



Predicting plastic anisotropy using crystal plasticity and Bayesian neural network surrogate models



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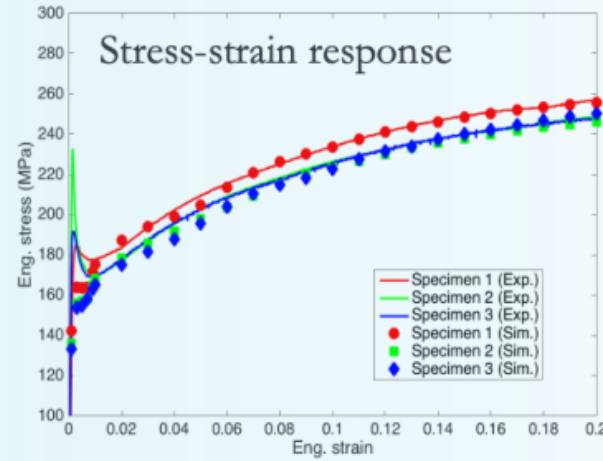
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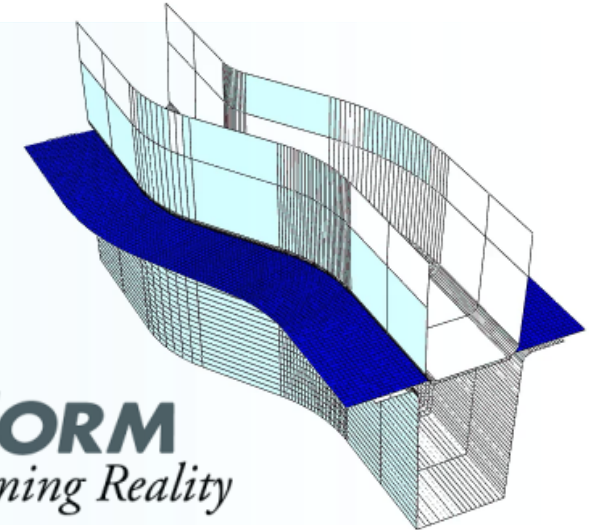


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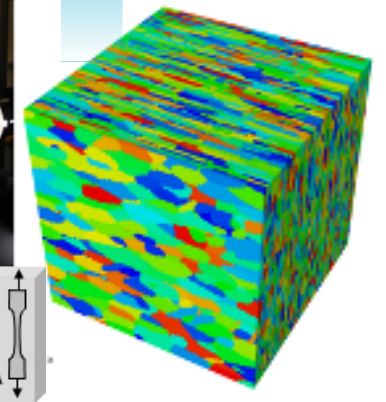
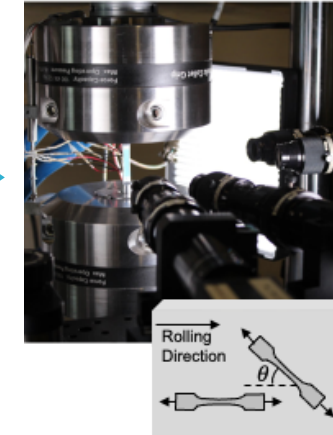
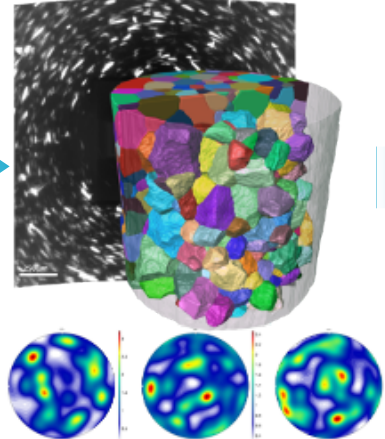
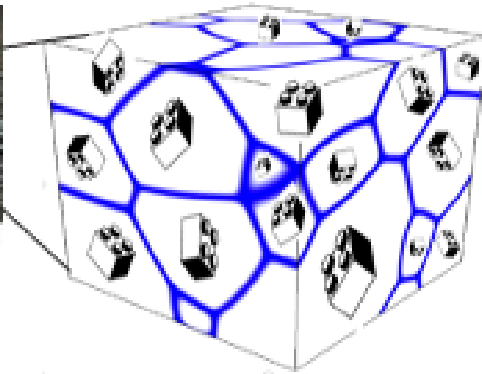
Structure-Property: Traditional Paradigm

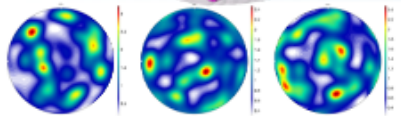
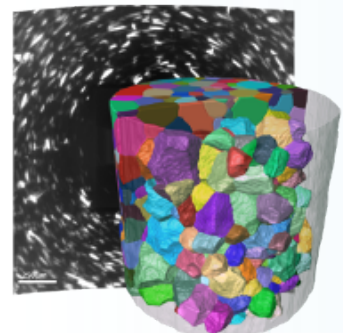


AUTOFORM
Forming Reality

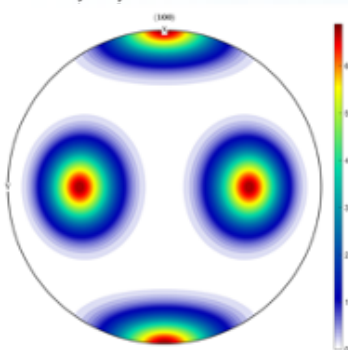


Calibrated Model Parameters
(e.g., Hill's quadratic anisotropic yield model constants)

