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A Dynamic Substructuring Technique for Coupling and Decoupling Structure-Cavity Systems

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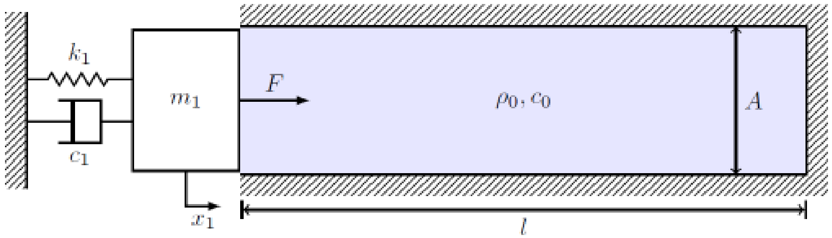
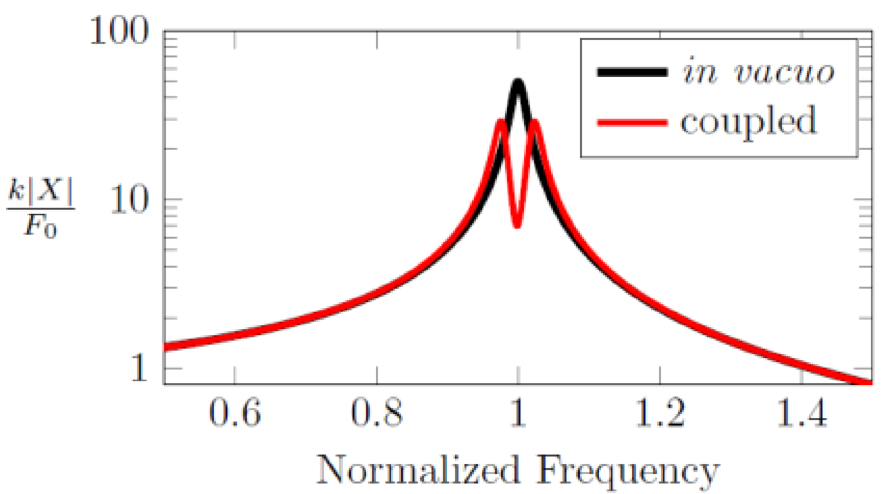
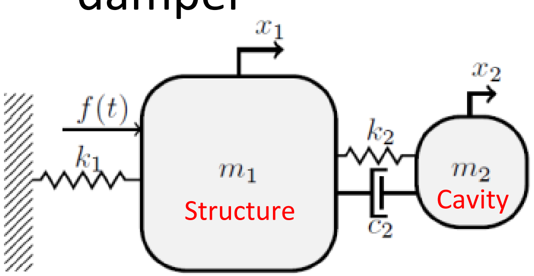
Coupled Structure/Cavity Systems

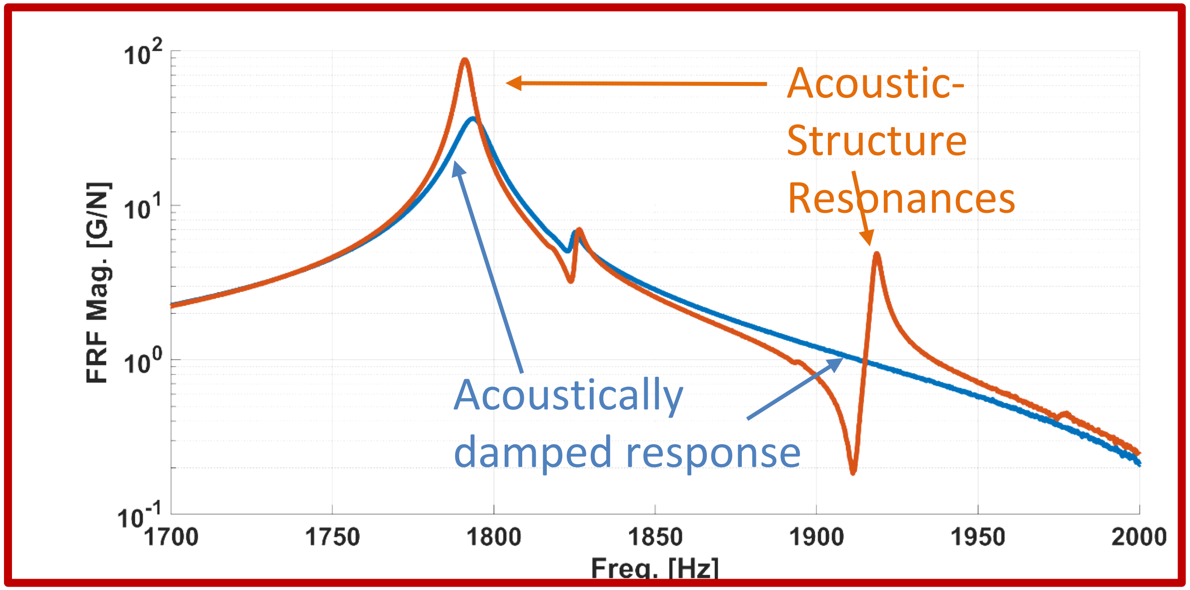
- **Examples:**
  - Fuel Tanks
  - Automobile cabins
  - Musical instruments
  - General hollow structures
- **Strong coupling requires:**
  - Near coincidence of uncoupled natural frequencies
  - Mode shape similarity
- **Effects:**
  - Split resonance peaks
    - Analogous to a tuned mass damper





