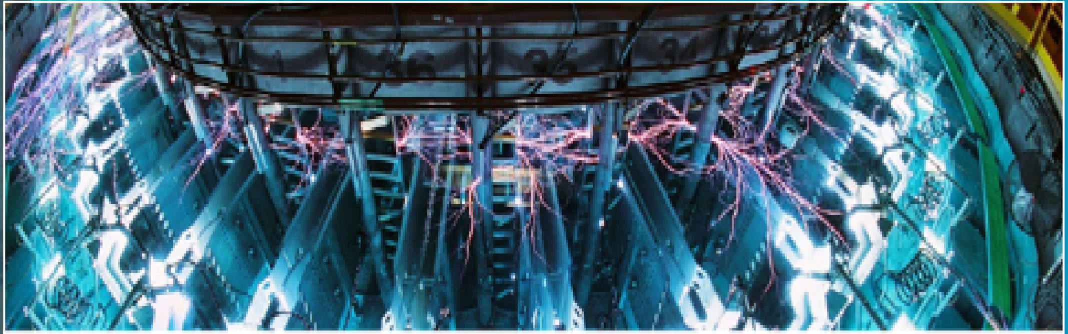
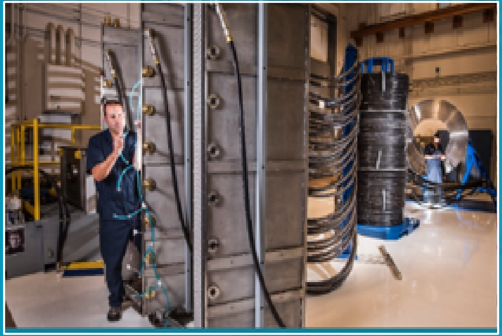
SAND2022-4567C

Assured Survivability & Agility with Pulsed Power (ASAP) Mission Campaign





Sandia National Laboratories

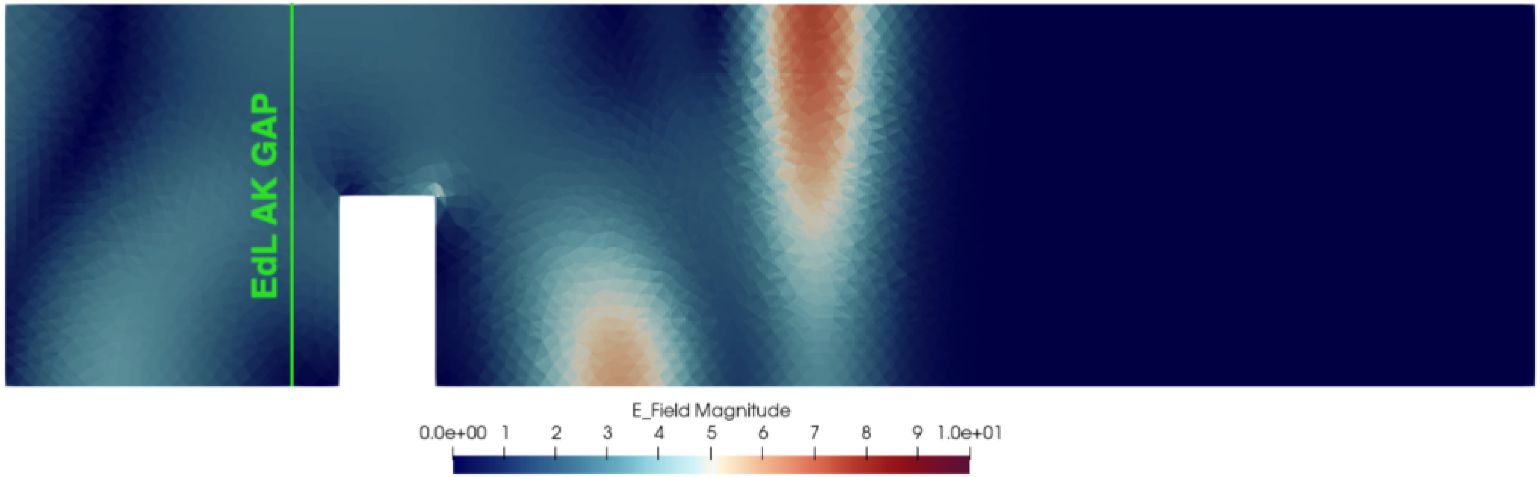
Coupling Powerflow to Targets Simulations

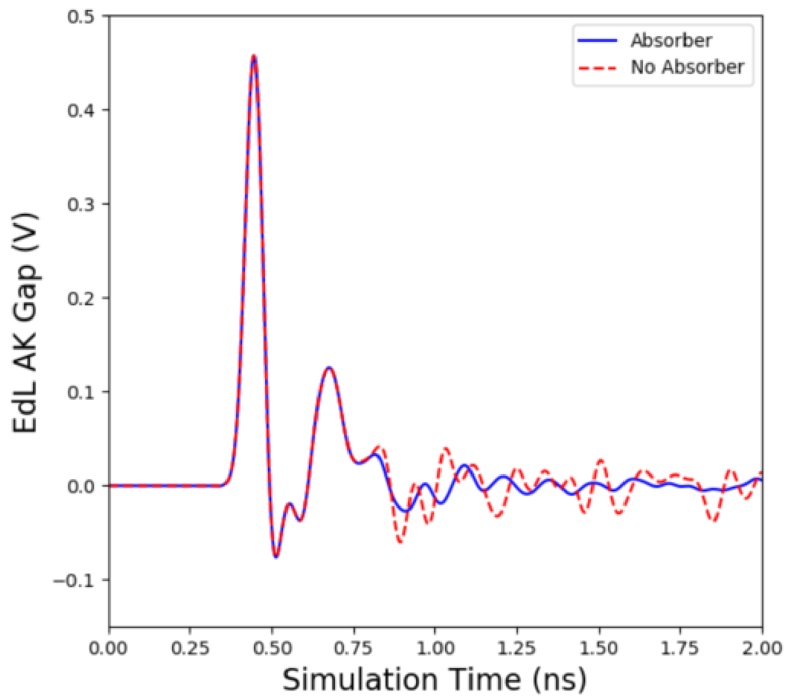
Plasma Theory and Simulation, 1351
Duncan McGregor, David Sirajuddin, Edward Phillips

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

EMPIRE EM-TL Coupling

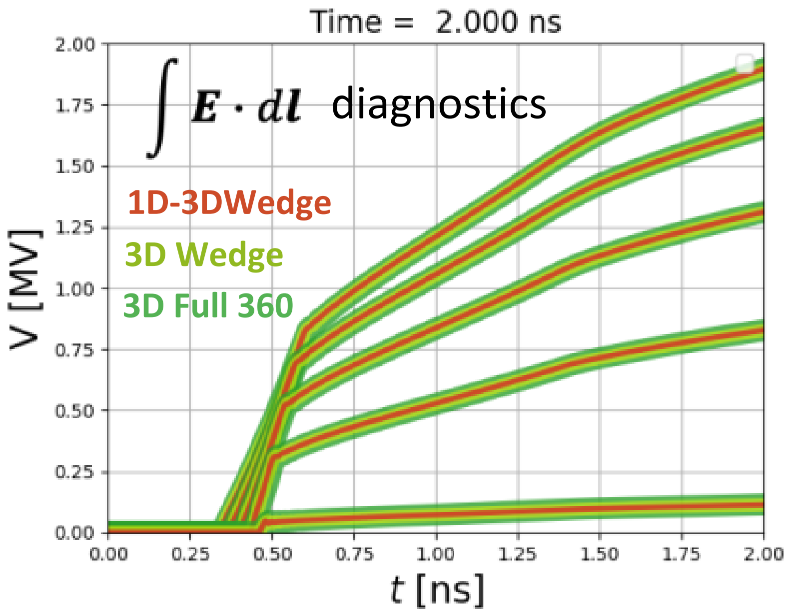
- **TEM Mode** assumption in TL domain
- **Variational** – couple through surface integrals – this allows it to apply to arbitrary meshes
- **Implicit coupling** – introduces no new stability constraints – we developed an efficient linear solver to handle implicit solves
- **Self-consistent** – coupling is based directly on the assumptions used to derive the TL model– we enforce continuity of voltage and current at the interface in the sense of a projection

