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## Finite Element Model Levelling for Material Model Calibration using Digital Image Correlation

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# Background and Motivation

## Background:

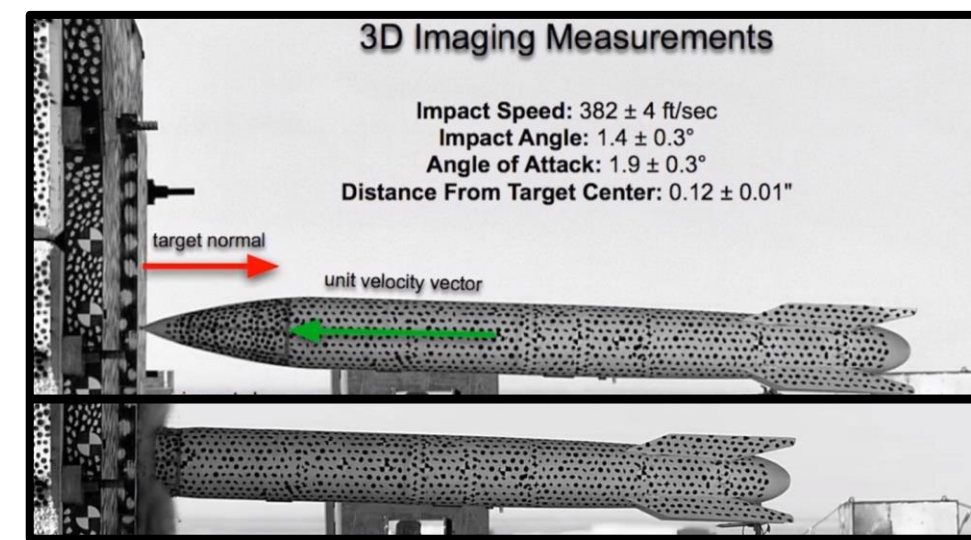
- Computational simulation to reduce expensive experimentation.
- Simulation accuracy is crucial, requiring robust calibration.
- Accuracy is enabled by robust measurements via digital image correlation (DIC) and inverse parameter identification techniques such as finite element model updating (FEMU).

## Problem:

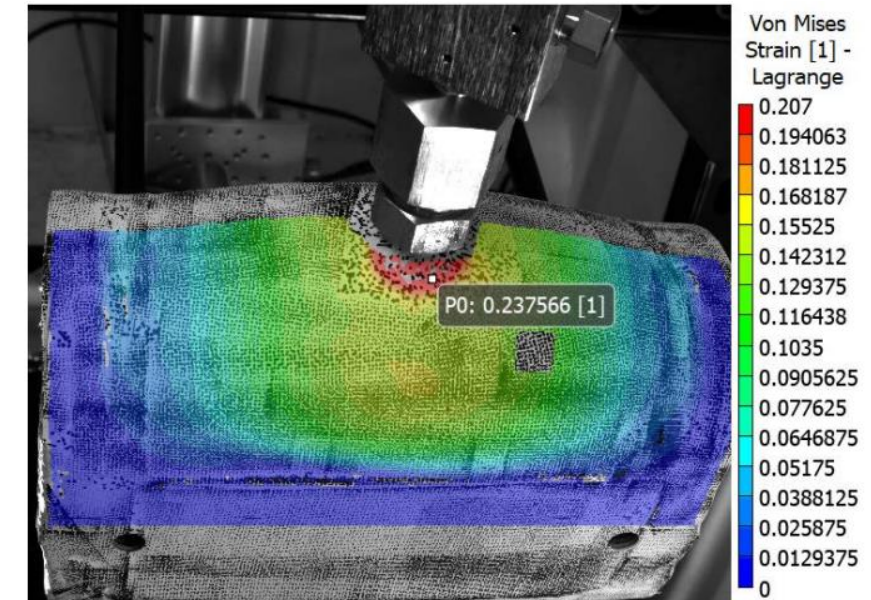
- DIC-measured strains suffer from a filtering bias caused by estimating the average strain across an area called a **virtual strain gage (VSG)**.

## Goal:

- We seek to account for the mismatch between the strains calculated through FEA and measured via DIC for the purpose of material model calibration.



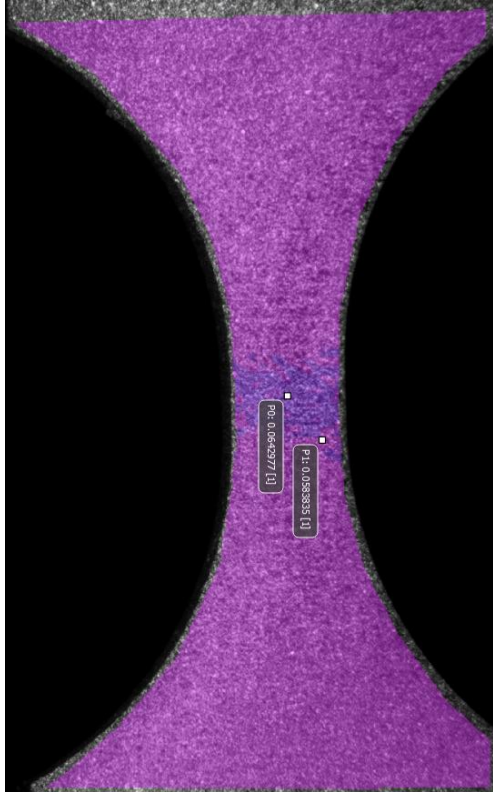
Sandia National Labs Ballistic Rocket Sled Test  
(December 2015)



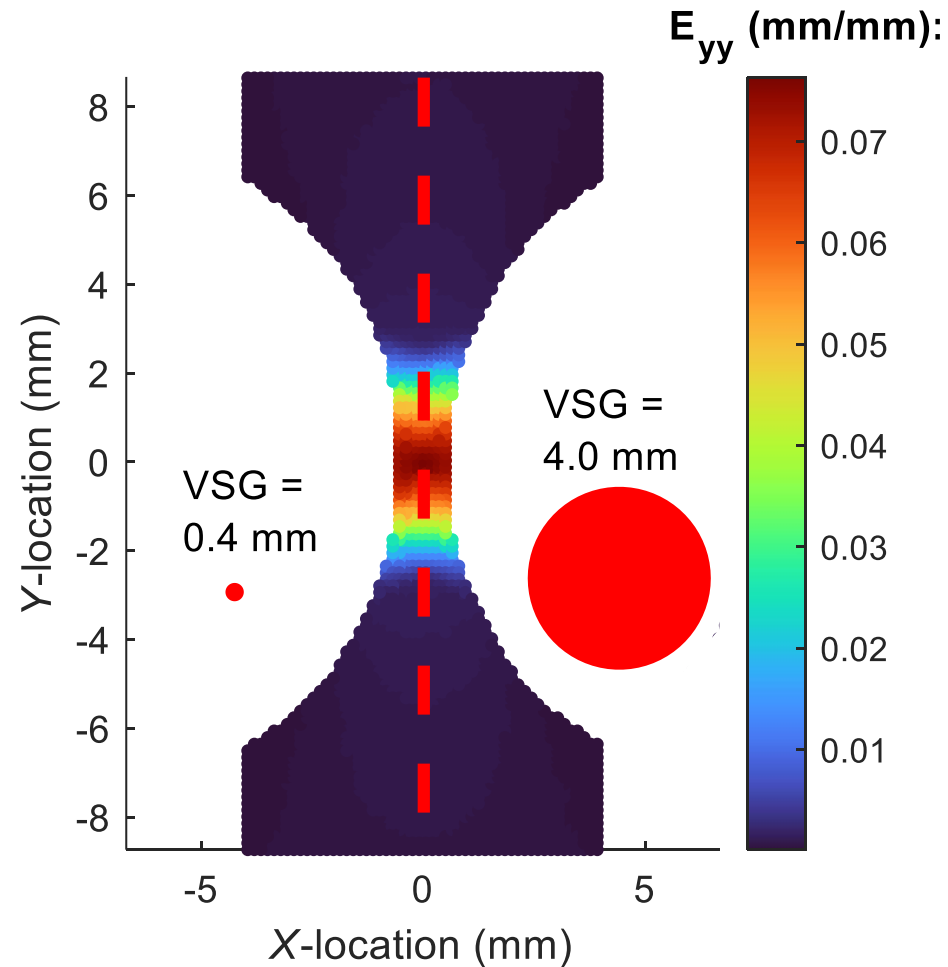
Lance, BW, & Carlson, MD. "Compact Heat Exchanger Semi-Circular Header Burst Pressure and Strain Validation."  
Proceedings of the ASME Turbo Expo 2019