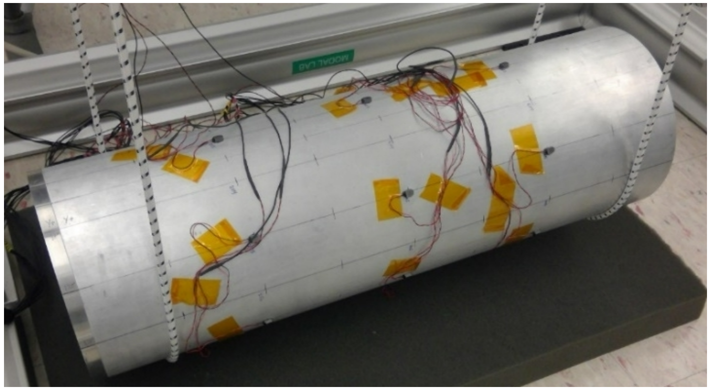


Motivation

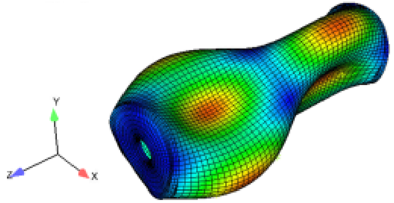


Experimental FRF

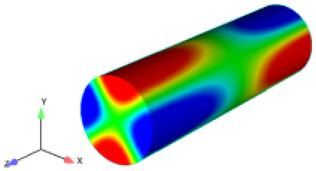
- Coupling can cause additional resonances in experimental FRFs
- Since FEMs typically assume the structure is *in vacuo*, coupling confounds test-analysis correlation
- Simply adding acoustic damping to the cavity does not recover the *in vacuo* response



Cylindrical test article



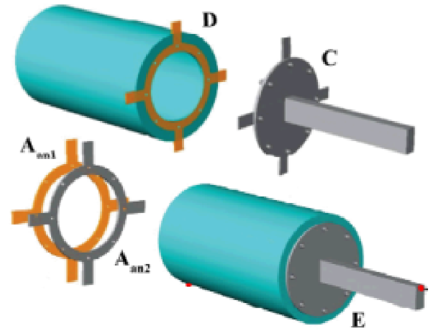
Structural component mode



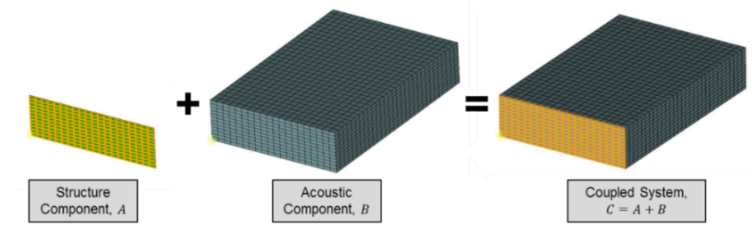
Acoustic component mode

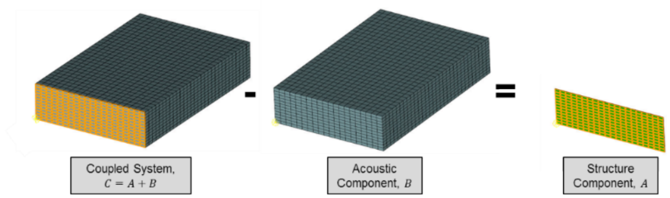
Idea

- Remove the acoustic part of the coupled FRFs using an uncoupled model of the cavity
    - Inspired by transmission simulator method[1]
- **First:** establish a generalized coordinate assembly component mode synthesis method (GCA-CMS)[2] for structure/cavity coupling problem
- **Then:** Adapt the GCA-CMS for the decoupling problem



D+C-A=E





[1]R. L. Mayes and M. Arviso, "Design studies for the Transmission Simulator Method of Experimental Dynamic Substructuring," in *International Seminar on Modal Analysis (ISMA2010)*, Lueven, Belgium, 2010.
[2]D. de Klerk, D. J. Rixen and S. N. Voormeeren, "General Framework for Dynamic Substructuring: History, Review, and Classification of Techniques," *AIAA Journal,* vol 46(5), pp 1169-1180, 2008.

