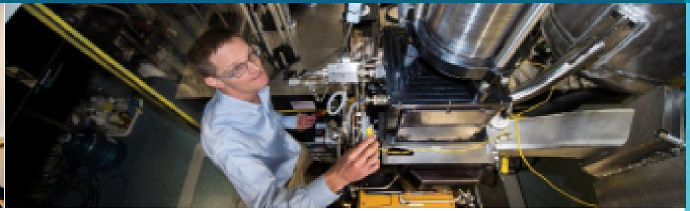
SAND2021-14146C

Sandia National Laboratories

Grid Tailored Reduced-Order Models for Steady Hypersonic Aerodynamics



PRESENTED BY

Patrick Blonigan

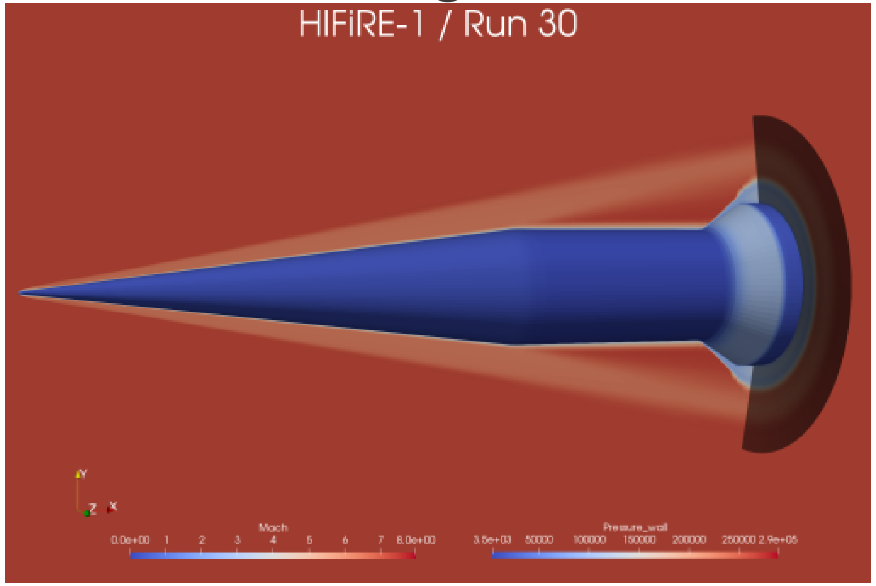
Collaborators: David Ching, Marco Arienti, Francesco Rizzi, and Jeff Fike

U.S. DEPARTMENT OF ENERGY NNSA

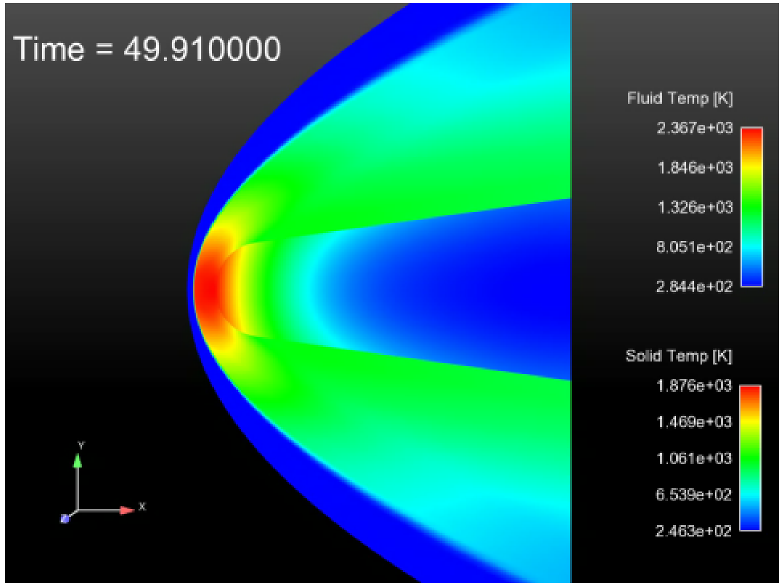
Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

RANS simulations are crucial for hypersonic vehicle analysis and design



Mach number and pressure contours for the HIFiRE-1
(Courtesy M. Howard, SNL)



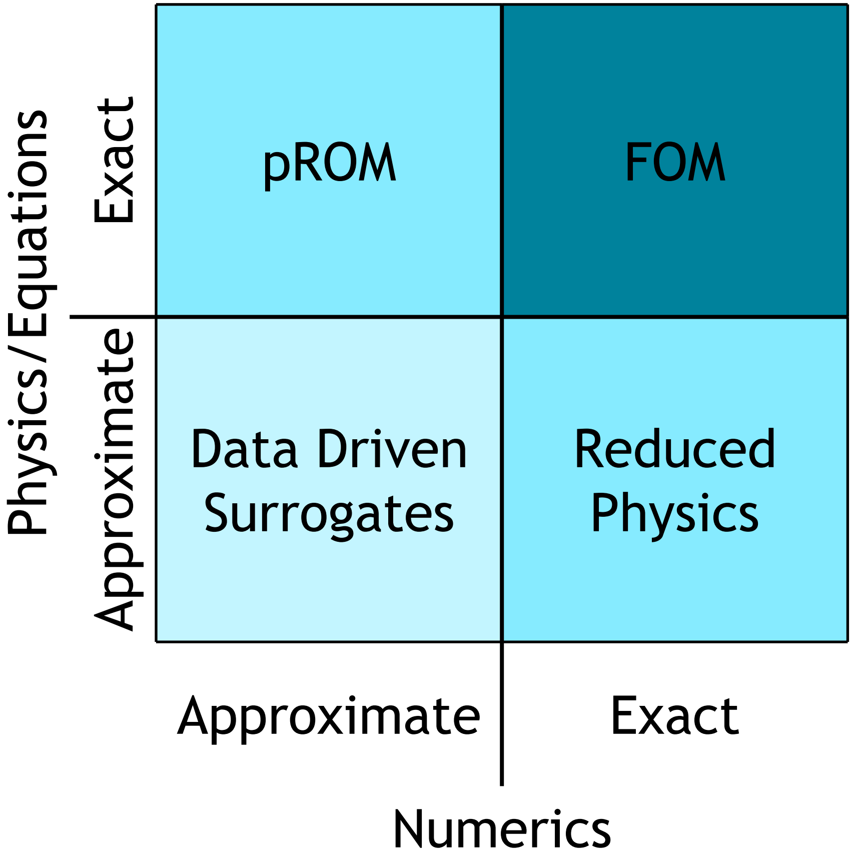
Temperature of a slender body in hypersonic flow
(Courtesy M. Howard, SNL)

- RANS is our large-scale, nonlinear full-order model (FOM).
  - High cost: requires hundreds or thousands of CPU hours.
- High cost creates a "**computational barrier**" to the application of many-query and/or time -critical problems:
  - **Many-Query**: Design Optimization, Model Calibration, Uncertainty Propagation
  - **Time-Critical**: Path Planning, Model Predictive Control, Health Monitoring

A surrogate model is needed to break the computational barrier!

