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ASSESSING MAGNETIC INSULATION IN A CROSSED-FIELD GAP

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MOTIVATION AND INTRODUCTION

Crossed-field devices (CFDs), which have an external magnetic field perpendicular to the electric field, are important in many applications, including high power microwaves and directed energy. One critical quantity for characterizing CFDs is the Hull cutoff magnetic field (HC), which corresponds to the maximum magnetic field for an electron emitted from the cathode to reach the anode [1]. CFDs with magnetic fields above the HC are referred to as magnetically insulated. Theory demonstrated that placing ions in the gap increased the distance that an emitted electron traveled from the cathode [2]. Subsequent simulations demonstrated the loss of magnetic insulation for sufficiently high pressures [3].

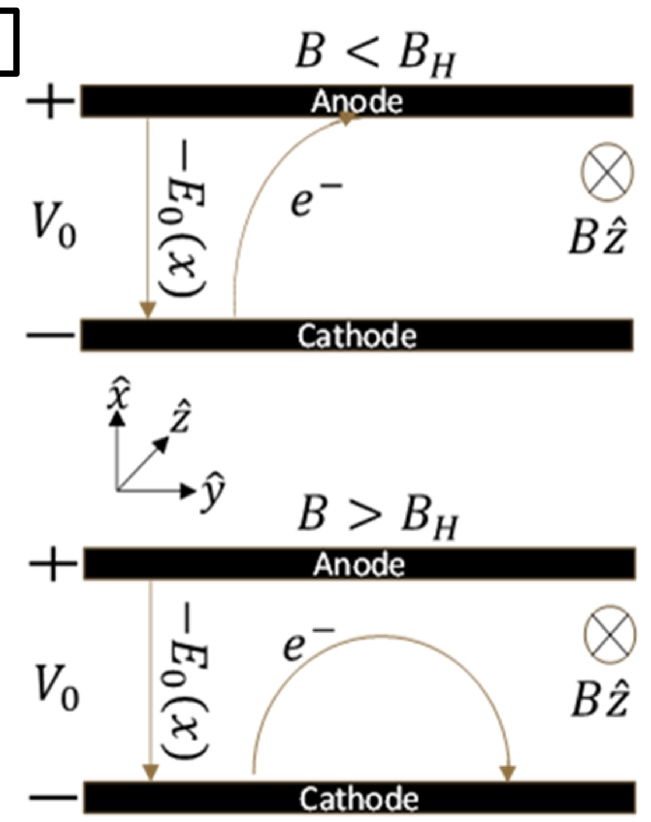
In this presentation, we theoretically assess the motion of electrons emitted from a cathode in the presence of a crossed-magnetic field and account for collisions by electron mobility, analogous to previous assessments of electron emission in a nonmagnetic diode [4]. To address this, we perform the following:

(1) Present the changes in the HC based on how mobility changes electron velocity across the gap.

(2) Create simulations using the 1D/3v (one-dimension in space, three-dimensions in velocity) particle-in-cell code PDP1, which was used in the previous studies [3], and use to compare to theory.

Implications to magnetic insulation for practical devices will be discussed.

Fig. 1



DATA

- Equations are analyzed using Wolfram Mathematica Eqs. (1)-(2) and Fig. 2
- Properties utilized in Eq. (2) are provided by the Navy Aegis Radar System
- Nondimensional variable values are based upon previously defined nondimensional parameters [4]

Figure 1. Hull Cutoff magnetic field is demonstrated along with the associated electric field, applied voltage, gap distance, and applied magnetic field. Future equations and figures take into account the effect of collisions within the gap distance on the Hull Cutoff value.

Eq.(1)

Derivation of Hull Cutoff magnetic field position and velocity equations with collisional effects

- Lorentz Force Law:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

- Using Newton's 2nd Law yields two differential equations:

$$\ddot{x} = \frac{eE_x}{m} - \frac{eB_z}{m}\dot{y}, \ddot{y} = \frac{eB_z}{m}\dot{x}$$

where $eB_z/m = \Omega$ is the cyclotron frequency. Solving gives:

$$x(t) = \frac{eE_x}{m\Omega^2}(1 - \cos\Omega t), x'(t) = \frac{eE}{m\Omega^2}\sin\Omega t$$

- Hull Cutoff Boundary Conditions are applied at the transit time τ:

$$x(\tau) = D, v_x = x'(\tau) = 0$$

- Hull Cutoff Magnetic Field is defined:

$$B_H = \sqrt{\frac{2mV}{eD^2}}$$

- After introducing collisions, the differential equations are solved for $x(t)$ and $x'(t)$

