SAND2022-7632C

Multilevel Monte Carlo derivative-free optimization under uncertainty of wind power plants

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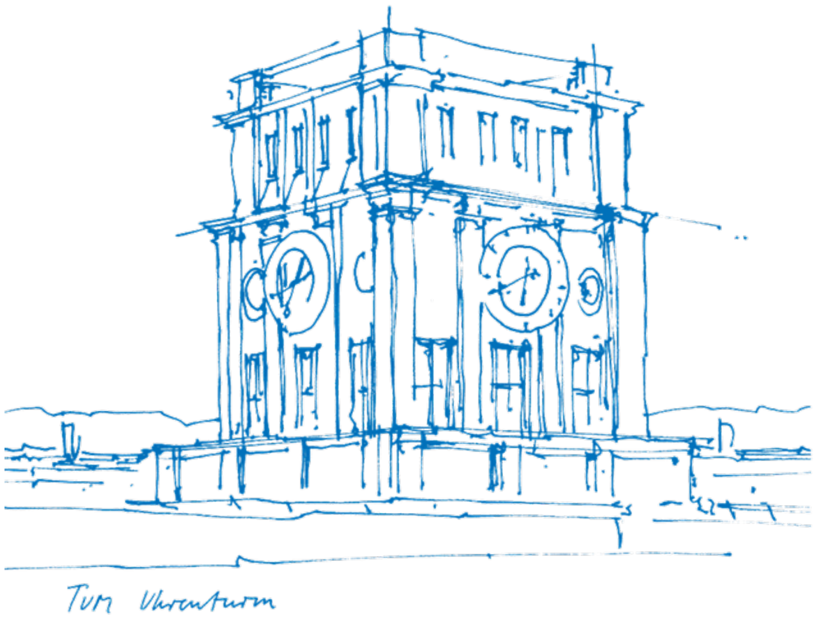
[4]Massachusetts Institute of Technology, USA

ECCOMAS Congress 22, Oslo, June 05-09, 2022

MS128A

Dakota Software for Optimization, Uncertainty Quantification and Model Calibration

June 8th, 2022



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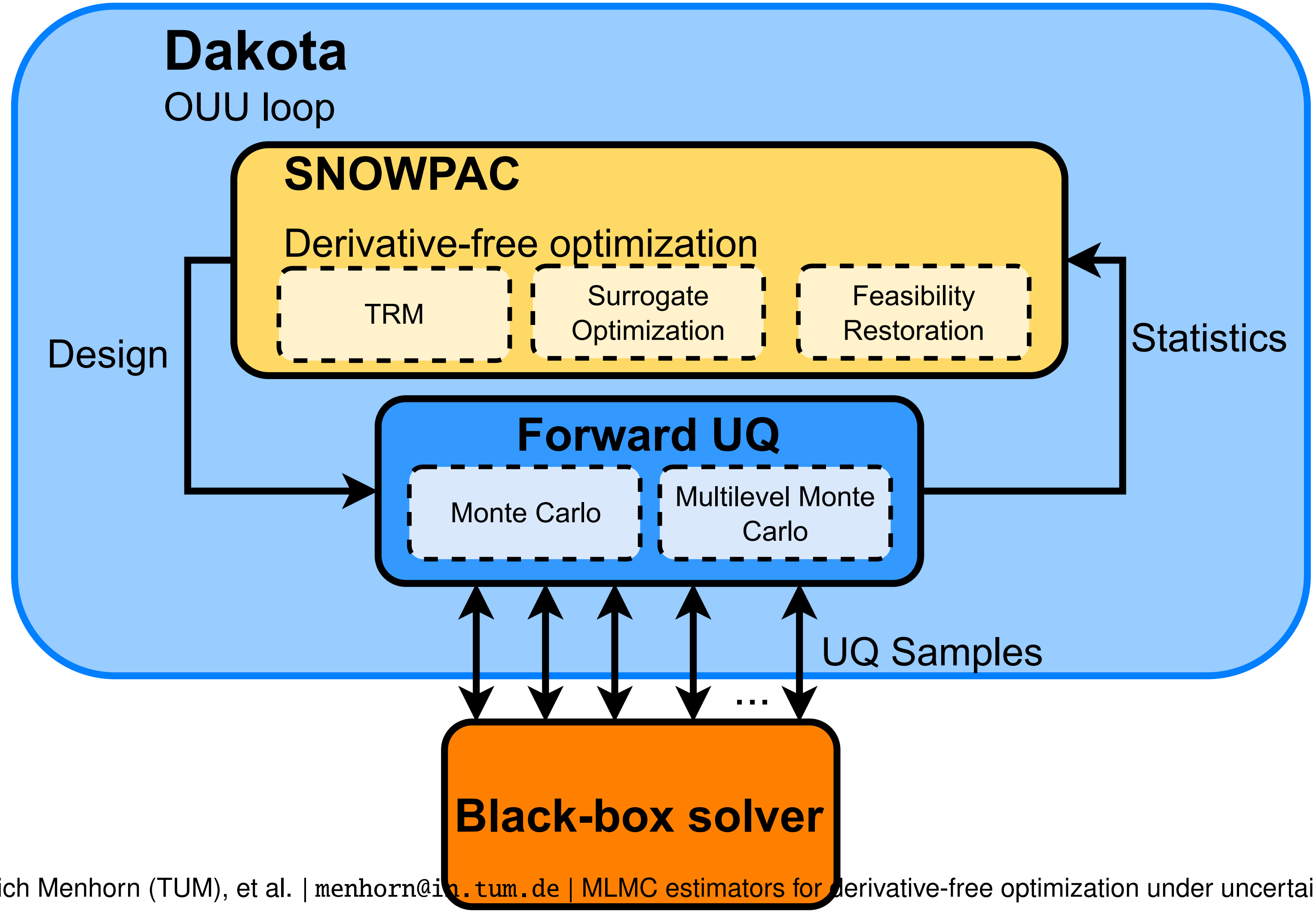
Wake steering in Dakota with SNOWPAC



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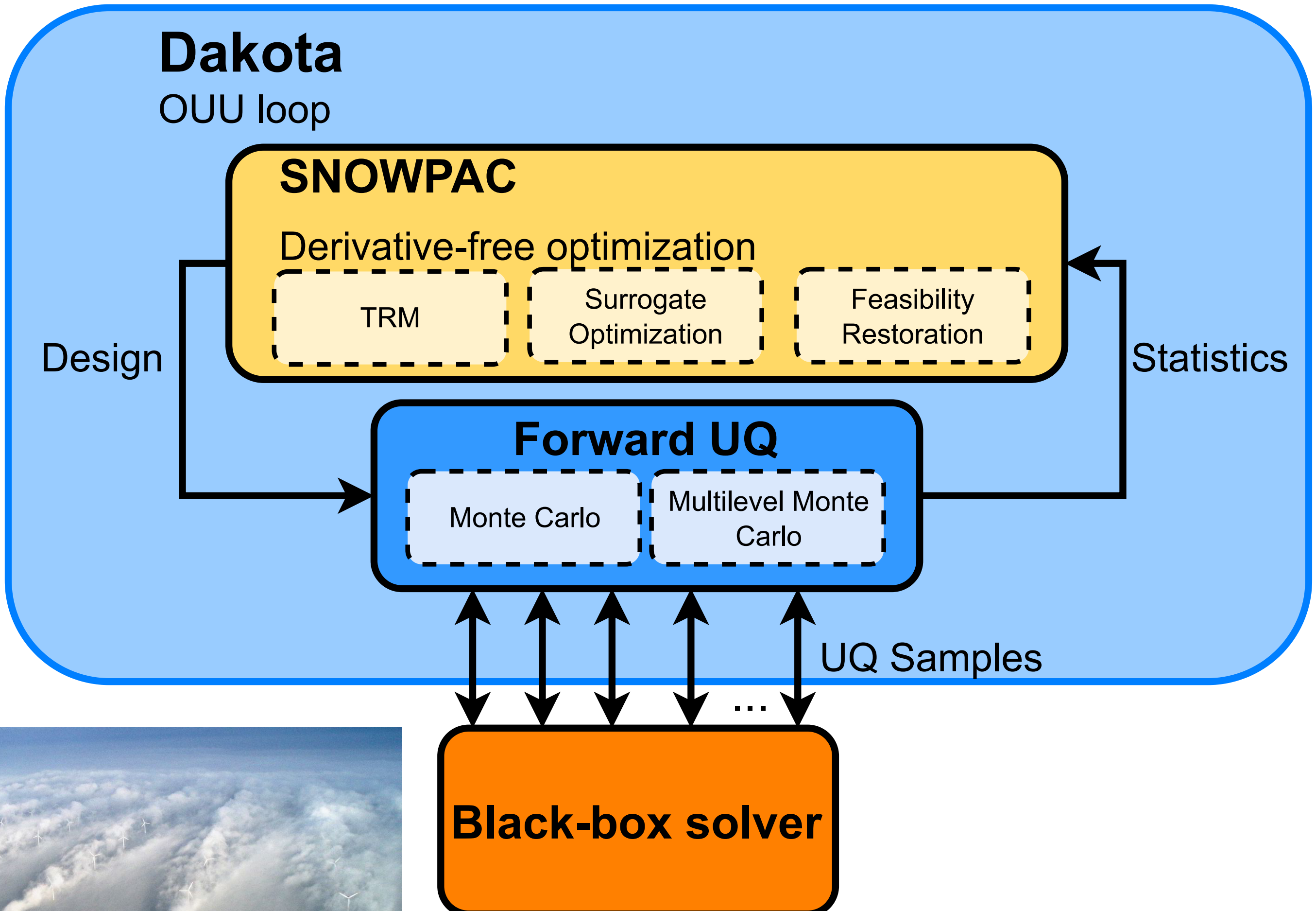

*Figure from https://www.rechargenews.com/wind/will-wind-wake-slow-industrys-ambitions-offshore-/2-1-699430

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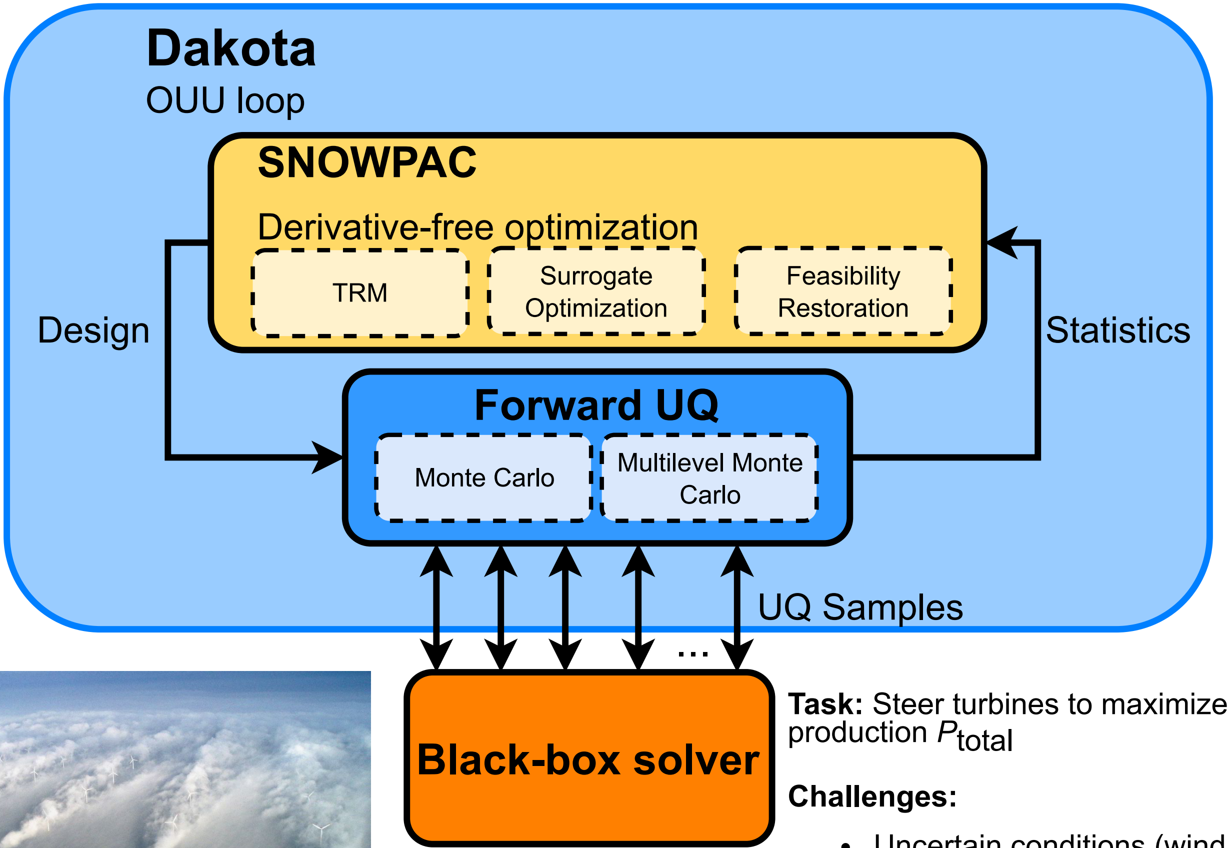

