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Mesh Convergence for Coupled Problems using Immersed Methods

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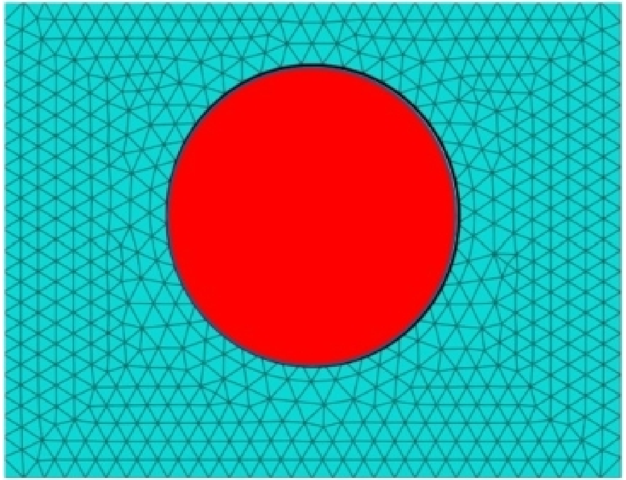
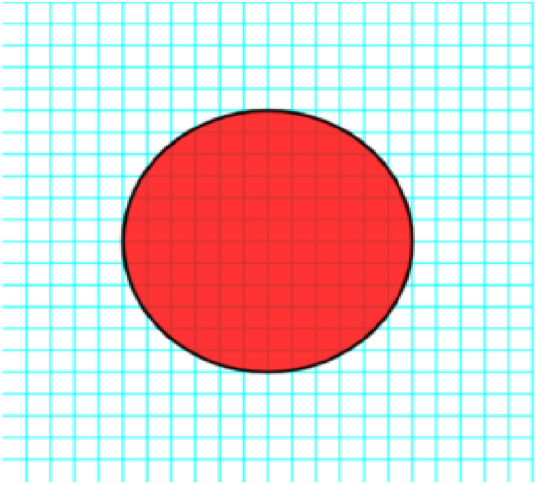
Conforming vs non-comforming meshing techniques



Conforming mesh method:

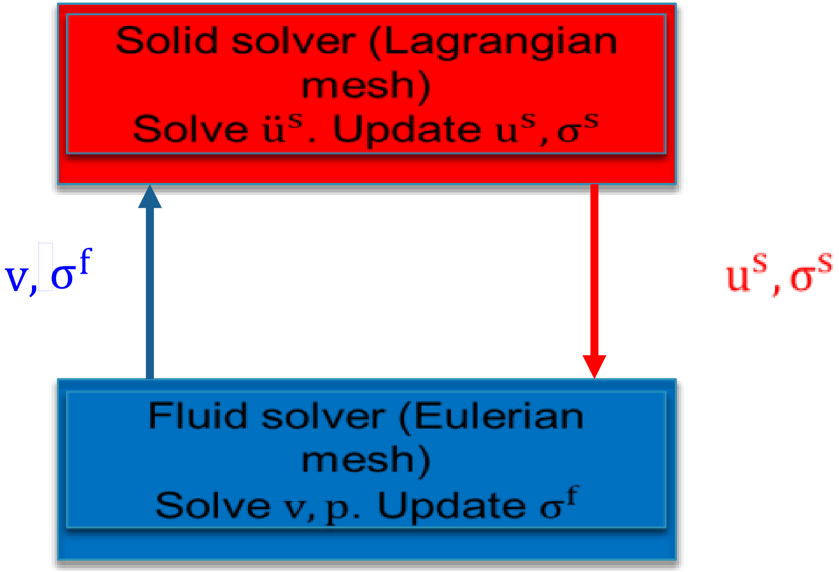
- Mesh conforming at interface
- Mesh movement incorporated into governing equations
- Needs remeshing



Non-conforming mesh method:

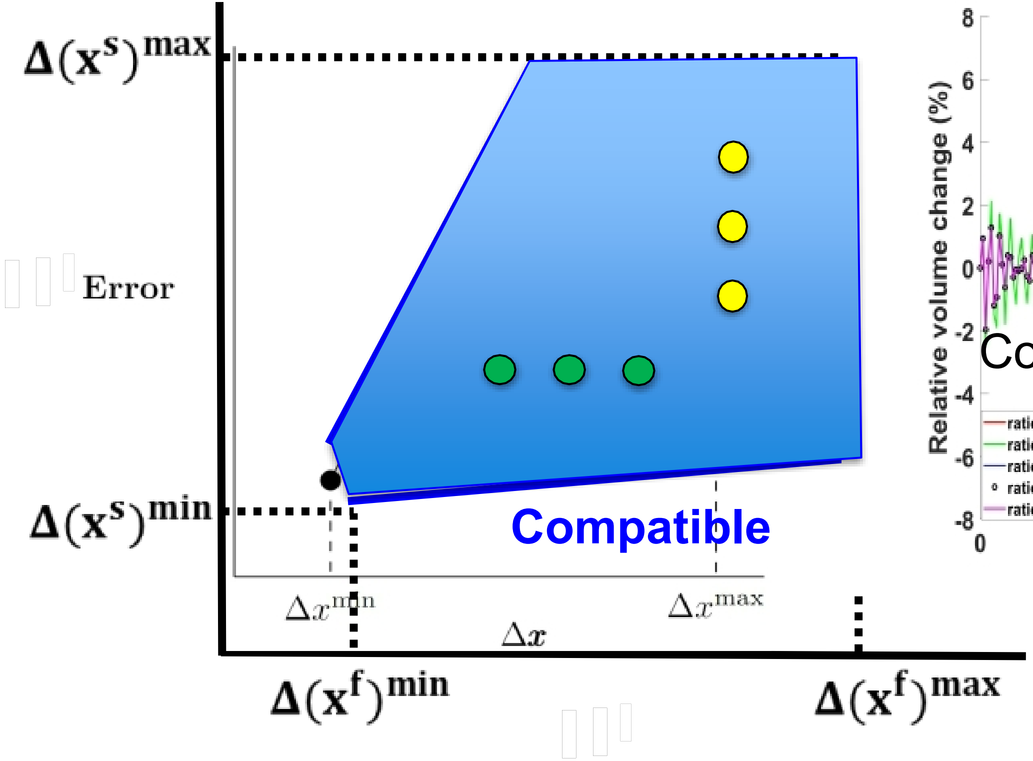
- Two independent meshes
- Data exchange through interpolation
- No remeshing, inexact interface representation

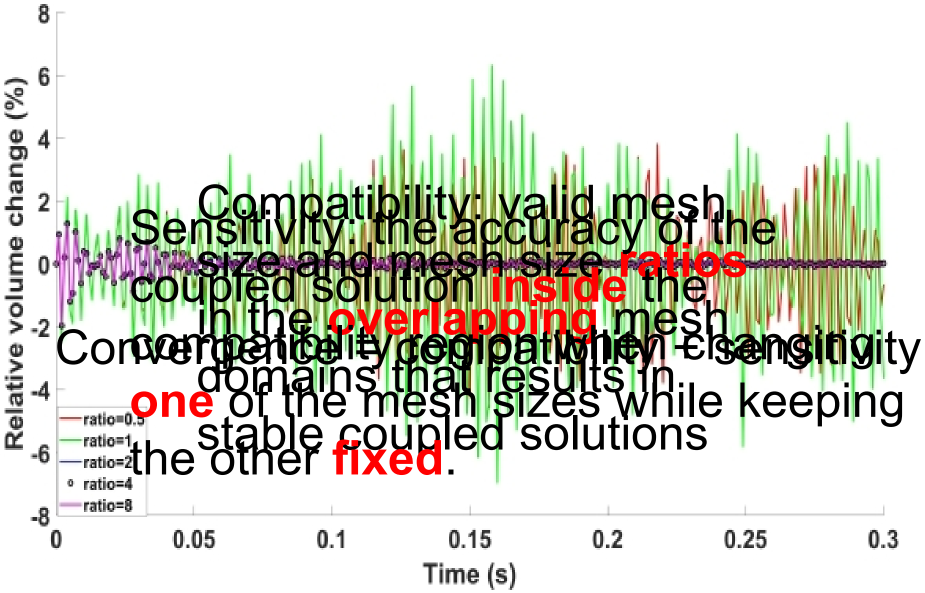
A generic algorithm for fully-coupled non-boundary-fitted meshing FSI methods