Dimensional:

$$x(t) = \frac{1}{(e^2 + m^2\mu^2\Omega^2)^2} e \exp\left(-\frac{et}{m\mu}\right) Ex\,\mu\left(-e^2\exp\left(\frac{et}{m\mu}\right)m\,\mu + \exp\left(\frac{et}{m\mu}\right)m^3\mu^3\Omega^2 + e^3\exp\left(\frac{et}{m\mu}\right)t + e\exp\left(\frac{et}{m\mu}\right)m^2\mu^2\omega^2 t + e^2 m\mu\cos(\Omega t) - m^3\mu^3\omega^2\cos(\Omega t) - 2e\,m^2\mu^2\Omega\sin(\Omega t)\right)$$

Nondimensional:

$$\bar{x}(\bar{t}) = \frac{e^{-\frac{\bar{t}}{\bar{\mu}}}\bar{\mu}\bar{V}\left(-e^{\frac{\bar{t}}{\bar{\mu}}}\bar{\mu} + e^{\frac{\bar{t}}{\bar{\mu}}}\bar{\mu}^3 + e^{\frac{\bar{t}}{\bar{\mu}}}\bar{t} + e^{\frac{\bar{t}}{\bar{\mu}}}\bar{\mu}^2\bar{t} + \bar{\mu}\cos\bar{t} - \bar{\mu}^3\cos\bar{t} - 2\bar{\mu}^2\sin\bar{t}\right)}{\bar{d}(1 + \bar{\mu}^2)^2}$$

PROGRESS AND ACHIEVEMENTS

- Utilized Mathematica to recover collisional Hull Cutoff with various mobility
- Used properties from the Navy Aegis Radar System to verify Hull Cutoff recovery
- Derived equations for electron position and velocity [Eqs. (1) and (2)]
- Analyzed mobility within both dimensional and nondimensional position and velocity equations to understand sinusoidal relationship with mobility values
- Confirmed time-dependent nondimensional position and velocity equations
- Created plots using the nondimensional code to analyze sinusoidal relationships with different mobilities, voltages, and gap distances [Fig. 2]
- Continue to characterize magnetic insulation as a function of mobility using different mathematical methods and concepts
- Currently attempting to determine a nondimensional mobility below which magnetic insulation decreases (or vanishes) – note red terms in Eq. (1).

Equation 2. Numerically calculate the modified Hull Cutoff magnetic field using Mathematica. Typical values from the Navy Aegis Radar System.

Eq.(2)

$$e = 1.602 \times 10^{-19}\,\text{C}, m = 9.11 \times 10^{-31}\,\text{kg}$$

$$E_x = \frac{V}{d}, V = 12000\,\text{V}, d = 0.00216\,\text{m},$$

$$B_{guess} = \sqrt{\frac{2mV}{ed^2}}, \Omega_2 = \frac{eB_{guess}}{m},$$

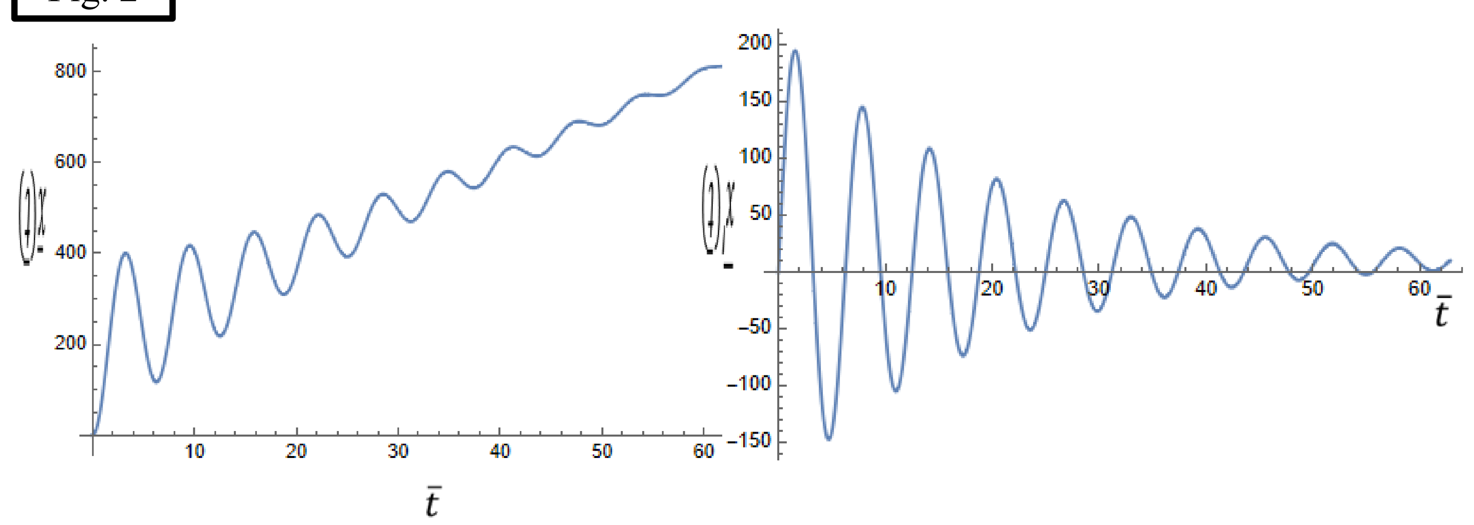
$$\Omega = \frac{eB}{m}, \mu = 20\,\text{m}^2\text{V}^{-1}\text{s}^{-1}$$