Task: Steer turbines to maximize total power production P_{total}

Challenges:

- Uncertain conditions (wind speed/angle)
- Complex, computationally expensive CFD
- No gradients (assumption)

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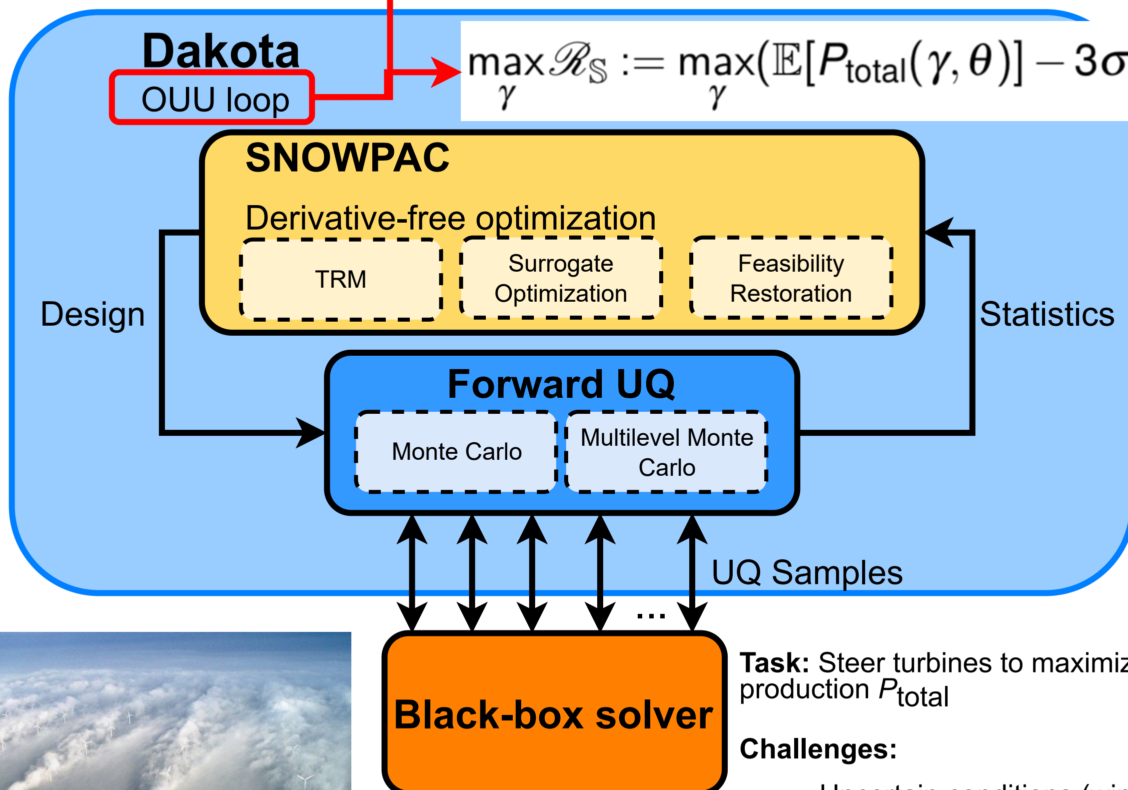
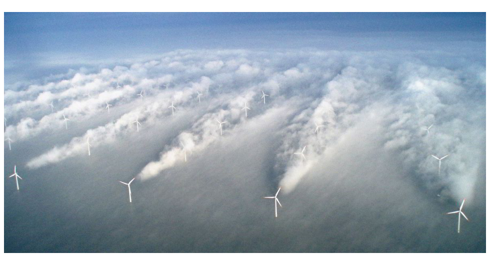
$$\max_{\gamma} \mathscr{R}_{\mathbb{E}} := \max_{\gamma} \mathbb{E}[P_{\text{total}}(\gamma, \theta)]$$

$$\max_{\gamma} \mathscr{R}_{\mathbb{S}} := \max_{\gamma} (\mathbb{E}[P_{\text{total}}(\gamma, \theta)] - 3\sigma[P_{\text{total}}(\gamma, \theta)])$$





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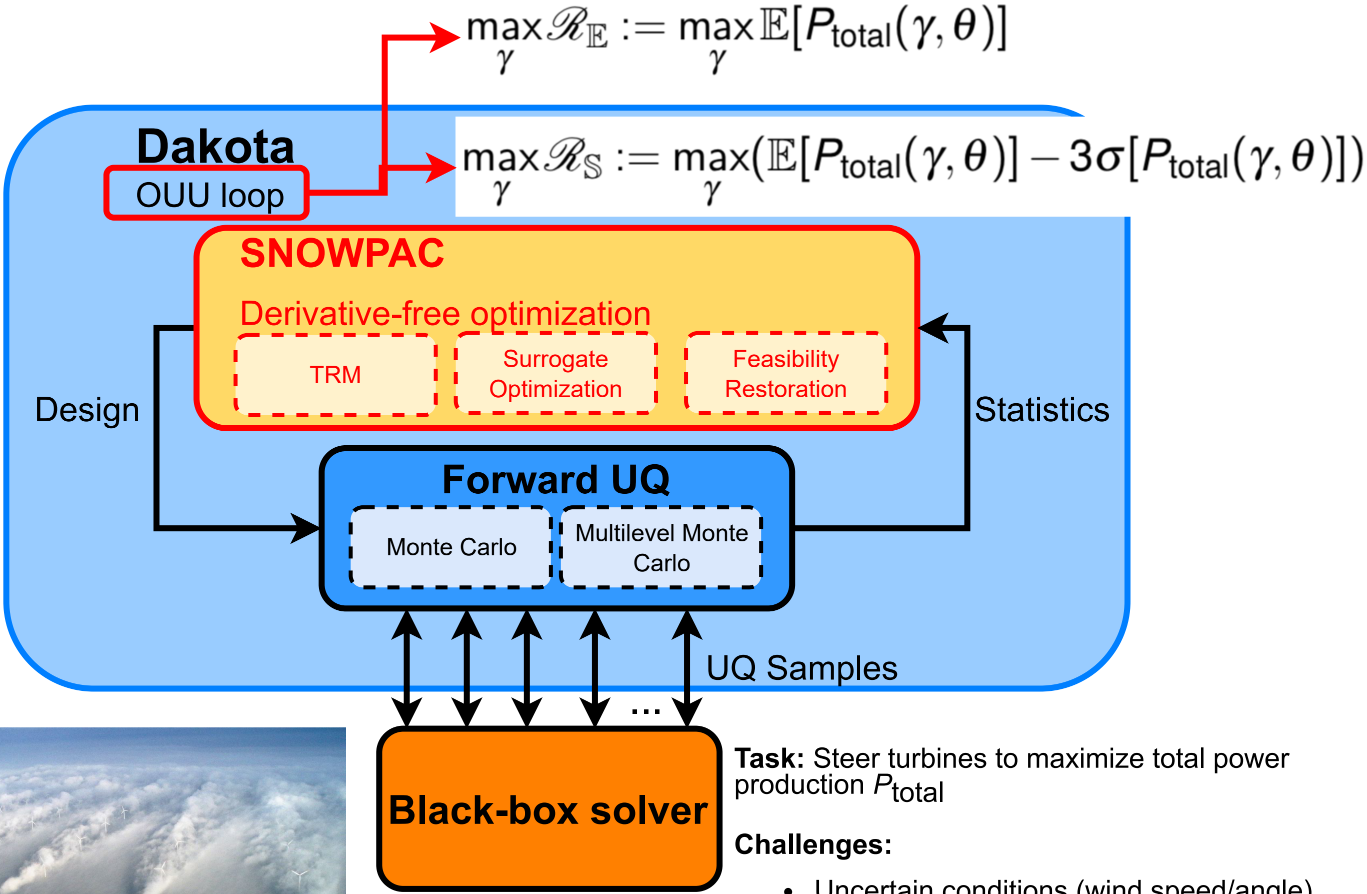

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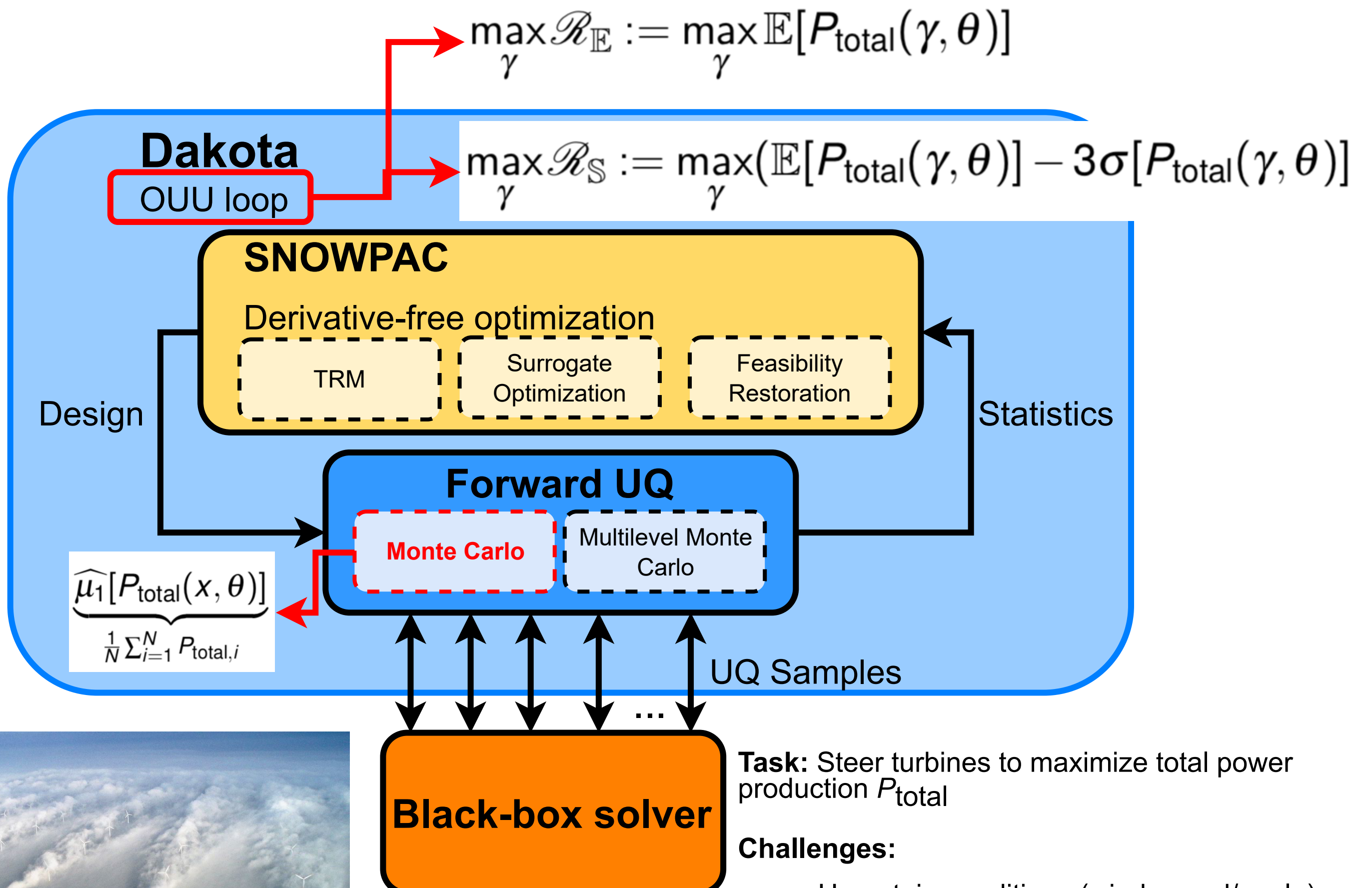

