SAND2022-6213C

INCORPORATING ELECTRON INJECTION VELOCITY INTO THE TRANSITION FROM FIELD EMISSION TO SPACE-CHARGE LIMITED EMISSION WITH COLLISIONS

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This material is based upon work supported by the Sandia National Laboratories (SNL) and a Purdue Doctoral Fellowship. SNL is managed and operated by NTESS under DOE NNSA contract DE-NA0003525.

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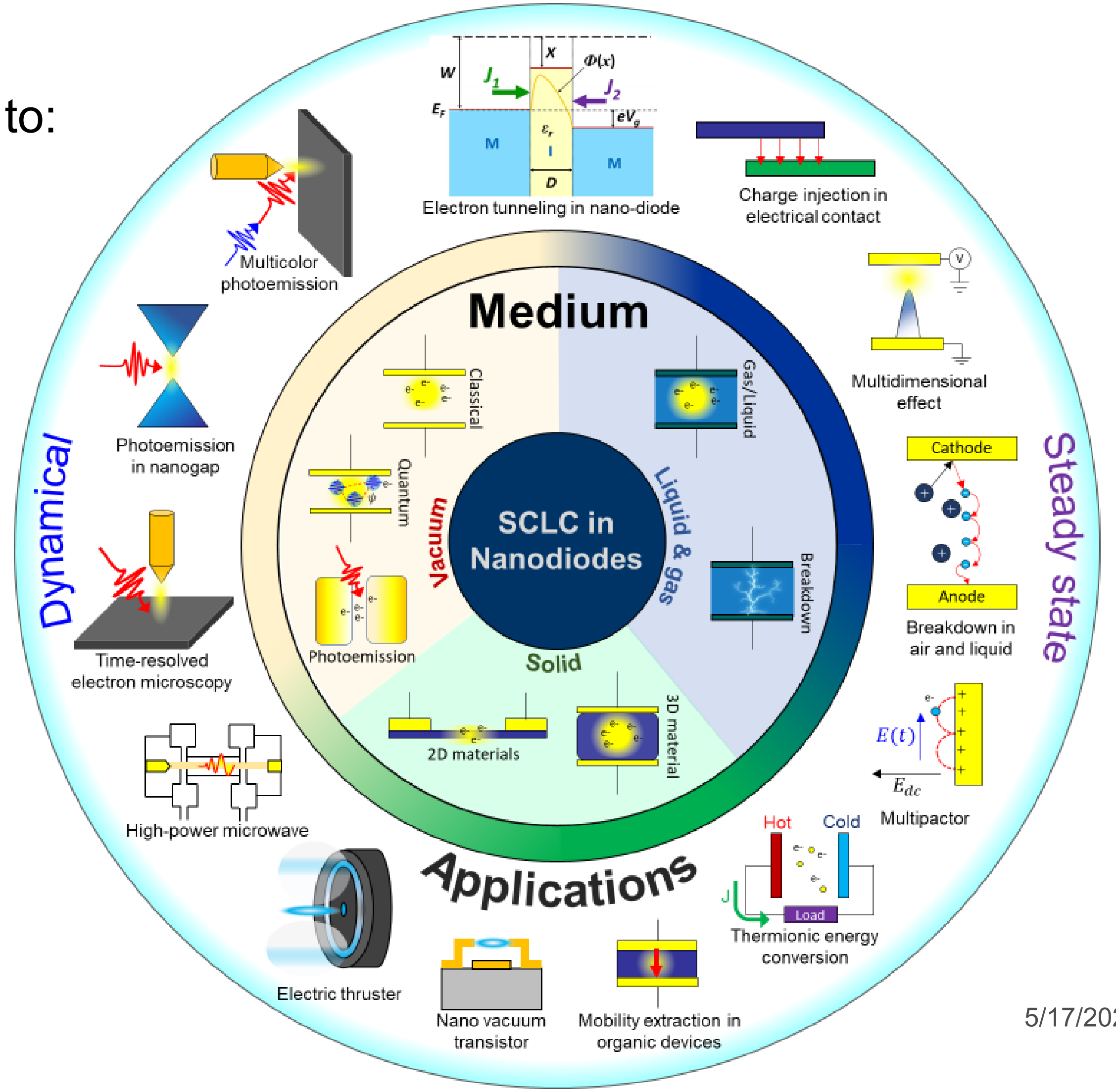
Introduction

Emission limits can be significant for various applications.

