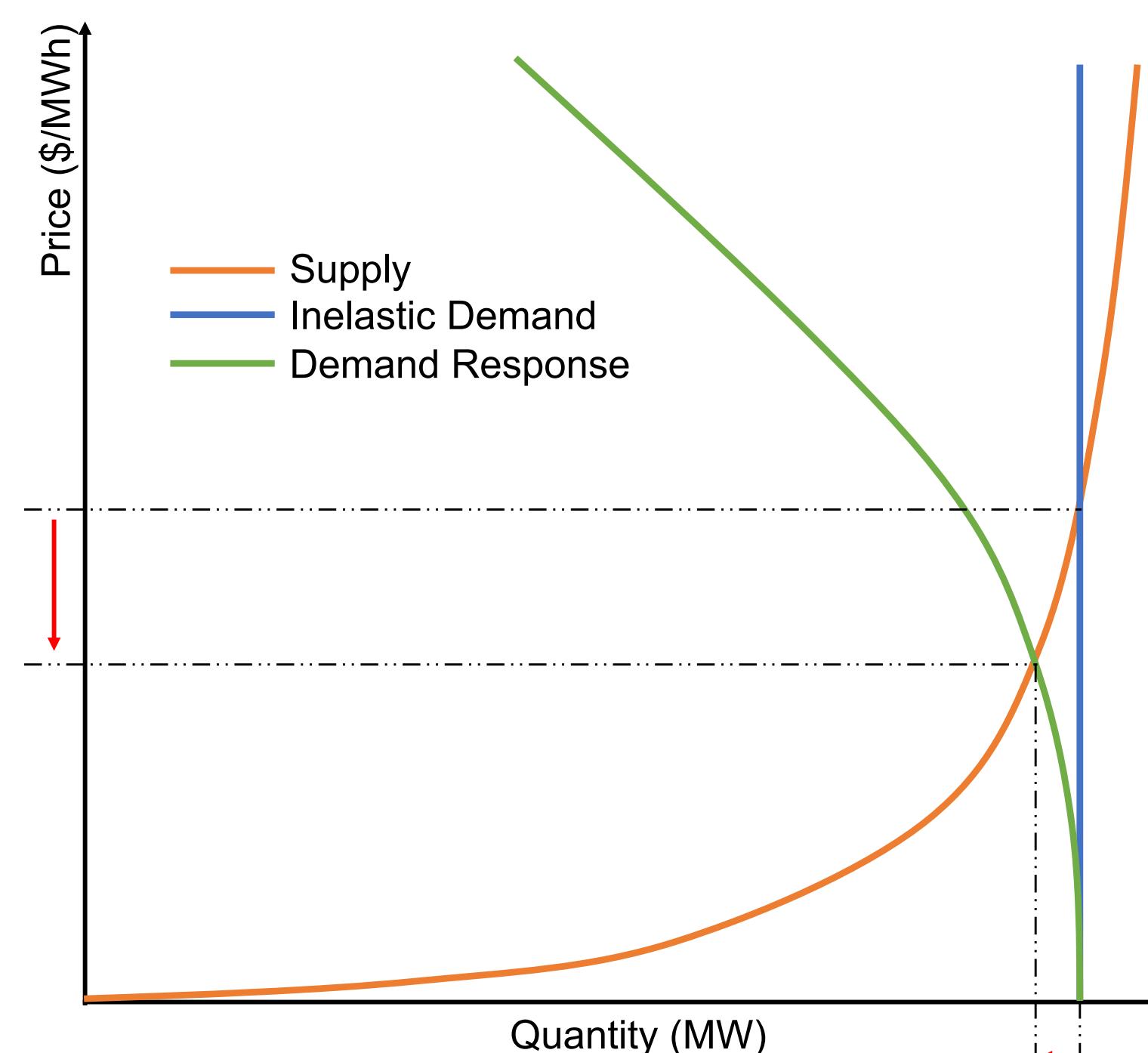


# Evaluating Demand Response (DR) Opportunities for Power Systems Resilience Using MILP and MINLP Formulations

## Motivation

- Catastrophic events result in \$18-70 billion/year in consequences and recovery costs in the U.S.
- Large industrial consumers can improve power grid resilience by modulating their load during these extreme events



## Goal of Our Study

Quantifying the tradeoff between required capital investment in the grid and the benefits of potential demand response (DR) partnerships can help process facilities make operational or investment decisions in support of these arrangements.