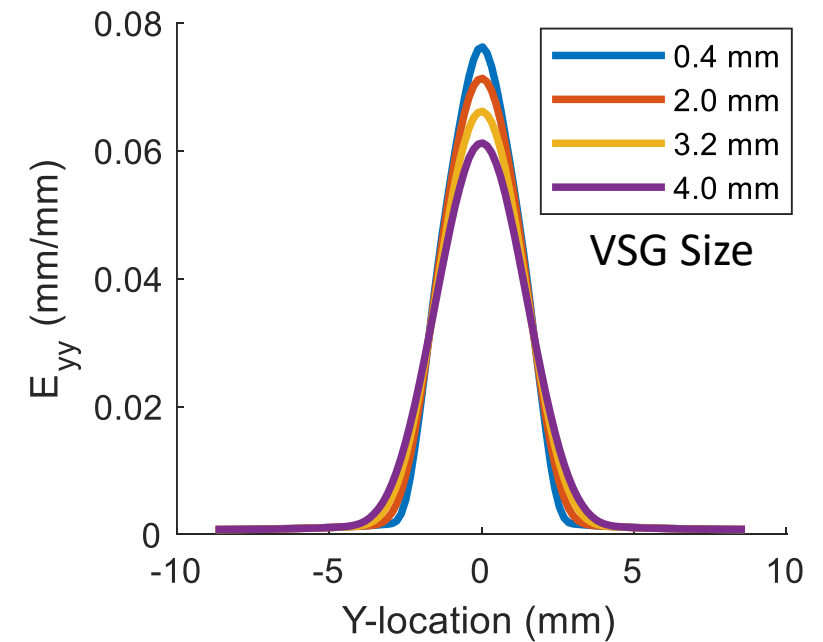
# DIC experiences length-scale dependent filtering biases



304L hourglass specimen  
subjected to uniaxial tension

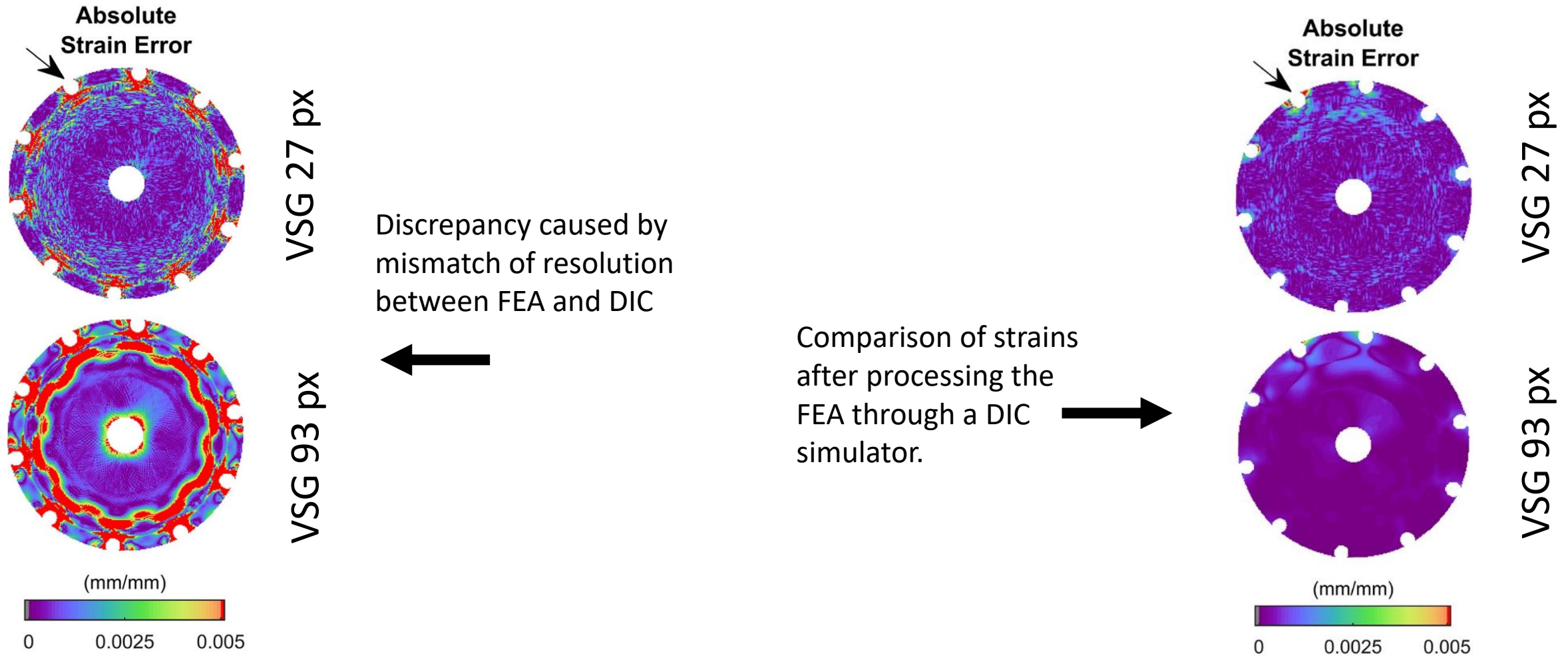


DIC results for a synthetic  
experiment.



