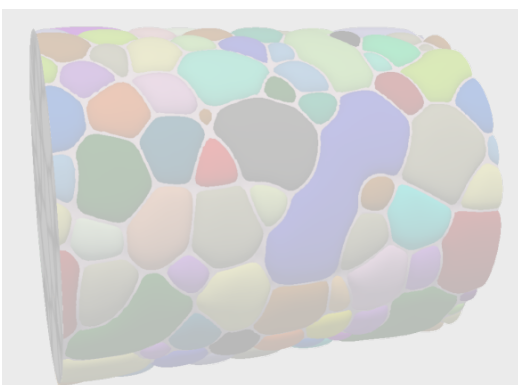


$$v_n = M\gamma H \quad \gamma = \Delta F - \Gamma \left. \frac{\partial f_{mix}}{\partial c} \right|_{eq}$$

$$\frac{\partial \mathbf{c}}{\partial t} = \nabla \cdot \left[\mathbf{M}_c \nabla \left(\frac{\delta \mathcal{F}_{tot}}{\delta \mathbf{c}} \right) \right]$$



Mesoscale Modeling of Grain Boundaries: From Interdiffusion Processes to Alloy Segregation

Fadi Abdeljawad, Paul Kotula, S. Parasad and Stephen M. Foiles

Sandia National Laboratories, Albuquerque, NM USA



*Exceptional
service
in the
national
interest*



U.S. DEPARTMENT OF
ENERGY

Office of
Science

Sandia National Laboratories, a multi-mission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

■ Nanocrystalline Materials

- Thermal instability
- Grain boundary (GB) segregation as route to mitigate grain growth processes
- GB segregation isotherms and GB phase transitions
- Simulations results

■ Kirkendall processes: Accounting for GBs

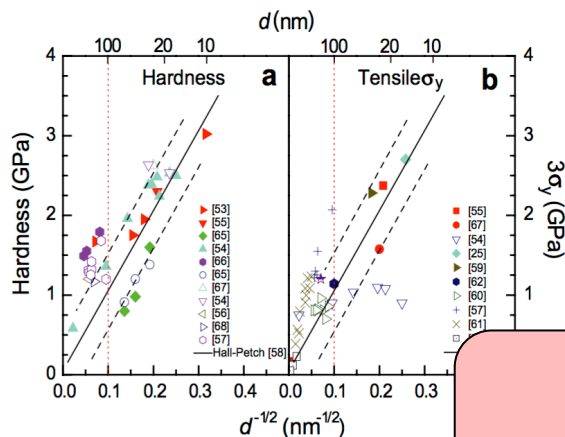
- Diffusion in substitutional binary alloys: Kirkendall effect
- A model to account for GBs as sources/sinks for vacancies
- Preliminary results

■ Concluding remarks

Grain Boundary Segregation in Nanocrystalline Alloys

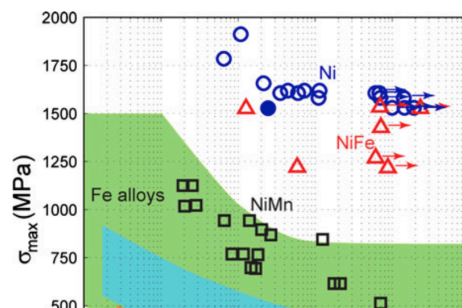
Nanocrystalline (NC) Metals

Hardness and Yield

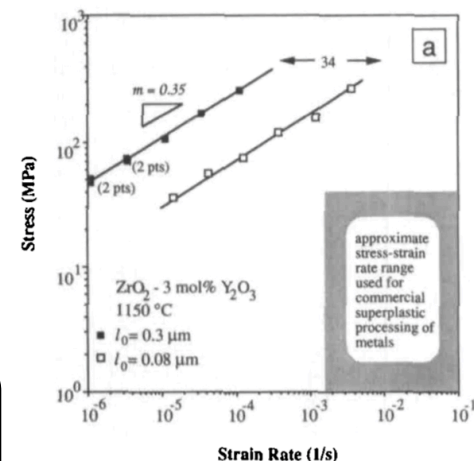


Dao *et al.*, Acta Mater. (2007)

Fatigue



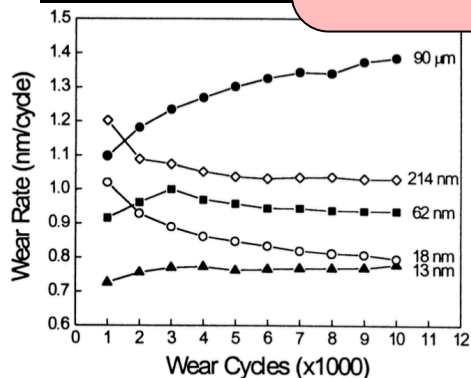
Strain Rate



Weertman *et al.*, MRS Bull. (1999)

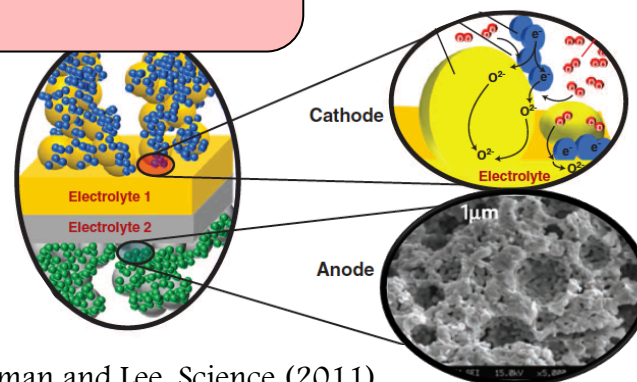
But ...

Wear (electrical)



Jeong *et al.*, Scripta Mat. (2001)

Solid oxide fuel cells



Wachsman and Lee, Science (2011)

Suzuki *et al.*, Science (2009)

The Case for NC Metals

■ **Problem:** High density of grain boundaries (GBs)

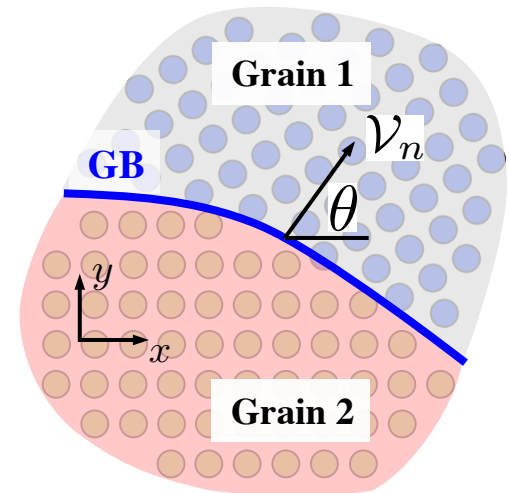
- Excess free energy: $\Delta\mathcal{F} = \Delta(\gamma A)$
- Grain-growth and homogenization processes

$$\mathcal{V}_n = M_{gb} \gamma_{gb} \mathcal{K}$$

Herring, Phys. Powd. Metall. (1951)

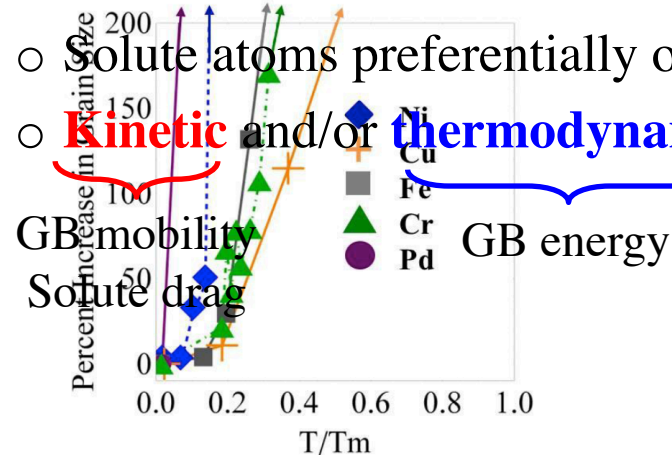
Burke and Turnbull, Prog. Metal. Phys. (1952)

Mullins, J. App. Phys. (1956)

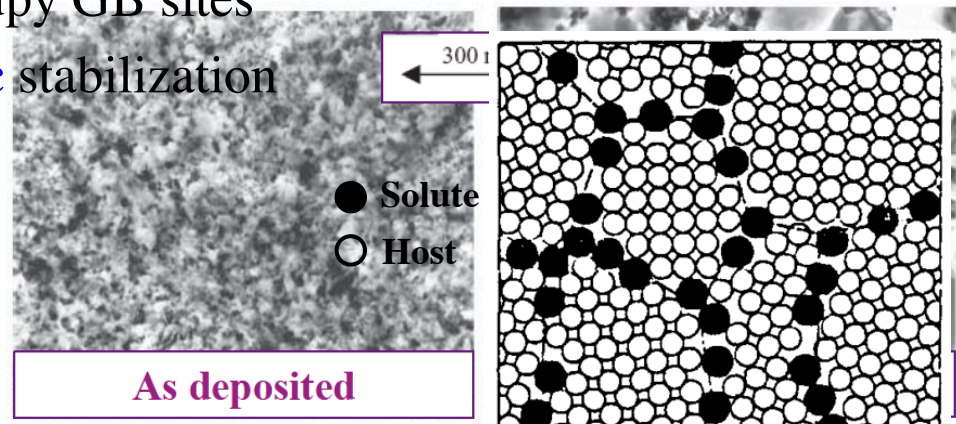


■ **Possible solution:** Solute segregation to GBs

- Solute atoms preferentially occupy GB sites
- **Kinetic** and/or **thermodynamic** stabilization



Rampant grain growth in pure Ni
Structure no longer NC after 20 min.