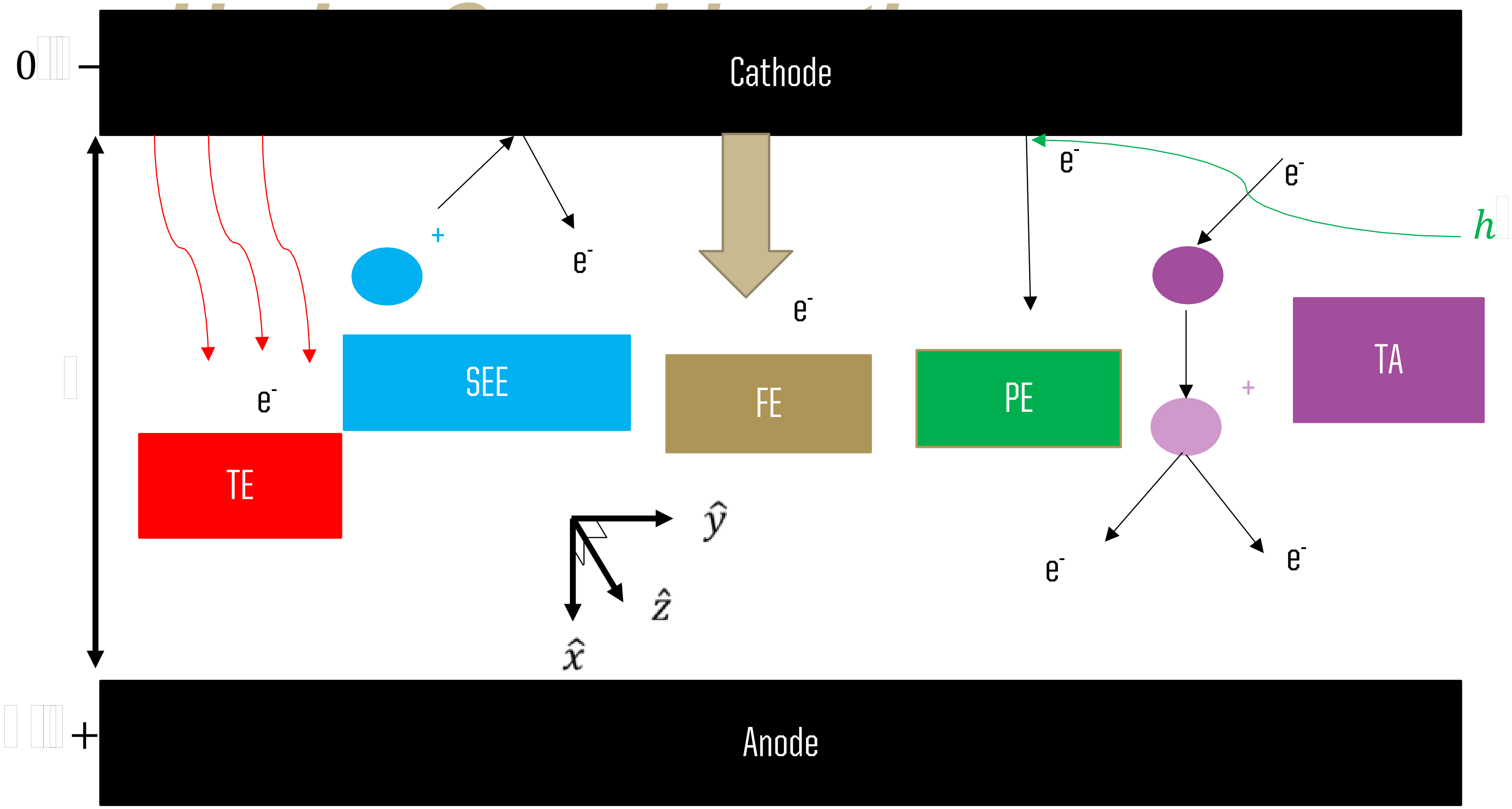
Diode applications include but are not excluded to:

- Directed energy systems
- High power vacuum systems
- Time-resolved electron microscopy
- X-ray systems

P. Zhang, Y. S. Ang, A. L. Garner, Á. Valfells, J. W. Luginsland, and L. K. Ang, J. Appl. Phys. **129**, 100902 (2021).



Introduction: Emission Mechanisms



A. L. Garner, G. Meng, Y. Fu, A. M. Loveless, R. S. Brayfield II, and A. M. Darr, J. Appl. Phys. **128**, 210903 (2020).

S. A. Lang, A. M. Darr, and A. L. Garner, J. Vac. Sci. Technol. B **39**, 062808 (2021).

Nexus Theory: Emission Equations

Electron Emission Single Mechanisms

Emission Type	Name	Equation	Dimensionless Equation
Field Emission	Fowler-Nordheim (FN)	$J_{FN} = AE_s^2 exp(-B/E_s)$	$J_{FN} = \bar{E}^2 \exp(-1/\bar{E})$
Space Charge Limited Emission	Vacuum: Child-Langmuir (CL)	$J_{CL} = \left(\frac{4\sqrt{2}}{9D^2}\right)(V^{3/2})$	$J_{CL} = \left(\frac{4\sqrt{2}}{9}\right)\frac{\bar{V}^{3/2}}{\bar{D}^2}$
	Pressure: Mott-Gurney (MG)	$J_{MG} = \frac{9}{8}\mu\epsilon_0\epsilon_r\frac{V^2}{D^3}$	$J_{MG} = \frac{9}{8}\frac{\bar{\mu}\bar{V}^2}{\bar{D}^3}$
Thermionic Emission	Richardson-Laue-Dushman (RLD)	$J_{RLD} = A_{RLD}T^2 \exp\left(-\frac{\Phi}{k_B T}\right)$	$J_{RLD} = \bar{T}^2 \exp\left(-\frac{1}{\bar{T}}\right)$

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S. A. Lang, A. M. Darr, and A. L. Garner, J. Vac. Sci. Technol. B **39**, 062808 (2021).

Unification of FE and SCL with Collisions

Previous work considered two emission models (FN-CL[1] and MG-CL[2]) to derive connections between all three (FE-MG-CL[3])

First Principles: $\frac{d^2\phi}{dx^2} = \frac{\rho}{\epsilon_0}$; ▯ = ▯▯;

$$m\frac{dv}{dt} = e\frac{d\phi}{dx} - \frac{ev}{\mu}$$

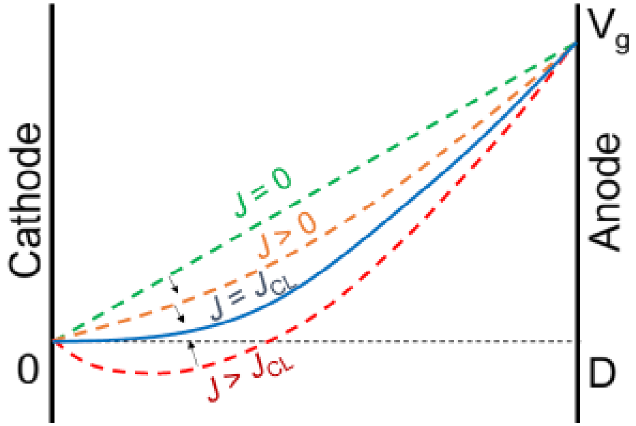
Guiding Equations: $\bar{J} = \frac{d^2\bar{v}}{d\bar{t}^2} + \frac{1}{\bar{\mu}}\frac{d\bar{v}}{d\bar{t}}$; $\bar{J}_{FN} = \bar{E}^2 e^{-1/\bar{E}}$