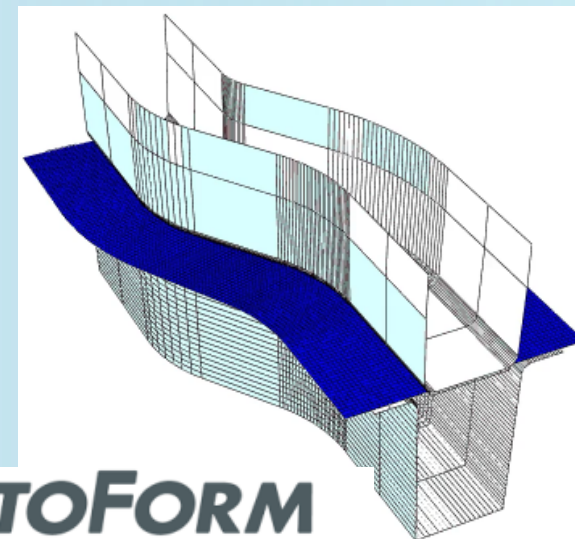




Polycrystalline Texture



AUTOFORM
Forming Reality



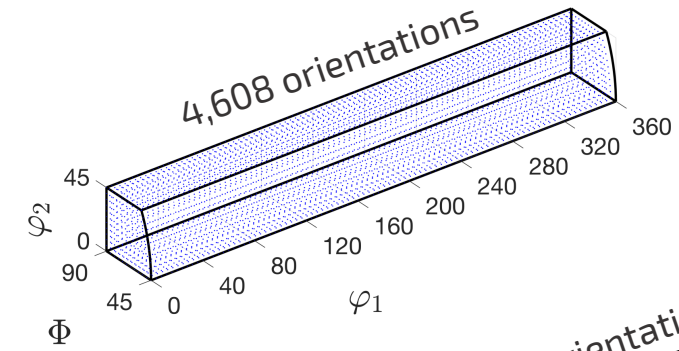
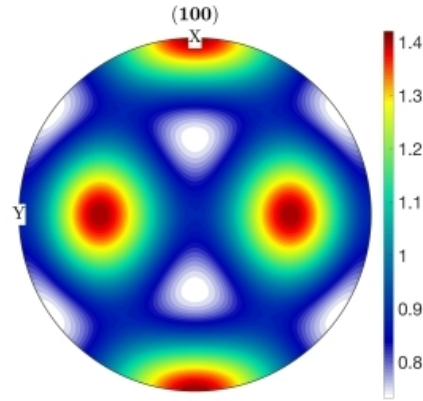
Calibrated Model Parameters
(e.g., Hill's quadratic anisotropic yield model constants)

Surrogate Model: Generation of Dataset

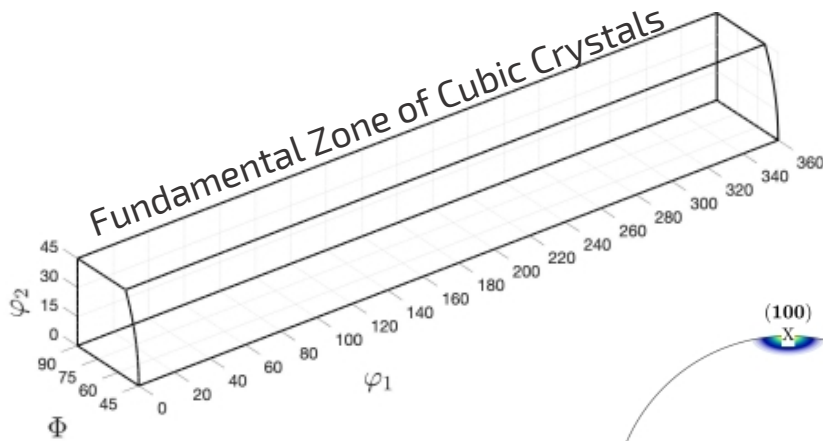
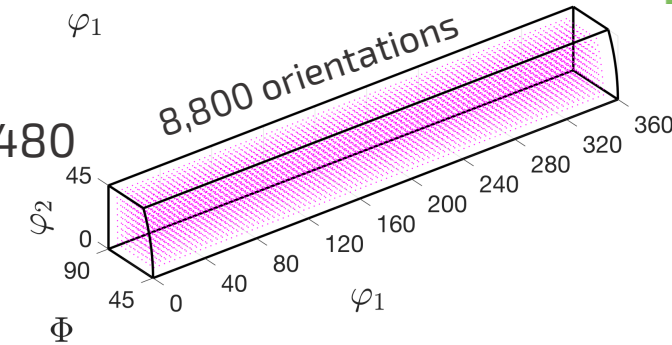
Apply 9 levels of spread every 5° from 0° to 45°



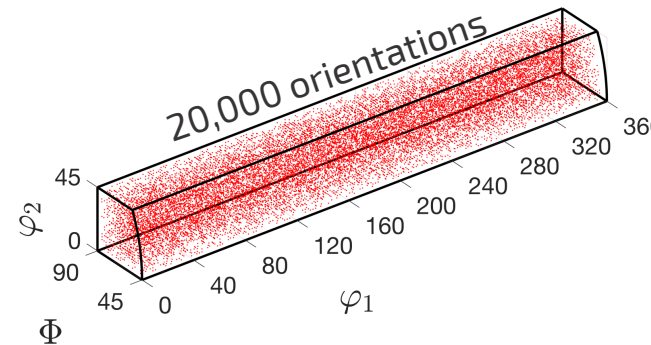
Pole figure with spread of 35°



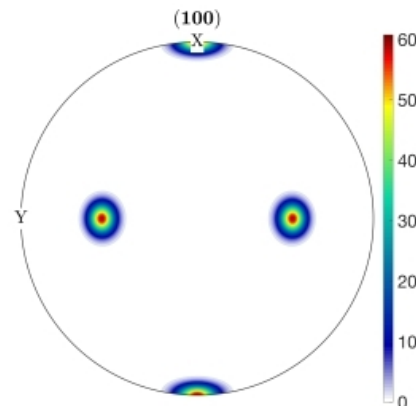
Training Textures: 54,480



Testing Textures: 20,000



Pole figure with spread of 5°



Perform crystal plasticity to calibrate Hill's Constants for each texture

$$\mathbf{L}^p = \dot{\mathbf{F}}^p \mathbf{F}^{p-1} = \sum_{\alpha=1}^12 \dot{\gamma}^\alpha \mathbf{s}_0^\alpha \otimes \mathbf{n}_0^\alpha$$

$$F = \frac{1}{2} \left[-\left(\frac{\sigma_0}{\sigma_{xx}}\right)^2 + \left(\frac{\sigma_0}{\sigma_{yy}}\right)^2 + \left(\frac{\sigma_0}{\sigma_{zz}}\right)^2 \right]$$

$$G = \frac{1}{2} \left[\left(\frac{\sigma_0}{\sigma_{xx}}\right)^2 - \left(\frac{\sigma_0}{\sigma_{yy}}\right)^2 + \left(\frac{\sigma_0}{\sigma_{zz}}\right)^2 \right]$$

$$H = \frac{1}{2} \left[\left(\frac{\sigma_0}{\sigma_{xx}}\right)^2 + \left(\frac{\sigma_0}{\sigma_{yy}}\right)^2 - \left(\frac{\sigma_0}{\sigma_{zz}}\right)^2 \right]$$

$$L = \frac{1}{2} \left[\left(\frac{2\sigma_0}{\sigma_{yz}}\right)^2 - G - H \right]$$

$$M = \frac{1}{2} \left[\left(\frac{2\sigma_0}{\sigma_{zx}}\right)^2 - H - F \right]$$