H. Murdoch, Phil. Mag. (1993)

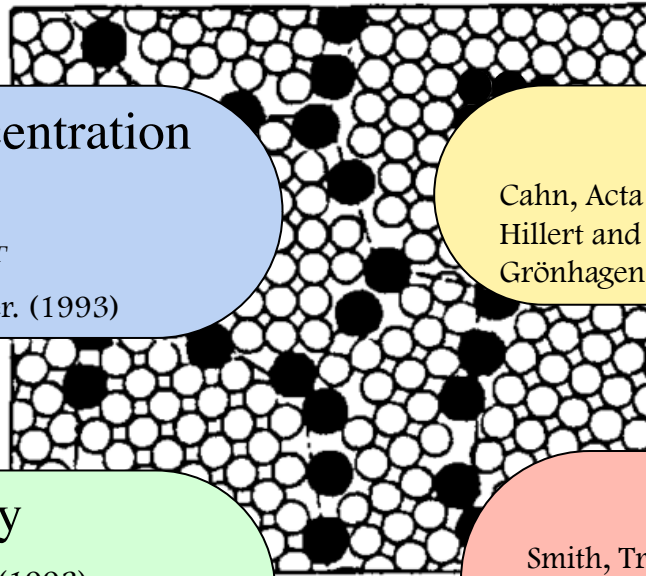
Mat. Sci. and Eng. A 539 (2012), Nanostruct. Mat. 6 (1995)

J. Phys. Chem. C 111 (2007)

Key Questions

- Predict role of GB solute segregation on **microstructural evolution**
- Leverage information from lower length scales
- **Detangle** thermodynamic aspects from kinetic ones
- Use the framework in the **design** of NC metallic alloys

Phase field framework



GB energy vs. concentration

$$\Gamma = - \left(\frac{\partial \gamma_{gb}}{\partial \mu} \right)_{P,T}$$

Weissmüller, Nanostruct. Mater. (1993)

Solute drag

Cahn, Acta Metall. (1962)

Hillert and Sundman, Acta Metall. (1976)

Grönhagen and Ågren, Acta Mater. (2007)

GB anisotropy

Udler and D. Seidman, Phys. Stat. Sol. (1992)

Seki *et al.*, Acta Metall. Mater. (1991)

Hashimoto *et al.*, Acta Metall. (1984)

Zener pinning

Smith, Trans. AIME **175** (1948)

Hellman and M. Hillert, Scand. J. Met. (1975)

Chen *et al.*, J. Am. Ceram. Soc. (1998)

Moelens *et al.*, Acta Mater. (2005)

Phase Field Formalism

Free energy

- $c(\mathbf{r}, t)$: alloy concentration
- $\Phi_i(\mathbf{r}, t)$, $\{i = 1, \dots, n_\phi\}$: grain order parameters

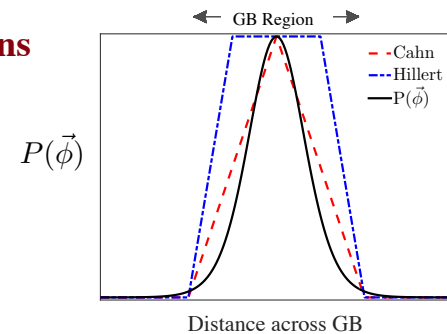
$$\mathcal{F}_{tot}[c, T] = \int d\mathbf{r} \left[f_{loc}(c, T) + \underbrace{\sum_i^{n_\phi} \frac{\epsilon^2}{2} |\nabla \phi_i|^2 + \frac{\kappa^2}{2} |\nabla c|^2}_{\text{Gradient terms for GBs and compositional damins}} \right]$$

Local energy density

Gradient terms for
GBs and compositional damins

$$f_{loc}(c, T) = P(\vec{\phi}) f_{gb}(c, T) + [1 - P(\vec{\phi})] f_b(c, T)$$

$P(\vec{\phi})$: interpolates free energy between bulk and GB

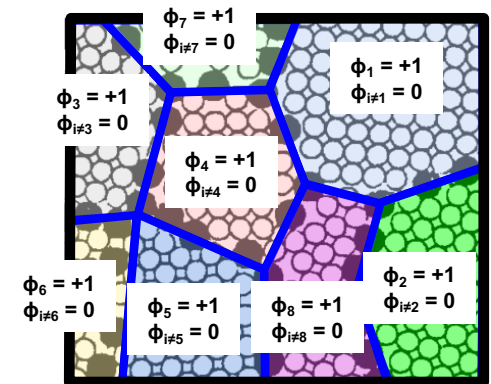


Dynamics

$$\frac{\partial c}{\partial t} = \nabla \cdot \left[M_c \nabla \left(\frac{\delta \mathcal{F}_{tot}}{\delta c} \right) \right] \quad \begin{array}{l} \text{Mass conservation} \\ \text{Cahn-Hilliard Eq.} \end{array}$$

$$\frac{\partial \phi_i}{\partial t} = -L_i \left(\frac{\delta \mathcal{F}_{tot}}{\delta \phi_i} \right) \quad \begin{array}{l} \text{Gradient flow} \\ \text{Allen-Cahn Eq.} \end{array}$$

● Solute
○ Host



GB Segregation Isotherm

$$f_i^{mix}(c, T) = \mathbf{G_i^B}c + \mathbf{G_i^A}(1 - c) + \frac{RT}{V_m} \left[c \ln c + (1 - c) \ln(1 - c) \right] + \mathbf{\Omega_i}c(1 - c), \quad i = gb, bulk$$

Free energy of pure states

Heat of mixing

A : Solvent
B : Solute atoms
 c_∞ : Alloy concentration

Bulk: G_b^A, G_b^B, Ω_b
GB: $G_{gb}^A, G_{gb}^B, \Omega_{gb}$

$$\frac{c_{gb}}{1 - c_{gb}} = \frac{c_\infty}{1 - c_\infty} \exp \left[\frac{\left\{ (G_{gb}^A - G_b^A) - (G_{gb}^B - G_b^B) - (\Omega_{gb} - \Omega_b) \right\} + 2c_{gb}\Omega_{gb} - 2c_\infty\Omega_b}{(RT/V_m)} \right]$$

Langmuir-McLean isotherm*

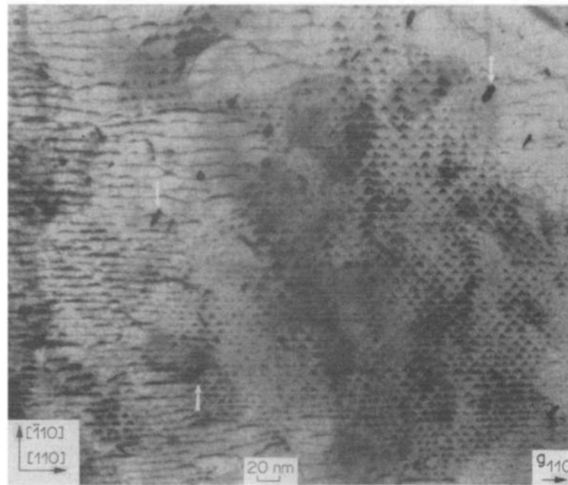
Fowler-Guggenheim isotherm**

* McLean (1957)

** Fowler and Guggenheim(1939)

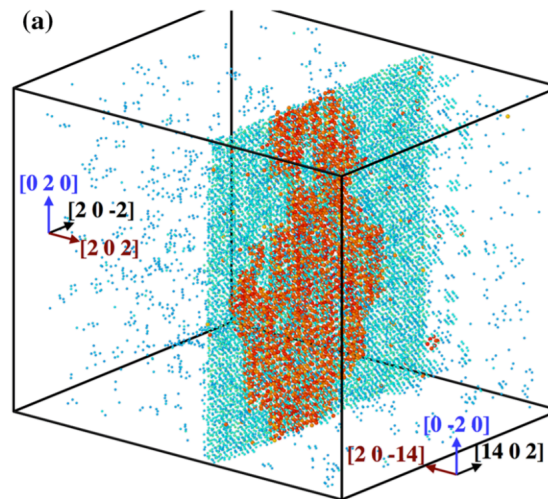
GB Segregation Isotherm

1.5° about $[001]$, Fe-1.03 at.% Au



Sickafus and Sass, Acta Metall. (1987)

$\Sigma 5$, Pt-15 at.% Au



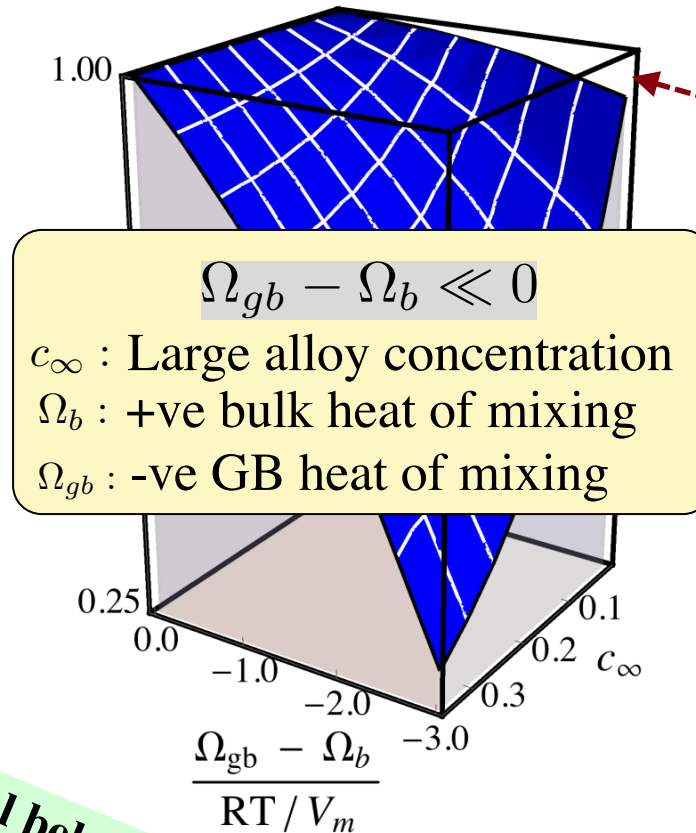
O'Brien et al. J. Mater. Sci. (2017)