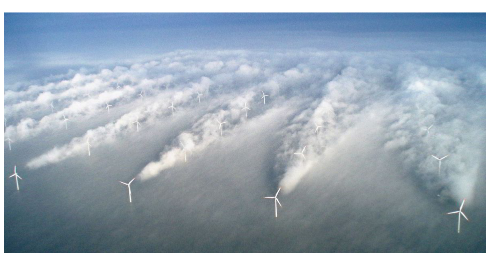

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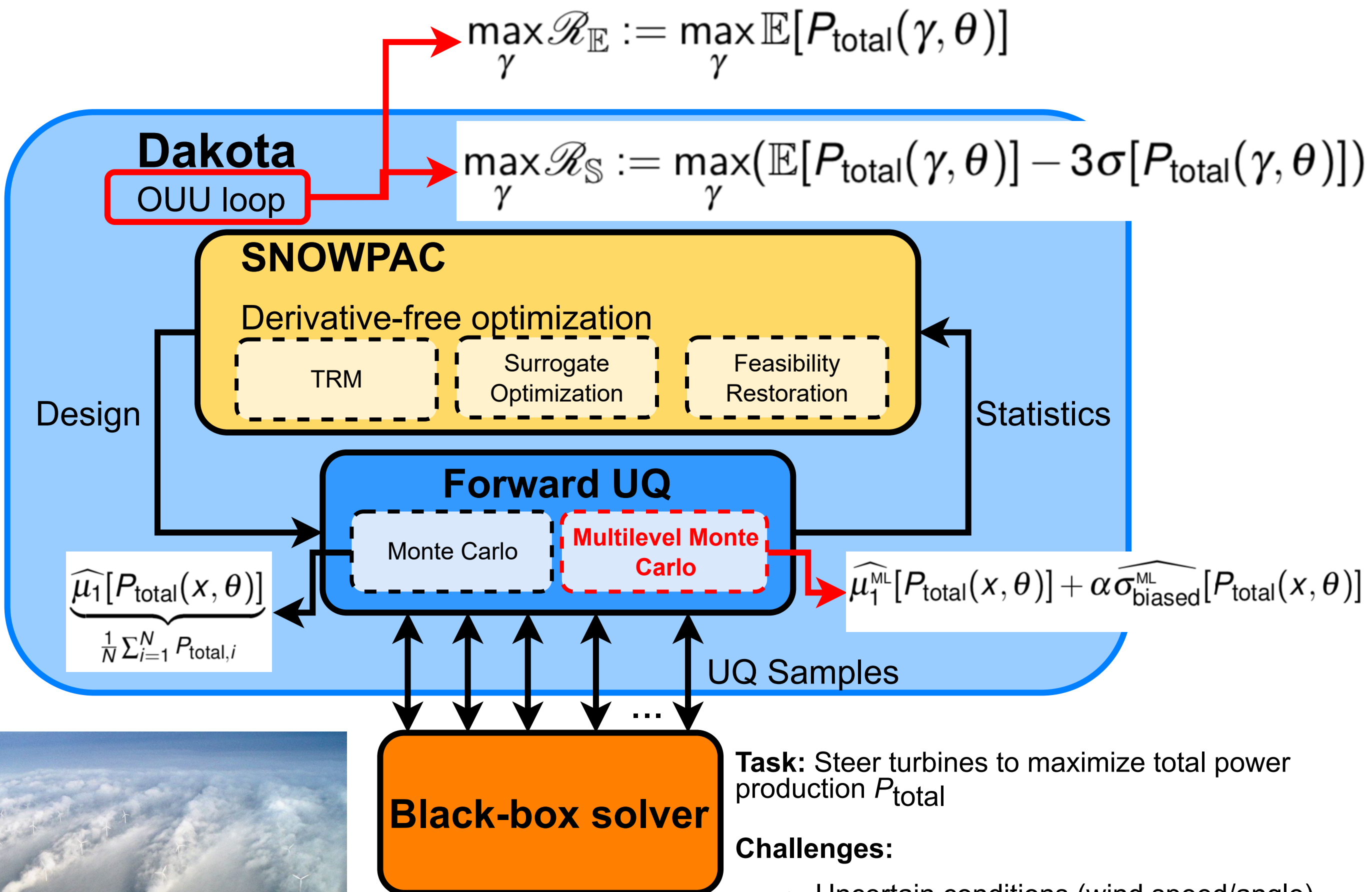

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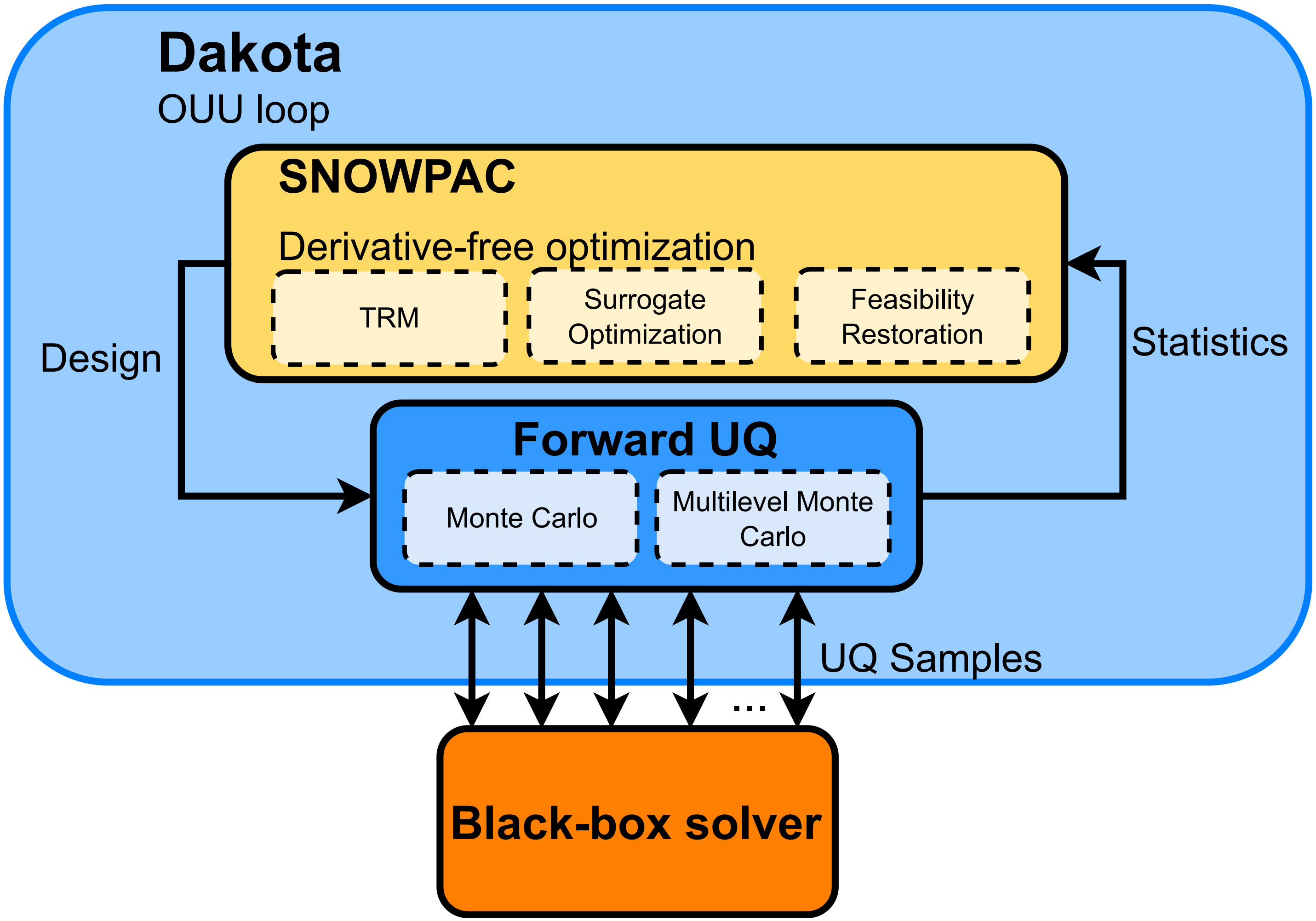
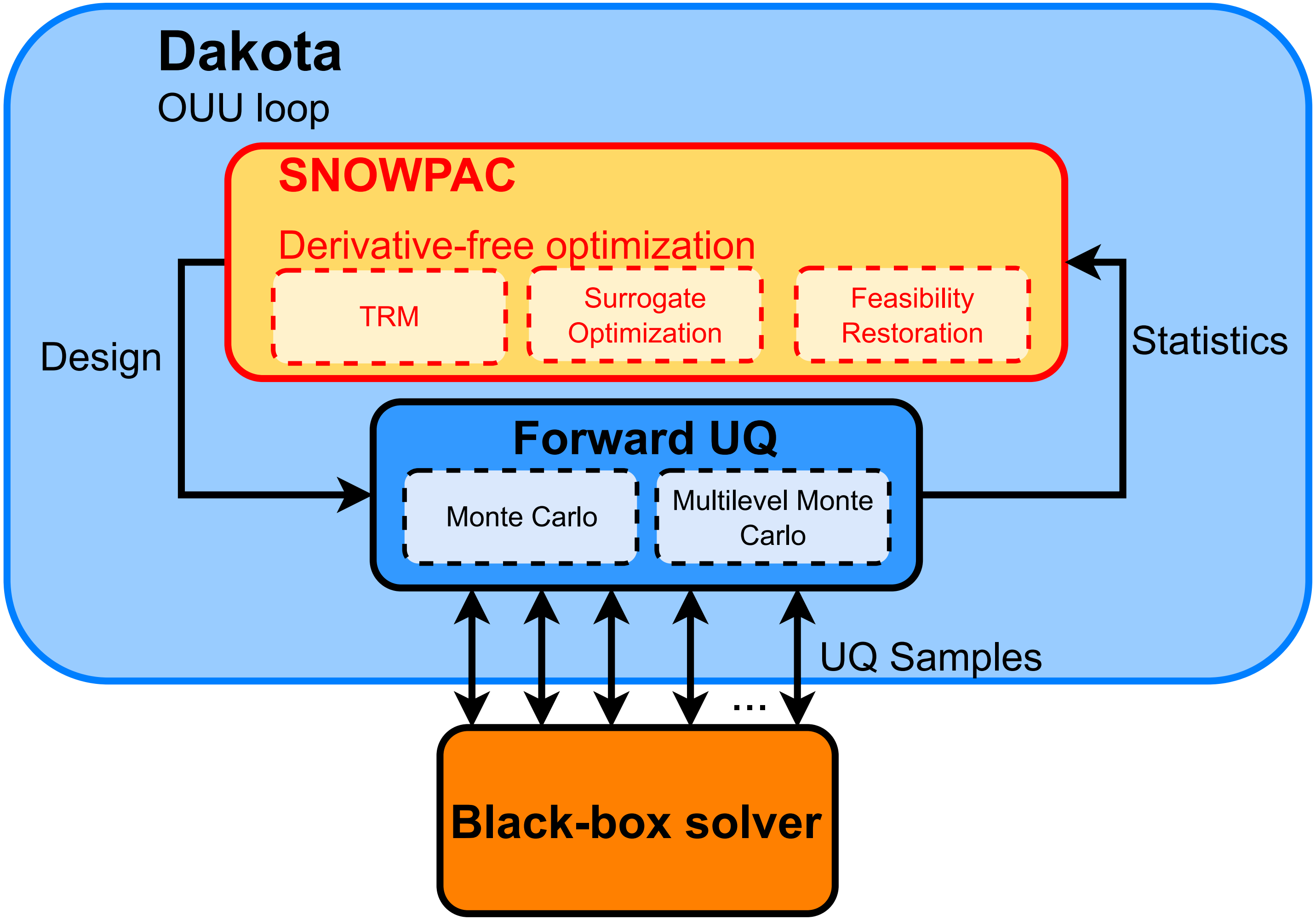