Coupling Procedure

Assemble disjoint system in generalized coordinates:

$$\begin{bmatrix} [I_A] & [0] \\ [0] & [I_S] \end{bmatrix} \begin{Bmatrix} \ddot{q}_A \\ \ddot{q}_S \end{Bmatrix} + \begin{bmatrix} [\omega_A^2] & [0] \\ [0] & [\omega_S^2] \end{bmatrix} \begin{Bmatrix} q_A \\ q_S \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Transformation between physical and generalized coordinates:

$$\begin{Bmatrix} u_A \\ u_S \end{Bmatrix} = \begin{bmatrix} [\nabla \Phi_A] & [0] \\ [0] & [\Phi_S] \end{bmatrix} \begin{Bmatrix} q_A \\ q_S \end{Bmatrix}$$

$[\nabla \Phi_A]$ Normalized gradient of acoustic modes. Modes normalized according to:

$$\nabla \Phi_j = \frac{\nabla \phi_j}{\sqrt{m_j}} \quad \text{where} \quad m_j = \rho_f \int_V \nabla \phi_j \cdot \nabla \phi_j dV$$

$[\Phi_S]$ Mass normalized structural modes

Coupling Procedure (cont'd)

Constraints enforced in terms of physical displacements:

$$[a]\begin{Bmatrix} u_A \\ u_S \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

where $[a]$ is the Boolean constraint matrix. In terms of generalized coordinates:

= 0 for rigid wall modes, so they can't be used exclusively as a basis

$$[a]\begin{bmatrix} [\nabla\Phi_A] & [0] \\ [0] & [\Phi_S] \end{bmatrix}\begin{Bmatrix} q_A \\ q_S \end{Bmatrix} = [\hat{a}]\begin{Bmatrix} q_A \\ q_S \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Using Lagrange multipliers, the complete set of EOMs is:

$$\begin{bmatrix} [I_A] & [0] \\ [0] & [I_S] \end{bmatrix}\begin{Bmatrix} \ddot{q}_A \\ \ddot{q}_S \end{Bmatrix} + \begin{bmatrix} [\omega_A^2] & [0] \\ [0] & [\omega_S^2] \end{bmatrix}\begin{Bmatrix} q_A \\ q_S \end{Bmatrix} = [\hat{a}]^T\{\lambda\}$$

$$[\hat{a}]\begin{Bmatrix} q_A \\ q_S \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Coupling Procedure (cont'd)

Need to find a set of unconstrained DOFs. This set is related to the constrained set by the coordinate transformation

$$\begin{Bmatrix} q_A \\ q_S \end{Bmatrix} = [B]\{\xi\}$$

From the constraint equations:

$$[\hat{a}][B]\{\xi\} = \{0\}$$ ⇨ $[B]$ is the null space of $[\hat{a}]$

The coupled natural frequencies and modes can then be calculated from

$$\left([\hat{K}] - \omega^2[\hat{M}]\right)\{\phi\} = \{0\}$$

where

$$[\hat{M}] = [B]^T \begin{bmatrix} [I_A] & 0 \\ 0 & [I_S] \end{bmatrix} [B] \qquad [\hat{K}] = [B]^T \begin{bmatrix} [\omega_A^2] & 0 \\ 0 & [\omega_S^2] \end{bmatrix} [B]$$

Decoupling Procedure

Disjoint system

$$\begin{bmatrix} -[I_A] & 0 \\ 0 & [I_C] \end{bmatrix} \begin{Bmatrix} \ddot{q}_A \\ \ddot{q}_C \end{Bmatrix} + \begin{bmatrix} -[\omega_A^2] & 0 \\ 0 & [\omega_C^2] \end{bmatrix} \begin{Bmatrix} q_A \\ q_C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Constraint equation in generalized coordinates:

$$[a] \begin{bmatrix} [\nabla \Phi_A] & 0 \\ 0 & [\Phi_C] \end{bmatrix} \begin{Bmatrix} q_A \\ q_C \end{Bmatrix} = [\hat{a}] \begin{Bmatrix} q_A \\ q_C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

The decoupled natural frequencies and modes can then be calculated from

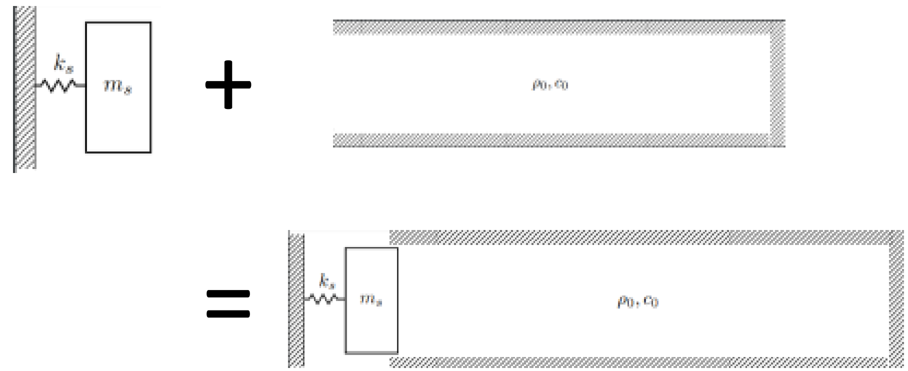
$$\left([\hat{K}] - \omega^2 [\hat{M}] \right) \{\phi\} = \{0\}$$

