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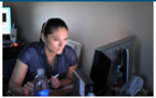
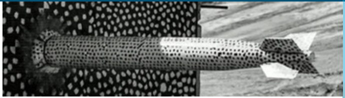
Block Preconditioning for Magnetic Confinement Fusion Relevant Resistive MHD Simulations





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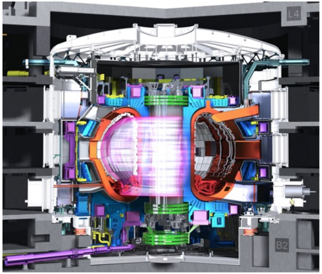
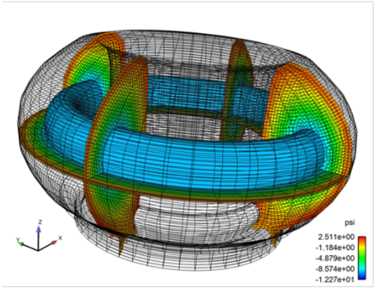
Tokamak Simulation

- Achieve temperatures of 100M deg K (6x Sun temp.)
- Energy confinement times $\mathcal{O}(1-10)$ min.



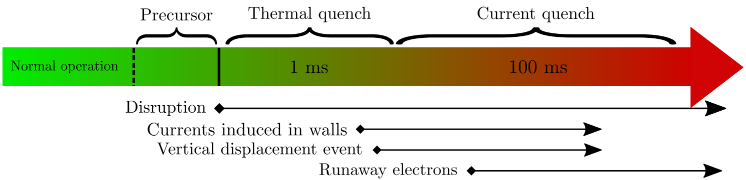


- Plasma disruptions can cause a breakdown of the magnetic field surface structure
  - loss of plasma confinement, plasma interacts with wall
  - huge thermal energy loss (thermal quench)
  - possible discharge of very large electrical currents (20MA) into structure
- ITER can sustain only a limited number of significant disruptions/instabilities

A vertical displacement event

Definition

Disruption event in Tokamak devices with sudden loss of plasma confinement and vertical movement towards wall.



1. Fast temperature drop $\Longrightarrow$ change in MHD equilibrium, $\mathbf{j} \times \mathbf{B} \approx 0$ $\Longrightarrow$ loss of vertical position control.
2. Temperature drop $\Longrightarrow$ resistivity increase $\Longrightarrow$ plasma current drop + ohmic to runaway current conversion.
3. Plasma current drop $\Longrightarrow$ magnetic field rearrangement, i.e. VDE.
4. VDE $\Longrightarrow$ Induce large electromagnetic force in the walls with halo current.

Compressible visco-resistive MHD

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \tag{1}$$

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot \left[(\rho \mathbf{u} \otimes \mathbf{u}) + pI - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})I - \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \right] - \mathbf{j} \times \mathbf{B} = \mathbf{0}, \tag{2}$$

$$\rho c_v \frac{\partial T}{\partial t} + \rho c_v \mathbf{u} \cdot \nabla T - \nabla \cdot (\kappa \nabla T) + p(\nabla \cdot \mathbf{u}) - \eta \|\mathbf{j}\|^2 + \gamma[T - T_0]_+ = 0, \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot \left[\mathbf{u} \otimes \mathbf{B} - \mathbf{B} \otimes \mathbf{u} - \frac{\eta}{\mu_0}(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \psi \right] = \mathbf{0}, \tag{4}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{5}$$

plus appropriate boundary conditions.

Discretization

- First order cG.
- VMS (convective & saddle point stabilization).
- DCO on equation (1) & (2).
- Lagrange multiplier for $\nabla \cdot \mathbf{B} = 0$.

Block Linear System

Newton linearized stabilized finite element discretization

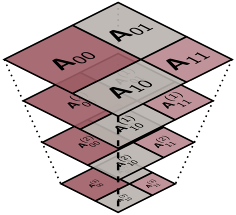
$$\begin{bmatrix} \mathbf{F_B} & \mathcal{B}_\mathbf{B}{}^T & \mathbf{Y}_\mathrm{nst} \\ \mathcal{B}_\mathbf{B} & \mathbf{L_r} & \mathbf{C}_\mathrm{nst} \\ \mathbf{Z} & & \mathbf{F}_\mathrm{nst} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{B} \\ \Delta \lambda \\ \Delta \mathbf{u}_{nst} \end{bmatrix} = \begin{bmatrix} -r_\mathbf{B} \\ -r_\lambda \\ -r_{\mathbf{u}_{nst}} \end{bmatrix}$$