Dakota
OUU loop
SNOWPAC
Derivative-free optimization
TRM
Surrogate Optimization
Feasibility Restoration
Design
Statistics
Forward UQ
Monte Carlo
Multilevel Monte Carlo
UQ Samples
...
Black-box solver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SNOWPAC*

OUU problem statement

$$\boxed{\mathscr{R}^*_\theta = \mathscr{R}^f(\mathbf{x}^*, \theta) = \min_{\mathscr{R}^c(\mathbf{x},\theta)\leq 0} \mathscr{R}^f(\mathbf{x}, \theta)}$$

Features of SNOWPAC:

0. **Extension of NOWPAC†**: Derivative-free nonlinear constraint optimization method using trust-regions (deterministic)

†F. Augustin, YMM, NOWPAC: A provably convergent derivative-free nonlinear optimizer with path-augmented constraints, 2015
*FM, F. Augustin, HJB, YMM, A trust-region method for derivative-free nonlinear constrained stochastic optimization. 2022, Submitted

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1. **Estimate robustness measures**: Use sampling, e.g.

$$\mathscr{R}_\theta^f = \mathbb{E}[f_\theta(\mathbf{x})] \approx R^f = \frac{1}{N}\sum_{i=1}^N f(\mathbf{x}, \theta_i) + \varepsilon_N$$

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2. **Implement new trust region management**: Account for noise ε_N in objective/constraint evaluations
3. **Introduce Gaussian process surrogates**: Mitigate effect of noise ε_N

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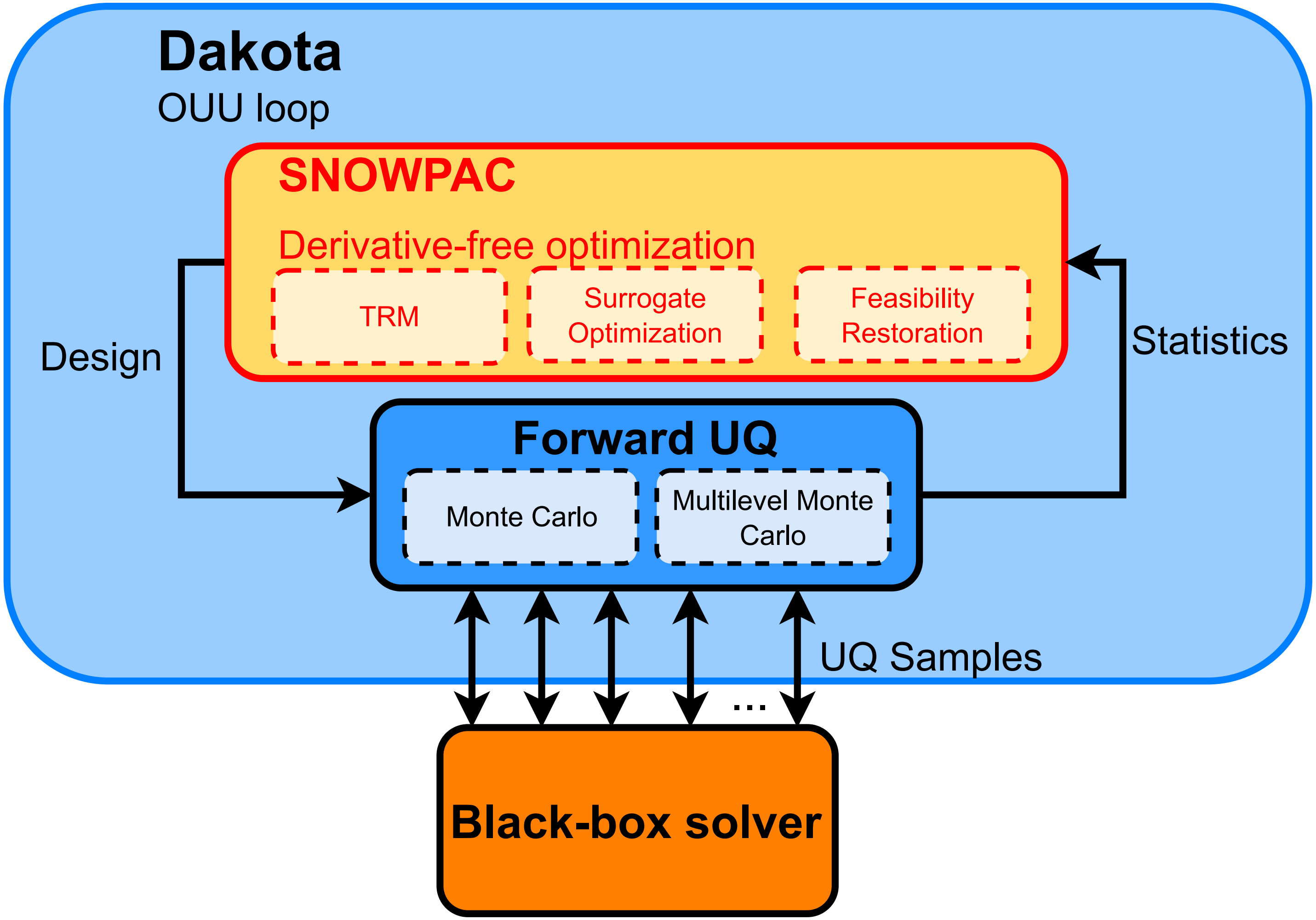
3. **Introduce Gaussian process surrogates**: Mitigate effect of noise ε_N
4. **Feasibility restoration mode**

[†] F. Augustin, YMM, NOWPAC: A provably convergent derivative-free nonlinear optimizer with path-augmented constraints, 2015
[*] FM, F. Augustin, HJB, YMM, A trust-region method for derivative-free nonlinear constrained stochastic optimization. 2022, Submitted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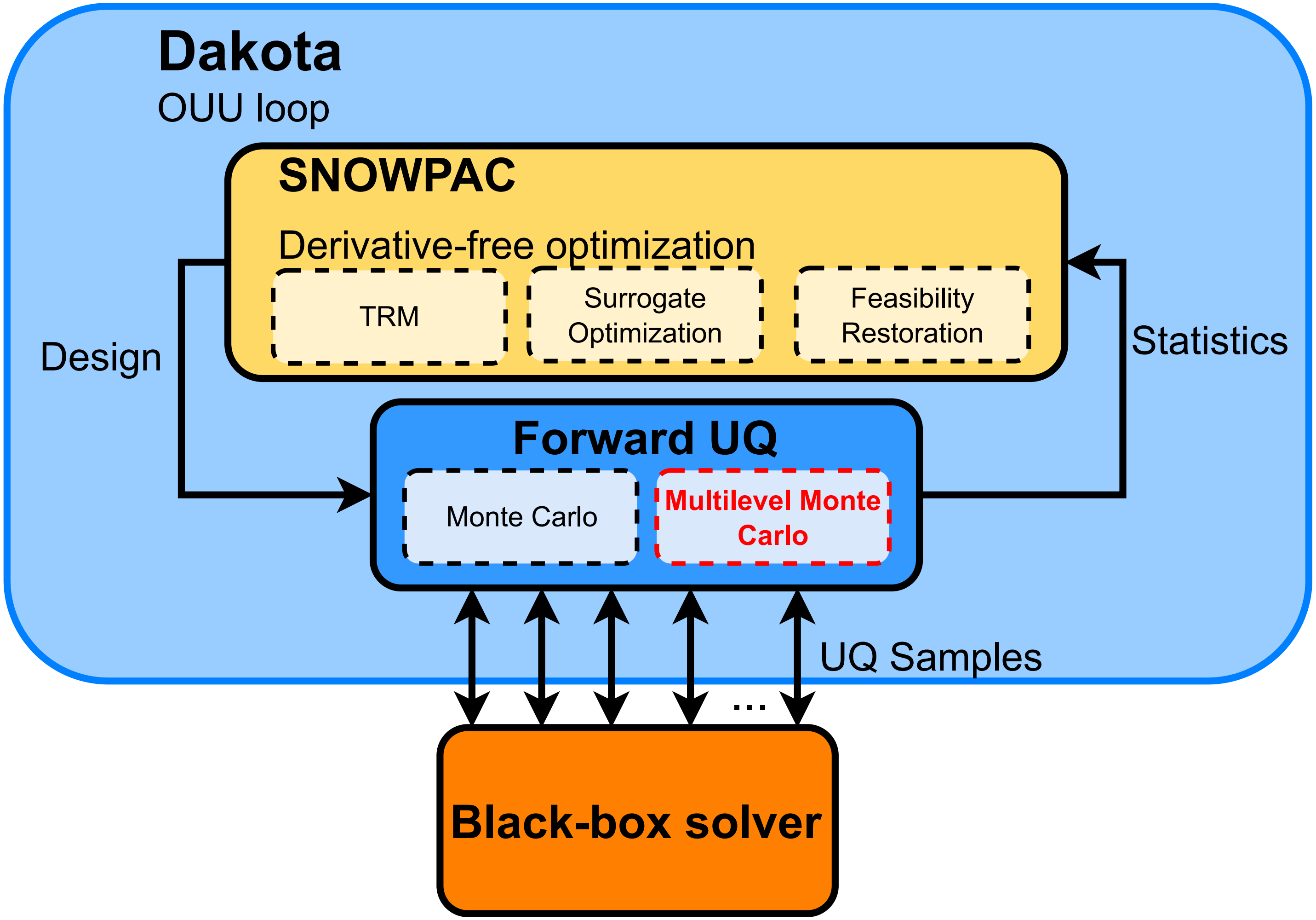
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MLMC estimator: Mean

Mean in OUU:

$$\min_x \mathscr{R}_{\mathbb{E}} \approx \min_x \widehat{\mu_1^{\mathrm{ML}}}[Q(x,\theta)]$$

*Giles, M.B., "Multilevel Monte Carlo methods," Acta Numerica, Vol.24, 2015, p.259–328

MLMC estimator: Mean

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- Estimator*:

$$\mathbb{E}[Q_L] = \mu_1^{\text{ML}}[Q_L] \approx \widehat{\mu_1^{\text{ML}}}[Q_L] = \sum_{\ell=0}^{L} \underbrace{\widehat{\mu_1}[Q^{(\ell)} - Q^{(\ell-1)}]}_{\text{telescopic sum}} = \sum_{\ell=0}^{L} \underbrace{\frac{1}{N_\ell} \sum_{i=1}^{N_\ell} (Q_i^{(\ell)} - Q_{i,\ell}^{(\ell-1)})}_{\text{estimator expansion}}, \quad Q_{i,0}^{(-1)} := 0$$

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- Sample allocation:

$$\min_{N_\ell^{\mathbb{E}}} \sum_{\ell=0}^{L} C_\ell N_\ell^{\mathbb{E}},$$

$$\text{s.t. } \mathbb{V}[\widehat{\mu_1^{\text{ML}}}] = \varepsilon^2, \text{ where } \mathbb{V}[\widehat{\mu_1^{\text{ML}}}] = \sum_{\ell=0}^{L} \mathbb{V}[\widehat{\mu_1^{(\ell)}} - \widehat{\mu_{1,\ell}^{(\ell-1)}}] = \sum_{\ell=0}^{L} \frac{\mathbb{V}[Q^{(\ell)} - Q^{(\ell-1)}]}{N_\ell}$$