GB energy $\gamma_{gb} = \gamma_{gb}(c, \Omega_{gb})$

- Gibbs adsorption eq.:

$$\gamma = \Delta F - \Gamma \left. \frac{\partial f_{mix}}{\partial c} \right|_{eq}$$

c_{∞} : Alloy concentration
 Ω_b : Bulk heat of mixing
 Ω_{gb} : GB heat of mixing



Dilute limit

Non-ideal behavior

Considerable
reduction in
GB energy

Concentration
away from
dilute limit

Abdeljawad *et al.*, Acta Mater. (2017)

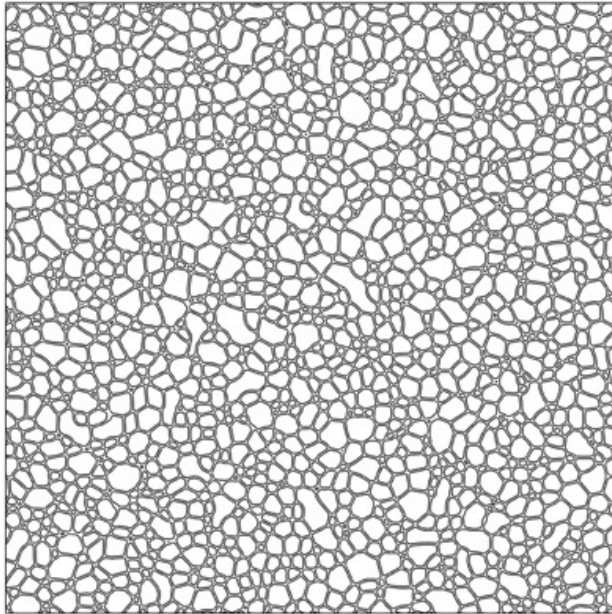
Murdoch *et al.*, Acta Mater. (2013)

Competing Processes

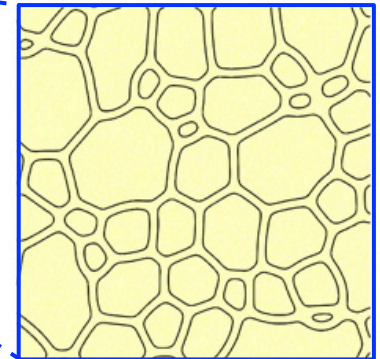
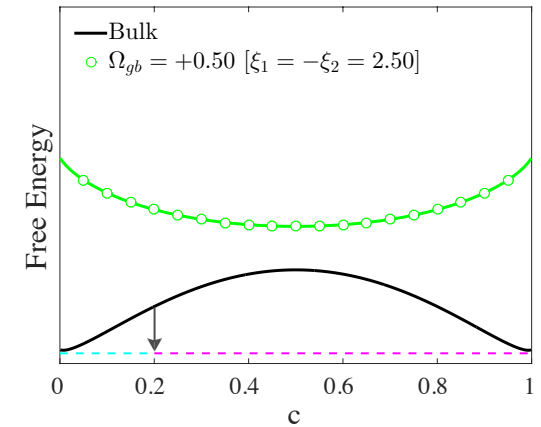
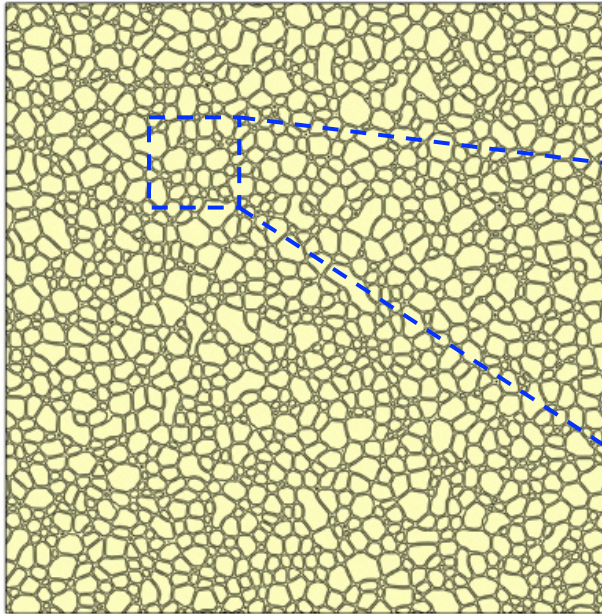
■ Polycrystalline systems

- Initially ($t=0$): **1560** grains
- $c(\mathbf{r}, t = 0) = \langle c \rangle = 0.2$

Normal grain growth
No segregation

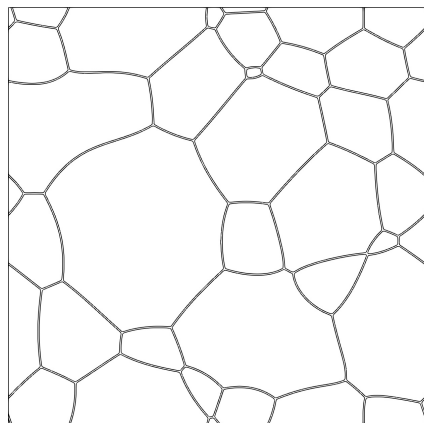


With GB segregation and
phase separation

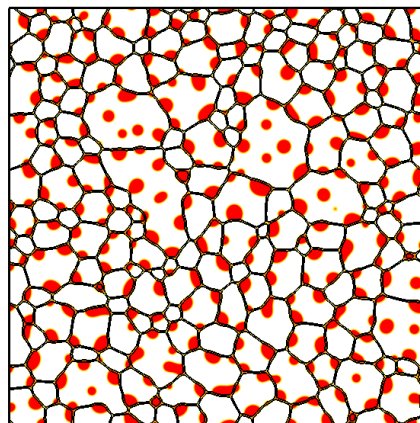


Role of GB Free Energy

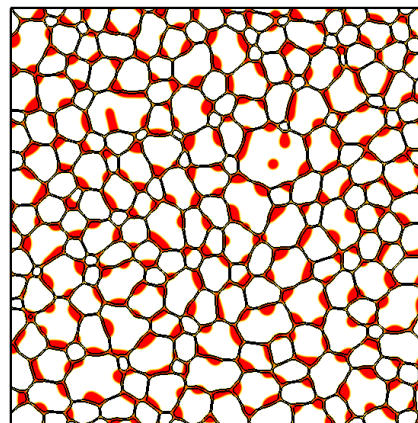
Normal grain growth
(No segregation)



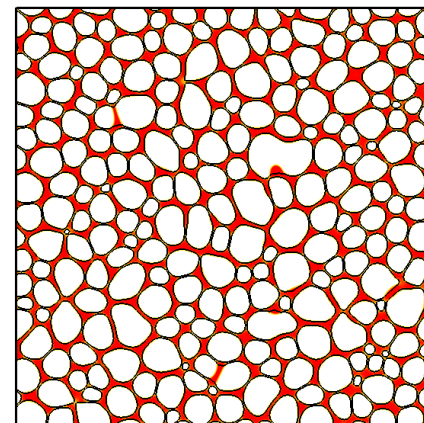
$$G_{gb}^B - G_b^B = 0.60$$



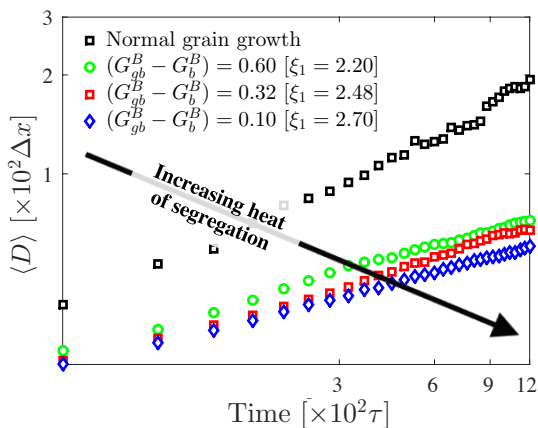
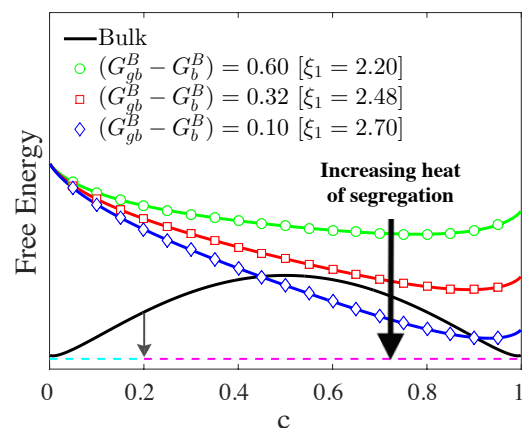
$$G_{gb}^B - G_b^B = 0.32$$



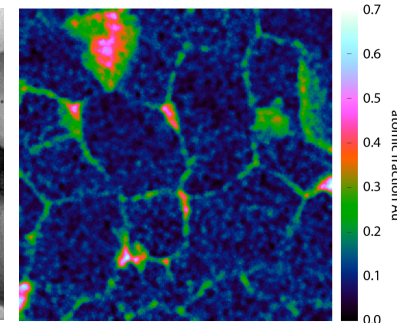
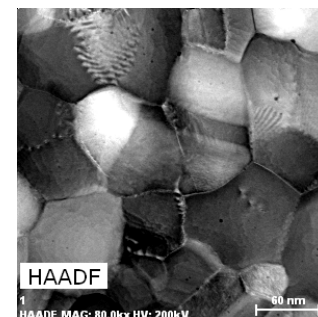
$$G_{gb}^B - G_b^B = 0.10$$



Increasing heat of segregation



Experimental: Pt-Au

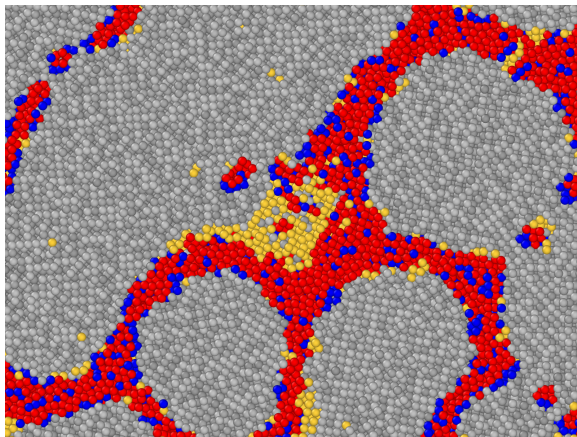


Abdeljawad *et al.*, Acta Mater. (2017)

