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SALSA3D: A Tomographic Model of the Compressional Wavespeed in the Earth's Mantle for Improved Seismic Event Location

S. Ballard[1], J. Hipp[1], A. Encarnacao[1], C. Young[1], M. Begnaud[2], S. Phillips[2] [1]Sandia National Laboratories, [2]Los Alamos National Laboratory

ABSTRACT

Several studies have shown that global 3D models of the compression wave speed in the Earth's mantle can provide superior first P travel time predictions at both regional and teleseismic distances. However, given the variable data quality and uneven data sampling associated with this type of model, it is essential that there be a means to calculate high-quality estimates of the path-dependent variance and covariance associated with the predicted travel times of ray paths through the model. In this presentation, we show a methodology for accomplishing this by exploiting the full model covariance matrix.

Typical SALSA3D global models may contain as many as 1/2 million grid nodes, so the challenge in calculating the covariance matrix is formidable: 0.9 TB storage for 1/2 of a symmetric matrix, necessitating an Out-Of-Core (OOC) blocked matrix solution technique. With our approach the tomography matrix, G, (which includes Tikhonov regularization terms) is multiplied by its transpose (G^TG) and written in a blocked sub-matrix fashion. We employ a distributed parallel solution paradigm that solves for $(G^TG)^{-1}$ by assigning blocks to individual processing nodes for matrix decomposition update and scaling operations. We first find the Cholesky decomposition of G^TG which is subsequently inverted. Next, we employ OOC matrix multiplication methods to calculate the model resolution and covariance matrices from $(G^TG)^{-1}$ and an assumed data covariance matrix. Given the model covariance matrix we solve for the travel-time covariance associated with arbitrary ray-paths by integrating the model covariance along dual ray paths that share a common source (setting the paths equal yields the variance for that path).

PROBLEM DEFINITION

The standard least squares tomography solution for p-wave slowness, s, is formulated given an m x n set of non-linear travel time path length weights, $A(s)$; a vector of n associated path residuals, d; an n x n Bayesian inferred prior model correlation matrix, ρ_m; and a vector of n uncertainty values for each node, σ_m. The Bayesian prior model parameters are used to constrain the solution in model regions possessing little or no data. This formulation can be written as

$$\begin{bmatrix} C_d^{-1/2} A(s_k) \\ \alpha \rho_m^{-1/2} \sigma_m^{-1} \end{bmatrix} \Delta s^{k+1} = \begin{bmatrix} C_d^{-1/2} (d - A(s_k) s_k) \\ 0 \end{bmatrix} \qquad s^{k+1} = \Delta s^{k+1} + s^k$$

Where C_d are the data variances associated with the travel time path weights, α is a damping parameter applied to ensure solution stability, and the non-linear solution is updated in an iterative manner (k) until convergence is obtained$(\Delta s \approx 0)$.

Applying standard solution techniques the prior model covariance, C_m, the posterior model covariance, $\widetilde{C}_m$, and the model resolution, R_m, can be discovered and written as

$$C_m = \sigma_m \rho_m \sigma_m \qquad \widetilde{C}_m = \left[A^T C_d^{-1} A + C_m^{-1}\right]^{1} \qquad R_m = \widetilde{C}_m A^T C_d^{-1} A = I - \widetilde{C}_m C_m^{-1}$$