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- Solution:

$$N_\ell^{\mathbb{E}} = \left\lceil \lambda \sqrt{\frac{\mathbb{V}[Q_\ell - Q_{\ell-1}]}{C_\ell}} \right\rceil, \text{ where } \lambda = \varepsilon^{-2} \sum_{\ell=0}^{L} \sqrt{\mathbb{V}[Q_\ell - Q_{\ell-1}] C_\ell}$$

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MLMC estimator: Standard deviation

Mean in OUU:

$$\min_x \mathscr{R}_{\mathbb{E}} \approx \min_x \widehat{\mu_1^{\text{ML}}}[Q(x,\theta)]$$

Standard deviation in OUU:

$$\min_x \mathscr{R}_{\mathbb{S}} \approx \min_x \widehat{\mu_1^{\text{ML}}}[Q(x,\theta)] + \alpha \widehat{\sigma_{\text{biased}}^{\text{ML}}}[Q(x,\theta)]$$

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MLMC estimator: Standard deviation

- Estimator:

$$\sigma[Q_L] = \sqrt{\mathbb{V}[Q_L]} \approx \sqrt{\widehat{\mu_2^{\text{ML}}}} := \widehat{\sigma_{\text{biased}}^{\text{ML}}}$$

where

$$\mathbb{V}[Q_L] \approx \widehat{\mu_2^{\text{ML}}}[Q_L] = \sum_{\ell=0}^{L} \underbrace{\widehat{\mu_2}[Q^{(\ell)}] - \widehat{\mu_2}[Q^{(\ell-1)}]}_{\text{telescopic sum}}$$

$$= \sum_{\ell=0}^{L} \underbrace{\frac{1}{N_\ell - 1}\left(\sum_{i=1}^{N_\ell}(Q_i^{(\ell)} - \widehat{\mu_1^{(\ell)}})^2 - (Q_{i,\ell}^{(\ell-1)} - \widehat{\mu_{1,\ell}^{(\ell-1)}})^2\right)}_{\text{estimator expansion}}$$

$$= \sum_{\ell=0}^{L} (\widehat{\mu_2^{(\ell)}} - \widehat{\mu_{2,\ell}^{(\ell-1)}}), \quad Q_{i,0}^{(-1)} := 0$$

† M. Pisaroni, S. Krumscheid, F. Nobile, Quantifying uncertain system outputs via the multilevel Monte Carlo method - Part I: Central moment estimation, 2020, JCP

MLMC estimator: Standard deviation

- Estimator:

$$\sigma[Q_L] = \sqrt{\mathbb{V}[Q_L]} \approx \sqrt{\widehat{\mu_2^{\mathrm{ML}}}} := \widehat{\sigma^{\mathrm{ML}}_{\mathrm{biased}}}$$

where

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$$= \sum_{\ell=0}^{L} (\widehat{\mu_2^{(\ell)}} - \widehat{\mu_{2,\ell}^{(\ell-1)}}), \quad Q_{i,0}^{(-1)} := 0$$

- Sample allocation:

$$\min_{N_\ell^\sigma} \sum_{\ell=0}^{L} C_\ell N_\ell^\sigma,$$

$$\text{s.t. } \mathbb{V}[\widehat{\sigma^{\mathrm{ML}}_{\mathrm{biased}}}] = \varepsilon^2, \text{ where } \mathbb{V}[\widehat{\sigma^{\mathrm{ML}}_{\mathrm{biased}}}] \approx \frac{1}{4}\frac{\mathbb{V}[\widehat{\mu_2^{\mathrm{ML}}}]}{\widehat{\mu_2^{\mathrm{ML}}}} \text{ (Delta Method)}$$

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- Solve this via numerical optimization or via an analytic approximation[†].

[†] M. Pisaroni, S. Krumscheid, F. Nobile, Quantifying uncertain system outputs via the multilevel Monte Carlo method - Part I: Central moment estimation, 2020, JCP

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MLMC estimator: Scalarization

Mean in OUU:

$$\min_x \mathscr{R}_{\mathbb{E}} \approx \min_x \widehat{\mu_1^{\text{ML}}}[Q(x,\theta)]$$

Scalarization in OUU:

$$\min_x \mathscr{R}_{\mathbb{S}} \approx \min_x \widehat{\mu_1^{\text{ML}}}[Q(x,\theta)] + \alpha \widehat{\sigma_{\text{biased}}^{\text{ML}}}[Q(x,\theta)]$$

MLMC estimator: Scalarization

- Estimator:

$$\mathbb{S}[Q_L] := \mathbb{E}[Q_L] + \alpha\sigma[Q_L] \approx \widehat{\mu_1^{\text{ML}}} + \alpha\widehat{\sigma_{\text{biased}}^{\text{ML}}} := \widehat{\zeta}^{\text{ML}}$$

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- Sample allocation:

$$\min_{N_\ell^{\mathbb{S}}} \sum_{\ell=0}^{L} C_\ell N_\ell^{\mathbb{S}},$$

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- Variance of scalarization:

$$\begin{aligned}\mathbb{V}[\widehat{\zeta^{\text{ML}}}] &= \mathbb{V}[\widehat{\mu_1^{\text{ML}}} + \alpha\widehat{\sigma_{\text{biased}}^{\text{ML}}}] \\ &= \underbrace{\mathbb{V}[\widehat{\mu_1^{\text{ML}}}]}_{\text{known}} + \alpha^2 \underbrace{\mathbb{V}[\widehat{\sigma_{\text{biased}}^{\text{ML}}}]}_{\text{known}} + 2\alpha \underbrace{\mathbb{C}\text{ov}[\widehat{\mu_1^{\text{ML}}}, \widehat{\sigma_{\text{biased}}^{\text{ML}}}]}_{\text{derived but not discussed tdy}}\end{aligned}$$

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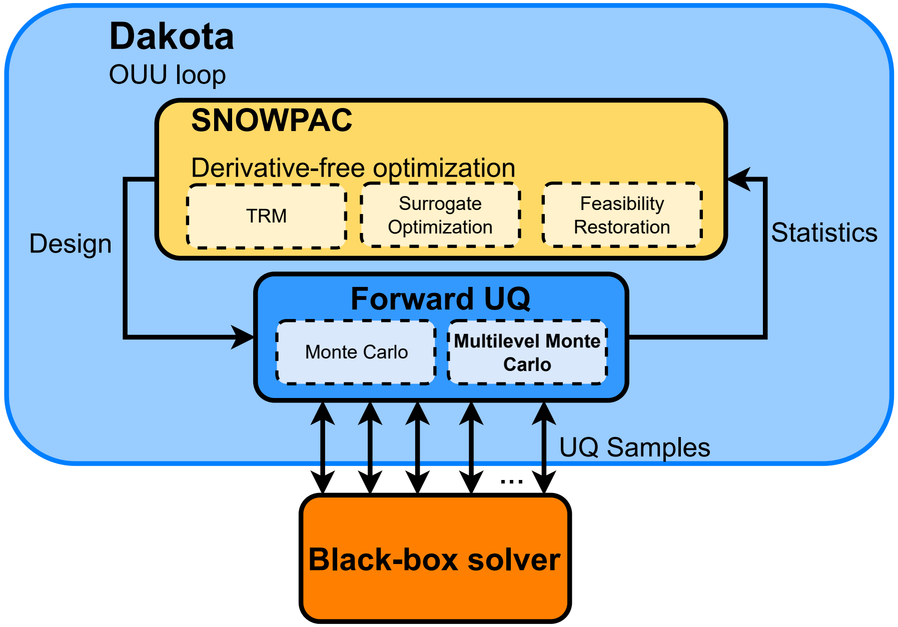
- Variance of scalarization:

$$\begin{aligned}\mathbb{V}[\widehat{\zeta^{\text{ML}}}] &= \mathbb{V}[\widehat{\mu_1^{\text{ML}}} + \alpha\widehat{\sigma_{\text{biased}}^{\text{ML}}}] \\ &= \mathbb{V}[\widehat{\mu_1^{\text{ML}}}] + \alpha^2\mathbb{V}[\widehat{\sigma_{\text{biased}}^{\text{ML}}}] + 2\alpha\mathbb{C}\text{ov}[\widehat{\mu_1^{\text{ML}}}, \widehat{\sigma_{\text{biased}}^{\text{ML}}}]\end{aligned}$$

- Solve this via numerical optimization or via an analytic approximation.

SNOWPAC in Dakota using MLMC



Algorithmic details:

- Iterative sample allocation
- Underrelaxation
- Choice of sample allocation strategy: Analytic approximation vs. numerical optimization
- Choice of covariance approximation: Pearson vs. Bootstrap vs. Correlation Lift

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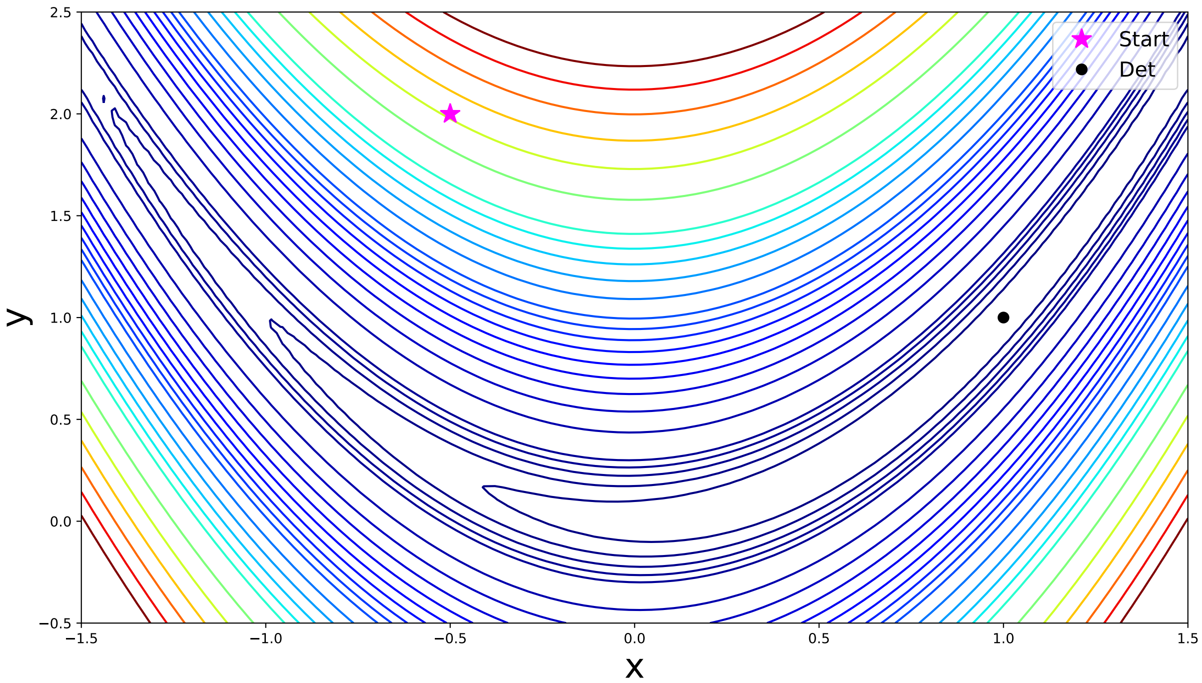
Rosenbrock

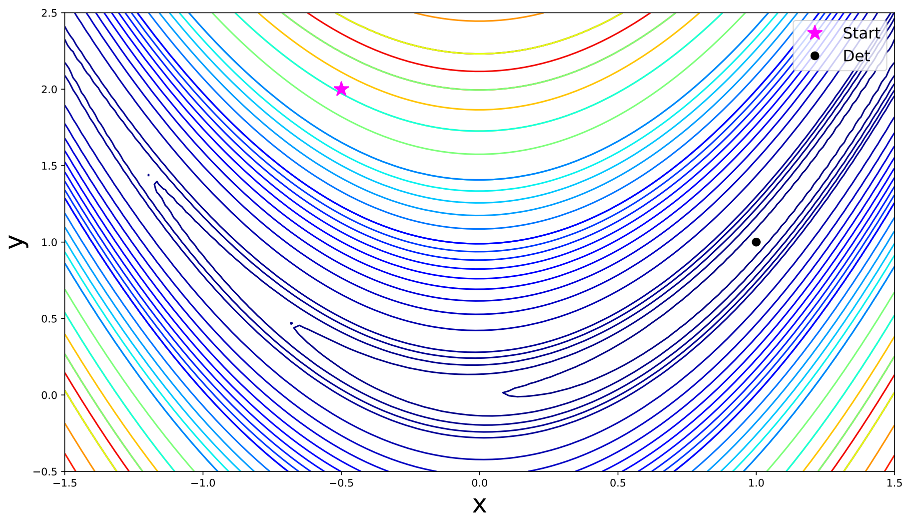
$$\min_{x,y} r(x,y) = \min_{x,y} 100(y - x^2)^2 + (1-x)^2$$



