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Non-intrusive coupling of multi-physics codes for Eulerian-Lagrangian solid-solid interaction using Immersed Finite Element Method

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- **Introduction**
- Review of modified Immersed Finite Element Method (mIFEM)
- Non-intrusive coupling of software
- Improvements in mIFEM algorithm
- Numerical test cases
- Summary and future work

Motivation



shock induced structural damage [1]

- **Simulation of hypervelocity impact and solid-solid interactions**
- **Applications:**
 - Ballistic penetration [2]
 - Geodynamics [3]
 - Spacecraft design [4]
- **Challenges:**
 - Non-linear wave propagation
 - Large plastic deformations, fragmentations

[1] https://commons.wikimedia.org/wiki/File:Shock_tube.JPG

[2] Shen Wei and Wu Cuisheng. Computer simulation for damage-failure process of composite plate under high-speed impact. Engineering fracture mechanics, 1992

[3] Francois E Heuzé. An overview of projectile penetration into geological materials. In International journal of rock mechanics and mining sciences & geomechanics abstracts, Elsevier, 1990

[4] MV Silnikov, IV Guk, AF Nechunaev, and NN Smirnov. Numerical simulation of hypervelocity impact problem for spacecraft shielding elements. Acta Astronautica, 2018

Motivation

Hydrocode: codes used to simulate **highly dynamic events** involving shocks

Lagrangian Hydrocodes

- **Nodes are attached to material points.**
- Explicit material boundaries
- Easier to apply interface conditions.
- Mesh distortion
- Need re-meshing/ element deletion strategies
- DYNA [1], Camacho and Ortiz [2]

Eulerian Hydrocodes

- **Materials are tracked across a fixed mesh.**
- Fixed mesh, large plastic deformations.
- interfaces are not tracked explicitly
- Benson [3], Udaykumar et. al. [4]

Coupled Lagrangian-Eulerian

- **Projectile: Lagrangian description**
- **Target: Eulerian description**
- **Leverage advantages of both approaches**
- Zapotec [5]

[1] JO Hallquist and RG Whirley. Dyna3d user manual, nonlinear dynamic analysis in three dimensions. Report UCID-19592, Rev. 5, 1989.

[2] GT Camacho and M Ortiz. Adaptive lagrangian modelling of ballistic penetration of metallic targets. Computer methods in applied mechanics and engineering, 1997

[3] DJ Benson. A multi-material eulerian formulation for the efficient solution of impact and penetration problems. Computational mechanics, 1995.

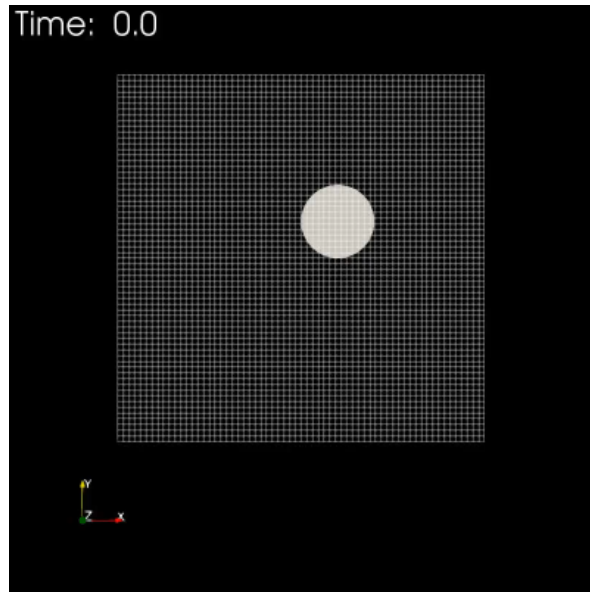
[4] HS Udaykumar, L Tran, DM Belk, and KJ Vanden. An eulerian method for computation of multimaterial impact with eno shock-capturing and sharp interfaces. Journal of Computational Physics, 2003

[5] Bessette et. al., Zapotec: A Coupled Euler-Lagrange Program for Modeling Earth Penetration (2002)

Overview

- Introduction
- **Review of modified Immersed Finite Element Method (mIFEM)**
- Non-intrusive coupling of software
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Background



- **Modified Immersed Finite Element Method (mIFEM)** [1]
 - Non-conforming mesh method, partitioned approach
 - Inspired from IB method [2]

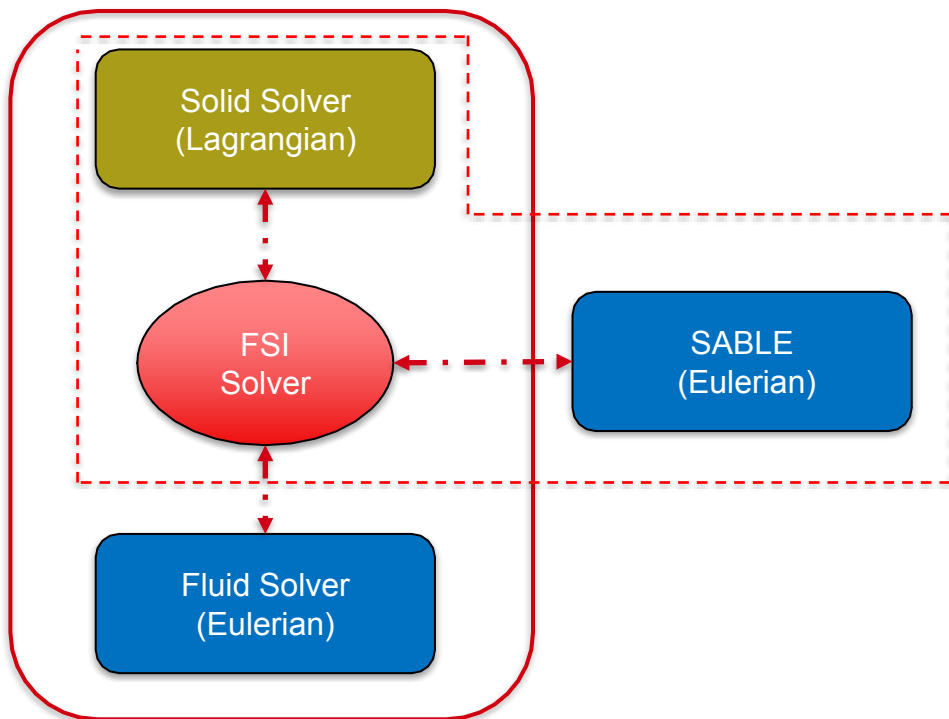
- **Comparison with classical IB:**
 - **Volume based** interpolation
 - **Solve solid dynamics** and enforce it on overlapping fluid
 - **Solid has constitutive relationship** and yields stress information

[1] Xingshi Wang and Lucy T. Zhang. Modified immersed finite element method. Computer Methods in Applied Mechanics and Engineering, 267:150–169, 2013.

[2] Charles S Peskin. The immersed boundary method. Acta Numerica, 11:479–517, 2002

Background

OpenIFEM



➤ **OpenIFEM** [1]:

- Opensource, C++
- Modular implementation of mIFEM

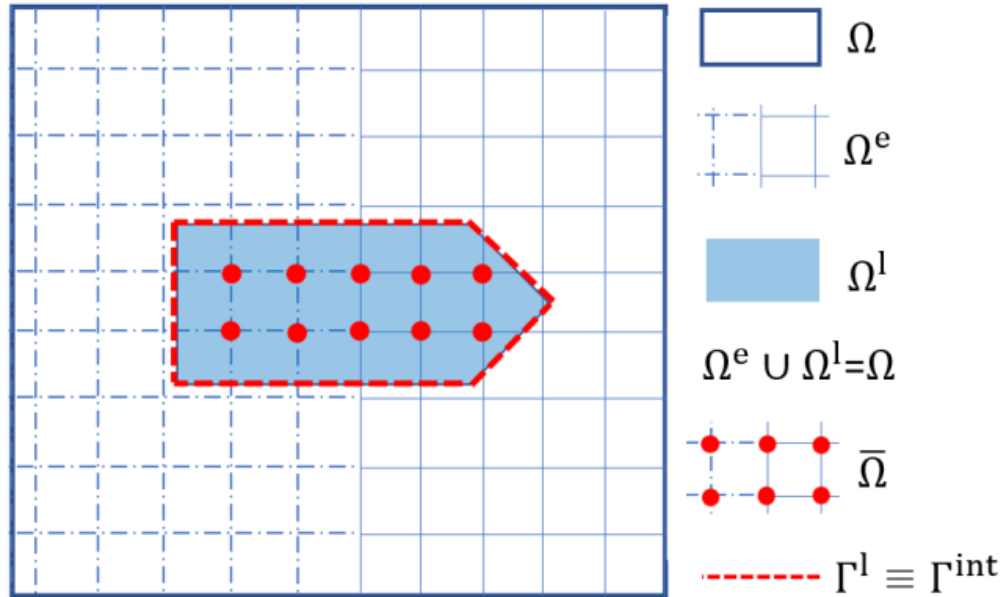
➤ **Objectives:**

- Couple a multi-material Eulerian shock physics code **SABLE** [2] with **OpenIFEM** for simulation of solid-solid impacts.
- Couple OpenIFEM and SABLE non-intrusively.
- Provide framework for coupling two hydrocodes.

[1] <https://github.com/OpenIFEM>

[2] KM Hays et al. A users guide to sable 2.0: The sandia automated boolean logic evaluation software. Sandia National Lab. (1996)

mIFEM Algorithm: Domain decomposition



mIFEM Algorithm: Formulation

virtual work done by the Eulerian material

virtual work done by the Lagrangian material

$$\int_{\Omega^e} \delta v_i \left(\rho^e \frac{Dv_i^e}{Dt} - \sigma_{ij,j}^e - \rho^e g_i \right) d\Omega + \int_{\Omega^l} \delta v_i \left(\rho^l \frac{Dv_i^l}{Dt} - \sigma_{ij,j}^l - \rho^l g_i \right) d\Omega = 0$$

$$\int_{\Omega} \delta v_i \left(\rho^l \frac{Dv_i^e}{Dt} - \sigma_{ij,j}^e - \rho^l g_i \right) d\Omega + \int_{\Omega^l} \delta v_i \left(\rho^l \left(\frac{Dv_i^l}{Dt} - \frac{Dv_i^e}{Dt} \right) - (\sigma_{ij,j}^l - \sigma_{ij,j}^e) \right) d\Omega$$

