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# Coupled poro-elasto-plasticity of geomaterials: Simulation and validation

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# Outline

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# Introduction

- Modeling coupled processes is necessary to study sustainable subsurface energy activities, including carbon sequestration and geothermal recovery [1]
- Governing equations of poroelasticity: (1) and (2)
  - Fixed stress scheme to incorporate multiple modules of software, reduce computational cost [2]
- Sandia **Kayenta** [3] is a generalized plasticity model to include any form of inelastic material response, including quasi-brittle phenomena
  - Calibration with experimental data

$$\left( K + \frac{1}{3} G \right) \frac{\partial \epsilon_{kk}}{\partial x_i} + G \nabla^2 u_i = \alpha \frac{\partial p}{\partial x_i} - b_i \quad (1)$$

$$\alpha \frac{\partial \epsilon_{kk}}{\partial t} + S \frac{\partial p}{\partial t} = \nabla \cdot \left( \frac{k}{\mu} \nabla p \right) \quad (2)$$

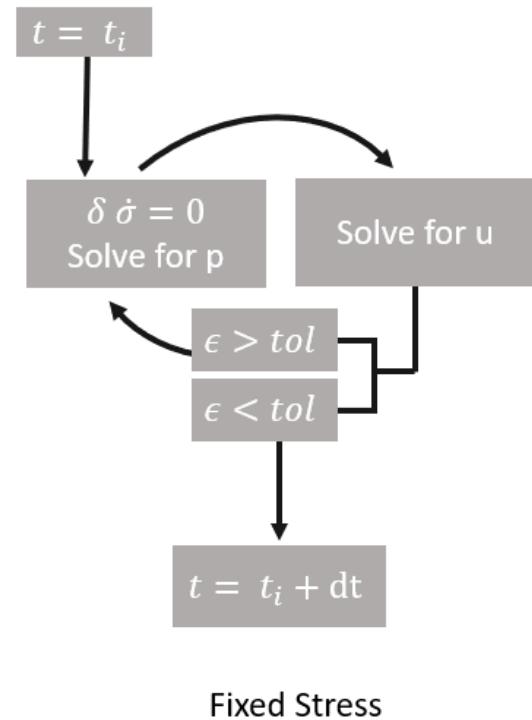
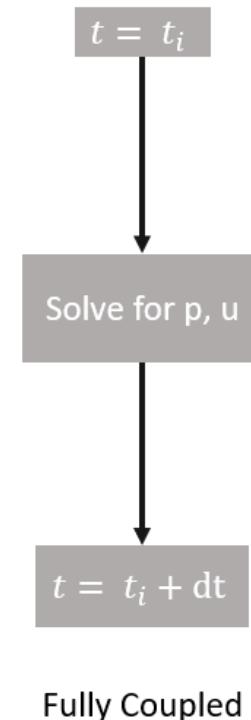


Choens et al., 2019 [10]

$K$  – bulk modulus  
 $G$  – shear modulus  
 $\epsilon_{kk}$  – volumetric strain  
 $x_i$  – coordinate reference frame  
 **$u$  – displacement**  
 $\alpha$  – Biot's coefficient  
 **$p$  – pore pressure**  
 $b_i$  – body forces  
 $t$  – time  
 $S$  – storativity  
 $k$  – intrinsic permeability  
 $\mu$  – fluid viscosity

# Methodology

- Sandia Sierra Multiphysics toolkit
  - Thermal/Fluid mechanics module: Sierra/Aria [4]
  - Solid mechanics module: Sierra/SM [5]
- Poromechanics problems (coupled)- Fixed stress scheme: set rate of total mean stress as constant from the solution at the previous iteration
  - Implement fixed stress scheme into Sierra/Aria and Sierra/SM using Sierra/Arpeggio [6]
- Solid mechanics problems (not coupled) - Sierra/SM [6]
- Verify implementation of plasticity through comparison with 1D and 2D analytical solutions [7,9]
- Extend to validation with experimental data of borehole breakout testing



Schematic of coupling schemes over a single time step. The fixed stress scheme iterates based on comparison of error,  $\epsilon$ , with tol, the global residual tolerance



