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Modeling of diffusion in complex fractured porous media by using hierarchical material properties

Summary

The variable length scales of fractures and the heterogeneity of the surrounding porous rock impose high computational costs and excessive mesh refinement in fractured porous media modeling. To overcome these challenges, we utilize the hierarchical finite element method (*Hi*-FEM) that has been previously developed to model the electric potentials in complex geologic environments. By employing the hierarchical basis functions in the finite element analysis, *Hi*-FEM enables us to represent the material properties on each dimensional component of a 3D unstructured tetrahedral mesh and inherently allow the fracture-rock interactions. Here, the application of *Hi*-FEM is extended for the physics of transient fluid flow and heat conduction. The diffusion equation is solved in the Laplace domain and the time-domain solutions are obtained by using the numerical inverse Laplace transform. We consider a set of benchmark models to test *Hi*-FEM's accuracy and basin-scale rock mass models with embedded complex fracture networks to evaluate its robustness and computational performance. Results show that *Hi*-FEM is computationally economical and numerically robust even for basin-scale simulations without the need of coupling or transfer mechanisms.

Hierarchical Finite Element Method (*Hi*-FEM)

The transient flow in 3D porous media is given by the diffusion equation and can be solved in the Laplace domain,

$$-\nabla\cdot(\mathbf{K}\cdot\nabla h)+\mathrm{S}_s\frac{\partial h}{\partial t}=q_f \longrightarrow -\nabla\cdot(\mathbf{K}\cdot\nabla\bar{h})+p\mathrm{S}_s\bar{h}=\bar{q}_f$$

where $\mathbf{K}$ is a rank-2 conductivity tensor; h is hydraulic head, S_s is specific storage, q_f is volumetric flow rate, t is time, p is the complex-valued Laplace parameter and the superscript bar denotes the transformed variables.

Hi-FEM has been previously developed to model electrical conduction in geologic media and represents material properties at each dimensional component of a 3D tetrahedral finite element mesh (Weiss, 2017):

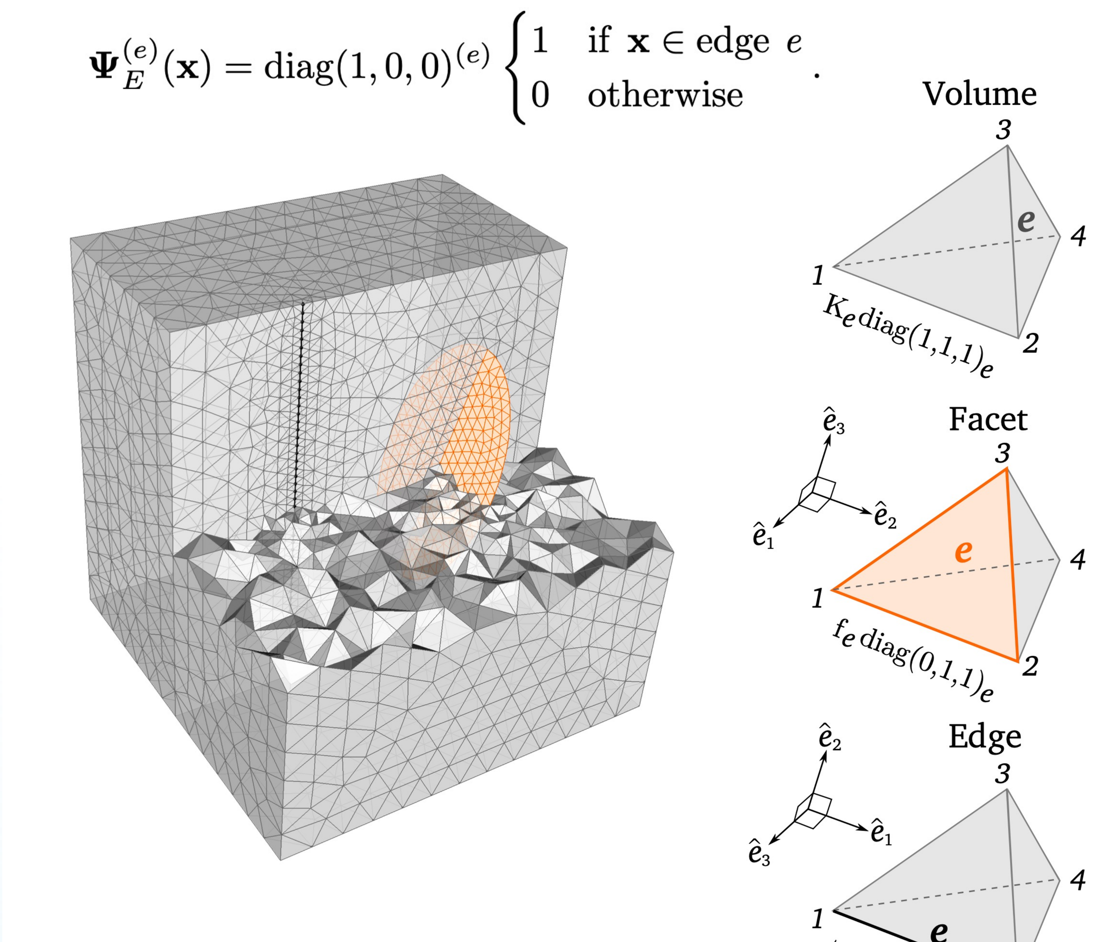
$$\mathbf{K}(\mathbf{x})=\sum_{e=1}^{n_V}\mathrm{K}^{(e)}\mathbf{\Psi}_V^{(e)}(\mathbf{x})+\sum_{e=1}^{n_F}\mathrm{f}^{(e)}\mathbf{\Psi}_F^{(e)}(\mathbf{x})+\sum_{e=1}^{n_E}\mathrm{t}^{(e)}\mathbf{\Psi}_E^{(e)}(\mathbf{x})$$