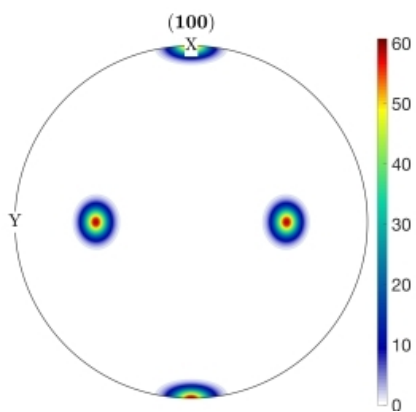
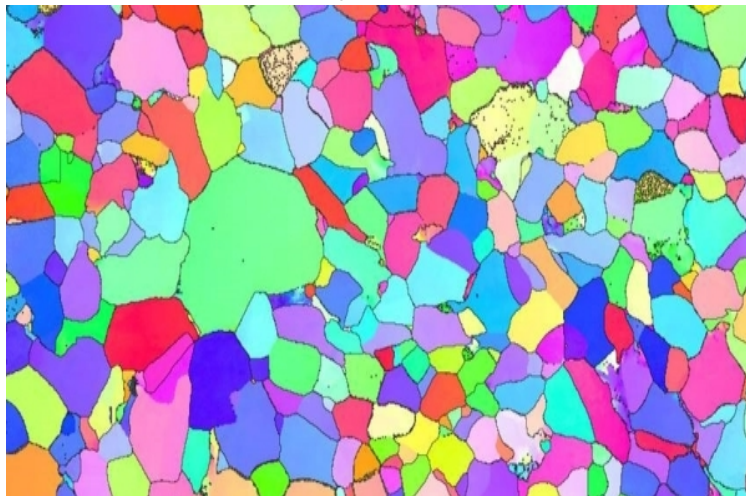
$$N = \frac{1}{2} \left[\left(\frac{2\sigma_0}{\sigma_{xy}}\right)^2 - F - G \right]$$

Apply random spread between 1° and 50°

Surrogate Model: Obtaining Fingerprint Descriptor



Colors denote the crystal lattice orientation



$f_s(g)$ is the probability distribution of the orientation of the crystal lattice.

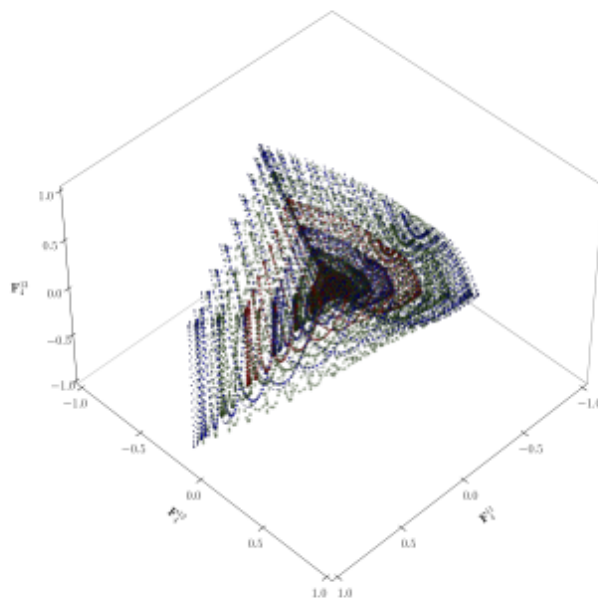
Fourier Series Representation:

$$T_l^{mn}(g) = T_l^{mn}(\varphi_1, \Phi, \varphi_2) = e^{im\varphi_1} P_l^{mn}(\Phi) e^{im\varphi_2}$$

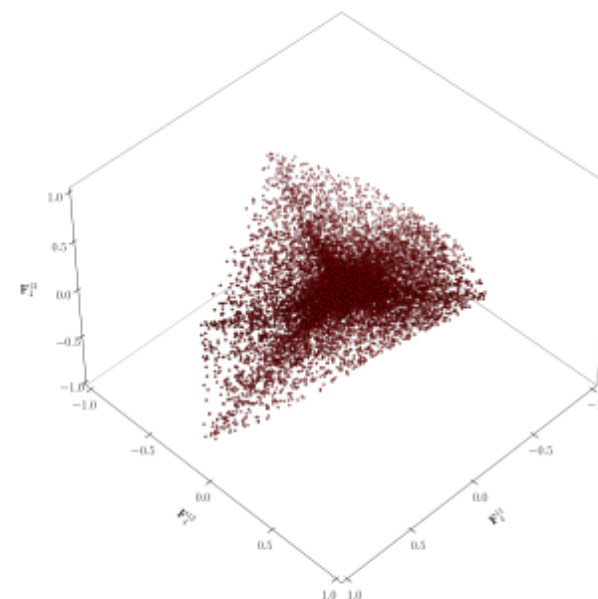
$$f_s(g) = \sum_{\mu, n, l} F_{ls}^{\mu n} T_l^{\mu n}(g),$$

- Orthogonal Basis functions basis functions
- Customized to account for symmetry.