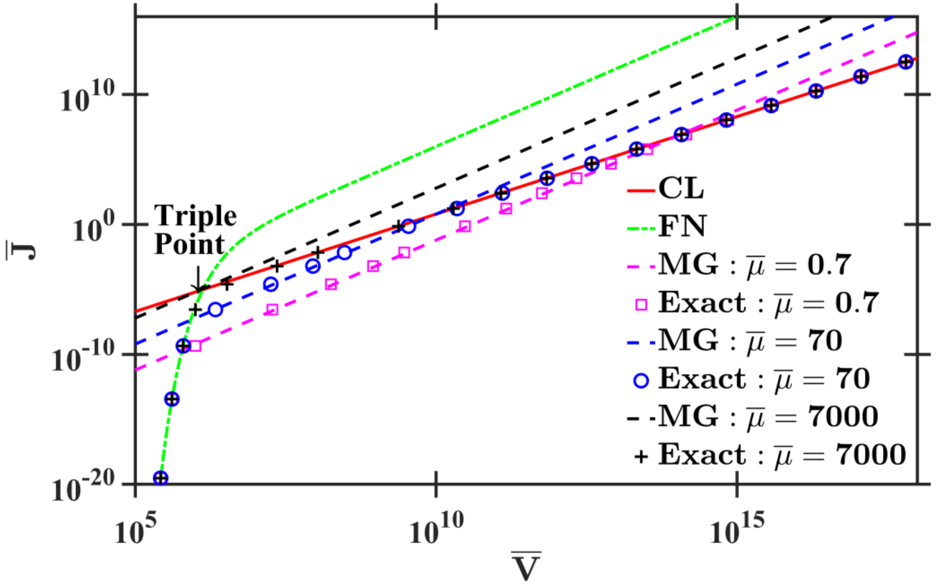
Nondimensionalizing and enforcing boundary conditions gives:

Full Solution!

$$\bar{V} = \frac{\bar{v}(\bar{t})^2}{2}\Bigg|_0^{\bar{T}} + \int_0^{\bar{T}} d\bar{t}\,\frac{\bar{v}(\bar{t})^2}{\bar{\mu}}$$

P. Zhang, A. Valfells, L. K. Ang, J. W. Luginsland, and Y. Y. Lau, Appl. Phys. Rev. **4**, 011304 (2017).





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[1] Y. Y. Lau, Y. Liu, and R. K. Parker, Phys. Plasmas **1**, 2082-2085 (1994).
[2] M. S. Benilov, Plasma Sources Sci. Technol. **18**, 014005 (2008).
[3] A. M. Darr, A. M. Loveless, and A. L. Garner, Appl. Phys. Lett. **114**, 014103 (2019).

Unification of SCLC with Thermionic Emission

Poisson's equation: $\nabla^2\phi = \frac{d^2\phi}{dx^2} = \frac{\rho}{\epsilon_0}$

Energy balance: $\frac{1}{2}mv^2 = e\phi + \frac{1}{2}mv_i^2 = e\phi + \frac{1}{2}k_B T$

Continuity: $J = \rho v$

General-Thermal-Field: $J_{GTF}(F,T) = A_{RLD}T^2N(n,s)$

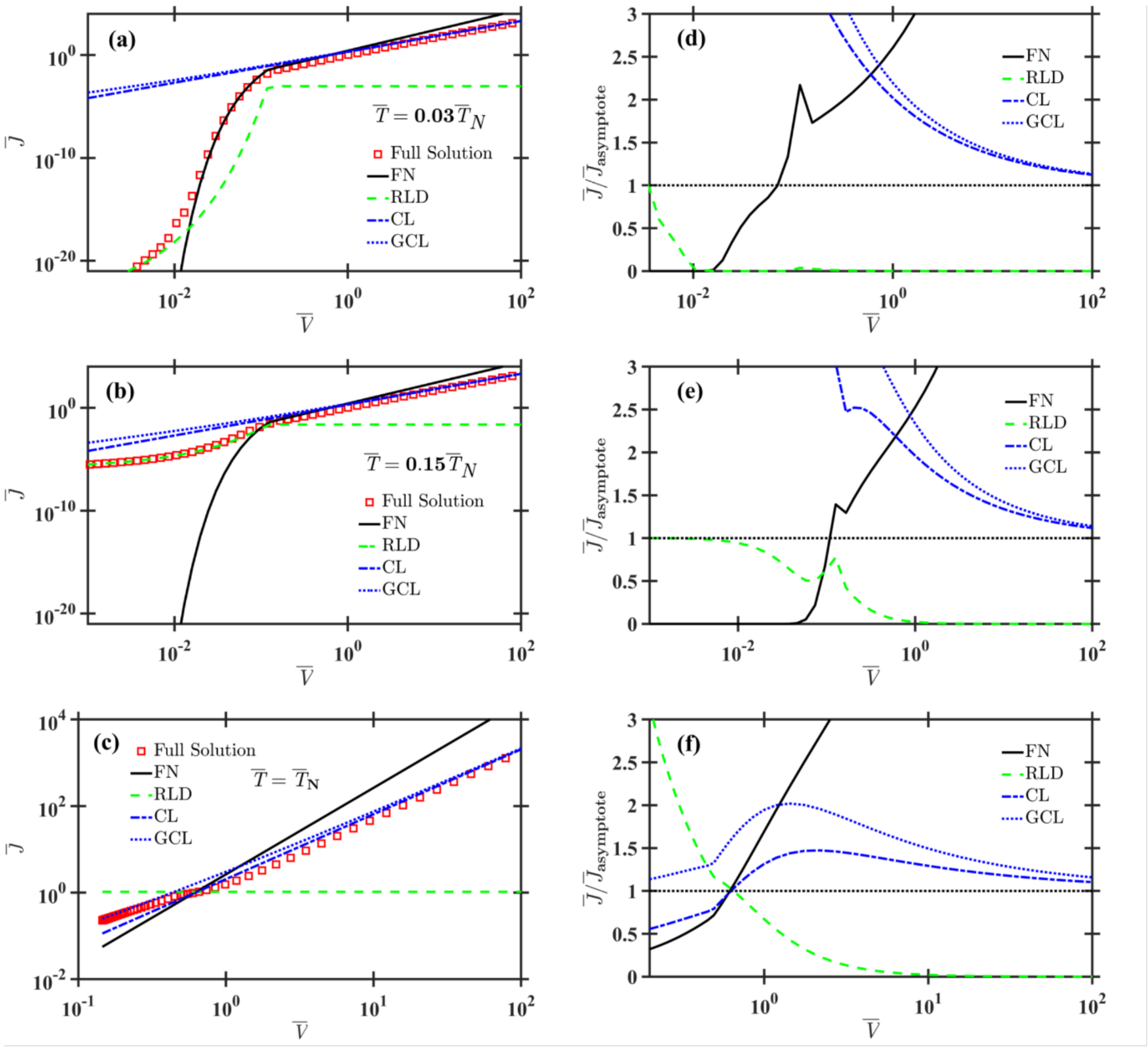
where $A_{RLD} = (emk_B^2)/(2\pi^2\hbar^3)$