# Kayenta Material Model [3]

- Constitutive model that generates a differentiable yield surface
- Models inelasticity, including phenomena such as microcracking, pore collapse
- Can be used to generate a simpler yield surface, such as von Mises, or calibrated to extensively experimental data
- Failure envelope:

$$F_f = a_1 - a_3 e^{-a_2 I_1} + a_4 I_1$$

Table 1. Mechanical and hydrological properties of geomaterials in simulations<sup>1,2</sup>

	1D Benchmark	2D Benchmark and wellbore breakout	
Material	Saline Aquifer [1]	Mancos shale, soft [10]	Mancos shale, stiff [10]
$\alpha$	1	-	-
$\phi_0$	0.15	-	-
$\nu$	0.2	0.2	0.2
$k$ (m <sup>2</sup> )	3.E-14	-	-
Yield function	Drucker-Prager	Tresca	Tresca
K (GPa)	1.11	13.4	168
G (GPa)	0.833	5.6	70
A1	6.12e6	160e6	160e6
A4	0.149	-	-

<sup>1</sup> All geomaterials in this work are modeled with isotropic material properties.

<sup>2</sup> For all materials, reference density of pore fluid is  $\rho=1$  g/cm<sup>3</sup>

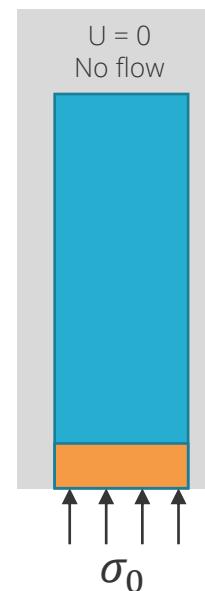
# Benchmark Problems for Verification

Evaluate through comparison with analytical solutions:

## 1. One-Dimensional (1D) Consolidation [7]

- Plasticity starts at the drainage boundary and proceeds towards the undrained end

