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Chiral-Symmetric Higher-Order Topological Phases Protected by Multipole Chiral Number Invariants

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CLEO – Chiral and Rotational Structures – FF3D.2

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Outline

- Brief review of known methods for classifying higher-order topological phases

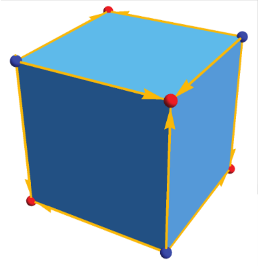
- Chiral-symmetric higher-order topological phases (HOTPs) in 2D
  - Multipole chiral numbers
  - In general, identify boundary obstructed phases

- Unexpected new phases, robust against disorder

Higher-order topological phases

topological insulators

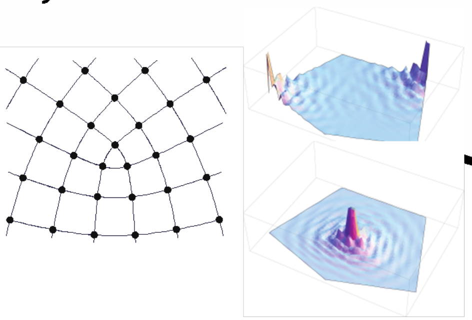
fractional charge



HIGHER-ORDER TOPOLOGICAL PHASES

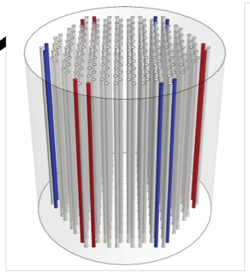
topological superconductors

Majorana bound states



topological photonics

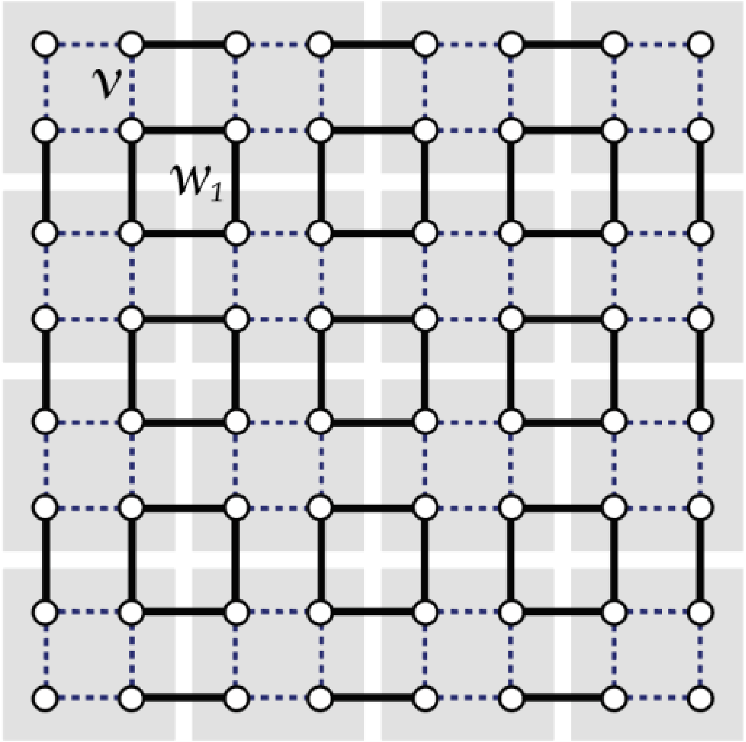
maximally confined mid-gap modes



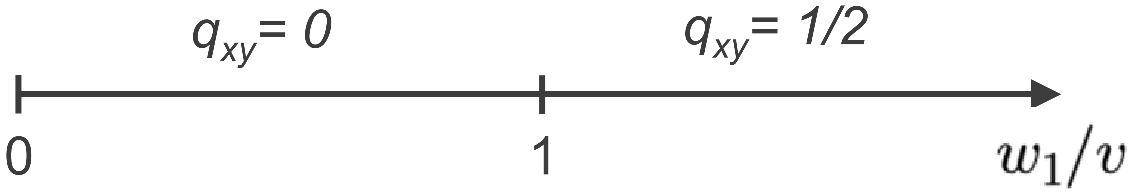
Wannier center picture:
(crystalline symmetries)
Fractional quantization of charge