with the following hierarchical rank-2 local basis functions

$$\mathbf{\Psi}_V^{(e)}(\mathbf{x})=\mathrm{diag}(1,1,1)^{(e)}\begin{cases}1 & \text{if } \mathbf{x}\in \text{volume } e\\ 0 & \text{otherwise}\end{cases},$$

$$\mathbf{\Psi}_F^{(e)}(\mathbf{x})=\mathrm{diag}(0,1,1)^{(e)}\begin{cases}1 & \text{if } \mathbf{x}\in \text{facet } e\\ 0 & \text{otherwise}\end{cases},$$

$$\mathbf{\Psi}_E^{(e)}(\mathbf{x})=\mathrm{diag}(1,0,0)^{(e)}\begin{cases}1 & \text{if } \mathbf{x}\in \text{edge } e\\ 0 & \text{otherwise}\end{cases}.$$



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Hereby, *Hi*-FEM allows us to simulate geologic models with important details at a wide range of length scales in a computationally cost-effective way.

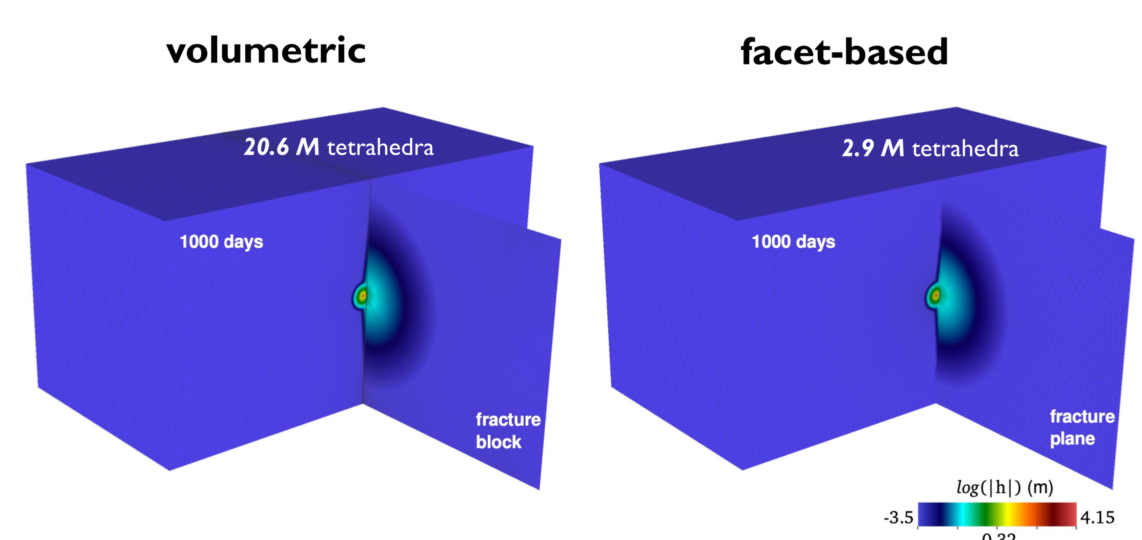
Fluid Flow

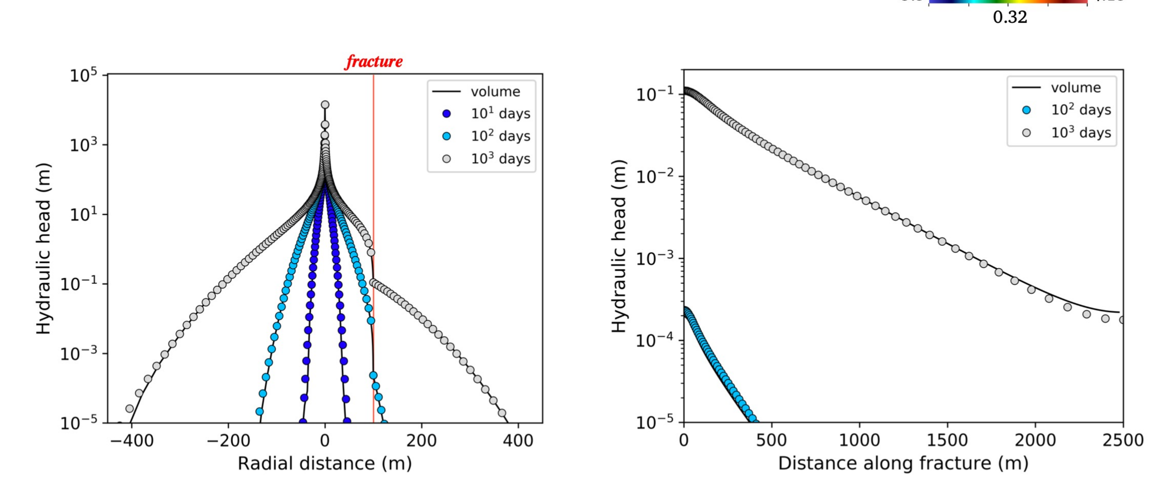
A vertical fracture model

We consider a fracture model to validate the equivalency of *Hi*-FEM's facet-based representation of fractures to a volumetric one. The fracture with a 1 m aperture cuts the 10 x 10 x 5 km porous host rock. The constant flow rate is 100 m³/day.

	Fracture	Host rock
Specific storage, S_s :	2x10^{-3} m^{-1}	2x10^{-4} m^{-1}
Hydraulic conductivity, K :	10^{-4} m/s	10^{-7} m/s
Hydraulic conductance, f :	10^{-4} m^2/s	
aperture :	1 m	

Hydraulic head distributions at 1000 days





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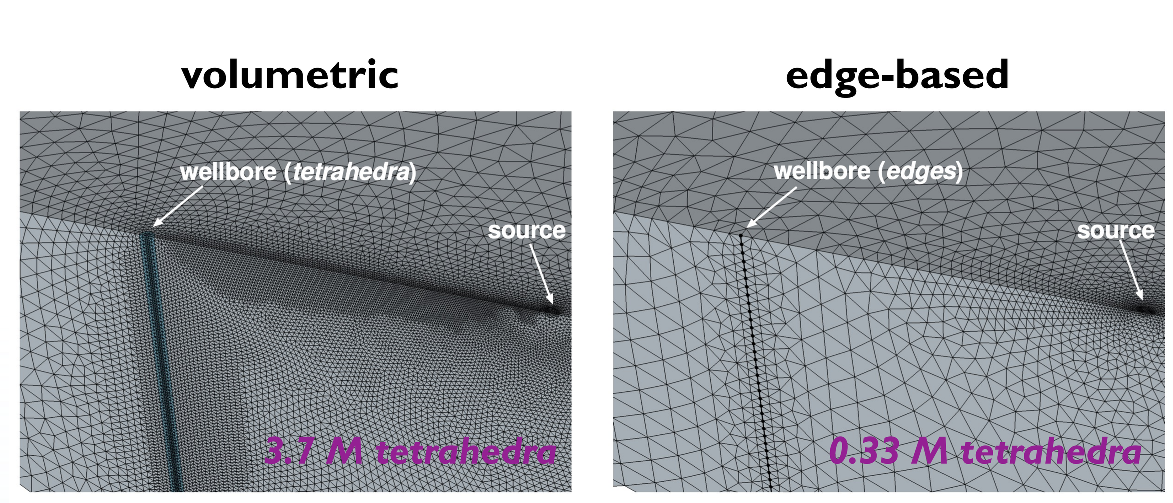
Heat conduction