Rosenbrock

$$\min_{x,y} \mathbb{E}[r(x,y) + l_2(Z)] \qquad \min_{x,y} \mathbb{E}[r(x,y) + l_2(Z)] + 3\sigma[r(x,y) + l_2(Z)]$$

$$l_2(Z_1, Z_2, Z_3) = \sin(Z_1) + a\sin(Z_2)^2 + bZ_3^4 \sin(Z_1) - a/2, \{Z_i \sim \mathscr{U}(-\pi, \pi)\}_{i=1}^3$$
$$l_1(Z_1, Z_2, Z_3) = \sin(Z_1) + 0.85a\sin(Z_2)^2 + bZ_3^4 \sin(Z_1) - 0.85a/2,$$
$$l_0(Z_1, Z_2, Z_3) = \sin(Z_1) + 0.6a\sin(Z_2)^2 + 9bZ_3^2 \sin(Z_1) - 0.6a/2.$$



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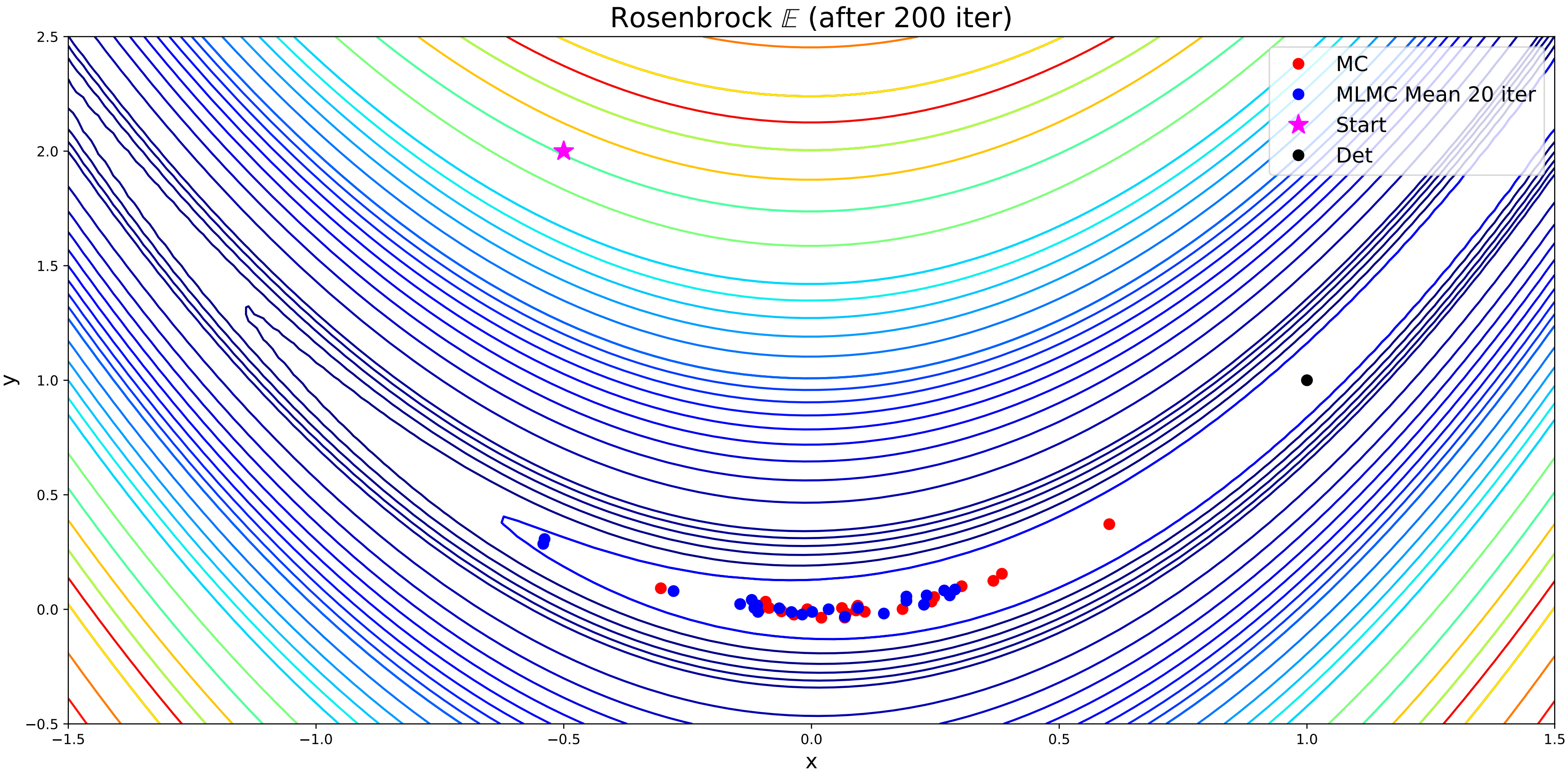
Rosenbrock results: Mean



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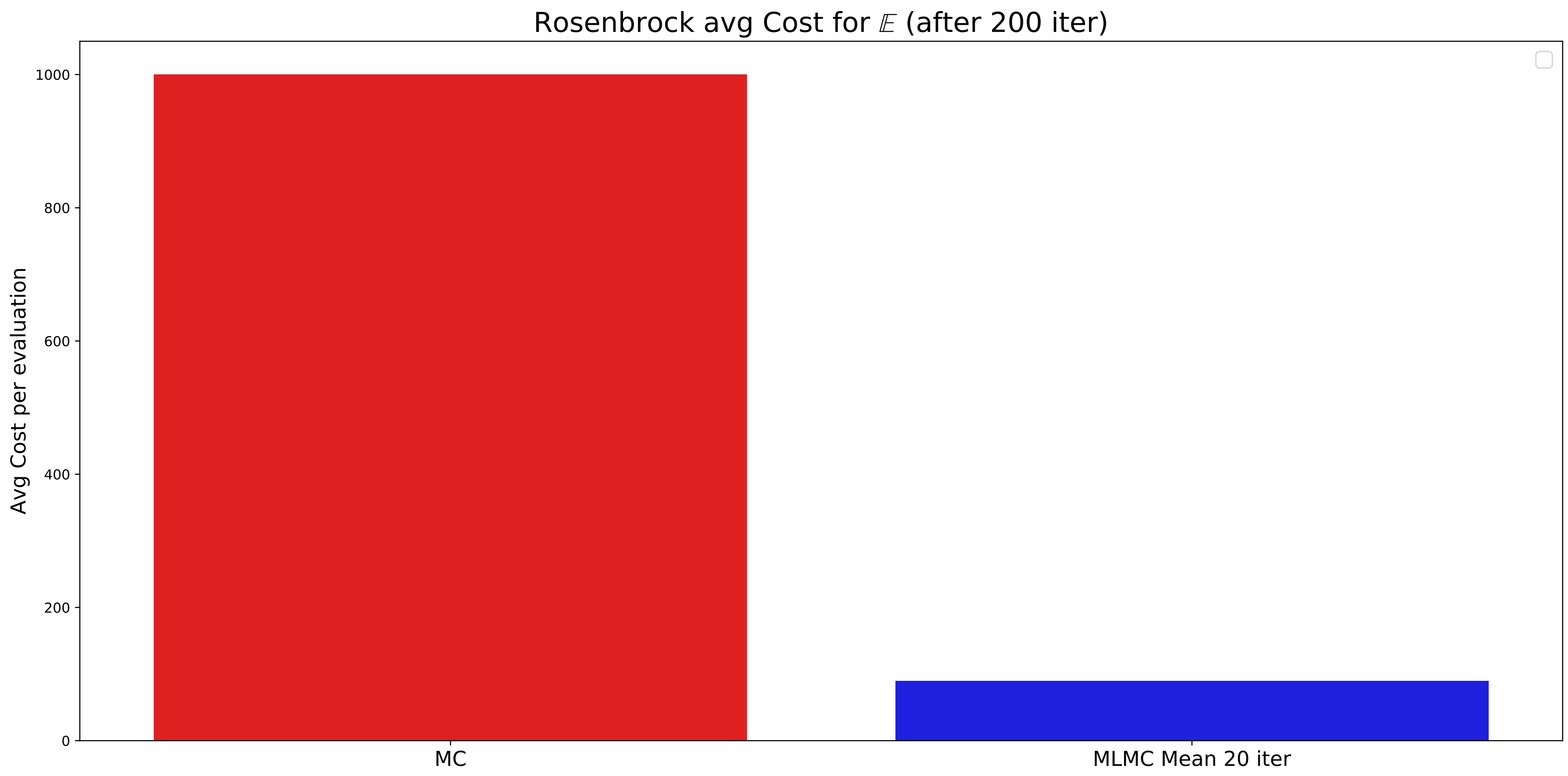

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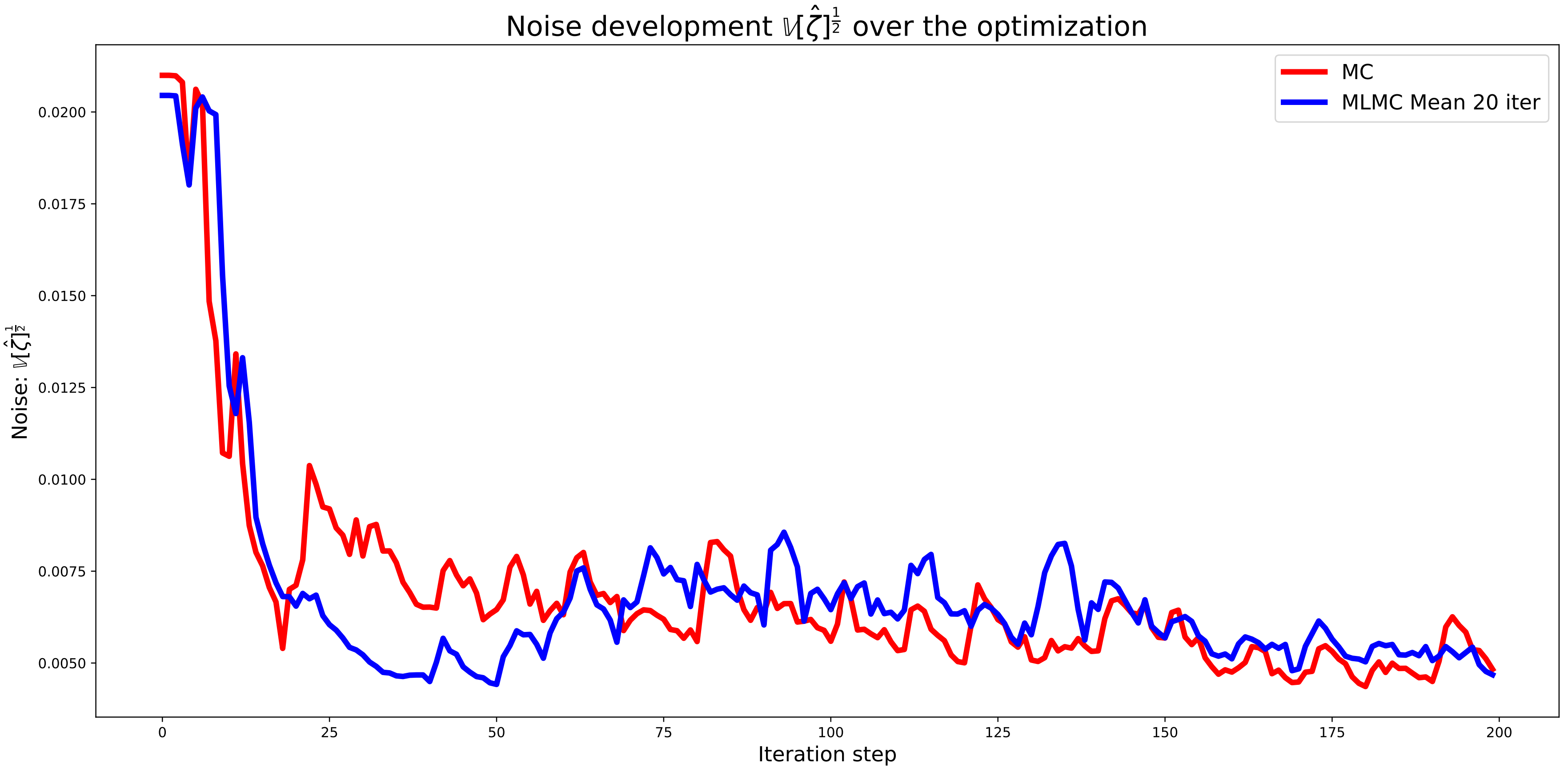
Rosenbrock results: Mean