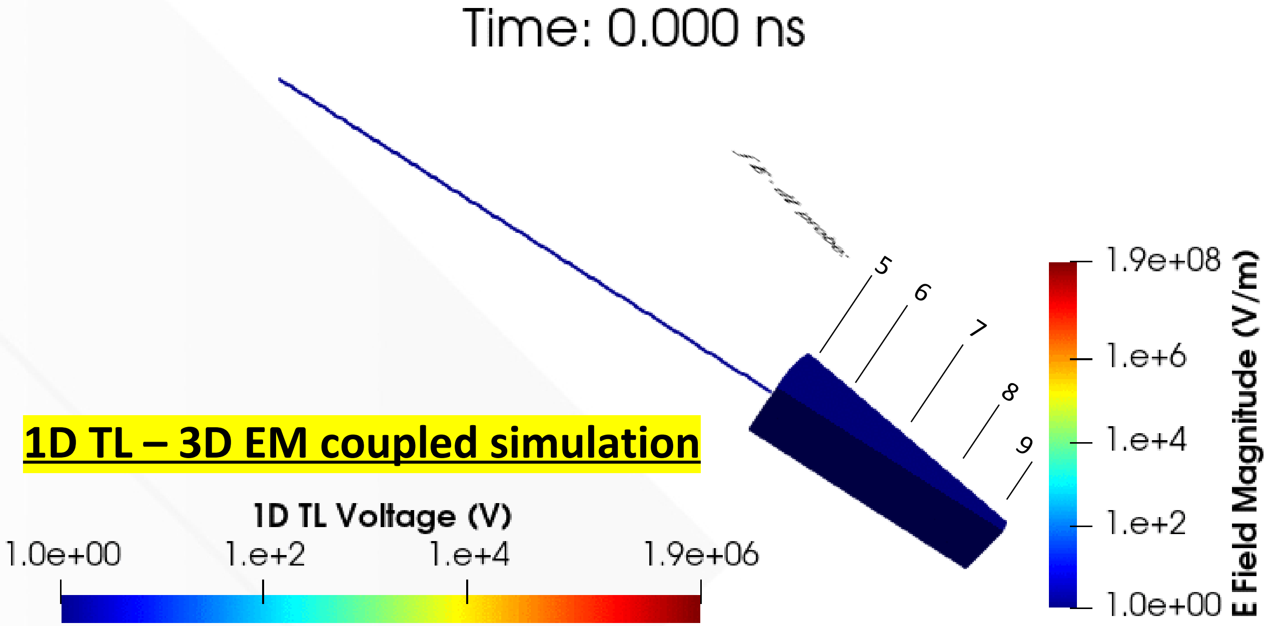




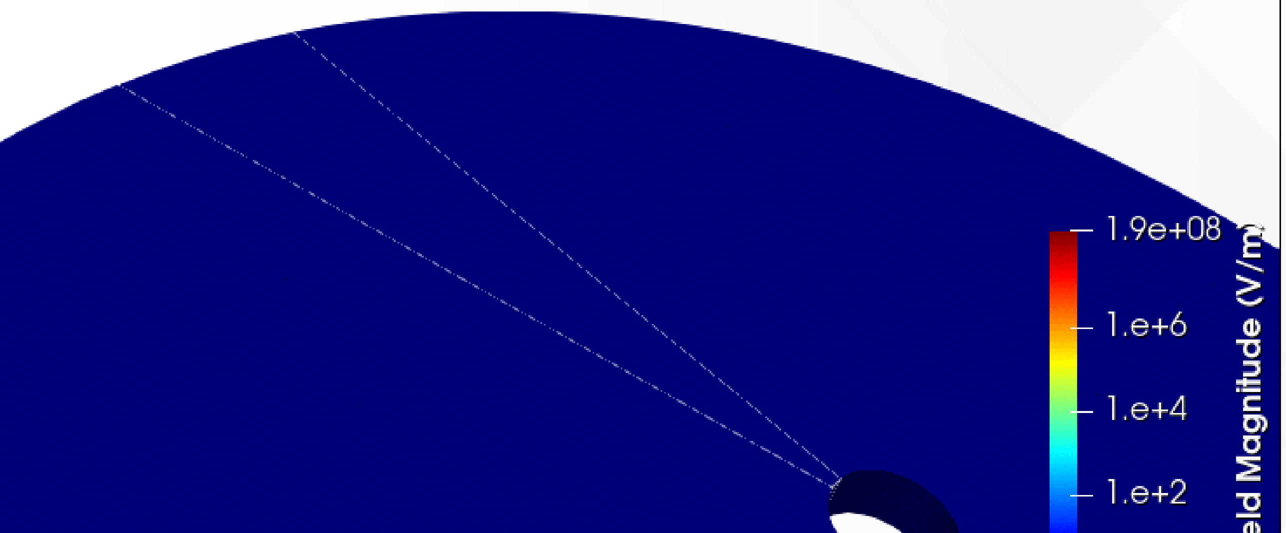
- We enforce voltage continuity via a constraint and a **Lagrange multiplier**
- Allows us to apply an additional boundary condition at the EM/TL interface
- We apply an absorbing BC at the interface that absorbs non-TEM modes
- **Reduces unphysical ringing** due to reflection of non-TEM modes