$$\boldsymbol{N(n,s)} \approx e^{-s}n^2\Sigma\left(\frac{1}{n}\right) + e^{-ns}\Sigma(n)$$

$$\Sigma(\boldsymbol{n}) \approx \frac{1+x^2}{1-x^2} + \left(\frac{\pi^2}{6} - 2\right)x^2$$

$$\boldsymbol{s} = \beta_F(E_M)(E_0 - \mu)$$

$$\boldsymbol{\beta_F} = \begin{cases} \beta_F(\mu+\varphi) \equiv \pi\sqrt{m\Phi y}/(\hbar F) & n < 1 \\ \beta_T & n = 1 \\ \beta_F(\mu) \equiv 2\sqrt{2m\Phi}t(y)/(\hbar F) & n > 1 \end{cases}$$



A. M. Darr, C. R. Darr, and A. L. Garner, Phys. Rev. Res. **2**, 033137 (2020).

Unification of FE and SCL with Collisions AND Injection Velocity

- Starting from two emission models (FN-CL[1] and MG-CL[2]) to derive connections between all three (FE-MG-CL[3])

First Principles: $$\frac{d^2\phi}{dx^2} = \frac{\rho}{\epsilon_0};\ \square = \square\square;\ m\frac{dv}{dt} = e\frac{d\phi}{dx} - \frac{ev}{\mu}$$

Guiding Equations: $$\bar{J} = \frac{d^2\bar{v}}{d\bar{t}^2} + \frac{1}{\bar{\mu}}\frac{d\bar{v}}{d\bar{t}};\ \bar{J}_{FN} = \bar{E}^2 e^{-1/\bar{E}}$$

Initial Conditions: $$\bar{x}(0) = 0, \color{red}{\bar{v}(0) = \bar{v}_0}, \frac{d\bar{v}}{d\bar{t}}(0) = \bar{E}$$

Resultant: $$\bar{x}(\bar{t}) = \bar{\mu}\left[(\bar{\mu}\bar{J} - \bar{E})\left(-\bar{\mu}e^{-\bar{t}/\bar{\mu}} - \bar{t} + \bar{\mu}\right) + \bar{J}\bar{t}^2/2\right] \color{red}{+ \bar{v}_0\bar{t}}$$
$$\bar{v}(\bar{t}) = \bar{\mu}\left[(\bar{\mu}\bar{J} - \bar{E})\left(e^{-\bar{t}/\bar{\mu}} - 1\right) + \bar{J}\bar{t}\right] \color{red}{+ \bar{v}_0}$$

Boundary Conditions: $$\bar{x}(\bar{T}) = \bar{D};\ \bar{\phi}(\bar{T}) = \bar{V}$$

Full Solution! $$\bar{V} = \left.\frac{\bar{v}(\bar{t})^2}{2}\right|_0^{\bar{T}} + \int_0^{\bar{T}} d\bar{t}\,\frac{\bar{v}(\bar{t})^2}{\bar{\mu}}$$

Variable	Scaling Constant
Electric potential ϕ	$\frac{e\epsilon_0^2}{mA_{FN}{}^2}$
Current density J	$A_{FN}B_{FN}^2$
Position x	$\frac{e\epsilon_0^2}{mA_{FN}{}^2 B_{FN}}$
Time t	$\frac{\epsilon_0}{A_{FN}B_{FN}}$
Electron mobility μ	$\frac{e\epsilon_0}{mA_{FN}B_{FN}}$
Electric field E	B_{FN}
Velocity v	$\frac{e\epsilon_0}{mA_{FN}}$

[1] Y. Y. Lau, Y. Liu, and R. K. Parker, Phys. Plasmas **1**, 2082-2085 (1994).
[2] M. S. Benilov, Plasma Sources Sci. Technol. **18**, 014005 (2008).
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SCLC with Injection Velocity

CHILD-LANGMUIR

Original Resultant ($\bar{v}_0 = 0$):

$$\bar{J}_{CL} = \frac{4\sqrt{2}\bar{V}^{3/2}}{9\bar{D}^2}$$

Resultant w/ injection velocity ($\bar{v}_0 \neq 0$):

$$\bar{J}_{GCL} = \frac{4\sqrt{2}\bar{V}^{3/2}}{9\bar{D}^2}\left(\left(\sqrt{1+\frac{\bar{v}_0^2}{2\bar{V}}}-\sqrt{\frac{\bar{v}_0^2}{2\bar{V}}}\right)\left(\sqrt{1+\frac{\bar{v}_0^2}{2\bar{V}}}+2\sqrt{\frac{\bar{v}_0^2}{2\bar{V}}}\right)\right)^2 \quad **$$

MOTT-GURNEY

Original Resultant ($\bar{v}_0 = 0$):

$$\bar{J}_{MG} = \frac{9\bar{\mu}\bar{V}^2}{8\bar{D}^3}$$

Resultant w/ injection velocity ($\bar{v}_0 \neq 0$):

$$\bar{J}_{GMG} = \frac{9\bar{\mu}\bar{V}^2}{8\bar{D}^3}\left(\frac{1}{2}-\frac{2\bar{D}^2\bar{v}_0^2}{3\bar{\mu}^2\bar{V}^2}+\frac{1}{2}\left[1-\frac{16\bar{D}^4\bar{v}_0^4}{27\bar{\mu}^4\bar{V}^4}+\frac{64\bar{D}^3\bar{v}_0^3}{27\bar{\mu}^3\bar{V}^3}-\frac{8\bar{D}^2\bar{v}_0^2}{3\bar{\mu}^2\bar{V}^2}\right]^{1/2}\right)$$