We use Projection-based Reduced Order Models (pROMs)

- pROMs are “physics-based” surrogates
  - Results are **explainable**
  - Evaluating **known dynamics** rather than *learning* unknown dynamics
- *A priori* and *a posteriori* error bounds
  - **Quantifying the uncertainty of the pROM is critical** for Sandia’s missions
- Hypersonic aero surrogates: mostly kriging or reduced physics
  - **[Blonigan et al. 2021]: LSPG for hypersonic RANS solver**

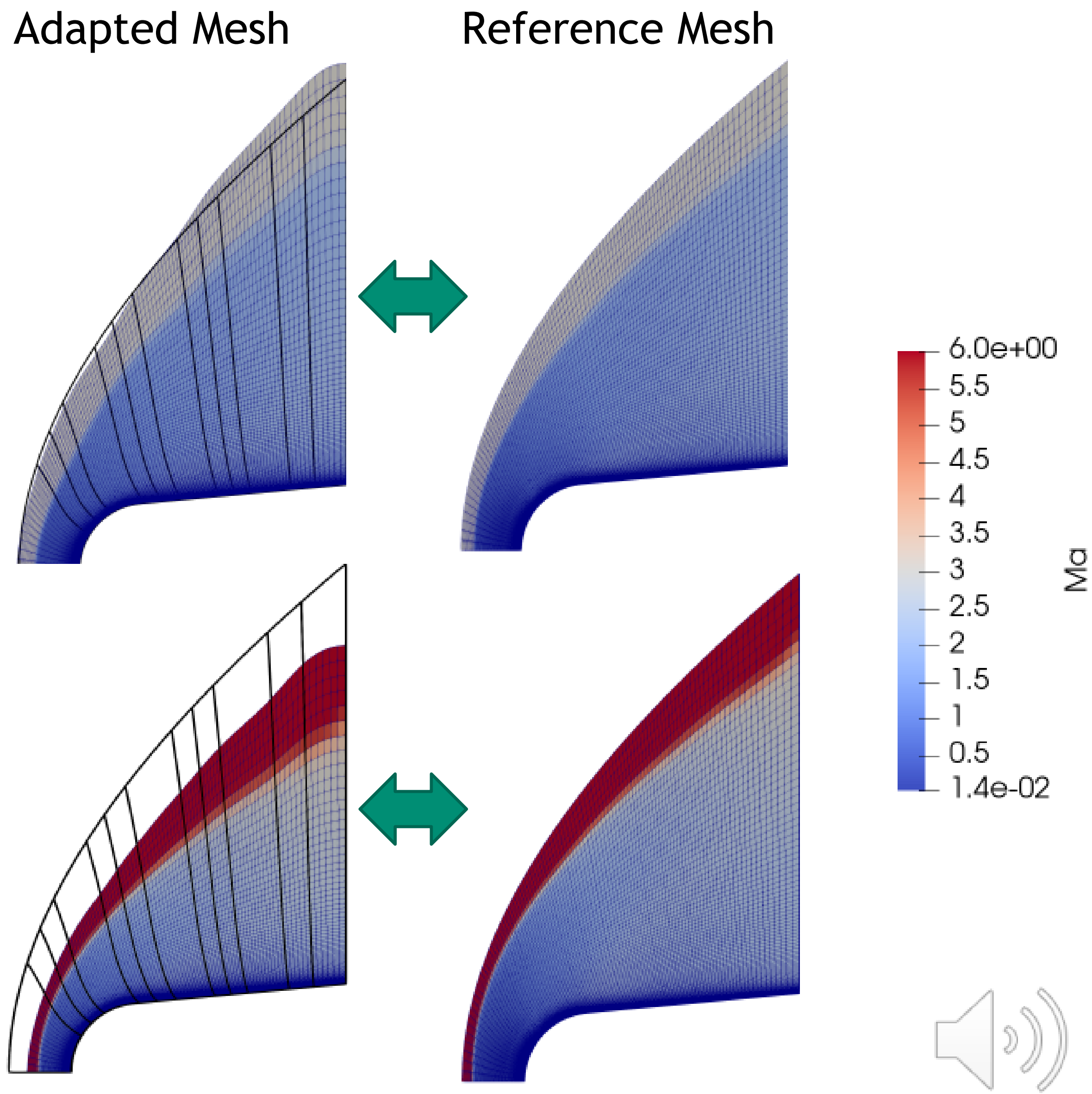


- Projection-based ROM implementations are very intrusive, difficult to set up
- Steady simulations do not provide much training data
- Linear basis representations are not well suited for shock-dominated flows

Our approach: leverage r-adaptation for improved basis quality

- We desire a basis that does not require much training data and can represent shock waves

- Idea: set up FOM to adapt mesh via grid-tailoring [Vinokur, 1983]
  - Bow shock wave is a fixed number of cells from the inlet
  - For FOM: Allows for greater robustness/accuracy when running at multiple inflow conditions and results in more accurate heat flux
  - For ROM: **R-adaptation provides mesh displacement data which can be used to create a mapping from a reference mesh to a tailored mesh**
    - Registration-based ROMs [Taddei 2020, Nair and Balajewicz 2019, Mojgani and Balajewicz 2020, Zahr et al. 2020, …].



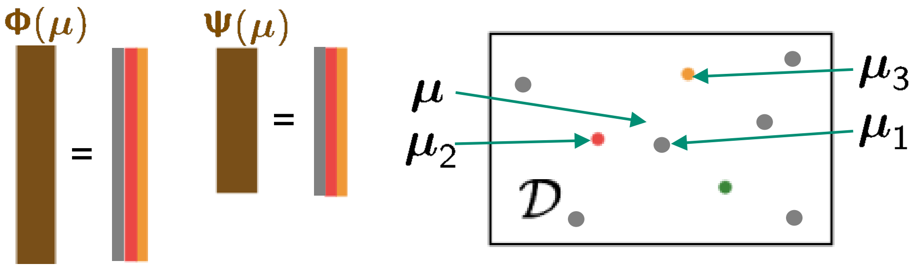
Our approach: perturb the mesh, then compute the pROM

- FOM Residual: $\mathbf{r}(\mathbf{x};\mathbf{z},\boldsymbol{\mu}) = \mathbf{0}$
  - **x**: state vector
  - **z:** nodal displacements for mesh
  - **μ:** input parameters

- Offline: solve FOM at various input parameters

- Online:
  1. Set up local basis for state, mesh displacements
  2. RBF interpolation over local basis to obtain mesh displacement
  3. RBF interpolation over local basis to obtain state initial guess
  4. Apply Least-squares Petrov—Galerkin (LSPG) projection with reduced basis on the **morphed mesh**

1. **Local Dictionary basis**

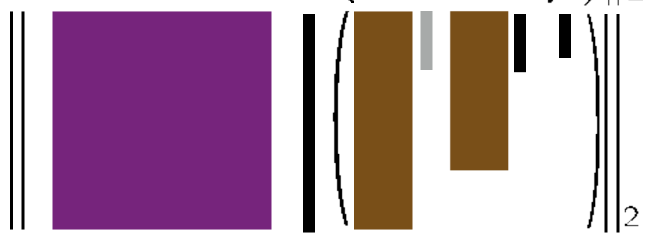


2. Interpolate for mesh deformation: $\hat{\mathbf{z}}(\boldsymbol{\mu}) = \sum_{i=0}^{N} \beta_i(\boldsymbol{\mu})\hat{\mathbf{z}}(\boldsymbol{\mu}_i) \Rightarrow \mathbf{z} = \boldsymbol{\Psi}(\boldsymbol{\mu})\hat{\mathbf{z}}(\boldsymbol{\mu})$