EMPIRE’s EM-TL Coupling: demonstration

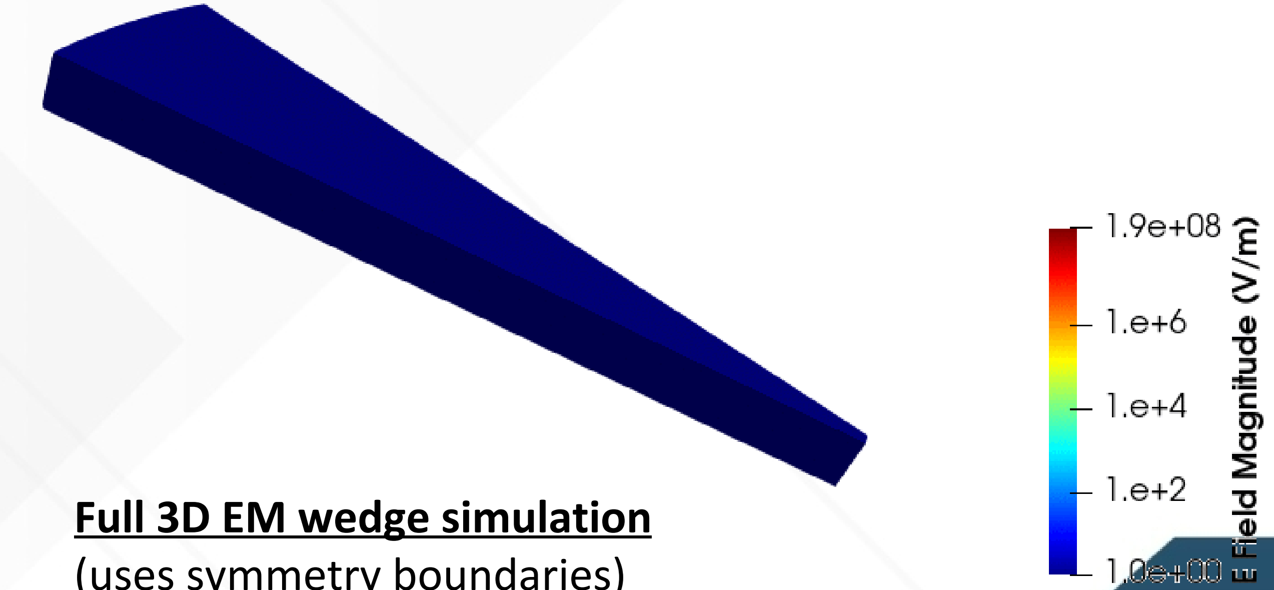




1D TL – 3D EM coupled simulation



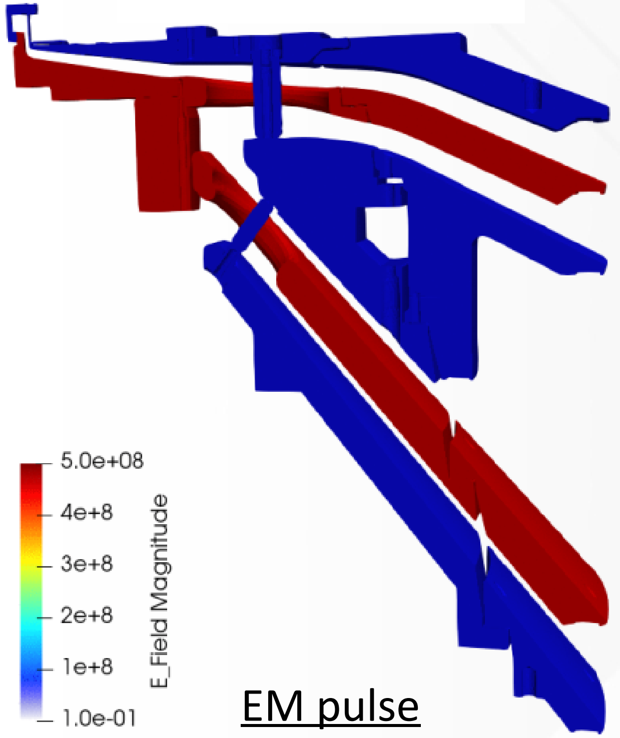
Full 3D EM simulation (closeup)



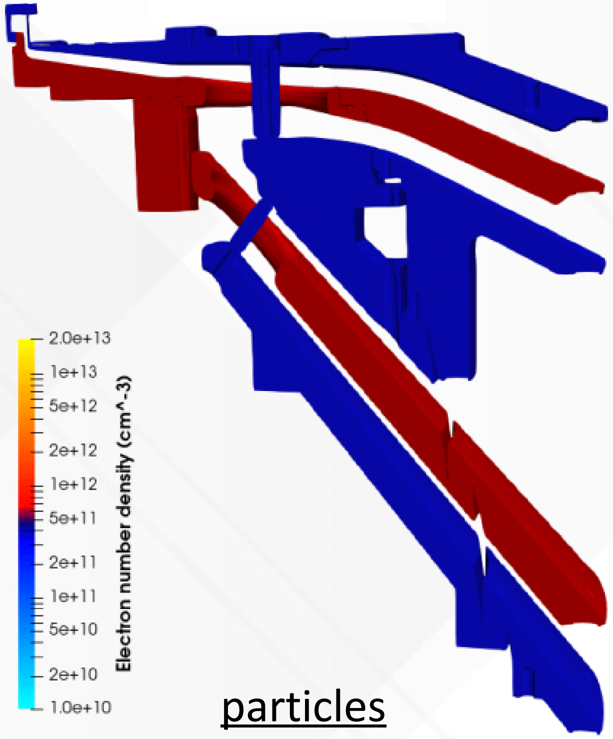
Full 3D EM wedge simulation
(uses symmetry boundaries)

TM wave contributions develop in high power accelerators

- Deviations from ideal (TEM) propagation arise from:
  - Changes along the EM transmission line geometry:
    - Small asymmetries: speed bumps, corners, holes, mating contacts between electrode plates
    - Large asymmetries: convolute, rapidly changing curvature near the load
  - Nonlinear mechanisms (e.g. plasma, gap closure, electrode melt)
  - Pulse shaping and machine jitter



