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Improving Digital Twins by Learning from a Fleet of Assets

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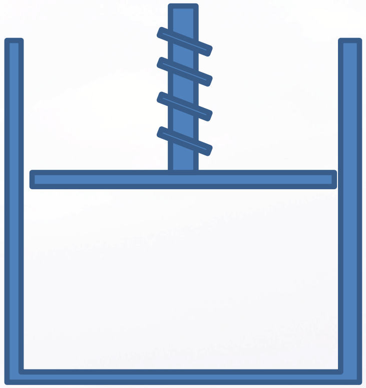
Motivation and Main Idea

Digital Twins

- Single asset:



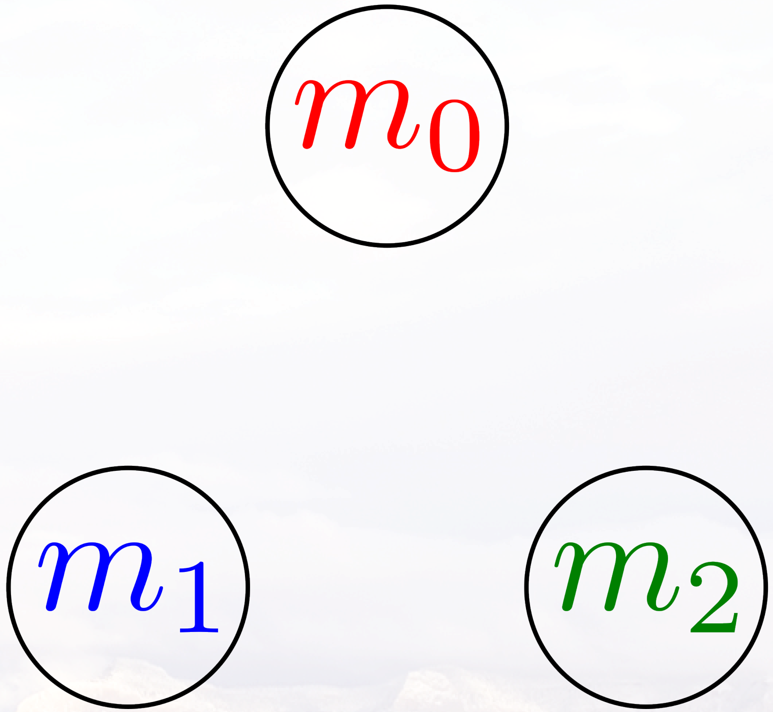
- Corresponding digital twin:

Fleet of Assets

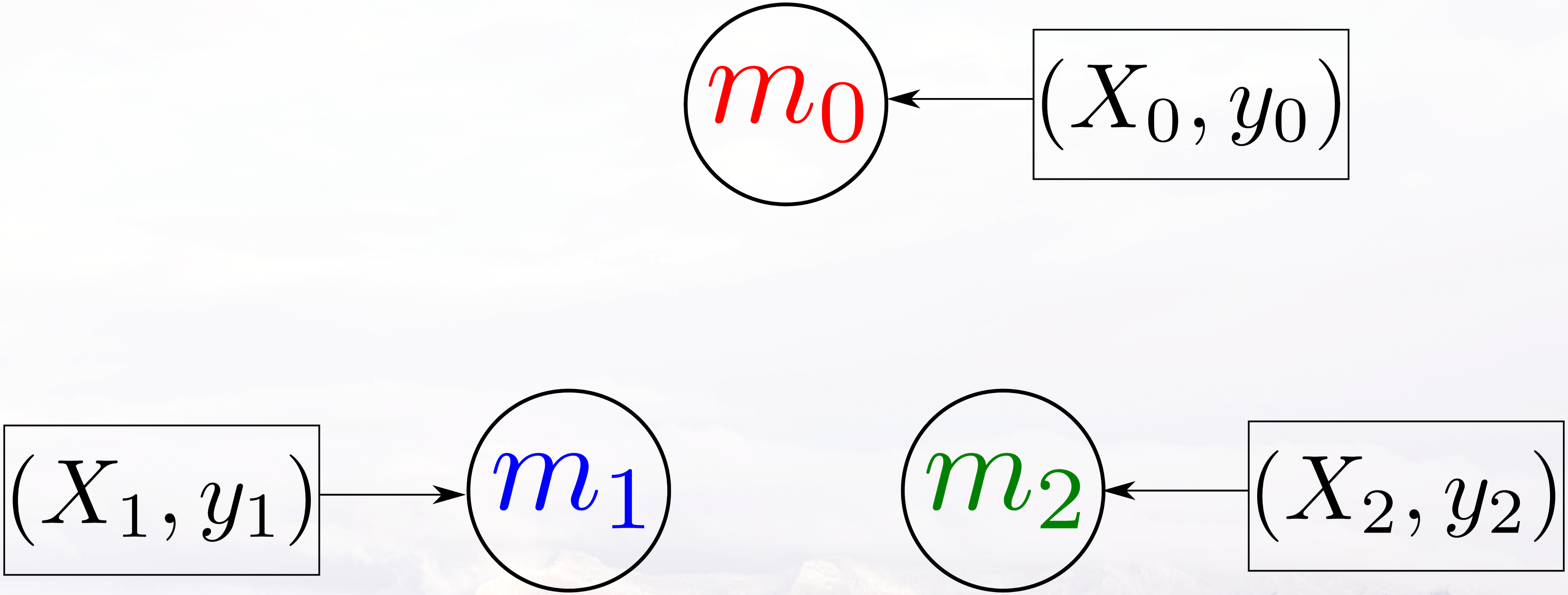
- Similar assets, different (unknown) operating conditions.

- Fuse information from the fleet to improve each individual digital twin.

Individual Digital Twins



Individual Digital Twins



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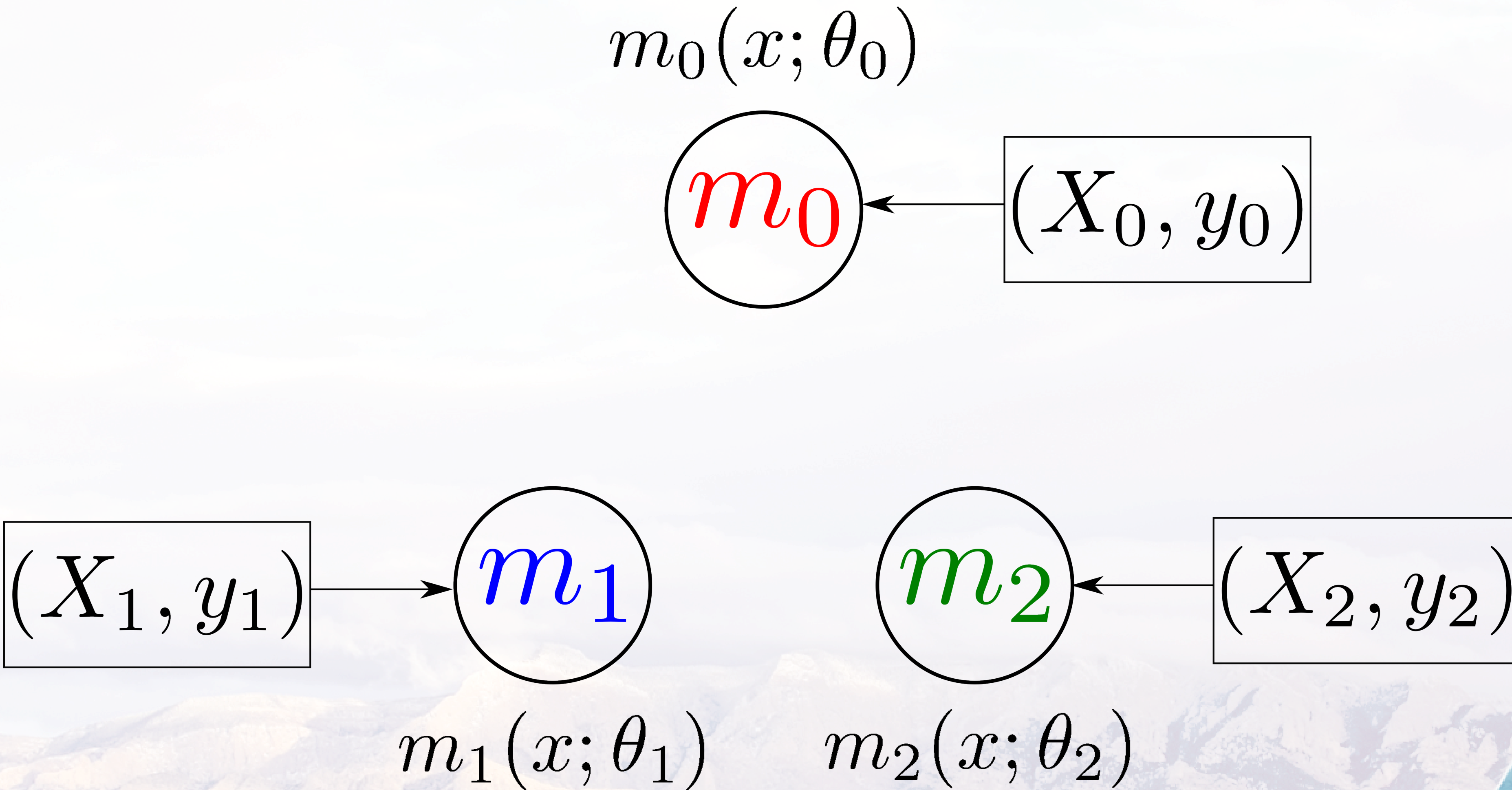
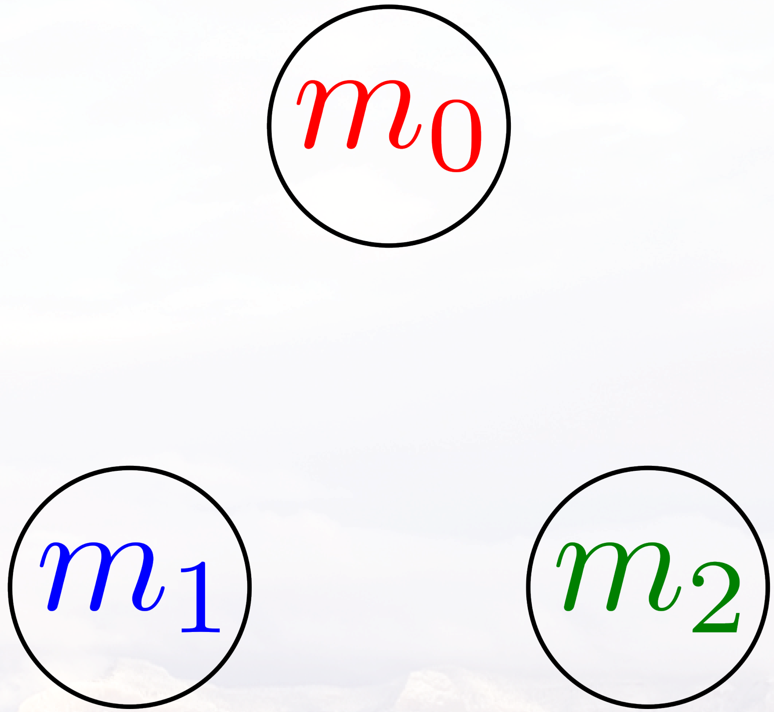