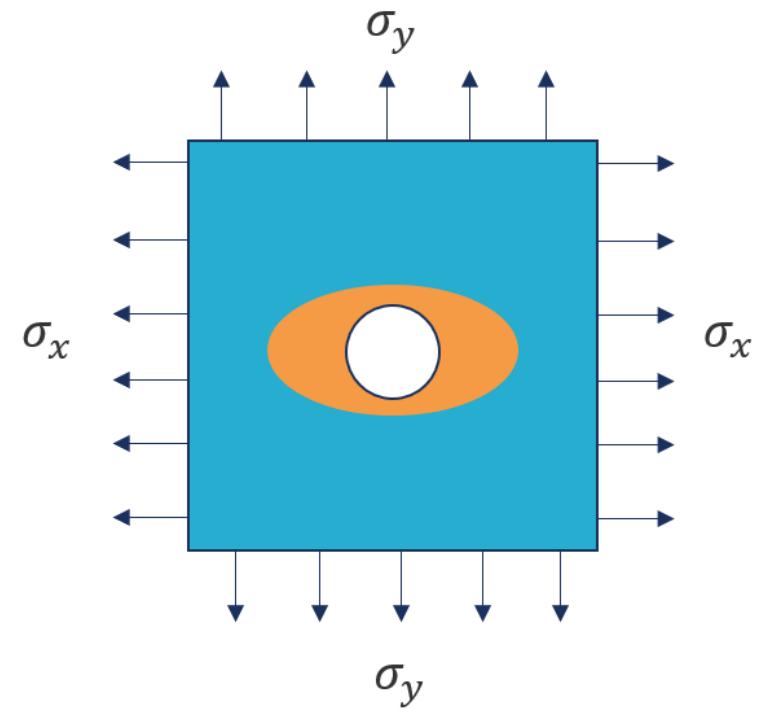
1D consolidation



## 2. Two-Dimensional (2D) Galin Plate [9]

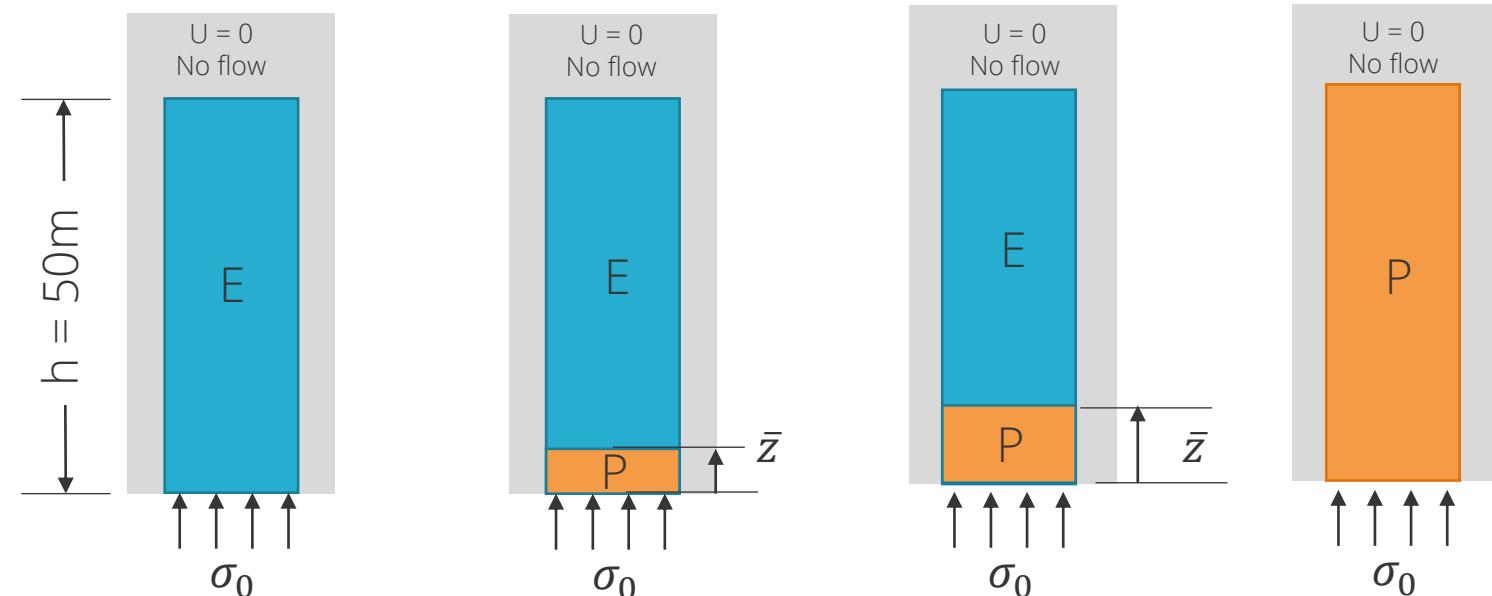
- Plasticity starts at the edges of the central hole and extends into the plate
- For the loading conditions in these analyses, plastic boundary is an ellipse