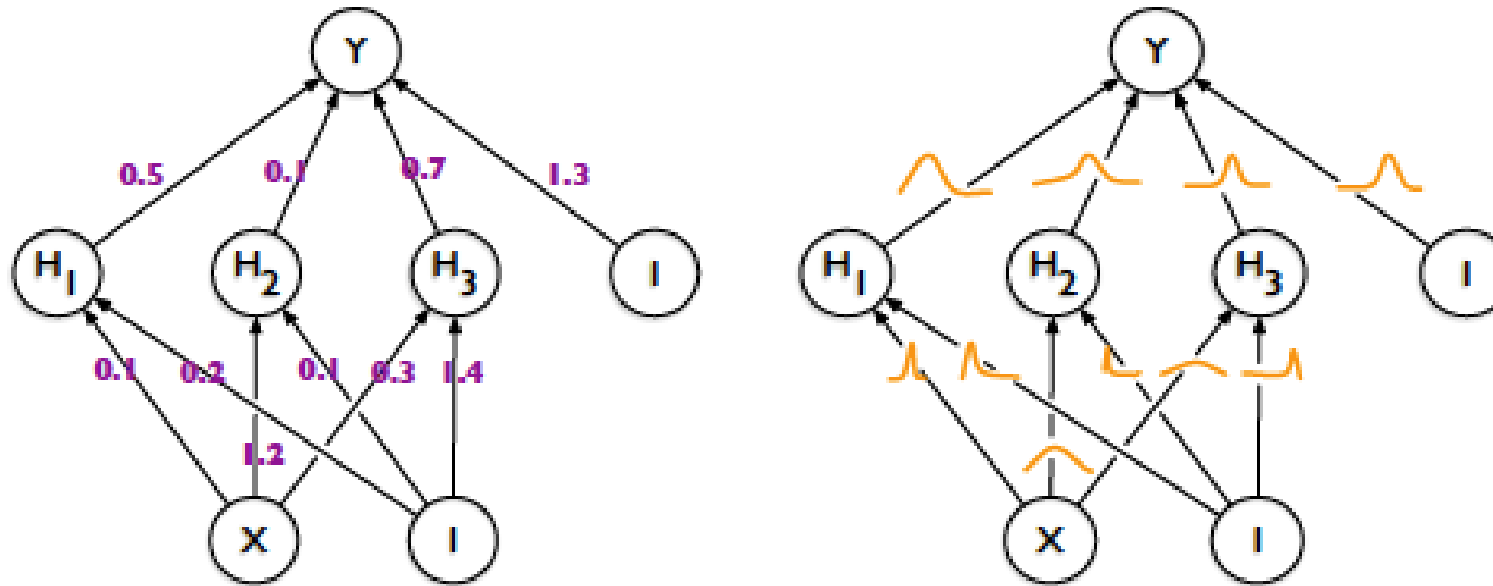
$$F_{ls}^{\mu n} = (2l + 1) \int_{FZ} f_s(g) T_l^{\mu n*}(g) dg$$



GSH representation of Training Textures

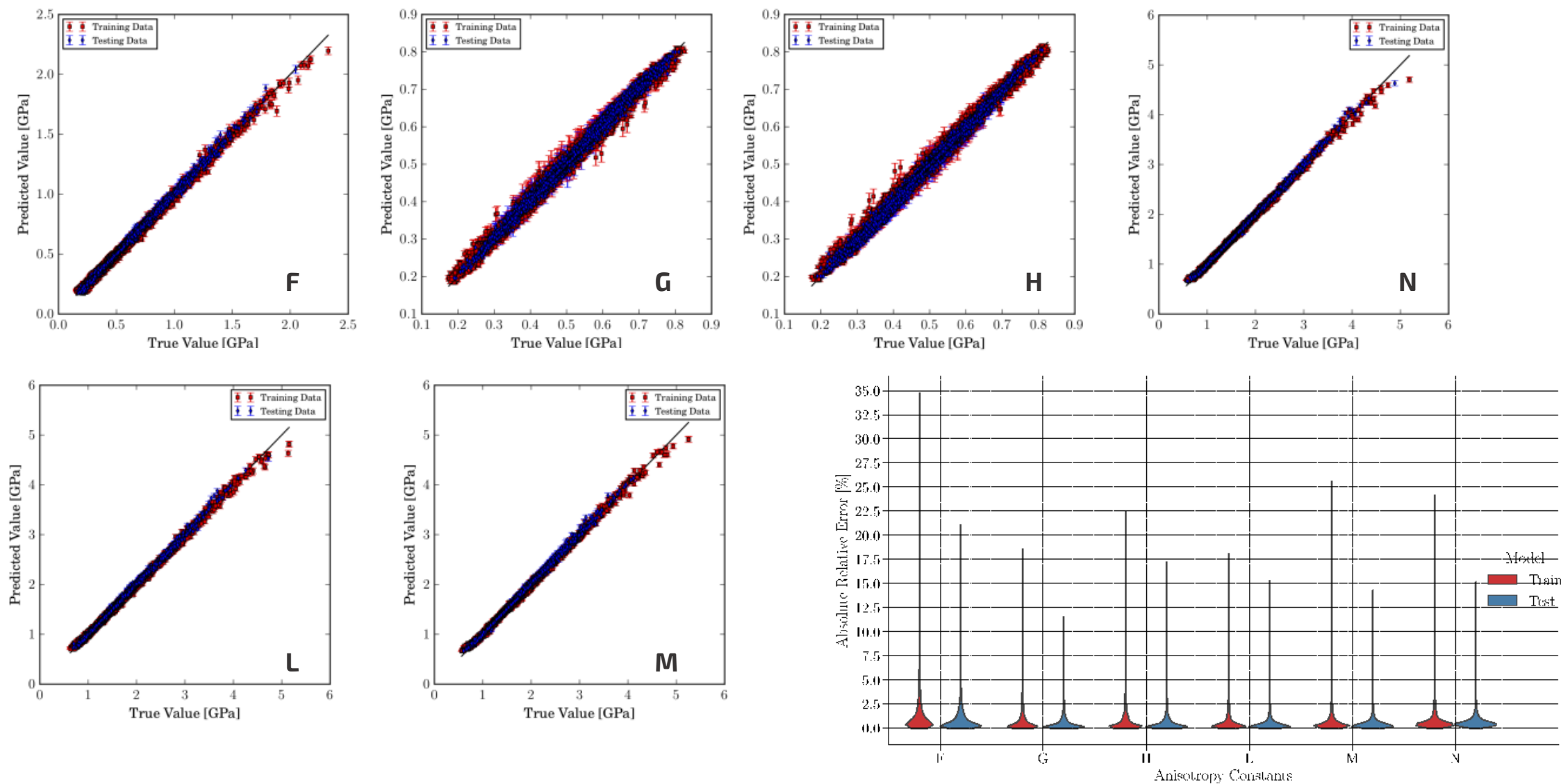


GSH representation of Testing Textures



- Monte Carlo Sampling of 50 steps to sample the distribution of weights to obtain a distribution of values.
- Architecture of 2 hidden layers with 81 nodes on each layer and sigmoid activation function to predict the output.
- Architecture trained to 5000 epochs.