Utilizing Mathematica to solve for time and magnetic field yields:

$$t \to 6.17618 \times 10^{-29}\,\text{s}, B_{H'} \to 0.171033\,\text{T}$$

Figure 2. Nondimensional electron position and velocity with a mobility value that demonstrates sinusoidal movement diminishing as time goes on.

Fig. 2



References

[1] Hull, AW 1921 "The effect of a uniform magnetic field on the motion of electrons between coaxial cylinders," Phys. Rev., 18, 31-57.

[2] Lau, YY, Luginsland JW, Cartwright KL, Haworth MD 2007 "Role of ions in a crossed-field diode," Phys. Rev. Lett., 98, 015002.

[3] Stutzman, BS, Luginsland JW 2010 "Loss of magnetic insulation in a crossed-field diode: Ion and collisional effects," IEEE Trans. Plasma Sci., 38, 2010-2015.

[4] Darr, AM, Loveless, AM, and Garner, AL 2019 "Unification of field emission and space charge limited emission with collisions," Appl. Phys. Lett., 114, 014103.

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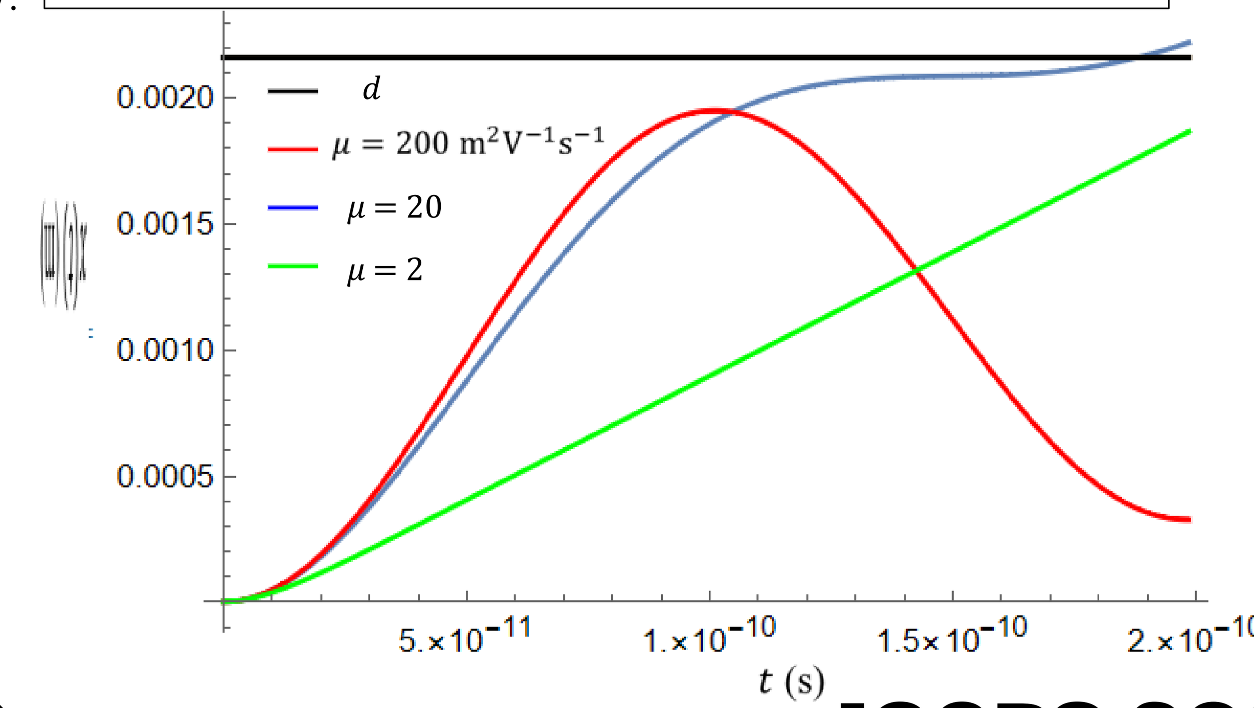
Figure 3. Differing dimensional electron trajectories based on varying mobility values as outlined in the legend below.



CONCLUSION AND FUTURE WORK

- Future work will use particle-in-cell simulations (XPDP1) to assess pressure dependence on Hull cutoff.
- Considerations to extend to fluid model (SOMAFOAM) in the future.
- Continue to utilize Mathematica and XPDP1 assess the modification in magnetic insulation with mobility/pressure changes.

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