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# A guardbanding method for managing false accept risk under process bias

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## Abstract

The Test Uncertainty Ratio (TUR) is often used to ensure false accept risk is minimal for a given measurement. The commonly used guidance requires either a TUR greater than 4, or appropriate guardbanding, to result in a global false accept probability of less than 2%. However, this guidance assumes the distribution of units under test is centered between the tolerance limits and fails to achieve 2% false accept probability when the product distribution is shifted toward one of the limits. A new guardbanding calculation is proposed that accounts for this potential bias in the product distribution. This guardband method may be applied when no assumptions should be made, and no information is available about the product distribution and works to ensure a 2% or lower false accept probability for all TURs (including TURs greater than 4) in the presence of modest product bias.

## 1. Introduction

Typical guardbanding methods, such as the root-sum-square (RSS) method [1] and the risk-managed guardbanding method [2], commonly referred to as "Method 6" because of its designation in NCSLI's handbook to ANSI/NCSL Z540 [3], are based on the Test Uncertainty Ratio (TUR). The TUR, calculated as the total span of the product tolerance divided by twice the expanded measurement uncertainty, serves as a proxy for calculating the global probability of false accept (PFA). The in-tolerance probability (ITP) describes the probability of any unit under test (UUT) being in tolerance regardless of inspection measurements, and when combined with TUR, allows for a simplified determination of PFA.

The relationship between PFA, TUR, and ITP is illustrated in Figure 1. The general guidance requiring a TUR greater than 4 ensures that PFA is less than 2% when ITP is greater than 80%. When a TUR is less than 4, the Method 6 guardband calculation guarantees a 2% maximum false accept rate at any ITP value.

However, using ITP to evaluate PFA in Figure 1 assumes that the distribution of UUTs is normal and centered between the tolerance limits; therefore, using TUR rules or TUR-based guardbanding only applies under normal and unbiased conditions. Based on

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observations of manufacturing processes and calibration inventory, this assumption is often violated.

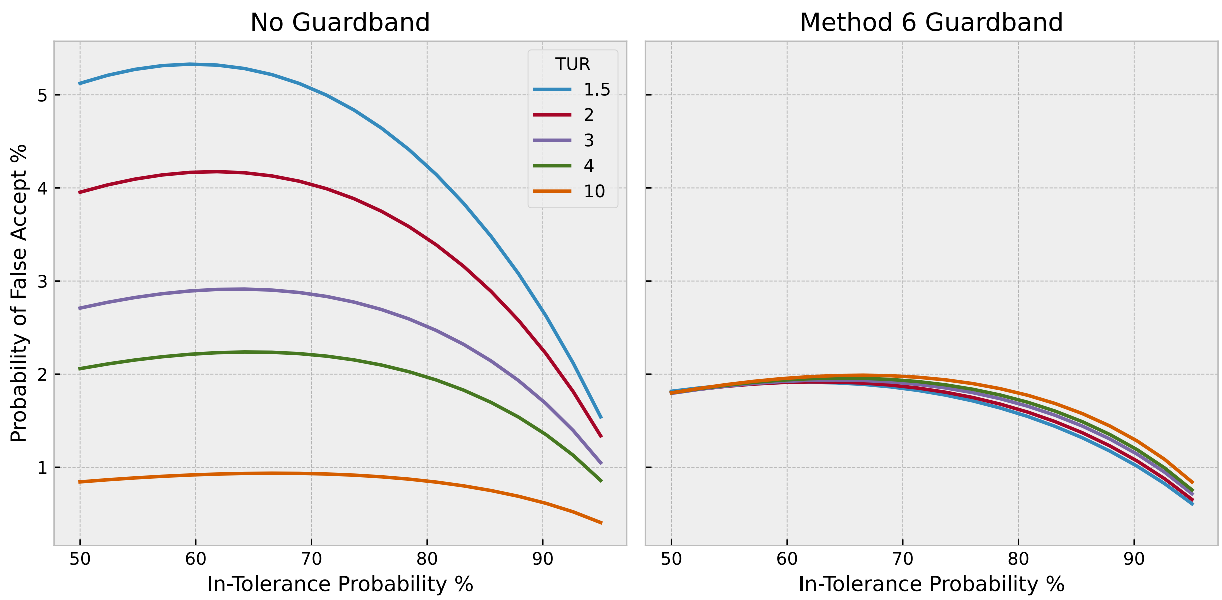


*Figure 1. PFA with no process bias.*

The Method 6 guardbanding formula was also derived assuming a normal process probability distribution centered between the upper and lower tolerance limits. If this distribution has any bias (no longer centered between the two limits), the PFA will be affected and guardbanding rules may not achieve the desired outcome. Bias, in this context, is defined as the shift in the process distribution mean away from the center of the tolerance zone, as a percentage of the tolerance. An unbiased distribution and a 75% biased distribution are illustrated in Figure 2.

