Approximate Model of Light Transport in Scattering Media for Computational Sensing in Fog and Tissue

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I. Abstract

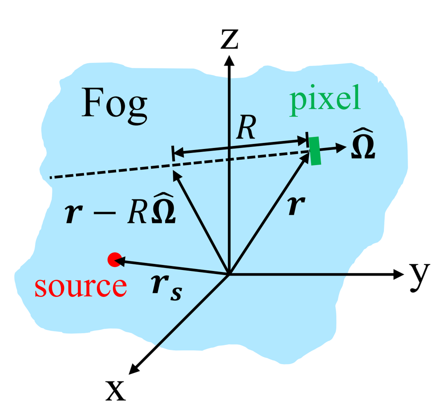
We present a computationally efficient approximate solution to the radiative transfer equation (RTE) that is applicable in weakly and diffuse scattering heterogeneous media. The model is in good agreement with experimental data acquired at the Sandia National Laboratory Fog Chamber Facility (SNLFC).

II. Theory

- For radiance I (W/m²/sr), absorption μ_a, scattering μ_s, and scattering phase function $f(\hat{\Omega}' \to \hat{\Omega})$, the RTE is [1,2,3]

$$\hat{\Omega} \cdot \nabla I(\boldsymbol{r}, \hat{\Omega}) + (\mu_a + \mu_s) I(\boldsymbol{r}, \hat{\Omega}) = \mu_s \int_{4\pi} d\hat{\Omega}' f(\hat{\Omega}' \to \hat{\Omega}) I(\boldsymbol{r}, \hat{\Omega}')$$

$$\hat{\Omega} \cdot \nabla I(\boldsymbol{r}, \hat{\Omega}) \exp[(\mu_a + \mu_s)\hat{\Omega} \cdot \boldsymbol{r}] + (\mu_a + \mu_s) I(\boldsymbol{r}, \hat{\Omega}) \exp[(\mu_a + \mu_s)\hat{\Omega} \cdot \boldsymbol{r}] = Q_s(\boldsymbol{r}, \hat{\Omega}) \exp[(\mu_a + \mu_s)\hat{\Omega} \cdot \boldsymbol{r}]$$



R is a distance along the line of sight, $\boldsymbol{r} = (x, y, z)$, and $\hat{\Omega}$ is a unit vector direction

$$\hat{\Omega} \cdot \nabla \{ I(\boldsymbol{r}, \hat{\Omega}) \exp[(\mu_a + \mu_s)\hat{\Omega} \cdot \boldsymbol{r}] \} = Q_s(\boldsymbol{r}, \hat{\Omega}) \exp[(\mu_a + \mu_s)\hat{\Omega} \cdot \boldsymbol{r}]$$

$$\hat{\Omega} \cdot \nabla \;\to\; \hat{\Omega} \cdot \frac{\partial}{\partial R} \hat{\Omega} \;\to\; \frac{\partial}{\partial R}$$

$$I(\boldsymbol{r}, \hat{\Omega}) \exp[(\mu_a + \mu_s)\hat{\Omega} \cdot \boldsymbol{r}] = \int_0^\infty dR\, Q_s(\boldsymbol{r} - R\hat{\Omega}, \hat{\Omega}) \exp[(\mu_a + \mu_s)\hat{\Omega} \cdot (\boldsymbol{r} - R\hat{\Omega})]$$

$$I(\boldsymbol{r}, \hat{\Omega}) = \int_0^\infty dR Q_s(\boldsymbol{r} - R\hat{\Omega}, \hat{\Omega}) \exp[-(\mu_a + \mu_s) R]$$

$$I(\boldsymbol{r}, \hat{\Omega}) = \int_0^\infty dR\, \mu_s \exp[-(\mu_a + \mu_s) R] \int_{4\pi} d\hat{\Omega}' f(\hat{\Omega}' \to \hat{\Omega}) I(\boldsymbol{r} - R\hat{\Omega}, \hat{\Omega}')$$