3. Interpolate for initial guess: $\hat{\mathbf{x}}^{IG}(\boldsymbol{\mu}) = \sum_{i=0}^{N} \alpha_i(\boldsymbol{\mu})\hat{\mathbf{x}}^{IG}(\boldsymbol{\mu}_i) \Rightarrow \mathbf{x}^{IG} = \boldsymbol{\Phi}(\boldsymbol{\mu})\hat{\mathbf{x}}^{IG}(\boldsymbol{\mu})$

4. Solve Residual minimization for $\hat{\mathbf{v}}$

$$\underset{\hat{\mathbf{v}}}{\text{minimize}} \left\| \mathbf{A}\, \mathbf{r}(\boldsymbol{\Phi}\hat{\mathbf{v}};\boldsymbol{\Psi}\hat{\mathbf{z}},\boldsymbol{\mu}) \right\|_2$$



Codes

Sandia Parallel Aerodynamics and Reentry Code (SPARC)

- Compressible CFD code focused on aerodynamics and aerothermodynamics in the Transonic and Hypersonic regimes
- Emphasis on performance portability
- See AIAA 2017-4407 for more details.

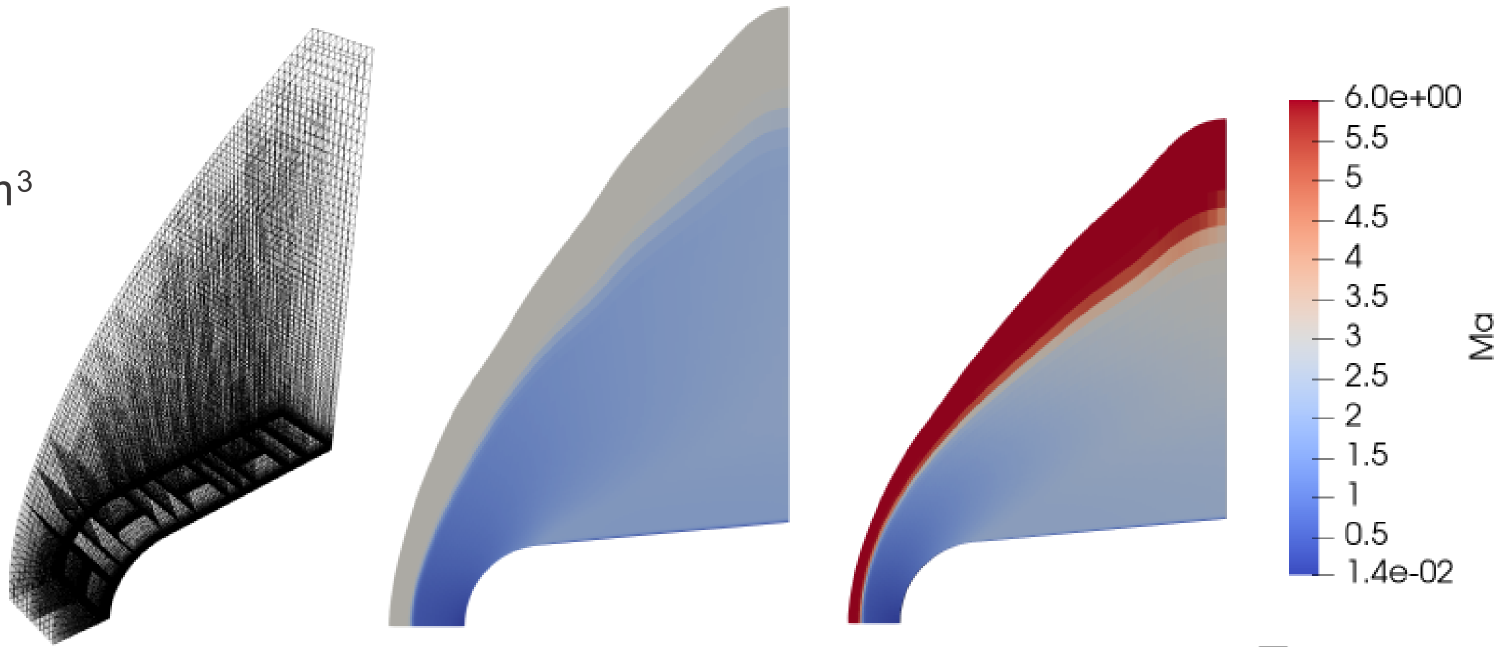
Pressio

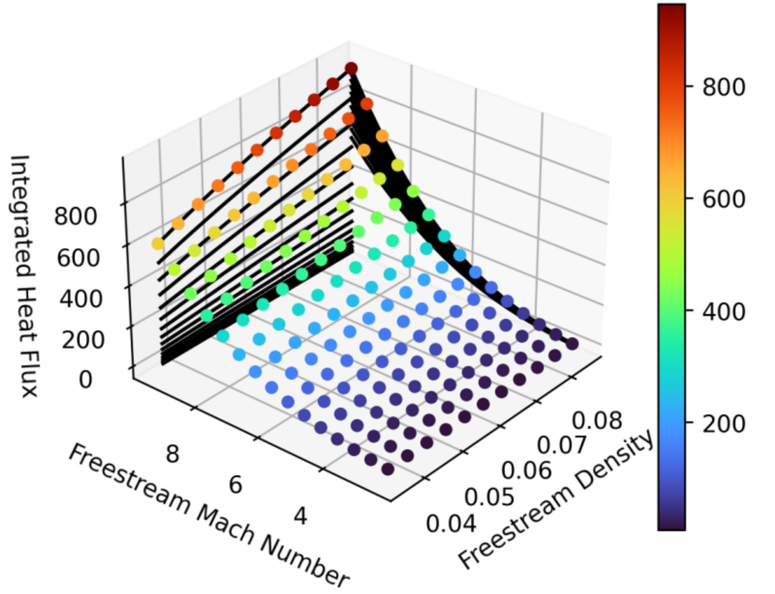
- Minimally intrusive API for model reduction
- Open source!