EM pulse

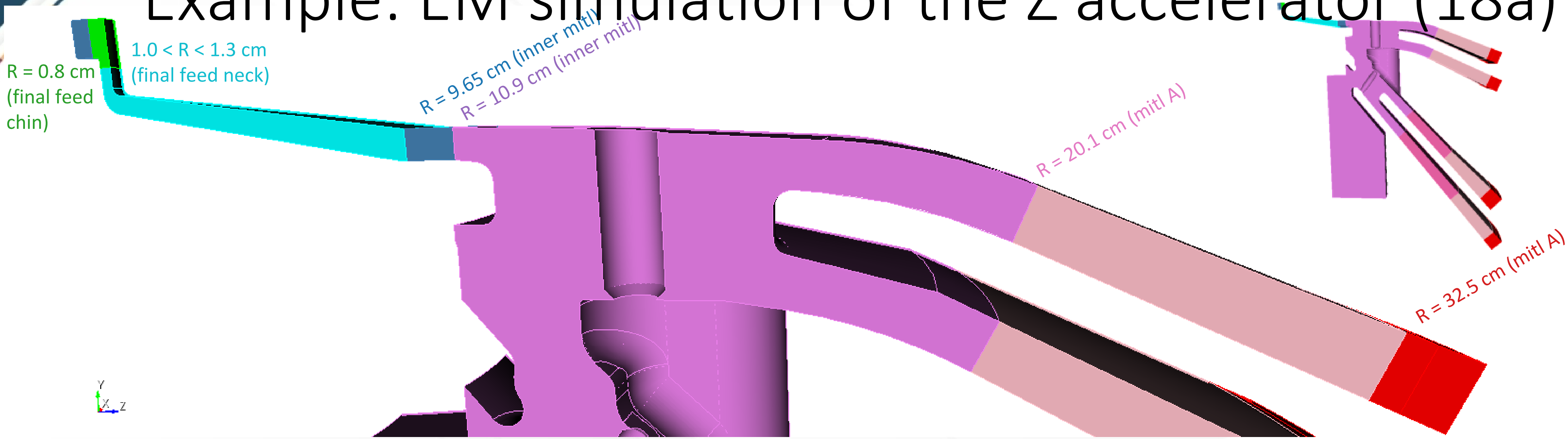


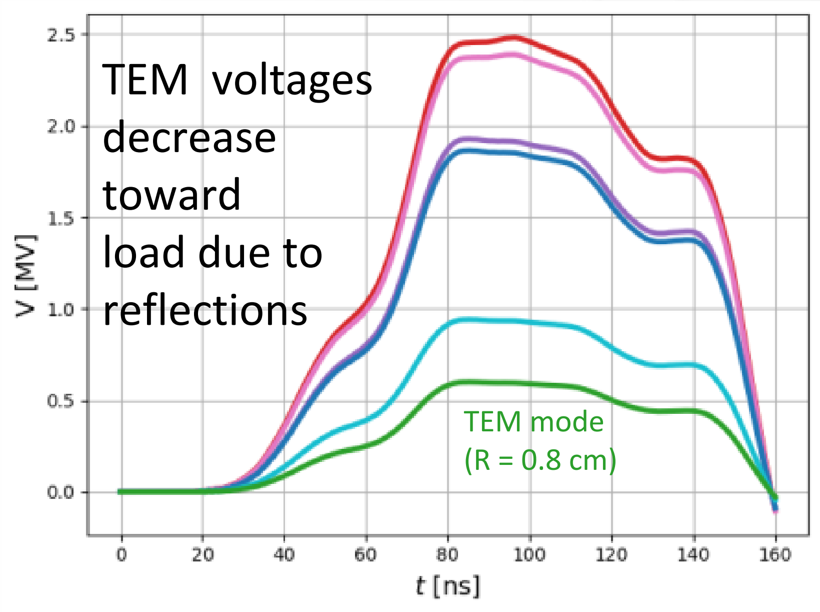
particles

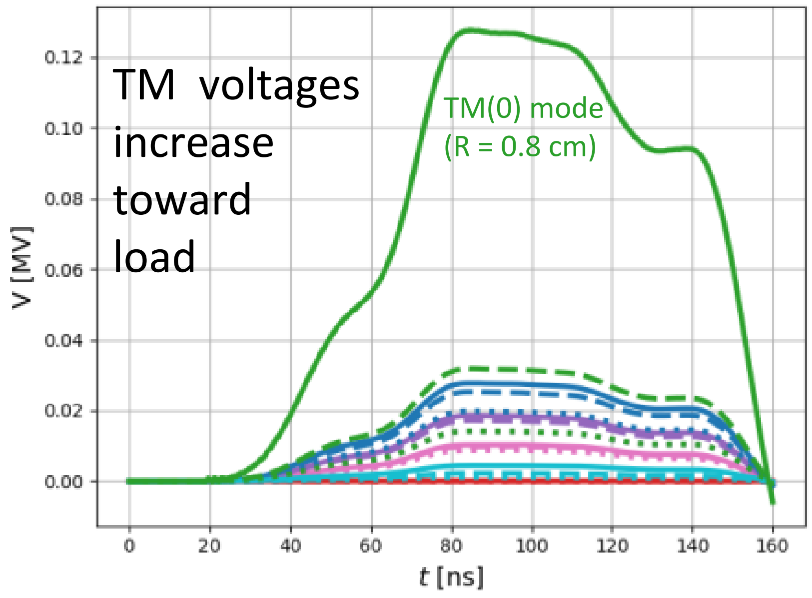
electrode heating

These effects alter the overall wave structure: TEM → TEM/TE/TM

Example: EM simulation of the Z accelerator (18a)



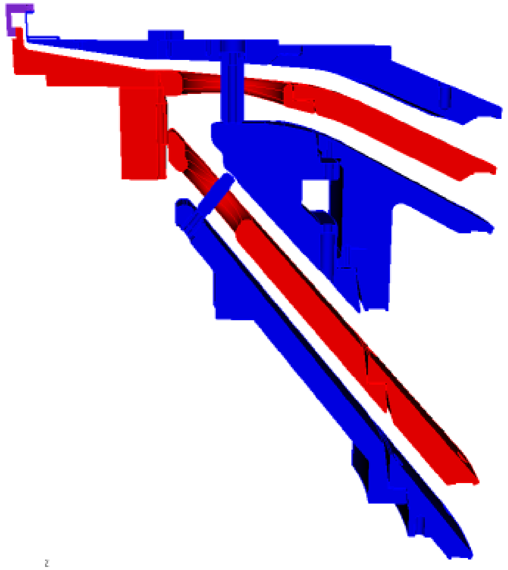


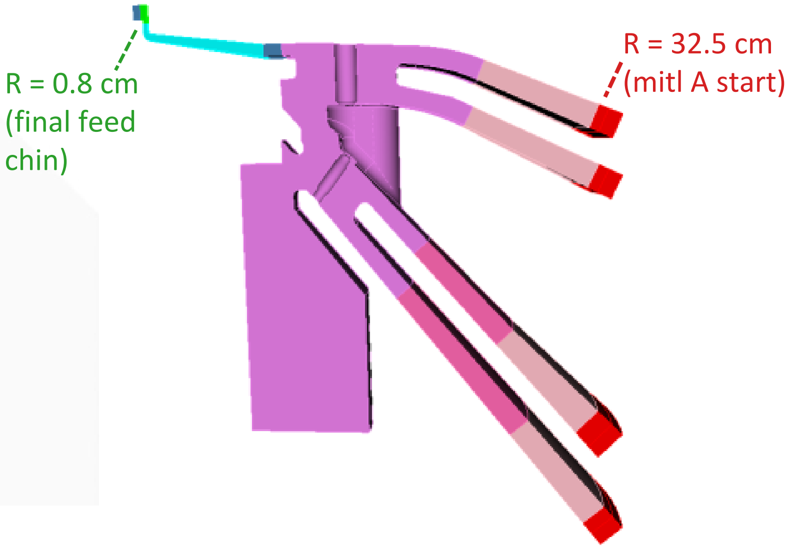


*curve colors match
With color-coded geometry above

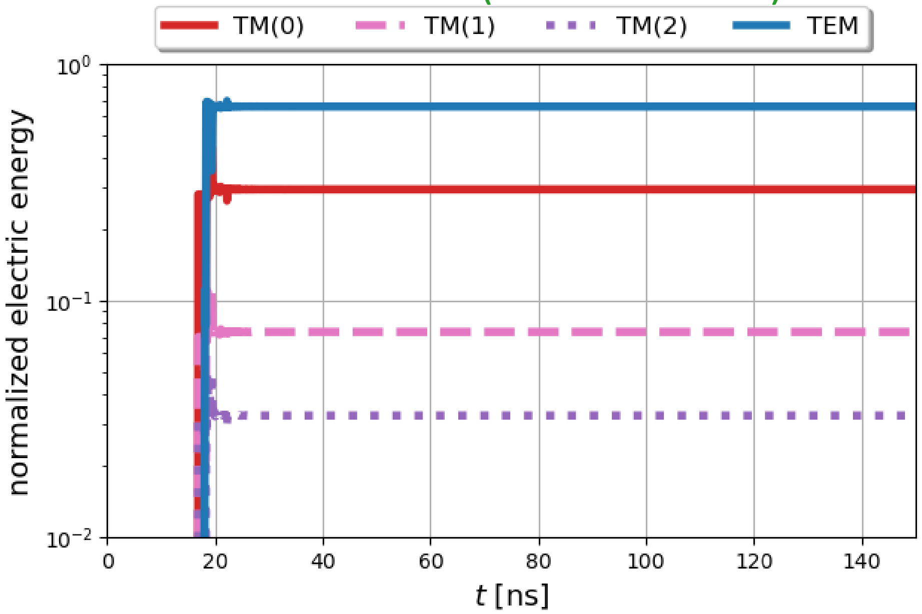
Solid – TM(0) mode
Dashed – TM(1) mode
Dotted – TM(2) mode

Example: EM simulation of the Z accelerator (18a)