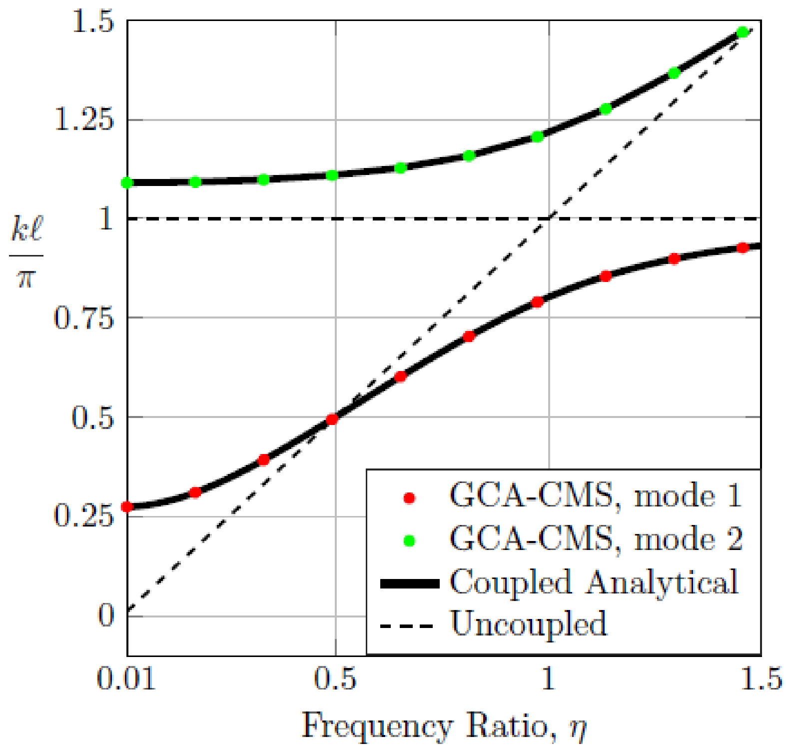
where

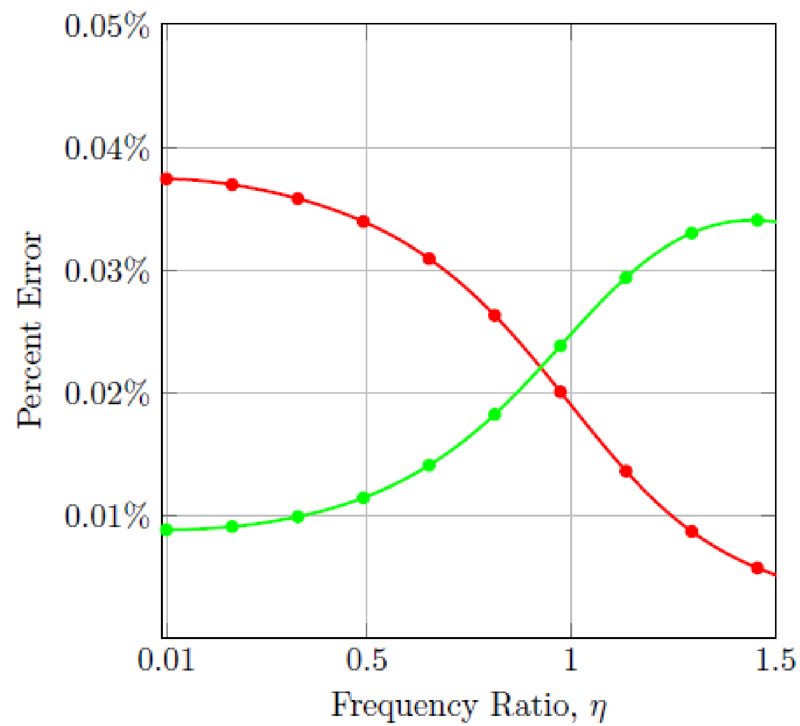
$$[\hat{M}] = [B]^T \begin{bmatrix} -[I_A] & 0 \\ 0 & [I_C] \end{bmatrix} [B] \qquad [\hat{K}] = [B]^T \begin{bmatrix} -[\omega_A^2] & 0 \\ 0 & [\omega_C^2] \end{bmatrix} [B] \qquad [B] = \mathrm{null}\left([\hat{a}] \right)$$

Example: Piston/Duct Coupling

- SDOF piston coupled to a 1D acoustic duct
- Coupled natural frequencies determined by:
    - Piston-to-fluid mass ratio
    - Uncoupled natural frequency ratio