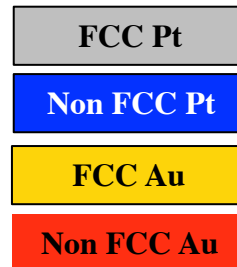
Observations

Atomistics: Pt-Au

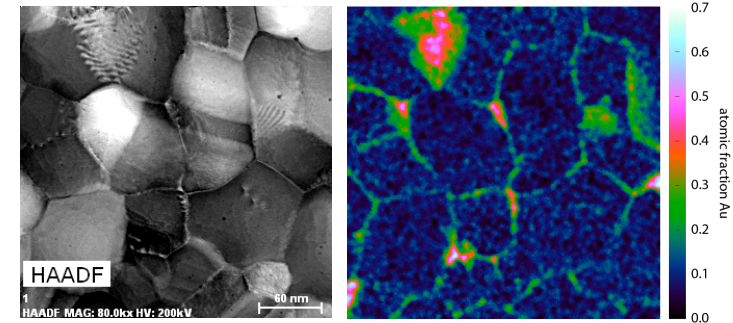
Hybrid MC/MD, $T = 775\text{K}$, 15 at.% Au



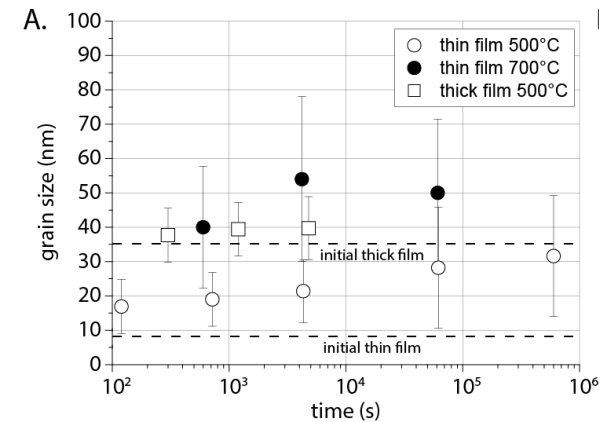
Foiles *et al.*, In preparation (2017)



Experimental: Pt-Au



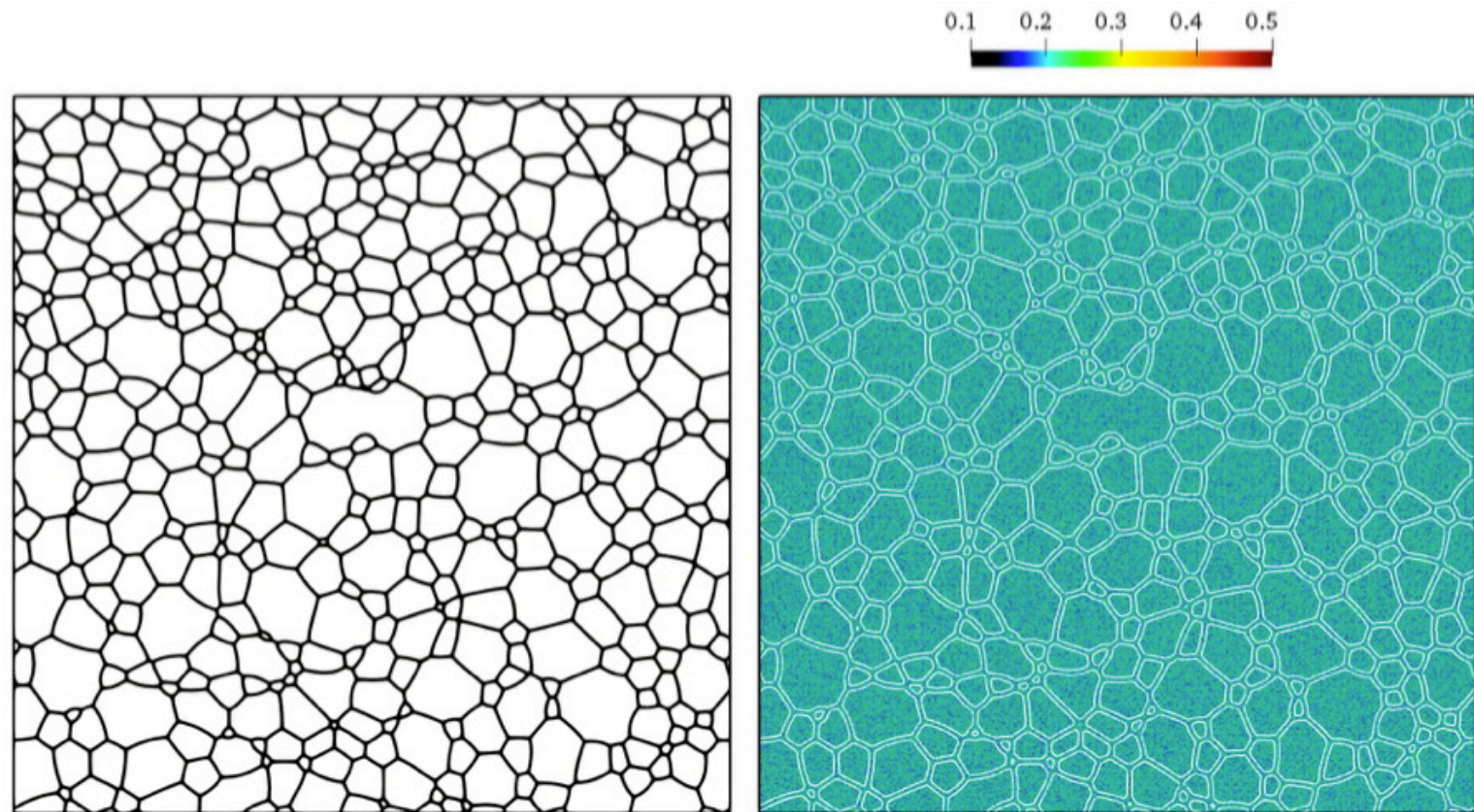
Abdeljawad *et al.*, Acta Mater. (2017)



Lu *et al.* (Under review in Scripta Mater.)

Future Work

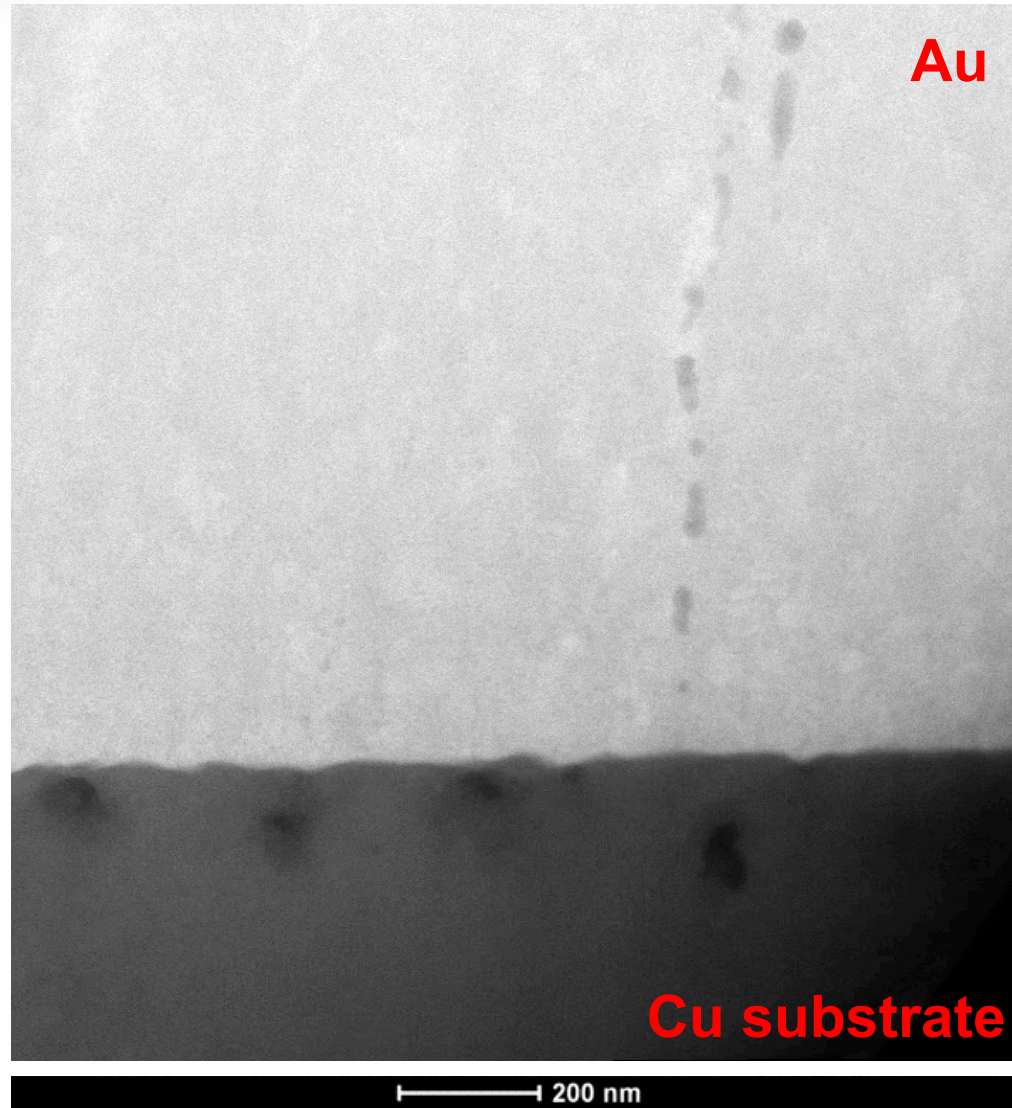
- Anisotropy in GB segregation
 - Dependency on GB geometric degrees of freedom