Hypothesis: Mesh size **ratio** is the dominant factor for convergence

Definition of mesh convergence for immersed methods





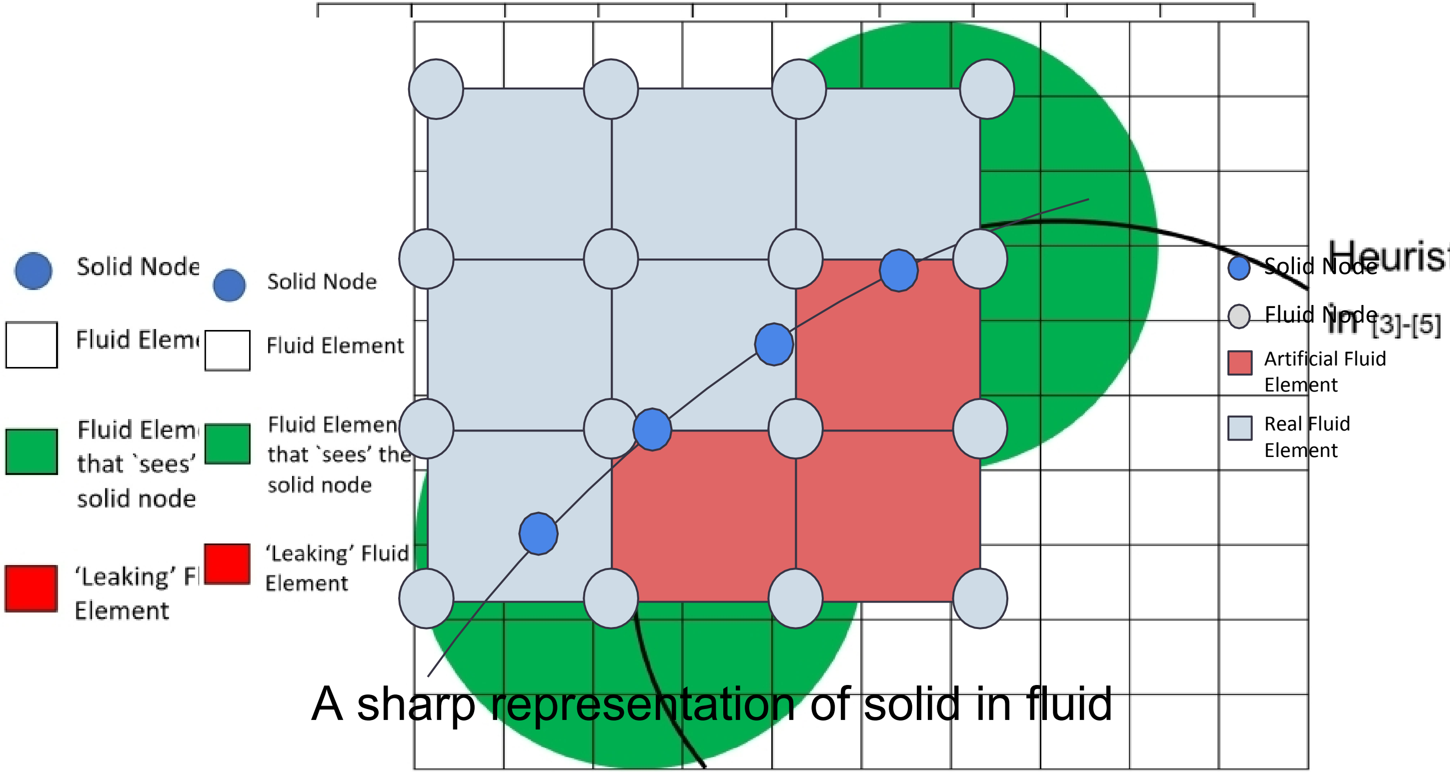
Compatibility: valid mesh size and mesh size **ratios** in the **overlapping** mesh domains that results in stable coupled solutions