- $\mathbf{F_B}$ - Magnetics terms
- $\mathbf{L_r}$ - Lagrange multiplier, VMS stabilization laplacian
- $\mathbf{F}_\mathrm{nst}$ - Momentum, Density, and Temperature terms

$$\mathbf{u}_{nst} = \begin{bmatrix} \mathbf{u} \\ \rho \\ T \end{bmatrix}$$

Results - Monolithic AMG

- Monolithic AMG preconditioned GMRES
  - Deal with elliptic diffusion operator stiffness
  - Not intended to deal with off-diagonal Alfven wave physics
- Relaxation: proc. based domain decomposition Schwarz
  - overlap 1 with ILU subsolve
- Increasing time step size, up to a multiple of Alfven CFL, $\mathrm{CFL}_a^{\max}$
  - $\mathrm{CFL}_a = \lambda\, \mathrm{d}t/h < \mathrm{CFL}_a^{\max}$
  - $\lambda = |\mathbf{u}| + |\mathbf{u}_A|$
  - $\mathbf{u}_A = |\mathbf{B}|/\sqrt{\rho\mu_0}$
- 663,984 dofs, 144 mpi ranks
- Linear solve to 10^{-12}, ensure correct physics



Results - Monolithic AMG

CFL_a^{max}	Linear Its. per non-Lin It.	Setup time per non-Lin It.	Solve time per non-Lin It.	Total Linear Time (Setup + Solve)
50	28.89	2.44	1.94	1909.15
100	75.01	2.43	4.97	2118.48
200	221.46	2.43	16.93	4493.42
400	236.16	2.45	18.34	5928.57

- Increasing iteration/solve time.
- Linear solve stagnates before we reach target CFL timescales
- Detrimental for scaling with mesh size

Operator Splitting

Approximately factor of 3×3 system into two 2×2 systems.

$$\mathcal{M}_{Split} = \begin{bmatrix} \mathbf{F_B} & & \mathbf{Y_{nst}} \\ & \mathbf{I} & \\ \mathbf{Z} & & \mathbf{F_{nst}} \end{bmatrix} \begin{bmatrix} \mathbf{F_B}^{-1} & & \\ & \mathbf{I} & \mathbf{C_{nst}} \\ & & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{F_B} & \mathcal{B}_\mathbf{B}^T & \\ \mathcal{B}_\mathbf{B} & \mathbf{L_r} & \\ & & \mathbf{I} \end{bmatrix}$$

- Groups magnetics and solenoid constraint
- Groups interaction between Lorenz force and convective term of magnetics
  - Develop Alfven wave propagation mode (fast hyperbolic time scale)

$$\mathcal{M}_{Split} = \begin{bmatrix} \mathbf{F_B} & \mathcal{B}_\mathbf{B}^T & \mathbf{Y_{nst}} \\ \mathcal{B}_\mathbf{B} & \mathbf{L_r} & \mathbf{C_{nst}} \\ \mathbf{Z} & \mathbf{Z}\,\mathbf{F_B}^{-1}\,\mathcal{B}_\mathbf{B}^T & \mathbf{F_{nst}} \end{bmatrix} \approx \mathcal{J} = \begin{bmatrix} \mathbf{F_B} & \mathcal{B}_\mathbf{B}^T & \mathbf{Y_{nst}} \\ \mathcal{B}_\mathbf{B} & \mathbf{L_r} & \mathbf{C_{nst}} \\ \mathbf{Z} & & \mathbf{F_{nst}} \end{bmatrix}$$

- Structural perturbation $\mathbf{Z}\,\mathbf{F_B}^{-1}\,\mathcal{B}_\mathbf{B}^T$ term is "small"

Cyr, Shadid, Tuminaro, Pawlowski, Chacón, *A new approximate block factorization preconditioner for two-dimensional incompressible (reduced) resistive mhd*, 2013.

Operator Splitting - Implementation

Block LU decomposition

$$\mathcal{M}_{Split}^{-1} \approx \left(\begin{bmatrix} \mathbf{F_B} & & \mathbf{Y_{nst}} \\ & \mathbf{I} & \\ \mathbf{Z} & & \mathbf{F_{nst}} \end{bmatrix} \begin{bmatrix} \mathbf{F_B}^{-1} & & \\ & \mathbf{I} & \\ & & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{F_B} & \mathcal{B}_\mathbf{B}{}^T & \\ \mathcal{B}_\mathbf{B} & \mathbf{L_r} & \\ & & \mathbf{I} \end{bmatrix} \right)^{-1}$$

$$\approx \left(\begin{bmatrix} \mathbf{I} & & \mathbf{Y_{nst}} \\ & \mathbf{I} & \\ & & \mathbf{F_{nst}} - \mathbf{Z}\,\mathbf{F_B}^{-1}\,\mathbf{Y_{nst}} \end{bmatrix} \begin{bmatrix} \mathbf{F_B} & \mathcal{B}_\mathbf{B}{}^T & \\ \mathcal{B}_\mathbf{B} & \mathbf{L_r} & \\ & & \mathbf{I} \end{bmatrix} \right)^{-1}$$

Murphy, Golub, Wathen, *A note on preconditioning for indefinite linear systems*, 2000.