Substitutional Diffusion in Polycrystals

Observations: Seeing is Believing

heating to 250°C for
40 minutes

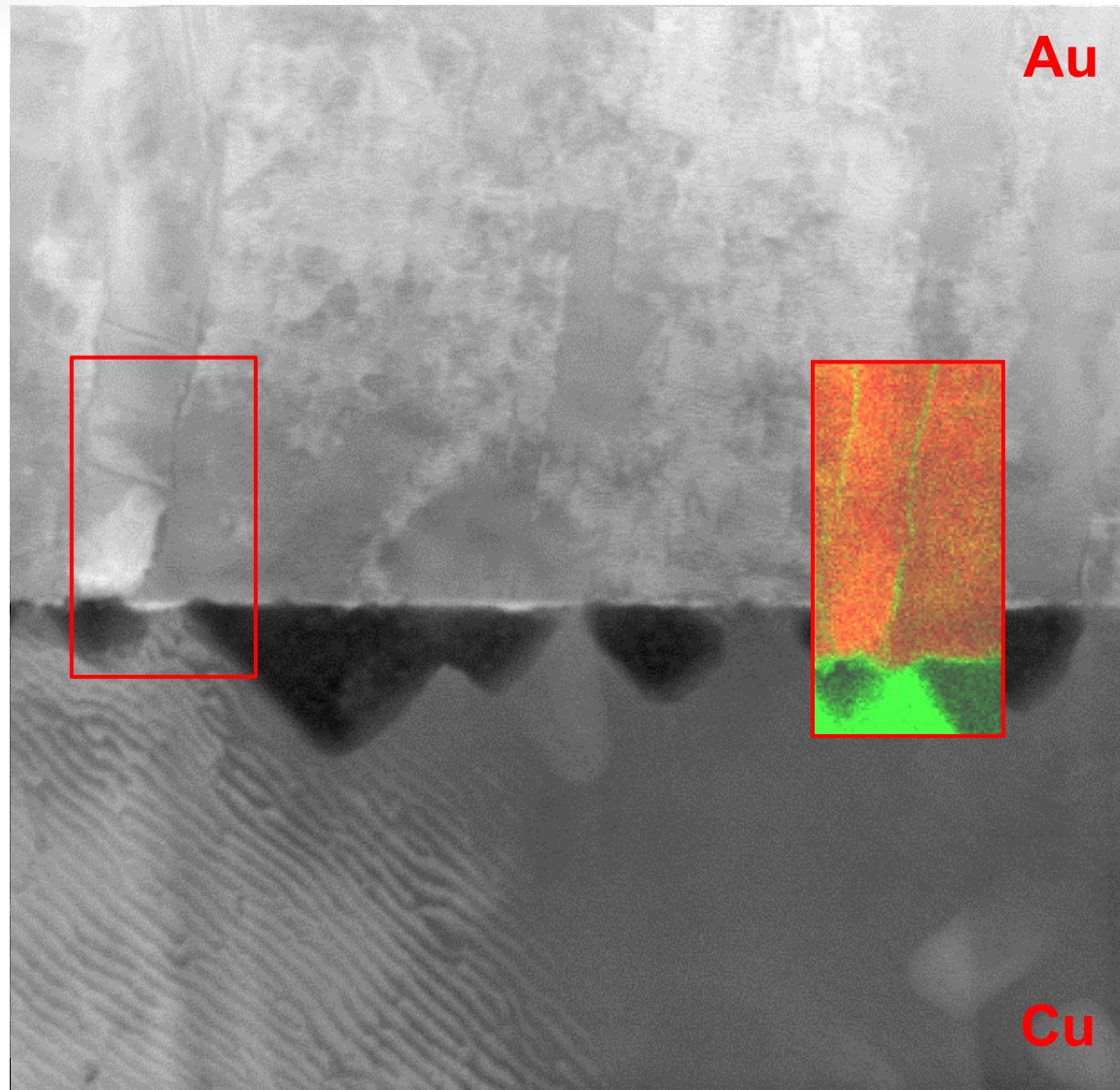


Each frame represents 2 minutes at 250C for a total heating of
about 40 minutes

Nano-grained electroplated Au

Paul Kotula (SNL, 2018)

Observations: Seeing is Believing

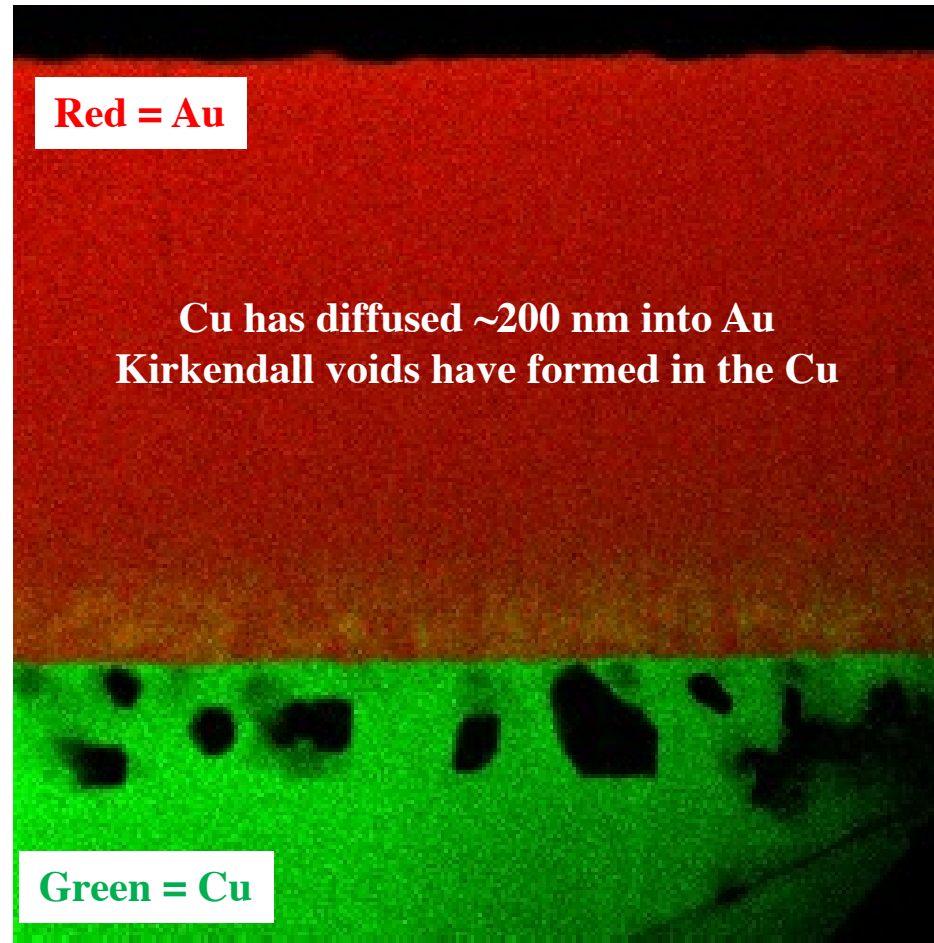


heating to 200°C for
10 minutes
Kirkendall voids

Paul Kotula (SNL, 2018)

Questions to Address

- 2) Kirkendall effect in cases when $\mu_v \neq \text{const}$ → GB/surface/dislocations: sources/sinks
Vacancy mediated transport, kirkendall voids



Paul Kotula (SNL, 2018)

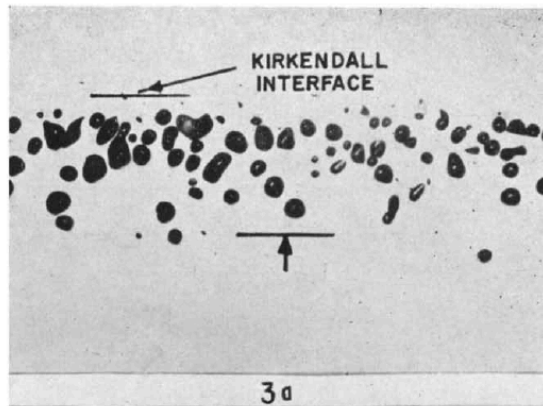
Kirkendall Effect

Kirkendall, Thomassen and Uethegrove, Trans. AIME 133 (1939)

Kirkendall, Trans AIME **147** (1942)

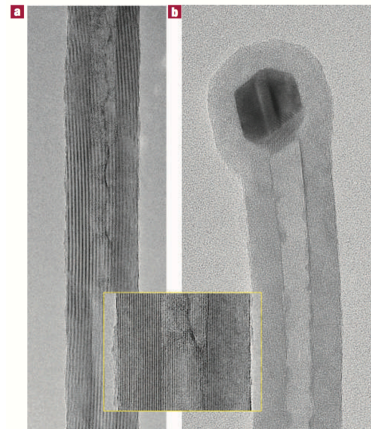
Darken, Trans. AIME 175 (1948)

Bardeen and Herring, Atom Movements (1951)



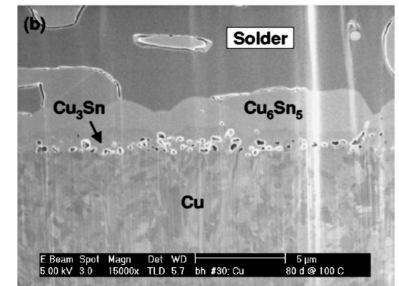
Balluffi, Acta Metall. **2** (1954)

Nanotube fabrication



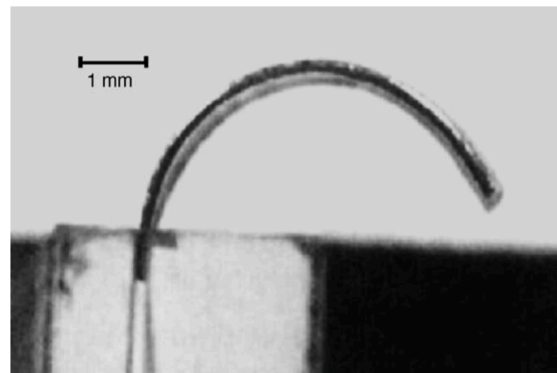
Fan et al., Nature Mater. **5** (2006)

Solder joints After 80 days at 100C



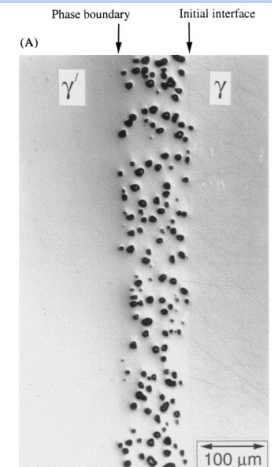
Zeng et al., J. App. Phys. **97**(2005)

Bending of Ti/Zr at 1223 K after 150 min.