2D Galin Plate



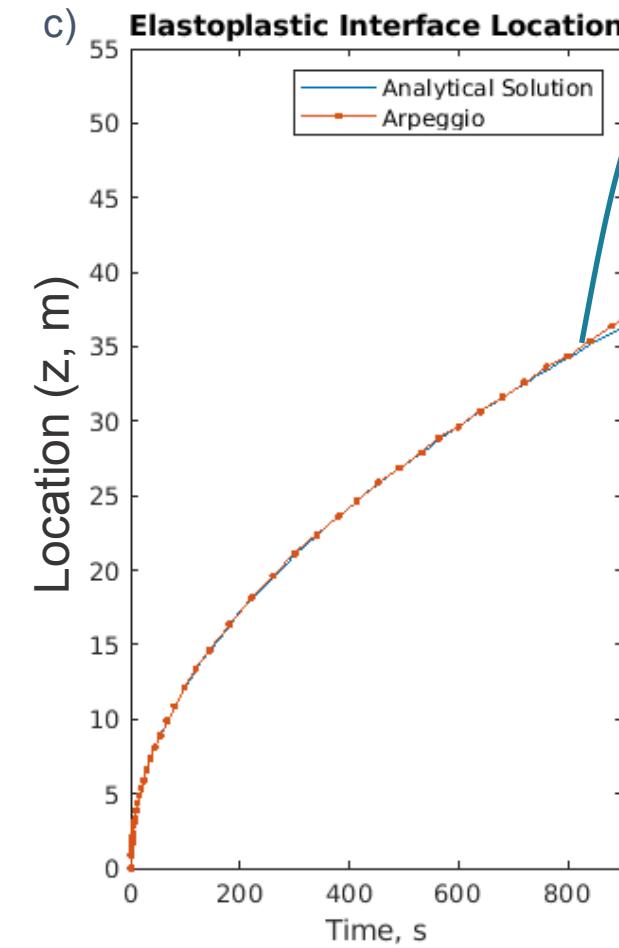
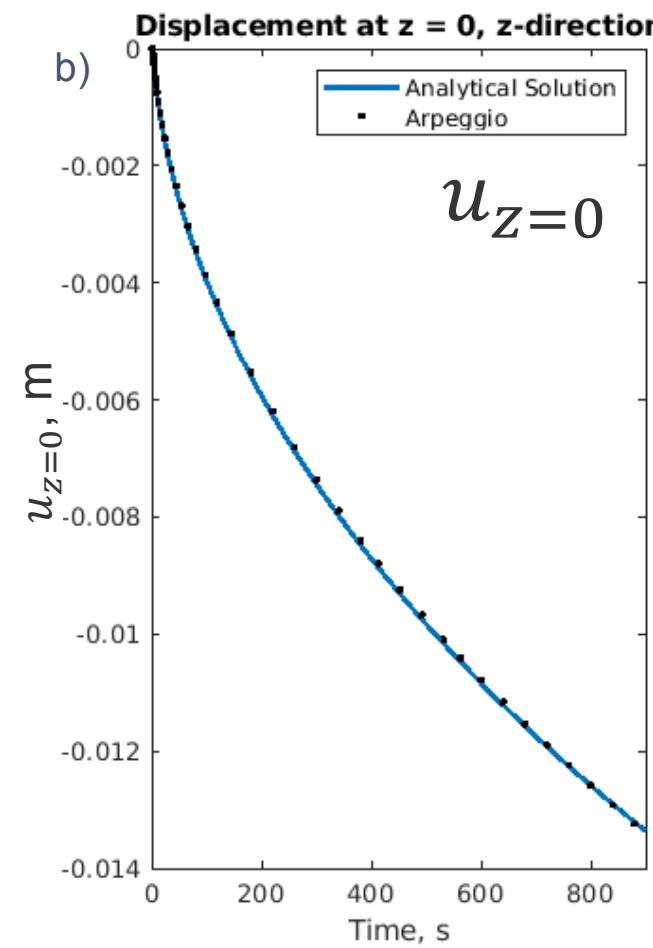
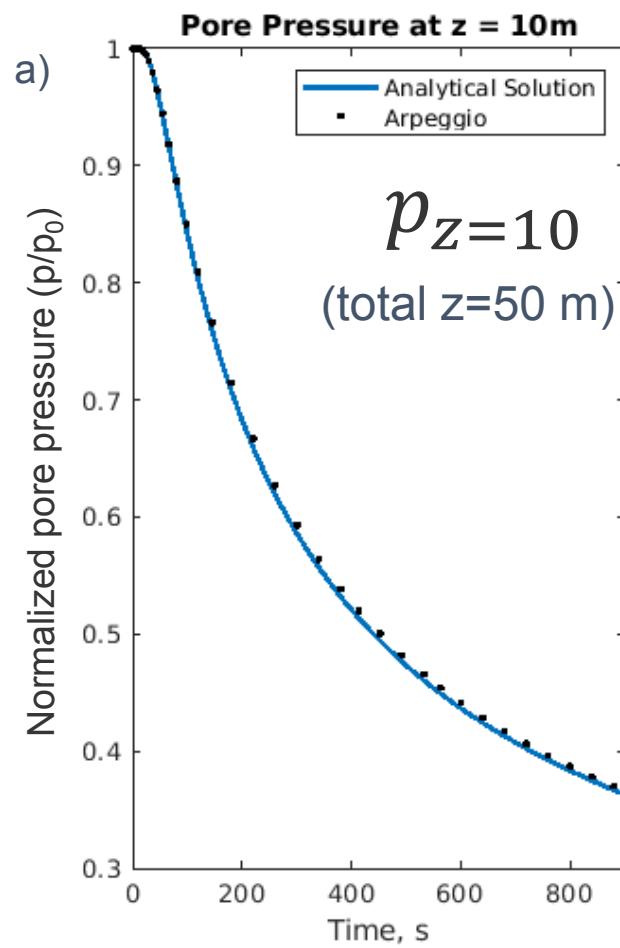
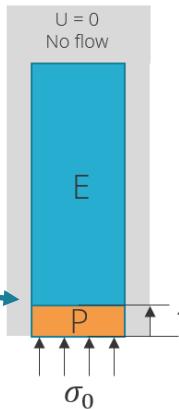
# 1D Elasto-plastic Consolidation