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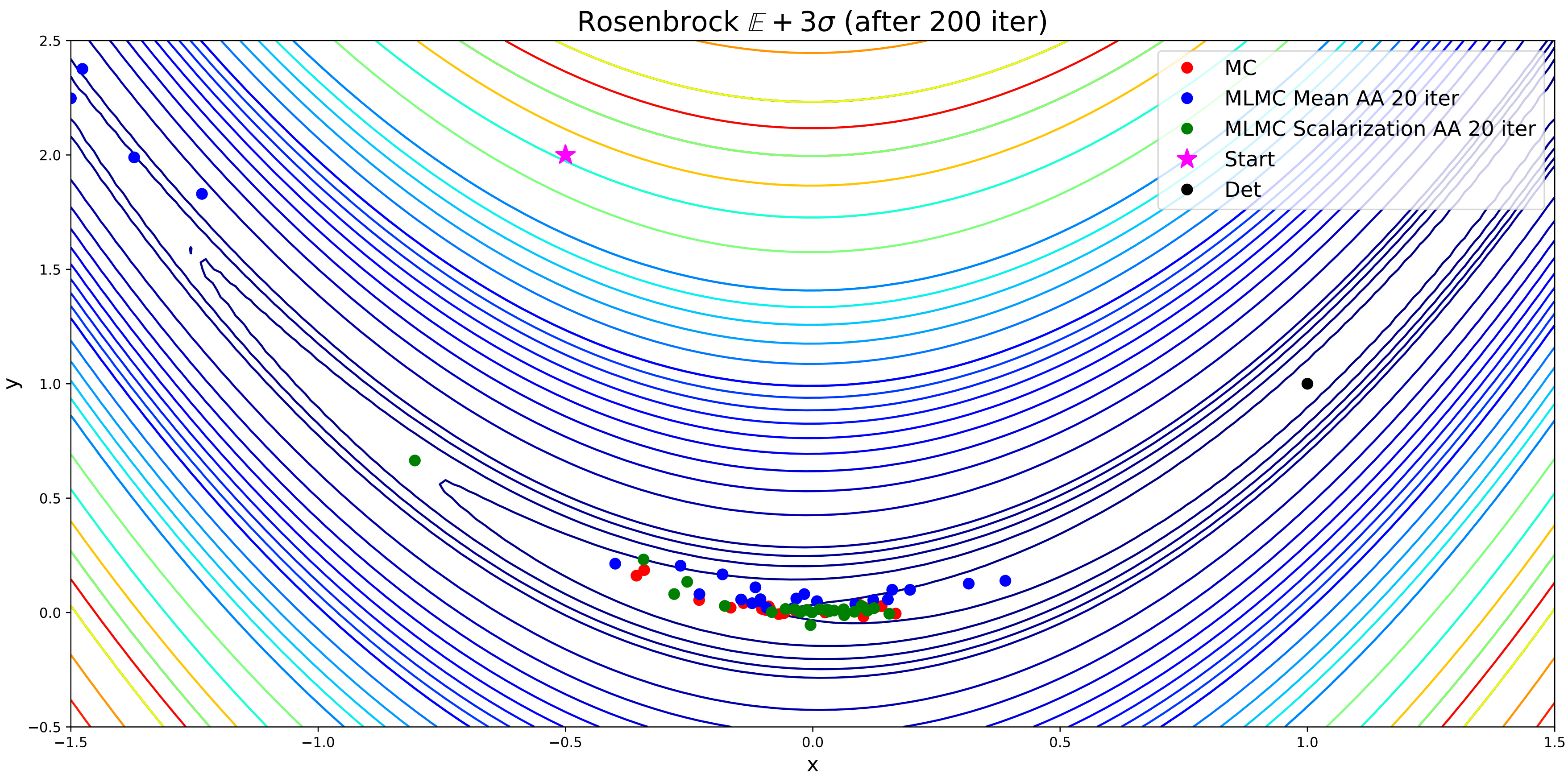
Rosenbrock results: Scalarization



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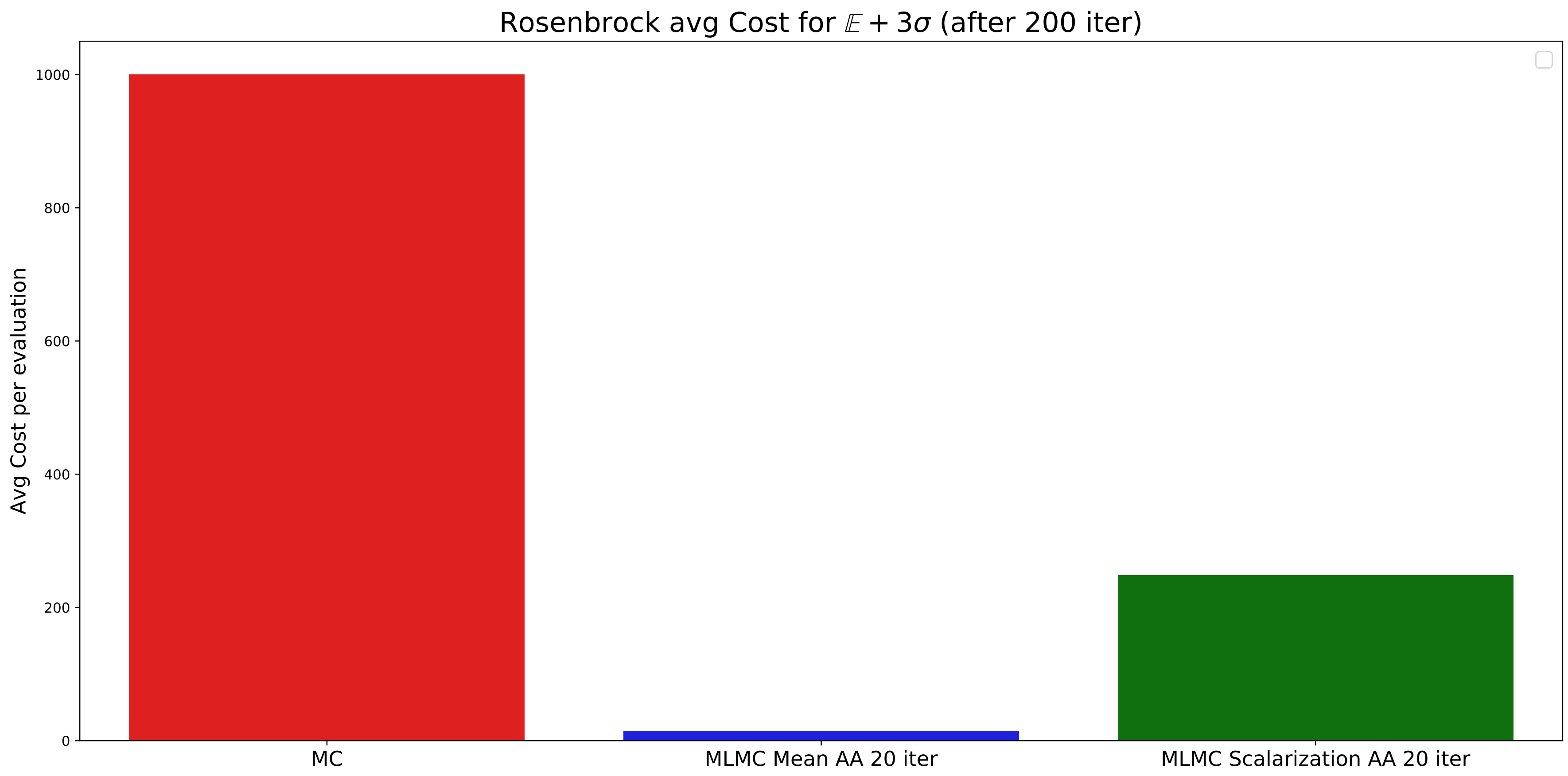

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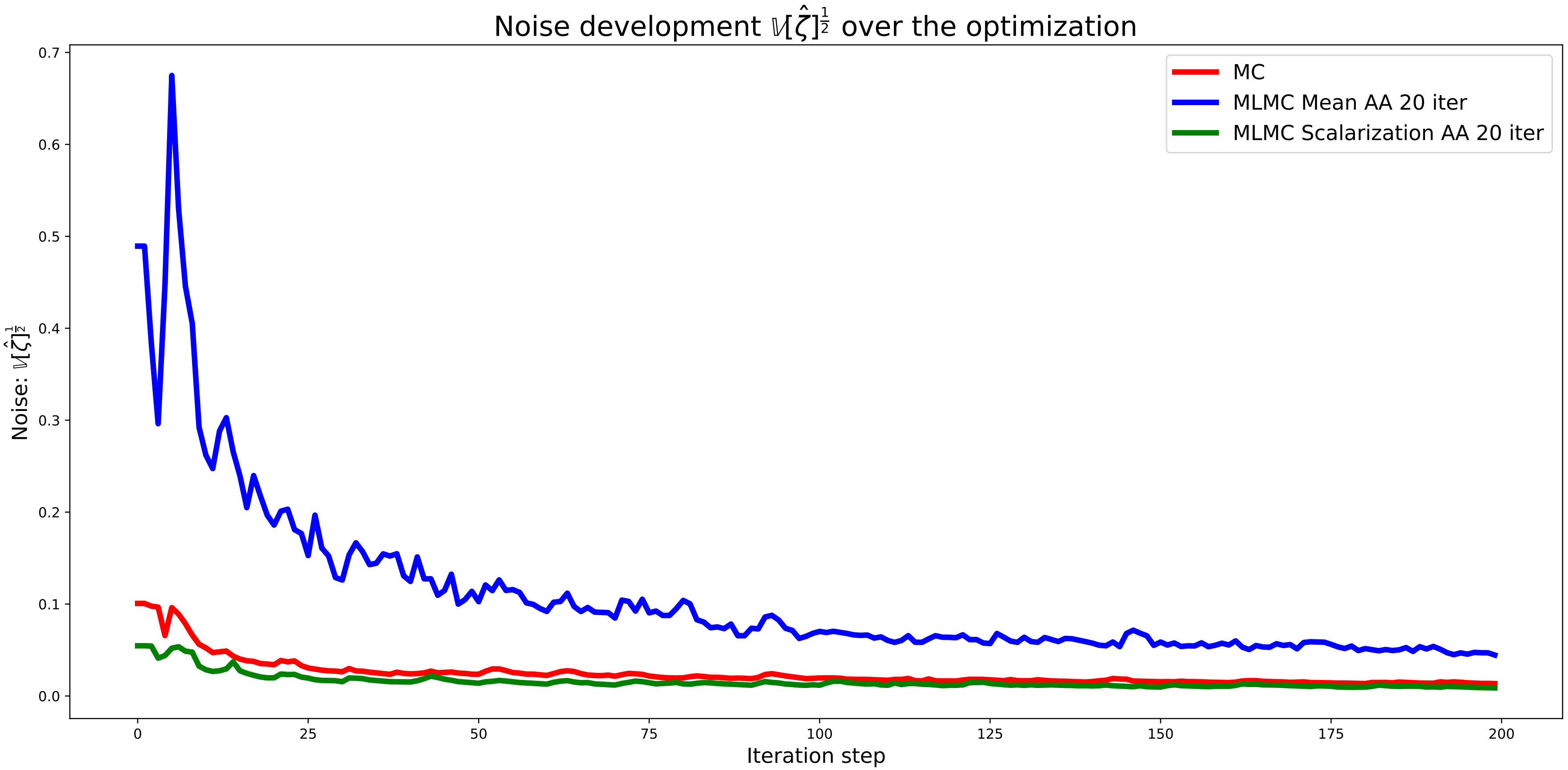
Rosenbrock results: Scalarization