https://github.com/Pressio

Test case: two-dimensional wedge geometry

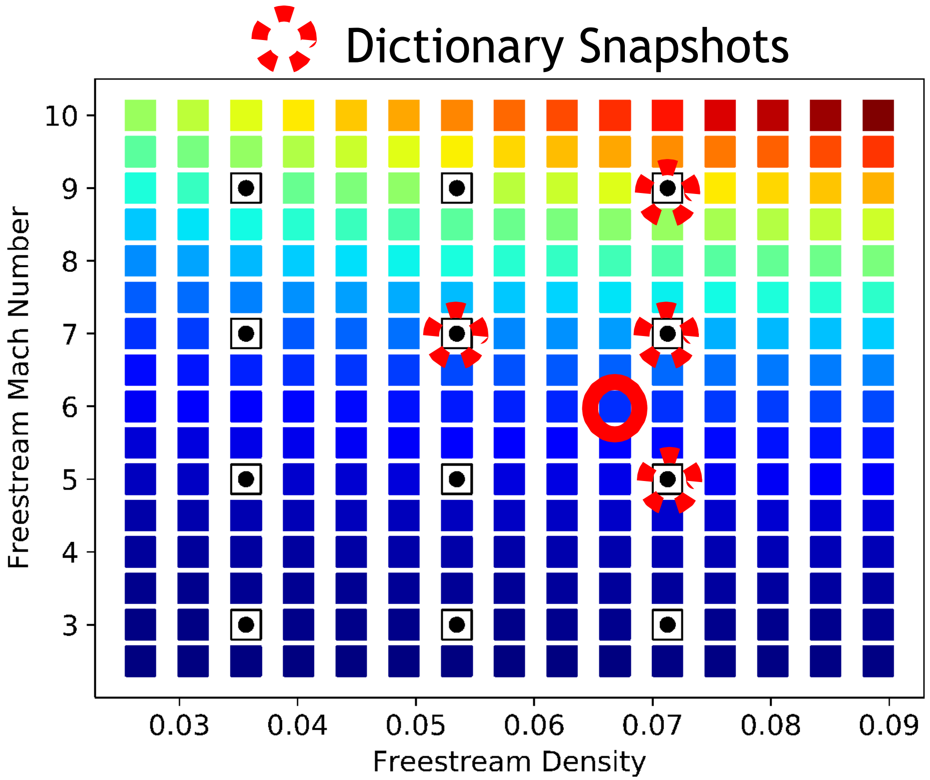
- Parameters:
  - Freestream Mach number: 3.0 to 9.5
  - Freestream Density: 0.03 to 0.09 kg/m^3
- FOM:
  - 100,000 cell mesh
  - Perfect Gas
  - Laminar flow
  - Steady-state
  - Run grid tailoring twice
- ROMs:
  - Basis constructed from 12 solutions
  - Mesh perturbed using local dictionary basis
  - LSPG with local dictionary basis
    - RBF for initial guess
    - Gauss-Newton with QR-decomposition
  - pROM is 27 to 50 times faster than FOM without hyper-reduction
- Quantities of Interest (QoIs):
  - Axial force
  - Integrated Wall Heat Flux