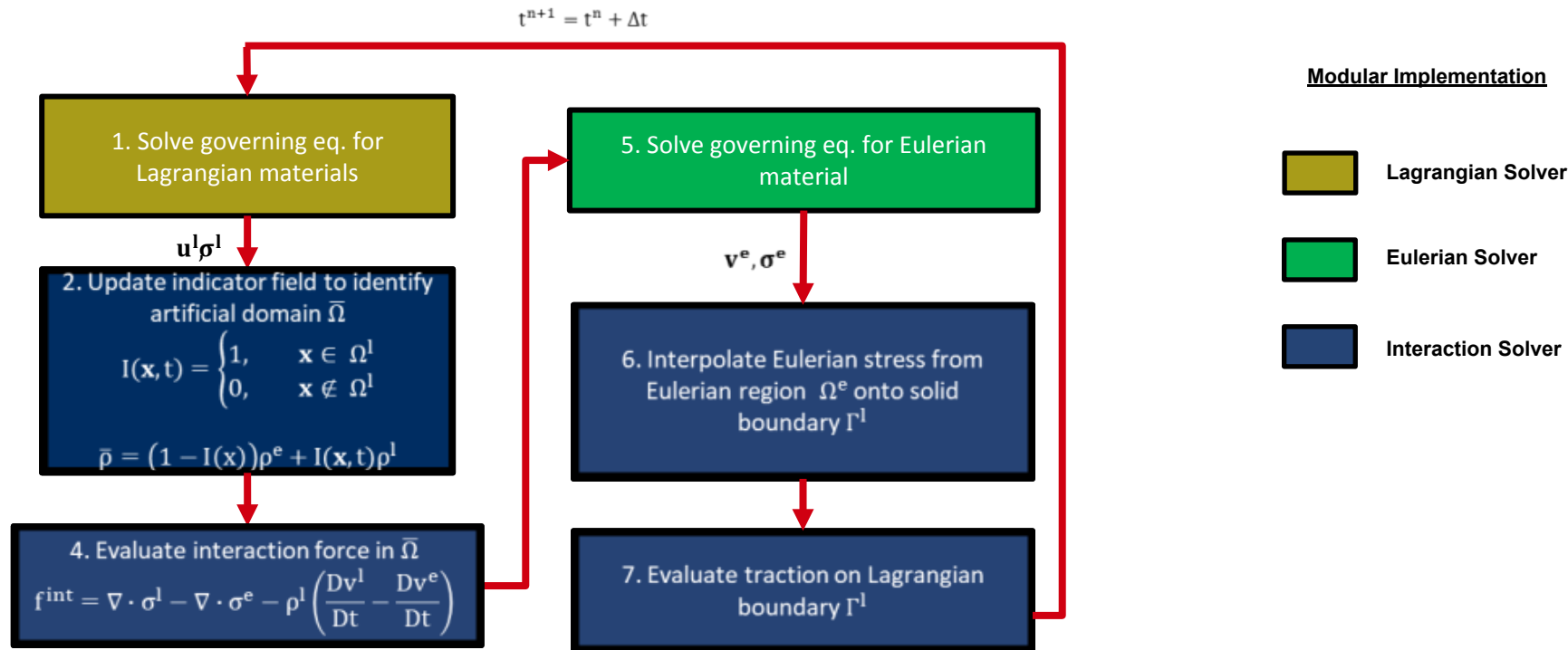
$$\bar{\rho} = \begin{cases} \rho^l & \text{in } \Omega^l \\ \rho^e & \text{in } \Omega^s \end{cases}$$

$$\int_{\Omega} \delta v_i \left(\bar{\rho} \frac{\partial v_i^e}{\partial t} + \bar{\rho} v_j^e v_{i,j}^e - \sigma_{ij,j}^e - \bar{\rho} g_i \right) d\Omega = \int_{\Omega^l} \delta v_i F_i^{\text{int}} d\Omega$$

$$\begin{aligned}
 \rho^l u_{,tt} &= \sigma_{ij}^l && \text{in } \Omega^l \\
 u_i &= q_i = v_i^e \Delta t && \text{on } \Gamma^{\text{sq}} \\
 \sigma_{ij}^l n_j &= h_i = -\sigma_{ij}^e n_j && \text{on } \Gamma^{\text{sh}}
 \end{aligned}$$

$$\bar{\rho} \left(\frac{\partial v_i^e}{\partial t} + v_{j,j}^e v_i^e \right) = \sigma_{ij,j}^e + F_i^{\text{int}}(\mathbf{x}, t)$$

mIFEM Algorithm



Cheng J et al., OpenIFEM: A high performance modular open-source software fluid-structure interactions. (2019)

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OpenIFEM-SABLE Coupling

- Coupling via information exchange using MPI protocols

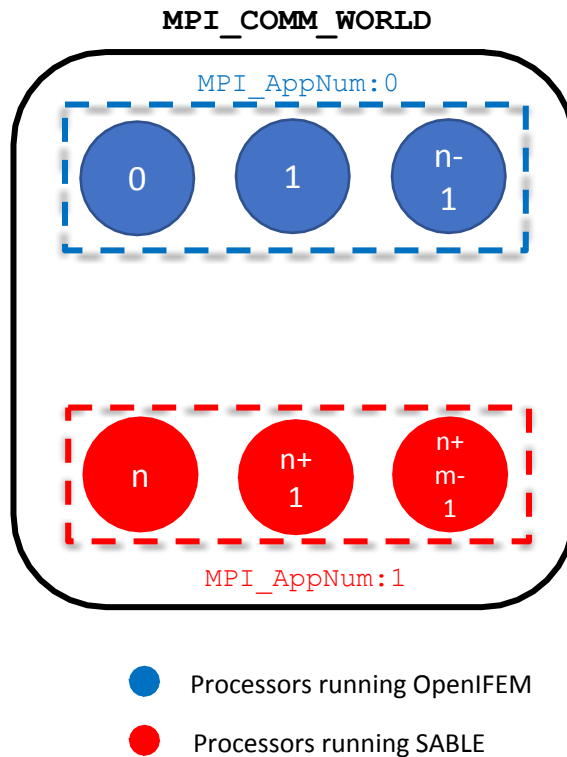
1. Launch OpenIFEM and SABLE:

- `mpirun -np n OpenIFEM : -np m SABLE`

2. Find total number of applications running simultaneously:

- Get `MPI_APPNUM` using the function

`MPI_Comm_get_attr`



OpenIFEM-SABLE Coupling

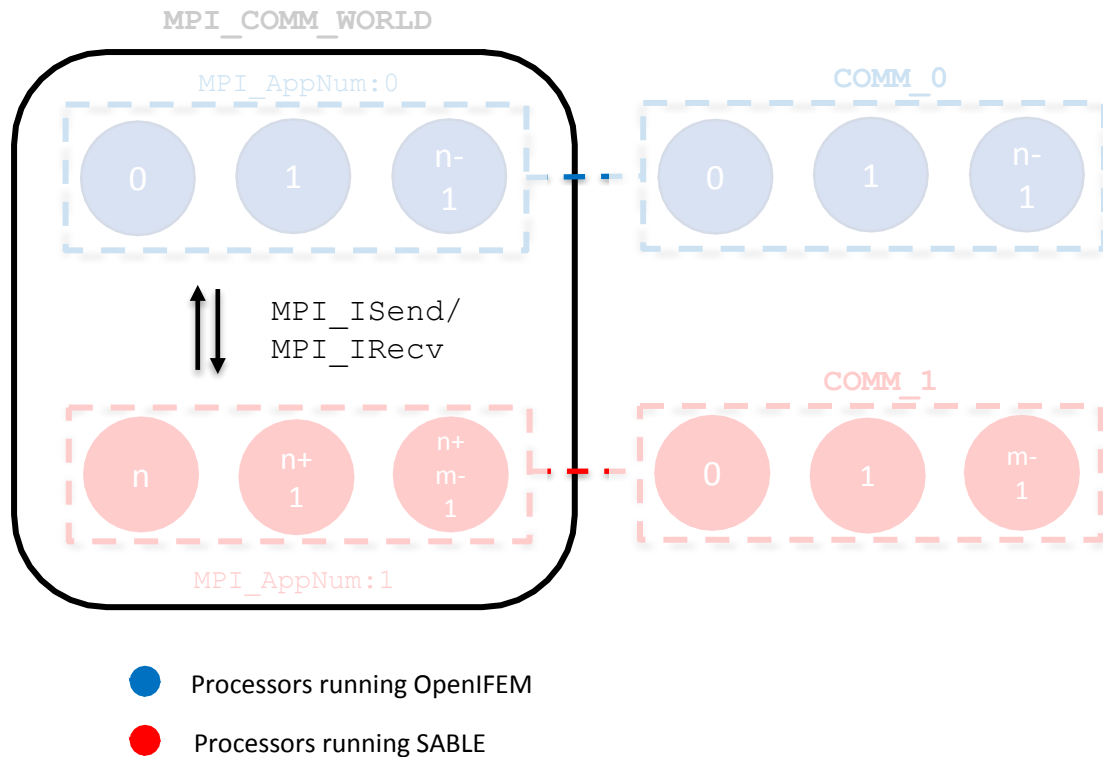
3. Split `MPI_COMM_WORLD` into two

communicators using function:

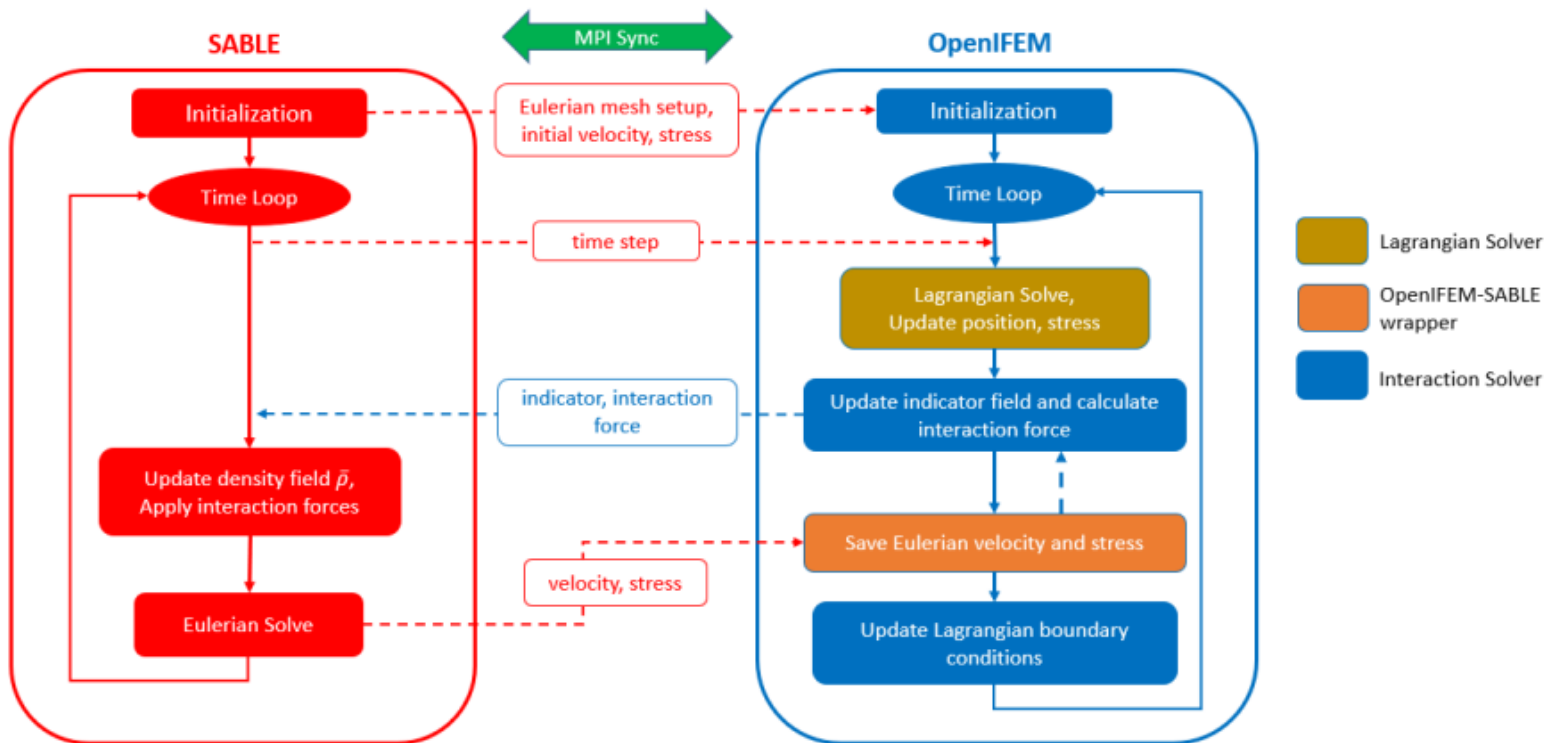
`MPI_COMM_split`

4. Switch to `MPI_COMM_WORLD` and
exchange data using `MPI_Isend` and

`MPI_Irecv`



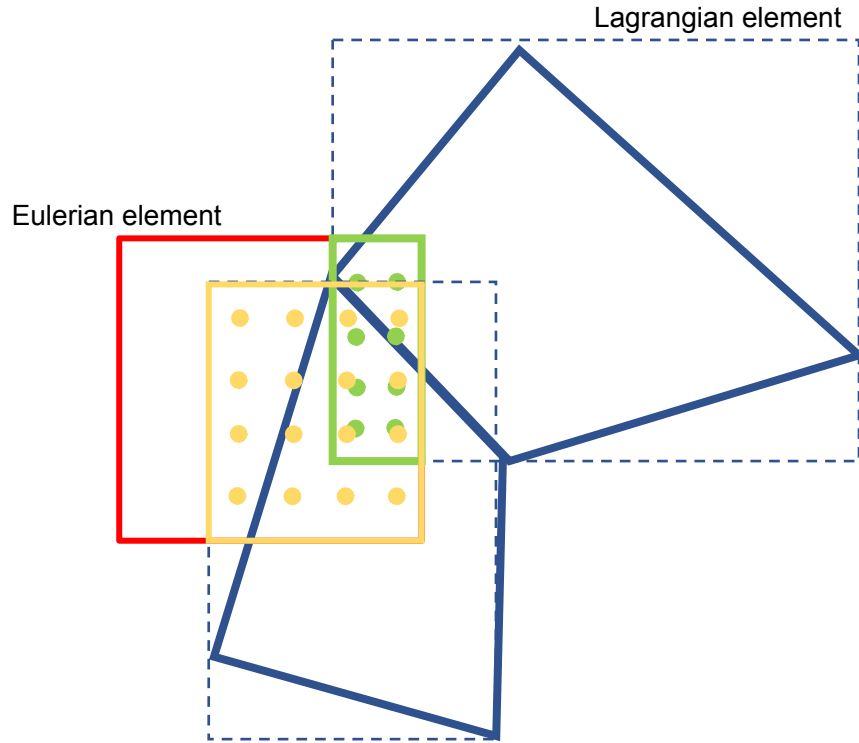
OpenIFEM-SABLE Coupling



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Improvements in mIFEM for solid-solid coupling



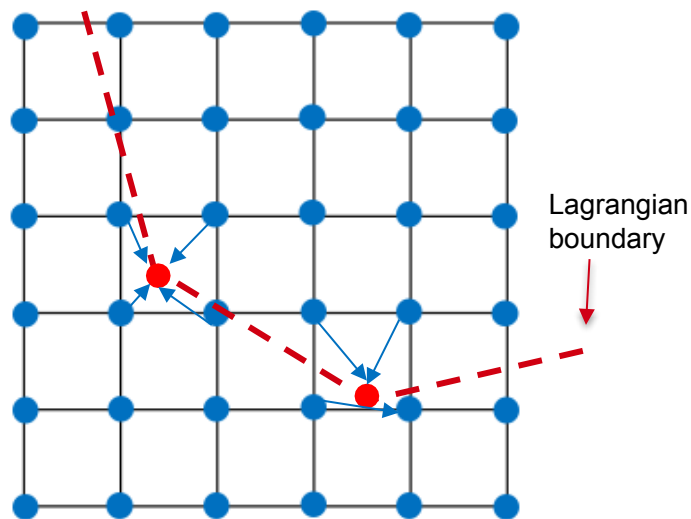
➤ Improved Interface

- Check intersection of each Lagrangian cell bounding box with Eulerian cell
- Sample points in bounding box.
- Check if the point is inside the selected element.
- Exact indicator from current Lagrangian cell

$$= \frac{\text{Area (intersection box)}}{\text{Area(cell)}} * \frac{\text{points inside solid cell}}{\text{total points sampled}}$$

- Repeat for all the elements.

Improvements in mIFEM for solid-solid coupling



➤ Added mass effect [1], [2]:

- Inertial effect of Eulerian solid
- Increase effective forces on Lagrangian solid
- Calculate lumped mass for Eulerian solid.
- Interpolate Eulerian nodal mass to Lagrangian boundary nodes

➤ Calculation of stable time step:

- SABLE: explicit hydrocode
- Stable time step $\Delta t = \alpha * f(h, C^E)$
- h : grid size, C^E : elastic wave speed for Eulerian material, $\alpha=0.85$
- Stable time step for coupled run:
 - $\beta = C^E / C^L$
 - $\Delta t = \beta * \alpha * f(h, C^E)$

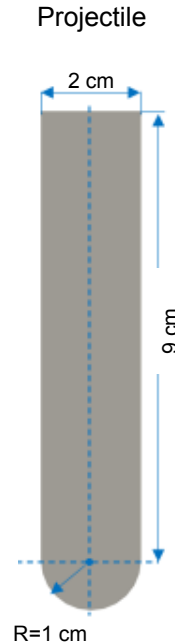
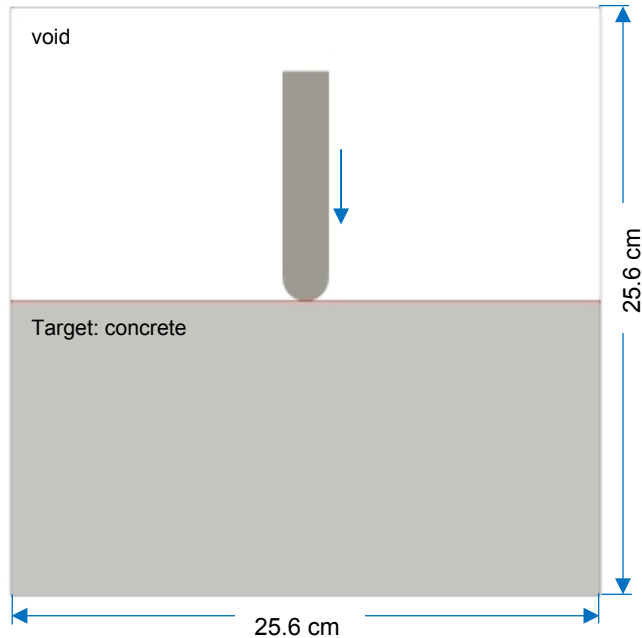
[1] Panton, R.L., Incompressible Flow, JohnWiley and Sons (1984)

[2] Brown et. al. Coupled Eulerian–Lagrangian methods for earth penetrating weapon applications, Sandia National Laboratories (2002)

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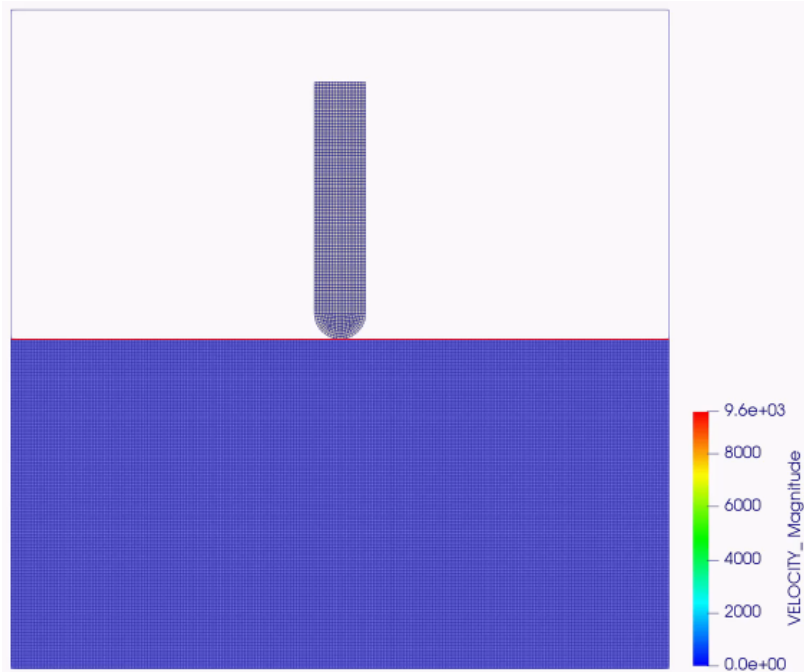
Test Case 1: Setup



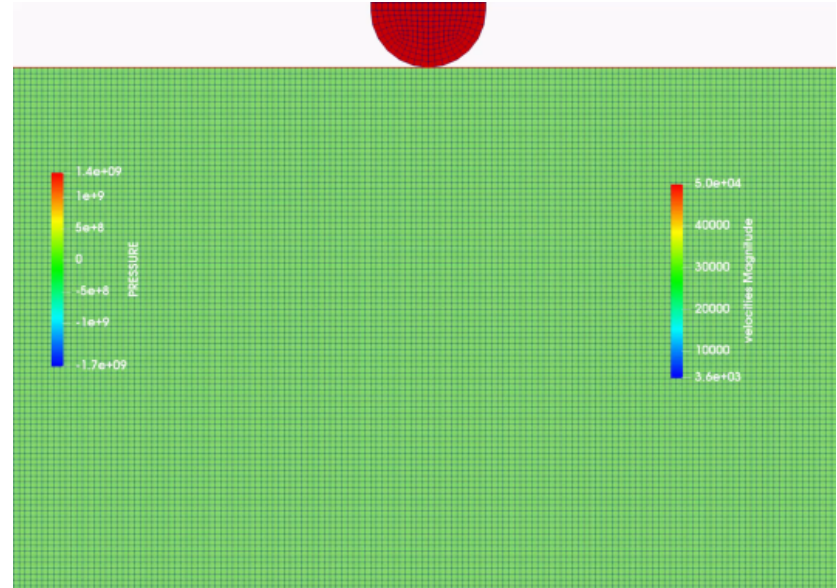
- **Eulerian (target):**
 - Material: void + concrete
 - Concrete:
 - Material mode: elastic-plastic
 - Compressive strength: 23 Mpa
 - Density: 2.03 g/cm^3
 - BCs: no displacement at all sides
 - Mesh: $dx=dy=0.1 \text{ cm}$
- **Lagrangian (projectile):**
 - Material: 4340 steel
 - Material model: linear elastic
 - $E: 20 \text{ GPa}$ $\nu: 0.28$
 - Density: 7.8 g/cm^3
 - Mesh: $dx=dy=0.1 \text{ cm}$
 - Initial velocity: **-50,000 cm/s**
- Simulation time: $5e-4 \text{ s}$

Test Case 1: Results

Target velocity (cm/s)