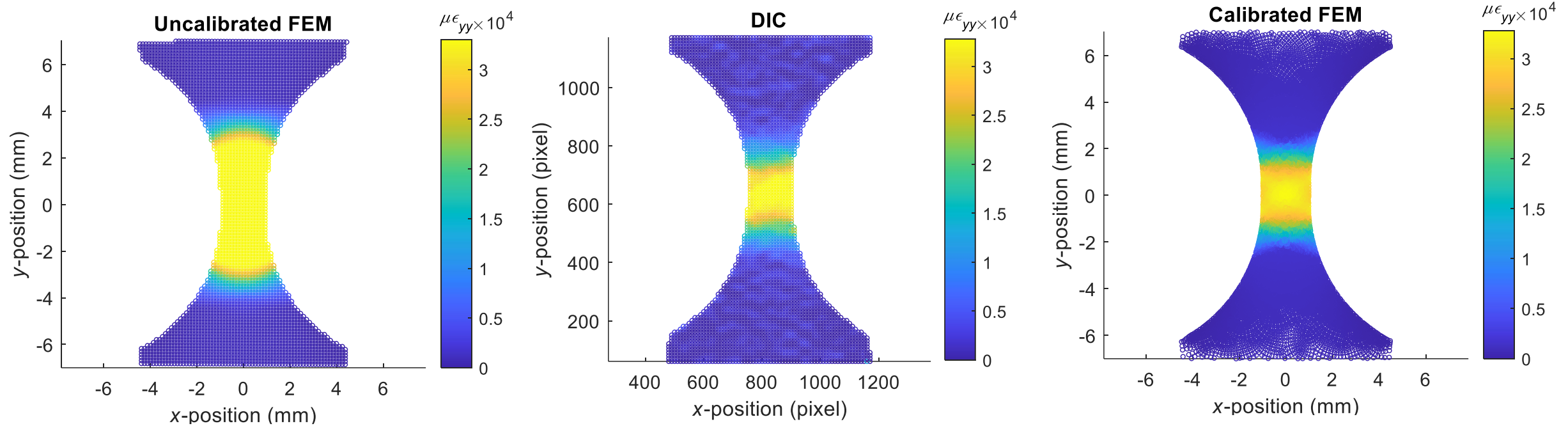
Sampled linecuts of the y-strain.  
Larger VSG sizes correspond to  
strain attenuation

# The filtering causes a mismatch between FEA and DIC



Lava P, Jones EMC, Wittevrongel L, Pierron F, (2020) Validation of finite-element models using full-field experimental data: Levelling finite-element analysis data through a digital image correlation engine. Strain 56(4): e12350. <https://doi.org/10.1111/str.12350>

# Finite Element Model Updating (FEMU)



## FEMU Objective Function

$$\chi_u^2(\{p\}) = \frac{1}{N_\epsilon} \sum_{i=1}^{N_\epsilon} (\{\epsilon_m\} - \{\epsilon_c\})^2$$

$$\min_p \chi_u^2(\{p\})$$

## Newton-Raphson Optimization

$$\delta p = [H^{i-1}]^{-1} [S]^t (\{\epsilon_m\} - \epsilon_c(x, \{p^{i-1}\}))$$

$\epsilon_m$ : the DIC measured strain

$\epsilon_c$ : the FEM calculated strain

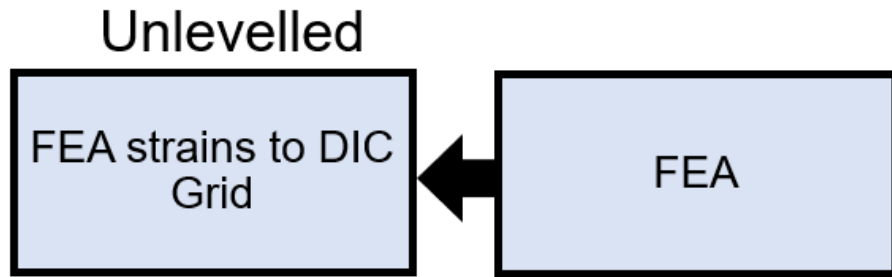
$N_\epsilon$ : the number of strain measurements

$[H]$ : the Hessian matrix  $[H^i] = [S^i]^t [S^i]$

$[S]$ : the sensitivity matrix  $[S] = \frac{\partial(\epsilon_{FEM}^i)}{\partial p_j}$

Mathieu, F., Leclerc, H., Hild, F. et al. Estimation of Elastoplastic Parameters via Weighted FEMU and Integrated-DIC. Exp Mech 55, 105–119 (2015). <https://doi.org/10.1007/s11340-014-9888-9>

# Direct of interpolation of FEA strain (ie unlevelled) for comparison is ill-advised



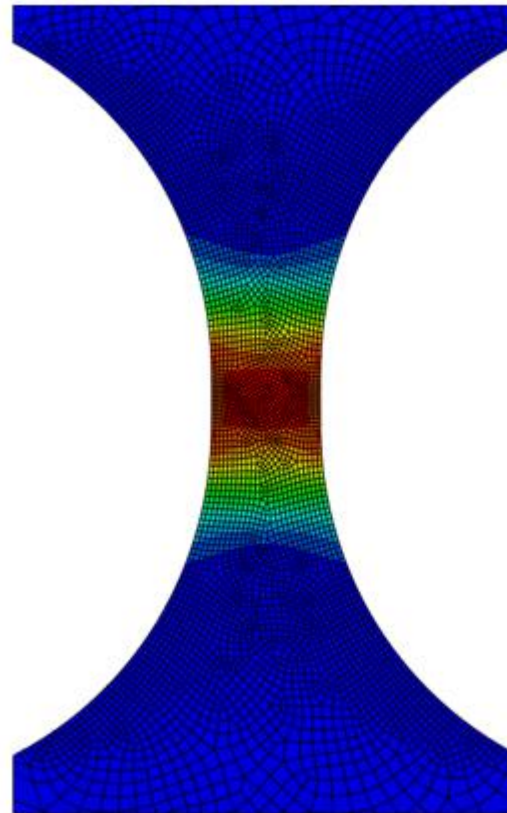
Strains computed using different:

- Calculation method
- Spatial resolution
- Tensor!

Logarithmic Strain (ABAQUS)

$$\boldsymbol{\varepsilon} = \ln(\sqrt{\mathbf{B}}) = \ln(\sqrt{\mathbf{F} \cdot \mathbf{F}^T})$$