Network from the Fleet of Assets



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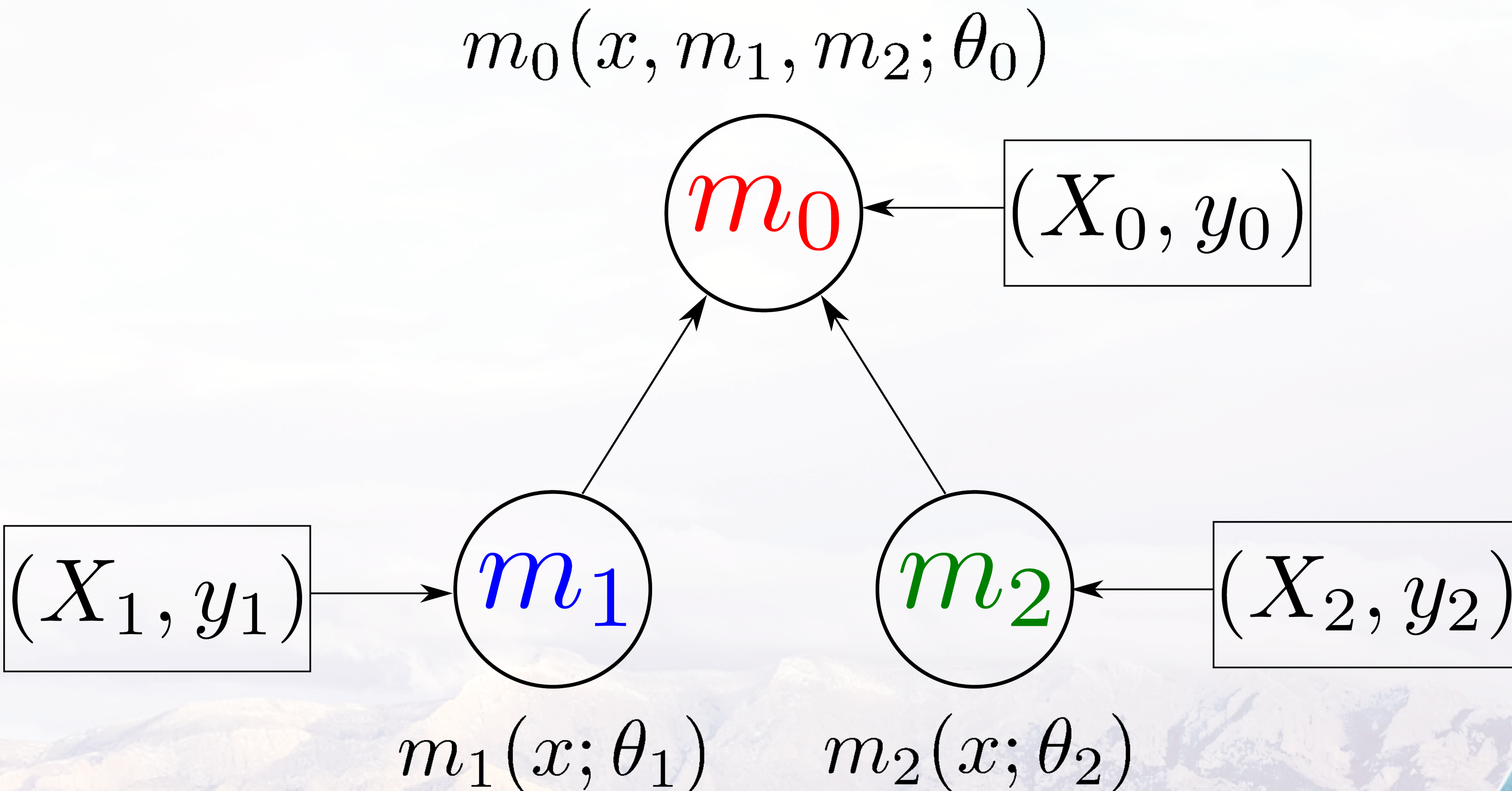

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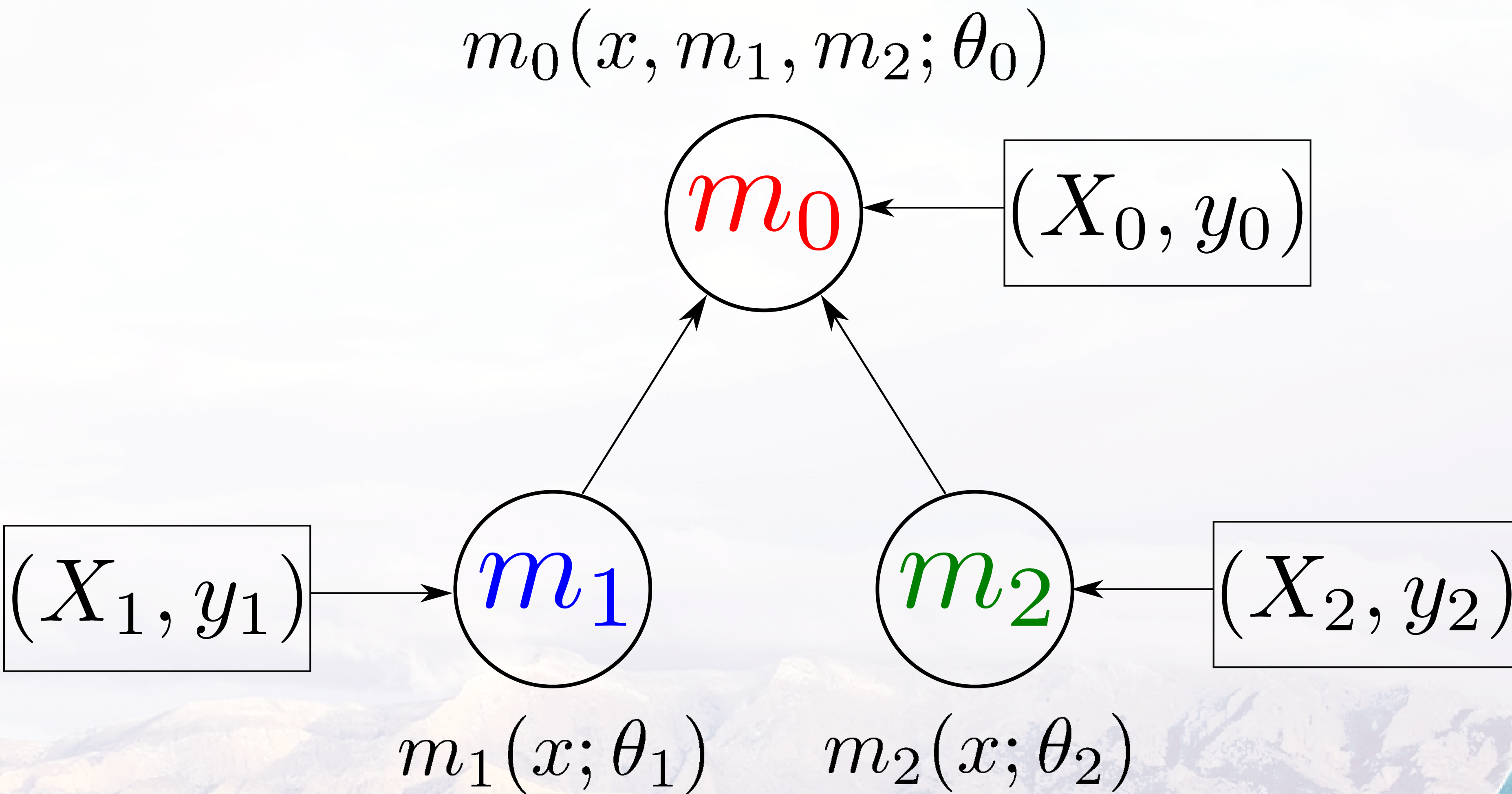
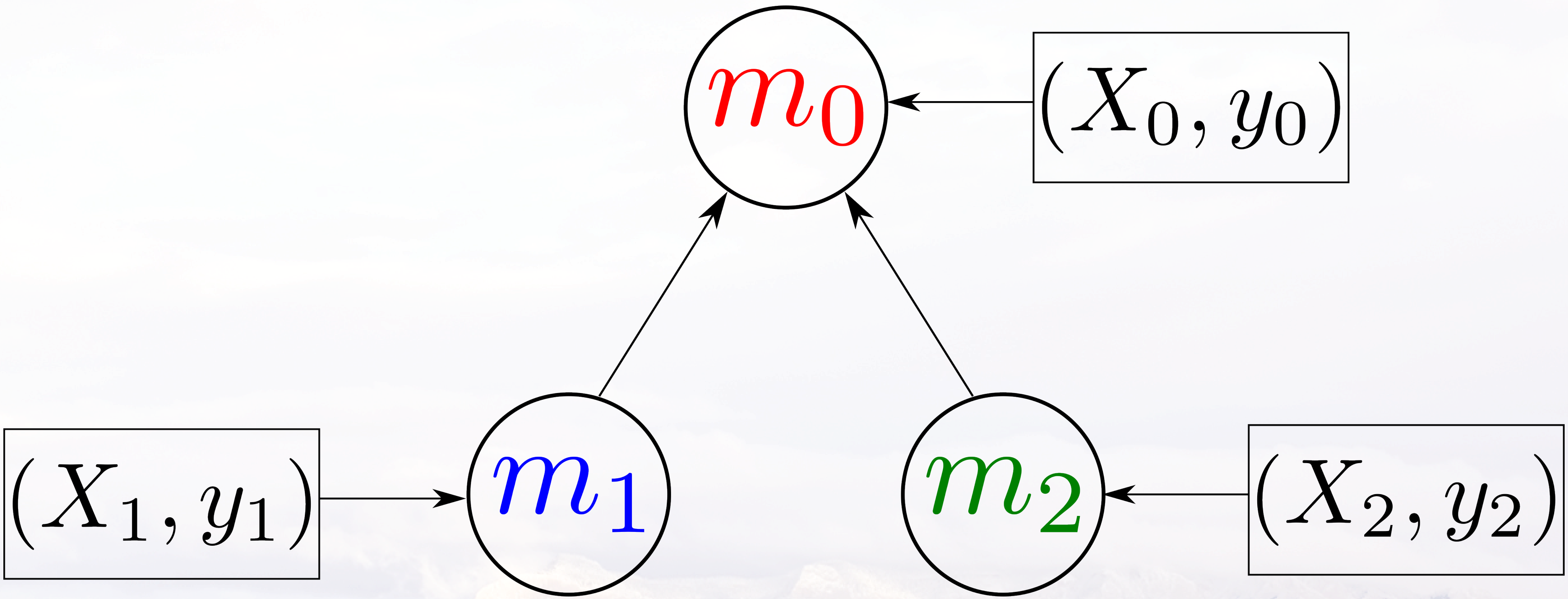


- We can repeat this procedure for any asset of interest (AoI) in the fleet.