Surrogate Model: Validating the developed Model

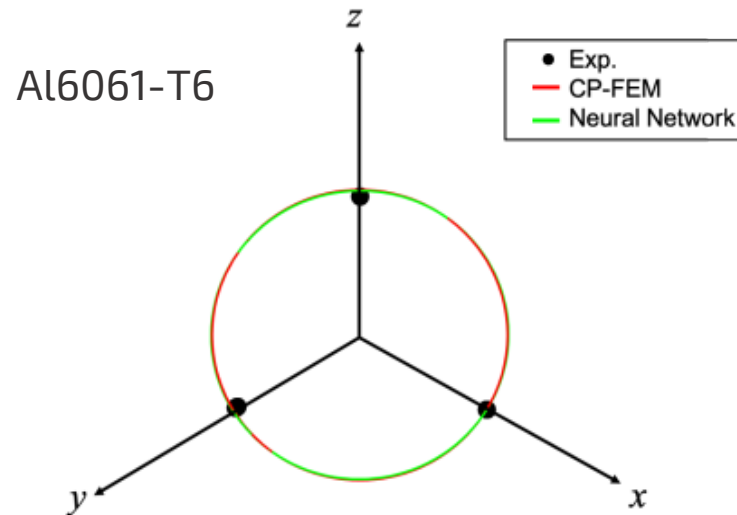


Surrogate Model: Experimental Validation



Parameterizing Hill's quadratic anisotropic yield model:

$$f = F(\sigma_{yy} - \sigma_{zz})^2 + G(\sigma_{zz} - \sigma_{xx})^2 + H(\sigma_{xx} - \sigma_{yy})^2 + 2(L\sigma_{yz}^2 + M\sigma_{zx}^2 + N\sigma_{xy}^2)$$

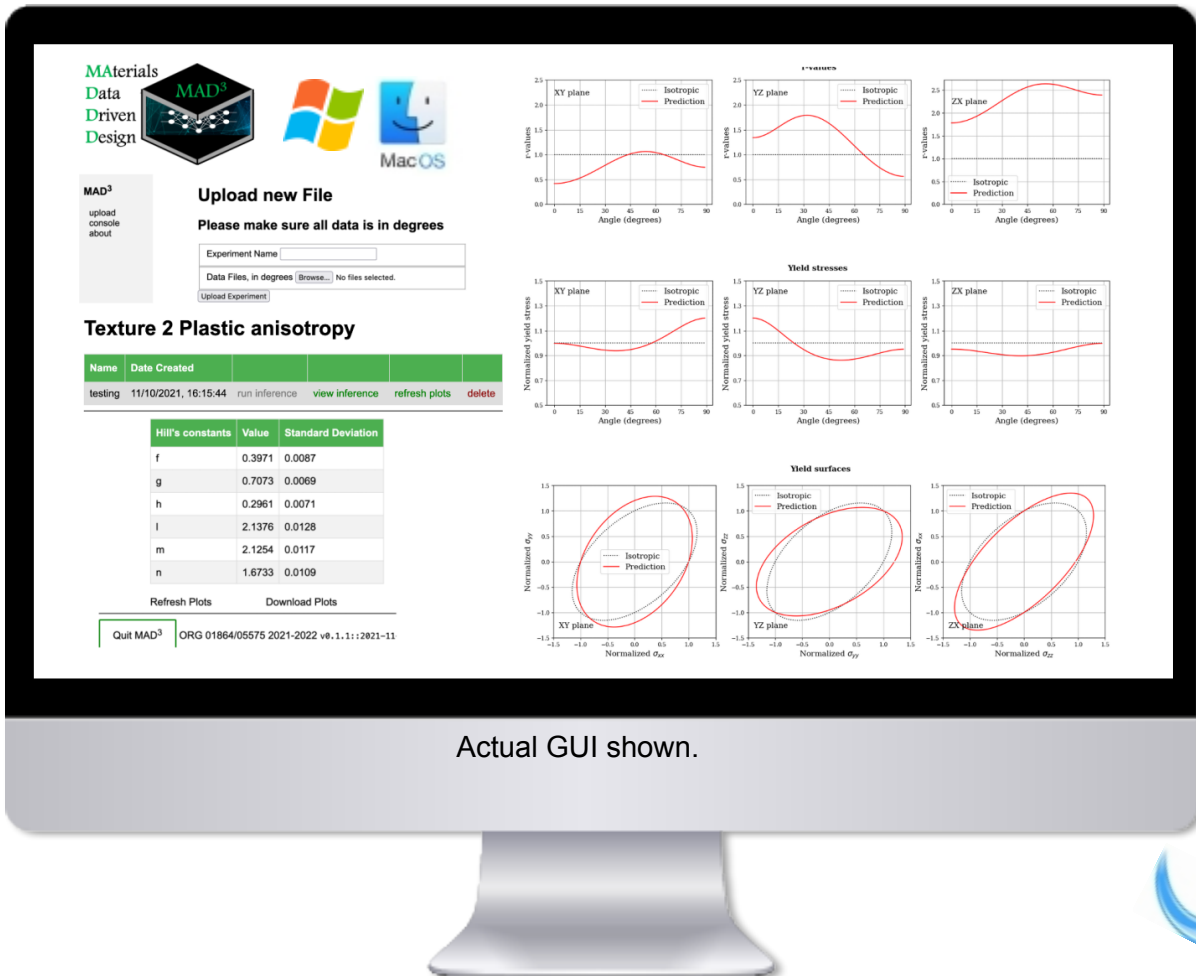


π -plane yield surfaces of Al6061

Al6061-T6	F	G	H	L	M	N	TIME
Experiments	0.6097	0.5495	0.4061	-	-	-	3-6 months
Crystal plasticity-FE	0.5268	0.5261	0.4739	1.5258	1.4788	1.7604	~10 h in HPC
Neural Network predictions	0.5298 ± 0.0013	0.5369 ± 0.0010	0.4631 ± 0.0010	1.5735 ± 0.0017	1.5296 ± 0.0013	1.6548 ± 0.0015	<1 sec.

