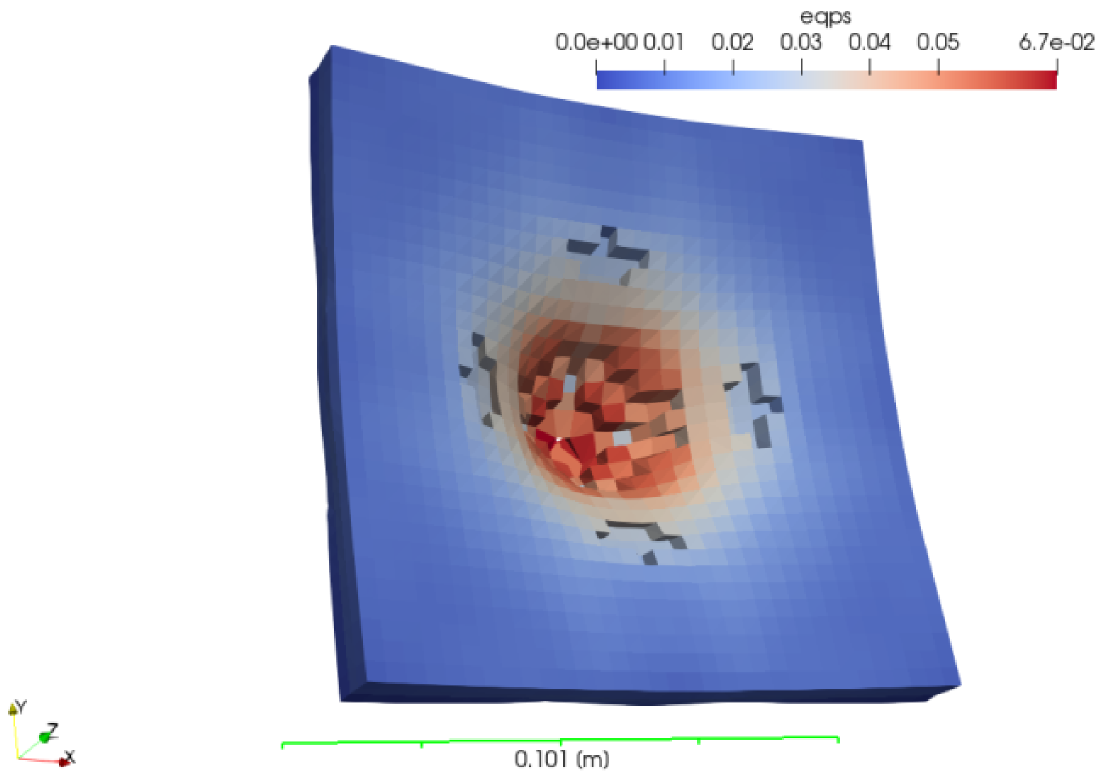


Demonstration in a plate puncture problem
Isotropic Vs. Anisotropic Failure (with a critical failure eqps of 0.12)