A. M. Darr, C. R. Darr, and A. L. Garner, Phys. Rev. Research **2, 033137 (2020).

Mott-Gurney versus General Mott-Gurney

MOTT-GURNEY

$$\bar{\mu} \to 0$$
$$\bar{v}(\bar{T}) \approx \bar{\mu}(\bar{E} + \bar{J}\bar{T})$$
$$\bar{x} \approx \bar{\mu}\left(\bar{E}\bar{T} + \frac{\bar{J}\bar{T}^2}{2}\right)$$

$$\bar{x}[\bar{T}] = \bar{D}$$
$$\bar{T} = \frac{\bar{E}}{\bar{J}}\left(-1 + \sqrt{\frac{2\bar{D}\bar{J}}{\bar{\mu}\bar{E}^2} + 1}\right) \approx \frac{\bar{E}}{\bar{J}}\left(\sqrt{\frac{2\bar{D}\bar{J}}{\bar{\mu}\bar{E}^2}}\right)$$

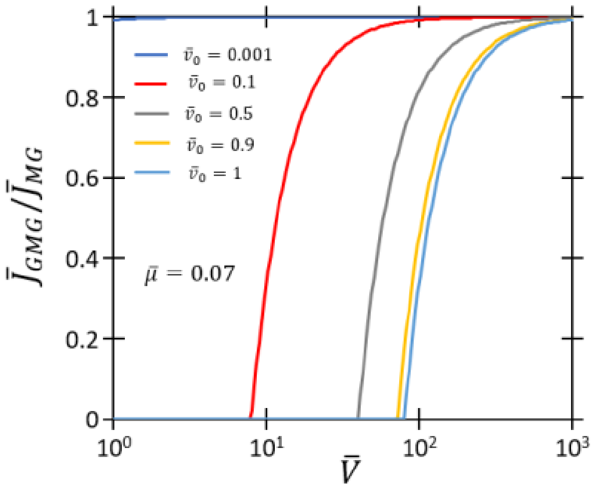
$$\bar{J}_{MG} = \frac{9\bar{\mu}\bar{V}^2}{8\bar{D}^3}$$

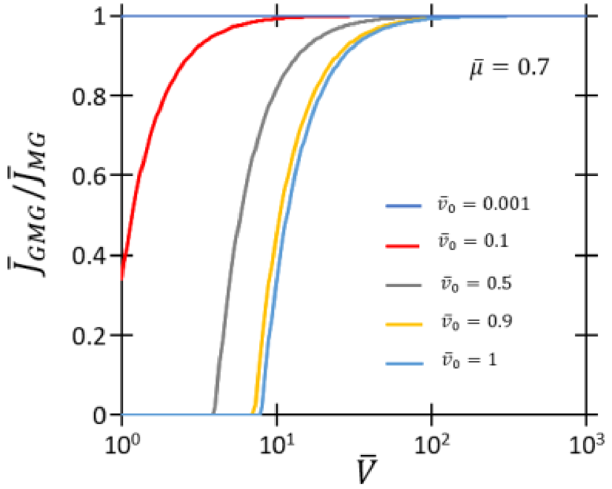
GENERAL MOTT-GURNEY

$$\bar{\mu} \to 0$$
$$\bar{v}(\bar{T}) \approx \bar{\mu}(\bar{E} + \bar{J}\bar{T}) + v_0$$
$$\bar{x} \approx \bar{\mu}\left(\bar{E}\bar{T} + \frac{\bar{J}\bar{T}^2}{2}\right) + v_0\bar{T}$$

$$\bar{x}[\bar{T}] = \bar{D}$$
$$\bar{T} = \frac{\bar{E}}{\bar{J}}\left(-1 - \frac{v_0}{\bar{E}\bar{\mu}} + \sqrt{\frac{2\bar{D}\bar{J}}{\bar{\mu}\bar{E}^2} + \frac{(\bar{E}\bar{\mu} + v_0)^2}{\bar{\mu}^2\bar{E}^2}}\right)$$
$$\approx \frac{\bar{E}}{\bar{J}}\left(-\frac{v_0}{\bar{E}\bar{\mu}} + \sqrt{\frac{2\bar{D}\bar{J}}{\bar{\mu}\bar{E}^2} + \frac{v_0^2}{\bar{\mu}^2\bar{E}^2}}\right)$$

$$\bar{J}_{GMG} = \frac{9\bar{\mu}\bar{V}^2}{8\bar{D}^3}\left(\frac{1}{2} - \frac{2\bar{D}^2\bar{u}_0^2}{3\bar{\mu}^2\bar{V}^2} + \frac{1}{2}\left[1 - \frac{16\bar{D}^4\bar{u}_0^4}{27\bar{\mu}^4\bar{V}^4} + \frac{64\bar{D}^3\bar{u}_0^3}{27\bar{\mu}^3\bar{V}^3} - \frac{8\bar{D}^2\bar{u}_0^2}{3\bar{\mu}^2\bar{V}^2}\right]^{1/2}\right)$$





Relativistic velocity is when the velocity is a significant fraction of the speed of light. Therefore, the dimensionless $\bar{v}_0$ must be set below 2.1996 to avoid the relativistic regime (2.1996 correlates to 0.1×10^8 m/s). For reference, $\bar{v}_0$ of 1 is 4.5463×10^6 m/s