ABAQUS Logarithmic Y-Strain

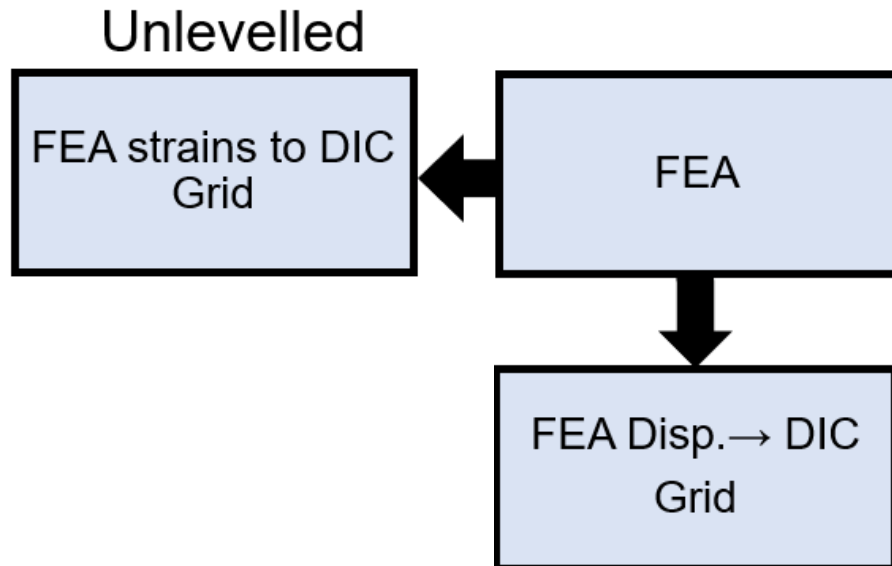


DIC Step Size



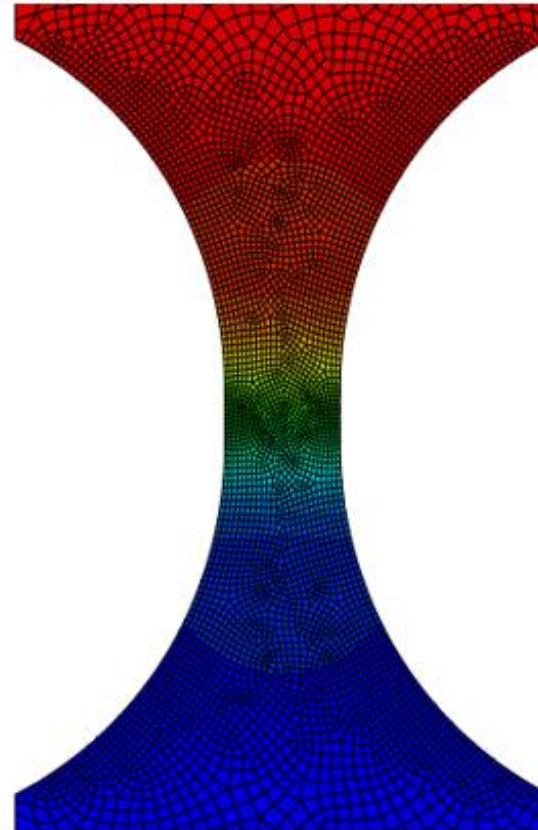


# Direct interpolation of displacement is better



The FEM data is uncorrupted with only small errors due to registering to DIC grid.

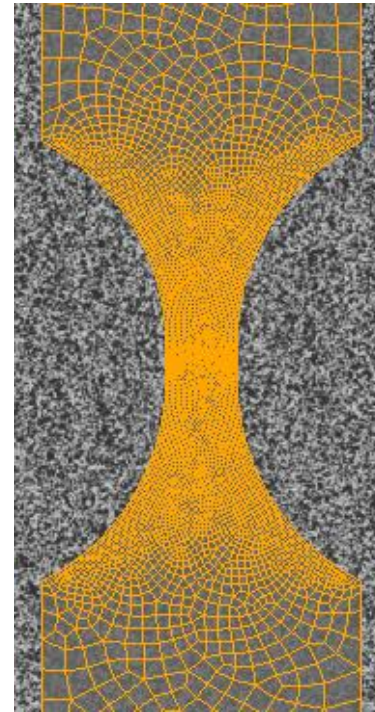
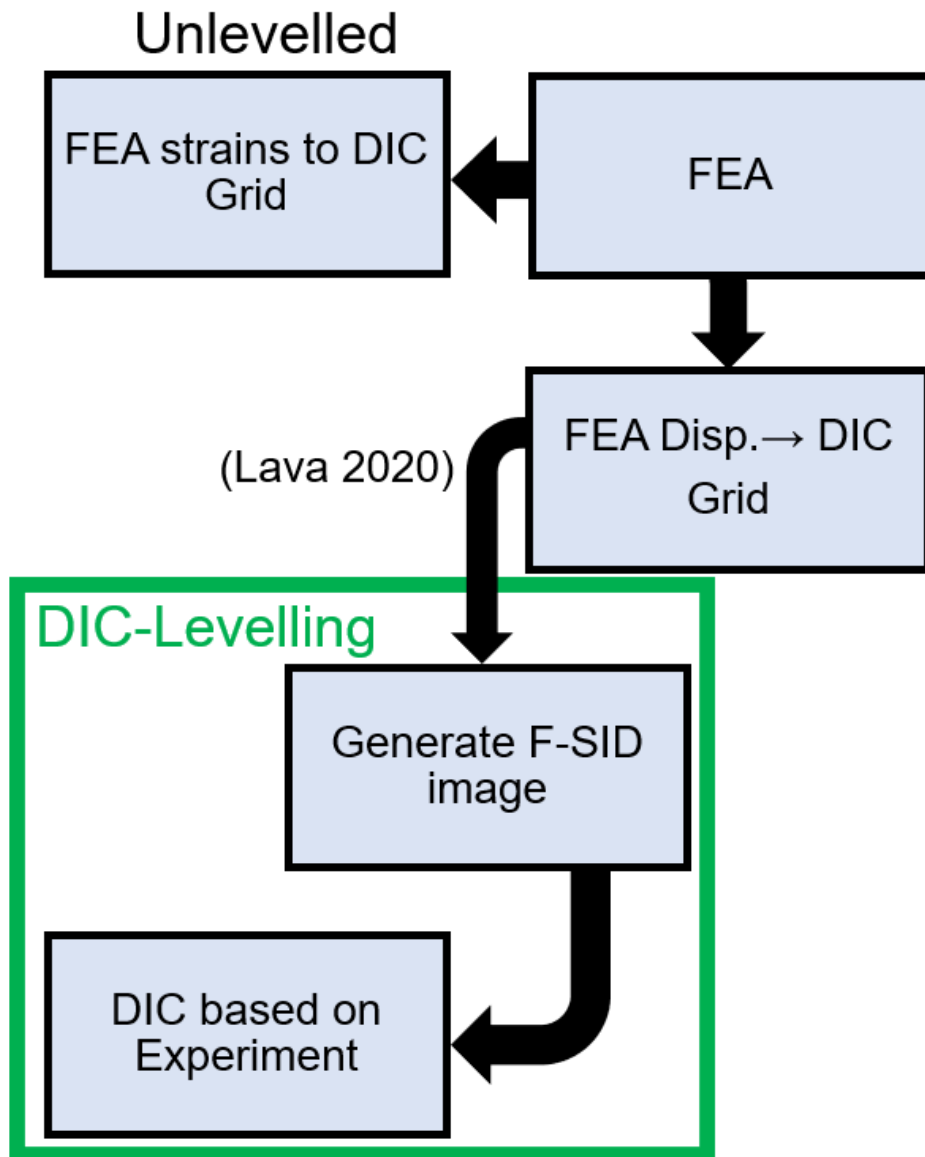
ABAQUS V-Displacement