Sensitivity: the accuracy of the coupled solution **inside** the compatibility region when changing **one** of the mesh sizes while keeping the other **fixed**.

Convergence = compatibility + sensitivity

Check the solid volume/area conservation [1-3]

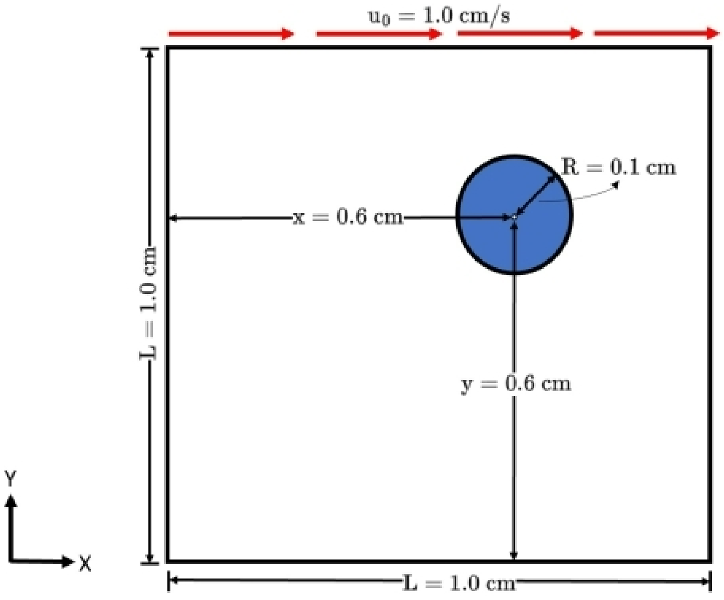
Explicit solid representation



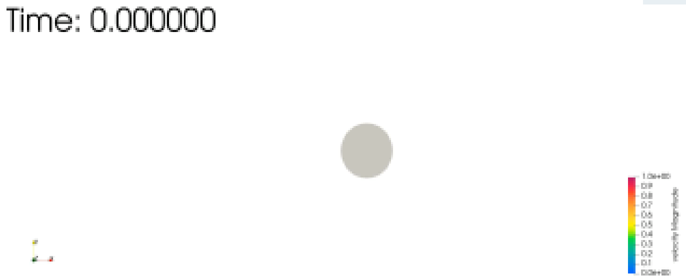
Heuristic $\frac{\Delta x^s}{\Delta x^f}$ = 1:2 is used in [3]-[5]

A sharp representation of solid in fluid

Numerical Example I: 2-D deformable solid disk driven by cavity flow[6]