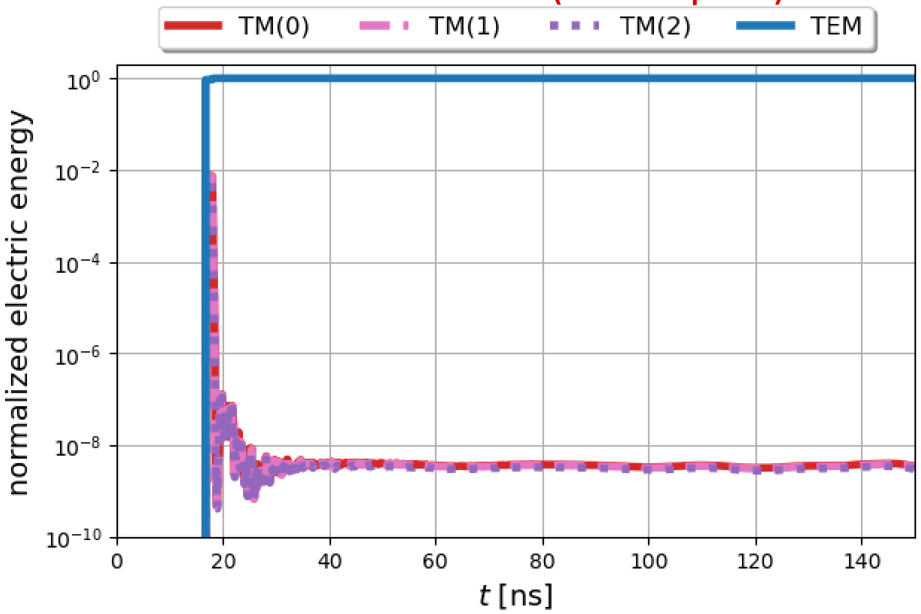




R = 0.8cm (final feed chin)

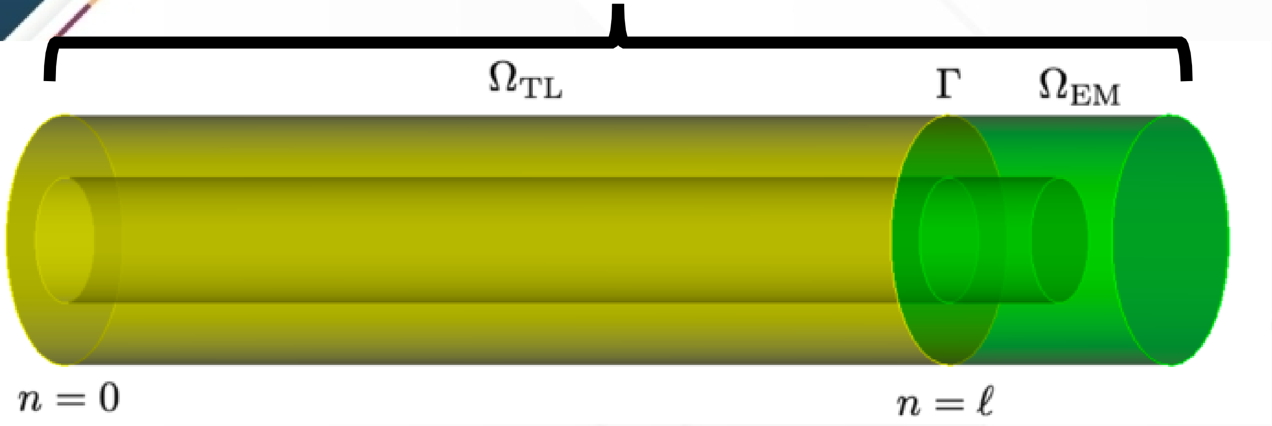


R = 32.5 cm (mitl A port)

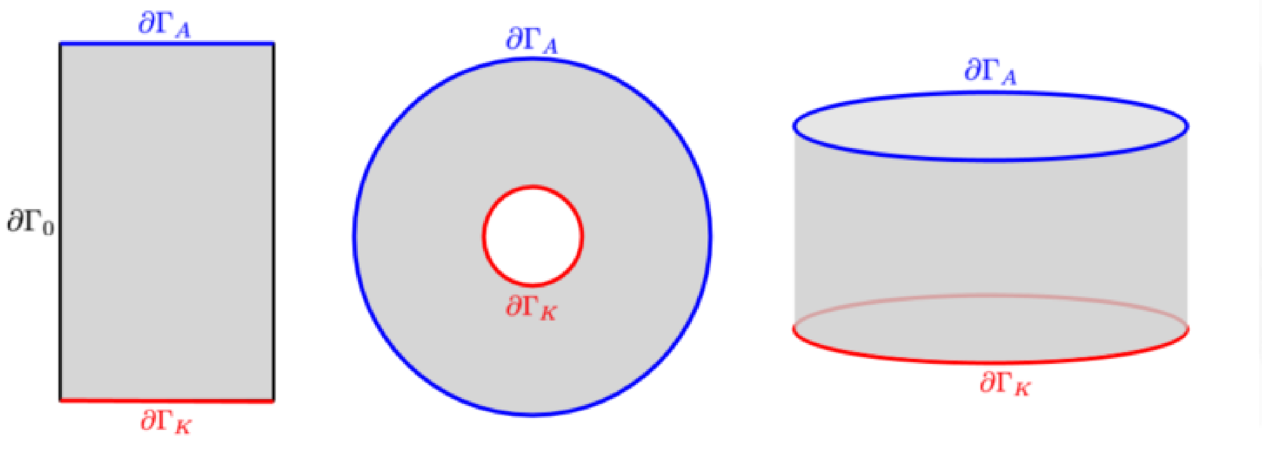


The energy content is nearly 100% TEM near the port (right plot) but develops a significant TM contribution near the load

Abstract Modeling Problem



Example EM-TL Coupling Interfaces



Maxwell's Equations

$$\begin{cases} \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} - \mathbf{curl}\ \mathbf{H} = \mathbf{0} \\ \frac{\partial \mathbf{B}}{\partial t} + \mathbf{curl}\ \mathbf{E} = \mathbf{0} \\ \text{div}\ \mathbf{D} = \rho \\ \text{div}\ \mathbf{B} = 0 \end{cases}$$

Simple Dielectric

$$\begin{cases} \mathbf{D} = \epsilon \mathbf{E} \\ \mathbf{B} = \mu \mathbf{H} \end{cases}$$

Homogeneous BCs

$$\begin{cases} \mathbf{E} \times \mathbf{n} = \mathbf{0} & \text{on conductors} \\ \mathbf{H} \times \mathbf{n} = \mathbf{0} & \text{on symmetry} \end{cases}$$

$\mathbf{J}$ is data, we'll assume its zero in TL domain

Transverse Magnetic Mode Definition

IF ϵ, μ constant $\mathbf{J} \equiv \mathbf{0}$

Eigenvalue problem

$$\begin{cases} -\Delta_\tau \varphi_{\mathrm{TM},j} = k_j^2 \varphi_{\mathrm{TM},j} \\ \left(\frac{1}{|\Gamma|} \int_\Gamma |\varphi_{\mathrm{TM},j}|^2 \, dA \right)^{1/2} = 1 \\ \varphi_{\mathrm{TM},j}|_{\Gamma_A} = 0 \\ \varphi_{\mathrm{TM},j}|_{\Gamma_K} = 0 \\ \nabla_\tau \varphi_{\mathrm{TM},j} \cdot \mathbf{m}|_{\Gamma_S} = 0 \end{cases}$$

Field Profiles

$$\mathbf{E}_{\mathrm{TM},\tau,j} = -\nabla_\tau \varphi_{\mathrm{TM},j}$$
$$E_{\mathrm{TM},n,j} = k_j \varphi_{\mathrm{TM},j}$$
$$\mathbf{H}_{\mathrm{TM},\tau,j} = \frac{\mathbf{curl}_\tau \varphi_{\mathrm{TM},j}}{k_j^2 |\Gamma|}$$