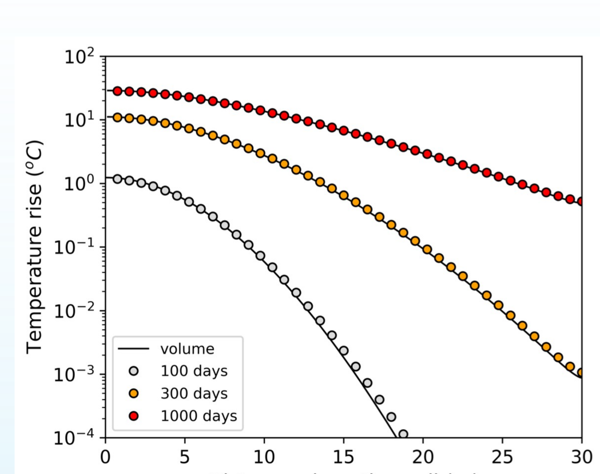
A wellbore model

We can simulate heat conduction by simply replacing hydraulic properties with thermal properties in the diffusion equation ($\mathbf{K}\rightarrow\mathbf{k}$, $S_s\rightarrow\rho c$, h $\rightarrow$ T, *see table below*). Here, we demonstrate the accuracy of *Hi*-FEM for heat conduction by considering a metallic well casing model.

A steel vertical well casing is located in a 10 km x 10 km x 5 km granite rock. The wellbore has a length of 30 m and a radius of 0.2 m. A heat source with a constant power 10 kW is located 100 m away.

	Granite	*Steel casing*
Thermal conductivity (k), W/(m°C)	2.5	75
Specific heat capacity (c), J/(kg°C)	897	460.5
Density (ρ), kg/m^3	2600	7850





Temperature along the well at 100 days

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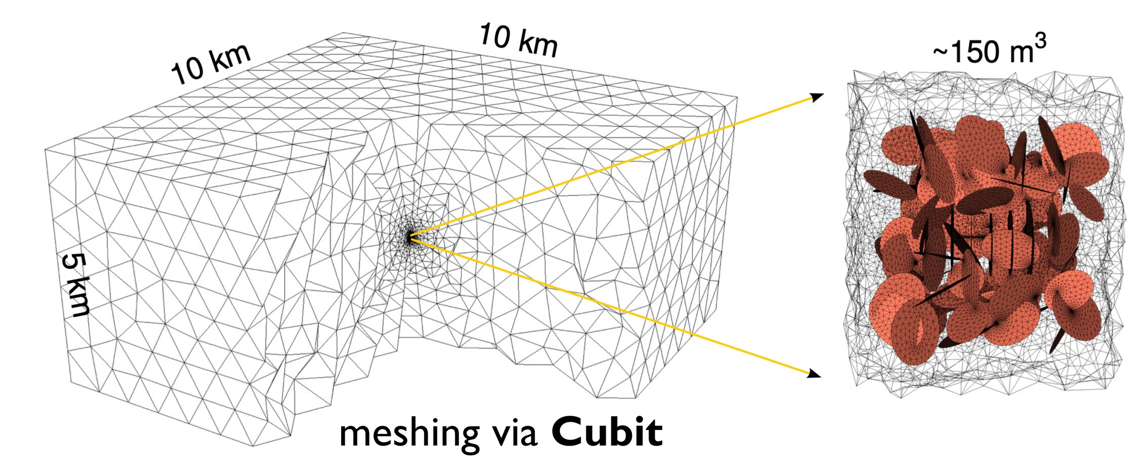
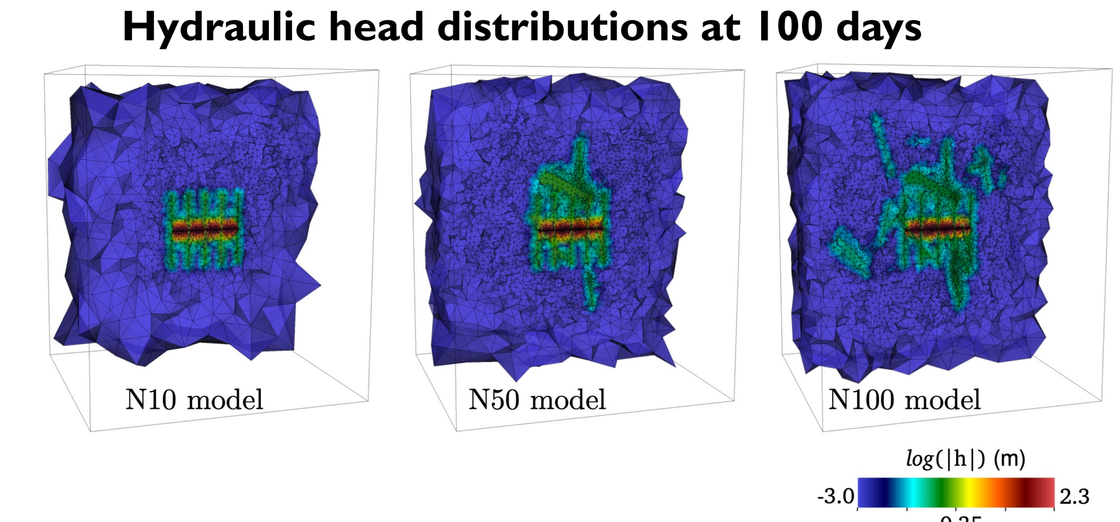
Fractured media