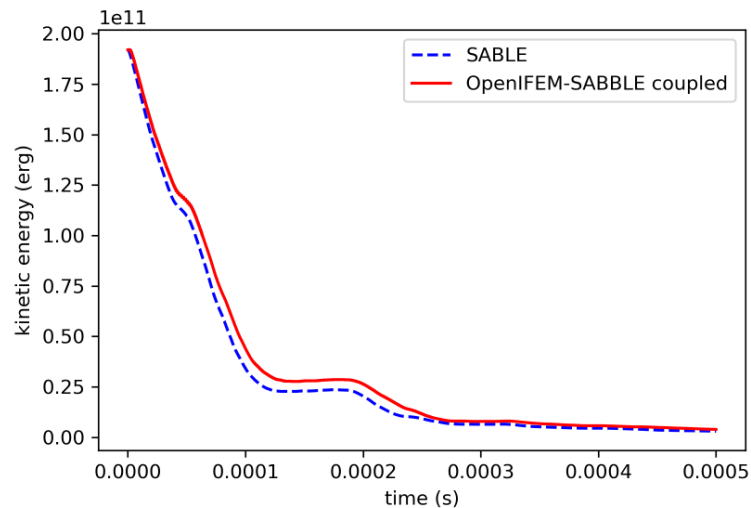
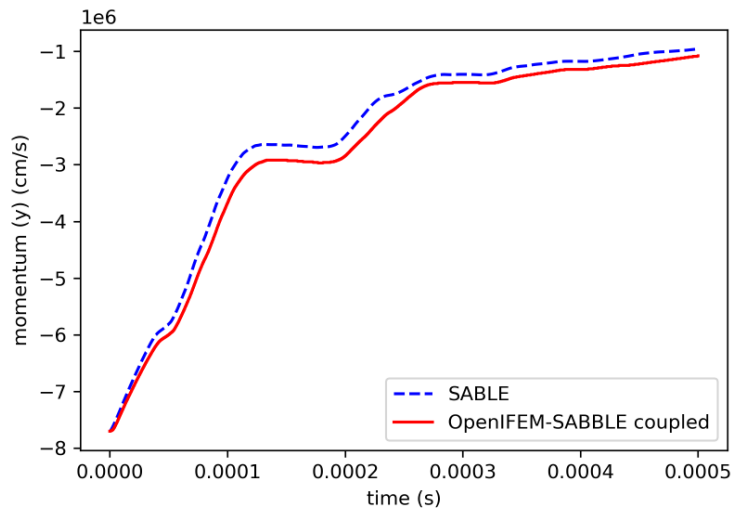


Target pressure (dyn/cm²) + Projectile velocity (cm/s)



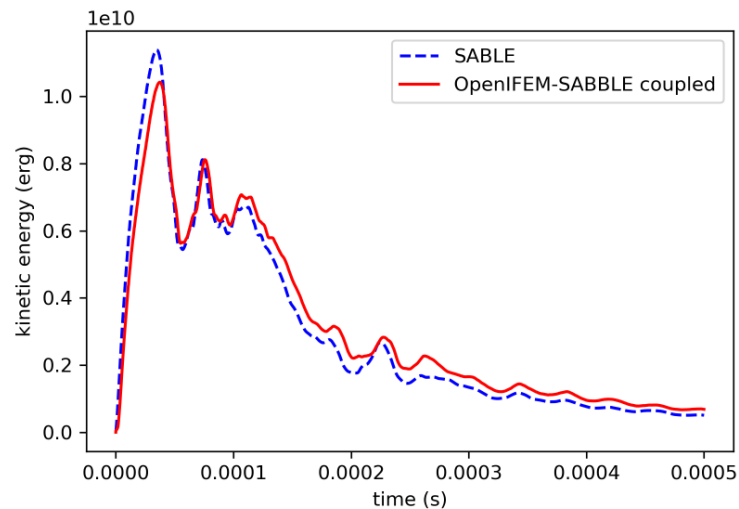
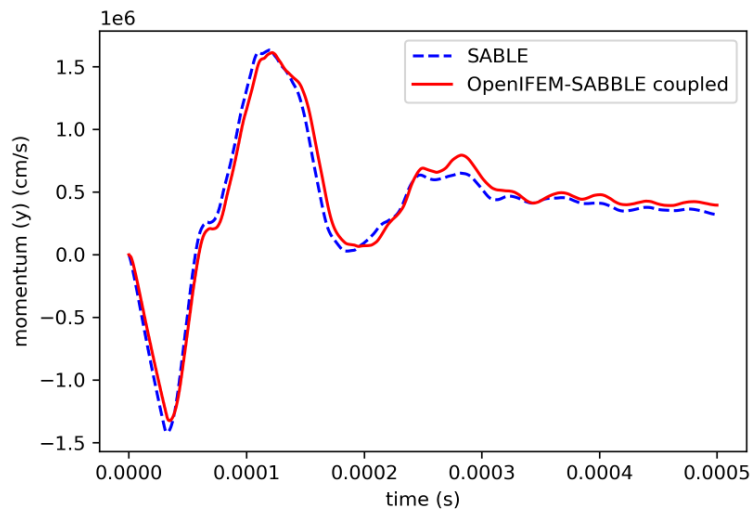
Test Case 1: Results

Projectile Results

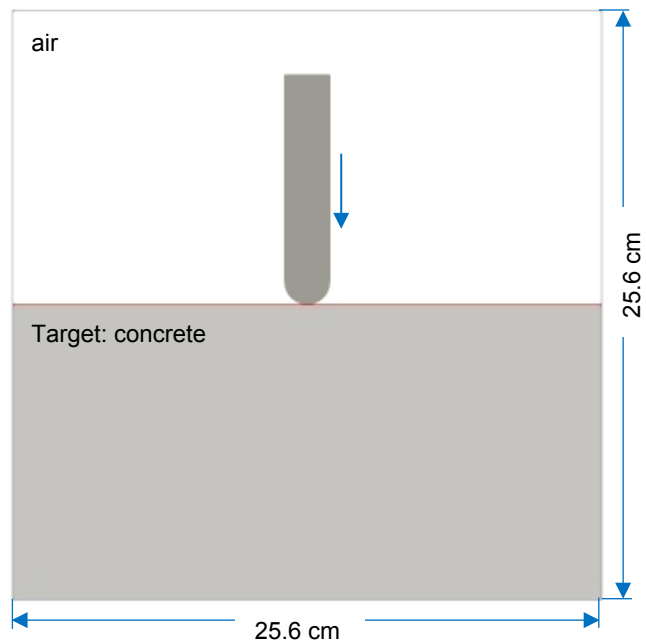


Test Case 1: Results

Target Results



Test Case 2: Setup



➤ Eulerian (target):

- Material: air + concrete
- Air:
 - Pressure: $1\text{e}6 \text{ dyn/cm}^2$
 - Density: $1.2\text{e-}3 \text{ g/cm}^3$
- Concrete:
 - Material mode: elastic-plastic
 - Compressive strength: 23 Mpa
 - Density: 2.03 g/cm^3
- BCs: no displacement at all sides
- Mesh: $dx=dy=0.1 \text{ cm}$

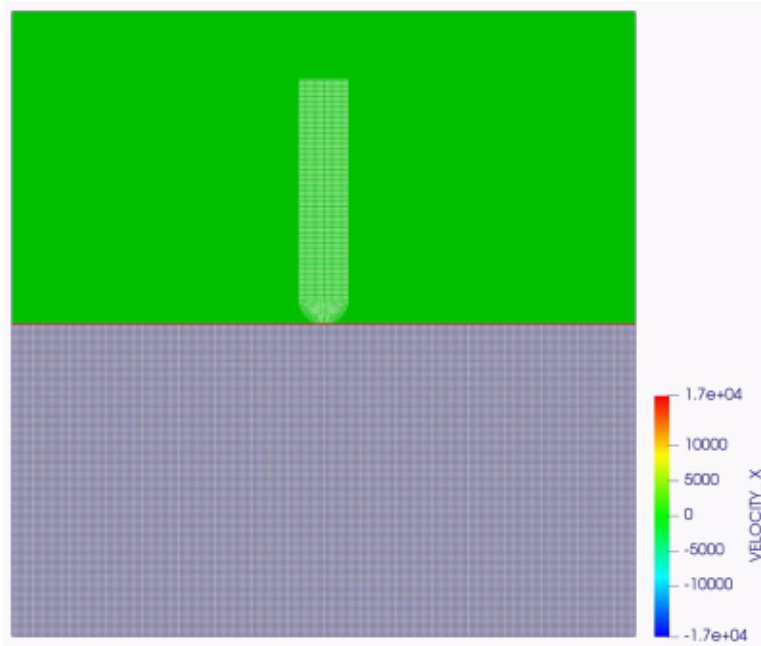
➤ Lagrangian (projectile):

- Material: 4340 steel
 - Material model: linear elastic
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 - Density: 7.8 g/cm^3
- Mesh: $dx=dy=0.1 \text{ cm}$
- Initial velocity: **-50,000 cm/s**

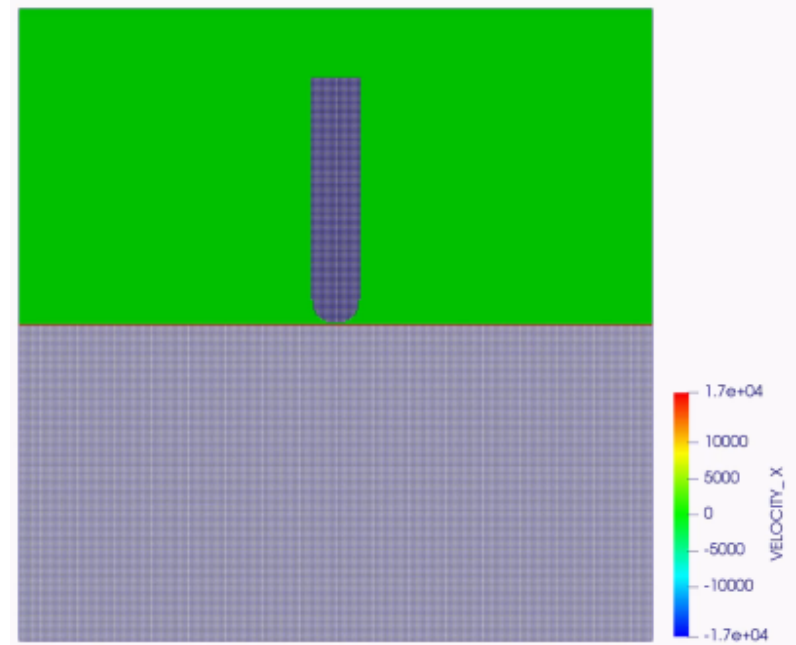
➤ Simulation time: $5\text{e-}4 \text{ s}$