Boundary mass domains:
(Particle-hole and/or chiral symmetries)
Single zero-energy states

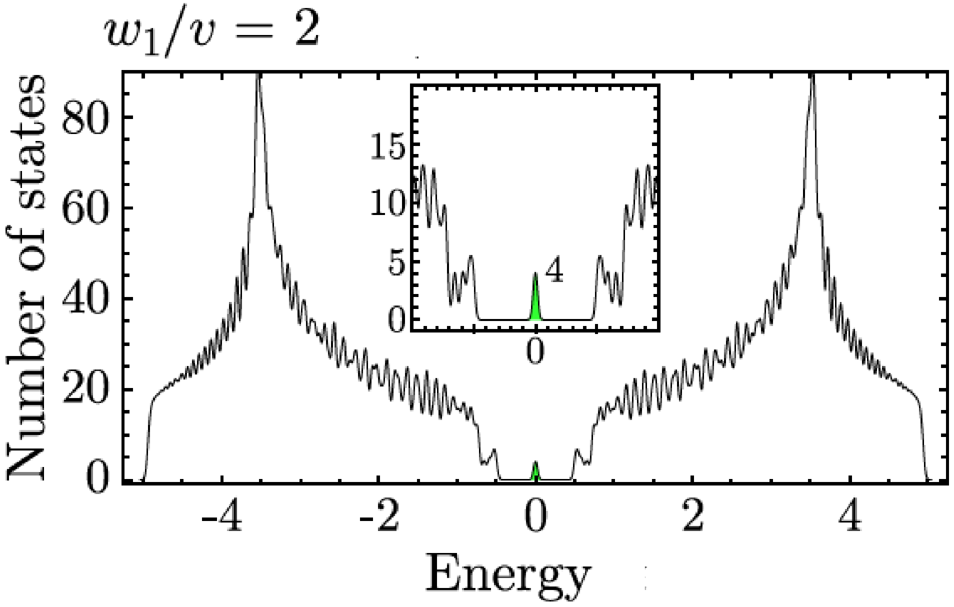
Chiral-symmetric quadrupole topological insulator

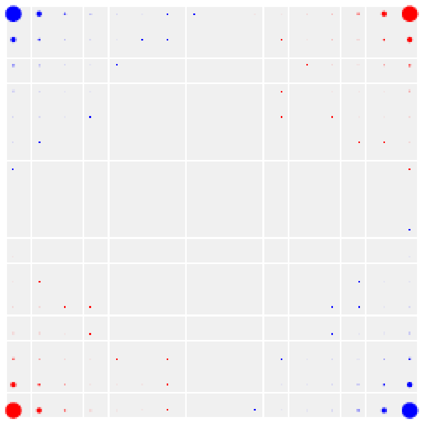


Tight-binding representation:
v: intra-cell hopping
w_1: inter-cell hopping
π flux per plaquette