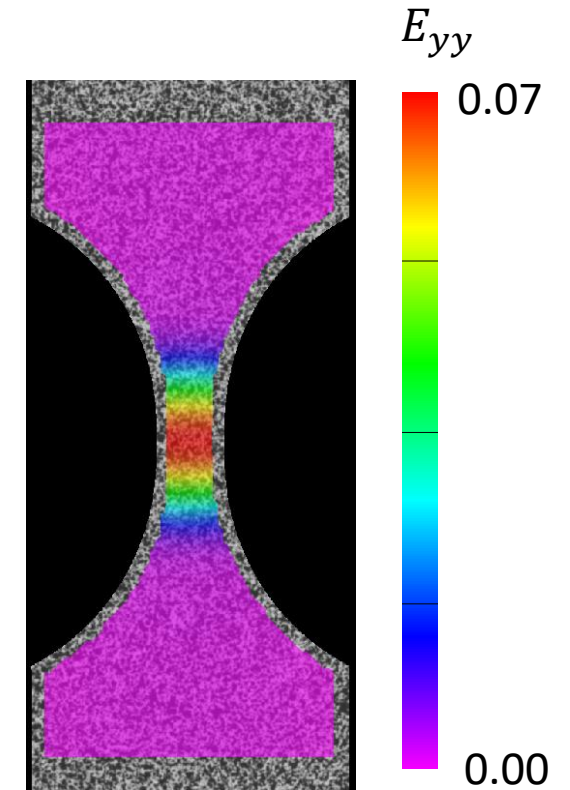
DIC Step Size



# DIC-Levelling corrects for DIC errors



Mesh to pattern alignment

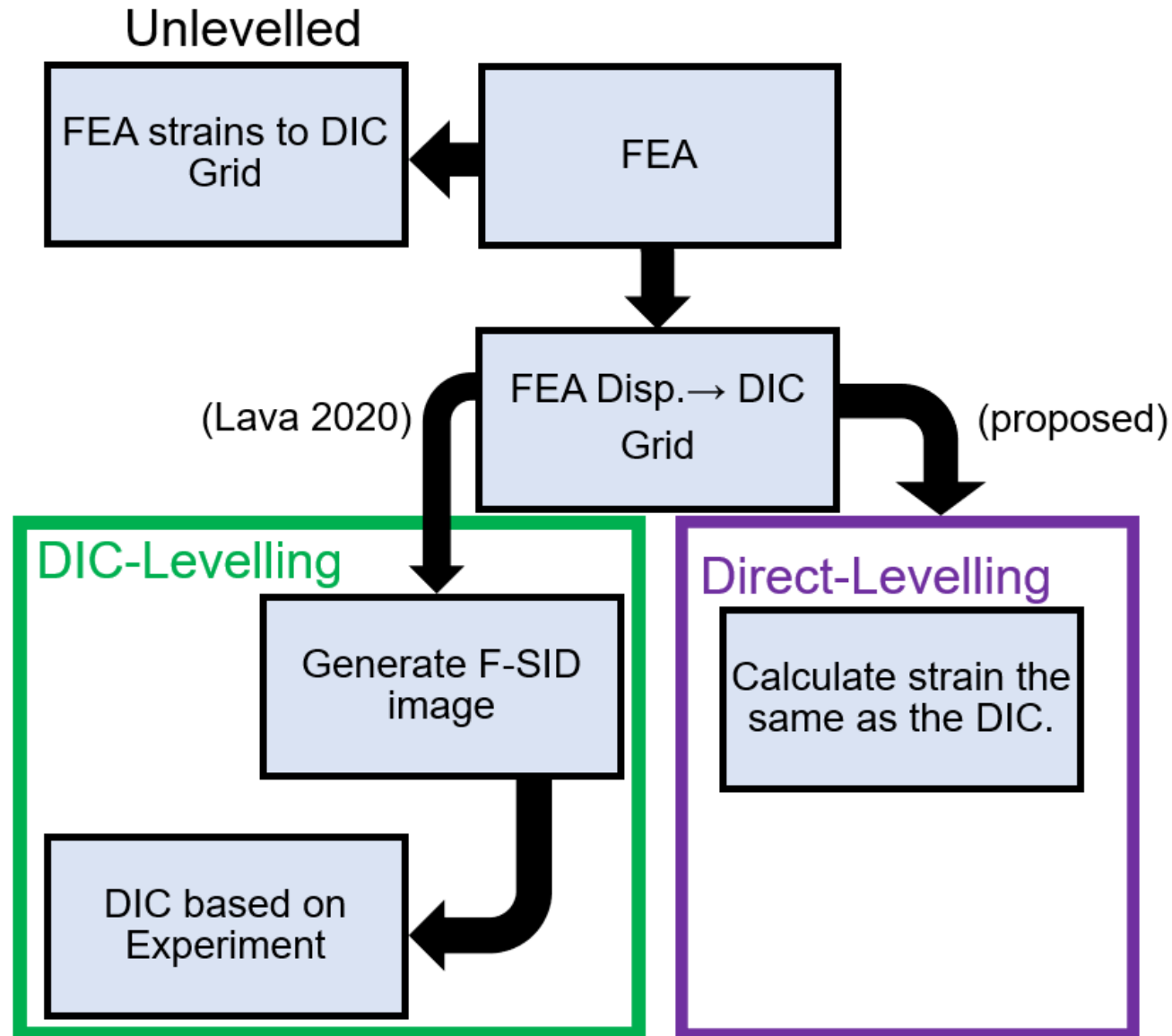


DIC Analysis of F-SID image

Rectifies issues with strain calculation, spatial resolution, and image based errors such as interpolation bias, image discretization, PIB. Does not account for image noise.

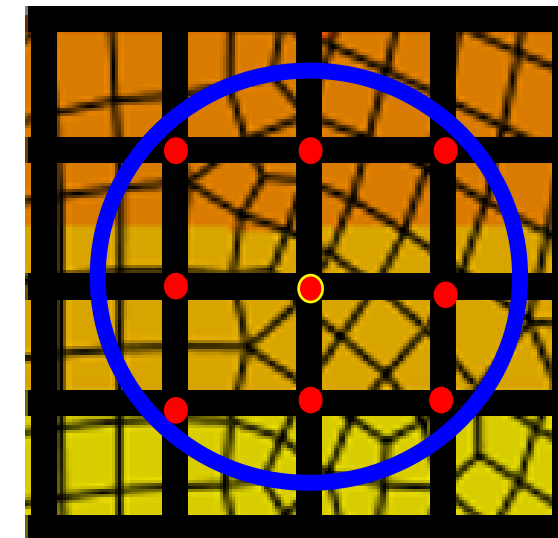


# Direct-levelling the FEA output as a simple solution



Calculate the Green-Lagrange Strain Tensor via the polynomial shape function method

$$E_{KL} = \frac{1}{2} \left( \frac{\partial x_j}{\partial X_K} \frac{\partial x_j}{\partial X_L} - \delta_{KL} \right)$$