Specialization to Gaussian Processes

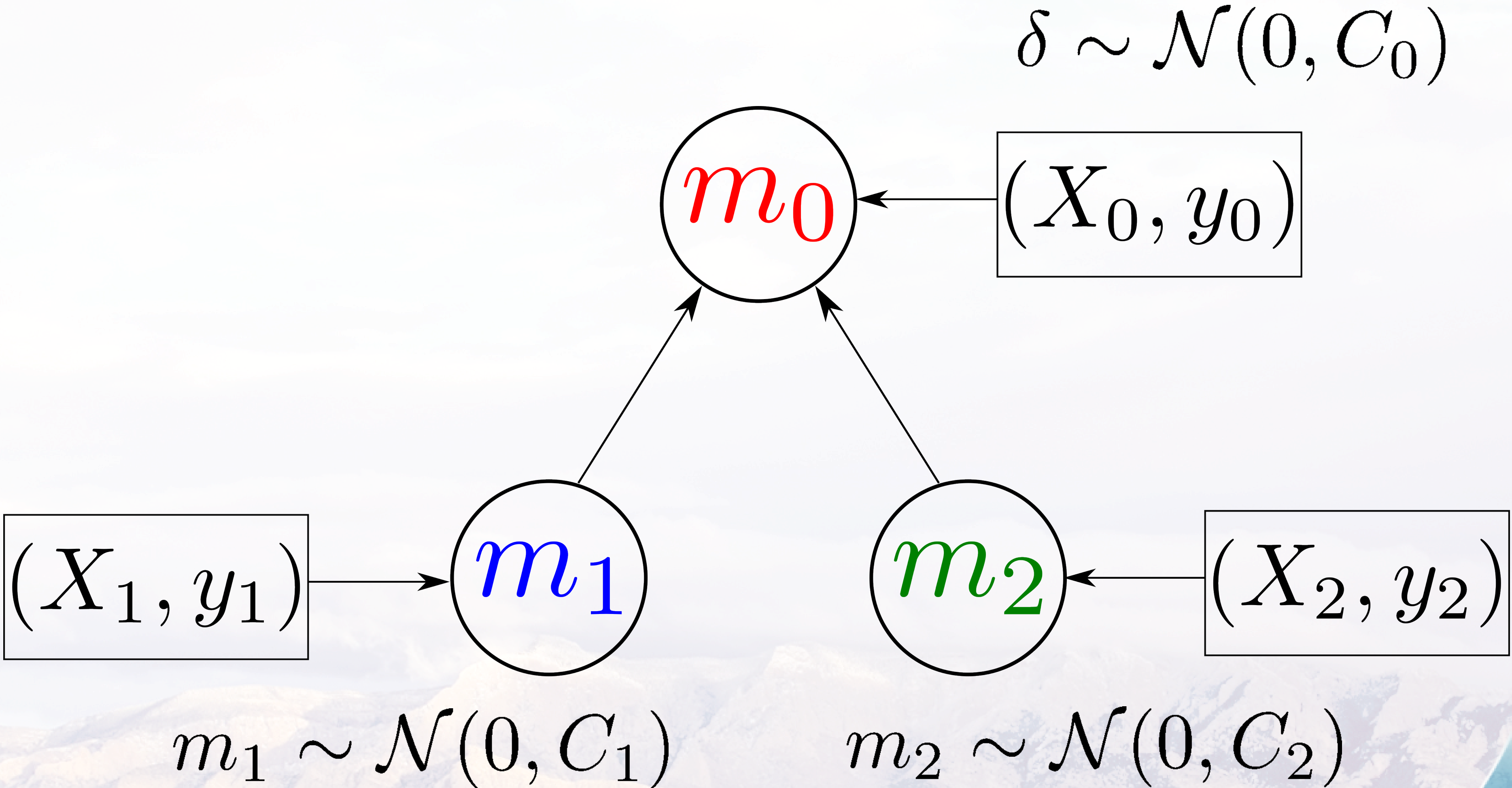
FA Network of Gaussian Processes



$$m_1 \sim \mathcal{N}(0, C_1) \qquad m_2 \sim \mathcal{N}(0, C_2)$$

- Introduce independent GP priors for the peers and a discrepancy GP.

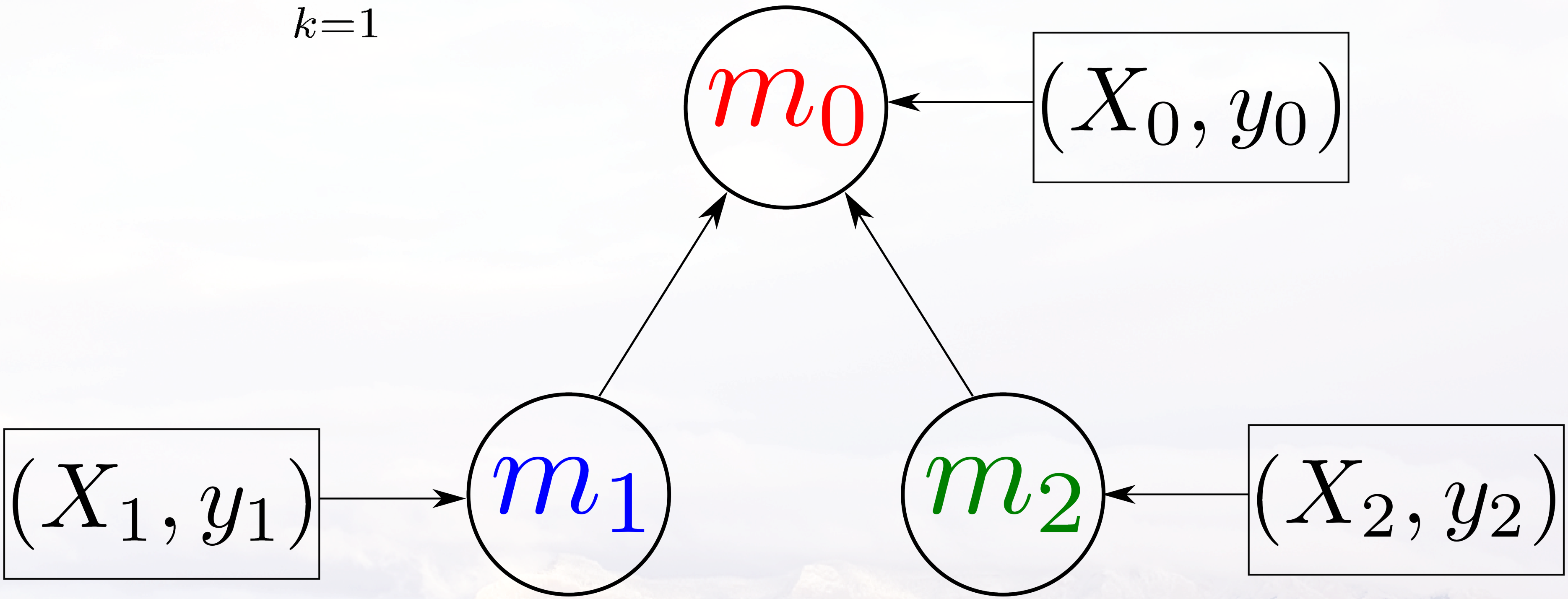
FA Network of Gaussian Processes



- Introduce independent GP priors for the peers and a discrepancy GP.

FA Network of Gaussian Processes

$$m_0(x) = \sum_{k=1}^{N_{\text{peer}}} \rho_k(x) m_k(x) + \delta(x) \qquad \delta \sim \mathcal{N}(0, C_0)$$



$$m_1 \sim \mathcal{N}(0, C_1) \qquad m_2 \sim \mathcal{N}(0, C_2)$$

- Treat ρ_k polynomial coefficients as hyperparameters and form a sparse prior for the covariance between asset GPs.

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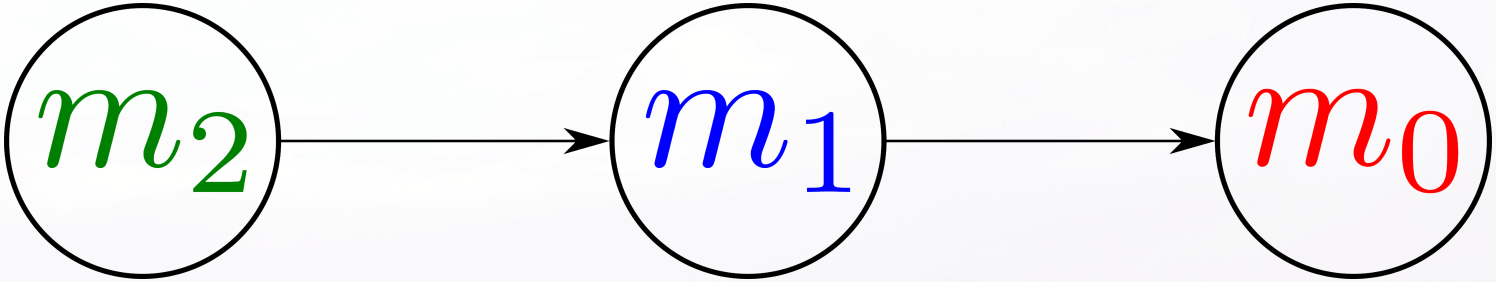
Related Work

- Hierarchical / auto-regressive GPs (Kennedy and O'Hagan, Le Gratiet and Garnier):



  - Requires nested training data sets and noiseless observations for the parent models.
  - Hierarchical ordering not appropriate for a fleet of assets.

- MF Nets (Gorodetsky et al.): Our work is an instance of a peer directed acyclic graph of Gaussian processes.

FA Network of GPs via Co-Kriging

- The covariance matrix has a block structure that is determined by computing the covariance between the approximations $m_k(\boldsymbol{x}),\ k=1,\dots N_{\text{peer}}$ and $m_0(\boldsymbol{x}) = \sum_{k=1}^{N_{\text{peer}}} \rho_k(\boldsymbol{x}) m_k(\boldsymbol{x}) + \delta(\boldsymbol{x})$:

$$\mathbb{E}\left[m_k m_k\right] = C_k(X_k, X_k), \quad k = 1, \dots N_{\text{peer}},$$

$$\mathbb{E}\left[m_0 m_k\right] = \left[\mathbb{1}_{N_0} \otimes \boldsymbol{\rho}_k(X_k)\right] \odot C_k(X_0, X_k), \quad k = 1, \dots N_{\text{peer}},$$

$$\mathbb{E}\left[m_0 m_0\right] = C_0(X_0, X_0) + \sum_{k=1}^{N_{\text{peer}}} \left[\boldsymbol{\rho}_k(X_0) \otimes \boldsymbol{\rho}_k(X_0)\right] \odot C_k(X_0, X_0).$$

- The matrices $C_k(X_i, X_j),\ k = 1, \dots, N_{\text{peer}}$ are obtained by evaluating the kernels for $m_k(\boldsymbol{x})$ at all pairs of points in X_i, X_j.

- Similarly, the matrix $C_0(X_0, X_0)$ is computed by evaluating the kernel for the discrepancy GP $\delta(\boldsymbol{x})$.

Hyperparameter Estimation

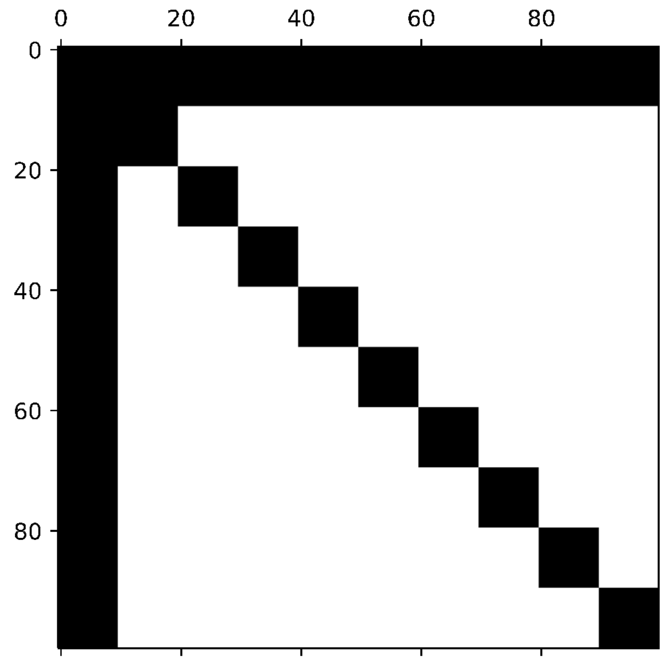
- Negative marginal log-likelihood:

$$\text{NLL} = \frac{1}{2}\log\left(\det\ \boldsymbol{C}\right) + \frac{1}{2}\boldsymbol{y}^T\boldsymbol{C}^{-1}\boldsymbol{y} + \frac{N}{2}\log\left(2\pi\right)$$

- Computational bottlenecks:
  - Cholesky factorization – $O\left(N^3\right)$.
  - NLL typically non-convex – multiple optimization runs from different starting points.
  - All-at-once and sequential approaches.