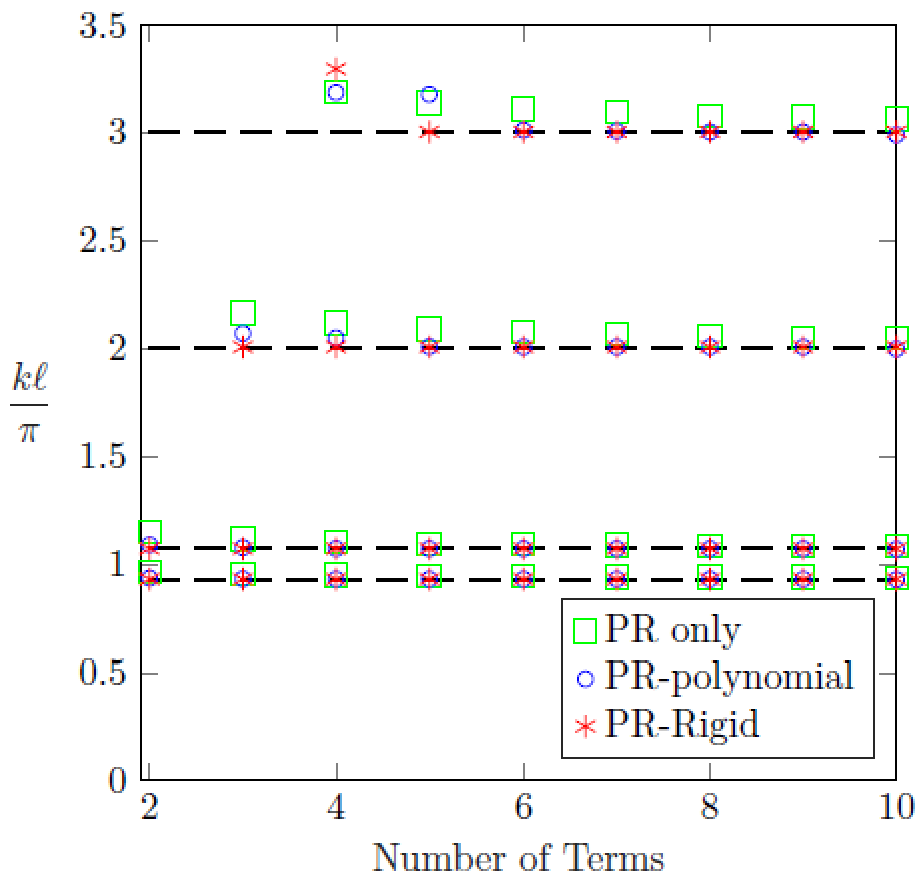




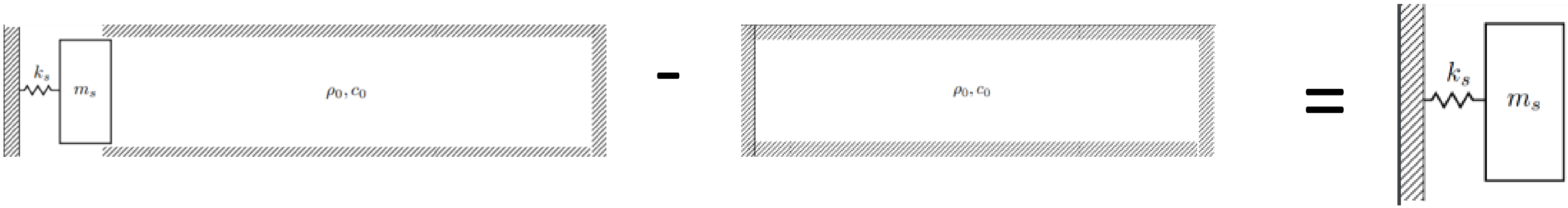


Piston/Duct Coupling
Choice of Acoustic Basis Functions

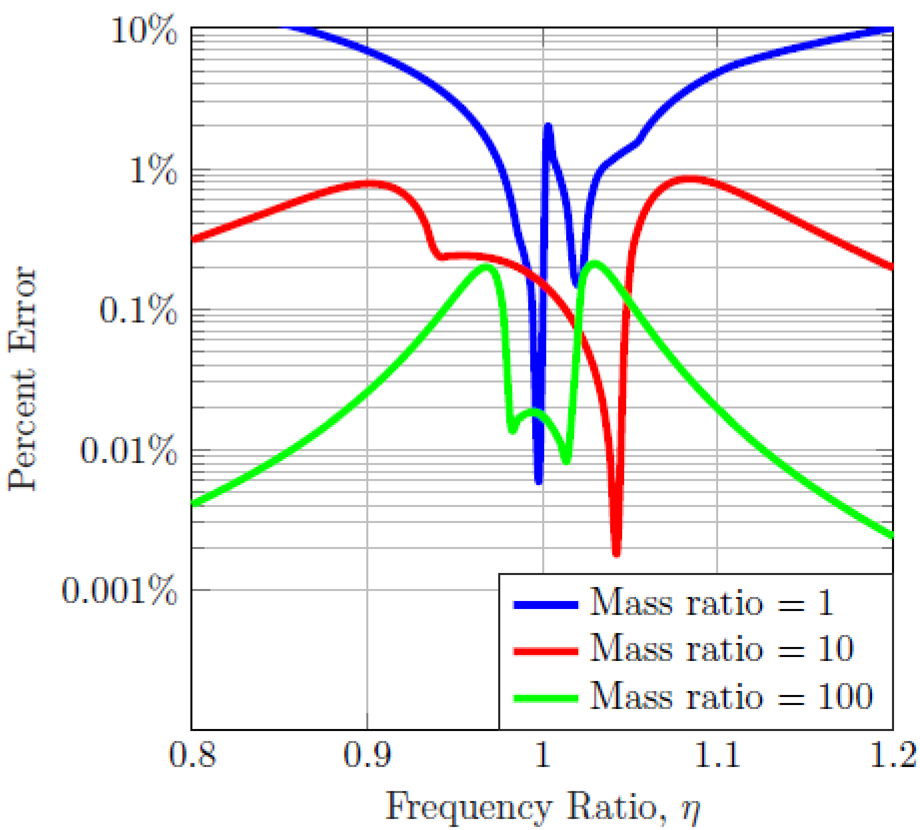
- Using a basis of pressure-release modes can lead to slow convergence
- Enriching basis with a few rigid wall modes or kinematically admissible polynomials can speed convergence