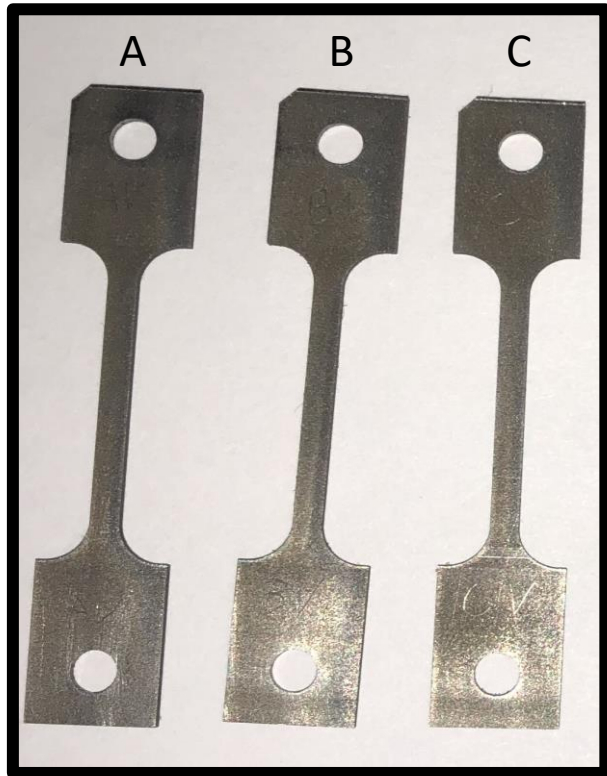


● Included points

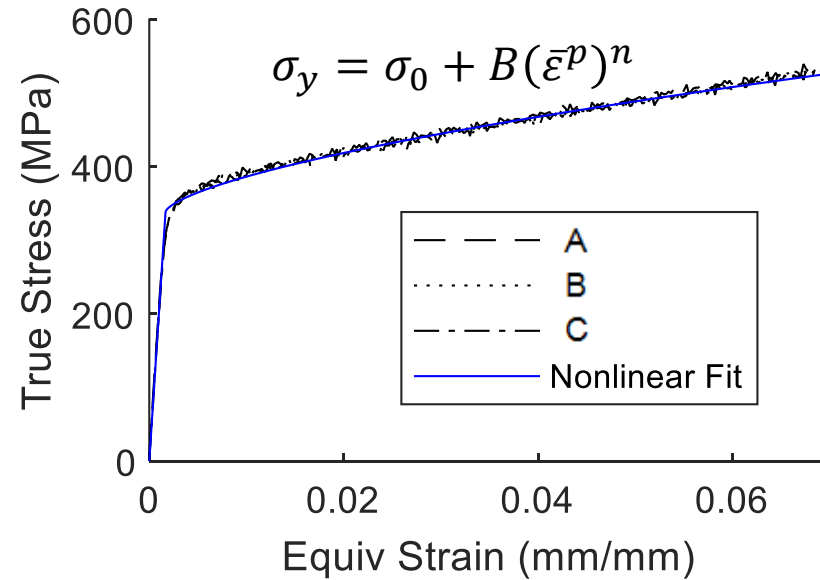
○ VSG

# Material Model for FE Model

304L stainless steel tensile specimens



Material hardening is given by the power law:



Material model parameters

$E = 200 \text{ GPa}$

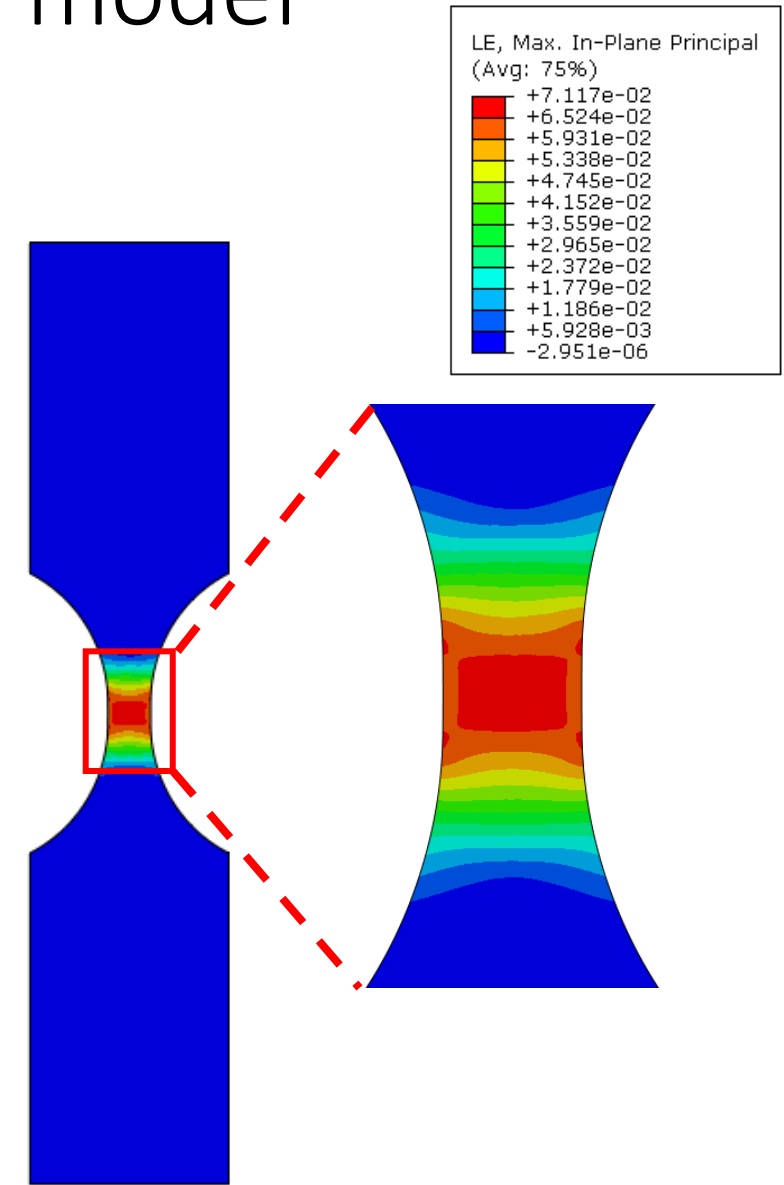
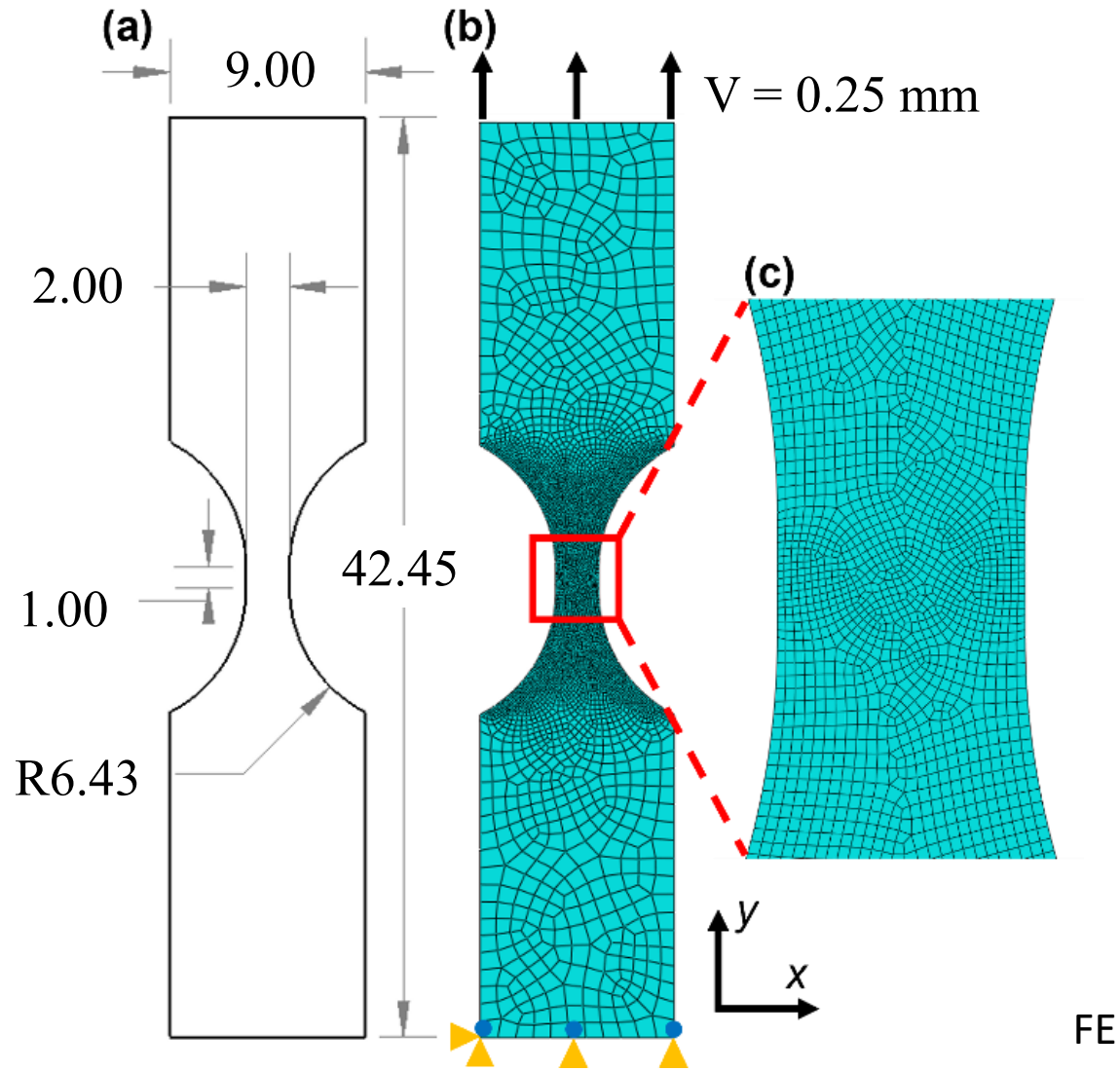
$\nu = 0.29$