Density of states in the q_{xy} = 1/2 phase

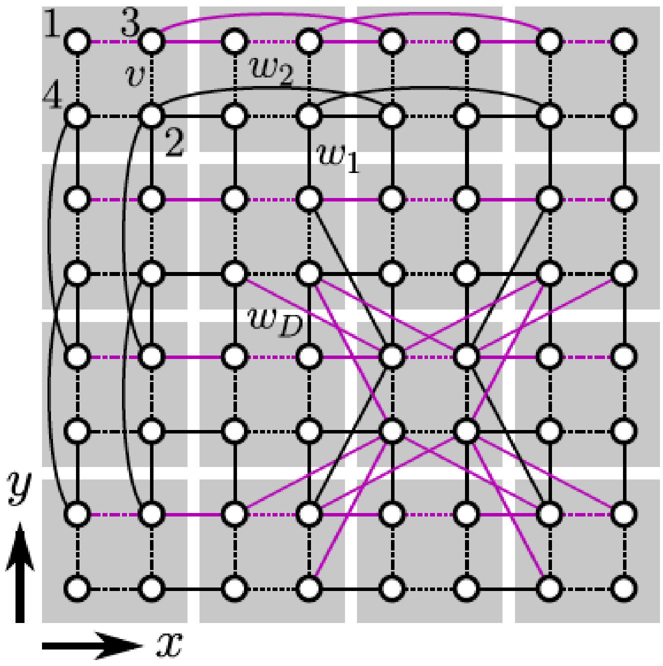


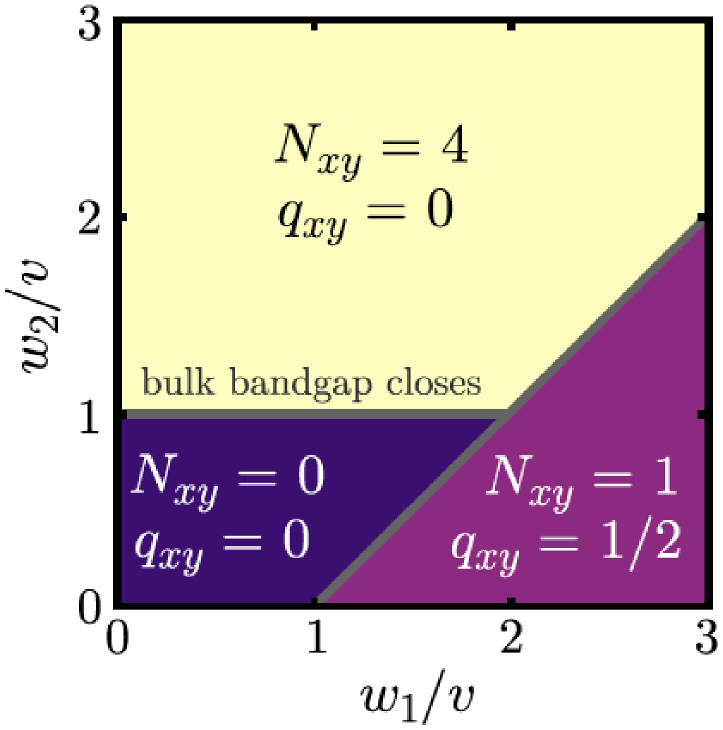


zero-energy states

Benalcazar, Bernevig, and Hughes, *Science* **357**, 61 (2017)

Chiral-symmetric higher-order topo. insulators in 2D





What happens with long-range hoppings?

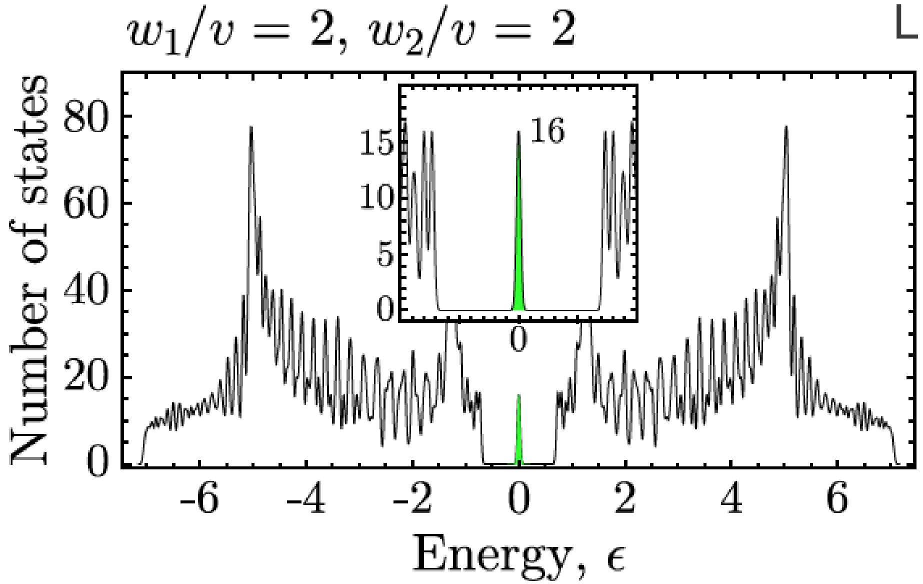
- Start with quadrupole topological insulator (QTI)
- ➢ Add horizontal/vertical long-range hopping
- ➢ Add diagonal long-range hopping

Phase diagram of the model:

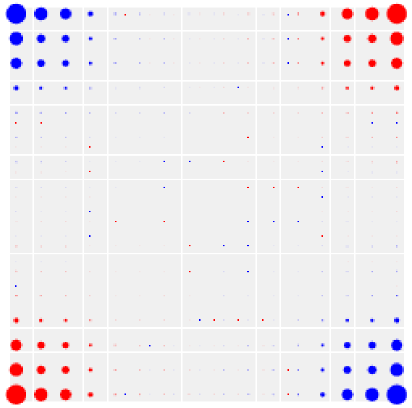
- Trivial phase (no corner states)
- Quadrupole phase (1 state per corner)
- $N_{xy} = 4$ phase (4 states per corner)

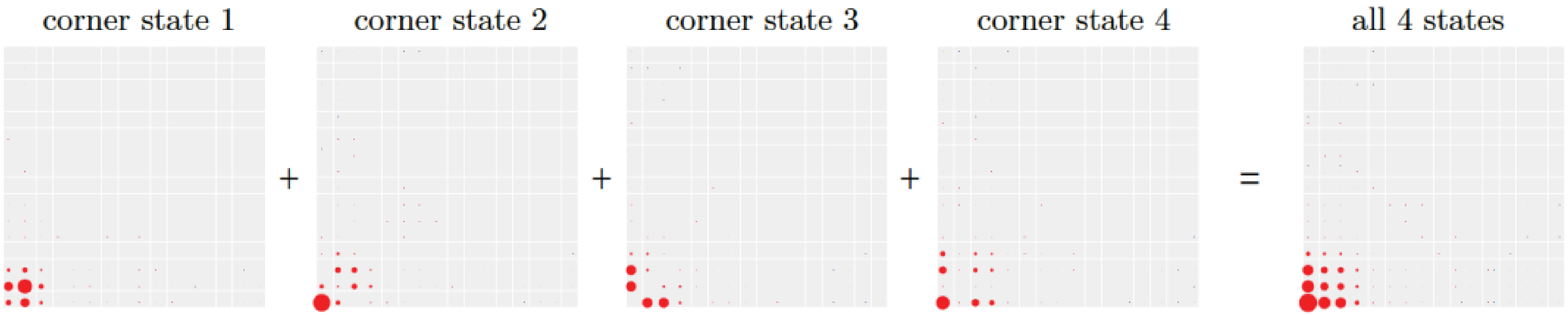
Chiral-symmetric higher-order topological phases

Density of states in $N_{xy} = 4$ phase



LDOS at zero-energy

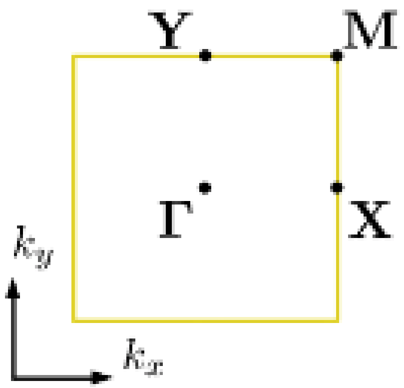




What are the invariants of chiral-symmetric HOTPs?

Not predicted by symmetry-indicator invariants

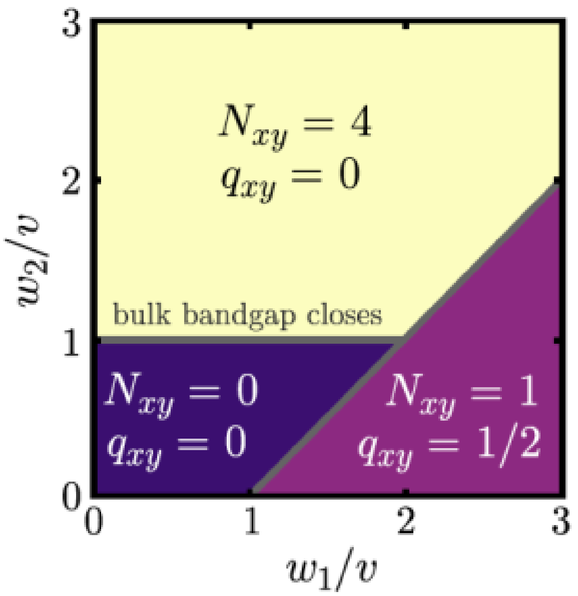
Bradlyn et al. , *Nature* **547**, 298 (2017)
Po et al., *Nat. Comm.* **8**, 50 (2017)



In a C_4-symmetric Brillouin zone, define:

$$[M_j] = \text{\# of states with evals } e^{i\pi(2j-1)/4} \text{ at } \mathbf{M} - \text{\# of states with evals } e^{i\pi(2j-1)/4} \text{ at } \mathbf{\Gamma}$$