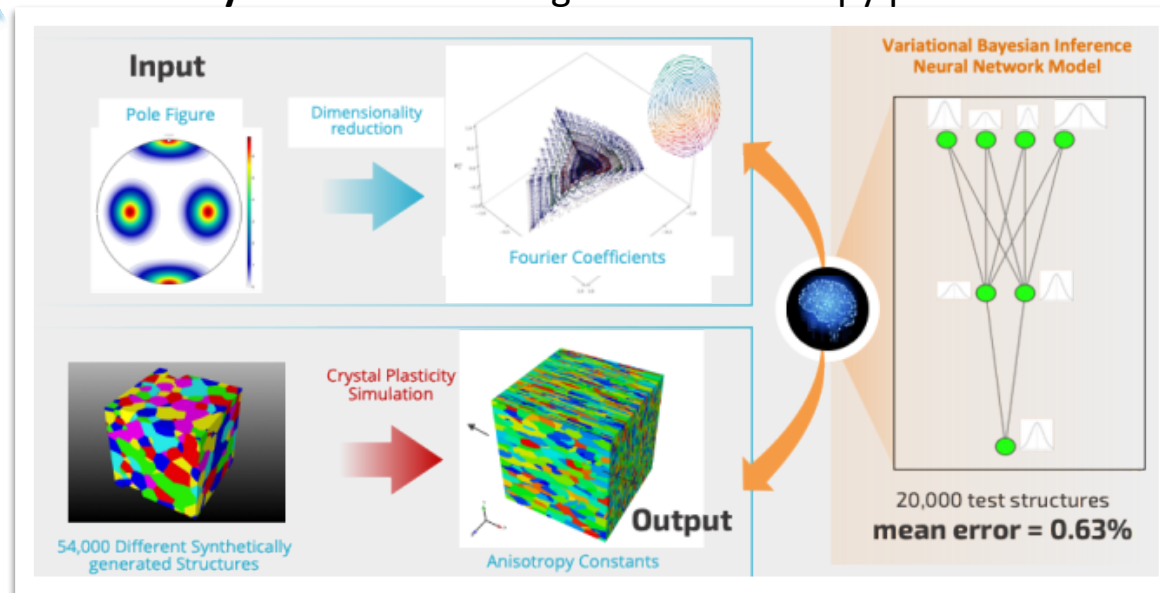
Obtain Parameterized Constants 36000 times faster than Crystal Plasticity.

Surrogate Model: GUI-based Software Solution



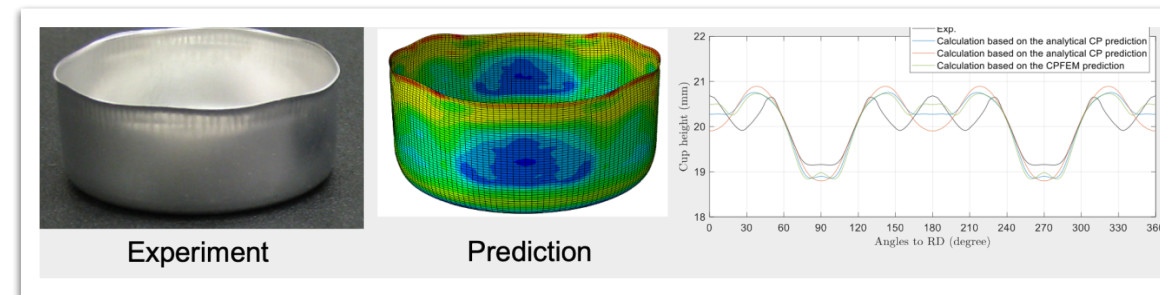
DOE Software Copyright Assertion (SCR#2683)

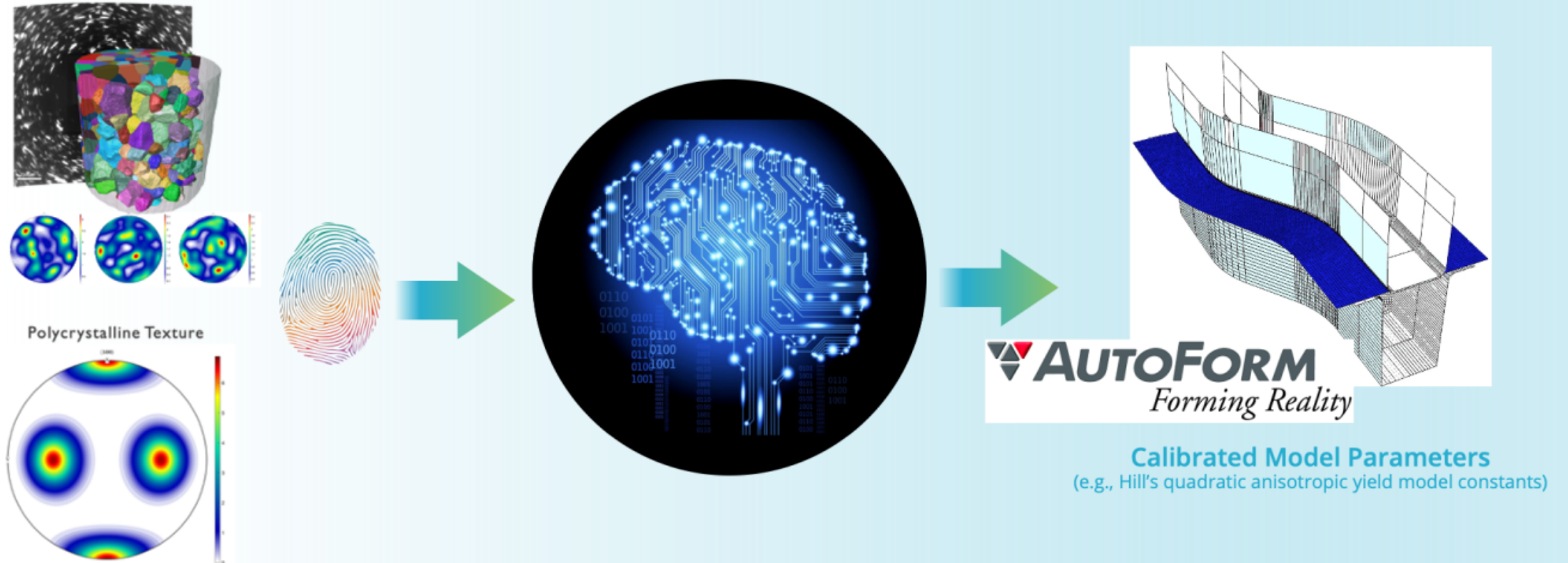
Theory: Machine Learning based anisotropy prediction



Montes de Oca Zapain et al., *Mater. Sci. Eng. A* (2022)

Application (e.g., metal forming analysis)





- Deep learning enabled us to establish an accurate and computationally efficient linkage between the underlying internal structure of the material and the resultant desired anisotropic constants while accounting for the uncertainty.