$\sigma_0 = 339 \text{ MPa}$

$B = 1.07 \text{ GPa}$

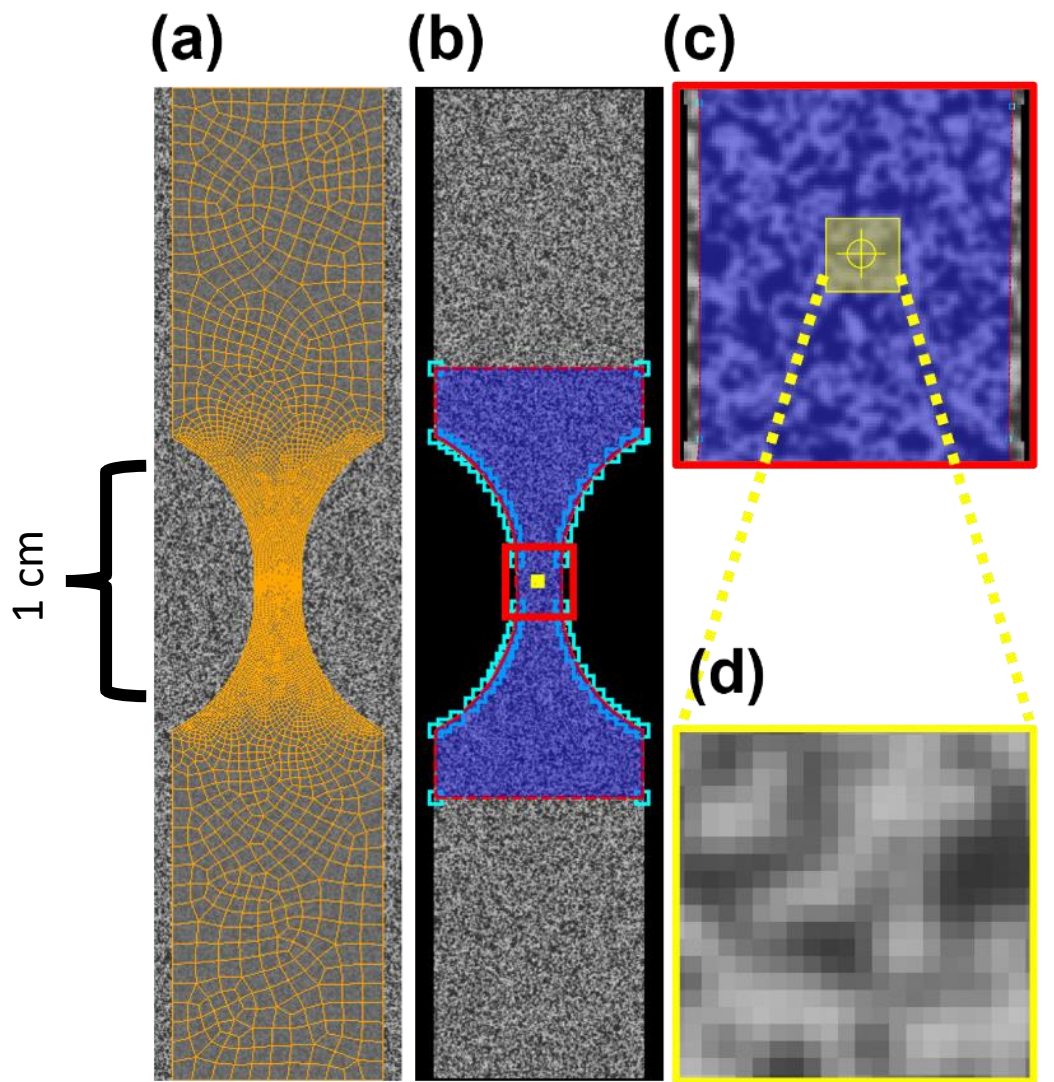
$n = 0.645$

# FEMU Calibration via direct-levelling: FE model

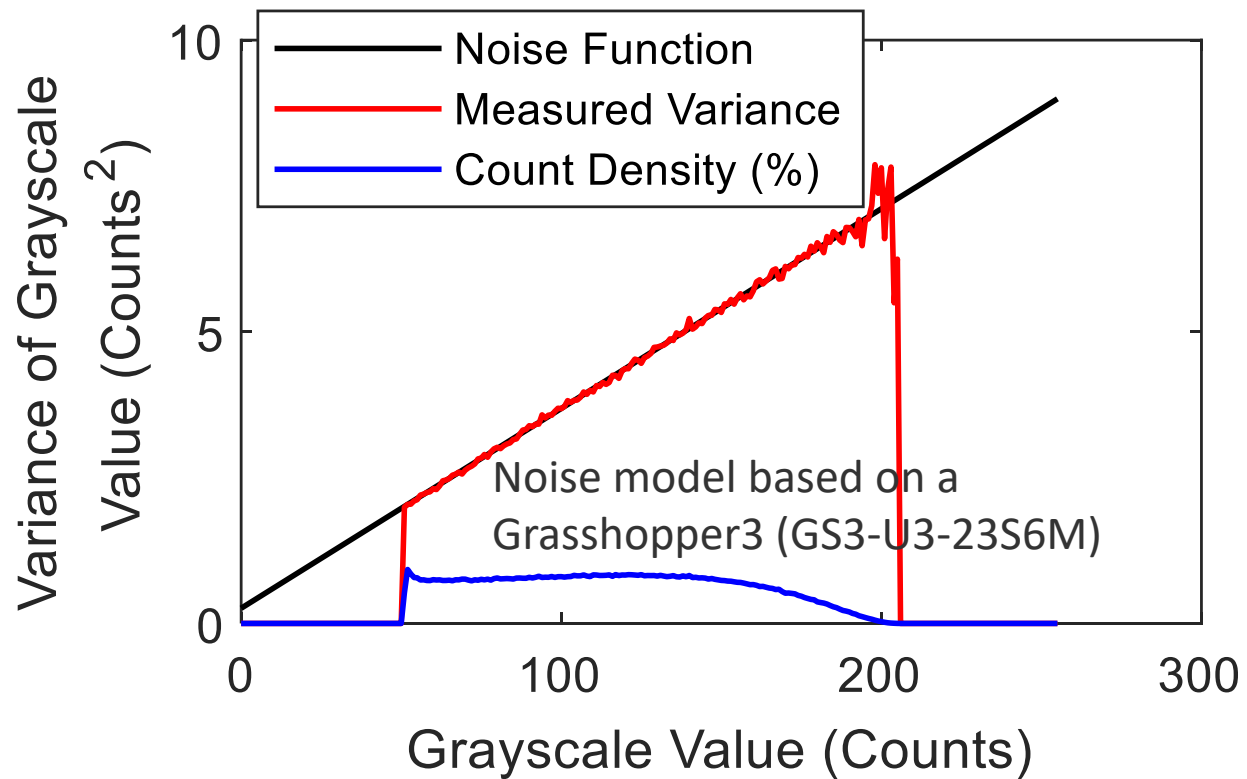




# Generation of Synthetic Images as “experimental” data



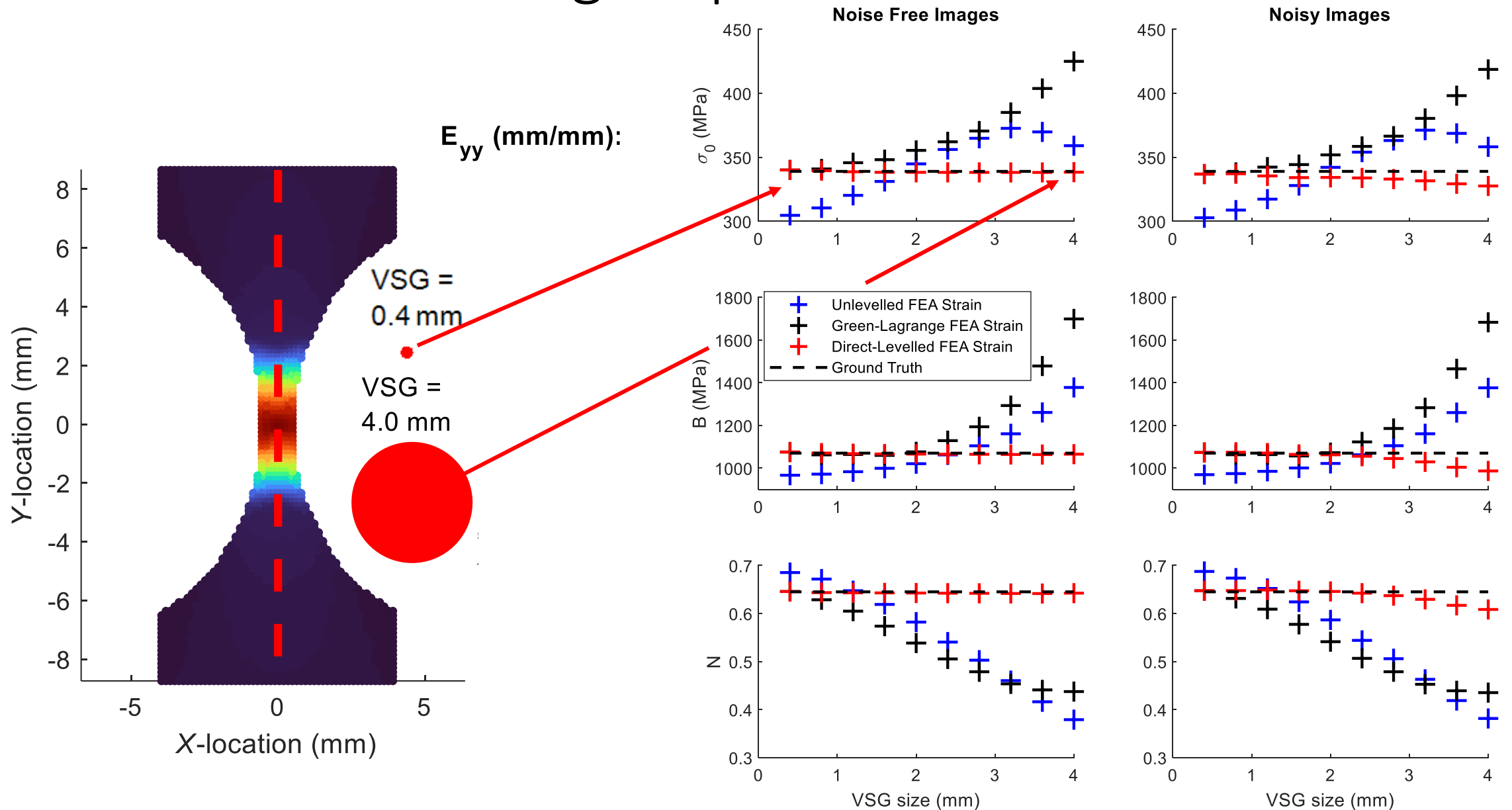
Deformation done by MatchID FEDEF module



DIC Settings

Subset Size	Step Size	Shape Function	Criterion	Image Interpolant
21 px	7 px	Quadratic	ANSSD	Bicubic Spl.

# Effect of direct-levelling on parameter identification



# Conclusions and Future Work

## Summary

- The errors due to the filtering effect of DIC can cause false errors when comparing DIC to FEA
- Properly levelling the FEM to the DIC results is important for an accurate calibration.
- Direct-levelling as opposed to full DIC-levelling is sufficient to obtain an accurate FEMU identification