- Assume weak angular dependence: $I(\boldsymbol{r} - R\hat{\Omega}, \hat{\Omega}) \approx \frac{1}{4\pi}\phi(\boldsymbol{r} - R\hat{\Omega}) + \frac{3}{4\pi}\boldsymbol{J}(\boldsymbol{r} - R\hat{\Omega}) \cdot \hat{\Omega}$
- Where $\phi = \int_{4\pi} d\hat{\Omega}\, I(\boldsymbol{r}, \hat{\Omega})$ is the fluence rate and $\mathbf{J} = \int_{4\pi} d\hat{\Omega}\, \hat{\Omega}\, I(\boldsymbol{r}, \hat{\Omega})$ is the flux density (W/m²)

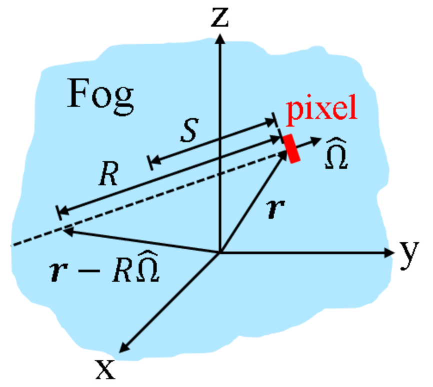
$$I(\boldsymbol{r}, \hat{\Omega}) = \int_0^\infty dR\, \mu_s \exp[-(\mu_s + \mu_a)R] \left[\frac{\phi(\boldsymbol{r} - R\hat{\Omega})}{4\pi} \int_{4\pi} d\hat{\Omega}' f(\hat{\Omega}' \to \hat{\Omega}) + \frac{3\boldsymbol{J}(\boldsymbol{r} - R\hat{\Omega})}{4\pi} \cdot \int_{4\pi} d\hat{\Omega}' \hat{\Omega}' f(\hat{\Omega}' \to \hat{\Omega}) \right]$$

$= 1$; $\approx g\hat{\Omega}$

$$I(\boldsymbol{r}, \hat{\Omega}) = \int_0^\infty dR\, \mu_s \exp[-(\mu_s + \mu_a)R] \left[\frac{1}{4\pi}\phi(\boldsymbol{r} - R\hat{\Omega}) + \frac{3g}{4\pi}\boldsymbol{J}(\boldsymbol{r} - R\hat{\Omega}) \cdot \hat{\Omega} \right]$$

anisotropic term, where g is the average cosine of the scattering angle

$$I(\boldsymbol{r}, \hat{\Omega}) = \int_0^\infty dR \left(\frac{\mu_s}{4\pi}\right) \exp[-(\mu_s + \mu_a)R] [\phi(\boldsymbol{r} - R\hat{\Omega}) + 3g\boldsymbol{J}(\boldsymbol{r} - R\hat{\Omega}) \cdot \hat{\Omega}]$$



S is a distance along the line of sight

- More generally, using the method of characteristics [1]:

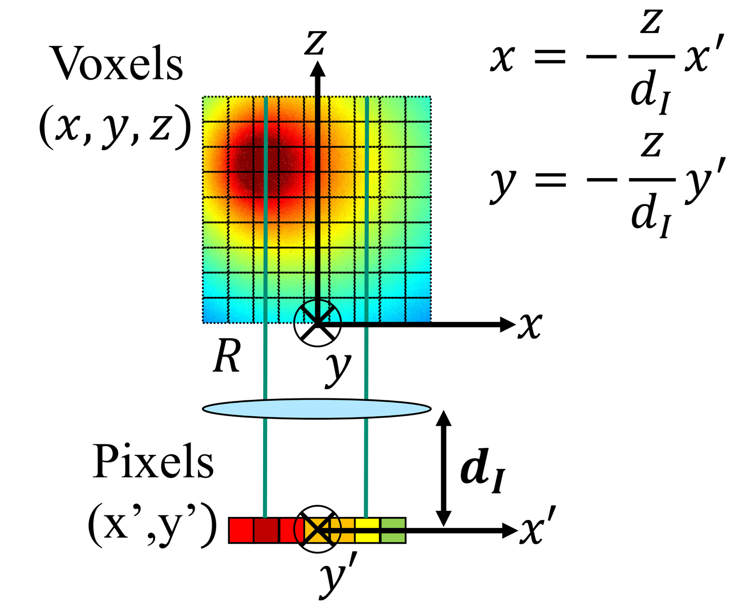
$$I(\boldsymbol{r}, t, \hat{\Omega}) = \int_0^\infty dR \frac{\mu_s(\boldsymbol{r} - R\hat{\Omega})}{4\pi} \exp[-\alpha(\boldsymbol{r}, \boldsymbol{r} - R\hat{\Omega})R] \left[\phi\left(\boldsymbol{r} - R\hat{\Omega}, t - \frac{R}{c}\right) + 3g\boldsymbol{J}\left(\boldsymbol{r} - R\hat{\Omega}, t - \frac{R}{c}\right) \cdot \hat{\Omega} \right]$$