General Mott-Gurney Asymptotic Solutions

Consider $\chi = \frac{\bar{D}\bar{v}_0}{\bar{\mu}\bar{V}}$, $\bar{\mu} \ll 1$

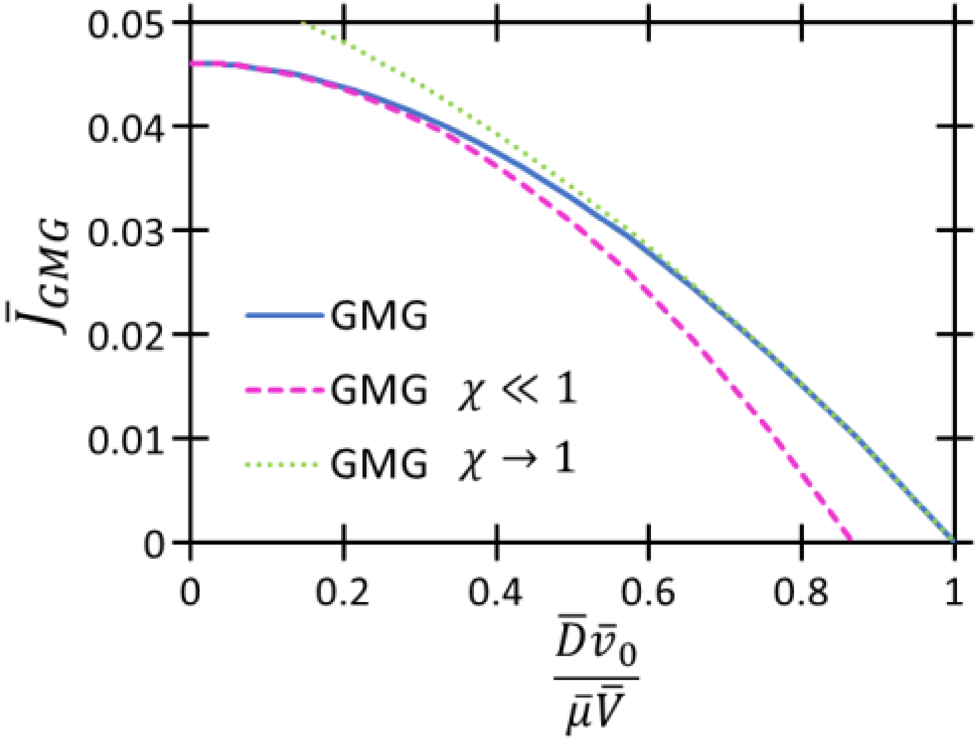
When $\chi \ll 1$ (but not 0) – low injection velocity

$$\bar{J}_{GMG} = \frac{9\bar{\mu}\bar{V}^2}{8\bar{D}^3}\left(\frac{1}{2} - \frac{2}{3}\chi^2 + \frac{1}{2}\left[1 - \frac{8}{3}\chi^2\right]^{1/2}\right) \approx \frac{9\bar{\mu}\bar{V}^2}{8\bar{D}^3}\left(1 - \frac{4}{3}\chi^2\right)$$

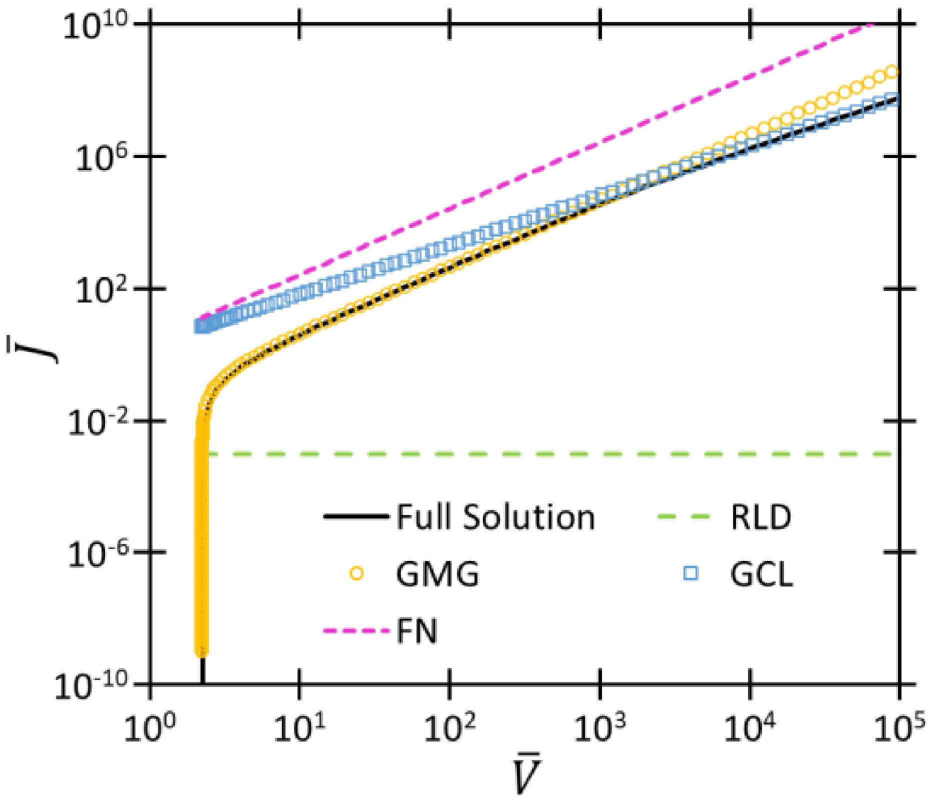
When $\chi \to 1$ – high injection velocity (but not relativistic)

$$\bar{J}_{GMG} \approx \frac{9\bar{\mu}\bar{V}^2}{8\bar{D}^3}\left(\frac{1}{2} - \frac{2}{3}\chi^2 + \frac{1}{6} - \frac{4}{9}(-1+\chi) + \frac{2}{27}(-1+\chi)^2\right)$$

$$\bar{J}_{GMG} \approx \frac{9\bar{\mu}\bar{V}^2}{8\bar{D}^3}\left(\frac{16}{9} - \frac{16}{9}\chi - \frac{16}{27}(-1+\chi)^2\right)$$