## Limitations

This method does not account for image-induced errors which we show is small compared to the filtering effects of DIC.

Identification errors using a DIC VSG of 0.8 mm

Levelling		Unlevelled	Direct-Levelled
$\sigma_0$	Noise Free	8.42%	-0.15%
$B$		9.19%	-0.03%
$n$		-4.12%	0.26%
$\sigma_0$	Noisy	8.92%	0.55%
$B$		8.96%	-0.39%
$n$		-4.43%	-0.40%

Identification errors have been reduced more than **10X** for the identification results using noisy images



# Supplemental Slides: Identification Errors

DIC Shape Fun.		Quadratic	Quadratic	Quadratic	Affine	Affine	Quadratic
DIC Step Size (pixels)		7	7	7	7	7	2
Levelling		UL	TL	SL	SL	SSL & SL	SL
$\sigma_0$	Noise Free	8.42%	-0.56%	-0.15%	0.76%	0.16%	0.04%
$B$		9.19%	0.76%	-0.03%	1.20%	0.40%	0.11%
$n$		-4.12%	2.56%	0.26%	2.03%	0.37%	0.21%
$\sigma_0$	Noisy	8.92%	0.24%	0.55%	1.28%	0.68%	0.40%
$B$		8.96%	0.70%	-0.39%	0.58%	-0.31%	-0.36%
$n$		-4.43%	2.13%	-0.40%	1.16%	-0.54%	-0.25%