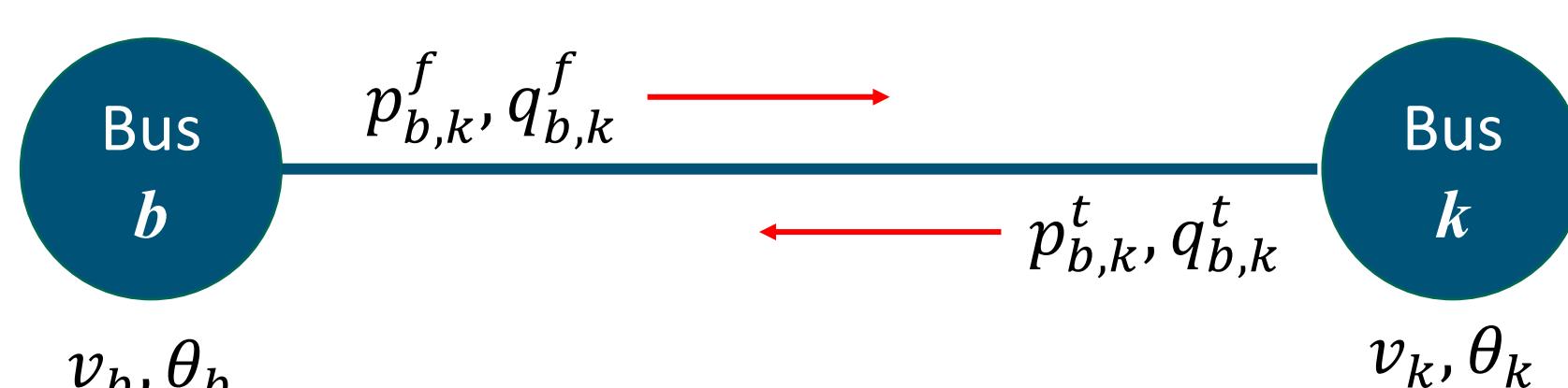
## Technical Approach

- Construct 100 weather-event scenarios with  $N$  line-outages per scenario
- Solve a stochastic programming approach to assess the value of DR compared to transmission line hardening for uncertain, extreme events
- Compare the AC power flow (nonconvex MINLP) with the DC approximation (MILP) formulation for transmission hardening versus DR contracts on an AC power grid

## AC Power Flow versus DC Power Flow Formulation

### AC Power Flow (ACPF) Formulation Nonconvexities:

$$\begin{aligned} p_{b,k}^f &= G_{b,k}^1 v_b^2 - v_b v_k (G_{b,k}^2 \cos(\theta_b - \theta_k) - B_{b,k}^2 \sin(\theta_b - \theta_k)) \\ p_{b,k}^t &= G_{b,k}^4 v_k^2 - v_b v_k (G_{b,k}^3 \cos(\theta_b - \theta_k) - B_{b,k}^3 \sin(\theta_b - \theta_k)) \\ q_{b,k}^f &= -B_{b,k}^1 v_b^2 + v_b v_k (B_{b,k}^2 \cos(\theta_b - \theta_k) + G_{b,k}^2 \sin(\theta_b - \theta_k)) \\ q_{b,k}^t &= -B_{b,k}^4 v_k^2 + v_b v_k (B_{b,k}^3 \cos(\theta_b - \theta_k) + G_{b,k}^3 \sin(\theta_b - \theta_k)) \end{aligned}$$



### DC Power Flow (DCPF) Formulation Assumptions:

- Resistance  $\ll$  Reactance (implies  $G \ll B$ )
- Nominal Voltage Magnitude ( $|V| \approx 1$  per-unit)
- Small Voltage Angle Difference ( $\sin(\theta_b - \theta_k) \approx \theta_b - \theta_k$ )

## Transmission Line Hardening versus Demand Response (DR):

$$\min |Y|_{true}$$

s.t.

### ACPF (or DCPF) Constraints and other Operational Limits

$$\begin{bmatrix} X_{s,l}^H \\ p_{b,k}^f, p_{b,k}^t, q_{b,k}^f, q_{b,k}^t \\ -\bar{\theta} \leq \theta_b - \theta_k \leq \bar{\theta} \end{bmatrix} \vee \begin{bmatrix} \neg X_{s,l}^H \\ p_{b,k}^f, p_{b,k}^t, q_{b,k}^f, q_{b,k}^t = 0 \end{bmatrix}$$