Cyr, Shadid, Tuminaro, Pawlowski, Chacón, *A new approximate block factorization preconditioner for two-dimensional incompressible (reduced) resistive mhd*, 2013.

Operator Splitting - Implementation

Block LU decomposition

$$\mathcal{M}_{Split}^{-1} \approx \left(\begin{bmatrix} \mathbf{F_B} & & \mathbf{Y}_{\text{nst}} \\ & \mathbf{I} & \\ \mathbf{Z} & & \mathbf{F}_{\text{nst}} \end{bmatrix} \begin{bmatrix} \mathbf{F_B}^{-1} & & \\ & \mathbf{I} & \\ & & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{F_B} & \mathcal{B}_\mathbf{B}{}^T & \\ \mathcal{B}_\mathbf{B} & \mathbf{L_r} & \\ & & \mathbf{I} \end{bmatrix} \right)^{-1}$$

$$\approx \left(\begin{bmatrix} \mathbf{I} & & \mathbf{Y}_{\text{nst}} \\ & \mathbf{I} & \\ & & \mathbf{F}_{\text{nst}} - \mathbf{Z}\,\mathbf{F_B}^{-1}\,\mathbf{Y}_{\text{nst}} \end{bmatrix} \begin{bmatrix} \mathbf{I} & & \\ \mathcal{B}_\mathbf{B}\,\mathbf{F_B}^{-1} & \mathbf{I} & \\ & & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{F_B} & \mathcal{B}_\mathbf{B}{}^T & \\ & \mathbf{L_r} - \mathcal{B}_\mathbf{B}\,\mathbf{F_B}^{-1}\,\mathcal{B}_\mathbf{B}{}^T & \\ & & \mathbf{I} \end{bmatrix} \right)^{-1}$$

Murphy, Golub, Wathen, *A note on preconditioning for indefinite linear systems*, 2000.

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Operator Splitting - Inverses

$$\mathcal{M}_{Split}^{-1} = \left(\begin{bmatrix} \mathbf{I} & & \mathbf{Y}_{\text{nst}} \\ & \mathbf{I} & \\ & & S_{nst} \end{bmatrix} \begin{bmatrix} \mathbf{I} & & \\ \mathcal{B}_{\mathbf{B}} \mathbf{F_B}^{-1} & \mathbf{I} & \\ & & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{F_B} & \mathcal{B}_{\mathbf{B}}{}^T & \\ & S_L & \\ & & \mathbf{I} \end{bmatrix} \right)^{-1}$$

SIMPLE-type Schur complement approximation

$$\mathbf{F}_{\text{nst}} - \mathbf{Z}\,\mathbf{F_B}^{-1}\,\mathbf{Y}_{\text{nst}} \approx S_{nst} := \mathbf{F}_{\text{nst}} - \mathbf{Z}(\text{absrowsum}(\mathbf{F_B}))^{-1}\,\mathbf{Y}_{\text{nst}}$$

$$\mathbf{L_r} - \mathcal{B}_{\mathbf{B}}\,\mathbf{F_B}^{-1}\,\mathcal{B}_{\mathbf{B}}{}^T \approx S_L := \mathbf{L_r} - \mathcal{B}_{\mathbf{B}}(\text{absrowsum}(\mathbf{F_B}))^{-1}\,\mathcal{B}_{\mathbf{B}}{}^T$$

Need to compute the inverses for S_{nst}, S_L, and $\mathbf{F_B}$.

- S_{nst} is the primary Alfven term

Cyr, Shadid, Tuminaro, Pawlowski, Chacón, *A new approximate block factorization preconditioner for two-dimensional incompressible (reduced) resistive mhd*, 2013.

Results - Operator Splitting Block Precond.

Operator Splitting Block Preconditioner

- Preconditioned GMRES, using the Operator Splitting Block Preconditioner
- Inverses (S_{nst}, S_L, and $\mathbf{F_B}$) computed with AMG
  - Relaxation: proc. based domain decomposition Schwarz, overlap 1 with ILUT subsolve
  - ILUT subsolve, threshold=0.01, fill=1.75
- Increasing time step size, up to a multiple of Alfven CFL, $\mathrm{CFL}_a^{\max}$
  - $\mathrm{CFL}_a = \lambda \, \mathrm{d}t / h < \mathrm{CFL}_a^{\max}$
  - $\lambda = |\mathbf{u}| + |\mathbf{u}_A|$
  - $\mathbf{u}_A = |\mathbf{B}| / \sqrt{\rho \mu_0}$
- 626,832 dofs, 36 mpi ranks
- Linear solve to 10^{-12}, ensure correct physics