Beke et al., Mater. Sci. Eng. A **387** (2004)

Superalloys Ni/Ni-14.7Al-9.9Ti (at.%)



Matan et al., Acta Mater. **46** (1998)

A Model Accounting for GBs

- Treat the system as a ternary alloy
 - Metal **A** (slow diffusor), Metal **B** (fast diffusor), and Vacancies **V**

- Irreversible thermodynamics

$$J_i = -L_{iA} \nabla \mu_A - L_{iB} \nabla \mu_B - L_{iV} \nabla \mu_V \quad i = A, B, V$$

- Convert $\nabla \mu_i$ to ∇C_i using alloy free energy (ideal alloy)

$$\begin{bmatrix} D_{AA} & D_{AB} \\ D_{BA} & D_{BB} \end{bmatrix} = \begin{bmatrix} L_{AA} & L_{AB} \\ L_{BA} & L_{BB} \end{bmatrix} \begin{bmatrix} \partial \mu_A / \partial C_A & \partial \mu_A / \partial C_B \\ \partial \mu_B / \partial C_A & \partial \mu_B / \partial C_B \end{bmatrix}$$

- Far from vacancy sources/sinks

$$J_V = -D_{VV} \nabla C_V - D_{VB} \nabla C_B$$

$$J_B = -D_{BV} \nabla C_V - D^* \nabla C_B$$

$$D_{VV} = D_{AA} + D_{BA}$$

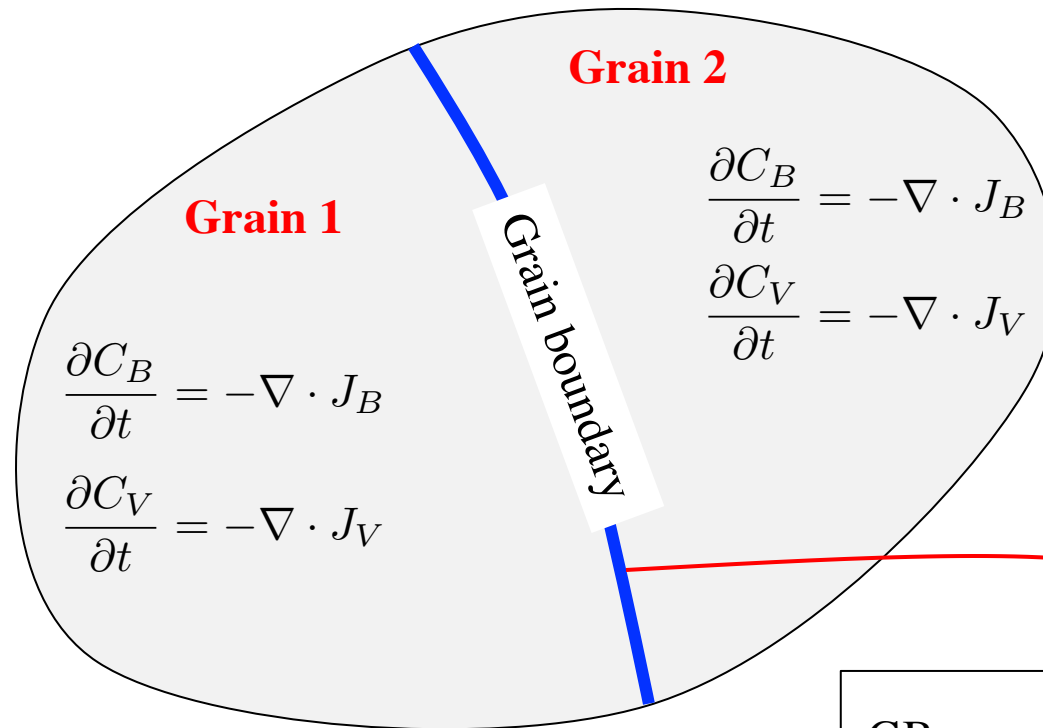
$$D_{VB} = D_{AA} - D_{AB} + D_{BA} - D_{BB}$$

$$D_{BV} = -D_{BA}$$

$$D^* = D_{BB} - D_{BA}$$

Accounting for GBs

- GBs are assumed as efficient sources/sinks for vacancies
 - Vacancy concentration at GBs is maintained at its equilibrium value C_V^{eq}
 - This introduces fluxes of vacancies into the bulk grains



$$J_V = -D_{VV}\nabla C_V - D_{VB}\nabla C_B$$

$$J_B = -D_{BV}\nabla C_V - D^*\nabla C_B$$

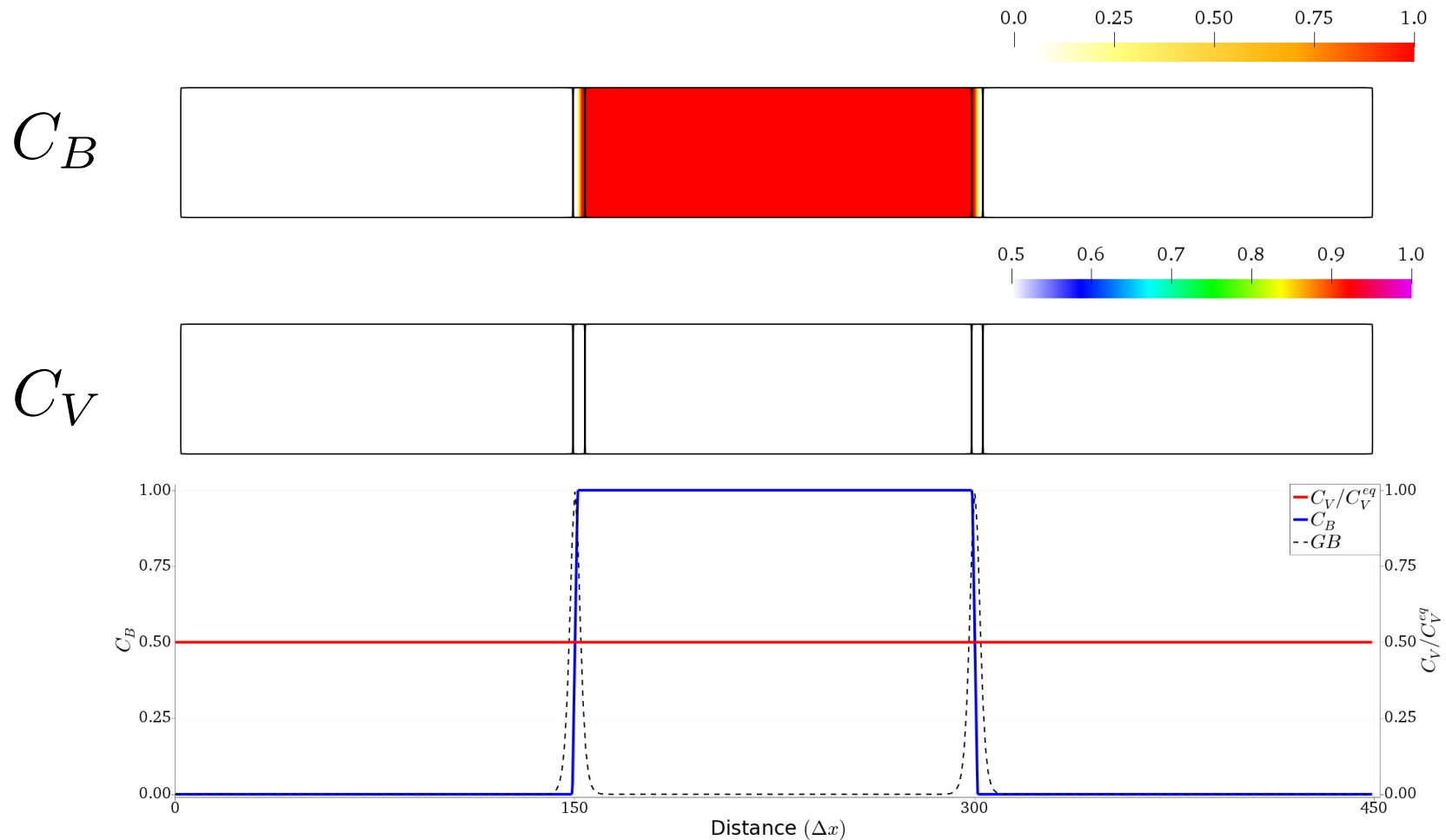
GB source/sink such that

$$C_V^{eq} = \text{const.} \rightarrow g = \frac{\nabla \cdot J_V}{1 - C_V^{eq}}$$

$$\frac{\partial C_B}{\partial t} = -\nabla \cdot J_B - \frac{C_B}{1 - C_V^{eq}} \nabla \cdot J_V$$