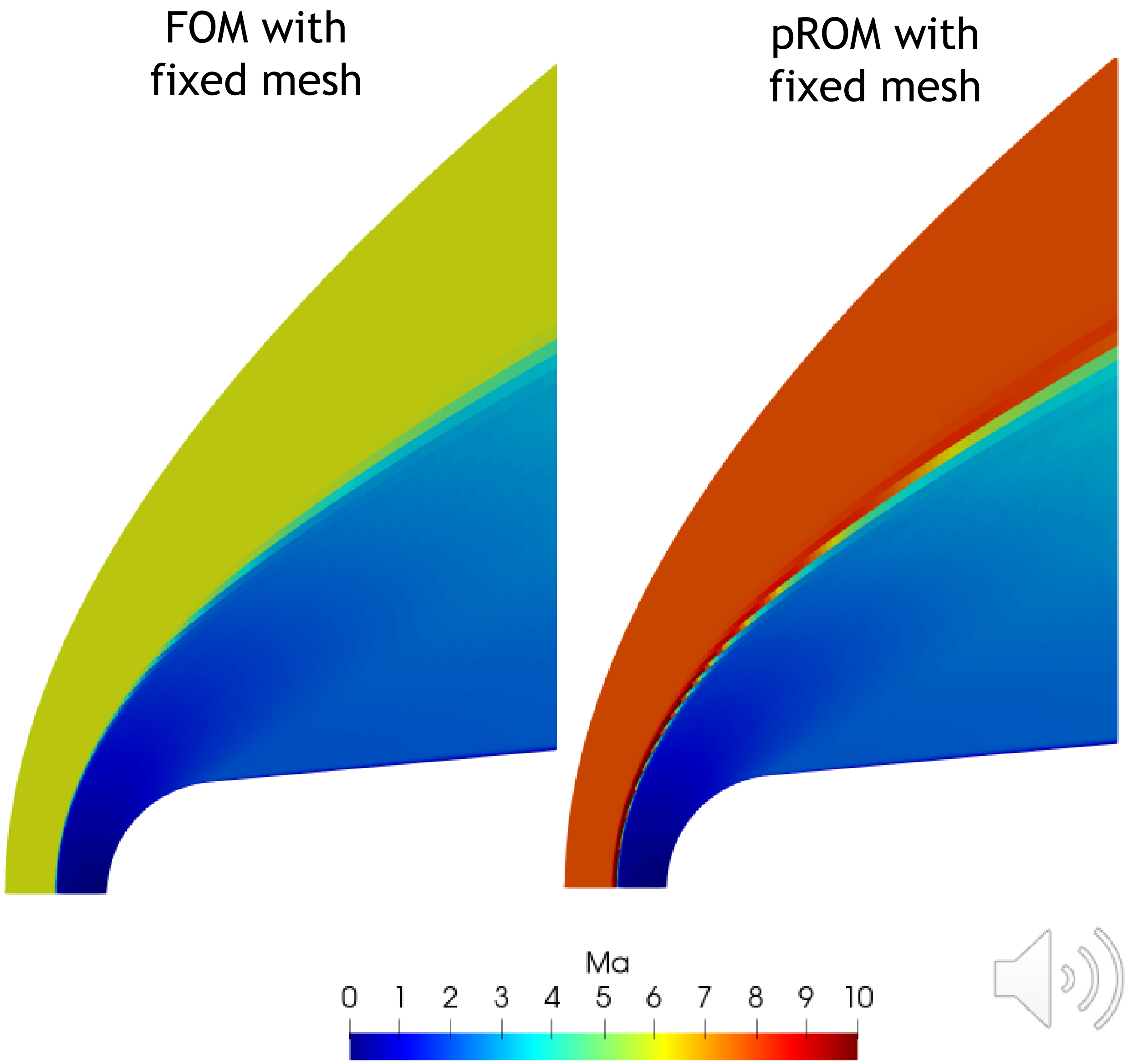


Mesh adaptation can prevent oscillations near shocks

Mach 6.0, Density 0.076 kg/m^3

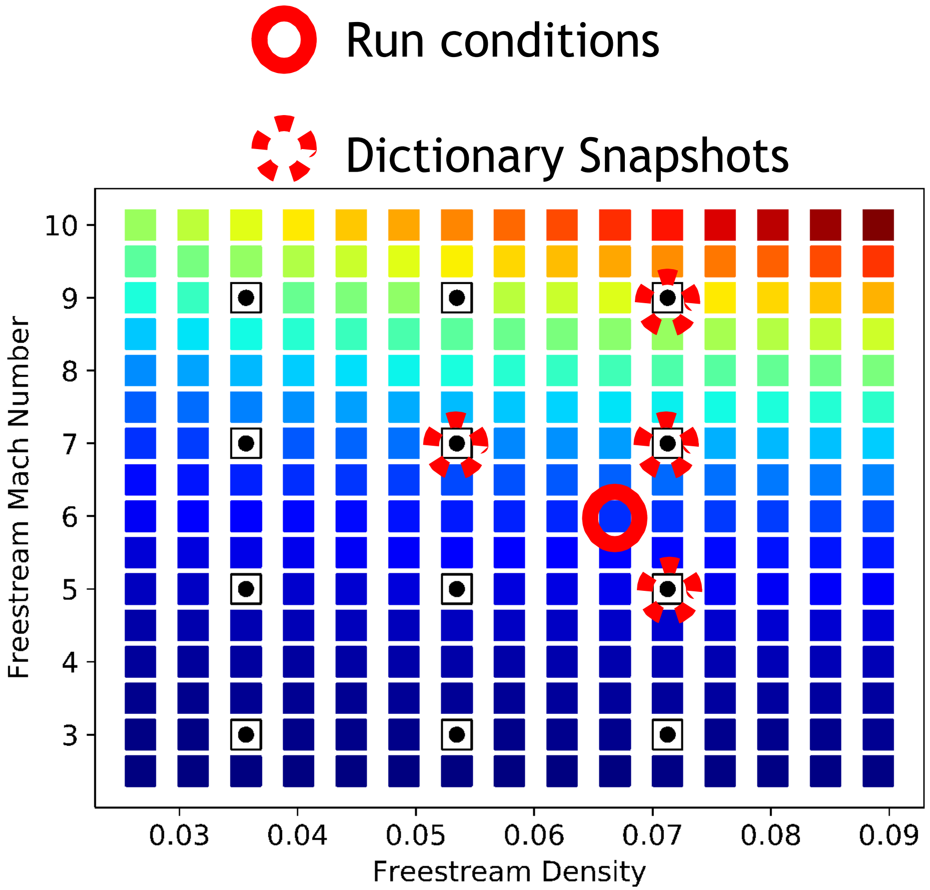


Large oscillation near shock without adaptation

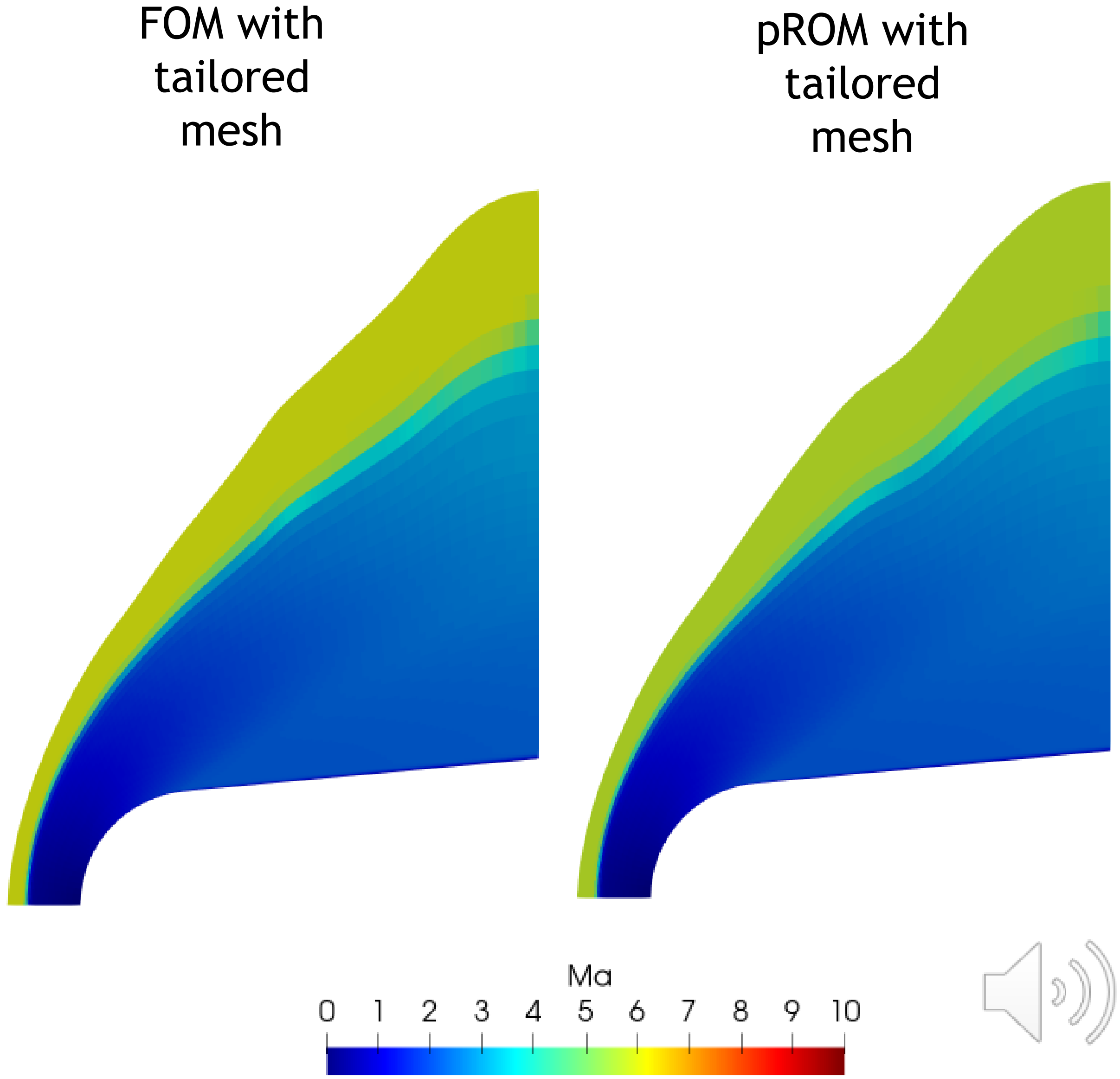