- Boundary and loading conditions
  - One-dimensional
    - Lateral displacement fixed at 0
    - Free z – displacement
  - Along lower boundary, pore fluid drainage
  - $\sigma_0 = 400MPa$ , elastoplastic
- Model details
  - Saline Aquifer material [1]
  - Drucker-Prager criteria
  - 300m height of column, 0.1m discretization in the z-direction
- Analytical solution from Liu et al. [6]
- Solution using Sierra/Arpeggio
  - Sierra/Aria for  $p$
  - Sierra/Solid Mechanics for  $u$



Schematic of 1-D Elastoplastic column, showing the plastic boundary,  $\bar{z}$ , as it gradually progresses along the column from the drainage boundary

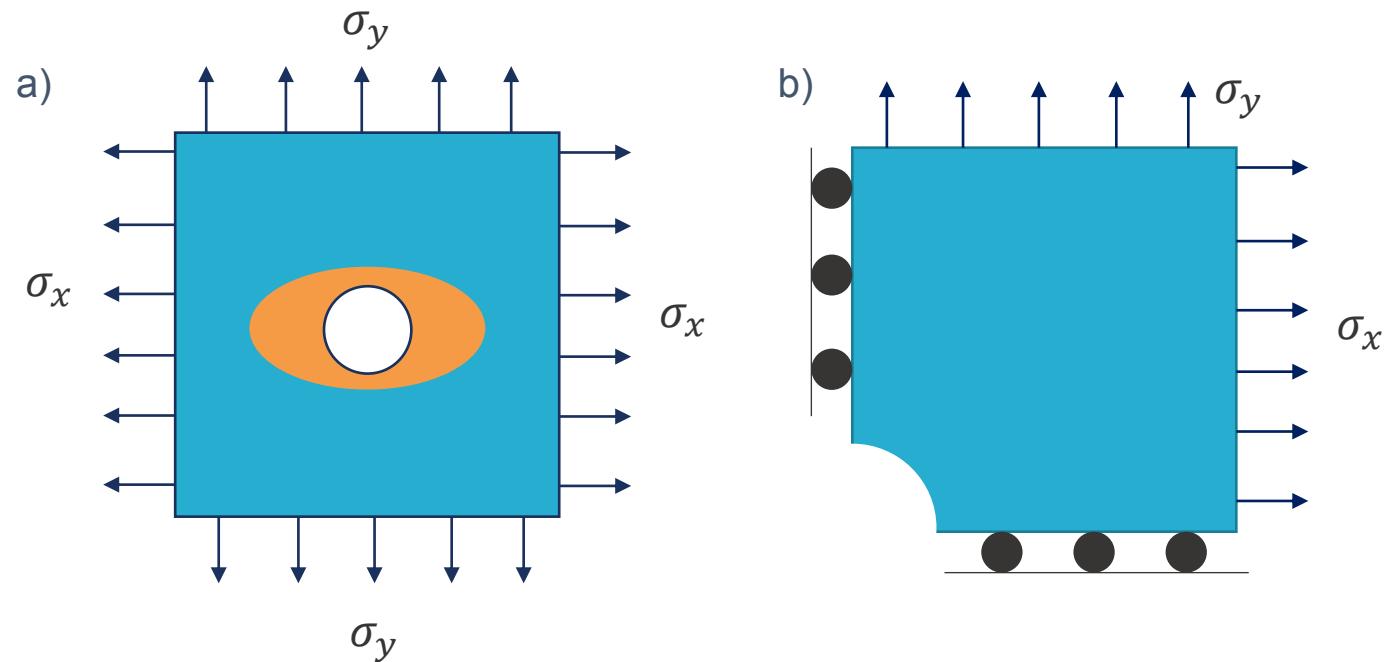
# Elasto-plastic, $\sigma_0 = 400MPa$



Plots comparing the Sierra/Arpeggio solution to the Analytical solution, a) Time history of the pore pressure at 10m from the drainage boundary, b) Time history of displacement at the drainage boundary ( $z = 0$ ), and c) Time history of the location of the elasto-plastic interface, showing its progression along the height of the column over time