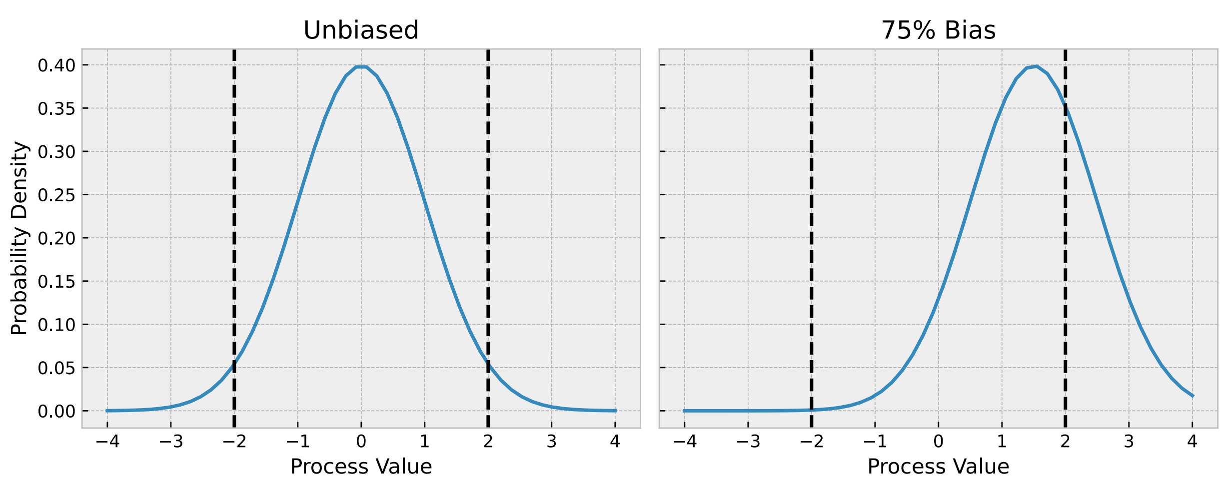


*Figure 2. Illustration of process bias.*

Once a process bias is introduced, as shown in Figure 3, a 2% PFA is no longer guaranteed by a TUR greater than 4 or by applying Method 6 guardbanding.

