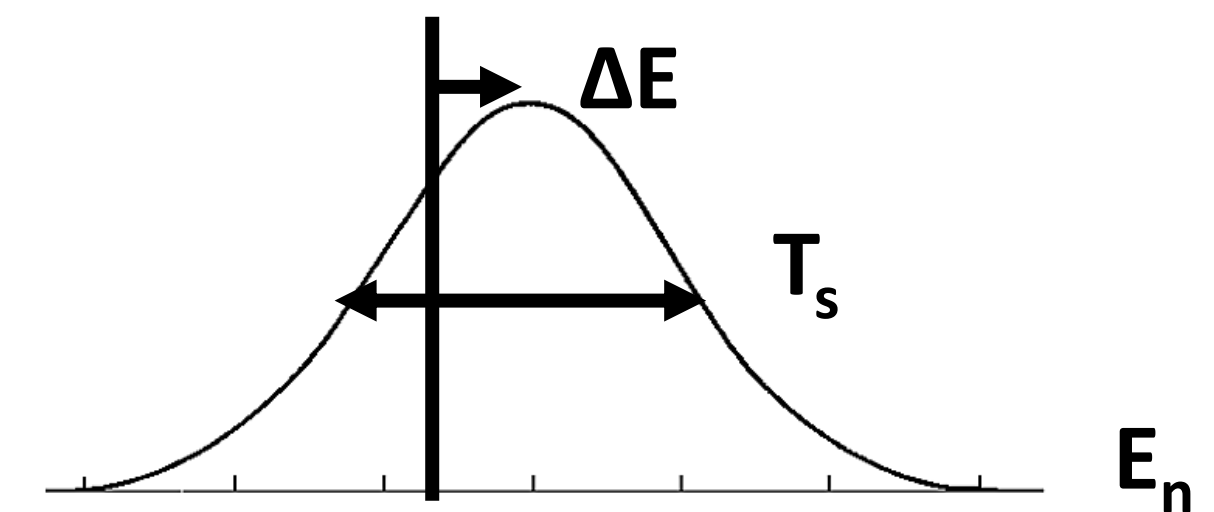


Constraints on Ion Velocity Distributions from Fusion Product Spectroscopy

The fundamentals of fusion product spectroscopy come from the reaction kinematics for the 2-to-2 body isotropic fusion reaction $1 + 2 \rightarrow 3 + 4$, the **relative velocity** and **Q-value** determine centre of mass frame product velocities and **centre of mass (CoM) velocities** introduce Doppler shifts to the lab frame outgoing particles:

$$E_3 = \frac{1}{2} m_3 v_{cm}^2 + \frac{m_4}{m_3 + m_4} (Q + K) + \cos \theta v_{cm} \sqrt{\frac{2m_3 m_4}{m_3 + m_4}} (Q + K) = \frac{1}{2} m_3 |\vec{u}_3 + \vec{v}_{cm}|^2$$

Summed effect of varying relative and centre of mass velocities \rightarrow



The above reaction kinematics describe the microscopic scale. Within a local picture, the spectral moments (mean shift and variance) are found from the ion velocity distributions. For isotropic* distributions:

$$\Delta E = \frac{m_4}{m_3 + m_4} \langle K \rangle + \frac{m_3}{2} \langle v_{cm}^2 \rangle + \dots$$

$$T_s = \frac{m_1 + m_2}{3} \langle v_{cm}^2 \rangle + \dots$$

Where $\langle \rangle$ define a reaction rate average. In the case of a stationary Maxwellian $T_s = T_i$ but this is a special case. Another unique property of single temperature Maxwellians is that they exhibit no correlation between K and v_{cm}^2 .