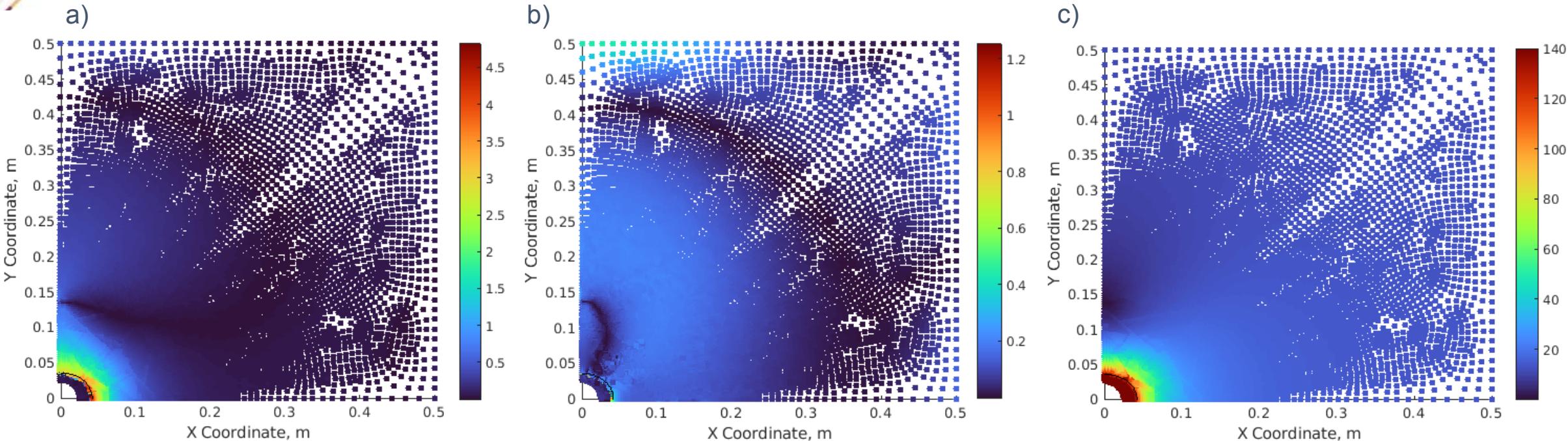
# 2D Galin Plate

- Problem information
  - $\sigma_x = -250 \text{ MPa}$  (Compression)
  - $\sigma_y = -275 \text{ MPa}$  (Compression)
  - Quarter model of  $1\text{m} \times 1\text{m}$  plate
  - Hole radius =  $0.025\text{m}$
  - Elliptical plastic zone
- Boundary conditions
  - Symmetric boundary conditions to model quarter of plate
  - Plane strain
- Modeling Details
  - Tresca Yield Criteria
  - 30,688 elements
- Analytical solution from Yarushina et al. [9]
- Sierra SM (no fluid flow)



a) Schematic of the Galin Plate problem, a 2D benchmark problem with plane strain conditions. The plastic zone forms around the central hole. For the given loading, the plastic zone will be in the shape of an ellipse. b) Schematic of the model used in this analysis, with symmetry boundary conditions imposed.