TM Telegrapher's Equations

$$\begin{cases} C_{\mathrm{TM},j} \frac{\partial V_{\mathrm{TM},\tau,j}}{\partial t} + \frac{\partial I_{\mathrm{TM},\tau,j}}{\partial n} = 0 \\ C_{\mathrm{TM},j} \frac{\partial V_{\mathrm{TM},n,j}}{\partial t} - k_j I_{\mathrm{TM},\tau,j} = 0 \\ L_{\mathrm{TM},j} \frac{\partial I_{\mathrm{TM},n,j}}{\partial t} + k_j V_{\mathrm{TM},n,j} + \frac{\partial V_{\mathrm{TM},\tau,j}}{\partial n} = 0 \end{cases}$$

TM Capacitance and Inductance per Length

$$C_{\mathrm{TM},j} = \int_\Gamma \epsilon |\mathbf{E}_{\mathrm{TM},\tau,j}|^2 \, dA = \epsilon k_j^2 |\Gamma|$$

$$L_{\mathrm{TM},j} = \int_\Gamma \mu |\mathbf{H}_{\mathrm{TM},\tau,j}|^2 \, dA = \mu (k_j^2 |\Gamma|)^{-1}$$

THEN

$$\mathbf{E} = (V_{\mathrm{TM},\tau,j} \mathbf{E}_{\mathrm{TM},\tau,j}, V_{\mathrm{TM},n,j} E_{\mathrm{TM},n,j})$$
$$\mathbf{H} = (I_{\mathrm{TM},\tau,j} \mathbf{H}_{\mathrm{TM},\tau,j}, 0)$$

ARE A SOLUTION TO MAXWELL'S EQUATIONS ON

$$\Omega_{\mathrm{TL}} = \Gamma \times [0, \ell]$$

WITH DISPERSION RELATIONSHIP

$$\epsilon \mu \omega^2 = k^2 + k_j^2$$

Modal Decomposition (finite)

An approximation of an arbitrary TM mode can be written as

$$\begin{cases} \mathbf{E}_\tau = V_{\mathrm{TEM},\tau}\mathbf{E}_{\mathrm{TEM},\tau} + \sum_{j=1}^{M} V_{\mathrm{TM},\tau,j}\mathbf{E}_{\mathrm{TM},\tau,j} \\ E_n = \sum_{j=1}^{M} V_{\mathrm{TM},n,j}\mathbf{E}_{\mathrm{TM},n,j} \\ \mathbf{H}_\tau = I_{\mathrm{TEM},\tau}\mathbf{H}_{\mathrm{TEM},\tau} + \sum_{j=1}^{M} I_{\mathrm{TM},\tau,j}\mathbf{H}_{\mathrm{TM},\tau,j} \\ H_n = 0 \end{cases}$$

These modes are orthogonal

We can evolve each of the modes independently from initial data using their respective telegrapher equations

Initial voltages and currents computed with projections onto the modes

$$V_{\mathrm{TEM},\tau}(n, t=0) = \frac{\int_\Gamma \mathbf{E}_0(\boldsymbol{\tau}, n), \mathbf{E}_{\mathrm{TEM},\tau}\ dA}{\|\mathbf{E}_{\mathrm{TEM},\tau}\|_\Gamma^2} \qquad I_{\mathrm{TEM},\tau}(n, t=0) = \frac{\int_\Gamma \mathbf{H}_0(\boldsymbol{\tau}, n) \cdot \mathbf{H}_{\mathrm{TEM},\tau}\ dA}{\|\mathbf{H}_{\mathrm{TEM},\tau}\|_\Gamma^2}$$

$$V_{\mathrm{TM},\tau,j}(n, t=0) = \frac{\int_\Gamma \mathbf{E}_0(\boldsymbol{\tau}, n) \cdot \mathbf{E}_{\mathrm{TM},\tau,j}\ dA}{\|\mathbf{E}_{\mathrm{TM},\tau,j}\|_\Gamma^2} \qquad I_{\mathrm{TM},\tau,j}(n, t=0) = \frac{\int_\Gamma \mathbf{H}_0(\boldsymbol{\tau}, n) \cdot \mathbf{H}_{\mathrm{TM},\tau,j}\ dA}{\|\mathbf{H}_{\mathrm{TM},\tau,j}\|_\Gamma^2}$$

$$V_{\mathrm{TM},n,j}(n, t=0) = \frac{\int_\Gamma \mathbf{E}_0(\boldsymbol{\tau}, n) \cdot \mathbf{n}E_{\mathrm{TM},n,j}\ dA}{\|E_{\mathrm{TM},n,j}\|_\Gamma^2}$$

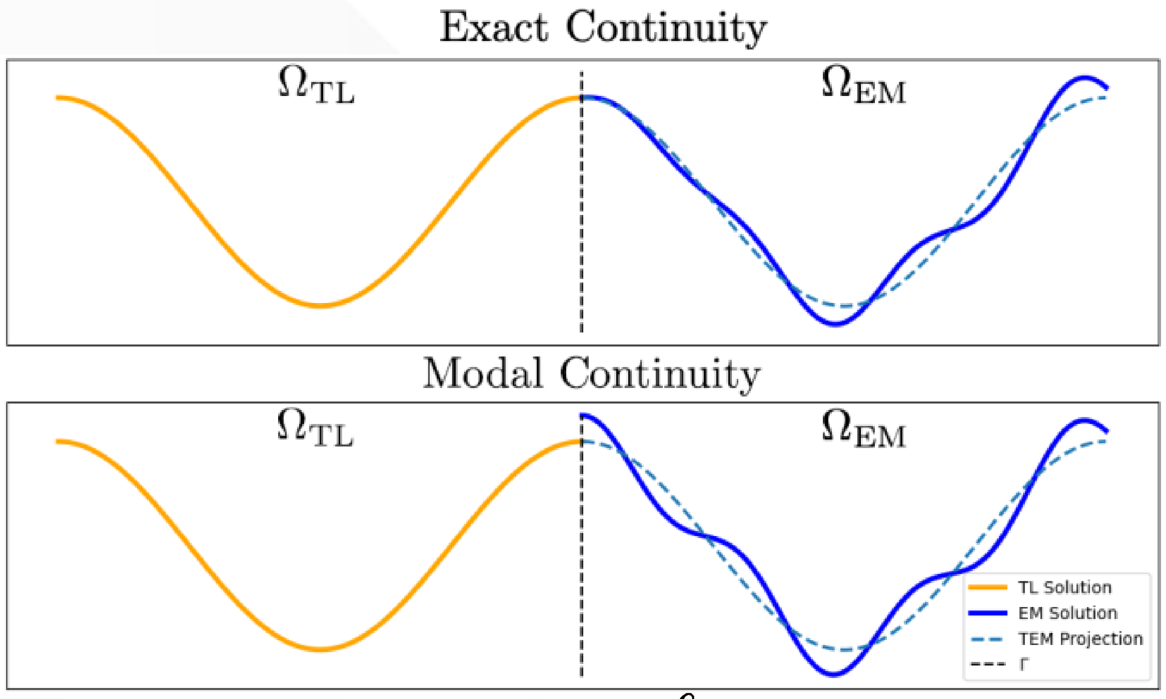
Coupling Strategy

For the coupling between EM and TL domains to be consistent we need tangent fields to "somehow agree" at the coupling interface.