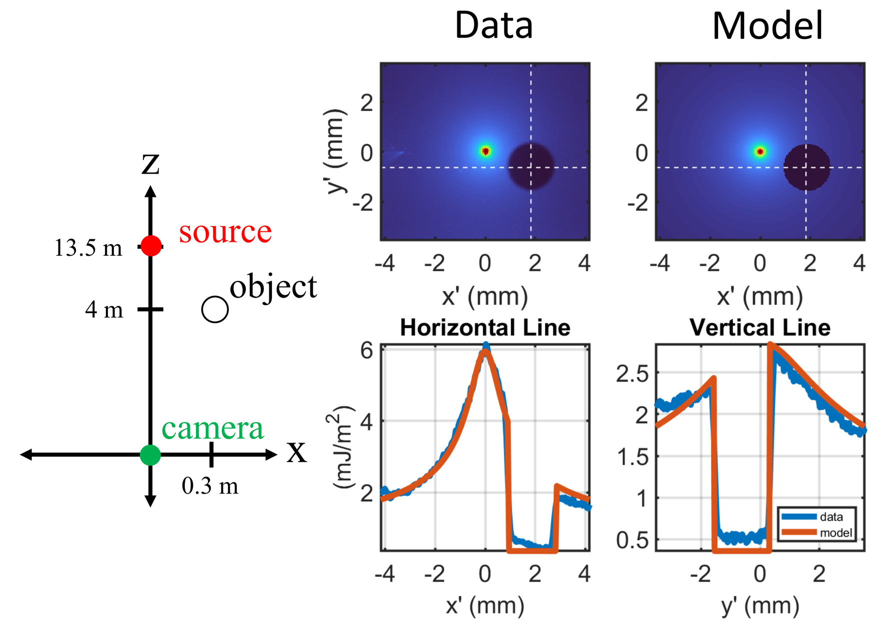
$$\alpha(\boldsymbol{r}, \boldsymbol{r}') = \int_0^R dS\, \mu_s\left(\boldsymbol{r} - S\frac{\boldsymbol{R}}{R}\right) + \mu_a\left(\boldsymbol{r} - S\frac{\boldsymbol{R}}{R}\right)$$

- Where α is the optical thickness accounting for spatial variations in μ_a and μ_s and $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'$
- Fluence rate (ϕ) and flux density (J) determined by diffusion equation