Wind plant application

Setup:

- five NREL 5 MW turbines (RD 126 m, HH 90 m)
- RANS model with actuator disk turbine representation using WindSE (https://github.com/NREL/WindSE)

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- Yaw angle design:

  $\{\gamma_i\}_{i=1}^{5} \in [-45^\circ, 45^\circ]$

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OUU Scalarization:

$$\max_{\{\gamma_i\}_{i=1}^{5}} \mathscr{R}_{\mathbb{S}} := \max_{\{\gamma_i\}_{i=1}^{5}} \mathbb{E}[P_{\text{total}}(\cdot)] - 3\sigma[P_{\text{total}}(\cdot)]$$

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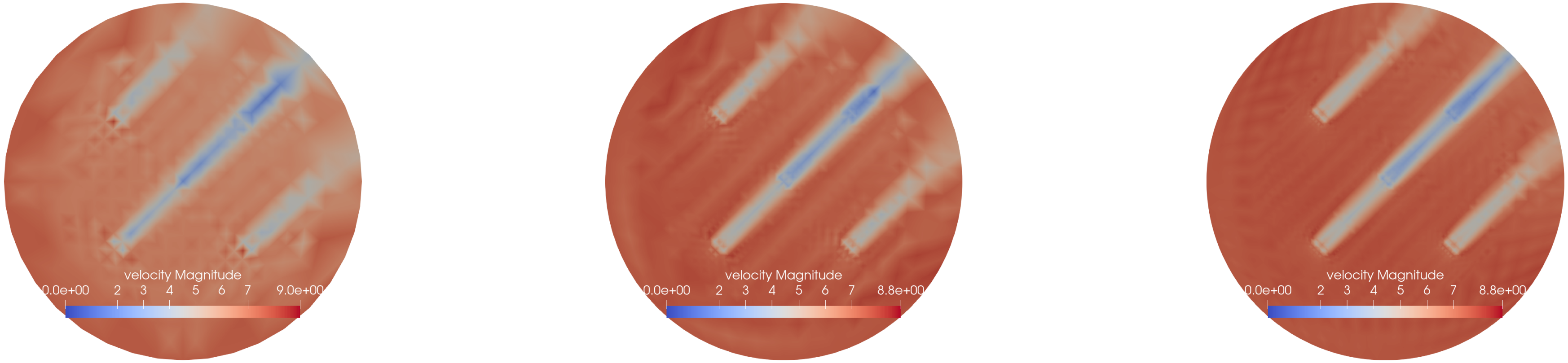

(a) COARSE (DoF 27760) (b) MEDIUM (DoF 130208) (c) FINE (DoF 376808)

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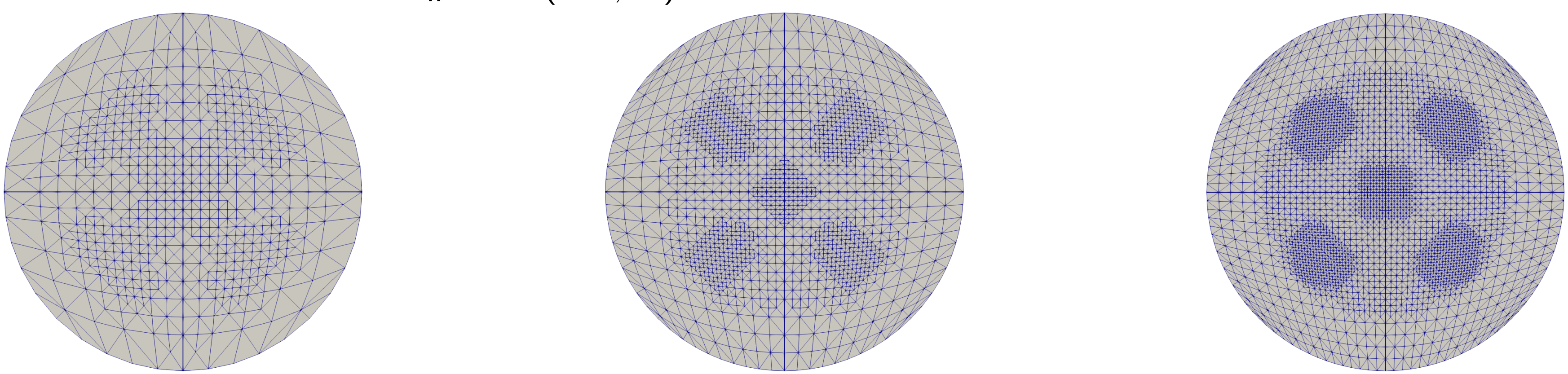
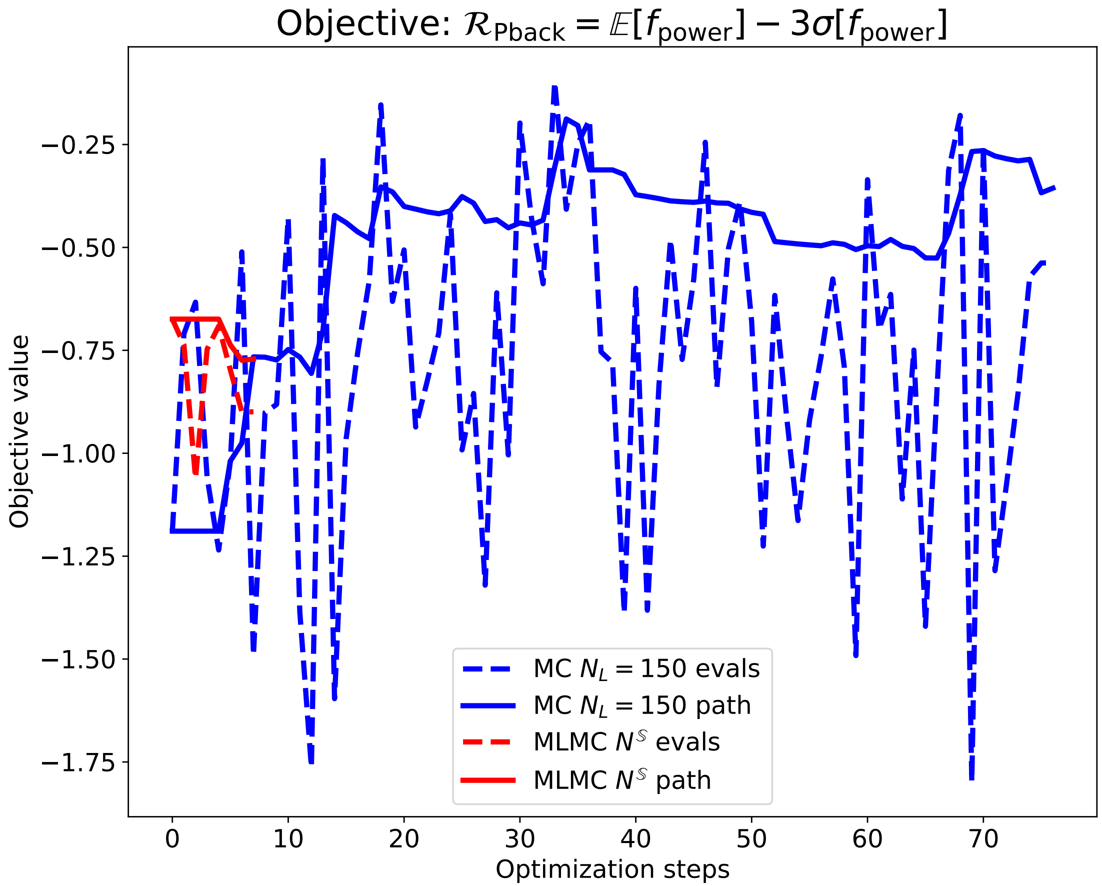


(a) COARSE (DoF 27760) (b) MEDIUM (DoF 130208) (c) FINE (DoF 376808)

Wake steering results



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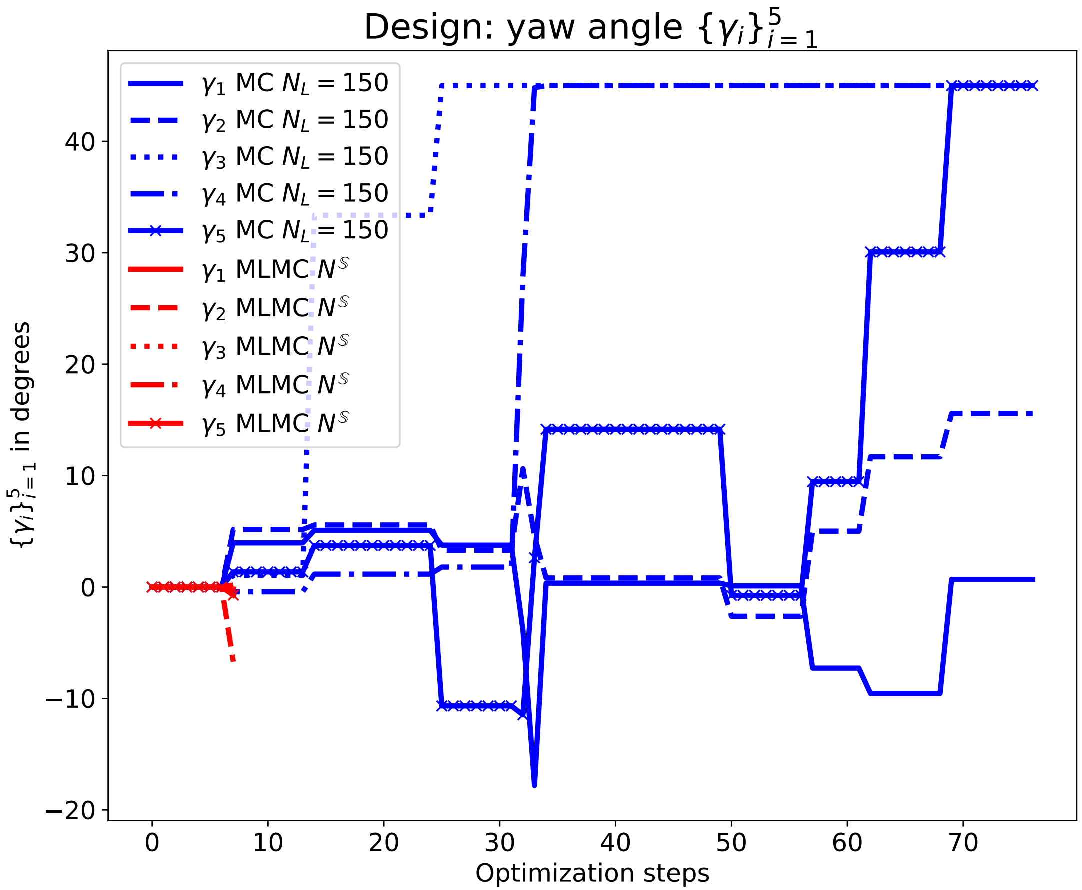
Wake steering results



Conclusion

⇒ New MLMC estimators for Standard Deviation and Scalarization coupled with **SNOWPAC** in **Dakota**.

⇒ Start of OUU results for wind application.

Future work and open questions:

- Short term: Figure out issues
- Long term: Introduce more uncertainty

Links:

- SNOWPAC: github.com/snowpac/snowpac
- Dakota: dakota.sandia.gov

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