Mesh adaptation can prevent oscillations near shocks

Mach 6.0, Density 0.076 kg/m^3



No oscillation near shock with adaptation



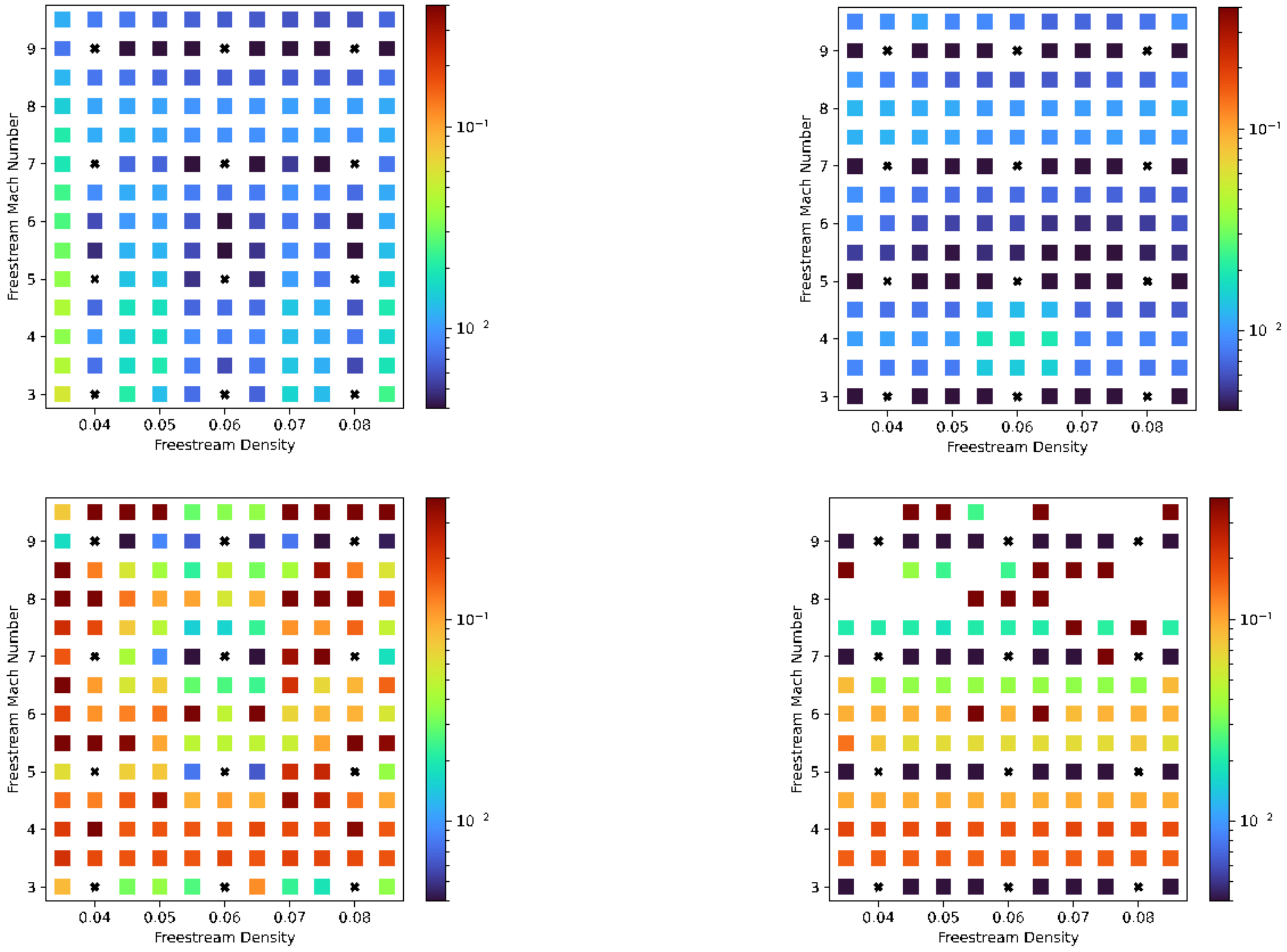
State L2 Error shows benefits of tailored mesh pROM