Test Case 2: Results

Air velocity x (cm/s): **OpenIFEM-SABLE**
coupled

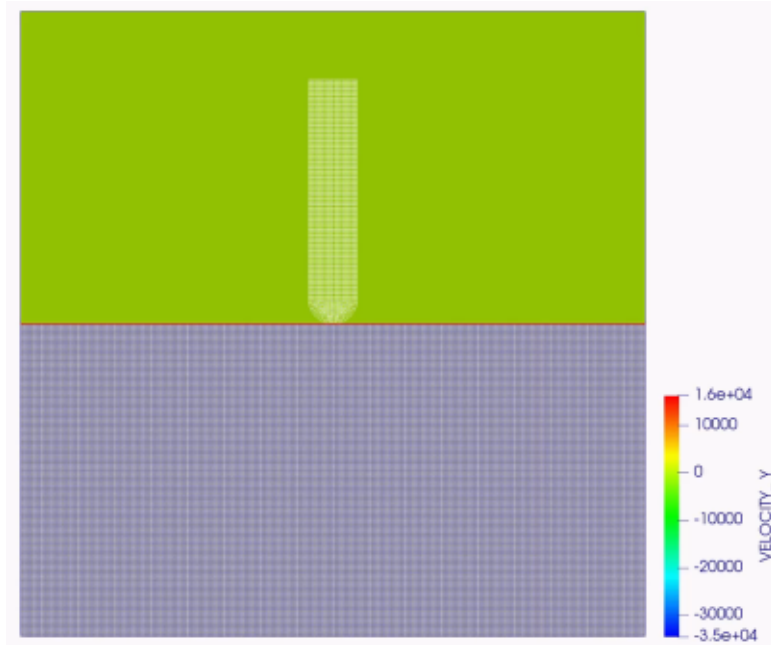


Air velocity x (cm/s): **SABLE**

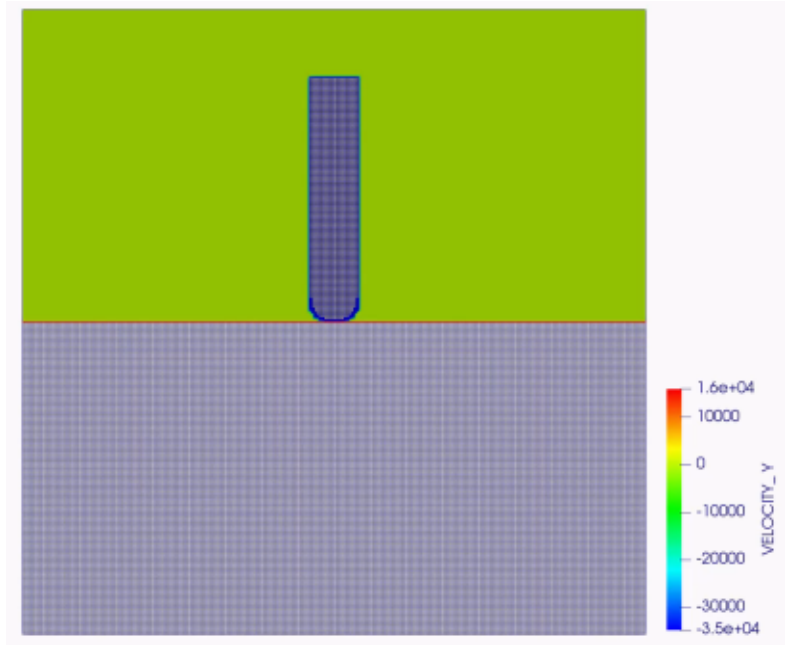


Test Case 2: Results

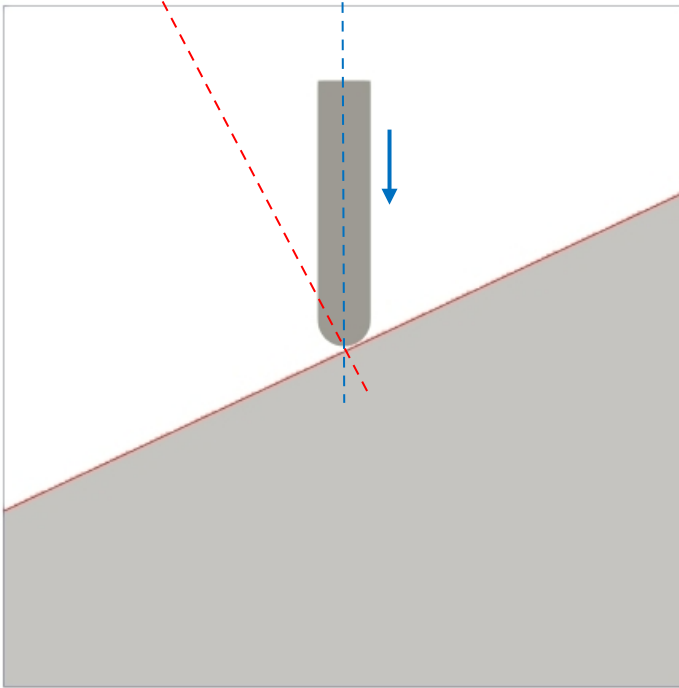
Air velocity y (cm/s): **OpenIFEM-SABLE**
coupled



Air velocity y (cm/s): **SABLE**



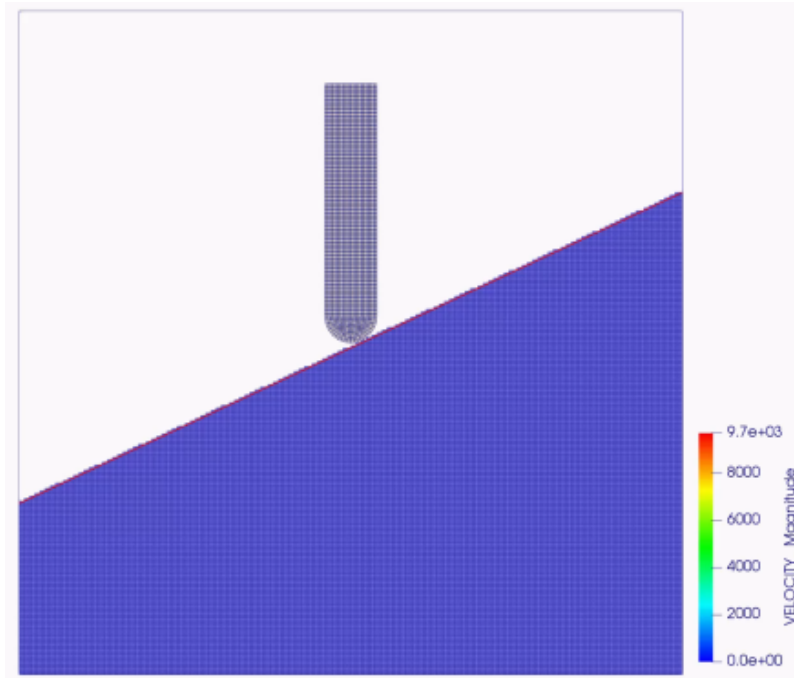
Test Case 3: Setup



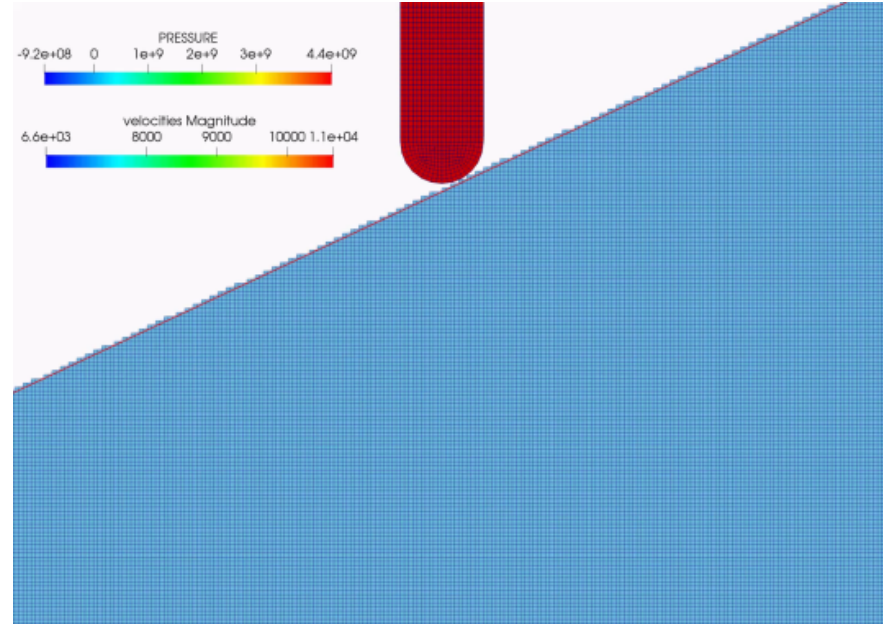
- **Oblique impact:** $\theta = 25^\circ$
- **Eulerian (target):**
 - Material: void + concrete
 - Concrete:
 - Material mode: elastic-plastic
 - Compressive strength: 23 Mpa
 - Density: 2.03 g/cm^3
 - BCs: no displacement at all sides
 - Mesh: $dx=dy=0.1 \text{ cm}$
- **Lagrangian (projectile):**
 - Material: 4340 steel
 - Material model: linear elastic
 - $E: 20 \text{ GPa}$ $\nu: 0.28$
 - Density: 7.8 g/cm^3
 - Mesh: $dx=dy=0.1 \text{ cm}$
 - Initial velocity: **-50,000 cm/s**
- Simulation time: $5e-4 \text{ s}$

Test Case 3: Results

Target velocity (cm/s)

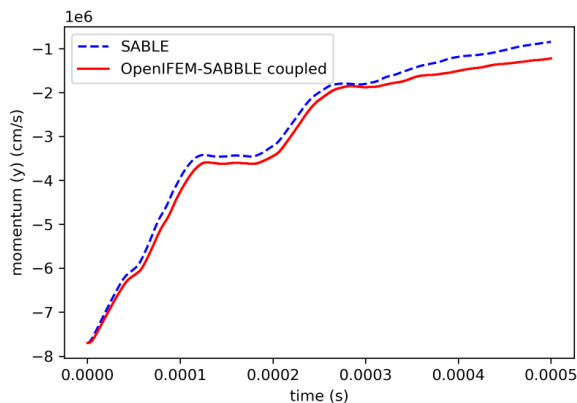


Target pressure (dyn/cm²) + Projectile velocity (cm/s)

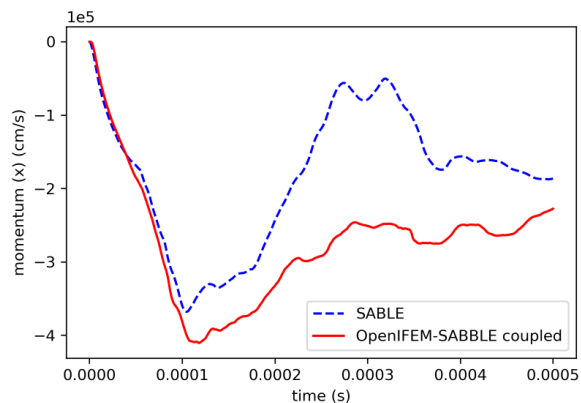


Test Case 3: Results

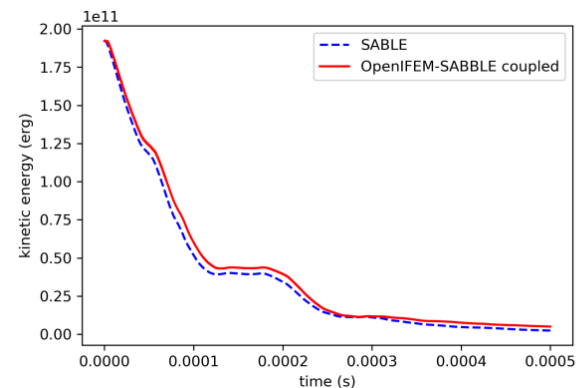
Projectile momentum (y)



Projectile momentum (x)