Linear Algebra for the FA Network of GPs

- The block covariance matrix is sparse.



- Efficient linear algebra techniques reduce the overall complexity of hyperparameter learning to $O(\max(N_k)^3)$.

Numerical Examples

Analytical Assets Example

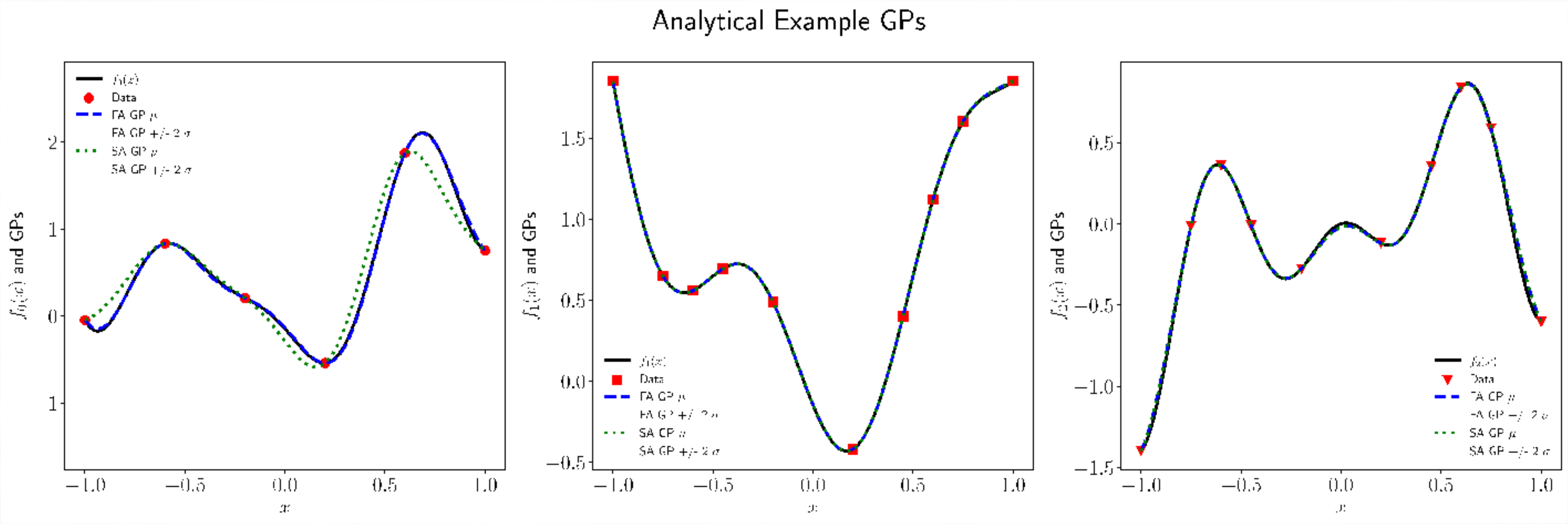
$$f_2(x) = 0.4x - x \sin\left(\frac{5\pi x}{2}\right)$$

$$f_1(x) = 2x^2 - \frac{1}{2}\sin\left(2\pi x + 0.3\right)$$

$$f_0(x) = -\frac{1}{2}x^3 \sin\left(\frac{\pi x}{2}\right) + f_1(x) + f_2(x)$$

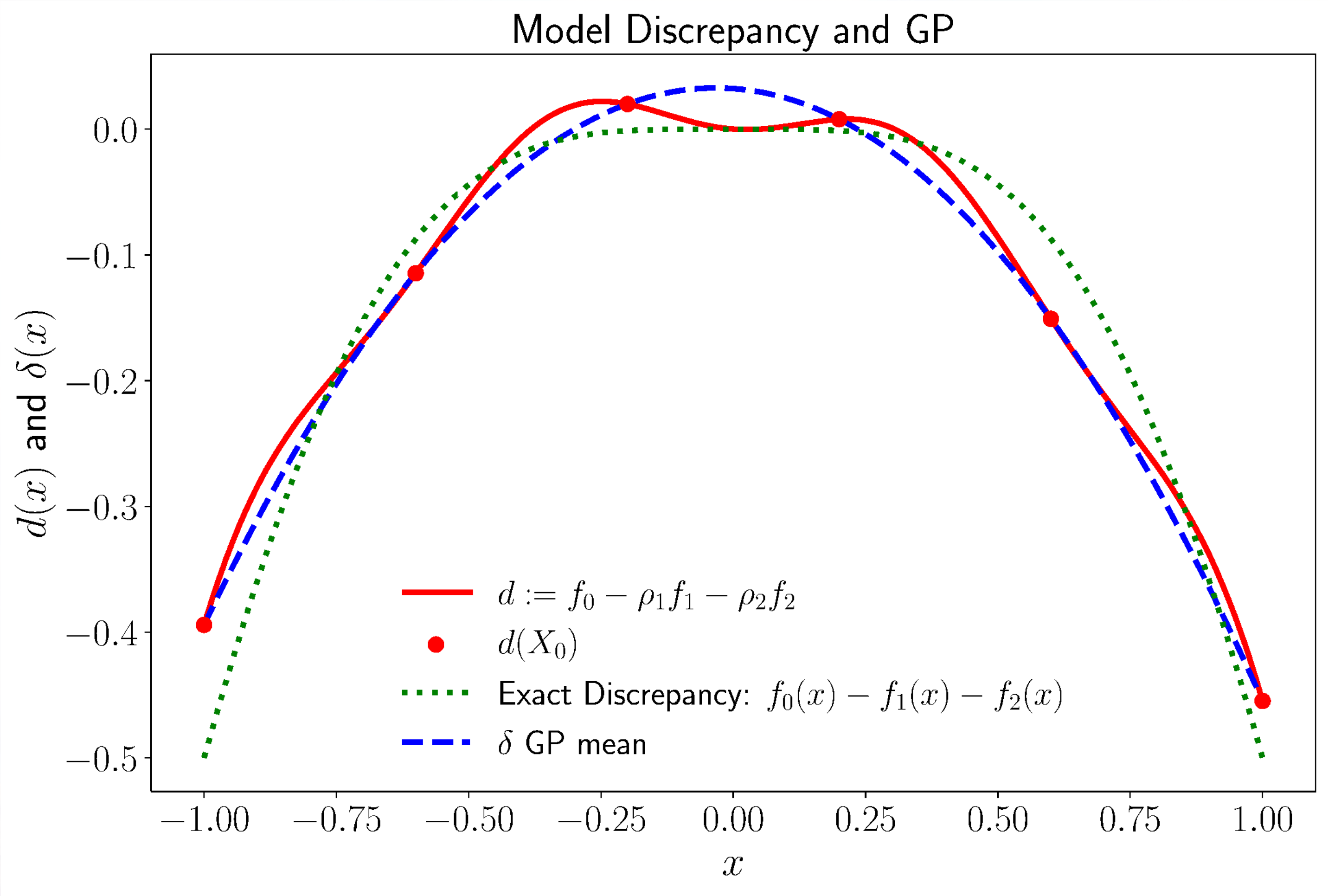
- Analytical functions for the assets that conform to the additive / multiplicative discrepancy structure.

Nested Samples Analytical DT GP Results



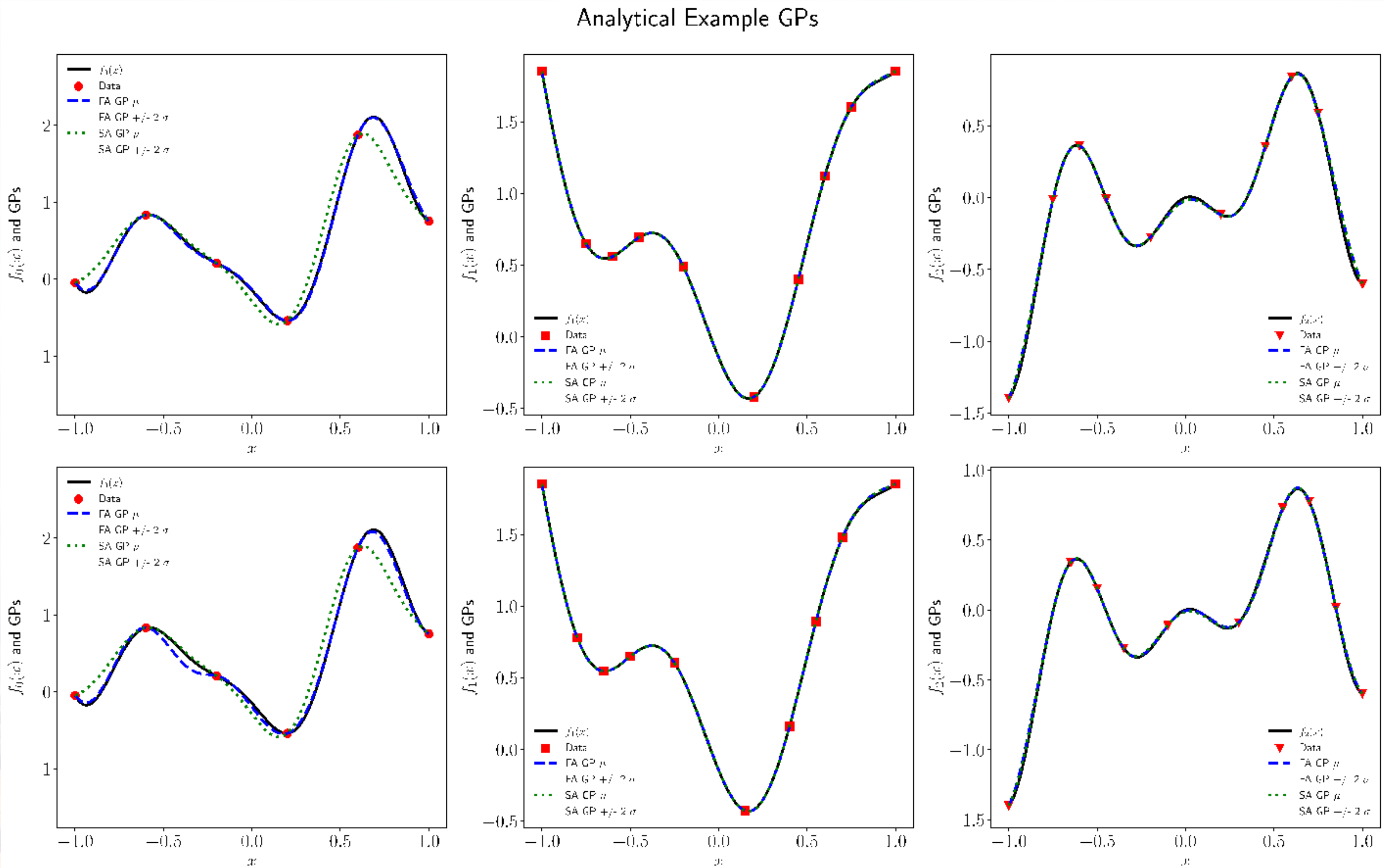
- Model 0 is the asset of interest (AoI) in these results.
- Fleet of assets (FA) vs single asset (SA).
- MLE results in $\rho_1 = 1.0, \rho_2 = 1.08.$