# Galin Plate - Comparison of Material Stiffness

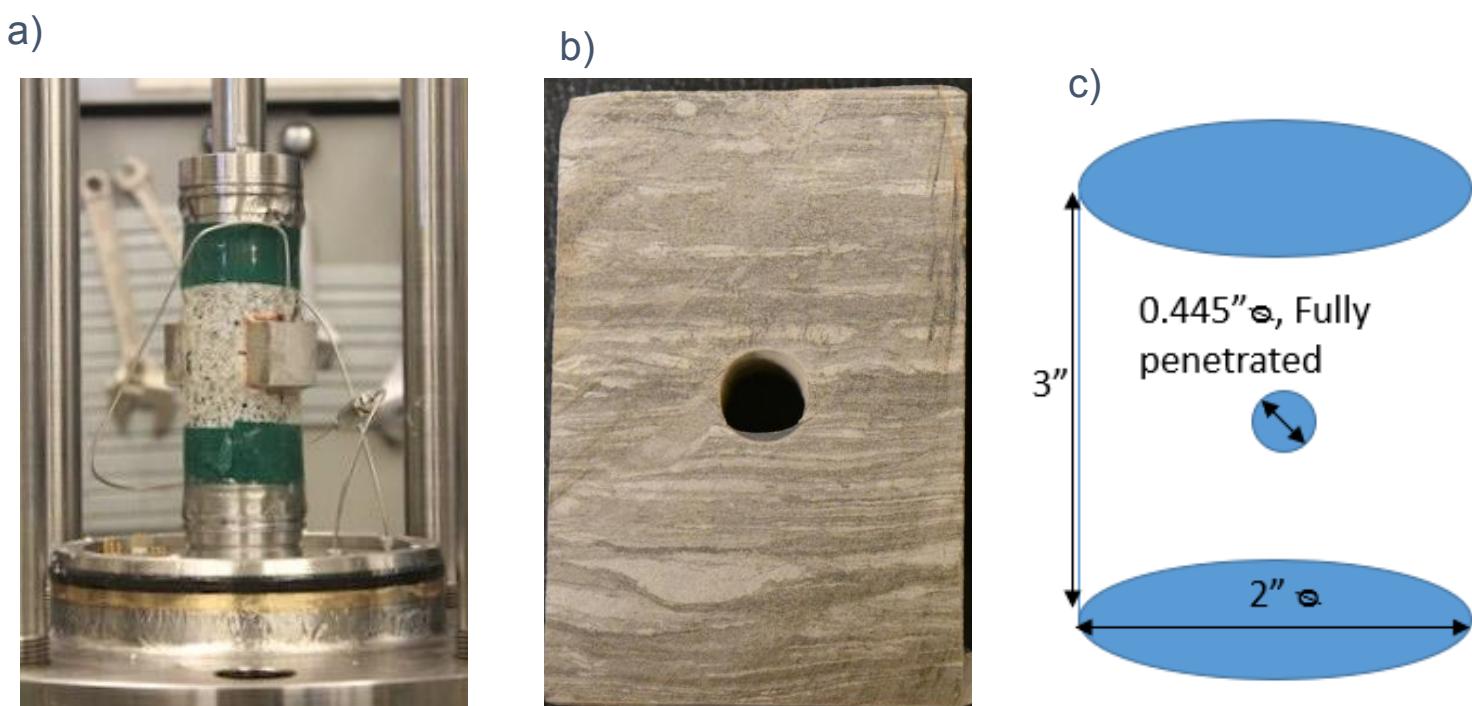


Scatter plots regarding maximum shear stress at the nodes, colored according to nodal magnitudes of a) percent error of soft material, b) percent error of stiff material, and c) analytical solution of maximum shear stress. In all three cases, the elliptical elasto-plastic boundary around the hole is shown with a black line.