Questions?



Generalized Spherical Harmonics



GSH Representation of Texture

$$f(g) = \sum_{\mu, n, l} F_l^{\mu n} T_l^{\mu n}(g)$$

$$T_l^{mn}(g) = T_l^{mn}(\varphi_1, \Phi, \varphi_2) = e^{im\varphi_1} P_l^{mn}(\Phi) e^{im\varphi_2}$$

GSH Representation of Single Crystal Texture

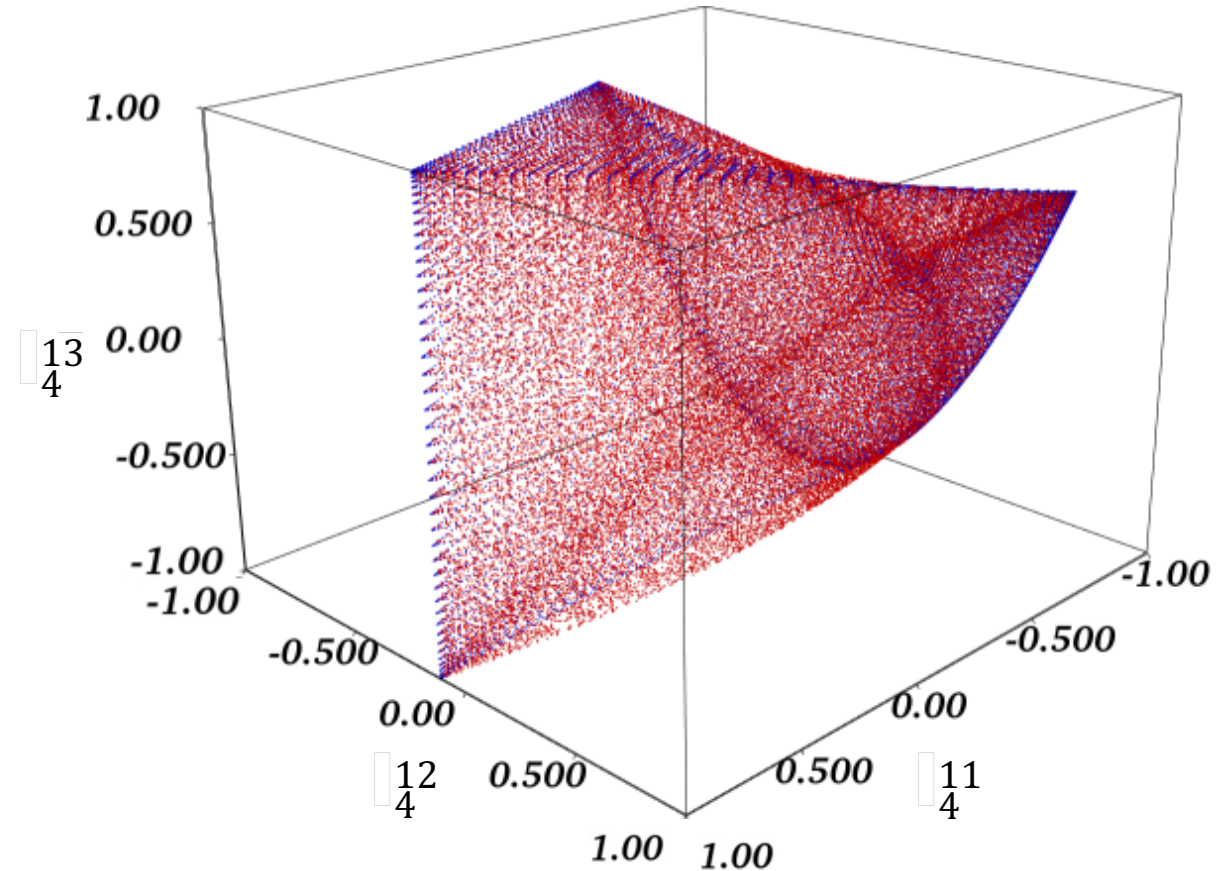
$$f(g) = \delta(g - g_0) = \sum_{\mu, n, l} F_l^{\mu n} T_l^{*\mu n}(g)$$