RBF Error

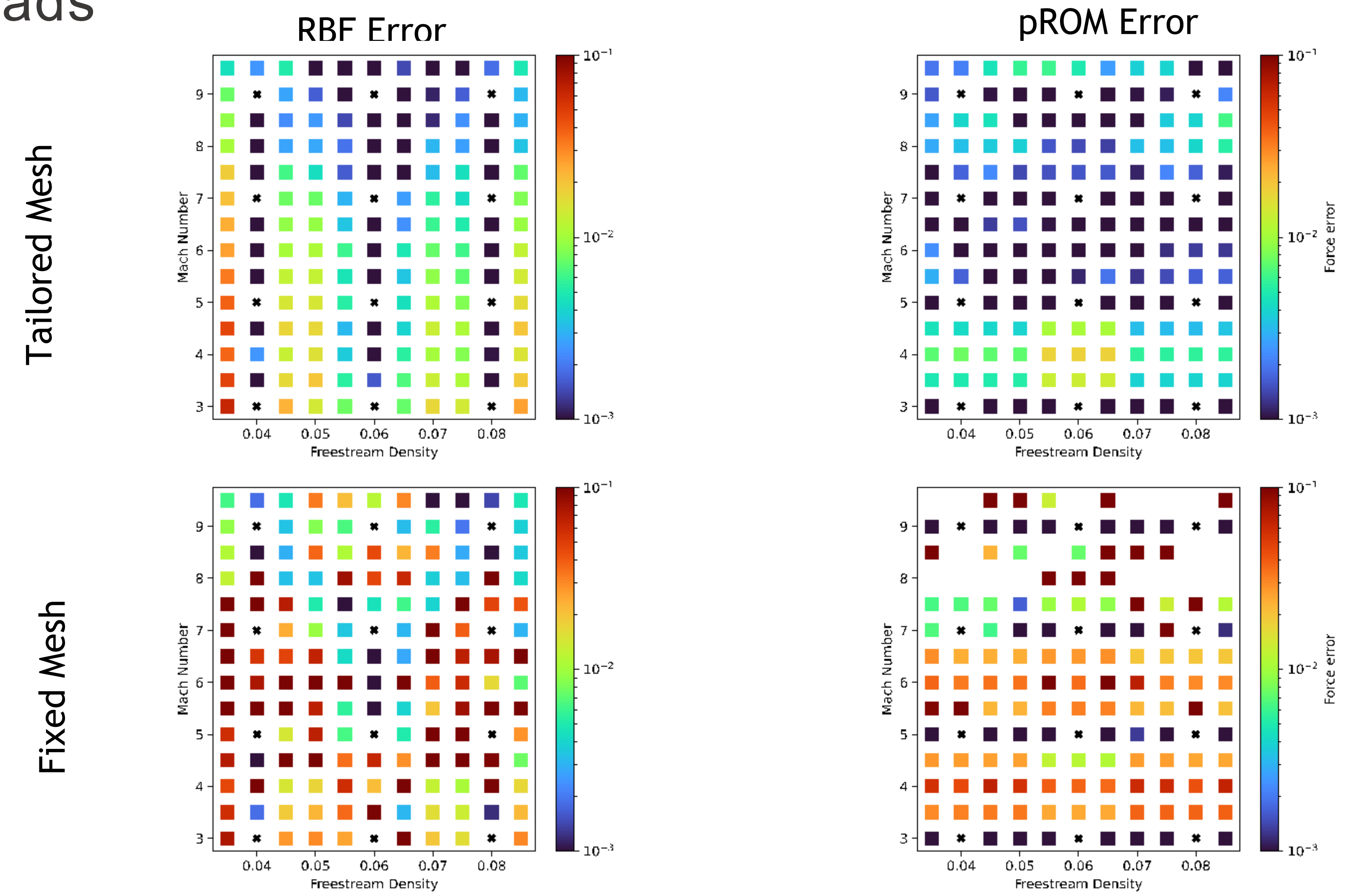
pROM Error

Tailored Mesh

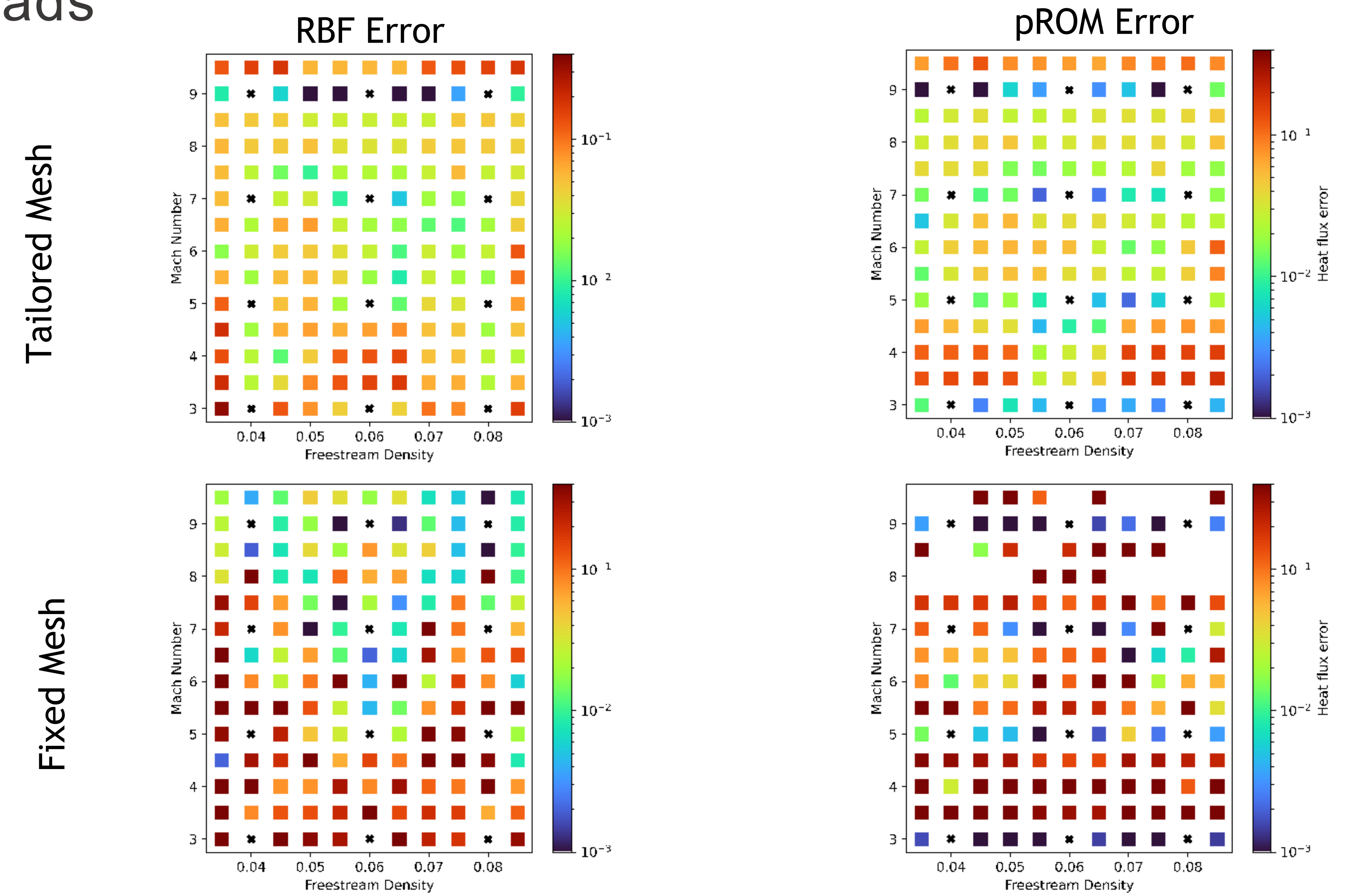
Fixed Mesh



Tailored mesh pROM computes accurate aerodynamic loads

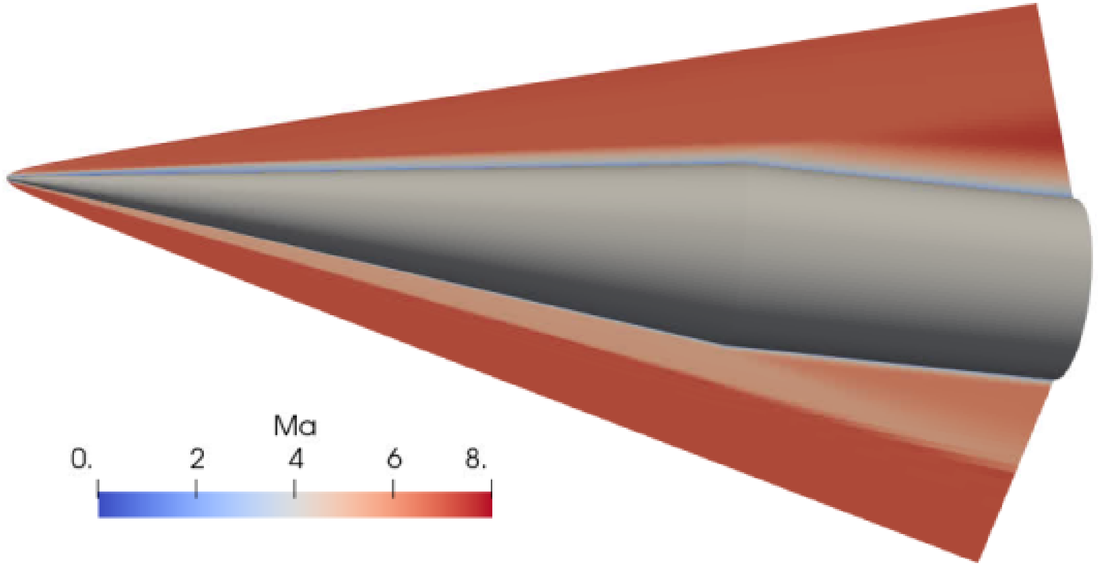


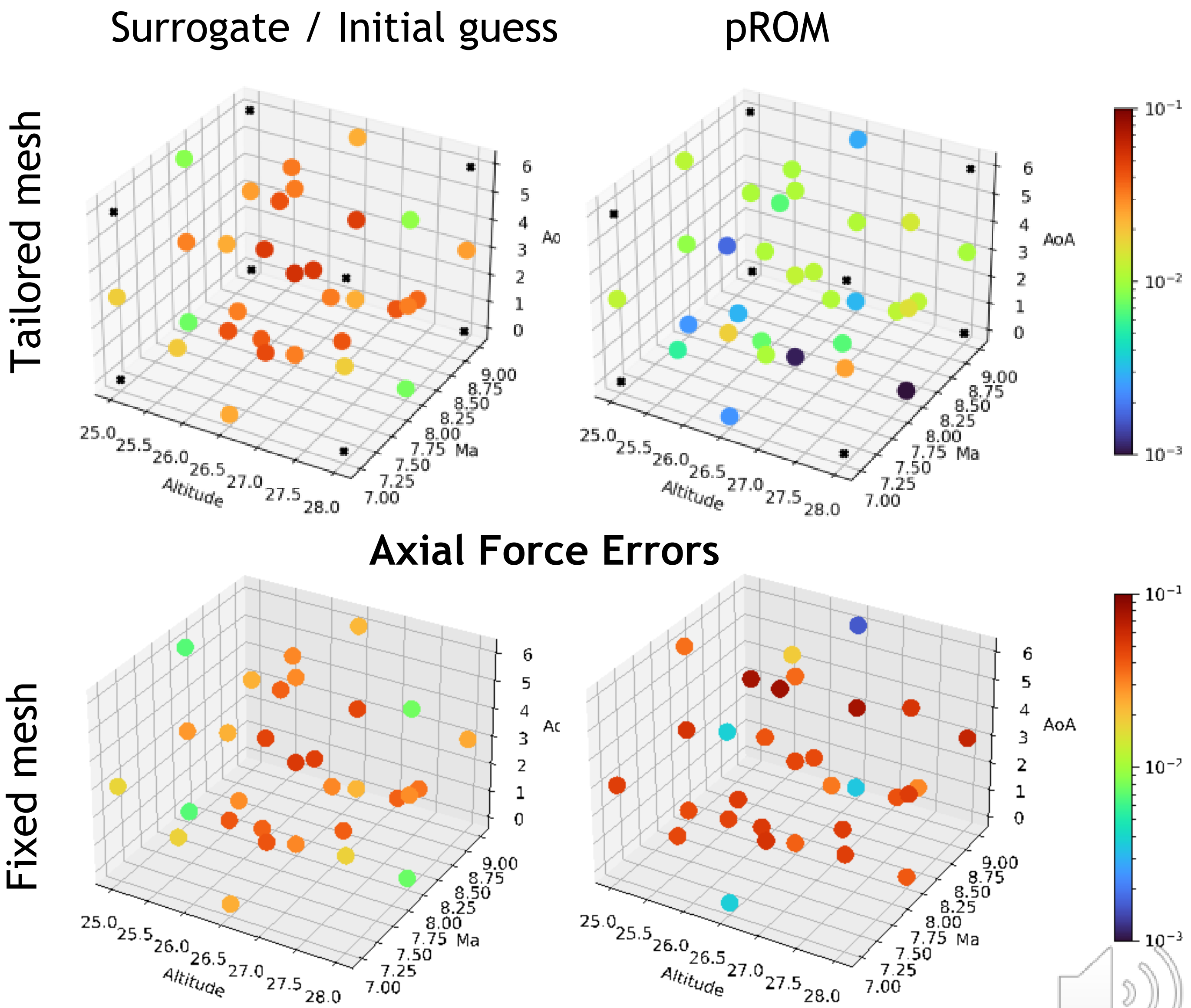
Tailored mesh pROM computes most accurate thermal loads