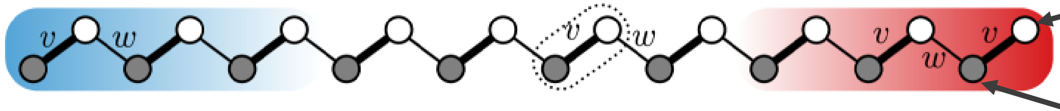
phase	irreps at $\mathbf{\Gamma}$	irreps at $\mathbf{M}$	$([M_1], [M_2], [M_3], [M_4])$
$N_{xy} = 0$	$\{e^{i3\pi/4}, e^{-i3\pi/4}\}$	$\{e^{i3\pi/4}, e^{-i3\pi/4}\}$	(0,0,0,0)
$N_{xy} = 1$	$\{e^{i3\pi/4}, e^{-i3\pi/4}\}$	$\{e^{i\pi/4}, e^{-i\pi/4}\}$	(1,-1,-1,1)
$N_{xy} = 4$	$\{e^{i3\pi/4}, e^{-i3\pi/4}\}$	$\{e^{i3\pi/4}, e^{-i3\pi/4}\}$	(0,0,0,0)



outside of the Wannier center / TQC framework

What are the invariants of chiral-symmetric HOTPs?

Under chiral symmetry: $\mathcal{H} = \begin{pmatrix} 0 & h \\ h^\dagger & 0 \end{pmatrix}$. Perform SVD: $h = U_A \Sigma U_B^\dagger$. Define $q = U_A U_B^\dagger$.



Topological invariant in 1D: $N_x = (1/2\pi \mathrm{i}) \int_{BZ} \mathrm{Tr} \left[q(k)^\dagger \partial_k q(k) \right]$ (winding number)

Recasting the 1D winding number in terms of sublattice dipole operators:

Lin, Ke, Lee, *Phys. Rev. B* **103**, 224208 (2021)

Consider the sublattice dipole operator

$P_x^{\mathcal{S}} = \sum_{R,\alpha \in \mathcal{S}} |R,\alpha\rangle \, \mathrm{Exp}(-\mathrm{i}2\pi R/L) \, \langle R,\alpha|$, where $\mathcal{S} = A, B$

By defining $\bar{P}_x^{\mathcal{S}} = U_{\mathcal{S}}^\dagger P_x^{\mathcal{S}} U_{\mathcal{S}}$

The winding number can be written as $N_x = (1/2\pi \mathrm{i}) \mathrm{TrLog}(\bar{P}_x^A \bar{P}_x^{B\dagger}) \in \mathbb{Z}$

Real-space formulation of a generalized 1D winding number

What are the invariants of chiral-symmetric HOTPs?

Multipole chiral numbers

Similarly, we can define the sublattice multipole moment operators:

$$Q_{xy}^{\mathcal{S}} = \sum_{\mathbf{R},\alpha\in\mathcal{S}} |\mathbf{R},\alpha\rangle \, \mathrm{Exp}\left(-\mathrm{i}\frac{2\pi xy}{L_x L_y}\right) \langle\mathbf{R},\alpha|$$

$$O_{xyz}^{\mathcal{S}} = \sum_{\mathbf{R},\alpha\in\mathcal{S}} |\mathbf{R},\alpha\rangle \, \mathrm{Exp}\left(-\mathrm{i}\frac{2\pi xyz}{L_x L_y L_z}\right) \langle\mathbf{R},\alpha|$$

resemble those in Wheeler, Wagner, Hughes, *Phys. Rev. B* **100**, 245135 (2019), but restricted to sublattice

Using them, we obtain the "multipole chiral numbers":