- For both materials, the stresses in the plates computed with Sierra S/M closely match the analytical solution
  - The model with the stiffer material is more accurate than the softer material
- The largest error in the Sierra simulation is at the boundary of the elasto-plastic transition, and is large compared to the error within the rest of the plate

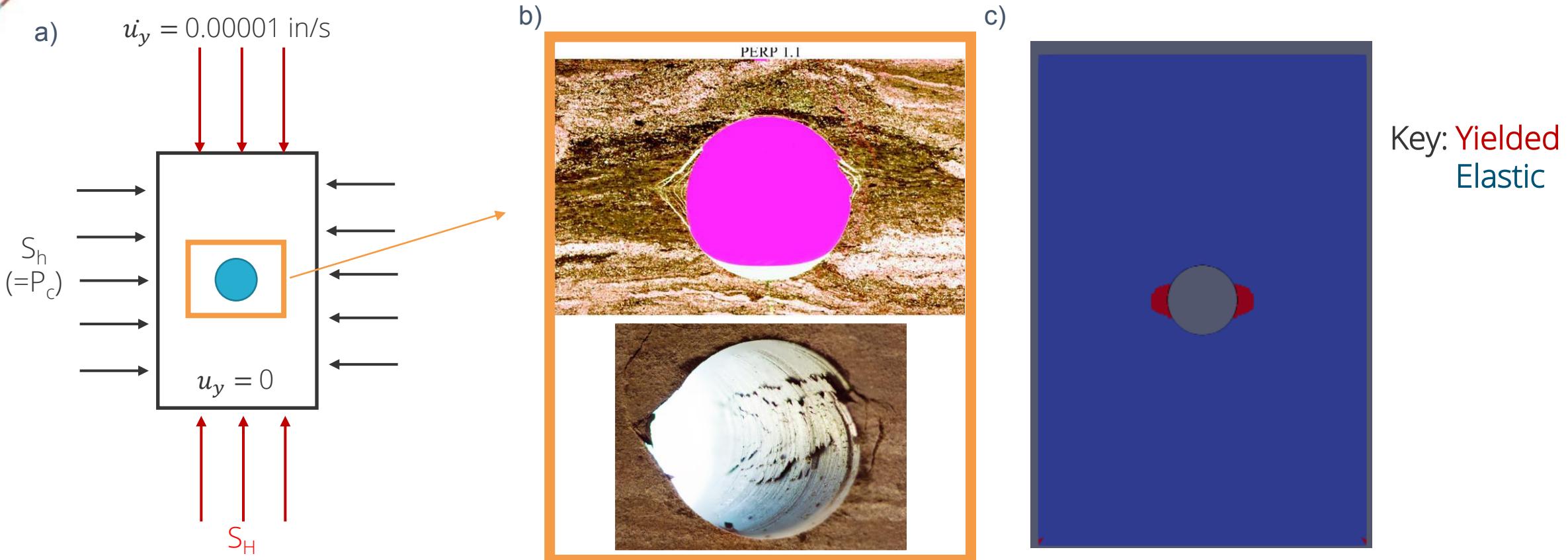
# Wellbore Breakout

- Problem information
  - $P_c = 2000 \text{ psi}$  (Compression)
  - Ambient pressure in hole
- Boundary conditions
  - Vertical deformation is displacement-controlled at  $u_y = 0.00001 \text{ in/s}$
  - 2D simplified model - plane strain
- Modeling Details
  - Tresca Yield Criteria
- Analytical solution from Yarushina et al. [9]
- Sierra SM (no fluid flow)



a) Photograph of wellbore breakout testing experimental setup from Choens et al., 2019 [10] b) Section of experimental specimen, c) schematic of experimental specimen, including dimensions

# Wellbore Breakout



a) 2D schematic of simplified model for wellbore breakout tests, b) Experimental results showing local damage at hole, from Choens et al. c) computational model results, with elements shaded according to localized yielding.

- Use simplified 2D model to qualitatively compare damage localized at hole
  - Agreement between the computational model (with Kayenta) and experimental results

# Conclusions and Future Work

- This work implemented plasticity into the fixed stress scheme for the Sandia Sierra Multiphysics toolkit
  - Accuracy of Kayenta constitutive model compared well with one-dimensional and two-dimensional benchmark problems
  - Larger errors are expected at areas with high stress concentrations and at boundaries
- Borehole breakout experiments were simulated with a simplified two-dimensional model using Sierra/SM
  - Localized damage patterns appear similar to experimental results, but further validation is necessary
- Future Work
  - Continue validation with wellbore breakout simulation, through comparison of stress-strain behavior
  - Develop computational modeling with explicit, meshed layers to simulate a variety of experimental orientations



Choens et al., 2019 [10]

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