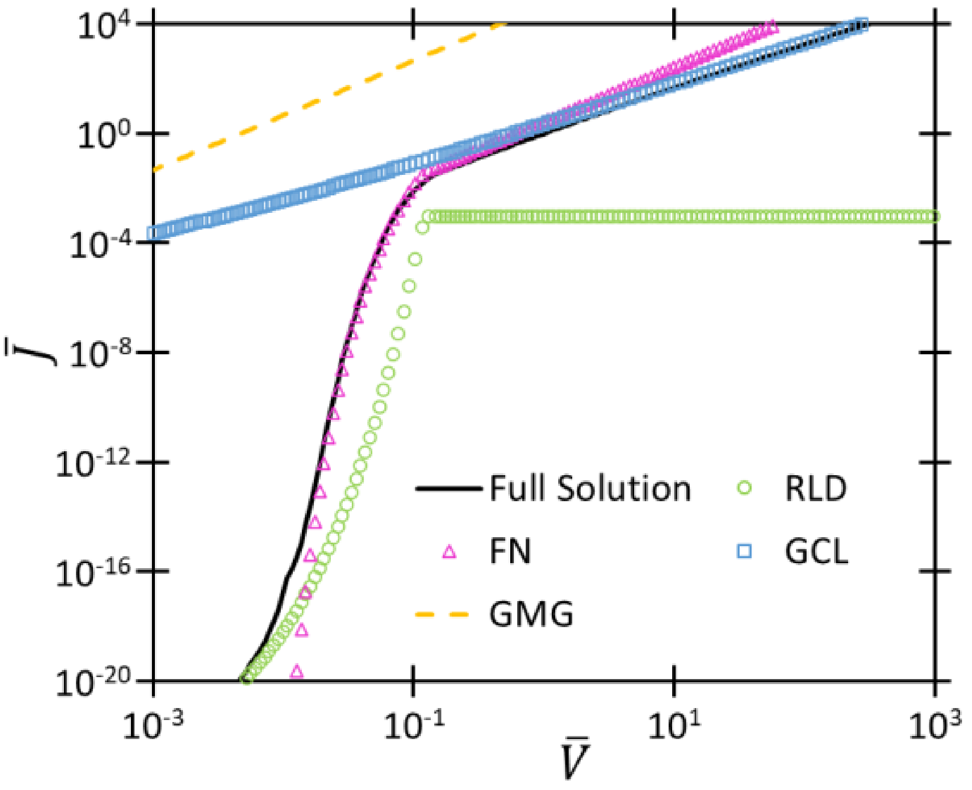


Nexus Theory: Correcting for Injection



$\bar{D} = 0.55, \bar{T} = 0.0204, \bar{\mu} = 0.007$

Demonstrates GMG to GCL



$\bar{D} = 0.55, \bar{T} = 0.0204, \bar{\mu} = 7000$

Demonstrates FN to GCL

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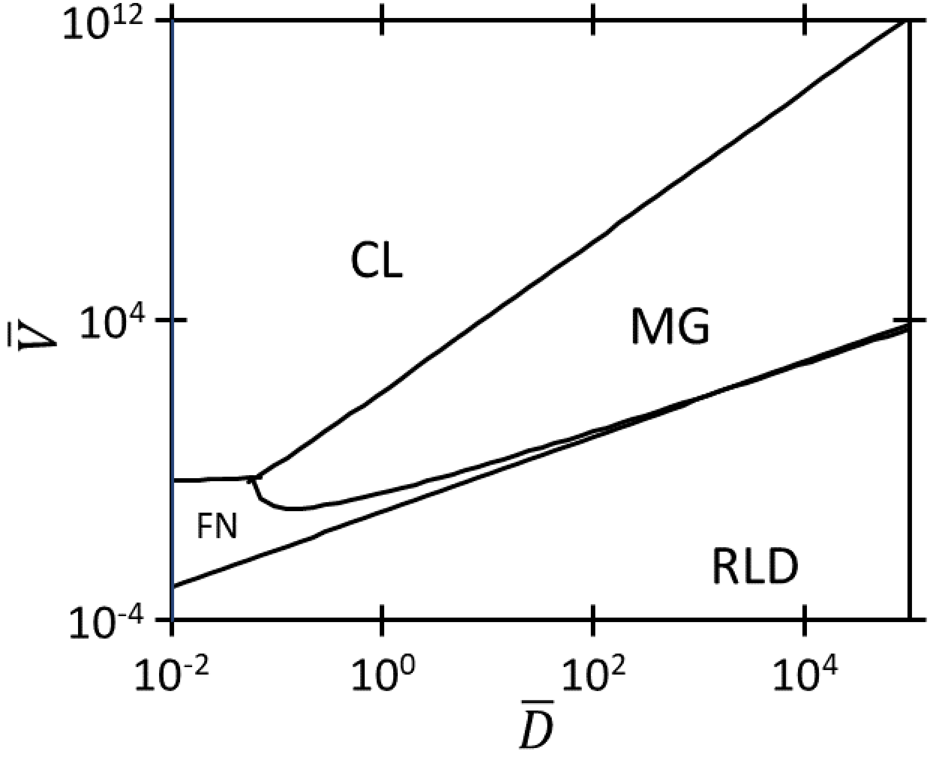
Results: $\bar{V}$ vs. $\bar{D}$ Nexus Phase Plot

Fixed Quantities

$\bar{\mu} = 0.05$

$\bar{T} = 7.5 \times 10^{-2}$

Incorporating $\bar{u}_0$ into the nexus theory eliminates the direct RLD to MG transition for the given gap range.