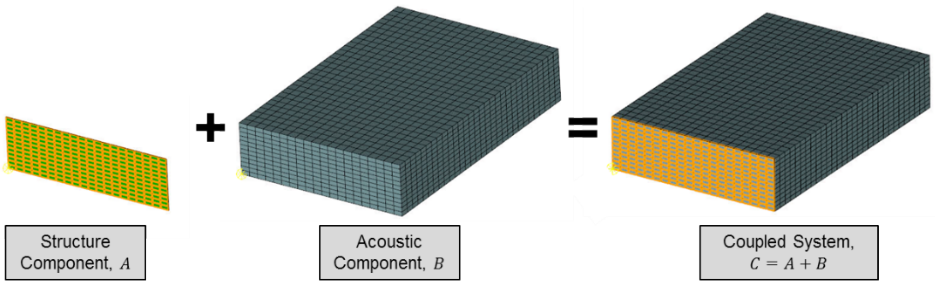
Example: Piston/Duct Decoupling

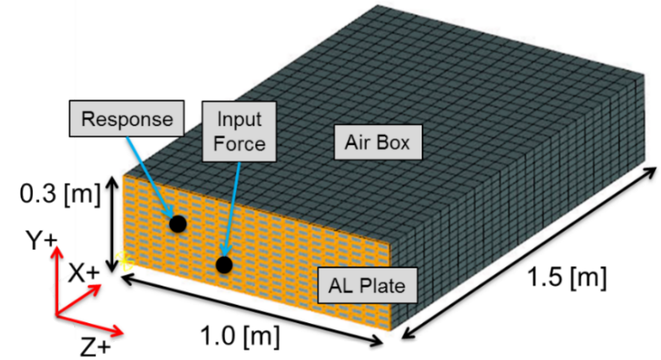


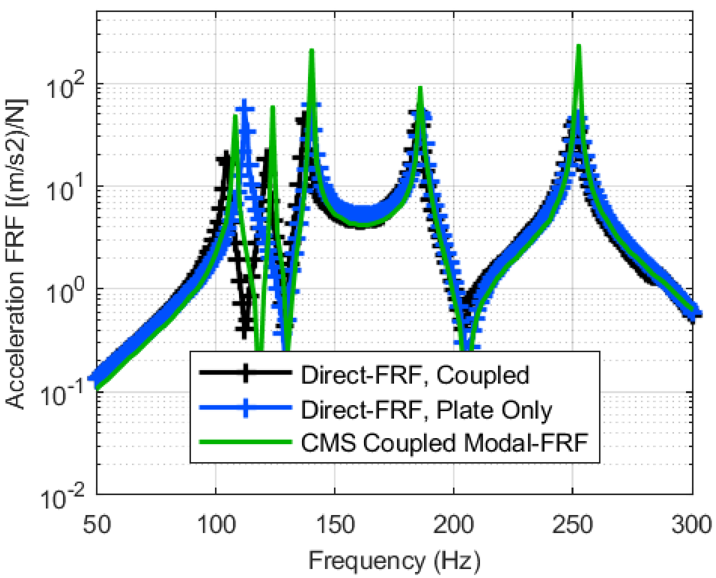
- Recovers accurate *in vacuo* natural frequencies when the system is well coupled
  - Accuracy improves with increasing structure-to-fluid mass ratio