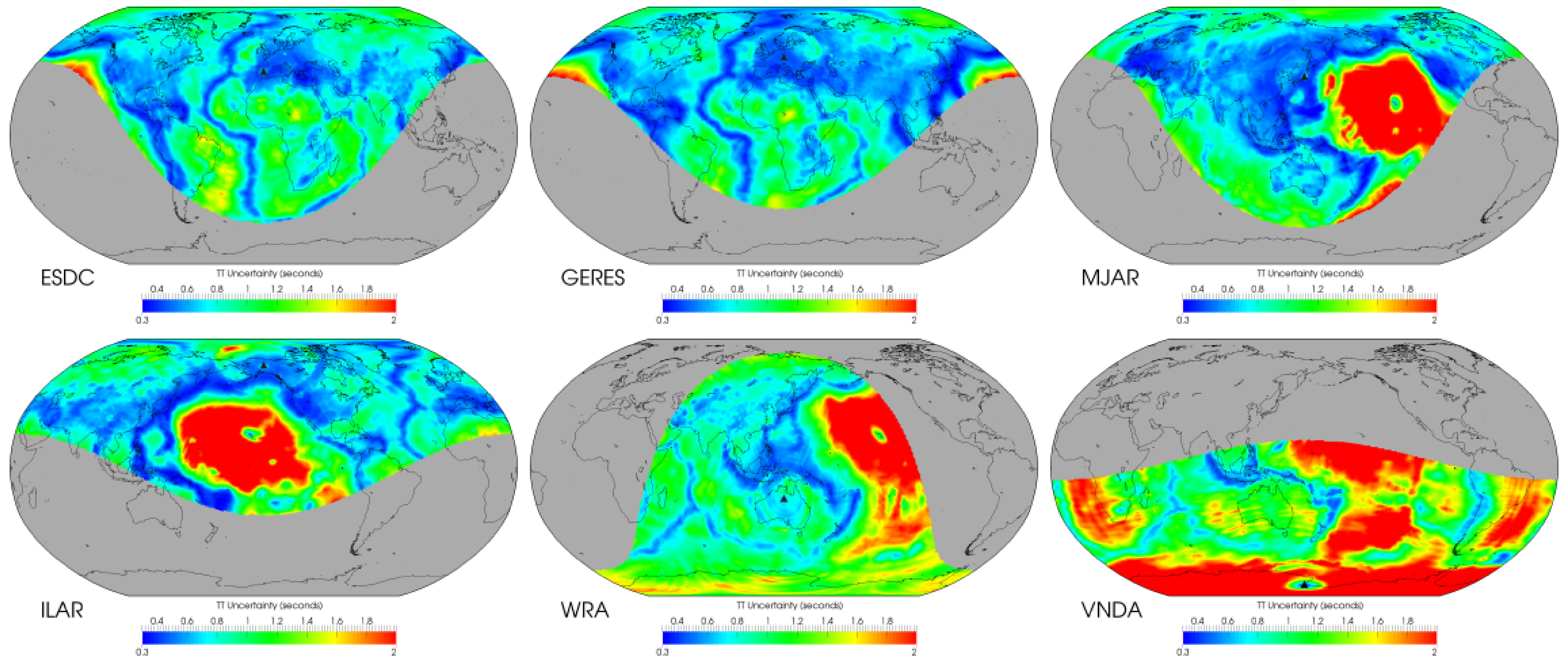
Given these definitions we can formulate the travel time and associated uncertainty of an arbitrary ray path, p, given its grid node vector of path length weights ($W_p = <w_{pj}>$) as

$$\widetilde{t}_p = \sum_{j=0} w_{pj} \widetilde{s}_j \pm \widetilde{\sigma}_p \qquad \widetilde{\sigma}_p = \sqrt{W(\widetilde{s}_m)\widetilde{C}_m W^T(\widetilde{s}_m) + W(s_m) C_m W^T(s_m)}$$

Here $W(\widetilde{s}_m)$ imply weights for nodes along the path p that lie in regions of the posterior model while $W(s_m)$ define weights for nodes along the path that lie in prior model regions for which slowness updates were not sought (the Crust).