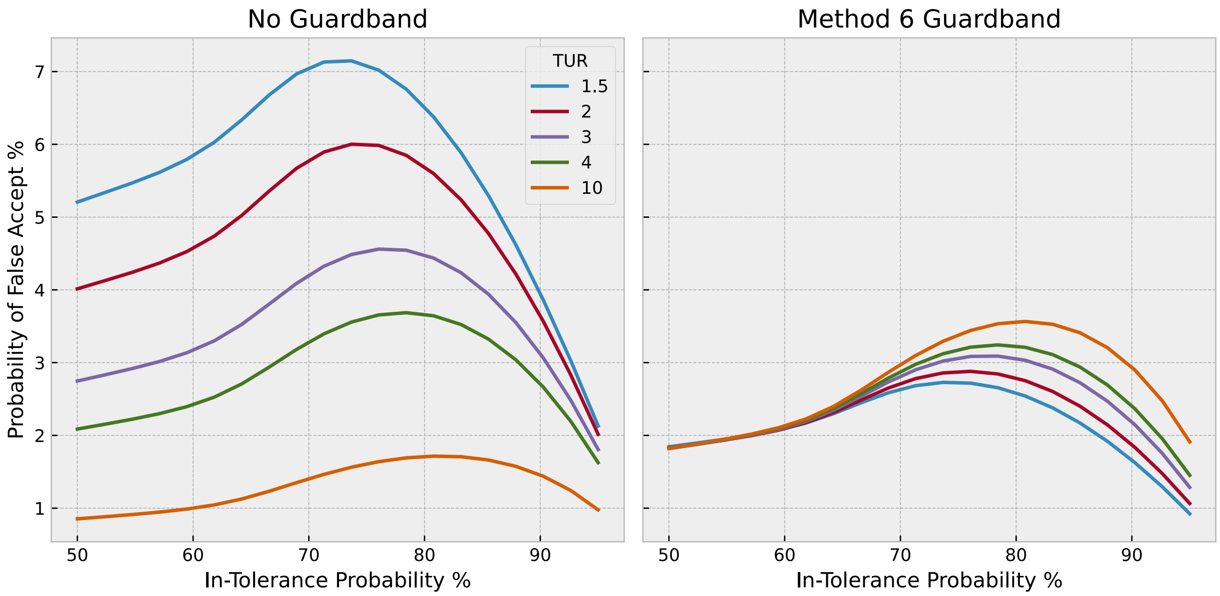


*Figure 3. Traditional guardbanding in the presence of a 75% process bias.*

Fortunately, a technique similar to Method 6 may be used to derive a guardbanding formula that results in a 2% PFA with some fixed amount of bias.

## 2. Derivation

The global PFA is calculated using a combination of the process distribution and the measurement uncertainty. For normal distributions, PFA is given by

$$PFA = \int_{-\infty}^{-T} \left( \int_{-A}^{A} \frac{1}{u_m\sqrt{2\pi}} e^{-\frac{1}{2u_m^2}(y-t)^2} dy \right) \frac{1}{u_0\sqrt{2\pi}} e^{-\frac{1}{2u_0^2}(t-y_0)^2} dt + \int_{T}^{+\infty} \left( \int_{-A}^{A} \frac{1}{u_m\sqrt{2\pi}} e^{-\frac{1}{2u_m^2}(y-t)^2} dy \right) \frac{1}{u_0\sqrt{2\pi}} e^{-\frac{1}{2u_0^2}(t-y_0)^2} dt$$

$T$ is the tolerance limit (assumed symmetric about zero), and $A$ is the guardbanded acceptance limit, where a Guardband Factor (GBF) is defined as $GBF = A/T$. The $u_m$ parameter is the standard measurement uncertainty and $u_0$ is the standard deviation of the process distribution. The $y_0$ term is the center of the process distribution, which captures the process bias. This equation is typically written with $y_0 = 0$ and thus assumes no bias is present.

In the typical unbiased case, $u_0$ is often estimated from

$$u_0 = \frac{T}{\Phi^{-1}\left(\frac{1+ITP}{2}\right)}$$

where ITP is the observed ITP found by sampling the process and $\Phi^{-1}$ is the inverse normal distribution function [4]. However, this calculation also assumes the distribution is unbiased. The ITP will have different contributions from out-of-tolerances above and below the limits. With a bias, ITP becomes:

$$ITP = \Phi(-T; y_0, u_0) + (1 - \Phi(\mathrm{T}; y_0, u_0))$$

where $\Phi(x; y_0, u_0)$ is the normal probability density function with mean $y_0$and standard deviation $u_0$ evaluated at $x$. Now it is not as easy to relate the observed ITP to a $u_0$ in the PFA calculation. Additionally, if the bias is 100% of the tolerance, the ITP is fixed at 50%, regardless of $u_0$. For these reasons, this derivation finds the worst-case PFA across a range of $u_0$, rather than a range of ITP when using Method 6, but because there is still a 1:1 relationship between $\Phi$ and $u_0$, the result will be the same.

The derivation of a guardband factor curve that works under bias is identical to the process used for Method 6 in Ref. [2], but does not assume that $y_0 = 0$. Determine $y_0$ by choosing a maximum allowable bias as a percent of $T$. Then, for a given TUR, determine $u_m$ and calculate PFA across the range of $u_0$ (or ITP). Find the $u_0$ that results in the worst-case PFA for this TUR, then calculate the guardband factor that brings the PFA down to 2%. This analysis must be completed with numerical minimization techniques, but when repeated over a range of TURs, results in a GBF versus TUR curve that ensures a 2% PFA for the given bias.

Guardband curves for bias values up to 90% are shown in Figure 4. The 0% bias curve is identical to Method 6. Interestingly, the 0%, 25%, and 50% curves are nearly identical (and are indistinguishable in this plot), meaning that Method 6 works well for biases up to 50%. This aligns with the conclusions in Ref. [5] that traditional guardbanding works adequately under modest bias.