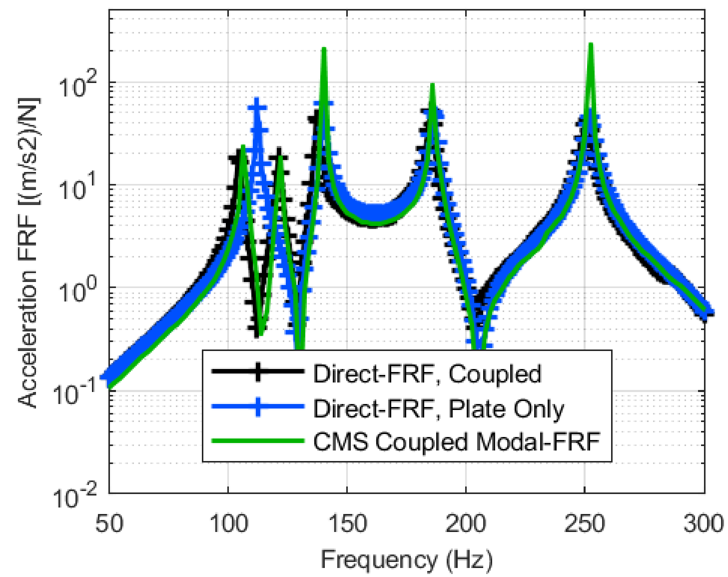
Example: Plate/Box Coupling







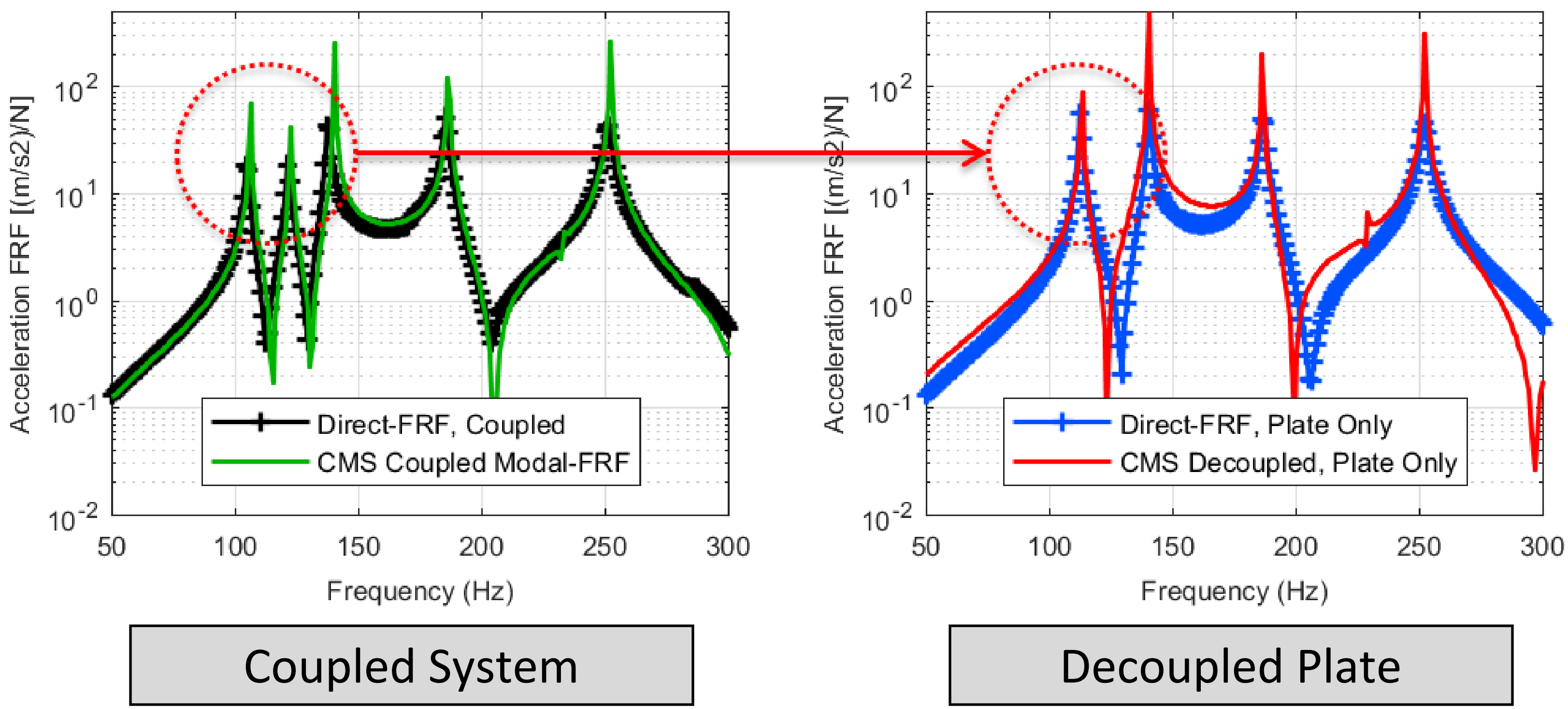
30 Plate + 10 Box Modes



50 Plate + 100 Box Modes

Example: Plate/Box Decoupling

- Coupling effect is removed and result approaches the truth plate-only FRF



Interface Reduction

- Often, the number of DOFs at fluid-structure interface is large
  - Coupling (or decoupling) problem quickly becomes over-constrained

- Possible to "soften" constraints by pre-multiplying by the pseudo-inverse of the acoustic mode matrix

$$[\nabla\Phi_A]^{\dagger}[a]\begin{bmatrix}[\nabla\Phi_A] & 0 \\ 0 & [\Phi_S]\end{bmatrix}\left\{\begin{array}{c} q_A \\ q_C \end{array}\right\} = \left\{\begin{array}{c} 0 \\ 0 \end{array}\right\}$$

- This approach worked well for first mode in plate/box example, but failed at higher modes

Approximate Decoupling Equation

The *in vacuo* natural frequency can be estimated using:

$$\omega_s = \omega_c \sqrt{\frac{\omega_r^2 - \omega_c^2 + \beta}{\omega_r^2 - \omega_c^2}}$$