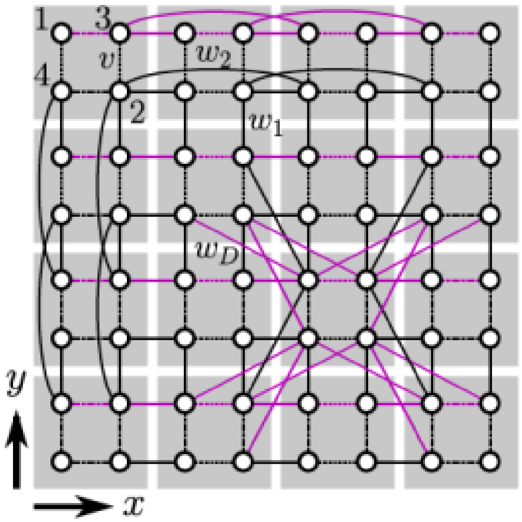
$$N_{xy} = \frac{1}{2\pi\mathrm{i}} \mathrm{TrLog}\left(\bar{Q}_{xy}^{A} \bar{Q}_{xy}^{B\dagger}\right) \in \mathbb{Z}$$

$$N_{xyz} = \frac{1}{2\pi\mathrm{i}} \mathrm{TrLog}\left(\bar{O}_{xyz}^{A} \bar{O}_{xyz}^{B\dagger}\right) \in \mathbb{Z},$$

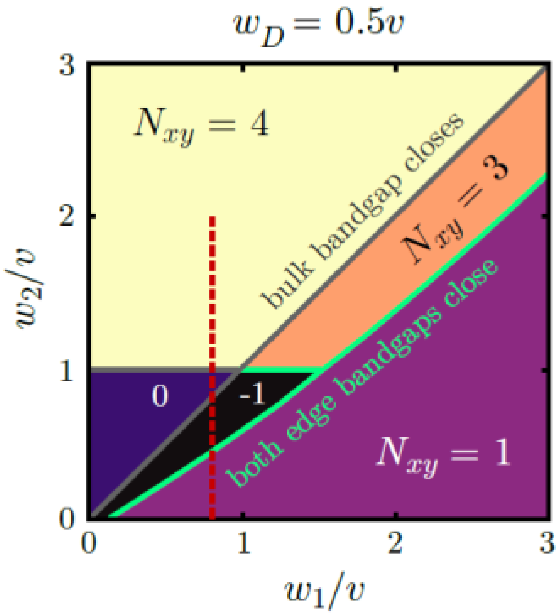
Integer invariant!

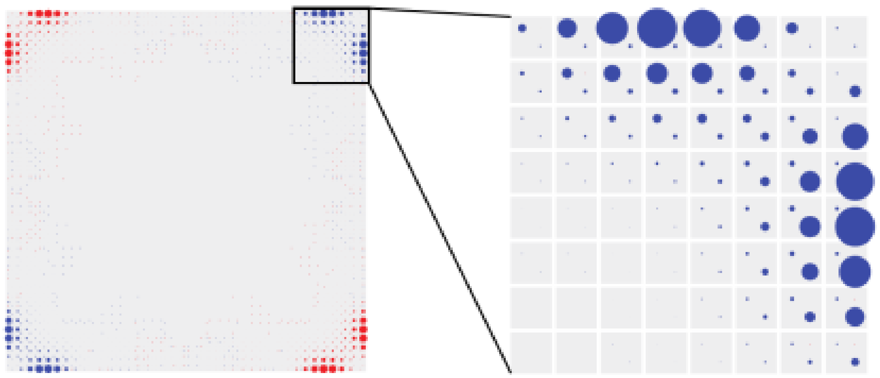
These invariants establish a higher-order bulk-boundary correspondence