Nested Samples Analytical DT GP Results



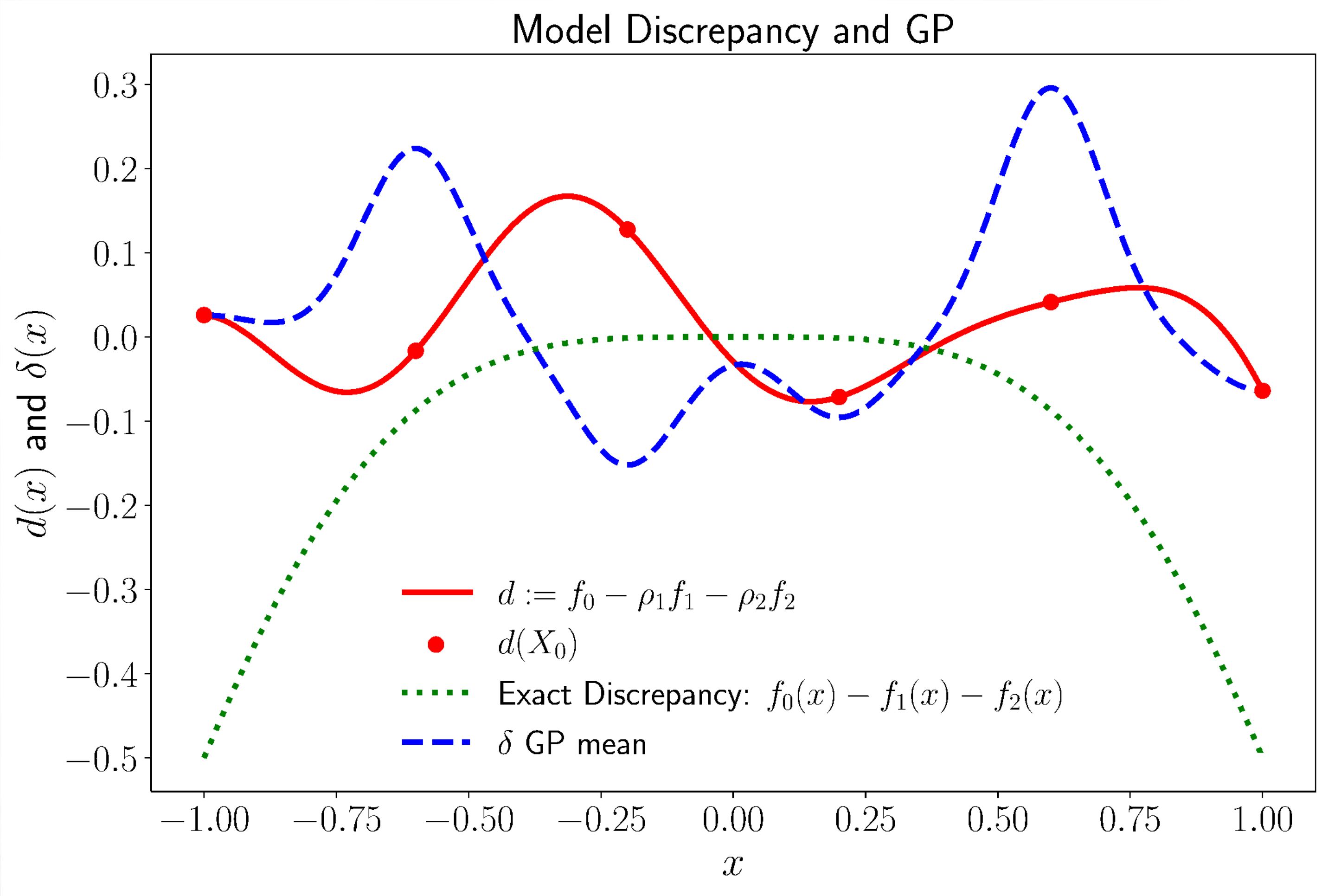
- The discrepancy GP approximates the discrepancy when the training data is nested and peer datasets are noiseless.

Non-nested Samples Analytical DT GP Results



- FA approach still performs better than single asset, but …

Non-nested Samples Analytical DT GP Results



- ... the discrepancy GP is doing something different.

Piston Model

- Benchmark problem for surrogate modeling: https://www.sfu.ca/~ssurjano/piston.html .

$$C(\boldsymbol{x}) = 2\pi \sqrt{\frac{M}{k + S^2 \frac{P_0 V_0}{T_0} \frac{T_a}{V^2}}}$$

$$V = \frac{S}{2k} \left(\sqrt{A^2 + 4k \frac{P_0 V_0}{T_0} T_a} - A \right)$$

$$A = P_0 S + 19.62 M - \frac{k V_0}{S}$$

$M \in [30, 60]$	piston weight (kg)
$S \in [0.005, 0.020]$	piston surface area (m^2)
$V_0 \in [0.002, 0.010]$	initial gas volume (m^3)
$k \in [1000, 5000]$	spring coefficient (N/m)
$P_0 \in [90000, 110000]$	atmospheric pressure ($\mathrm{N/m}^2$)
$T_a \in [290, 296]$	ambient temperature (K)
$T_0 \in [340, 360]$	filling gas temperature (K)

Piston Model: 1D Problem

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Input variable

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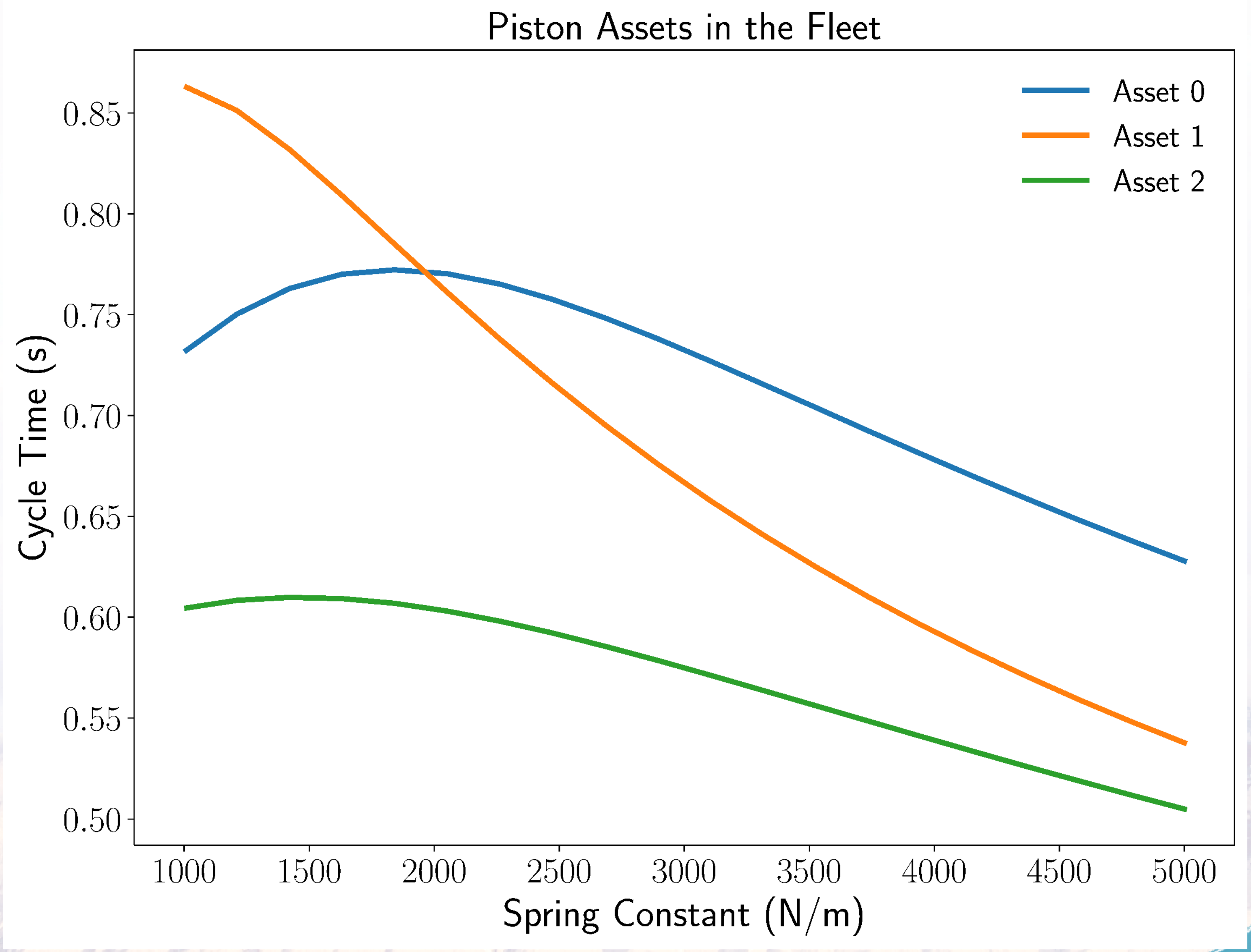
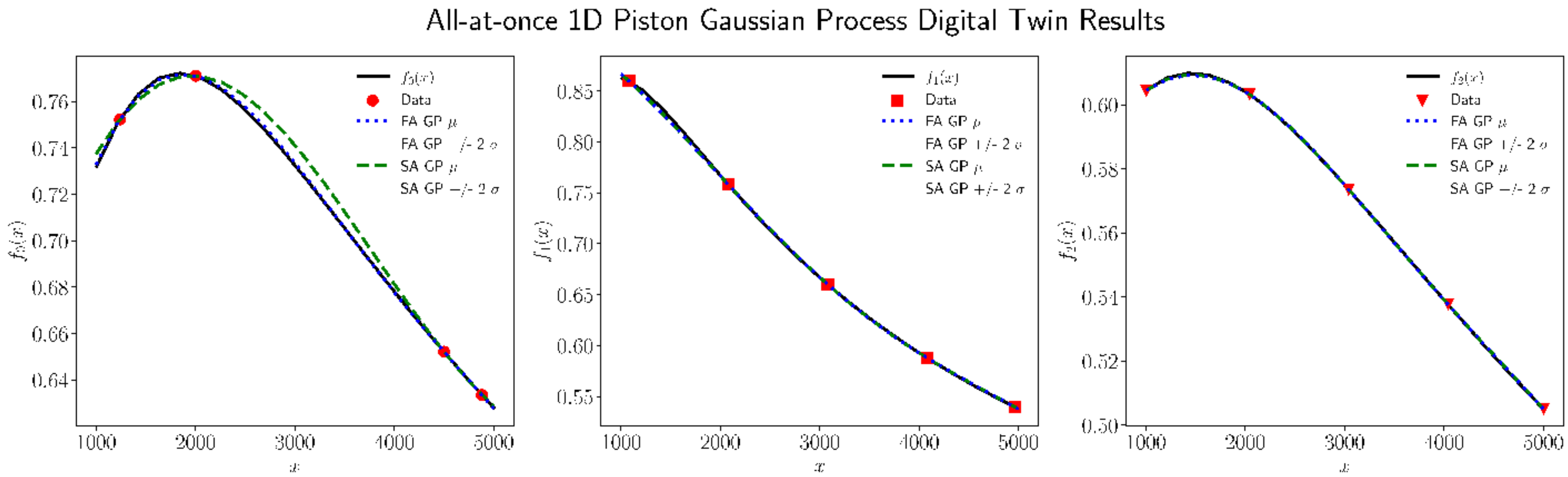
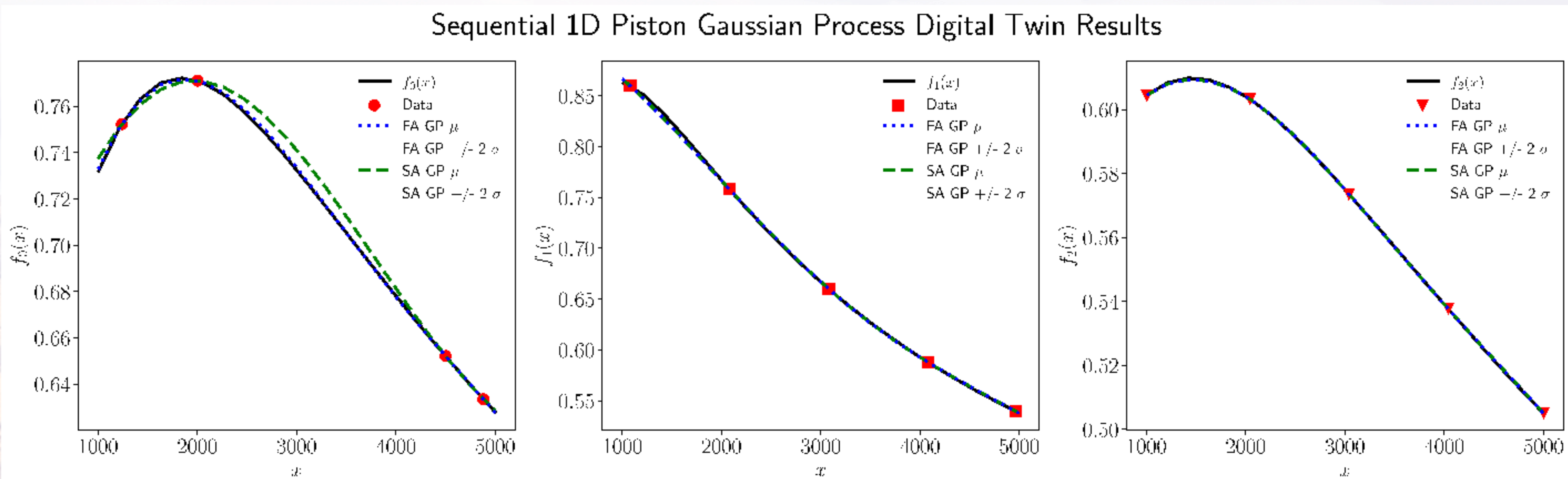
Piston Functions