The notion we impose is weaker than exact continuity and we call it "Modal Continuity"

$$\int_\Gamma \mathbf{E}(t) \cdot \mathbf{E}_{\mathrm{TM},\tau,j} \; dA = \|\mathbf{E}_{\mathrm{TM},\tau,j}\|_\Gamma^2 V_{\mathrm{TM},\tau,j}(\ell, t)$$

$$\int_\Gamma \mathbf{H}(t) \cdot \mathbf{H}_{\mathrm{TM},\tau,j} \; dA = \|\mathbf{H}_{\mathrm{TM},\tau,j}\|_\Gamma^2 I_{\mathrm{TM},\tau,j}(\ell, t)$$



$$n = \ell$$

The L2 projection of the EM fields onto the TM space at the coupling interface recovers the TL solution

Philosophical difference: The solution is TM in the TL domain vs. we don't track non-TM parts in the TL domain

Coupled Problem

$$(\mathbf{E},\mathbf{B},\boldsymbol{\lambda},\mathbf{V}_\tau,\mathbf{V}_n,\mathbf{I}_\tau)\in\mathbf{H}(\mathbf{curl},\Omega_{\mathrm{EM}})\times\mathbf{H}(\mathrm{div},\Omega_{\mathrm{EM}})\times\mathbb{R}^{M+1}\times[H^1(0,\ell)]^M\times[L^2(0,\ell)]^M\times[L^2(0,\ell)]^M:$$

$$\left\{\begin{array}{l}
\left\{\begin{array}{ll}
\displaystyle\int_{\Omega_{\mathrm{EM}}}\epsilon\frac{\partial}{\partial t}\mathbf{E}\cdot\boldsymbol{\Psi}+\mathbf{J}\cdot\boldsymbol{\Psi}-\mu^{-1}\mathbf{B}\cdot\mathbf{curl}\ \boldsymbol{\Psi}\ dV & \\
\displaystyle+\int_\Gamma\left(Z^{-1}\left(\mathrm{I}-\textstyle\sum_{j=1}^M\Pi_{\mathrm{TM},j}\right)(\mathbf{E})+\sum_{j=1}^M\lambda_j\mathbf{E}_{\mathrm{TM},\tau,j}\right)\times\mathbf{n}\cdot\boldsymbol{\Psi}\times\mathbf{n}dA=0 & \forall\boldsymbol{\Psi}\in\mathbf{H}(\mathbf{curl},\Omega_{\mathrm{EM}})\\
\displaystyle\int_{\Omega_{\mathrm{EM}}}\frac{\partial}{\partial t}\mathbf{B}\cdot\boldsymbol{\Phi}+\mathbf{curl}\ \mathbf{E}\cdot\boldsymbol{\Phi}\ dV=0 & \forall\boldsymbol{\Phi}\in\mathbf{H}(\mathrm{div},\Omega_{\mathrm{EM}})
\end{array}\right.\\
\left\{\theta\displaystyle\int_\Gamma\mathbf{E}_{\mathrm{TM},\tau,j}\times\mathbf{n}\cdot\mathbf{E}\times\mathbf{n}\ dA=\theta\|\mathbf{E}_{\mathrm{TM},\tau,j}\|_\Gamma^2V_{\mathrm{TM},\tau,j}(\ell)\quad\forall\theta\in\mathbb{R}\right.\\
\left\{\begin{array}{ll}
\displaystyle\int_0^\ell C_{\mathrm{TM},j}\frac{\partial}{\partial t}V_{\mathrm{TM},\tau,j}\varphi-I_{\mathrm{TM},\tau,j}\frac{\partial}{\partial n}\varphi\ dS+I_{\mathrm{TM},j}\varphi(\ell)=0 & \forall\varphi\in H^1(0,\ell)\\
\displaystyle\int_0^\ell C_{\mathrm{TM},j}\frac{\partial}{\partial t}V_{\mathrm{TM},n,j}\psi-k_jI_{\mathrm{TM},\tau,j}\psi\ dS=0 & \forall\psi\in L^2(0,\ell)\\
\displaystyle\int_0^\ell L_{\mathrm{TM},j}\frac{\partial}{\partial t}I_{\mathrm{TM},\tau,j}\psi+k_jV_{\mathrm{TM},n,j}\psi+\frac{\partial}{\partial n}V_{\mathrm{TM},\tau,j}\psi\ dS=0 & \forall\psi\in L^2(0,\ell)
\end{array}\right.\\
\left\{\begin{array}{l}
\displaystyle\Pi_{\mathrm{TM},j}(\mathbf{E})=\frac{\langle\mathbf{E}_{\mathrm{TM},\tau,j},\mathbf{E}\rangle_\Gamma}{\|E_{\mathrm{TM},\tau,j}\|_\Gamma^2}\mathbf{E}_{\mathrm{TM},\tau,j}\\
\displaystyle I_{\mathrm{TM},j}=\int_{\Omega_{\mathrm{EM}}}\epsilon\frac{\partial}{\partial t}\mathbf{E}\cdot\overline{\mathbf{E}}_{\mathrm{TM},\tau,j}+\mathbf{J}\cdot\overline{\mathbf{E}}_{\mathrm{TM},\tau,j}-\mu^{-1}\mathbf{B}\cdot\mathbf{curl}\ \overline{\mathbf{E}}_{\mathrm{TM},\tau,j}\ dV=\|\mathbf{H}_{\mathrm{TM},\tau,j}\|^{-2}\int_\Gamma\mathbf{H}_{\mathrm{TM},\tau,j}\cdot\mathbf{H}\ dA
\end{array}\right.
\end{array}\right.$$

Code Verification

3D TM Wave on a TL-EM domain

Theoretical Order of Accuracy (Simplex Mesh)

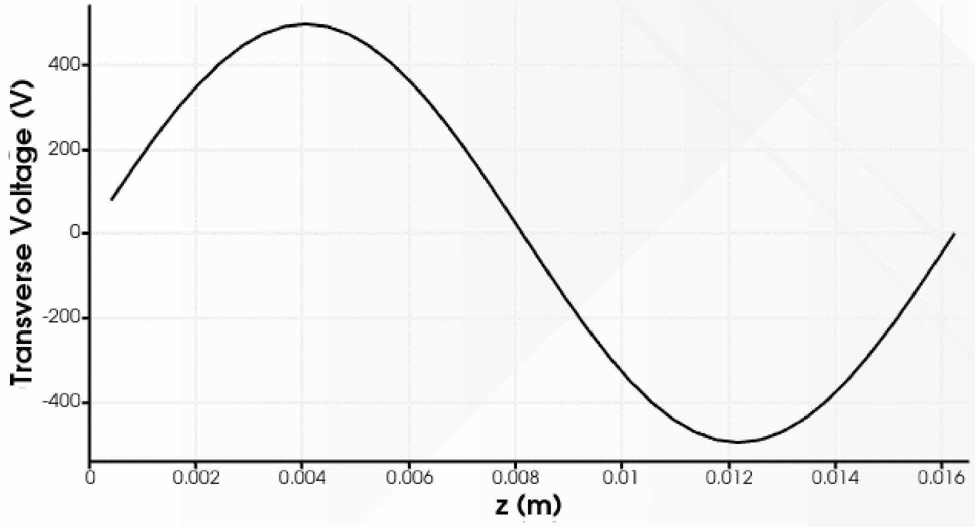
$$\mathcal{O}(\underbrace{\Delta n^2}_{\text{TL Space}} + \underbrace{h}_{\text{EM Space}} + \underbrace{k_c^2 h}_{\text{Boundary Profile}} + \underbrace{\Delta t^2}_{\text{Time}})$$

