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Dissipativity-based Voltage Control in Distribution Grids

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Abstract

We consider the problem of decentralized voltage control using reactive power provisioned by distributed energy resources within the power distribution grid. We assume that the reactance matrix of the grid is unknown and potentially time-varying. We present conditions for stability of the system when the reactive power at each inverter is set using a potentially heterogeneous droop curve. These conditions utilize energy dissipation requirements and can be naturally satisfied even when the reactance matrix is unknown by using an adaptive controller and when the reactance matrix is time-varying.

Problem Formulation

1. Radial distribution network with buses 0, 1, $\cdots$, n. The substation bus 0 has fixed voltage.
2. Voltage magnitudes $v(k)$ can be observed and reactive power injections $q(k)$ can be controlled.
3. The reactive power injections $q(k)$ specified at time step k will reach these values at $k+1$.
4. The voltages reach steady state before each time step.
5. Linearized power flow equations at 1 p.u. voltage result in [1]:

$$\Sigma_l: \quad \begin{aligned} q(k+1) &= u(k) \\ y(k) &= v(k) \end{aligned} \tag{1}$$

1. The voltage $v(k)$ and reactive power $q(k)$ satisfy $v(k) = Xq(k) + \bar{v}$,
2. X is a positive definite matrix that characterizes the reactance of the network,
3. $\bar{v}$ depends on the real power injections and the network parameters and is not controllable.

Motivation

1. Decentralized controllers are simple to implement but they can create oscillations.
2. System parameters including the reactance matrix X are often unknown.

Control Objective

To design $u(k)$ so that $v(k)$ locally asymptotically stabilizes to a desired set point $v^\star$.

$$\lim_{k\to\infty} v(k) \to v^*$$

1. The control input $u(k)$ should be a causal function of the output $y(0), \cdots, y(k)$.
2. It is a local controller, i.e., each input $u_i(k)$ depends only on the local voltages at bus i.
3. $u(k)$ must satisfy the inverter's limits to inject or absorb reactive power: $u(k) \in [q_{\min}, q_{\max}]$.
4. Note that the desired voltage is actually 1pu, i.e. $v^\star = 1$.

Methodology

1. *Dissipativity [4]* :We show that the distribution grid described by (1) is dissipative. We achieve this using the scattering transformation technique [5].
2. *Control Design*: Next we propose a new droop-like that controller and prove the asymptotic stability using the above said dissipativity property.
3. *Extremum Seeking Controller (ESC) [2]*: Used to estimate the desired reactive power setpoint and prove the stability of the overall system.

Dissipativity [4]

1. Denote the set of all feasible operating points by
$$\mathcal{C} = \{(q,v) \in \mathbb{R}^n \times \mathbb{R}^n | v = Xq + \bar{v}\}. \tag{2}$$
2. Let $(q^\star, v^\star) \in \mathcal{C}$ denote the desired operating point of the system Σ_l corresponding to the voltage set point $v^\star$ and the reactive power $v^\star = Xq^\star + \bar{v}$, with $q^\star \in [q_{\min}, q_{\max}]$.
3. Denote the incremental quantities $\Delta q(k) = q(k) - q^\star$, $\Delta u(k) = u(k) - u^\star$, and $\Delta v(k) = v(k) - v^\star$.
4. Denote
$$\nu(k) = \Delta v(k) + X\Delta u(k), \tag{3a}$$
$$\omega(k) = -\Delta v(k) + X\Delta u(k). \tag{3b}$$

Dissipativity

Lemma: The linearized system Σ_l is passive with respect to the input $\nu(k)$ and output $\omega(k)$ irrespective of how $u(k)$ is designed.

Controller

$$u(k) = \begin{cases} q_{\max} & v(k) < v_l \\ u^\star - \overline{K}\,(v(k) - v^\star) & v_l \le v(k) \le v_h \\ q_{\min} & v(k) > v_h, \end{cases} \tag{4}$$

Stability

Theorem: Consider Σ_l with the controller (4). Let $\overline{K}$ be a diagonal matrix that satisfies
$$K := (I + X\overline{K})^{-1}\left(X\overline{K} - I\right) < 0 \tag{5}$$
in the sense that $K + K^T$ is negative-definite. Then:

(i) The closed loop system is dissipative with respect to the supply-rate $w(\omega) := \omega^\top K \omega$.
(ii) The closed loop system is asymptotically stabilized to the desired operating point $(q^\star, v^\star)$.