Noiseless 1D Piston Results

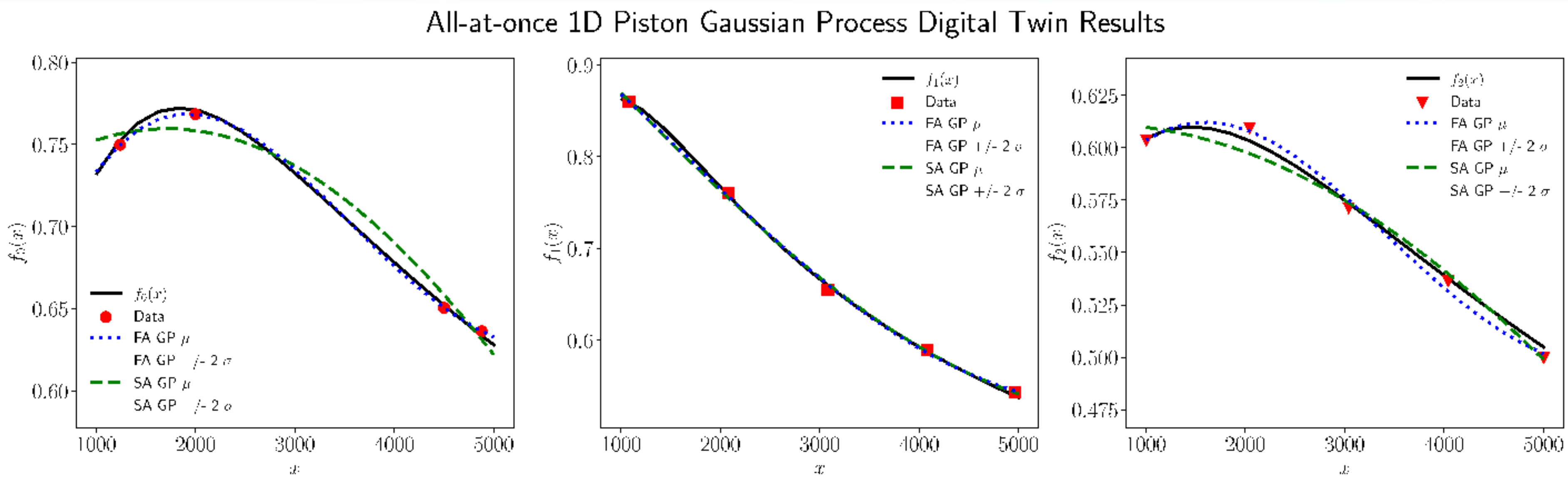
- All-at-once vs. sequential training of hyperparameters:

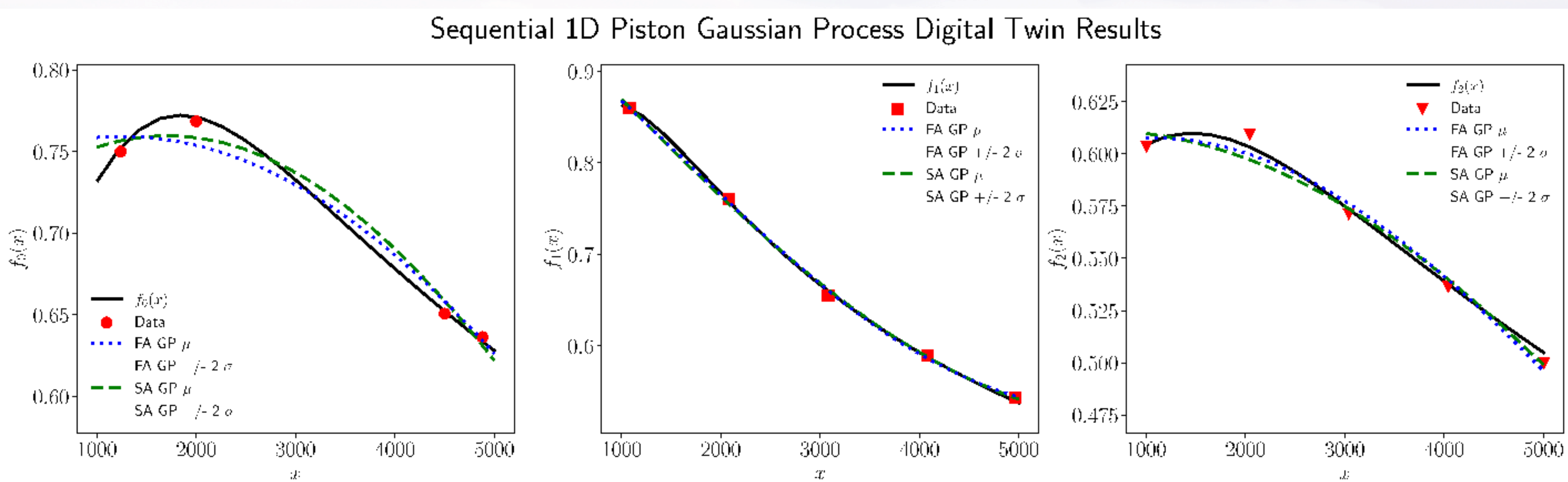




Noise-corrupted 1D Piston Results

- All-at-once vs. sequential training of hyperparameters:





Piston Model: 2D Problem

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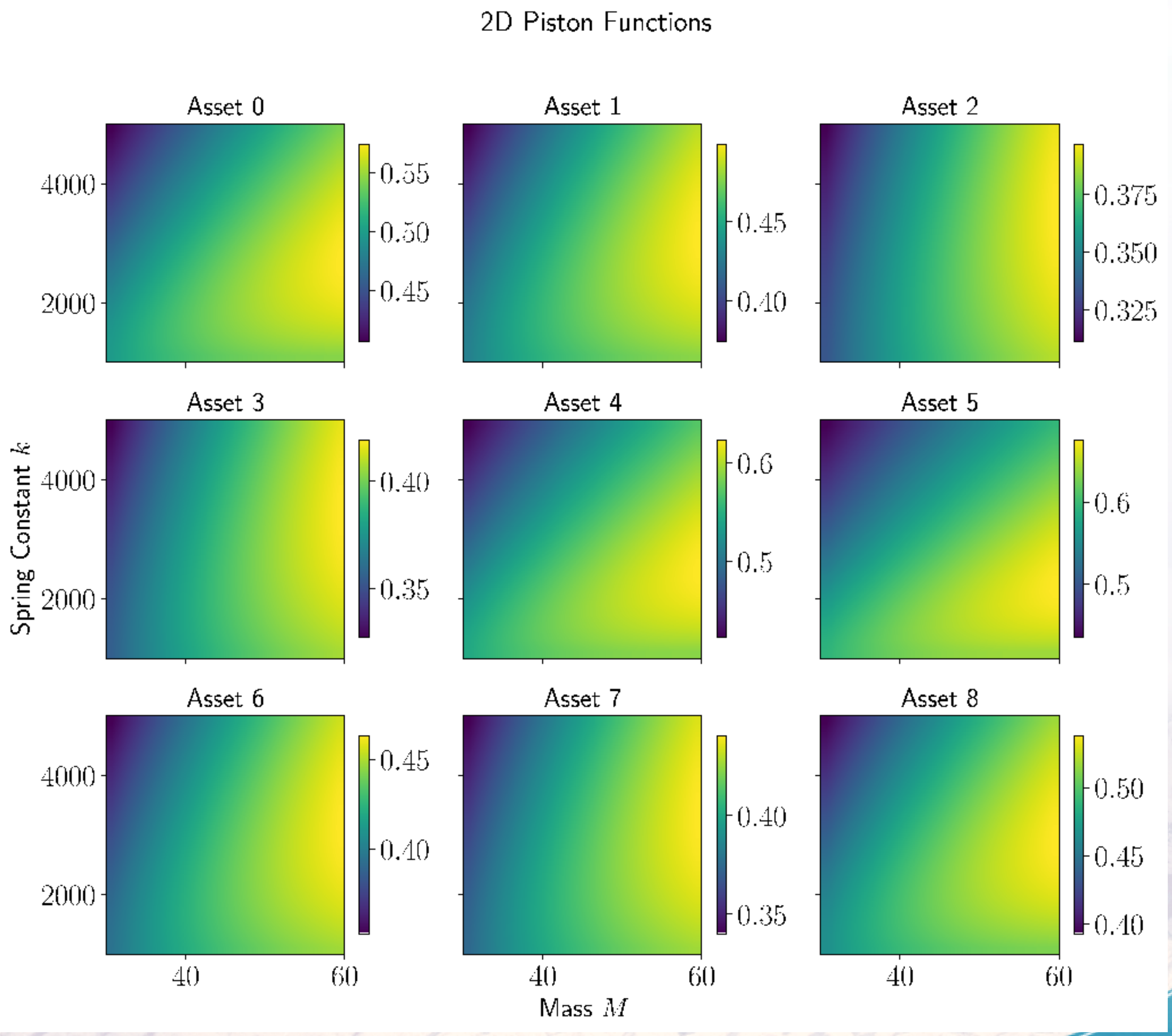