Line-outages per scenario  $s$

$Y_l \iff X_{s,l}^H$  Line selected for hardening

$$\begin{bmatrix} X_{s,b}^D \\ (1 - \Delta_b)P_b^L \leq p_{s,b}^L \leq P_b^L \end{bmatrix} \vee \begin{bmatrix} \neg X_{s,b}^D \\ p_{s,b}^L = P_b^L \end{bmatrix}$$

$X_{s,b}^D$  is false Buses in scenario  $s$  not candidates (or selected) for DR

$Z_b \iff X_{s,b}^D$  Buses selected for DR

$|Z|_{true} \leq N_z$  Limit on buses selected for DR

For increasing values of  $N_z$ , Computes minimum number of lines to harden to ensure feasible operations across all weather-event scenarios

## Solution Technique

### DCPF-Based Approach:

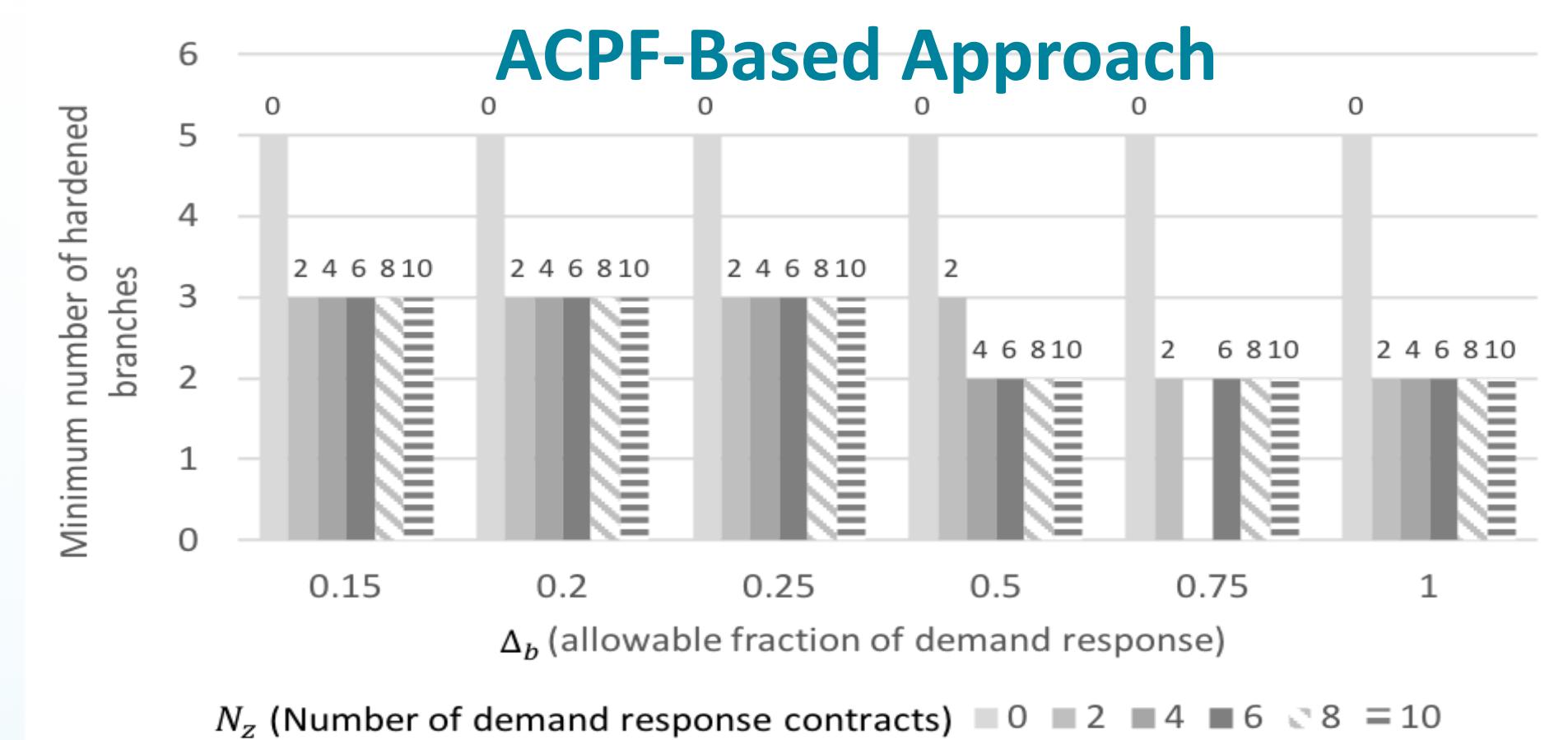
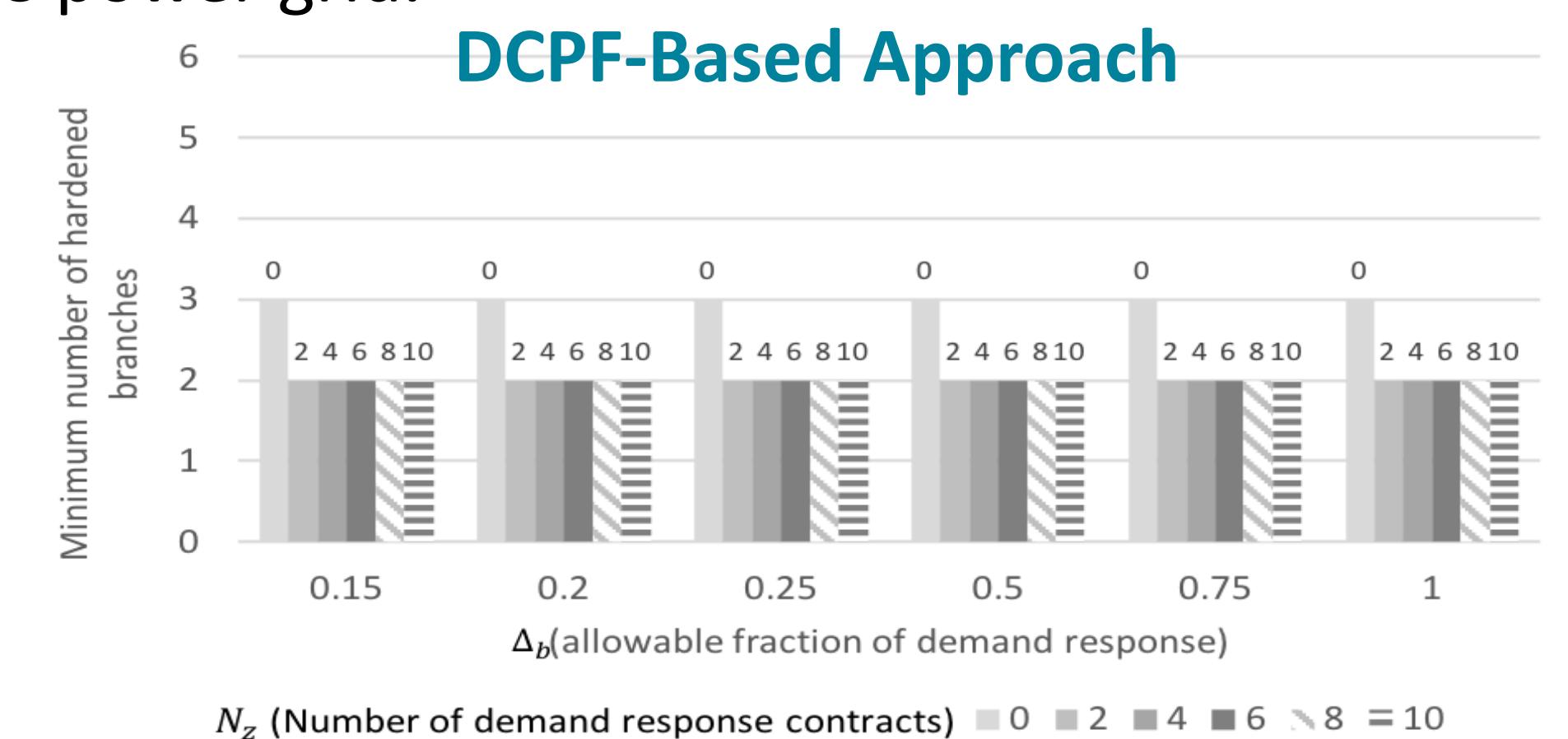
- MILP Formulation with Big-M Transformations
- Solve with Commercial B&B (Gurobi)
- Approximation is Solved Exactly

### ACPF-Based Approach:

- Nonconvex MINLP Formulation with Big-M
- Solve with Multi-Tree Approach
  - Lower-Bounding: MISOCP (Solve with Gurobi)
  - Upper-Bounding: NLP (Solve with IPOPT)
- If Zero Gap, then Formulation is Solved Globally

## Summary Results

DCPF-based approach results in solutions that are suboptimal and, at times, infeasible for operations on an AC power grid.



## Future Work

Improve Scalability of MINLP (see poster: "Decomposing Optimization Based Bounds Tightening via Graph Partitioning")

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