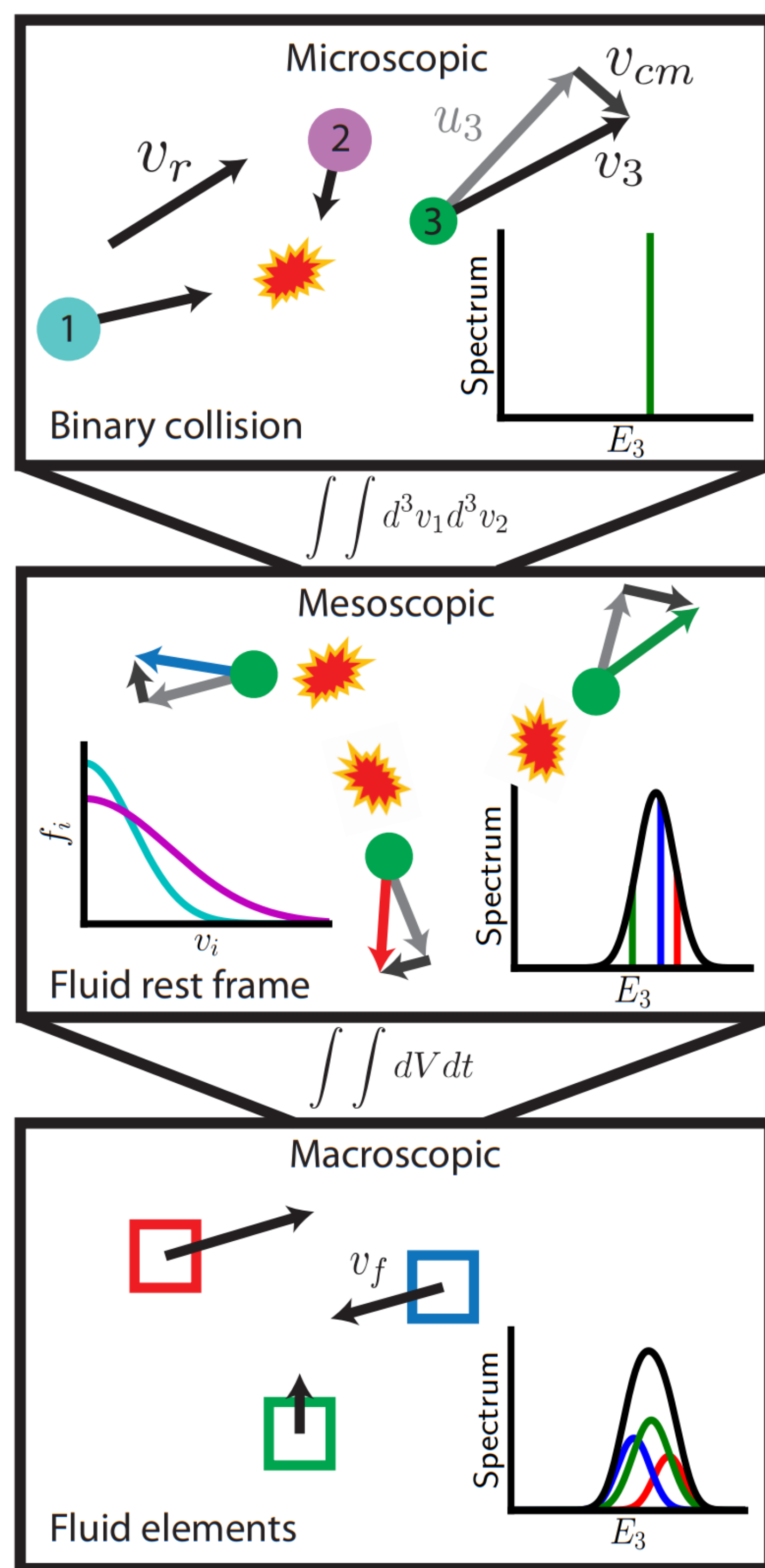
The fluid elements which make up the fusing plasma may have non-zero velocity themselves, if the mean particle velocity of the distribution is non-zero. Boosting the product spectra back to the lab frame introduces a Doppler shift:

$$\Delta E_m = \{\Delta E\} + m_3 v_0 \langle u_{||} \rangle \dots$$

$$T_{s,m} = \{T_s\} + (m_1 + m_2) \text{Var}(u_{||}) + \dots$$

Where $\{ \}$ defines a rest frame reaction rate average (for anisotropic distributions these moments are directionally dependent).

*moment expressions for anisotropic distributions have been derived, see pre-print