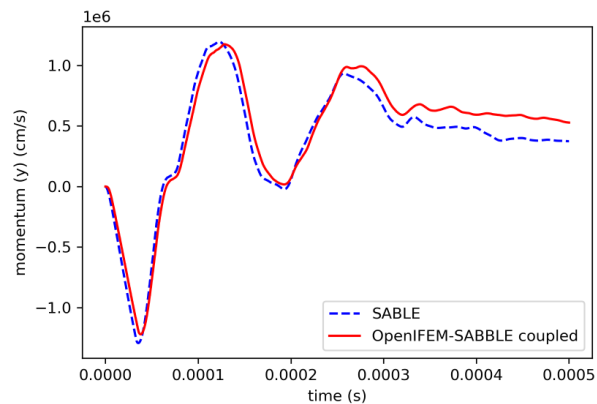


Projectile kinetic energy

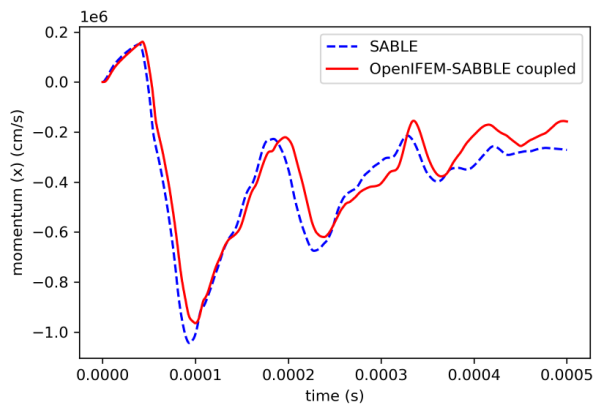


Test Case 3: Results

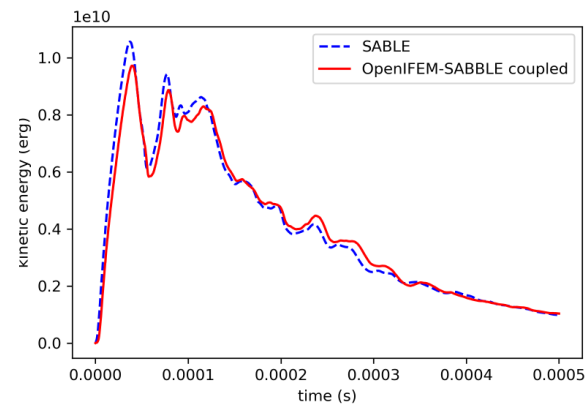
Target momentum (y)



Target momentum (x)

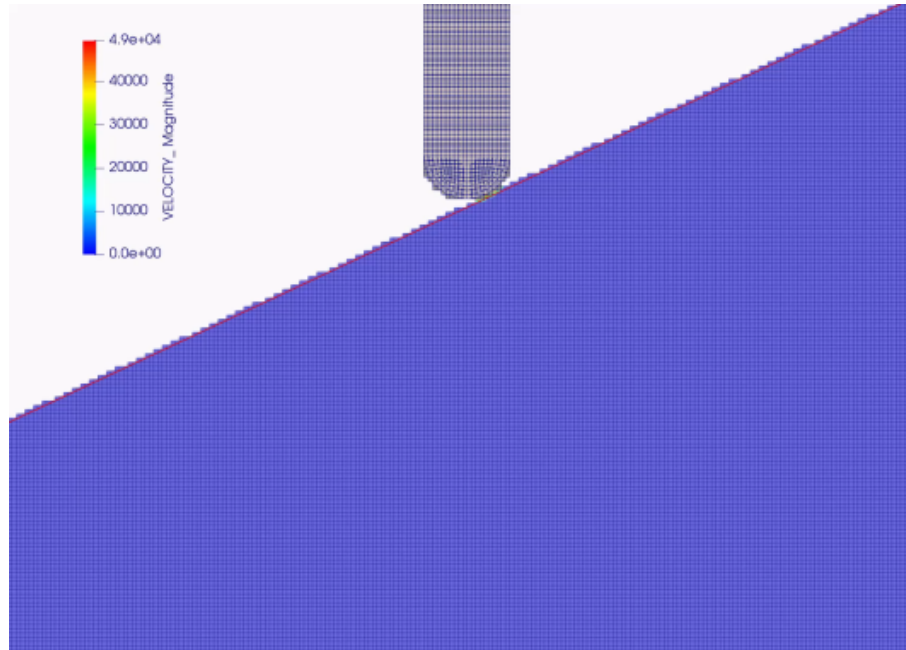


Target kinetic energy

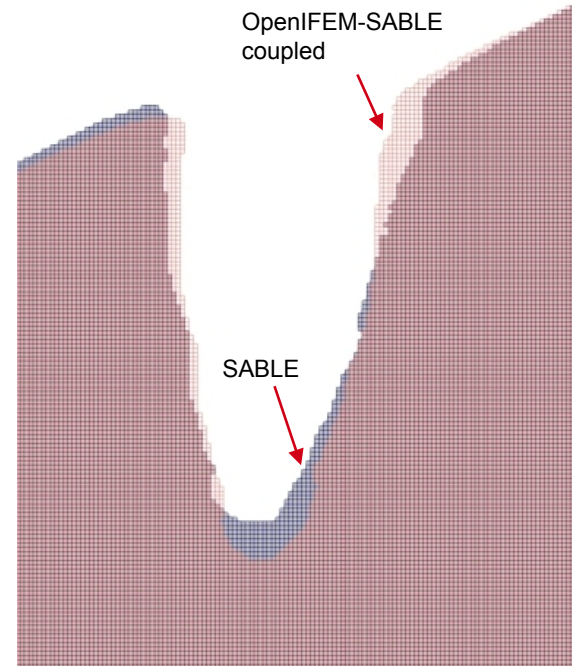


Test Case 3: Results

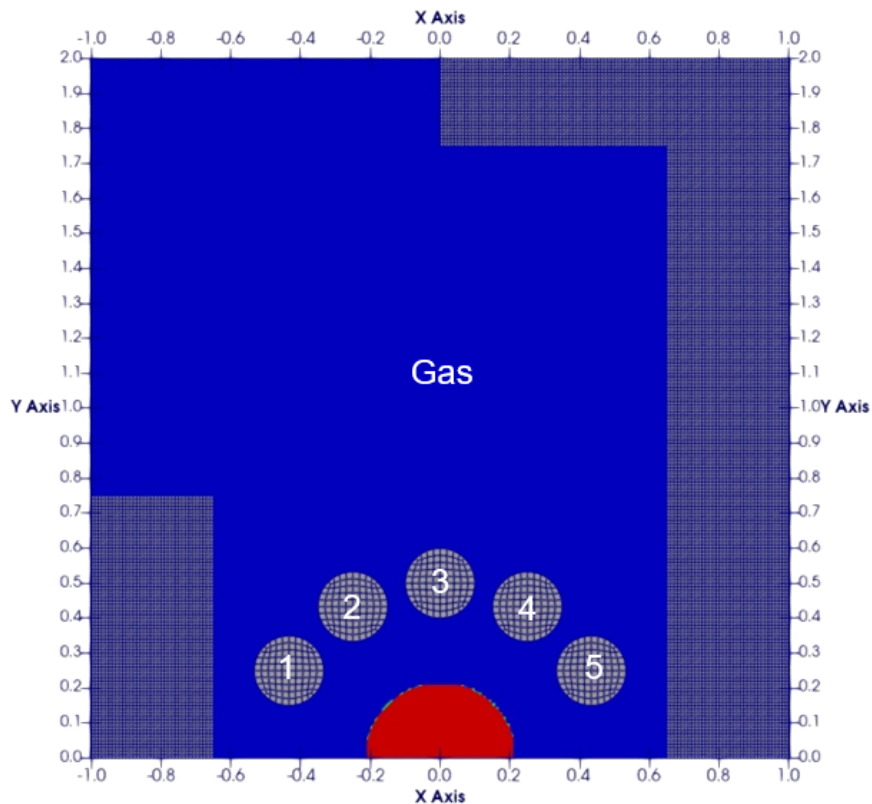
SABLE run: comparison with Lagrangian projectile



Target at $t=5e-4$ s



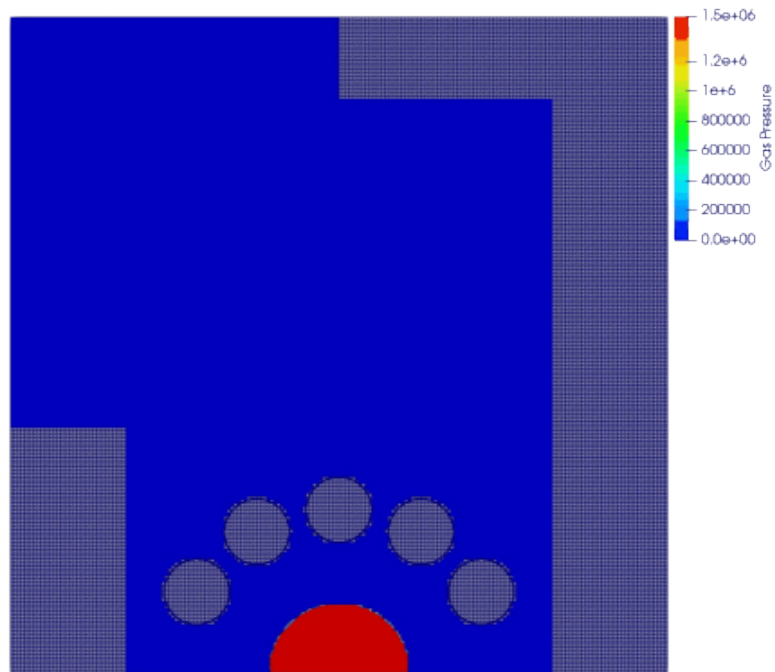
Test Case 4: Setup



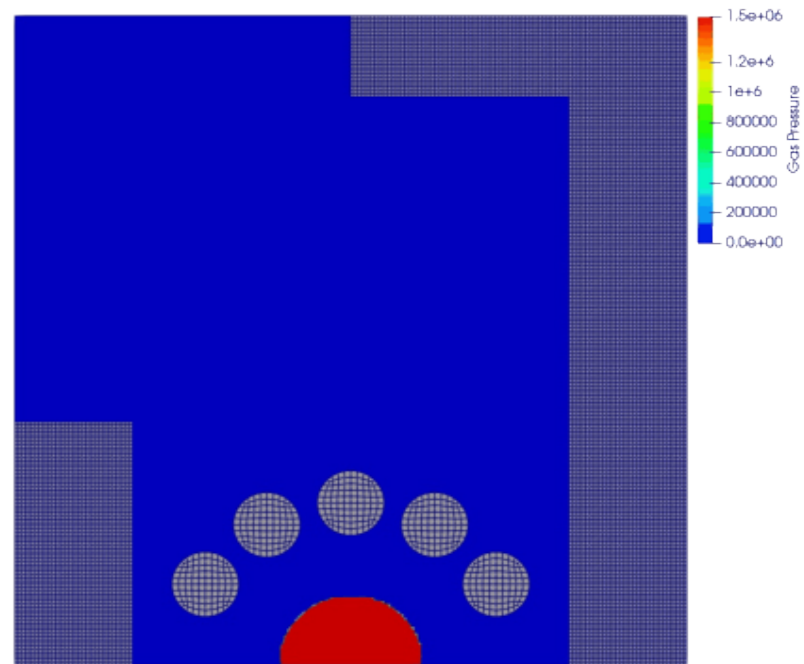
- **5 Plexiglass balls** loaded by a **high-pressure gas** with **moveable plexiglass walls**
- **Eulerian:**
 - **High-Pressure Gas Region**
 - $(x,y) = (0.0, 0.0)$, Radius = 0.3 cm
 - Initial Pressure: 10^8 dynes/cm²
 - **Low-Pressure Gas Region**
 - Initial Pressure: 10^{-5} dynes/cm²
 - Initial Density: 1 gm/cm³
- **Lagrangian:**
 - Plexiglass balls, Radius = 0.1 cm
 - Ball-1: $(x,y) = (-0.4330, 0.2500)$
 - Ball-2: $(x,y) = (-0.2500, 0.4330)$
 - Ball-3: $(x,y) = (0.0000, 0.5000)$
 - Ball-4: $(x,y) = (0.2500, 0.4330)$
 - Ball-5: $(x,y) = (0.4330, 0.2500)$
 - Plexiglass properties
 - Hyperelastic Neo-Hookean material
 - $c1 = 4.85e11$ dynes/cm² $c2 = 1.35e12$ dynes/cm²
 - Density: 1.18 gm/cm³
- Simulation time: $6e-4$ s

