SAND2022-8039D

Sandia National Laboratories

Optimization for Real-Time Dynamic Spatial Adaptivity

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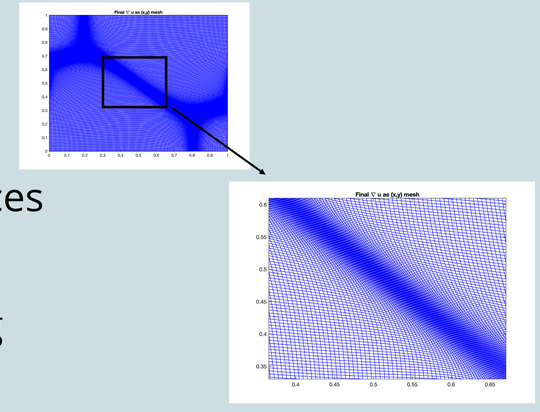
How do we accurately solve partial differential equations with complex dynamics?

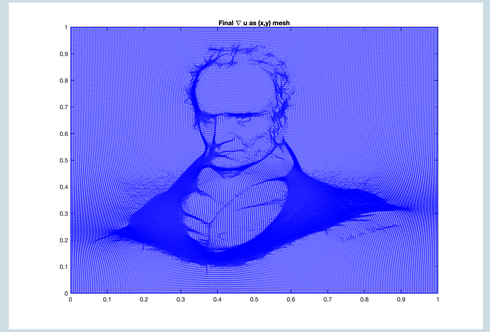
- Accurately modeling shock physics is crucial to several Sandia missions.
- Limited computational resources require algorithms that can accurately resolve physics without significantly increasing computational costs.



Adapted mesh with resolution to a sine wave and a zoomed picture by the wavefront.

Provide spatial adaptivity through a fixed number of mesh nodes

- Node locations determined by the solution to the Monge-Ampére equation [4].
- Use a low-order mixed finite element discretization [2].
- **Trust-region solver** [3] and multigrid methods are used to solve the resulting system accurately and efficiently.
- Algorithmic convergence in 20 iterations or less, multigrid method scales linearly in time.

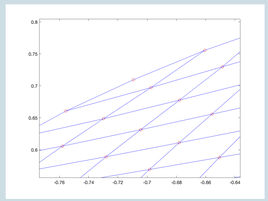


Mesh adaptivity using a bitmap as the indicator function

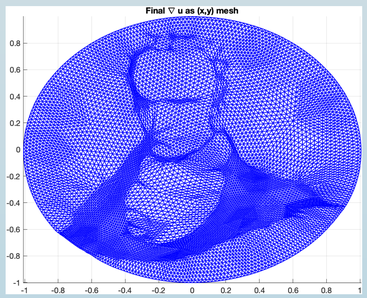
Results

- First illustration of convergence for a **low-order mixed finite element discretization** – allows for exploitation of fast solvers.
- Implementation of **multigrid** methods is crucial for scalability in 2D and 3D.
- Applicable to any convex domain.
- Exploring relevant applications in shocks, optical design, and inverse imaging.

Before: finite differences perform poorly on curved domains.

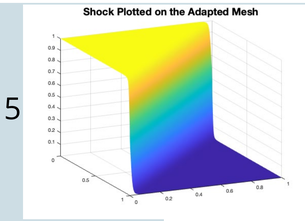


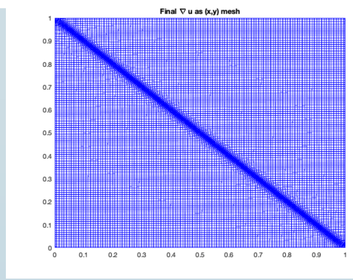
After: Our finite elements easily approximate a curved domain.



Impacts

- Journal article summarizing the results [1], 5 presentations (2 invited).
- Exemplar code incorporated into Intrelab (Intrepid for Matlab) in the Trilinos library.
- Results can lead to 2x improvement of PDE solution accuracy without adding mesh points. Crucial for application to **large-scale mission exemplars**.
- Plans to distribute work into generalized adaptivity software for PDEs in the Trilinos Rapid Optimization Library (ROL).





Mesh adaptation for a two-dimensional shock formed by solving Burger's equation.

Funded by the LDRD program under the Jill Hruby Postdoctoral Fellowship, FY20-22, 720K

LDRD LABORATORY DIRECTED RESEARCH & DEVELOPMENT

[1] Efficient optimization-based multigrid enabled solver for the Monge-Ampére equation. DiPietro, Ridzal. In prep. 2022. [2] A finite-element method for second order nonvariational elliptic problems, Kawecki et al. 2018. [3] A matrix-free trust region sqp method for equality constrained optimization. Ridzal et al. 2014. [4] Adaptivity with moving grids. Budd et al. 2009.

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