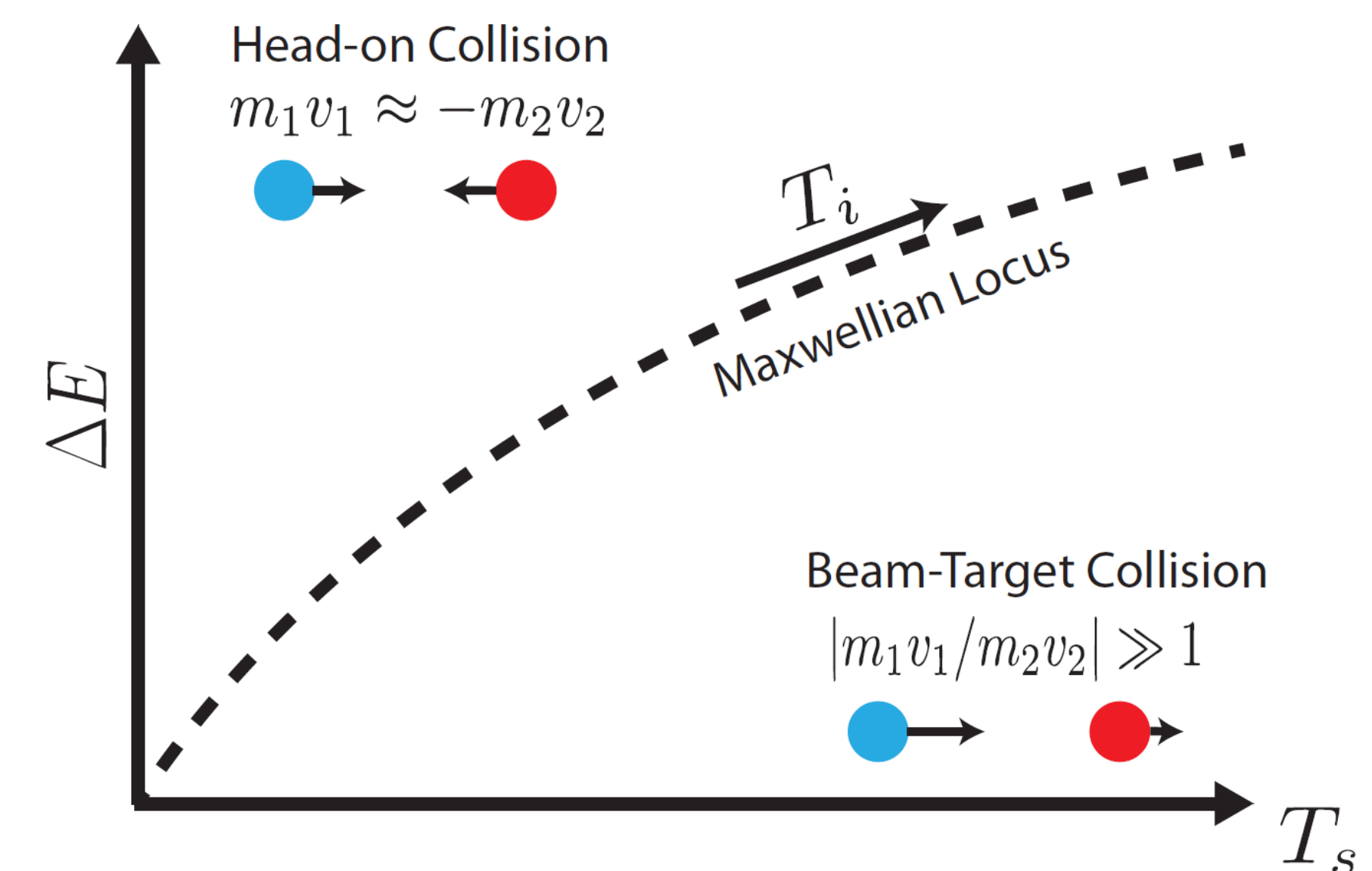


The spectral moments of fusion product spectra are directly related to the reactant kinematics. A large range in CoM velocities leads to Doppler broadening. High relative velocities, or relative kinetic energies $\langle K \rangle$, cause an upshift in the spectral mean. When considering a single reaction, momenta-matched collisions have zero CoM velocity at finite K . The opposite is then true for large momenta disparity in collisions, creating a beam-target like reaction.

Fluid velocities modify the spectral moments in a constrained manner. For the isotropic spectral moments, the spectral temperature is inflated.

Point 1 can only have been produced by anisotropic velocity distributions. Point 2 must have been produced by either isotropic and anisotropic distributions - the isotropic distributions must include a majority of momenta-matched reactions due to the proximity to the isotropic upper limit. Only point 3 could have been produced by hydrodynamic plasma. The exact positions of the constraint curves depend on the specific reaction.

We can use the coordinates in $(\Delta E, T_s)$ space to comment on the character of the collisions.



An upper limit on $\Delta E (T_s)$ for isotropic distributions is derived using velocity space shells with matched momenta