Isotropic



Anisotropic

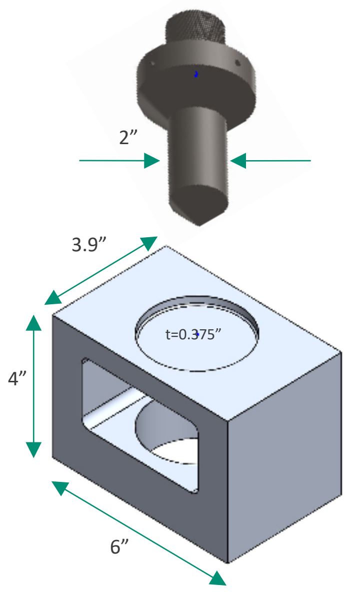


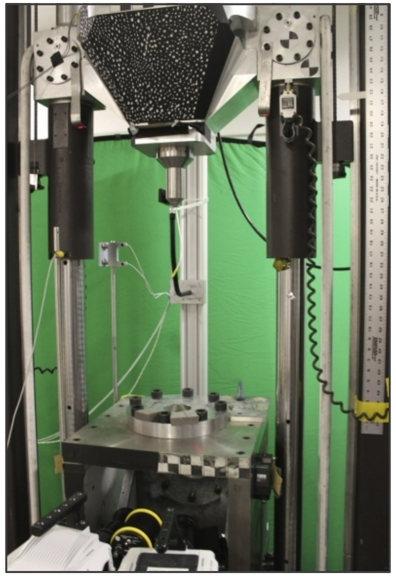
Future Validation Case

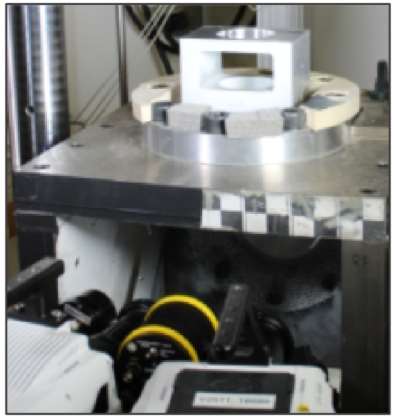
- 45° conical punch with a 0.25” diameter flat-end: 307 lb carriage and punch
- Base Material: 4”-thick Al 6061-T6 Plate

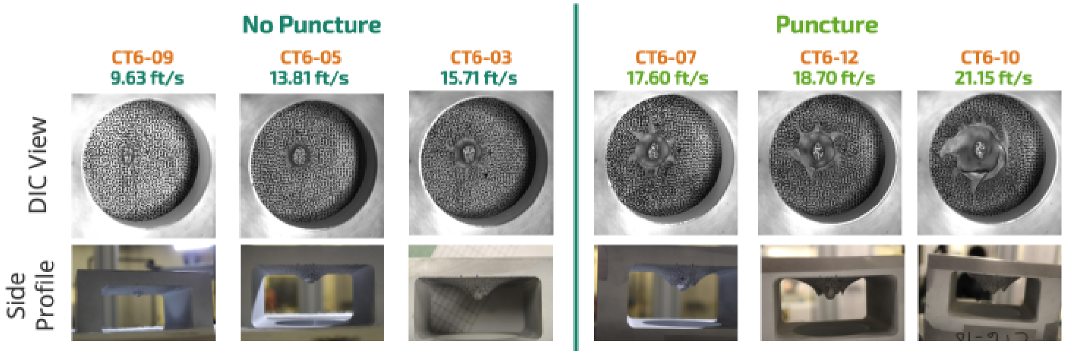
- Specimen Geometry (SFC Puncture Specimen C):
  - 4” (puncture direction) x 6” long x 3.9” wide
  - 2”x4” cutout and a 3” diameter bottom hole for viewing back side of puncture surface
  - Pocket: 3” diameter recessed hole, 0.25” deep with a local plate thickness of 0.375”

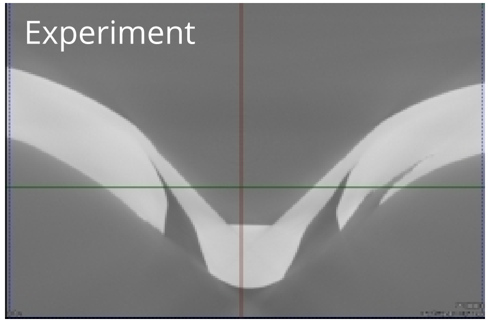
S. Kramer

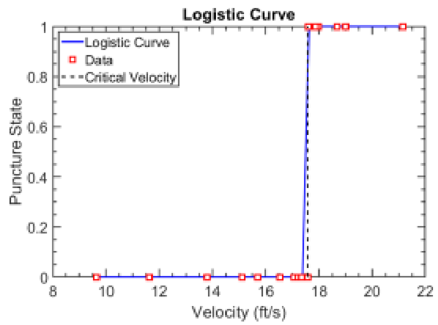












Critical Velocity = 17.6 ft/s (5.36 m/s)

Conclusions & Future Work

- Simple anisotropic concept with promising early demonstrations
- Questions remain about bounding of anisotropic weights, positive-semi-definiteness

- Presented two demonstration cases, calibrating to assumed model forms:
  - Aluminum metal characterization
  - A modular Hill Plasticity and modular anisotropic failure model were presented for homogenized laminate composites under large deformation cases
- The behavior is richer than isotropic failure simulations

- Next steps:
  - more comprehensive testing on 6061-T6 puncture problem
  - consider weighting methods equivalent to anisotropic failure surfaces