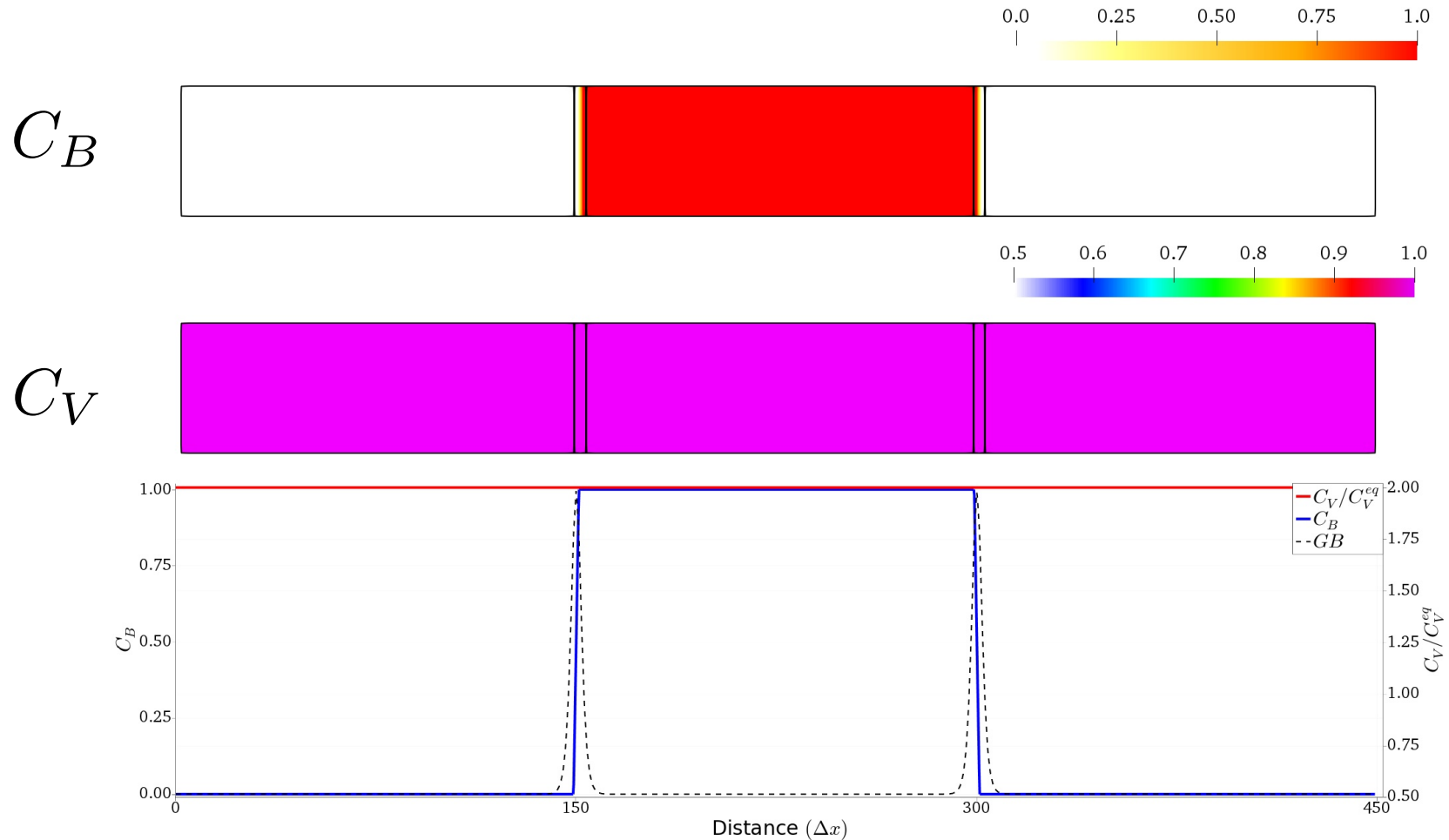
Results: Bi-crystals

- Initial condition: $C_V^{eq} > C_V(t = 0)$

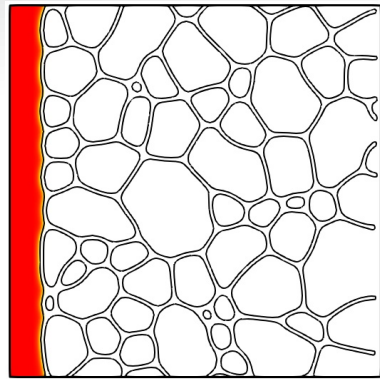


Results: Bi-crystals

- Initial condition: $C_V^{eq} < C_V(t=0)$



Results: Polycrystals



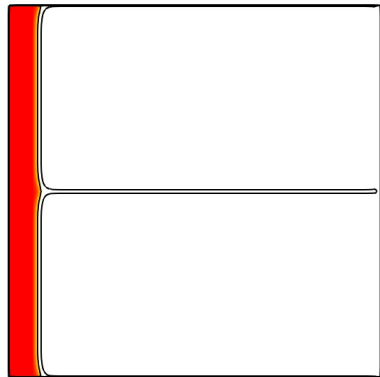
Equi-axed

c_B

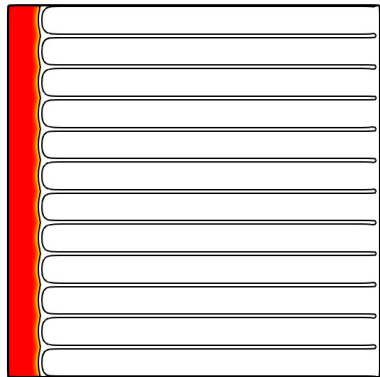


-  : Equi-axed
-  : 2-grain slab
-  : 12-grain slab

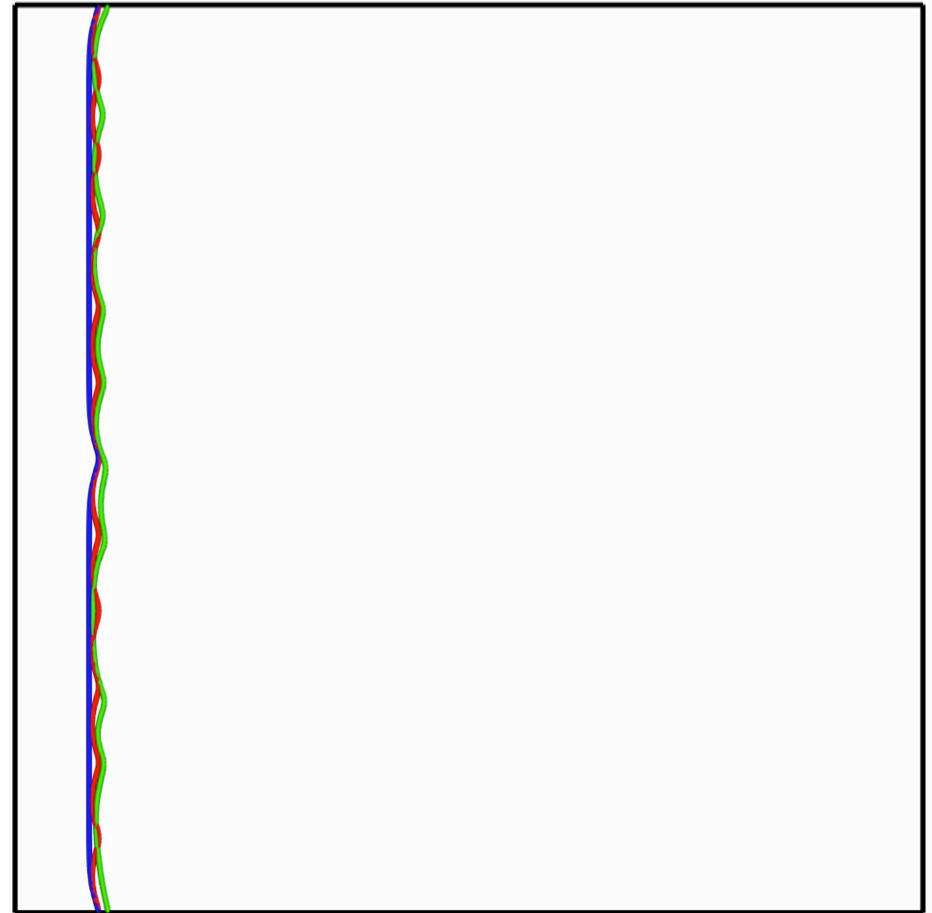
$c_B = 0.5$ iso-lines



2-grain slab



12-grain slab

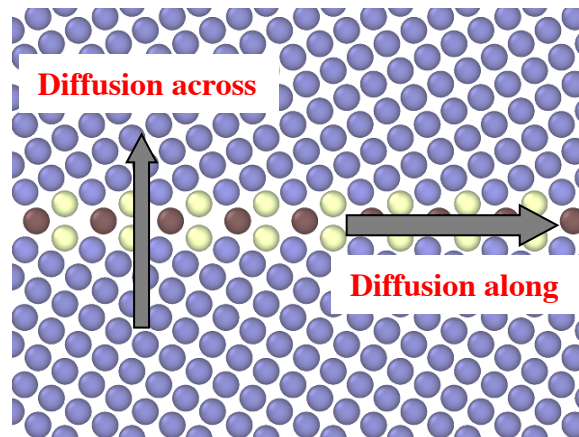


Future Work

- Atomistically informed description of D_{ij} through L_{ij}

$$L_{ij} = \frac{\langle (\sum_p \Delta \mathbf{R}_p^j(t)) \cdot (\sum_q \Delta \mathbf{R}_q^j(t)) \rangle}{2d\Omega t M k_B T}$$

- GB short circuit diffusion and anisotropy



Thank you

Publications relevant to this presentation

Kotula, Prasad, and Abdeljawad, Under preparation (2018)

Barr, Abdeljawad, Foiles, and Boyce, Under preparation (2018)

Abdeljawad, Lu, Argibay, Clark, Boyce and Foiles, Acta Mater. **127** (2017)

Abdeljawad, Medlin, Foiles, Hattar and Zimmerman, J. App. Phys. **119** (2016)

Abdeljawad and Foiles, Acta Mater. **101** (2015)

Acknowledgments

Funding:

Department of Energy (DOE) Basic Energy Sciences (BES)



Backup Slides

Competing Processes

■ Grain growth, GB segregation and phase separation

- Initially ($t=0$): **1560** grains
- $c(\mathbf{r}, t = 0) = \langle c \rangle = 0.2$ (unstable alloy)
- Vary GB free energies