Extremum Seeking Controller (ESC)

1. (4) requires the knowledge of the set point $u^\star$, which, in turn, requires the value of X
2. Following the theory of ESCs [2] the controller implemented at time k is of the form:
$$u(k) = \hat{u^\star}(k) - \overline{K}\,(v(k) - v^\star)\,, \tag{6}$$
where $\hat{u^\star}(k) \in \mathbb{R}^n$ denotes the current estimate of the unknown desired reactive power $u^\star$.

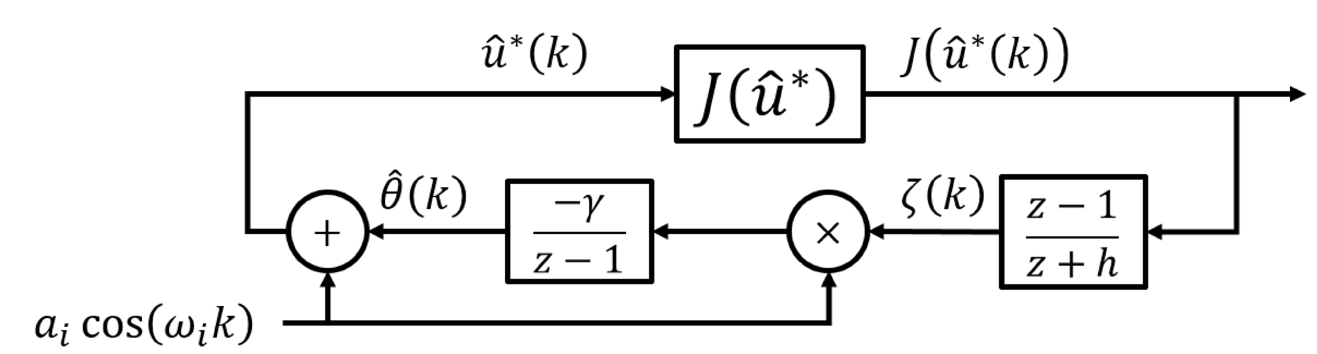

Figure 1. Proposed algorithm for identifying $\hat{u}^\star(k)$.

Since $u^\star$ corresponds to $q^\star$, we define a cost function $J(\hat{u^\star}(k))$ associated with $\hat{u^\star}(k)$ as
$$J(\hat{u^\star}(k)) = \|Xu(k) + \bar{v} - v^\star\|_2^2. \tag{7}$$

Stability under Time-Varying X

1. A time-varying X may not always satisfy (5). Thus, the stability proof above (and in similar works in the literature) will be violated.
2. Suppose X take matrices from a set $\{X_0, \cdots, X_n\}$. Let $K_i := (I + X_i\bar{K})^{-1}(X_i\bar{K} - I)$, and λ_i be the smallest eigenvalue of the matrix K_i and $\lambda_{\min} = \min_{i=1,\cdots,n} \lambda_i$.
3. Sufficient condition for stability: ensure that for every block of N steps, the nominal matrix X_0 is active at least m times such that $-m\lambda_0 - (N-m)\lambda_{\min} < 0$. λ_0 is the eigenvalue of K in (5).

Case Study: IEEE 13-bus test feeder system

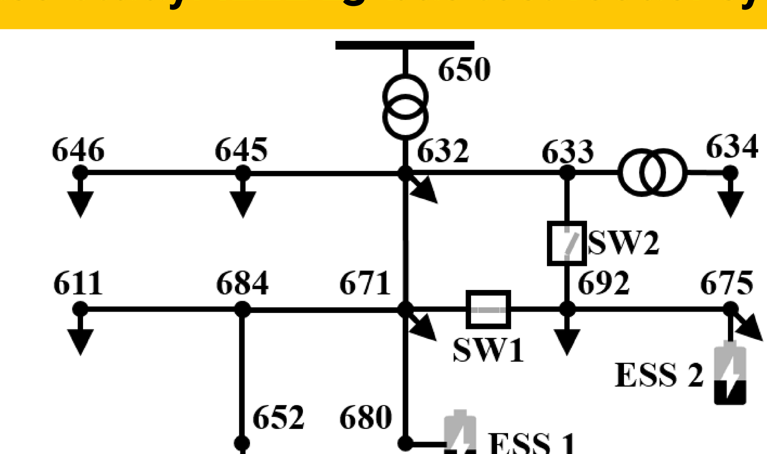

Figure 2. Modified test feeder with tow 3-phase 600 kW energy storage systems (ESSs) and switch SW2.