GSH Representation of Polycrystalline Texture

$$F_{polycrystal}^{\mu n} = \sum_j w_j F_{l,j}^{\mu n}$$

Volume fraction

GSH Representation of all Possible Orientations in the Cubic FZ



GSH Representation

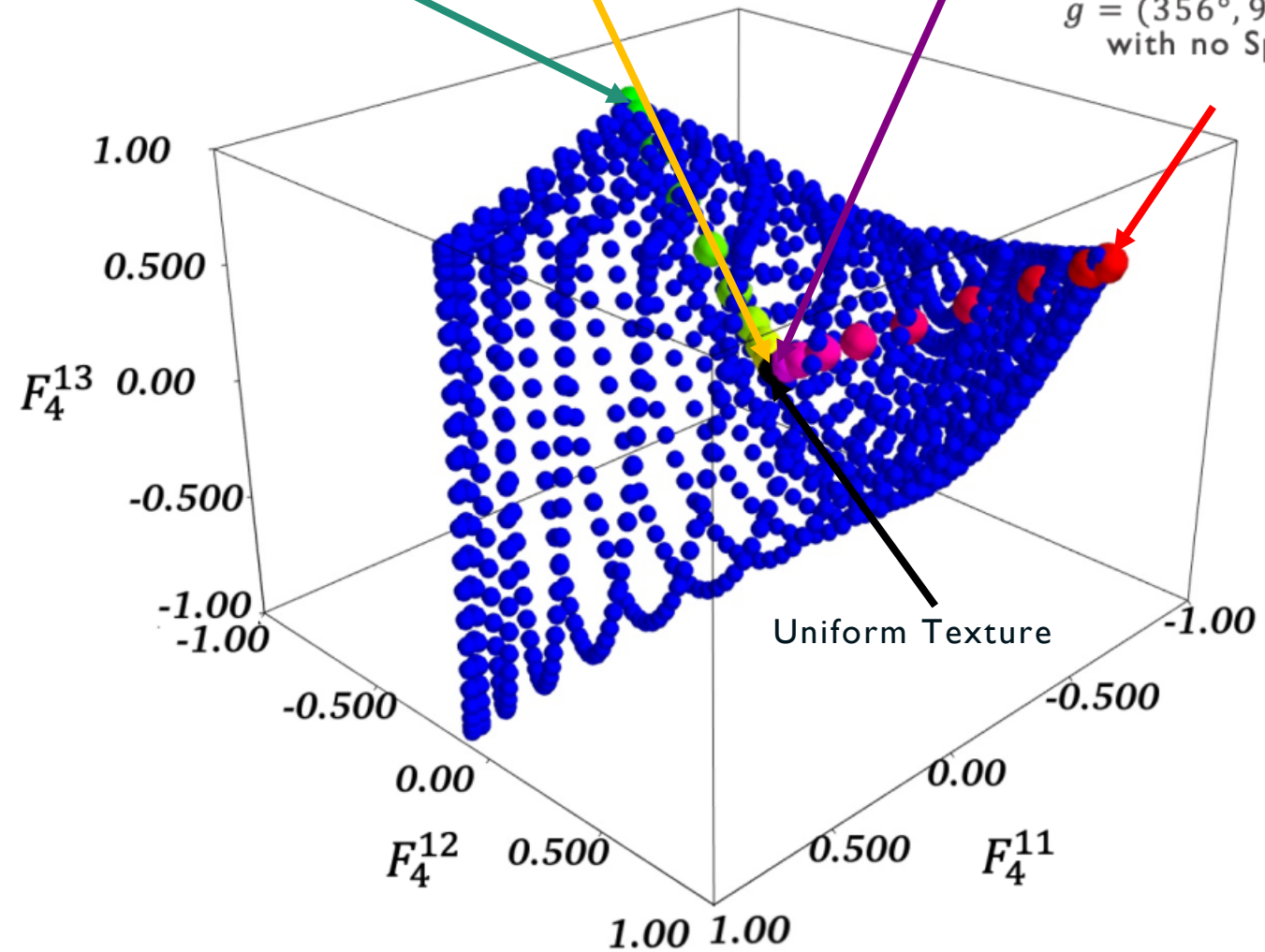


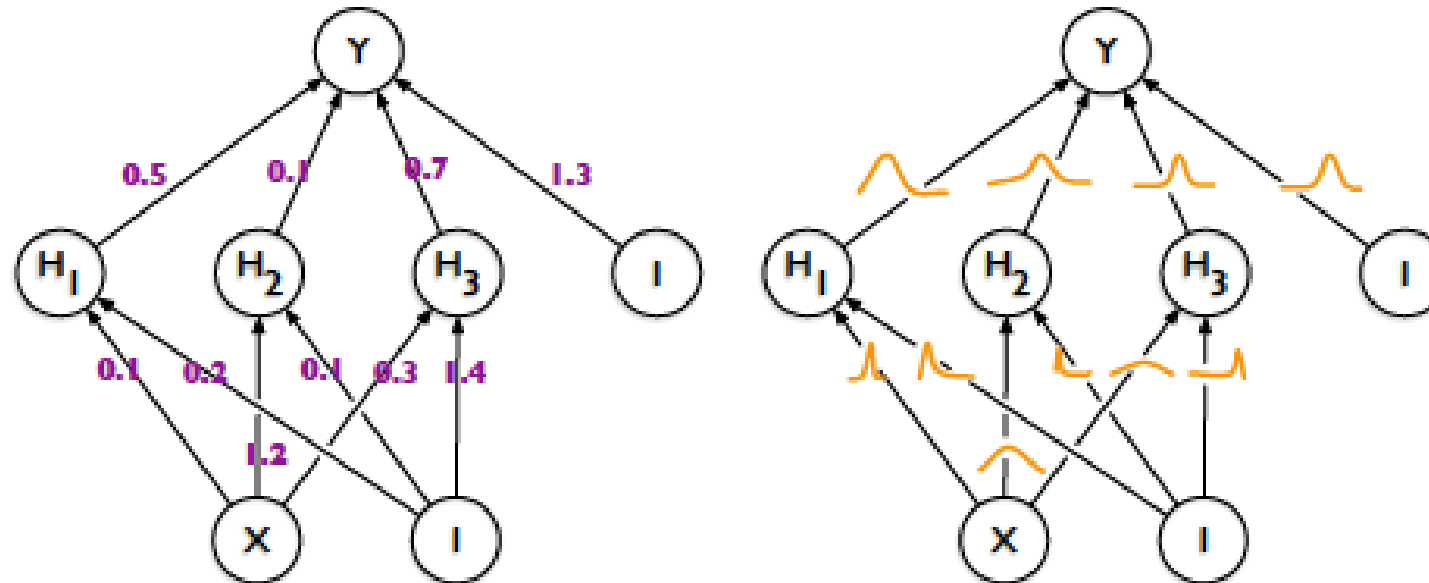
$g = (0^\circ, 45^\circ, 0^\circ)$
with no Spread

$g = (0^\circ, 45^\circ, 0^\circ)$
with 40° Spread

$g = (356^\circ, 90^\circ, 45^\circ)$
with 40° Spread

$g = (356^\circ, 90^\circ, 45^\circ)$
with no Spread





- Bayesian Neural Network that trains the parameters that describe the distribution of the weights and biases.
- Uses Monte Carlo Sampling (50 steps) to sample the distribution of weights to obtain a distribution of values.
- Enables the incorporation of uncertainty in the prediction.
- Architecture of 2 hidden layers to predict the output.