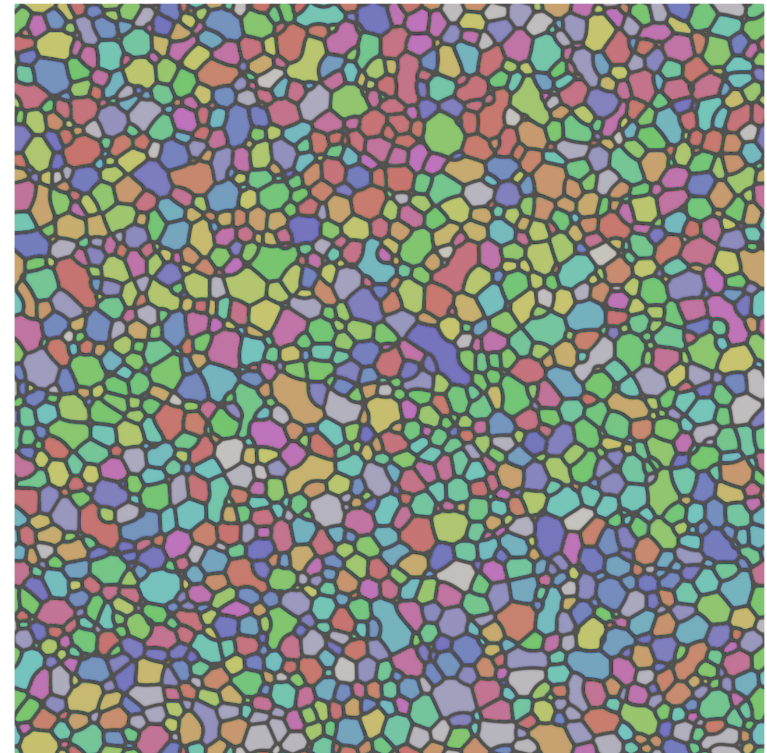
■ Metrics

- Average grain diameter $\langle D \rangle$
- Define solute partitioning factor

$$\Upsilon(t) = \frac{\int d\mathbf{r} \, c(\mathbf{r}, t) P(\vec{\phi})}{\int d\mathbf{r} \, c(\mathbf{r}, t)}$$

$$\Upsilon = \begin{cases} 0, & \text{No solute at GBs} \\ 1, & \text{GBs saturated with solute} \end{cases}$$

Initial ($t = 0$): Colors are for grain IDs



Relation to Solute Drag

- Sharp interface asymptotics (underway)

$$\mathcal{F}_{tot} = \int d\mathbf{r} \left[W f_{loc}(c, \phi, T) + \frac{\epsilon^2}{2} |\nabla \phi|^2 \right]$$

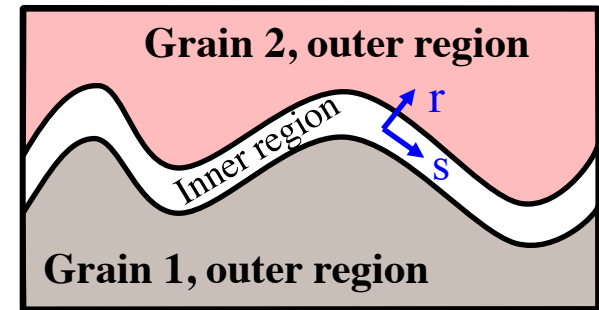
$$\frac{\partial \phi}{\partial t} = -L \left(\frac{\delta \mathcal{F}_{tot}}{\delta \phi} \right)$$

Normal grain growth

$$\mathcal{V}_n = M_{gb} \gamma_{gb} \mathcal{K} = L \epsilon^2 \mathcal{K}$$

With GB solute segregation

$$\mathcal{V}_n = \frac{L \epsilon^2}{1 + \left(\frac{LW}{D} \right) \frac{K_I}{K_{II}}} \mathcal{K} \simeq L \epsilon^2 \left(1 - \frac{LW}{D} \frac{K_I}{K_{II}} \right) \mathcal{K}$$



$$\phi = \phi_0 + \omega \phi_1 + \omega^2 \phi_2 + \dots$$

$$c = c_0 + \omega c_1 + \omega^2 c_2 + \dots$$

Provatas and Elder (2009)

Karma and Rappel, Phys. Rev. E **57** (1998)

$$K_I = \int_{-\infty}^{+\infty} \frac{\partial c_o}{\partial x} \int_0^x (c_o - c_\infty) dx' dx$$

$$K_{II} = \int_{-\infty}^{+\infty} \left(\frac{d\phi_0}{d\xi} \right)^2 d\xi$$

D: solute diffusivity

Young Man, in mathematics you don't understand things. You just get used to them

Relation to Solute Drag

- Phase field sharp interface asymptotics (underway)

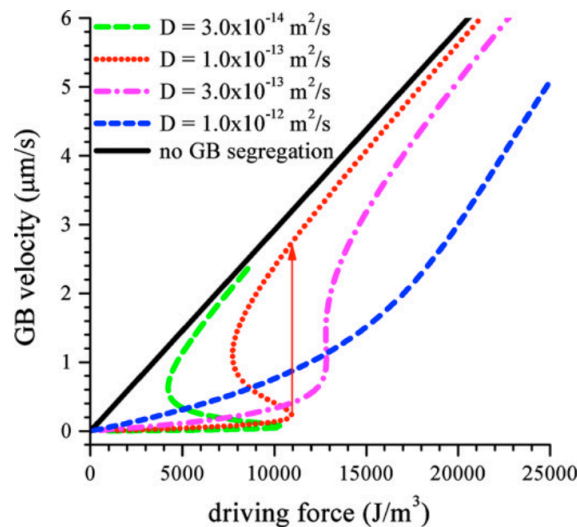
$$\mathcal{V}_n = \frac{L\epsilon^2}{1 + \left(\frac{LW}{D}\right) \frac{K_I}{K_{II}}} \mathcal{K} \simeq L\epsilon^2 \left(1 - \frac{LW}{D} \frac{K_I}{K_{II}}\right) \mathcal{K}$$

- Solute drag model [ideal, dilute alloys]

$$\mathcal{V}_n = M_{gb} \left(P - P^*(\mathcal{V}_n) \right)$$

$P = \gamma_{gb} \mathcal{K}$: Curvature driven flow

$P^*(\mathcal{V}_n)$: Drag pressure



Cahn, Acta Metall. (1962)

Hillert and Sundman, Acta Metall. (1976)

Gottstein and Shvindlerman (1999)

Kim and Park, Acta Mater. (2008)

An Active Area of Research

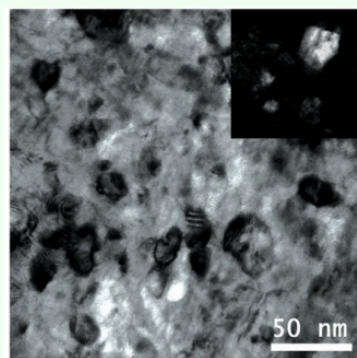
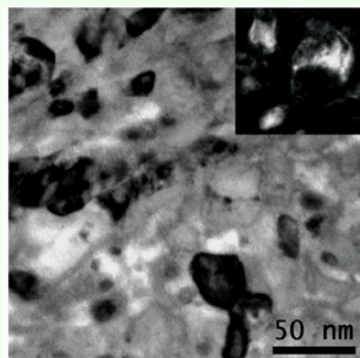
Sluggish growth dynamics

W-20 at.% Ti

Annealed

1 week at 1100°C

As milled



Chookajorn *et al.*, Science. (2012)

Chookajorn *et al.*, Acta Mater. (2014)

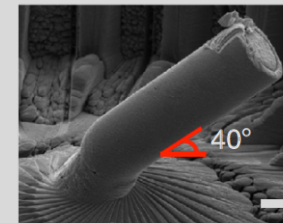
Pure Cu

$\epsilon_{\text{failure}} = 4.4 \pm 0.5\%$



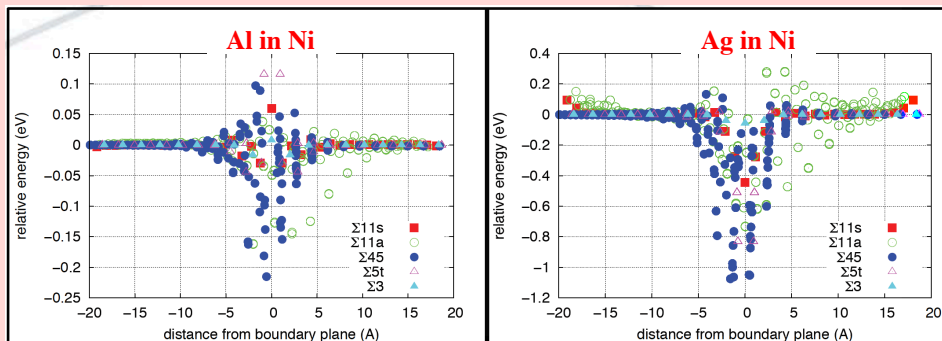
Cu-Zr (d = 45nm)

$\epsilon_{\text{failure}} = 56 \pm 2.5\%$



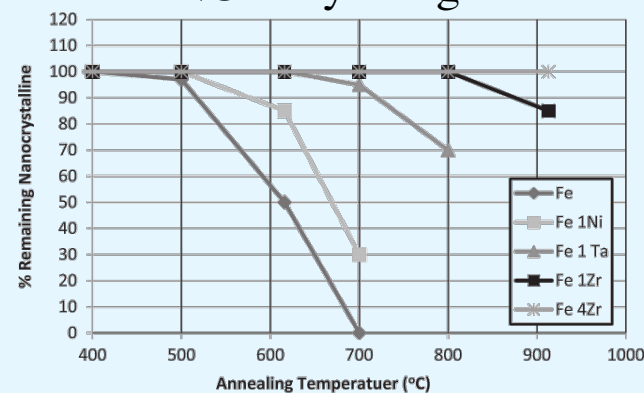
Khalajhedayati *et al.*, Nature Comm. (2016)

Dependency on solute and GB types



B. Ueberuaga, E. Martinez *et al.*, LANL (2016)

NC alloy design



Darling *et al.*, Mat. Sci. Eng. A (2011)