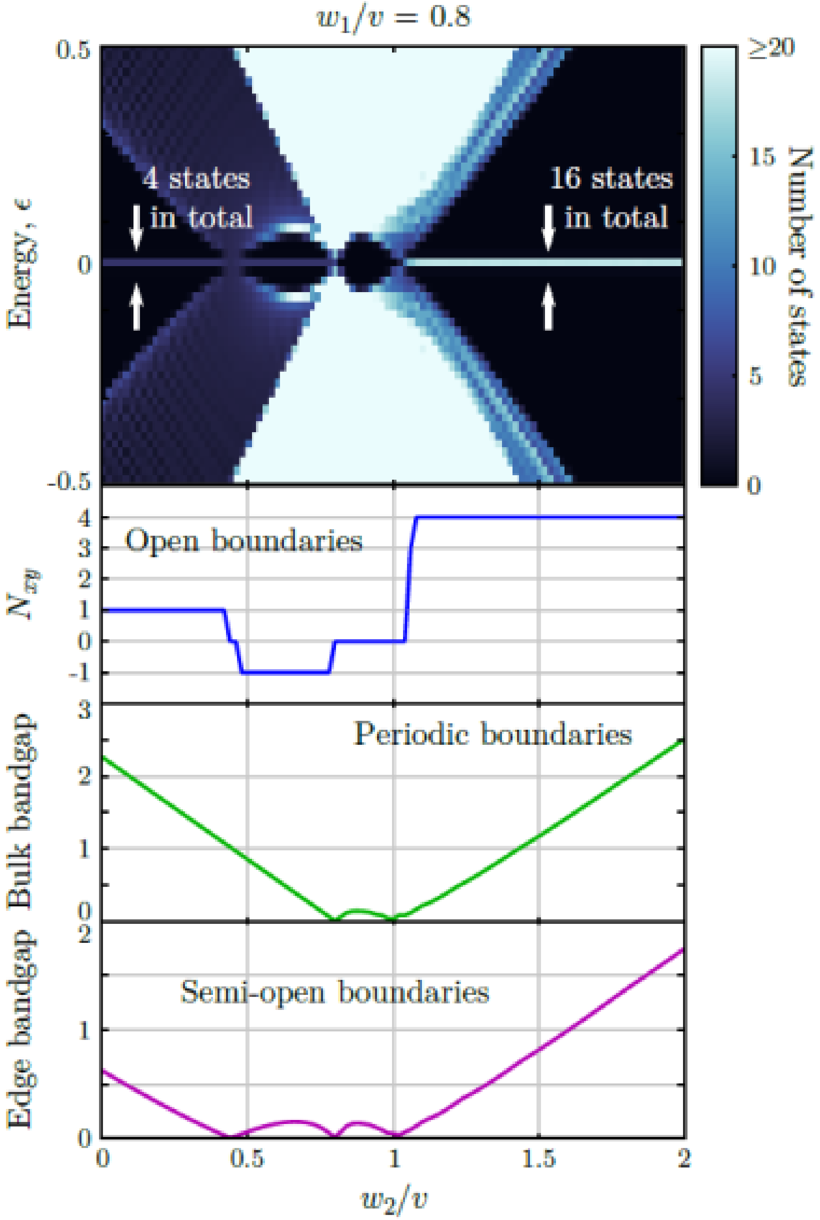
Strange new higher-order topological phases



- Add chiral-preserving diagonal hoppings
- New phases
- $N_{xy} < 0 \iff$ Corner states on "opposite" sublattice