We consider three fracture network models with 10, 50 and 100 fractures in a basin. We represent fractures as ellipses whose locations and orientations are randomly-distributed. The sizes are populated from uniform distribution U(10 m, 25 m). 5 additional ellipse fractures (20 m x 50 m) are located near a 40 m line source. The constant flow rate is 500 m³/day.

	Fracture network	Host rock
Specific storage, S_s :	2x10^{-3} m^{-1}	2x10^{-4} m^{-1}
Hydraulic conductivity, K :	10^{-4} m/s	10^{-9} m/s
Hydraulic conductance, f :	10^{-6} m^2/s	
aperture :	10^{-2} m	



Hydraulic head distributions at 100 days



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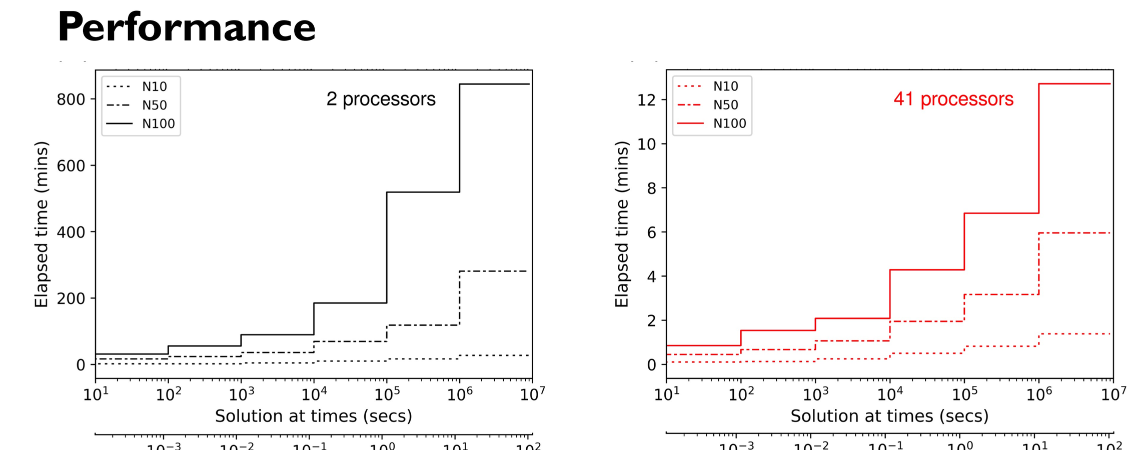
Performance



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The calculation times of the transient solutions of the fractured porous rock models that consist of 10, 50 and 100 fractures obtained by using 2 and 41 processors.

Conclusions

- We extend the previously developed *Hi*-FEM approach to model transient flow and heat conduction in complex fractured porous media.
- Our validations show that *Hi*-FEM produces accurate results for facet-based and edge based conductivity models compared to the fully volumetric solutions.
- The calculation times of large-scale fractured porous rock models indicate *Hi*-FEM's efficiency for complex geologic models.
- Our study suggests that *Hi*-FEM is a robust, high-fidelity modeling tool for modeling diffusion in fractured porous media with affordable computational cost.
- Our future work will further extend *Hi*-FEM's application to advective flow.

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References

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- Beskardes, G.D., Weiss, C.J., Kuhlman, K.L., Chang, K.W., (under review), Finite element modeling of diffusion in fractured porous media by using hierarchical material properties. Water Resources Research.
- *Finite-element meshes used for this analysis were generated by Cubit, available at http://cubit.sandia.gov*

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