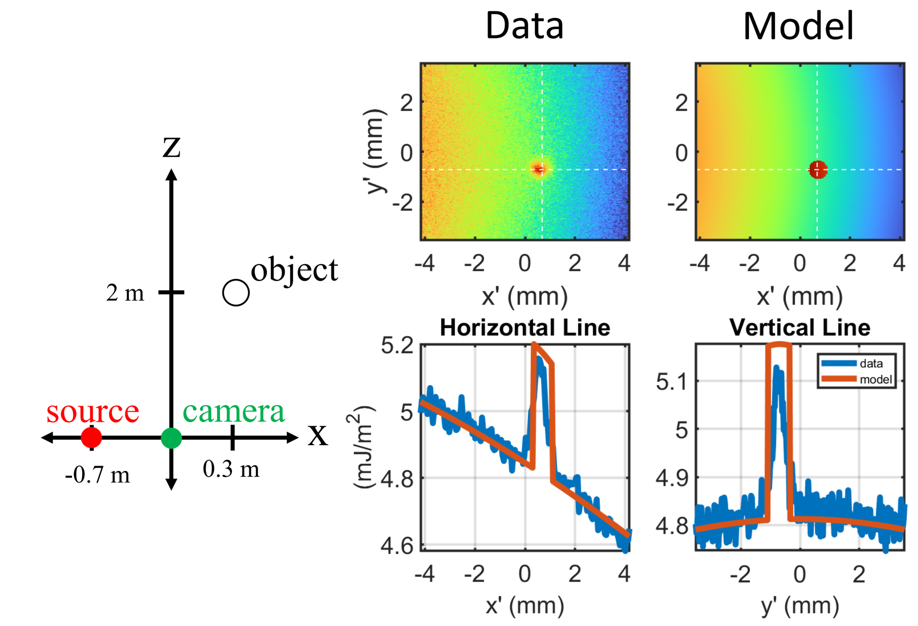
III. Applications

- Pinhole camera model allows validated with experimental data (transmission, reflection) [1,2]



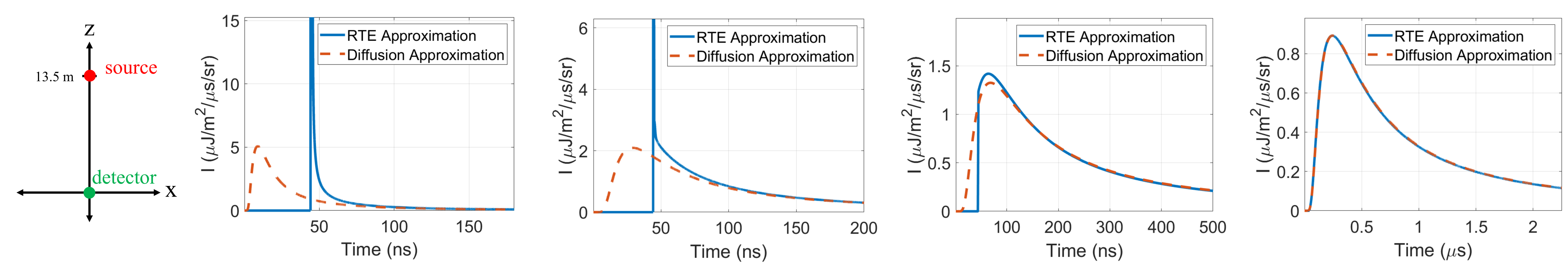


Measured Fog Optical Parameters:
$\mu_s = 0.24$ m⁻¹ $\mu_a = 0$ m⁻¹ $g = 0.8$



Measured Fog Optical Parameters:
$\mu_s = 1.0$ m⁻¹ $\mu_a = 0$ m⁻¹ $g = 0.8$

- Simulated homogeneous time-domain solution (13.5 m) shows improvement over diffusion equation



$\mu_s = 0.25$ m⁻¹ $\mu_a = 0$ m⁻¹ $g = 0.8$ | $\mu_s = 0.7$ m⁻¹ $\mu_a = 0$ m⁻¹ $g = 0.8$ | $\mu_s = 1.5$ m⁻¹ $\mu_a = 0$ m⁻¹ $g = 0.8$ | $\mu_s = 5$ m⁻¹ $\mu_a = 0$ m⁻¹ $g = 0.8$

IV. Conclusions

The proposed RTE approximation can have computational times comparable to the diffusion equation but describes light transport through weak scatter more accurately. In heavy scatter it matches the diffusion equation solutions. More work is needed to better account for the influence of singularities at the source positions. Code is available with [3].

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References

[1] G. I. Bell and S. Glasstone, "Nuclear Reactor Theory," US Atomic Energy Commission, Washington DC, (1970).

[2] B. Z. Bentz, B. J. Redman, J. D. van der Laan, K. Westlake, A. Glen, A. L. Sanchez, and J. B. Wright, *Optics Express,*. 29(9), 13231 (2021).

[3] B. Z. Bentz, C. A. Pattyn, J. D. van der Laan, B. J. Redman, A. Glen, A. L. Sanchez, K. Westlake, and J. B. Wright, *Optics Letters,*. 47(8), 2000 (2022).

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