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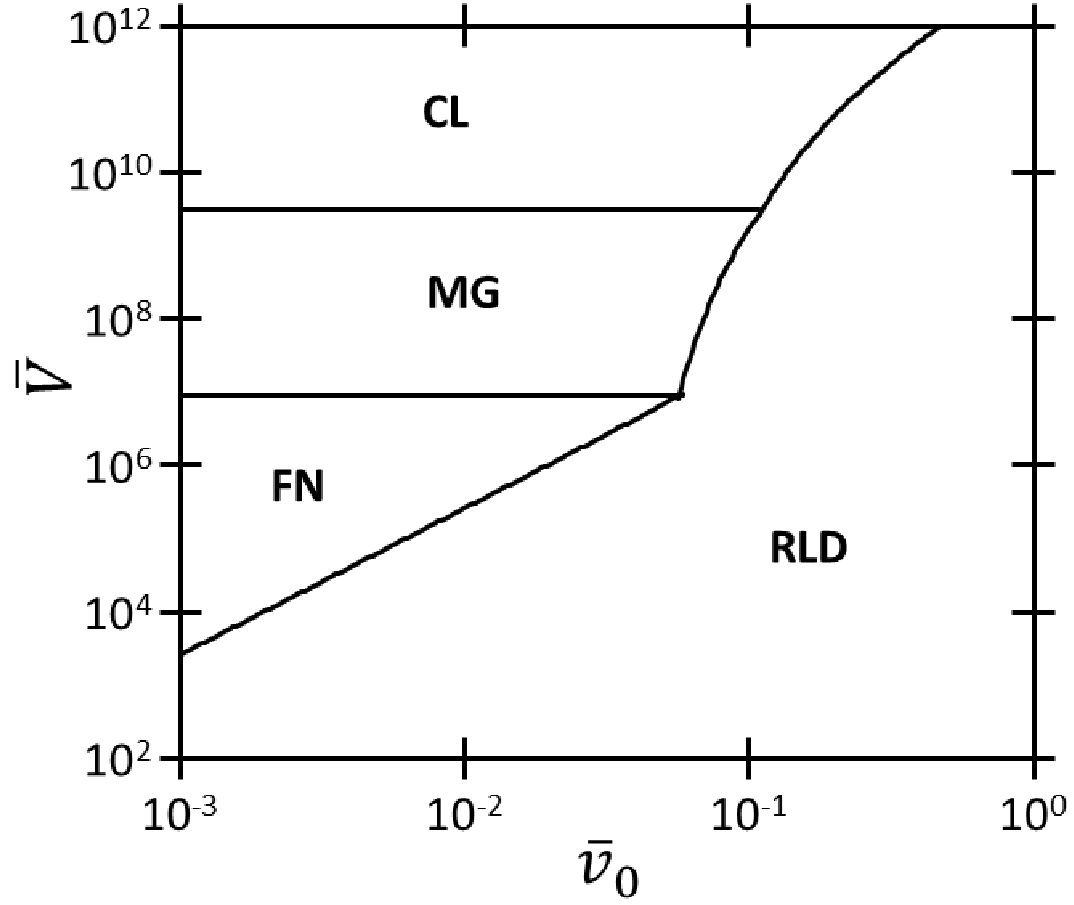
Results: $\overline{V}$ vs. $\overline{v}_0$ Nexus Phase Plot

Fixed Quantities

$$\bar{\mu} = 1000$$
$$\overline{D} = 10^8$$

Incorporating $\overline{u}_0$ into the nexus theory expands the Child-Langmuir (CL) and Mott-Gurney (MG) regimes.



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Conclusions and Future Work

Summary

- Expanded nexus theory to incorporate nonzero injection velocity into the SCLC regime
- Demonstrated asymptotic agreement with Fowler-Nordheim (FN), Mott-Gurney (MG), and Child-Langmuir (CL) in appropriate regimes
- **Future work:** Incorporate magnetic field perpendicular to electric field; consider crossed-field diode and include finding for general MG with injection velocity as the limit in the case of magnetic field going to zero.

THANK YOU!

We welcome you to direct questions to algarner@purdue.edu

Unification of Photo-, Thermionic, and Field Emission

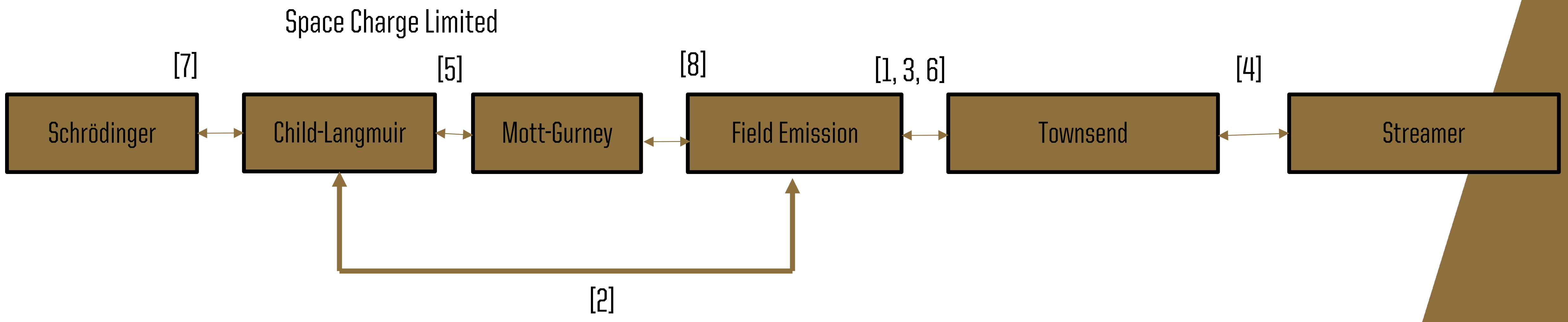
Nondimensionalization

$$\phi = \phi_0\bar{\phi}; J = J_0\bar{J}\,; x = x_0\bar{x}; t = t_0\bar{t}; \mu =, \mu_0\bar{\mu}; E = E_0\bar{E}\,; v = v_0\bar{v}; T = T_0\bar{T}$$

$$\phi_0 = \frac{e\epsilon_0^2}{mA^2}\ [\mathrm{V}];\ J_0 = AB^2\left[\frac{A}{m^2}\right]; x_0 = \frac{e\epsilon_0^2}{mA^2B}\ [m];$$

$$t_0 = \frac{\epsilon_0}{AB}\ [s]; \mu_0 = \frac{e\epsilon_0^2}{mAB}\left[\frac{m^2}{V\ s}\right]; E_0 = B\left[\frac{V}{m}\right]; v_0 = \frac{x_0}{t_0}\left[\frac{m}{s}\right]$$

Electron Emission Regime Connections



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