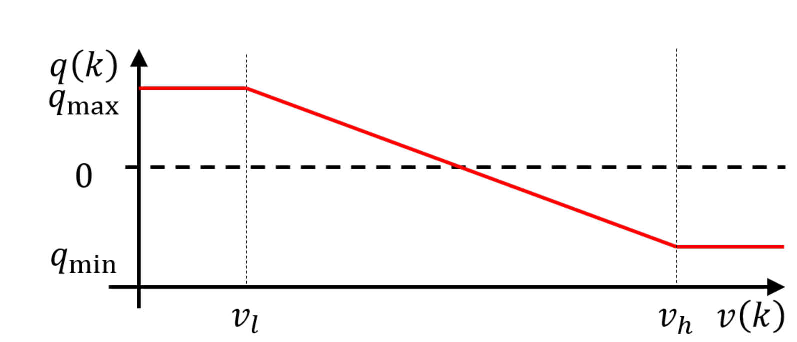

Figure 3. Voltage-reactive power control settings chosen within DER A range of standard IEEE 1547-2018.

Simulation in OpenDSS with time step of 1 s. SW1 opens and SW2 closes at t = 12h.

Instability: Oscillation with droop-controller

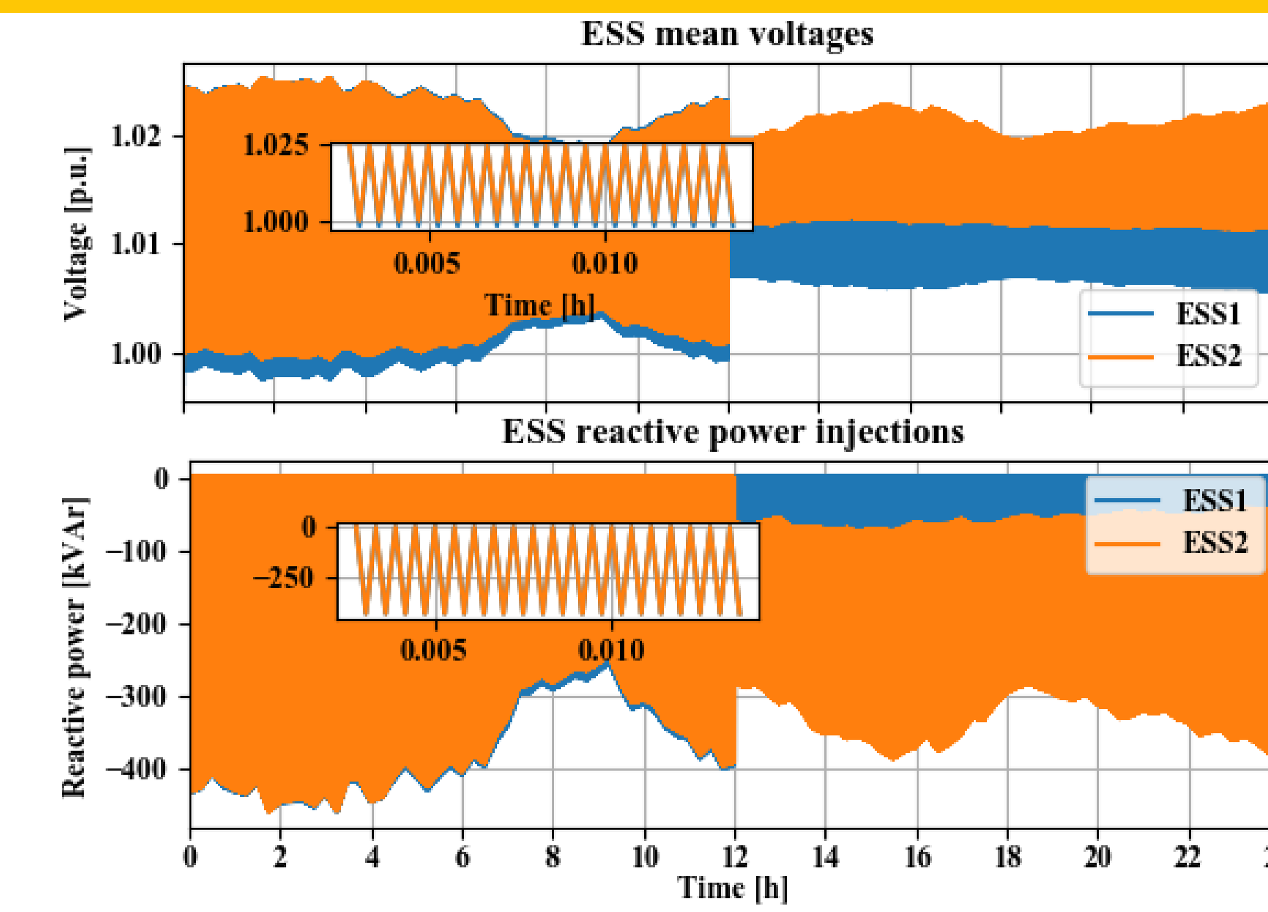

Figure 4. Similarly to [3], the volt/VAr droop controllers saturate at every time step causing oscillations.