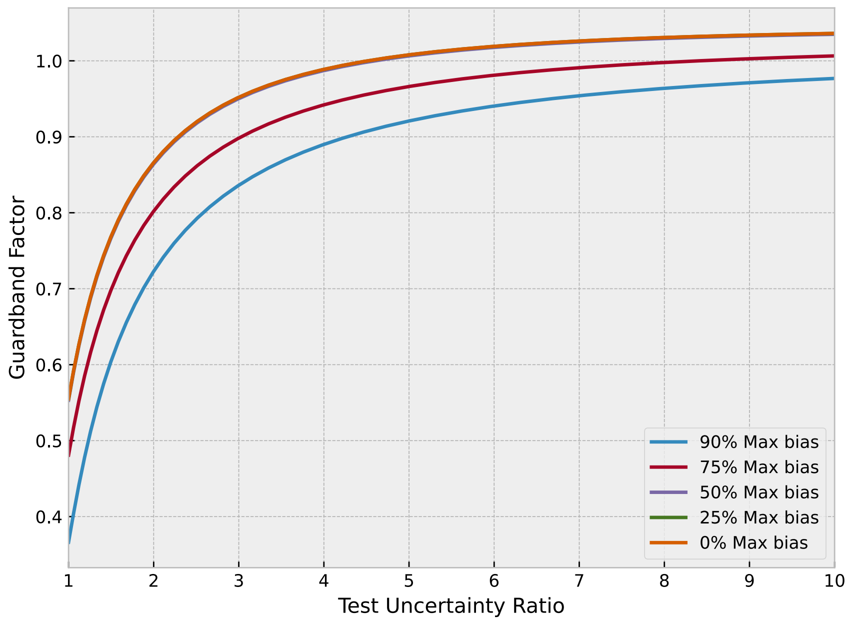


*Figure 4. Guardband factor curves that result in worst-case PFA of 2% under different bias conditions.*

Figure 5 illustrates the guardbanding curve when applied to a process distribution with 75% bias. The 75% maximum bias guardband curve is used to determine GBF for each TUR and is effective for bringing the PFA below 2% in all ITP conditions.

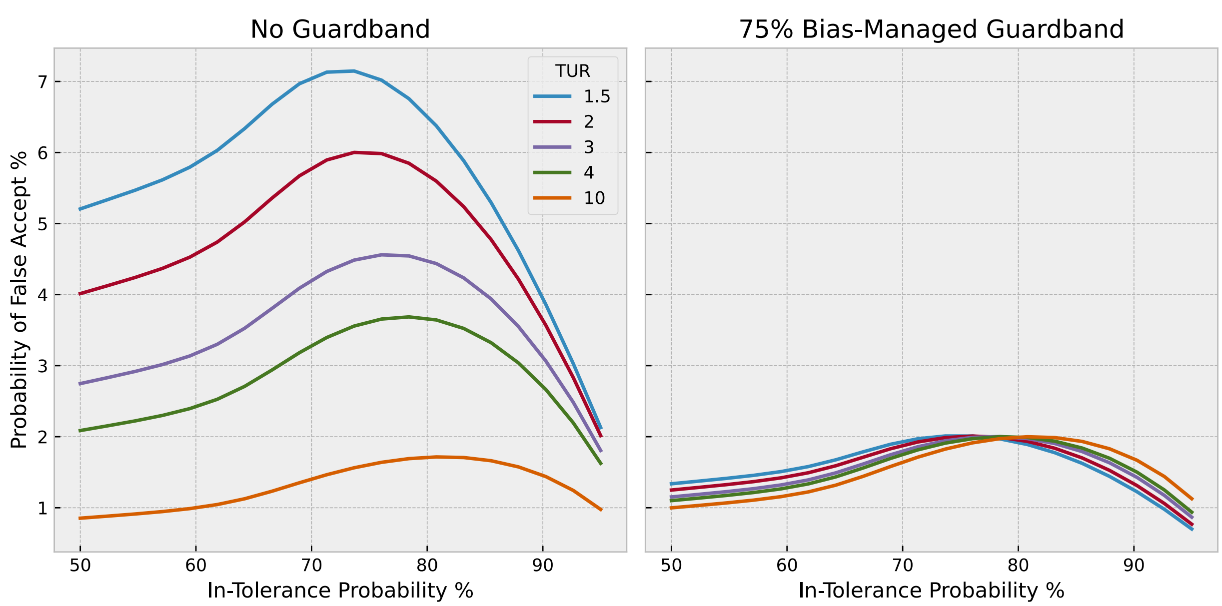


*Figure 5. PFA with 75% process bias and bias-managed guardband factor to control up to 75% process bias.*