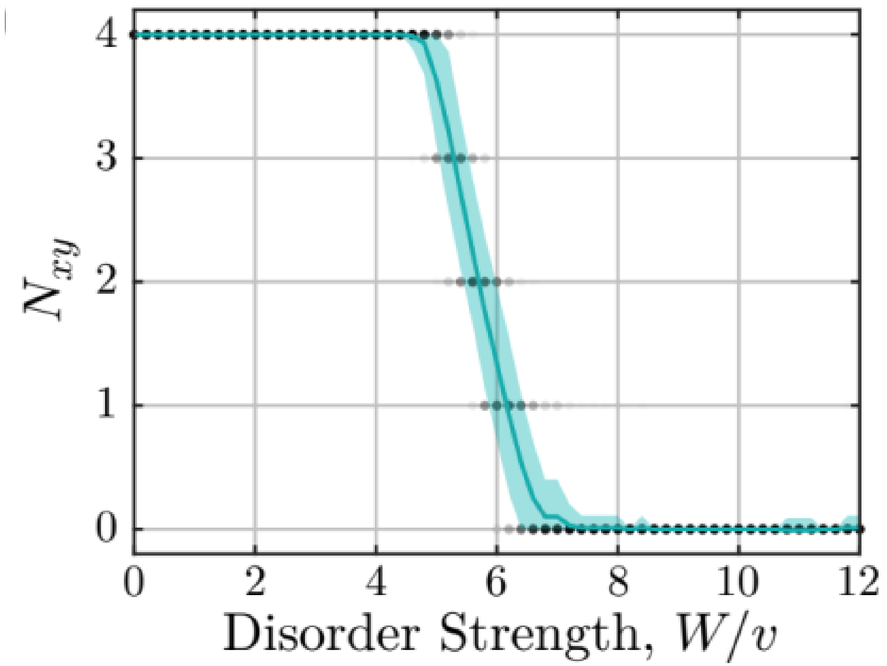


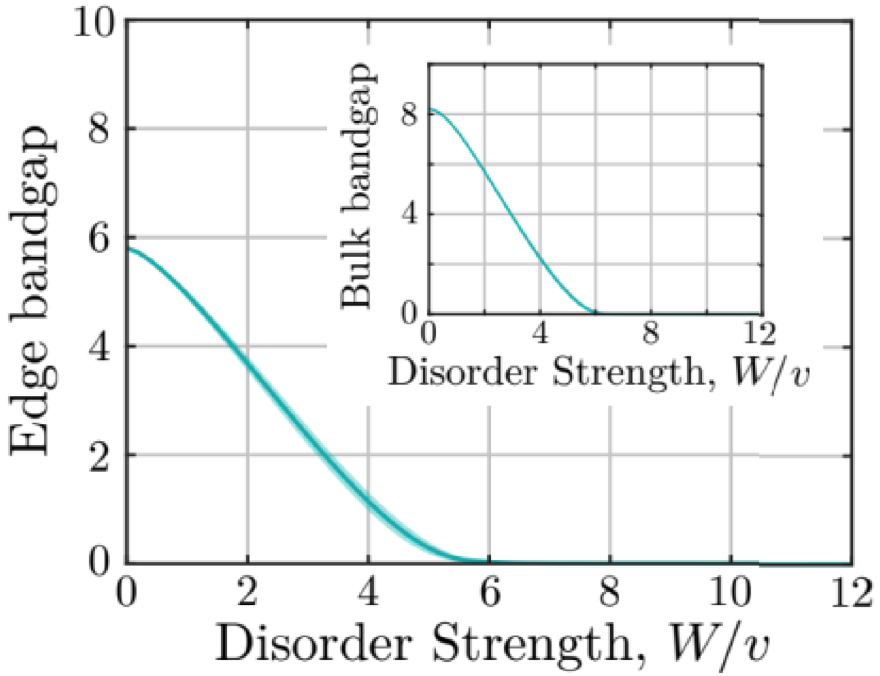


Robustness against disorder

Crystalline symmetries are not necessary for the protection of these phases

Phase transition into a localized phase (random disorder to nearest neighbor hoppings)



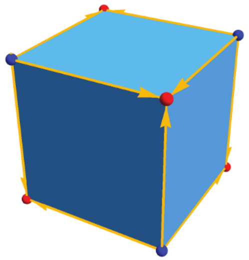


Higher-order topological phases

Wannier center picture

topological insulators

fractional charge



Multipole Chiral numbers

$$N_{xy} = \frac{1}{2\pi \mathrm{i}} \mathrm{TrLog}\left(\bar{Q}^{A}_{xy} \bar{Q}^{B\dagger}_{xy}\right) \in \mathbb{Z}$$

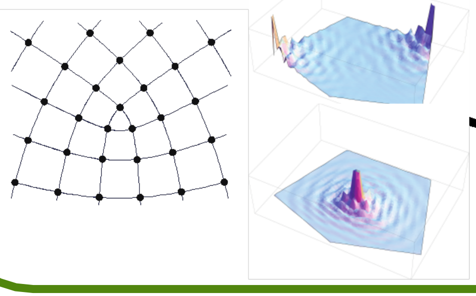
$$N_{xyz} = \frac{1}{2\pi \mathrm{i}} \mathrm{TrLog}\left(\bar{O}^{A}_{xyz} \bar{O}^{B\dagger}_{xyz}\right) \in \mathbb{Z},$$

HIGHER-ORDER TOPOLOGICAL PHASES

Boundary mass domains

topological superconductors

Majorana bound states



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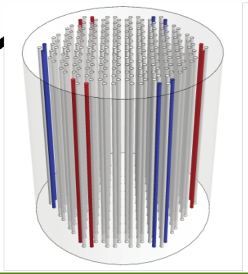

Summary

Benalcazar and Cerjan, *Phys. Rev. Lett.* **128**, 127601 (2022)

- Chiral symmetry protects more HOTPs than previously known
  - beyond those predicted by:
    - quantized multipole moments
    - TQC approaches

- These phases are protected by multipole chiral numbers
  - Bulk-boundary correspondence $\Rightarrow$ number of degenerate states at each corner

- Robust to (chiral-symmetry-preserving) disorder and do not require crystalline symmetries

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