Results - Operator Splitting

- Lundquist number $S = 3 \times 10^3$

CFL_a^{max}	Linear Its. per non-Lin It.	Setup Time per non-Lin it.	Solve Time per non-Lin It.	Total Time
50	26.19	12.66	7.96	9895.41
100	30.44	13.80	9.49	6396.47
200	36.96	14.59	11.68	4589.73
400	48.79	15.18	15.59	3762.11
800	102.39	15.66	33.40	3806.38

- Expensive setup time (Schur complement S_{nst} many non-zeros)
- Near $CFL_a^{max} = 500$, fluids CFL becomes important

Block Linear System - Alt. Ordering

Newton linearized stabilized finite element discretization

$$\begin{bmatrix} \mathbf{F}_{\text{nst}} & \mathbf{Z} & \\ \mathbf{Y}_{\text{nst}} & \mathbf{F}_{\mathbf{B}} & {\mathcal{B}_{\mathbf{B}}}^T \\ \mathbf{C}_{\text{nst}} & \mathcal{B}_{\mathbf{B}} & \mathbf{L}_{\mathbf{r}} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{u}_{nst} \\ \Delta \mathbf{B} \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} -r_{\mathbf{u}_{nst}} \\ -r_{\mathbf{B}} \\ -r_{\lambda} \end{bmatrix}$$

- $\mathbf{F}_{\mathbf{B}}$ - Magnetics terms
- $\mathbf{L}_{\mathbf{r}}$ - Lagrange multiplier, VMS stabilization laplacian
- $\mathbf{F}_{\text{nst}}$ - Momentum, Density, and Temperature terms

$$\mathbf{u}_{nst} = \begin{bmatrix} \mathbf{u} \\ \rho \\ T \end{bmatrix}$$

Operator Splitting - Inverses - Alt. Ordering

$$\mathcal{M}_{Alt}^{-1} = \begin{bmatrix} \mathbf{I} & & \\ & \mathbf{F_B} & \mathcal{B}_\mathbf{B}{}^T \\ & & S_L \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{I} & & \\ & \mathbf{F_B} & \\ & -\mathcal{B}_\mathbf{B} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{F}_{\text{nst}} & \mathbf{Z} & \\ & S_{mag} & \\ & & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{I} & & \\ -\mathbf{Y}_{\text{nst}}\,\mathbf{F}_{\text{nst}}{}^{-1} & \mathbf{I} & \\ & & \mathbf{I} \end{bmatrix}$$

SIMPLE-type Schur complement approximation

$$\mathbf{F_B} - \mathbf{Y}_{\text{nst}}\,\mathbf{F}_{\text{nst}}{}^{-1}\,\mathbf{Z} \approx S_{mag} := \mathbf{F_B} - \mathbf{Y}_{\text{nst}}(\text{absrowsum}(\mathbf{F}_{\text{nst}}))^{-1}\,\mathbf{Z}$$

$$\mathbf{L_r} - \mathcal{B}_\mathbf{B}\,\mathbf{F_B}{}^{-1}\,\mathcal{B}_\mathbf{B}{}^T \approx S_L := \mathbf{L_r} - \mathcal{B}_\mathbf{B}(\text{absrowsum}(\mathbf{F_B}))^{-1}\,\mathcal{B}_\mathbf{B}{}^T$$

Need to compute the inverses for $\mathbf{F}_{\text{nst}}$, S_L, S_{mag} and $\mathbf{F_B}$.

- S_{mag} is the primary Alfven term

Results - Operator Splitting - Alt

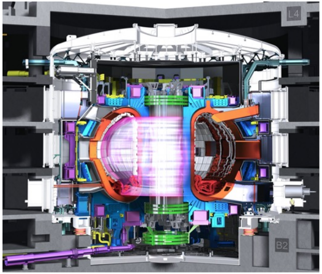
- Lundquist number $S = 3 \times 10^3$

$\text{CFL}_a^{\text{max}}$	Linear Its. per non-Lin It.	Setup Time per non-Lin it.	Solve Time per non-Lin It.	Total Time
50	33.84	7.27	6.49	6368.90
100	41.07	7.36	7.89	4081.69
200	51.64	7.46	9.96	2962.71
400	67.67	7.49	13.10	2435.92
800	95.25	7.54	18.67	2288.86

- Less expensive setup and solve due to sparser operators
- Higer initial linear iterations, better scaling with $\text{CFL}_a^{\text{max}}$

Future Work

- Refine mesh size
  - Resolve elliptic diffusion operator
  - Weak scaling
- Include off-diagonal flow/constraint coupling $\mathbf{C}_{nst}$ in block preconditioner
- Subblock solves and Relaxation
  - S_{nst}, S_{mag}, S_L, $\mathbf{F_B}$, $\mathbf{F}_{nst}$
- Heterogenous domain
  - Model magnetics outside of the plasma region



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