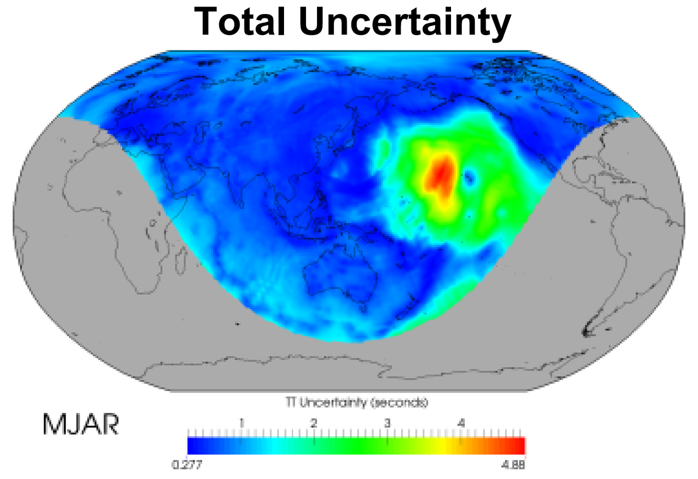
Uncertainty Results

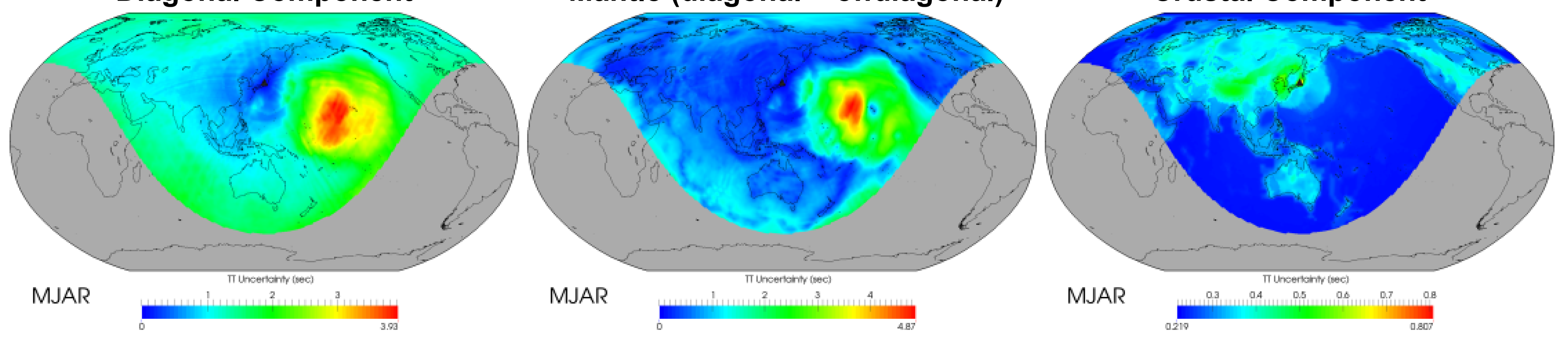
The panels below illustrate the travel time prediction uncertainty for 6 stations distributed around the world. The uncertainty estimates are fully path dependent and take into account the posteriori slowness variance and covariance of all model nodes 'touched' by the seismic ray from source to receiver.



Components of Uncertainty

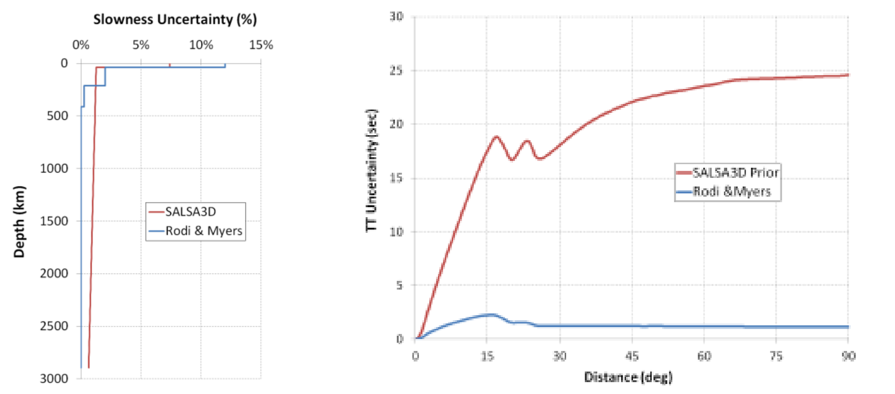
The maps below illustrate the various components of the travel time prediction uncertainty for station MJAR in Japan. Since the crustal nodes in our model are not modified by our tomographic process, the crust is not represented in the covariance matrix. We assume a crustal slowness uncertainty that adds the crustal component of uncertainty to all of out uncertainty results. The mantle component is the uncertainty computed from the model covariance matrix. The diagonal component is the uncertainty that would be computed if the off-diagonal terms of the model covariance are ignored. The total uncertainty illustrated to the right, is the sum of the mantle and crustal components. Comparison of the diagonal component with mantle component clearly indicates that a good handle on the off diagonal terms is very important.



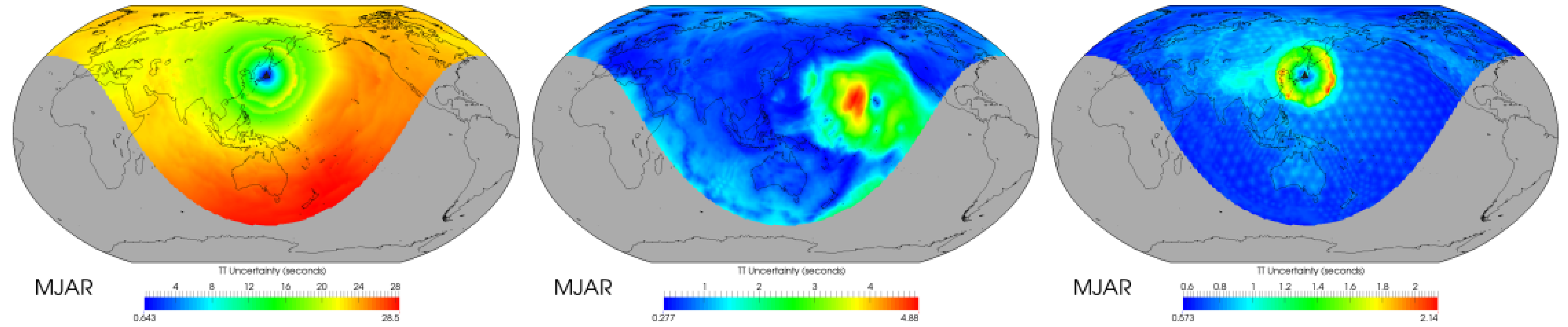


Comparison with Prior and with Rodi & Myers (2013)