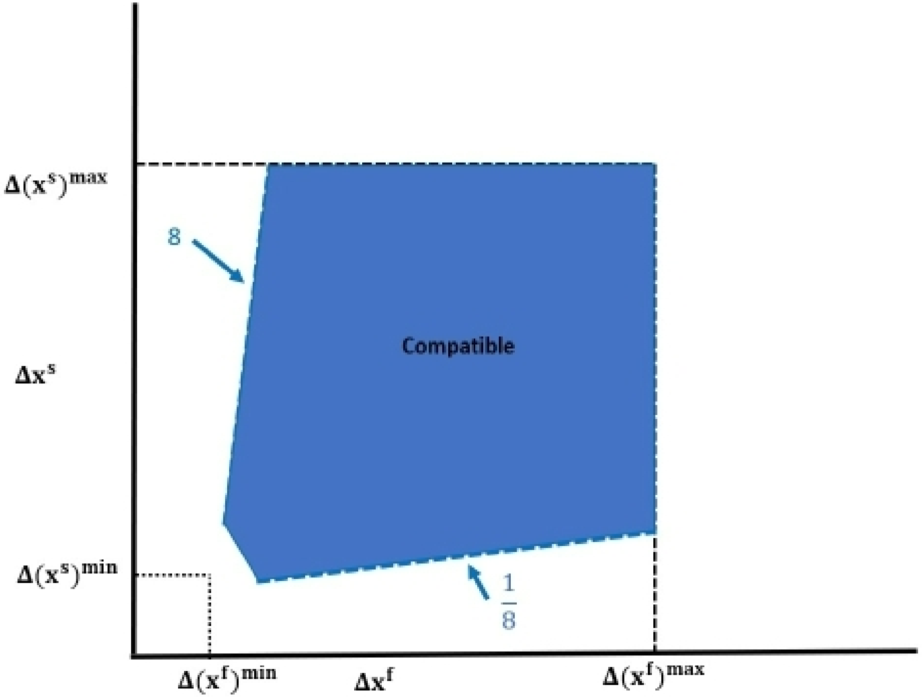
- **Fluid** (incompressible) constant velocity on the top, no-slip and no penetration on the other boundaries
- **Solid** (Neo-Hookean) free of constraints



Material properties

Fluid	ρ^f (g/cm^3)	1
	μ^f (dyne/(cm · s))	1
Solid	ρ^s (g/cm^3)	1
	C (dyne/cm^2)	1.72×10^4
	K (dyne/cm^2)	3.33×10^5

Numerical Example I: 2-D deformable solid disk driven by cavity flow



Fluid sensitivity tests	Solid sensitivity tests	Pure fluid tests
fluid velocity	fluid velocity	fluid velocity
solid KE over time	solid KE over time	
solid displacement	solid displacement	

Avg solid solution difference using half time step: 0.33%
Avg fluid solution difference using half time step: 0.22%

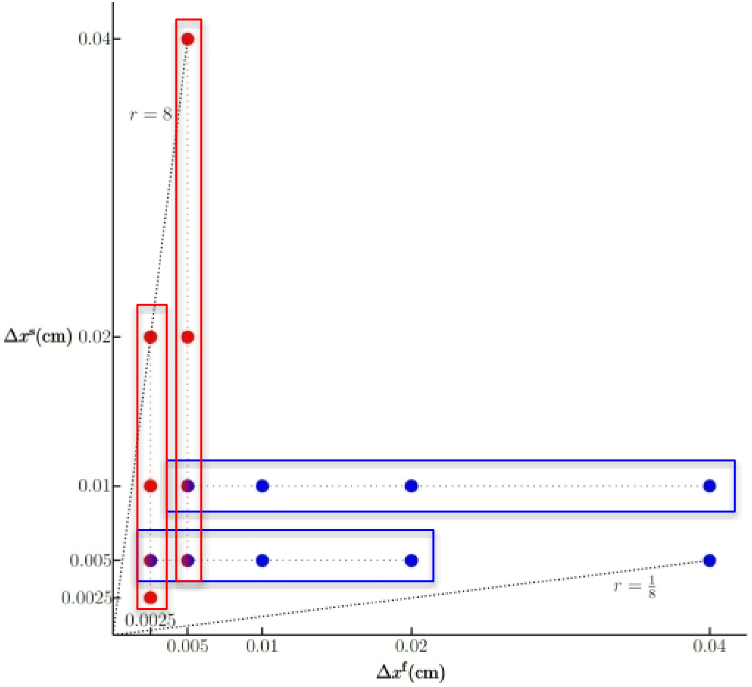
Extrapolation order of convergence using existing solutions [7]:

$$p = \frac{\ln\left(\frac{f_3 - f_2}{f_2 - f_1}\right)}{\ln(r)}$$

r is the mesh refinement ratio

Numerical Example I: 2-D deformable solid disk driven by cavity flow

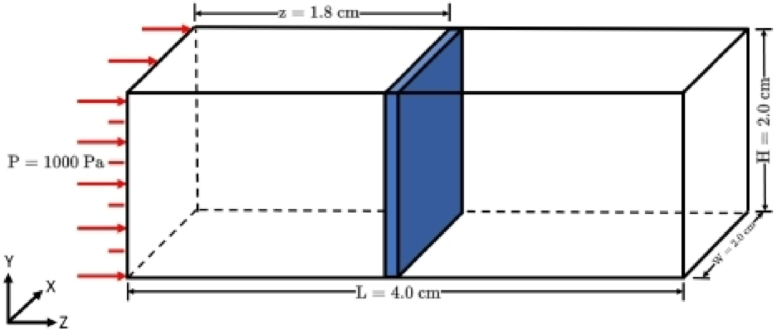
Fluid/solid sensitivity studies	Measurement	Convergence rate **coarse** mesh combinations (L1/L2/LInf)	Convergence rate **fine** mesh combinations (L1/L2/LInf)	pure fluid tests
fluid	fluid velocity	1.24/1.22/1.07 (0.97)	1.39/1.41/1.29 (0.87)	3.71/3.86/3.88/ (4.23)
fluid	solid displacement	1.31/1.34/1.66 (0.76)	1.28/1.28/1.36 (0.93)	N/A
fluid	solid KE	1.47 (time-avg) (1.16)	1.39 (time-avg) (1.01)	N/A
solid	fluid velocity	2.05/1.98/1.60 (1.66)	2.22/2.27/2.11 (2.18)	N/A
solid	solid displacement	1.86/1.90/2.00 (1.49)	1.90/1.92/2.01 (1.35)	N/A
solid	solid KE	2.25 (time-avg) (2.46)	2.22 (time-avg) (1.83)	N/A



Finer background mesh improves the convergence

Mesh size ratios dominates the convergence

Numerical Example II: 3-D elastic block under pressure wave

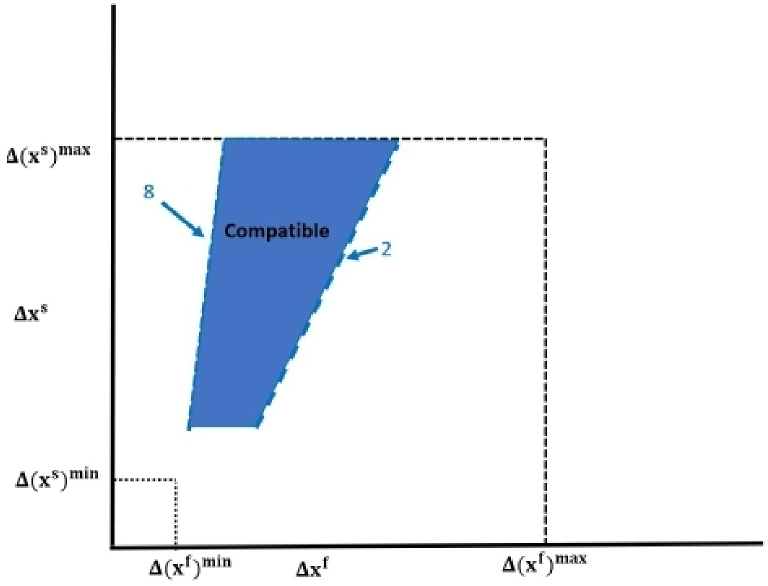


- **Fluid** (slightly compressible) constant pressure on the left, outflow on the right, no-slip on others
- **Solid** (Neo-Hookean) fully constrained on top and bottom, constrained in X on left and right surfaces, free for front and back surfaces

Material properties

Fluid	ρ^f (g/cm^3)	1.3×10^{-3}
	μ^f $(dyne/(cm \cdot s))$	1.8×10^{-4}
Solid	ρ^s (g/cm^3)	1
	C $(dyne/cm^2)$	1.72×10^4
	K $(dyne/cm^2)$	3.33×10^5