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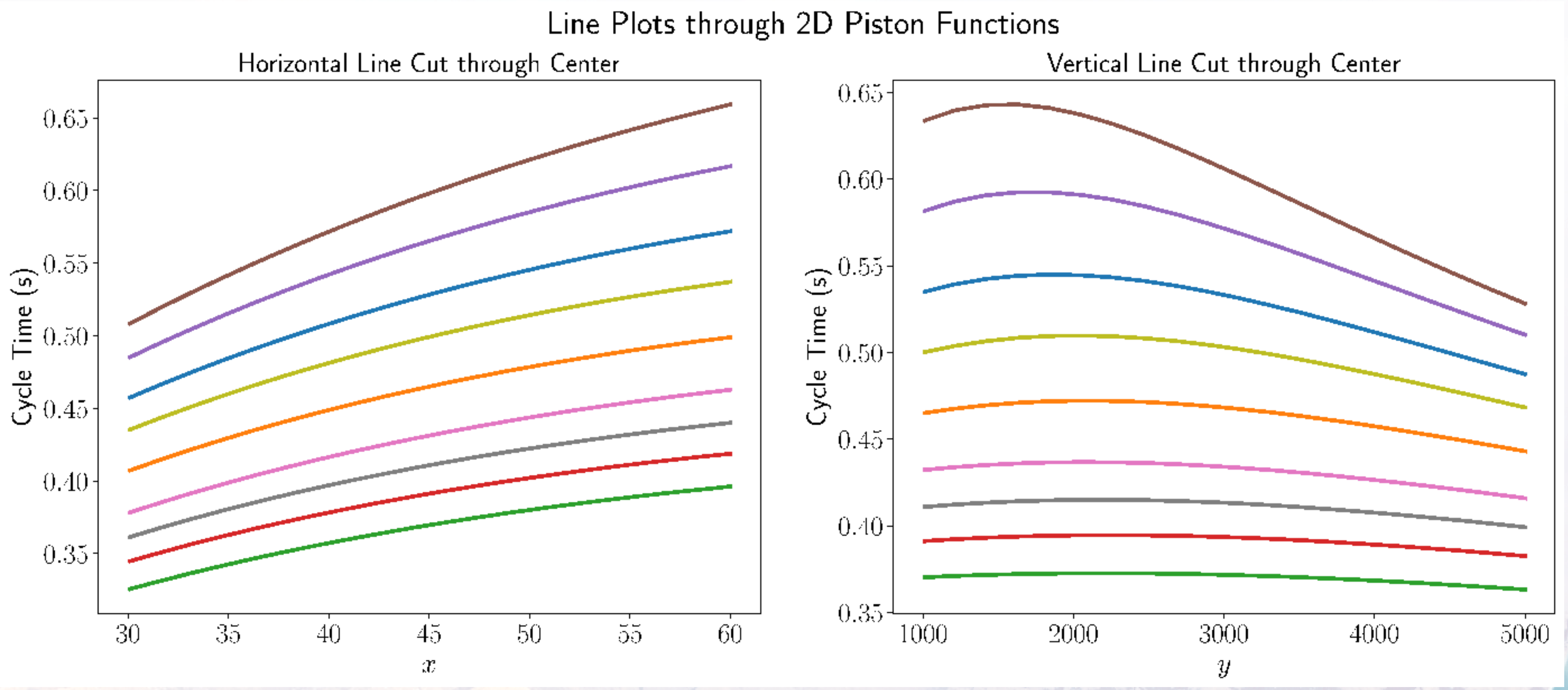

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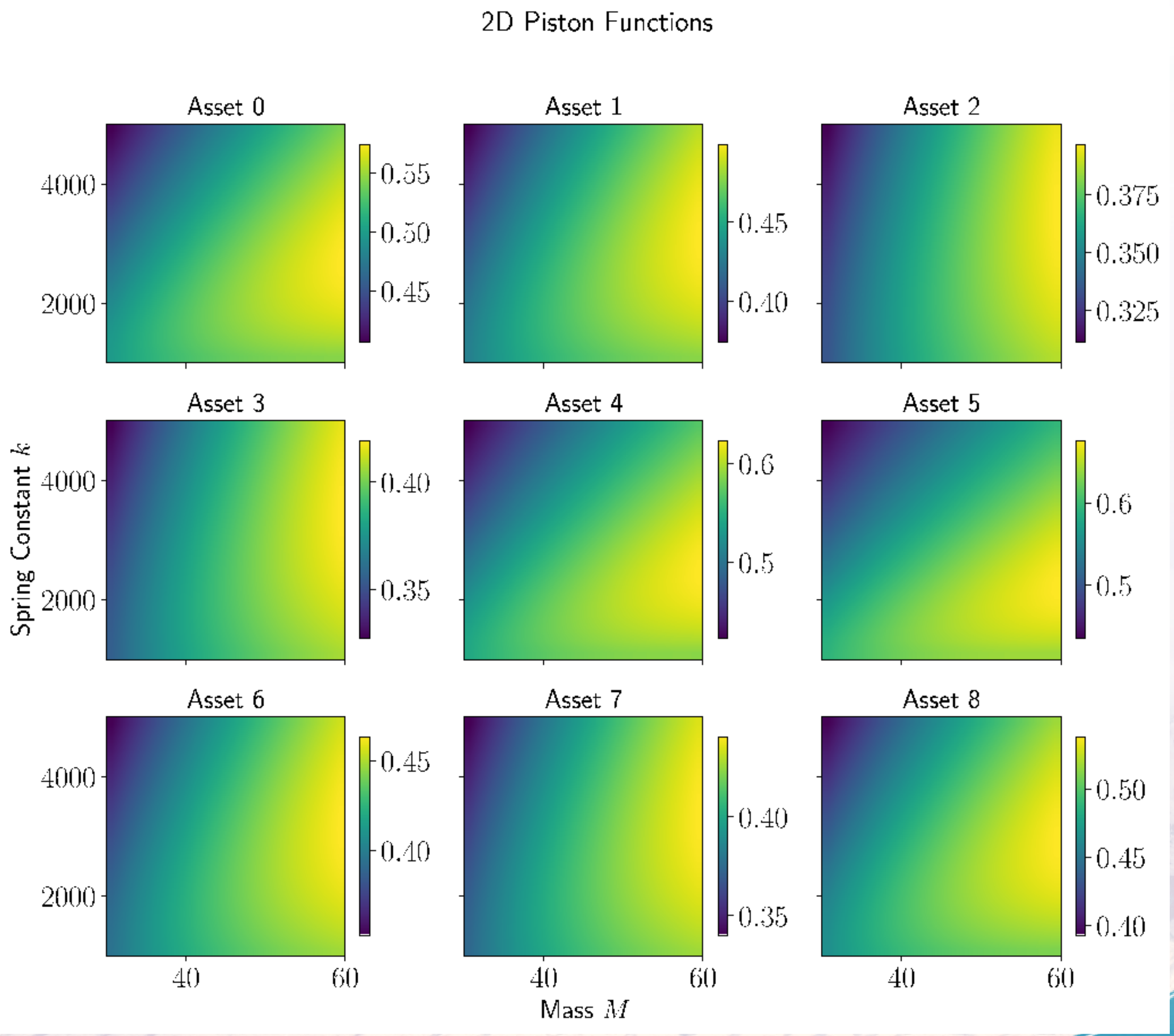

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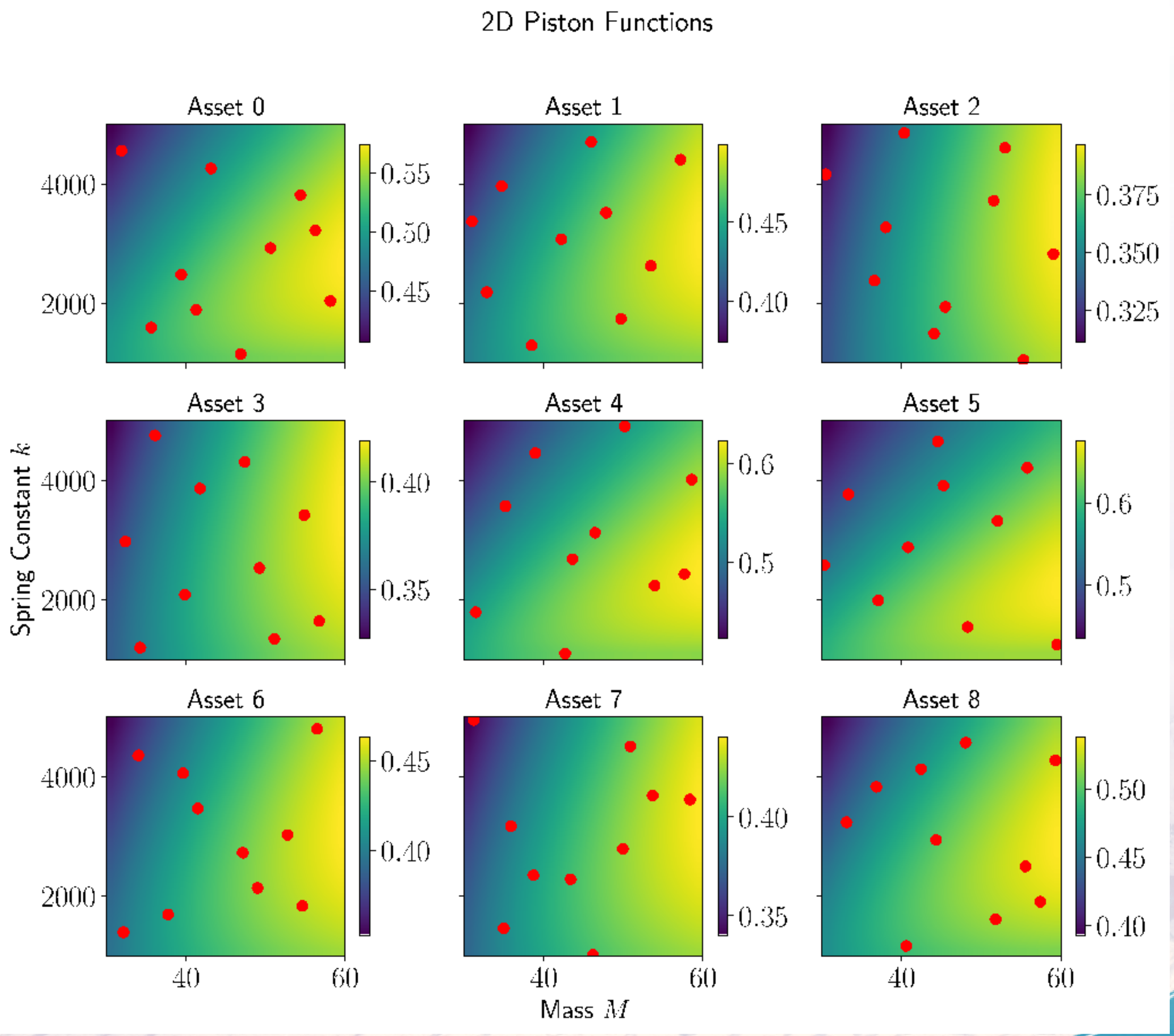