In the first two panels below the SALSA3D prior and posterior estimates of the travel time prediction uncertainty are compared. Note that the prior reaches values of 28 seconds, much larger than the maximum posterior value of 4.9 seconds. This indicates that the data are having a large impact on the computed uncertainties. Substantial reductions are observed even in aseismic areas like the middle of the Pacific Ocean because significant portions of the path from station MJAR to those regions still pass through parts of the model that are well calibrated by seismic data used in tomography.

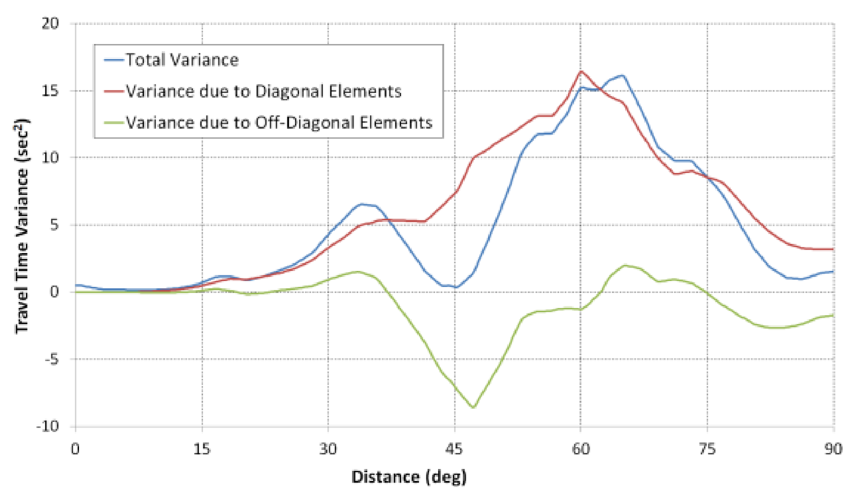


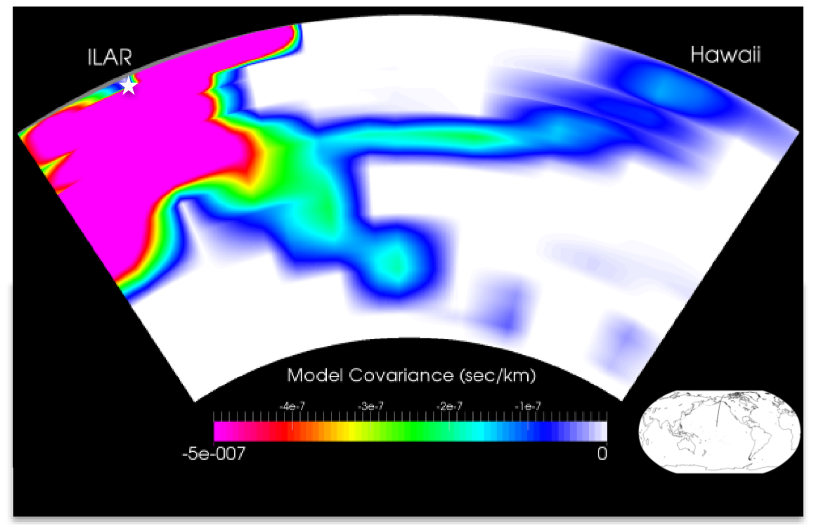
The final panel illustrates uncertainties computed through the posterior covariance of Rodi and Myers (2013). Their uncertainty results are distance dependent and do not reflect the effects of the data used in a tomographic inversion.



Significance of Off-Diagonal Terms

In the figures to the right, we illustrate the components of travel time prediction uncertainty along a transect that begins at station ILAR in Alaska, passes through the island of Hawaii at a distance fo 45° from the station, and then continues on into the southern Pacific Ocean. Out to a distance of 60°, the ray is traveling through parts of the mantle that are poorly sampled by the tomographic data set. The exception is the actual ray path from ILAR to the island of Hawaii, which is reasonably well sampled by the tomography data. In that distance range, substantial negative contribution from from the off-diagonal covariance component significantly reduces the travel time uncertainty estimates. Past 60°, the rays begin to penetrate the deep parts of the lower mantle which are well sampled by the rays traveling between seismic regions and stations in many parts of the circum-Pacific region, resulting in lower slowness variance in the deep mantle.





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