Results of Proposed Controller

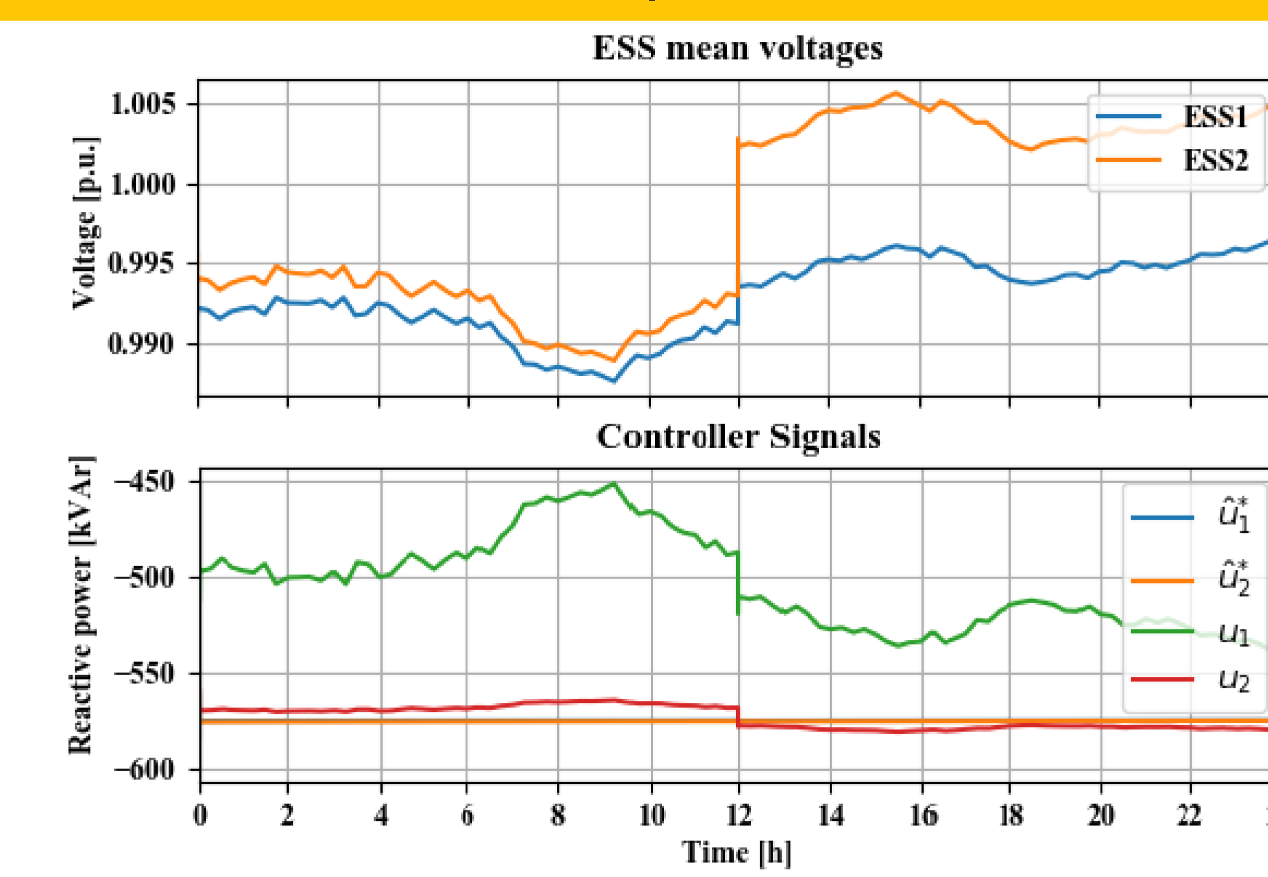

Figure 5. ESS voltages and power injections for $\bar{K} = diag\{10000, 1000\}$, $a_i = 0.1$, $\omega_i = \pi/2$, $\gamma_i = 0.027$, $h = 0.99$.

Conclusions and Future work

1. Unlike traditional droop-based approaches, the proposed dissipativity-based adaptive controller was able to provide voltage support without introducing oscillations into the distribution system when control gains were set according to the developed criterion. The effectiveness of the controller is currently limited by the saturation of the actuators, which could only be improved with higher ESS power capacity.
2. Future work includes extending this approach to distributed control of DERs. Identifying adversarial agents and how to mitigate their effect.

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