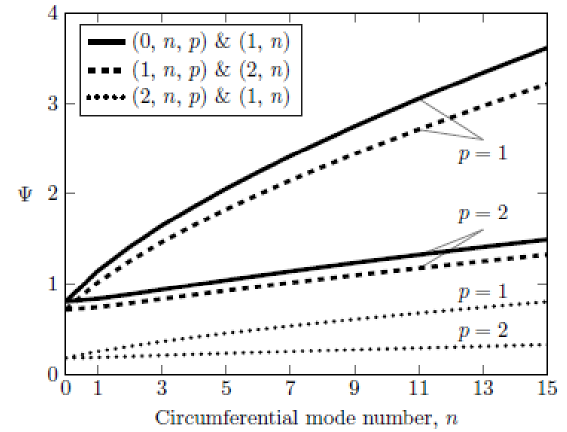
ω_c Coupled natural frequency

ω_r Rigid wall acoustic natural frequency

$\beta = \frac{\rho_0 c_0^2 A_F}{\rho_s V}\Psi$ Coupling strength parameter

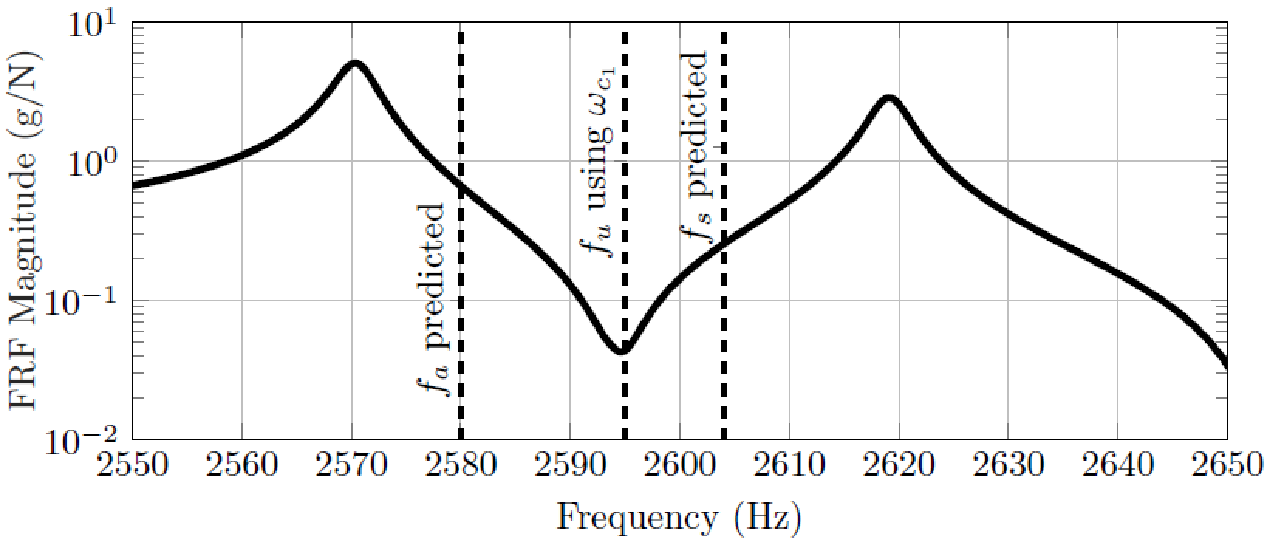
$\Psi = \frac{L^2}{\mu_s \mu_r}$ Non-dimensional component mode coupling number[3]

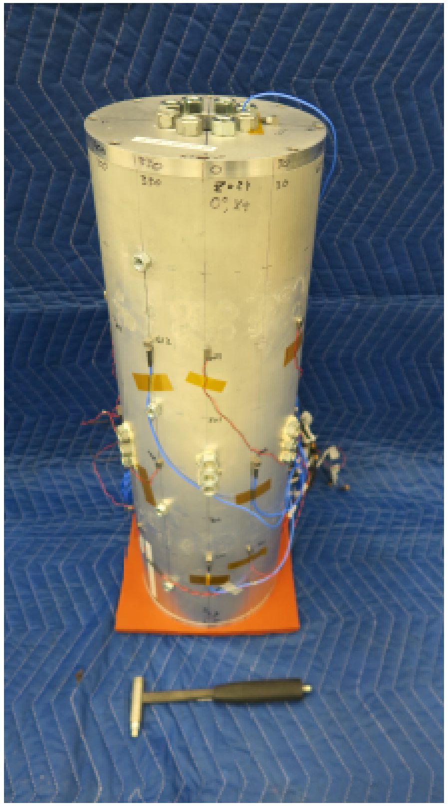
$L = \frac{1}{A_F}\int \phi_s \phi_r dA_F$ Coupling coefficient



[3]Davis, R. B. (2017). A simplified approach for predicting interaction between flexible structures and acoustic enclosures. *Journal of fluids and structures*, *70*, 276-294.

Decoupling Equation: Results





Predicted in vacuo frequency using approximate decoupling expression	Percent error relative to model prediction
2594 (Hz)	-0.37%

Conclusions

- **Desired Application:** Given coupled FRFs and uncoupled acoustic model, subtract off acoustic part to obtain the structure-only FRF
- A component mode synthesis approach was applied to coupling & decoupling problems
  - Demonstrated with:
    - Piston/duct
    - Plate/box
- Observed numerical issues when more than a few constraint equations are present
  - Better interface reduction approaches are needed
- Approximate expression shown to circumvent numerical issues