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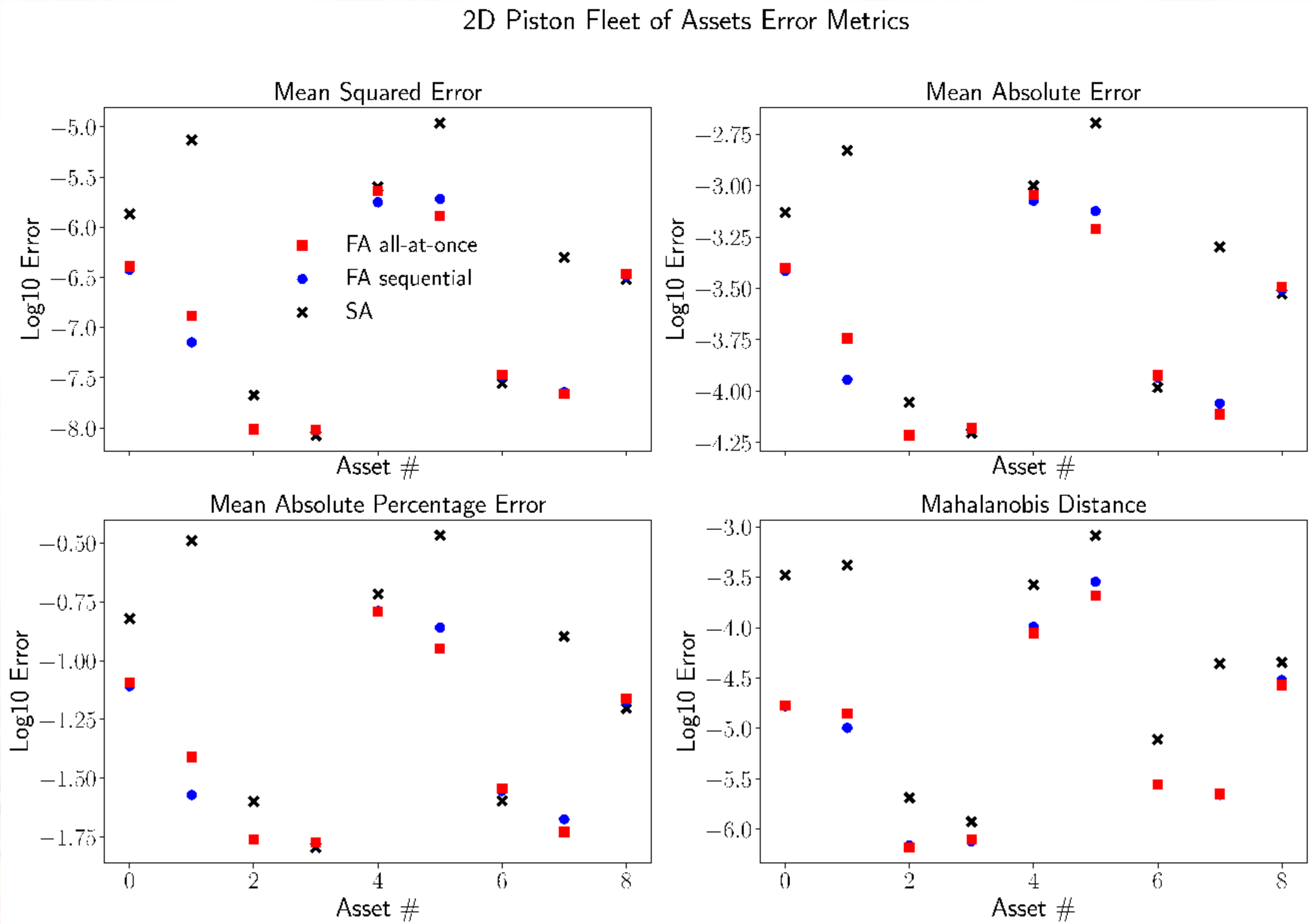

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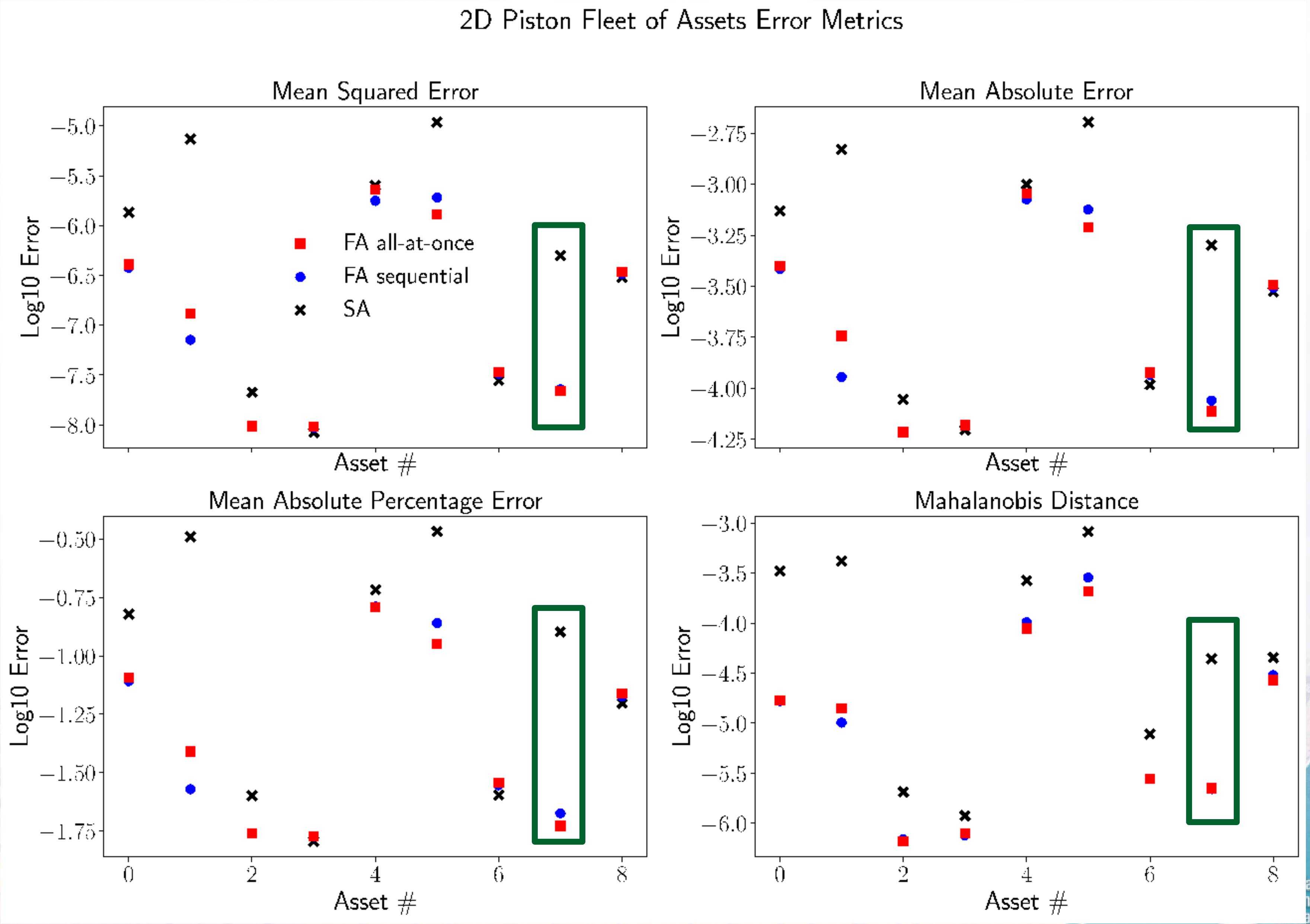

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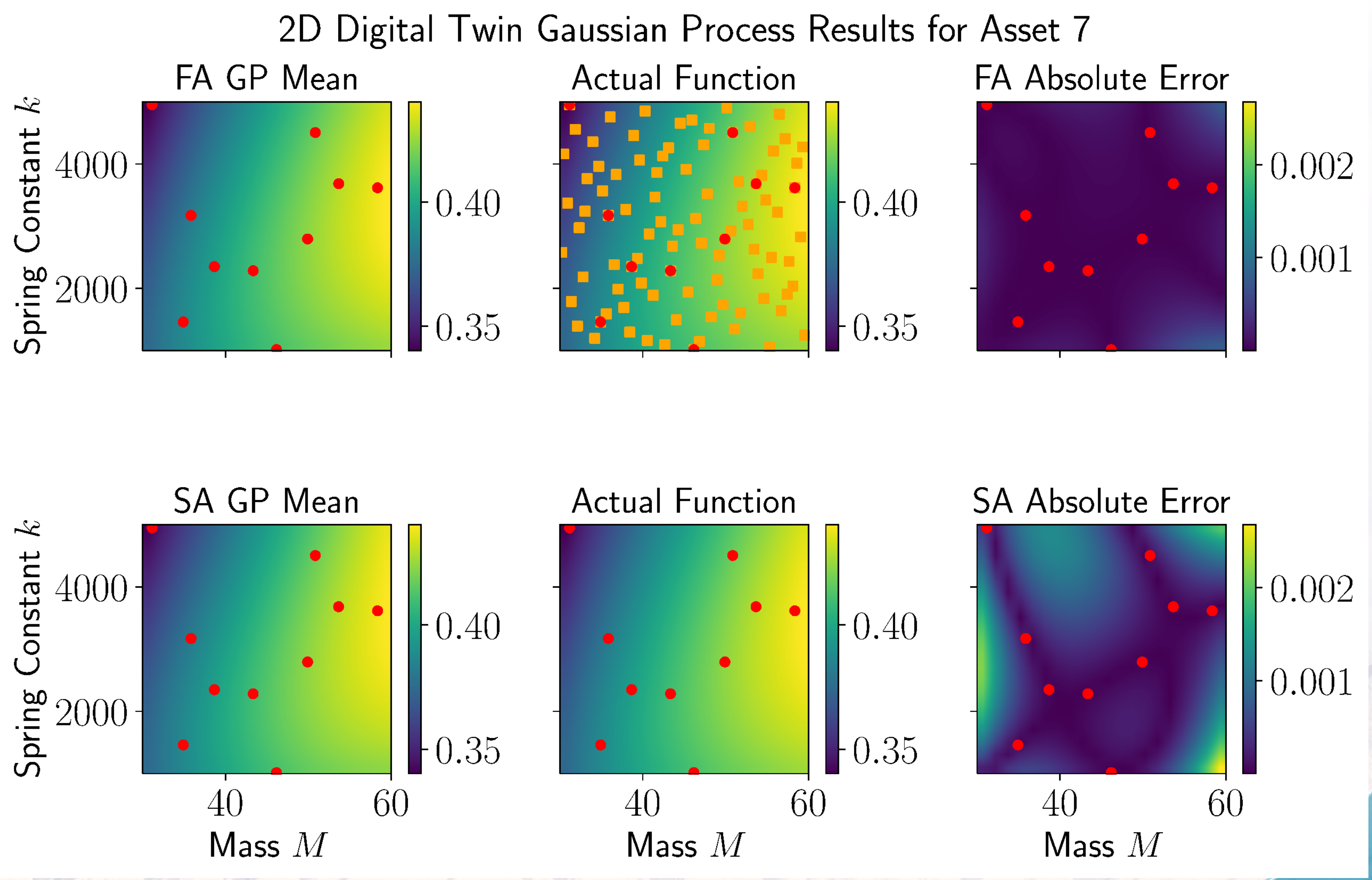

Conclusions

- Limited data (possibly non-nested, noisy) for each asset …
- … but the dataset for the entire fleet is rich and diverse.
- Demonstrated that fleet data can be used to increase prediction accuracy for any single asset.
- Efficient linear algebra is critical for reducing the cost of hyperparameter estimation.

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Thank you!