3D HIFiRE-1 case

Demonstrated grid-tailored ROM for 3D HIFiRE-1 with 3 input parameters



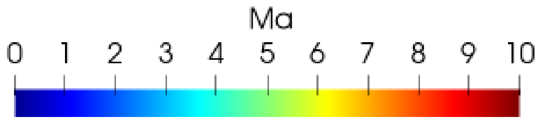


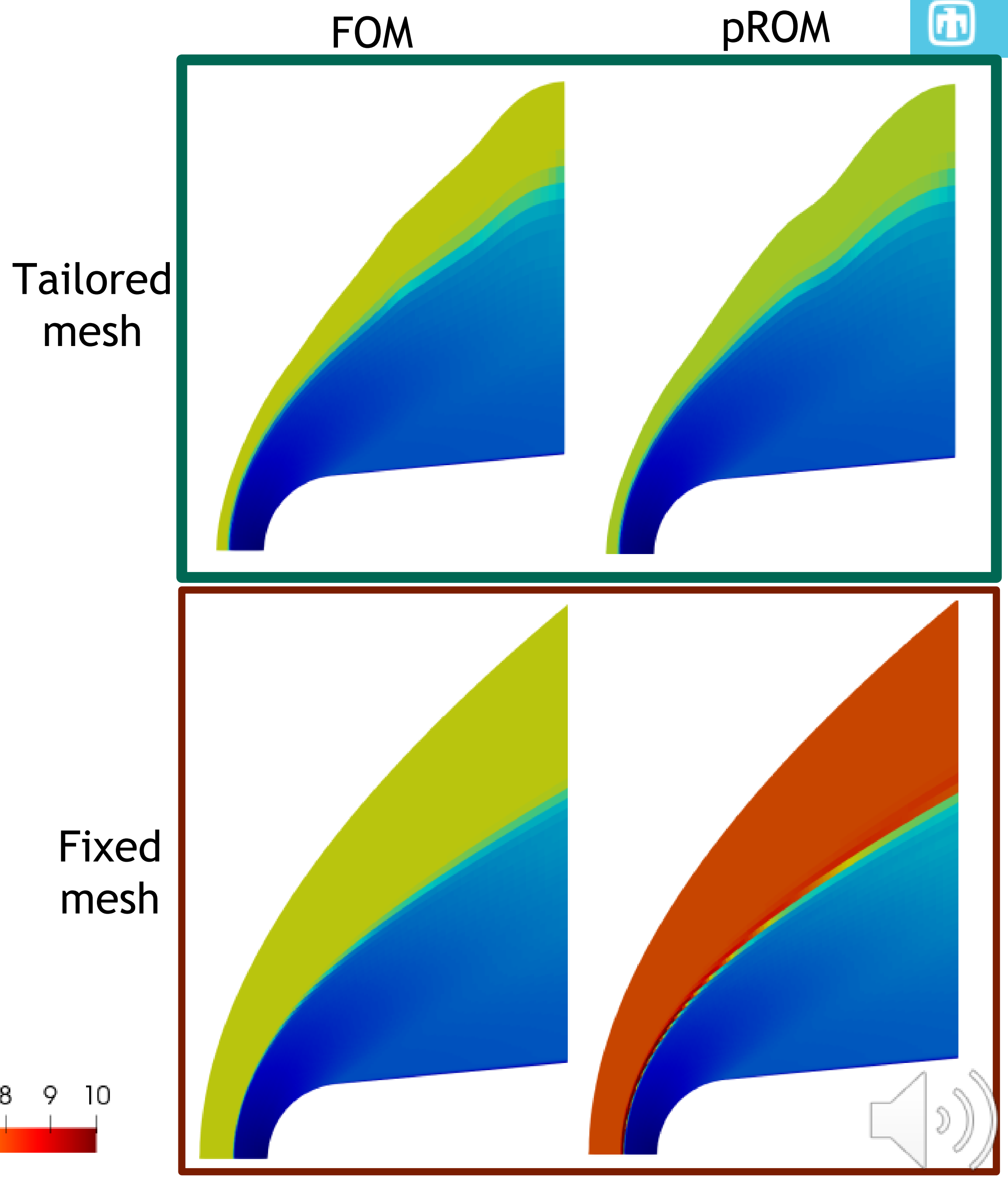
- Grid-tailored ROM also improved heat flux accuracy over initial guess

Grid tailored ROM works for 3D flight vehicle geometries

Conclusions

- pROM with an r-adapted mesh is more accurate than a pROM with a fixed mesh
- For a given training data set, pROM is more accurate than RBF snapshot interpolation
- Steady pROMs achieve speed-up even without hyper-reduction





Relevant Papers

- Blonigan P., Rizzi F., Howard M., Fike J., and Carlberg K., “Model Reduction for Steady Hypersonic Aerodynamics via Conservative Manifold Least-Squares Petrov–Galerkin Projection”, AIAA Journal 2021 59:4, 1296-1312
- Rizzi F., Blonigan P., and Carlberg K., “Pressio: Enabling projection-based model reduction for large-scale nonlinear dynamical systems,” Preprint: arXiv:2003.07798, 2021.
- https://github.com/Pressio
- Howard M., Bradley A., Bova S., Overfelt J., Wagnild R., Dinzl D., Hoemmen M., and Klinvex A., “Towards Performance Portability in a Compressible CFD Code,” 23rd AIAA Computational Fluid Dynamics Conference, AIAA Paper 2017-4407, 2017.
- Vinokur M., “On one-dimensional stretching functions for finite-difference calculations,” Journal of Computational Physics, Volume 50, Issue 2, 1983, Pages 215-234, ISSN 0021-9991
- Nair, N., Balajewicz, M., “Transported snapshot model order reduction approach for parametric, steady-state fluid flows containing parameter-dependent shocks”. Int. J. Numer. Methods Eng. 2019; 117: 1234– 1262.