Test Case 4: Results

Gas pressure (dyn/cm²): **SABLE**

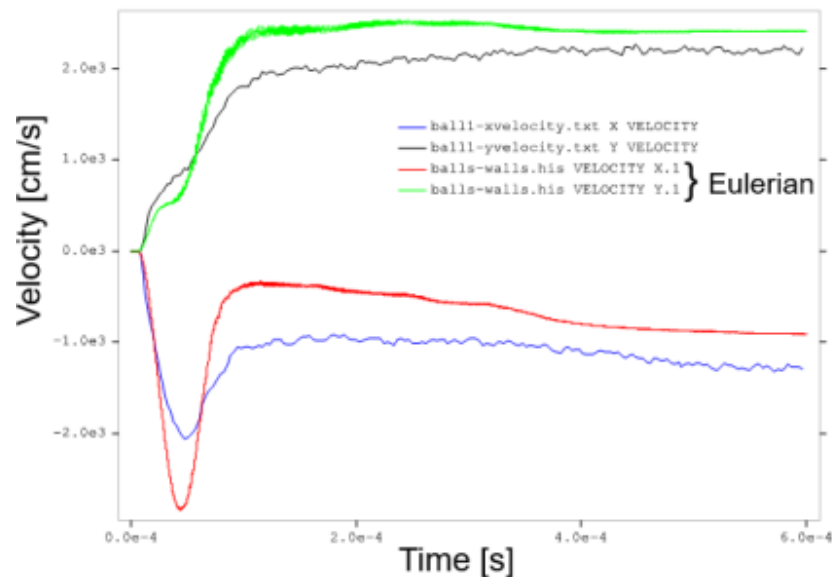
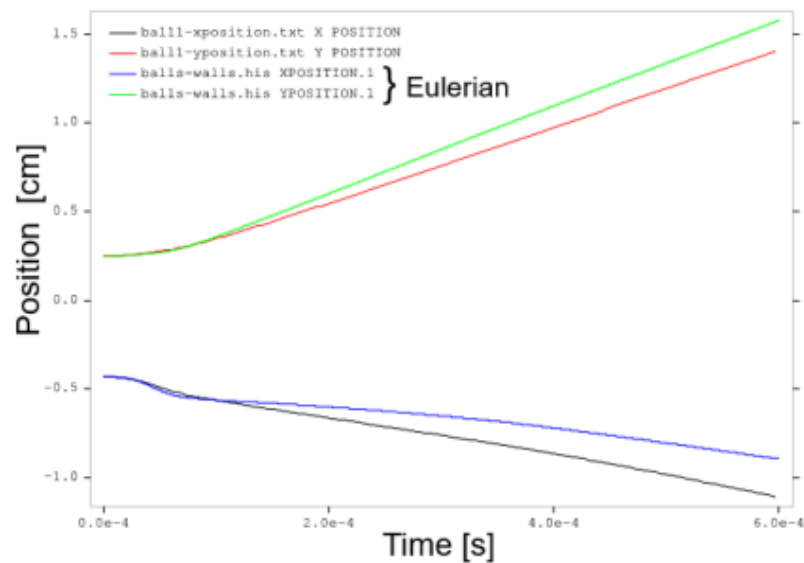


Gas pressure (dyn/cm²): **OpenIFEM-SABLE coupled**



Test Case 4: Results

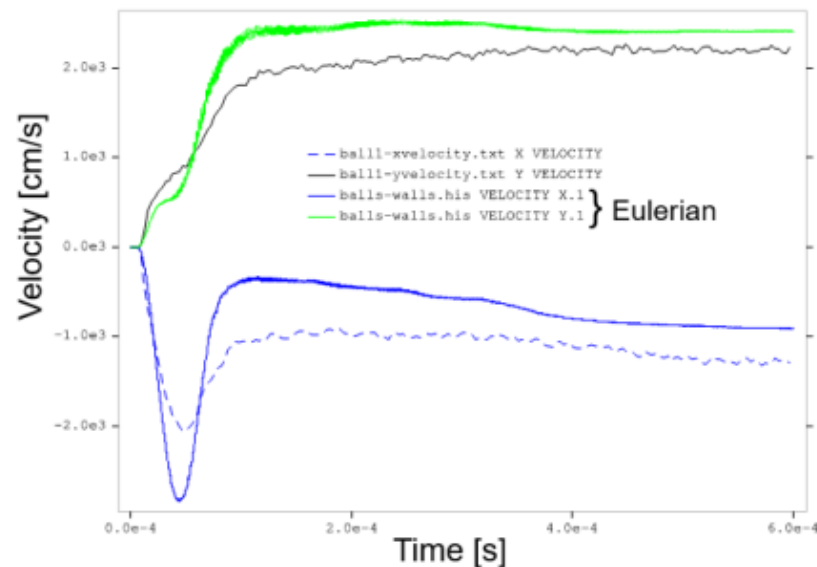
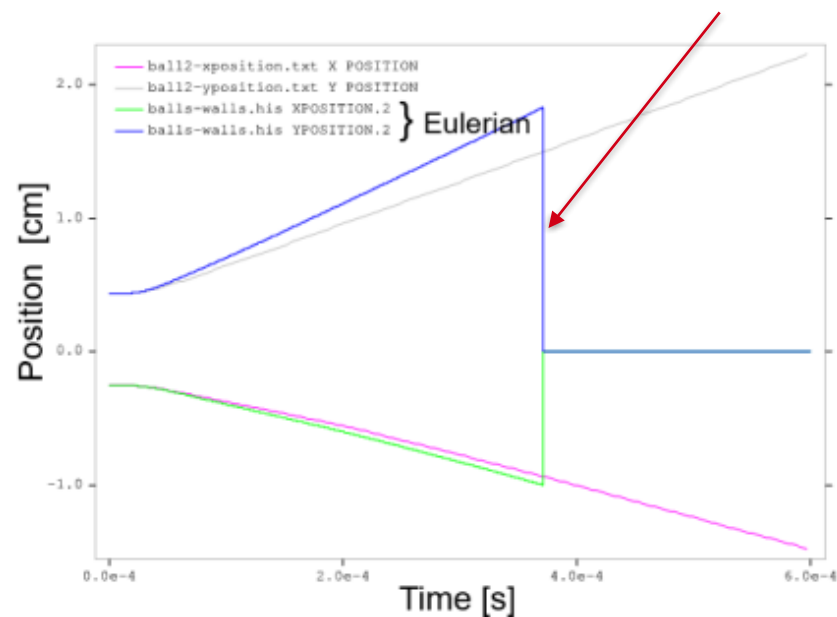
Ball-1:



Test Case 4: Results

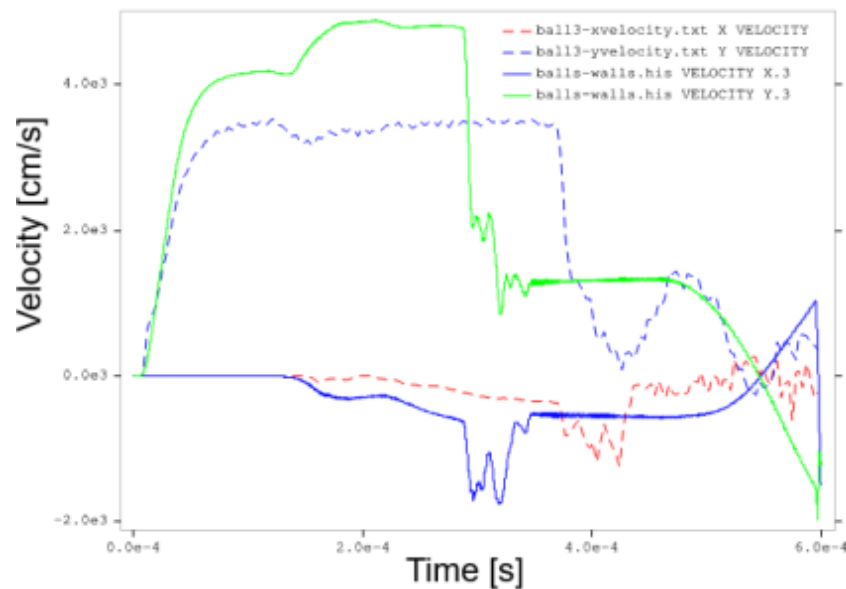
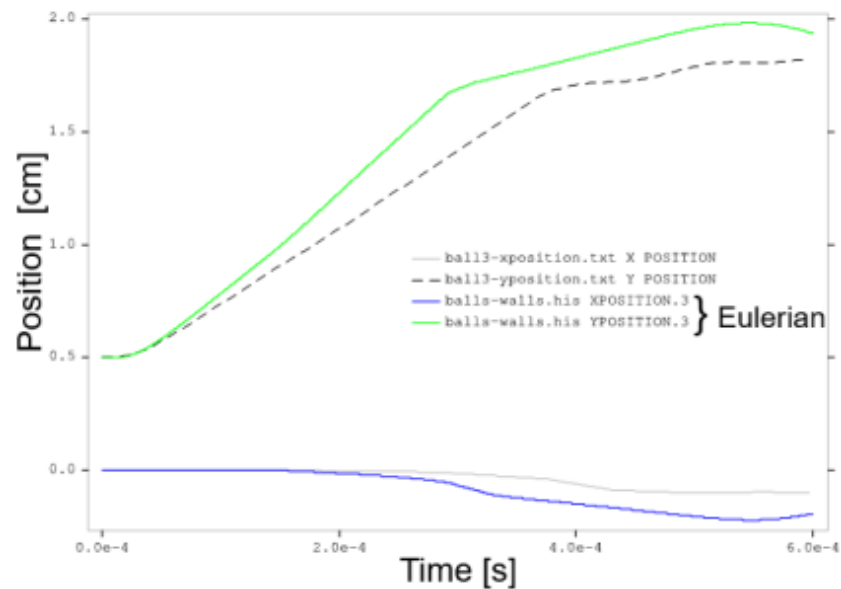
Ball-2:

Ball-2 Leaves the Eulerian Domain



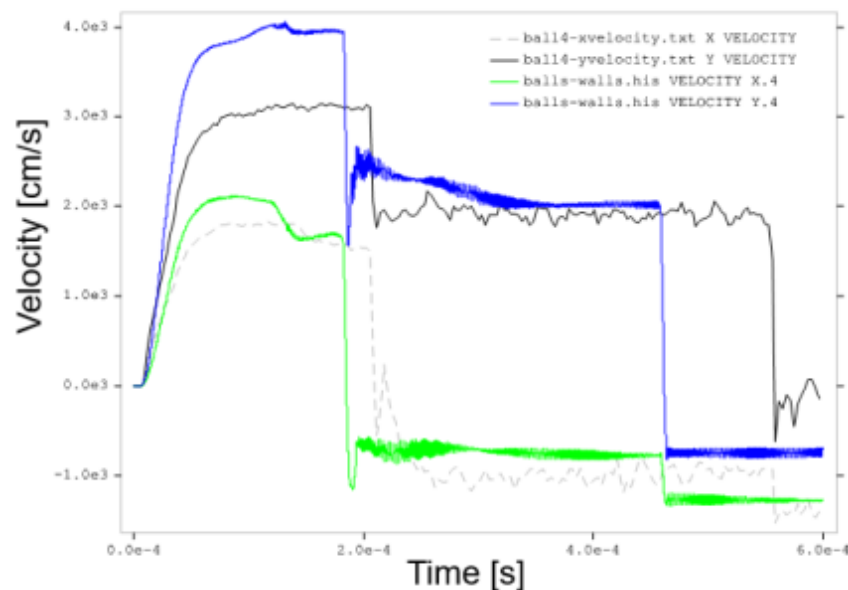
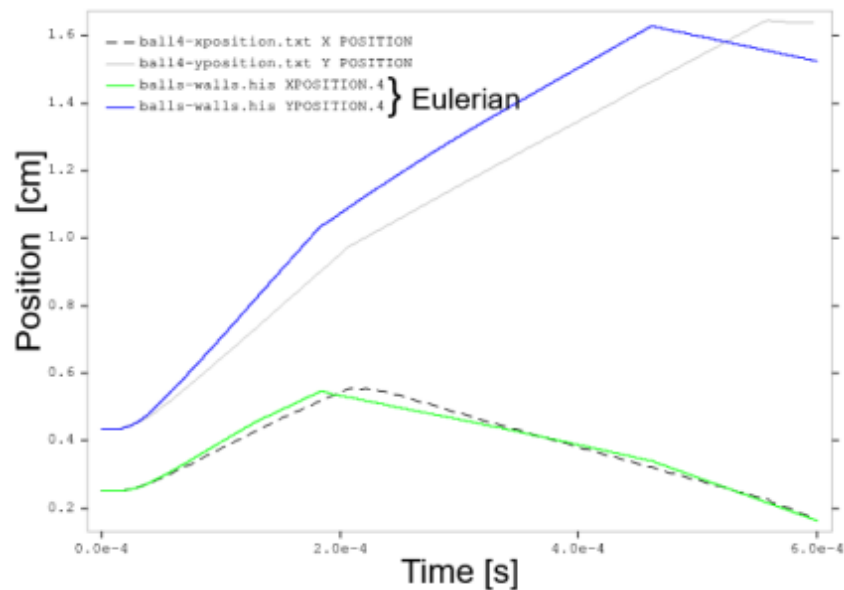
Test Case 4: Results

Ball-3:



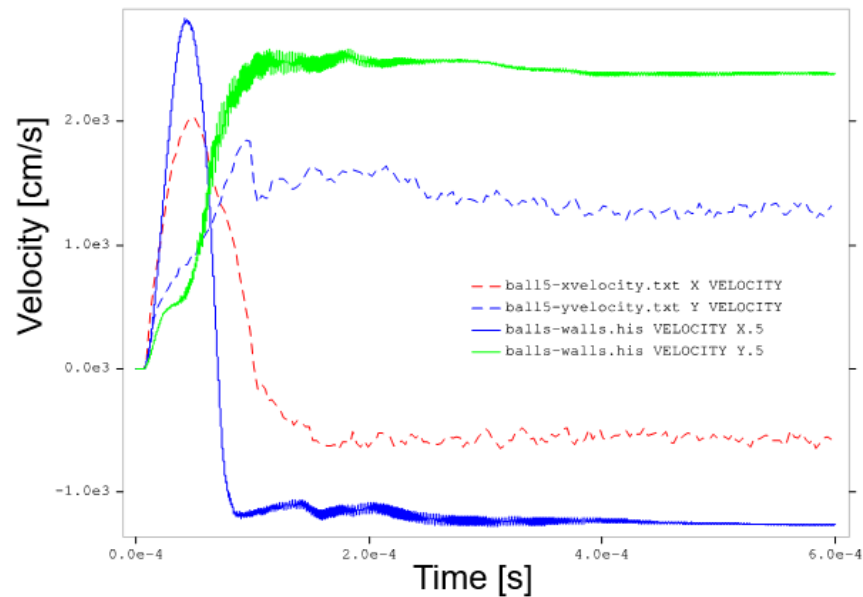
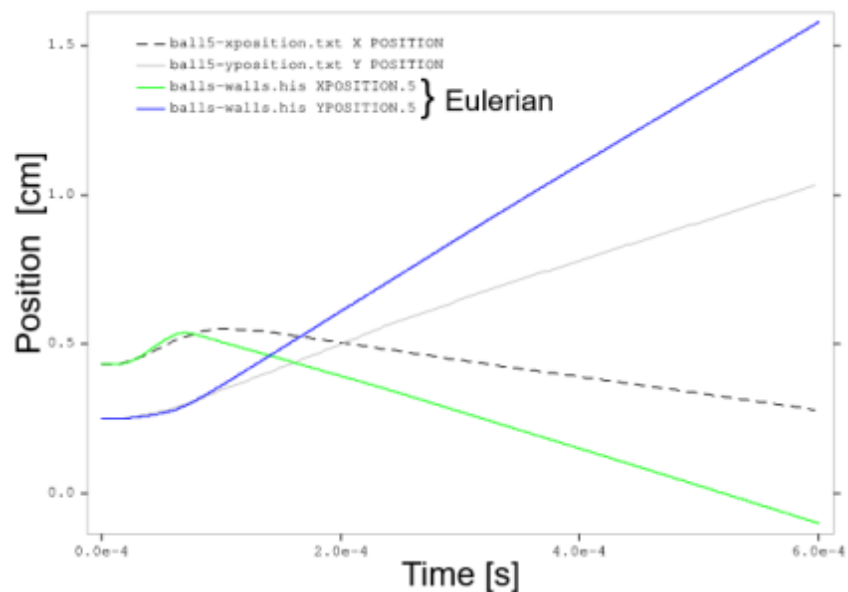
Test Case 4: Results

Ball-4:

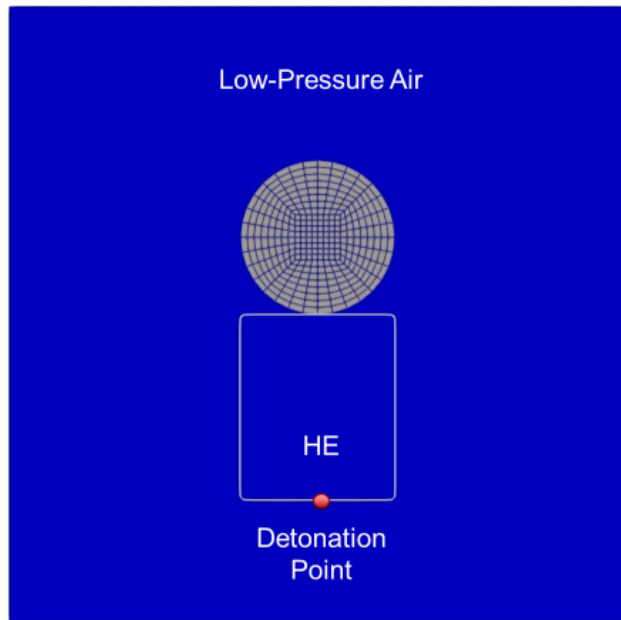


Test Case 4: Results

Ball-5:



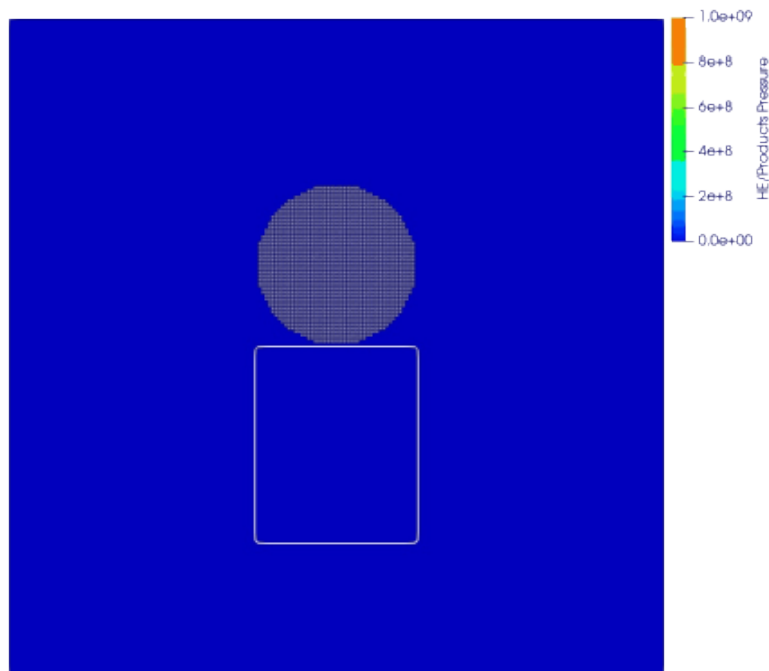
Test Case 5: Setup



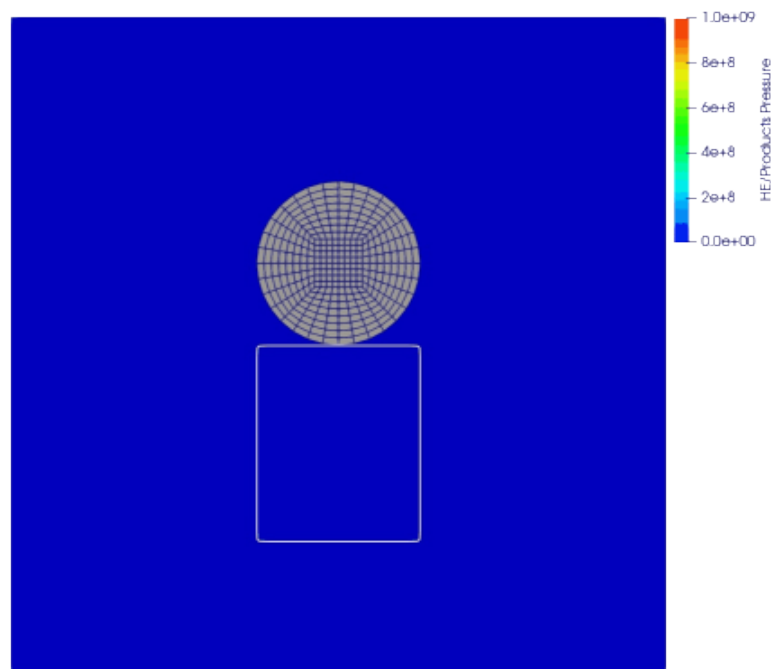
- **Eulerian Domain:**
 - $(-5.0, 0.0) \leq (x, y) \leq (15.0, 20.0)$ [cm]
 - Mesh size: $h = 0.1$ [cm]
 - Low-Pressure Air: $P = 10^{-5}$ [dynes/cm²]
 - High Explosive: $(2.5, 4.0) \leq (x, y) \leq (7.5, 10)$ [cm]
 - Tracer Location: $(x, y) = (4.5, 12.5)$ [cm]
- **Lagrangian Solid:**
 - Ball Position: $(x, y) = (4.5, 12.5)$
 - Linear Elastic material
 - Density: 7.8724 [g/cm³]
 - Young's Modulus: $E = 200 \times 10^{10}$ [dynes/cm²]
 - Poisson's ratio: $\nu = 0.28$
 - Mie Gruenesien EOS for Eulerian simulation

Test Case 5: Results

Gas pressure (dyn/cm²): **SABLE**



Gas pressure (dyn/cm²): **OpenIFEM-SABLE coupled**



Summary

- Develop framework for **simulation solid-solid interaction** and impact **using modified Immersed Finite Element Method**.
- **Non-intrusively couple** OpenIFEM and SABLE.
- Penetrator: **Lagrangian** description, Target: **Eulerian** description
- Future Work:
 - Extend coupling for **3D simulations**.
 - Investigate application of a **contact model**.
 - Validate coupling with **literature reported test cases**.

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