However, it is unrealistic to apply a guardband that accounts for large bias to all situations. This would be overly conservative and lead to a high rate of false rejects. It is reasonable to expect that as the TUR increases, a larger bias could occur while maintaining a reasonable ITP; whereas if a large bias was present with lower TURs, it would be evident by a large number of nonconforming UUTs. A worst-case bias inversely dependent on TUR is proposed to address the higher risk for bias as TURs increase. The proposed max bias is 100% minus the measurement uncertainty expressed as a percent of the tolerance. Equivalently,

$$bias_{max} = 1 - \frac{1}{TUR}.$$

This value was chosen to provide a reasonable tradeoff between false accept and reject and prevents the false reject probability from exceeding about 7% under a 90% ITP condition.

The guardband factor versus TUR curve was rederived using this maximum bias as a function of TUR, resulting in Figure 6, where the curve is shown alongside Method 6 and the RSS method curves. This method always generates a GBF less than 1, unlike Method 6, which can result in GBF greater than 1 that sets acceptance limits outside the tolerance limits.

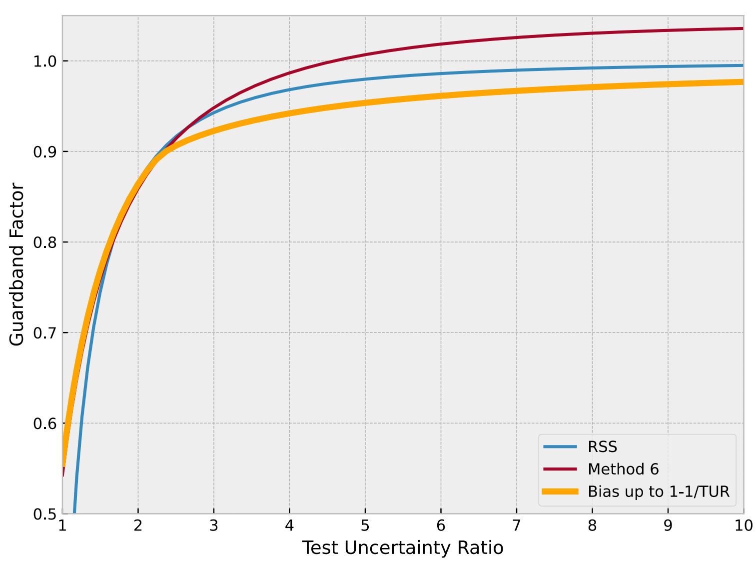


*Figure 6. Comparison of guardbanding curves.*

When applied to various TUR and ITP conditions under a bias of $1 - 1/TUR$, Figure 7 shows the results with this method keeping PFA < 2% for all cases.