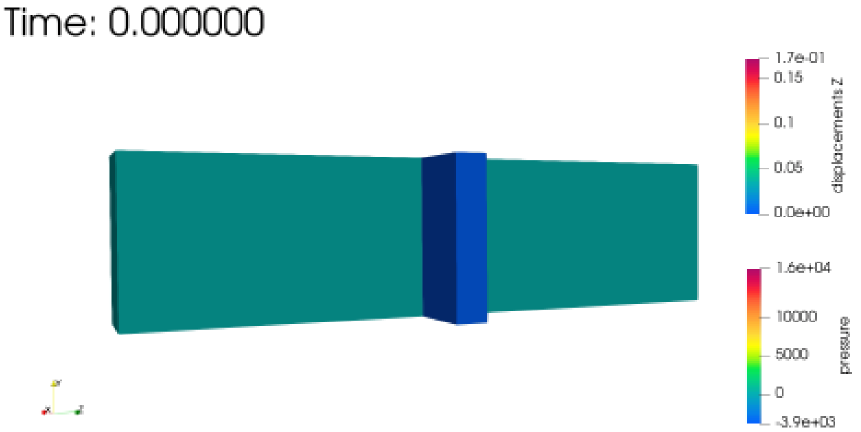
Numerical Example II: 3-D elastic block under pressure wave



Solid mesh size	Fluid mesh size	Convergence steps
0.1	0.1	3790
0.05	0.05	2970
0.025	0.025	1710

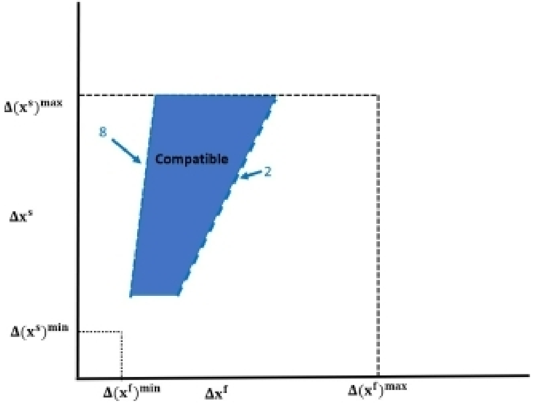
Observation: refine the mesh for incompatible mesh size ratios will accelerate the failure

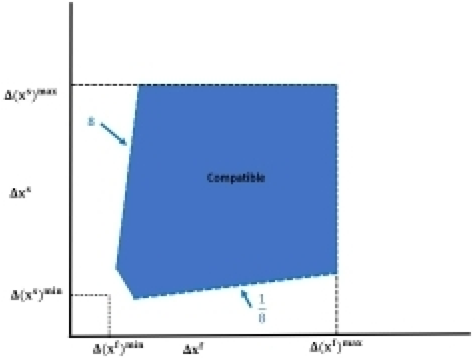
Numerical Example II: 3-D elastic block under pressure wave



Fluid/solid sensitivity studies	Measurement	convergence rate (L1/L2/LInf)
fluid	fluid velocity	1.46/1.60/1.11 (1.05)
fluid	solid displacement	0.98/1.02/0.83 (0.81)
fluid	solid KE	1.33 (time-avg) (1.40)
solid	fluid velocity	1.82/1.77/1.88 (1.72)
solid	solid displacement	1.34/1.13/1.42 (1.45)
solid	solid KE	1.47 (time-avg) (1.53)

Summary of mesh convergence studies





- ❑ Compatible mesh size **ratios** are **pre-requisite** to a convergence coupled solution
- ❑ FSI interpolation method, solid geometry, B.C., and deformation affect the **range** of the compatible mesh size ratios
- ❑ The mesh size ratio **dominates** the accuracy of the coupled FSI solutions

Summary

- Two-dimensional **mesh convergence** definition
- The need for the **mesh compatibility** study
- The dominant effect of the **mesh size ratio** on coupled solution convergence

References

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