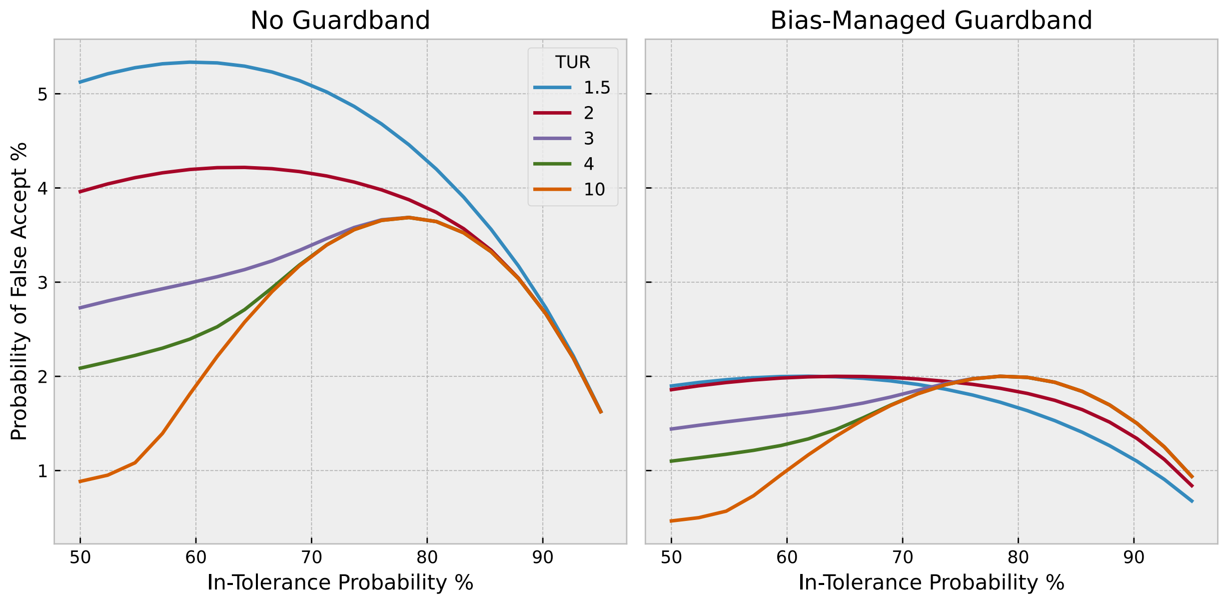


*Figure 7. PFA under bias-managed guardband with process bias of 1-1/TUR.*

## 3. Equation

In the previous section, the GBF versus TUR curve was numerically interpolated to find the guardband at each point. To be useful in realistic applications, an analytical function should be fit to the data, so the GBF may easily be calculated for any value of TUR. Through empirical and numerical curve fitting techniques, a reasonable fit was found using the guardband formula:

$$GBF = 1 - \frac{1.1}{5 \cdot TUR - 3}$$

This fit curve is shown in Figure 8. The numerator was adjusted from 1 to 1.1 to err on the conservative side and ensure 2% PFA is not exceeded based on minor curve fit residual errors and rounding of the coefficients. This guardband formula maintains 2% PFA or less for bias up to 1-1/TUR.

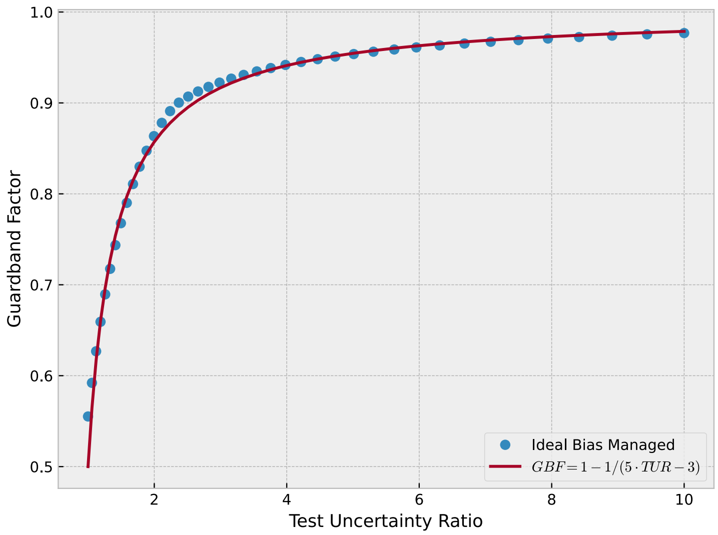


*Figure 8. Fitting a function to the ideal bias-managed guardband factor.*

Alternatively, fixed maximum biases can be used. To accommodate constant maximum bias values of 75% and 90%, guardband factor equations were derived as

- 75% maximum bias: $GBF = 1.04 - \exp\left(-1.24\ln(TUR) - 0.57\right)$
- 90% maximum bias: $GBF = 1.03 - \exp\left(-1.13\ln(TUR) - 0.40\right)$

using the same model form as the original Method 6 guardband equation.

## 4. Summary

When a bias is present in the UUT distribution, and that bias is greater than 50% of the tolerance, the PFA is adversely affected and traditional guardbanding methods are no longer adequate to ensure 2% PFA. The proposed guardbanding method, $GBF = 1 - 1.1/(5TUR - 3)$, ensures 2% PFA for biases up to 1-1/TUR.

This guardband method is ideal when a bias is known or suspected, or no assumptions can be made about the location of the product distribution, such as in a new operation. This method assures adequate PFA and works for all TURs above 1, including those above 4. However, if bias is known to be less than 50% of the tolerance, traditional guardbanding, such as Method 6 or RSS, remain adequate while avoiding potential over-guardbanding and unnecessarily increasing false rejects. Alternatively, if the UUT distribution is well-quantified, acceptance limits can be solved directly [6] using the PFA equation, without any reliance on TUR or ITP.

## Acknowledgement

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