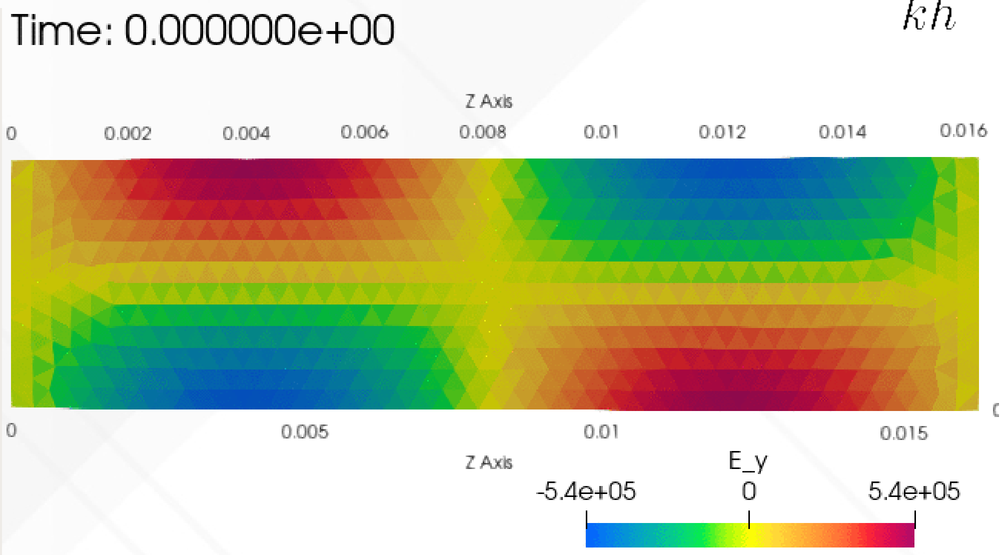
Error denotes a relative L^2 error for functions.

cell size [m]	PPP z	PPP c	E_z error	E_z rate	E_y error	E_y rate	B_x error	B_x rate
1.00E-03	16.2	8.5	3.15E-02	–	5.96E-02	–	4.73E-02	–
6.67E-04	24.3	12.7	1.41E-02	1.98	2.88E-02	1.79	2.58E-02	1.5
5.00E-04	32.5	16.9	9.86E-03	1.24	1.93E-02	1.4	1.85E-02	1.16
4.00E-04	40.6	21.1	7.50E-03	1.23	1.46E-02	1.24	1.43E-02	1.14

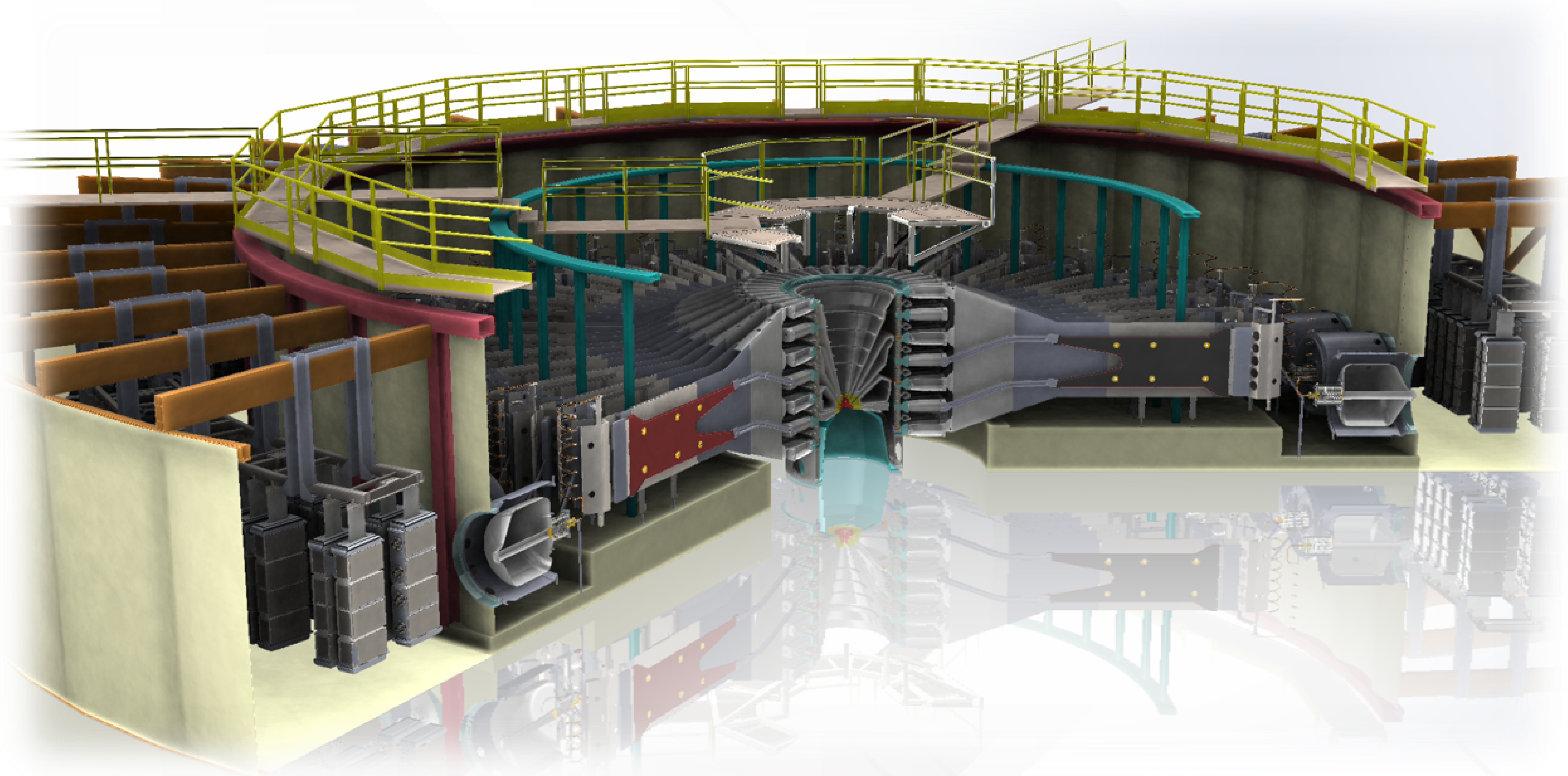
Rates are computed using a pairwise fit.

$$\text{PPP} = \frac{2\pi}{kh}$$





Saturn Exemplar



Saturn Exemplar

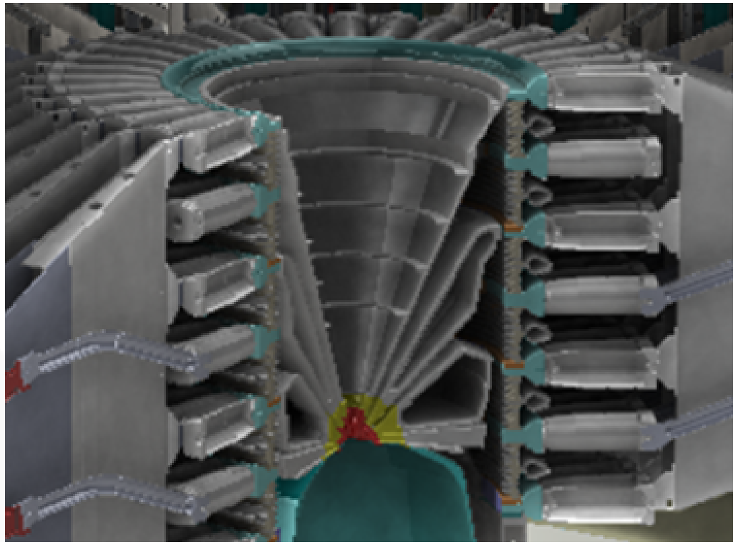
- Saturn's water convolute may introduce jitter to the EM Drive

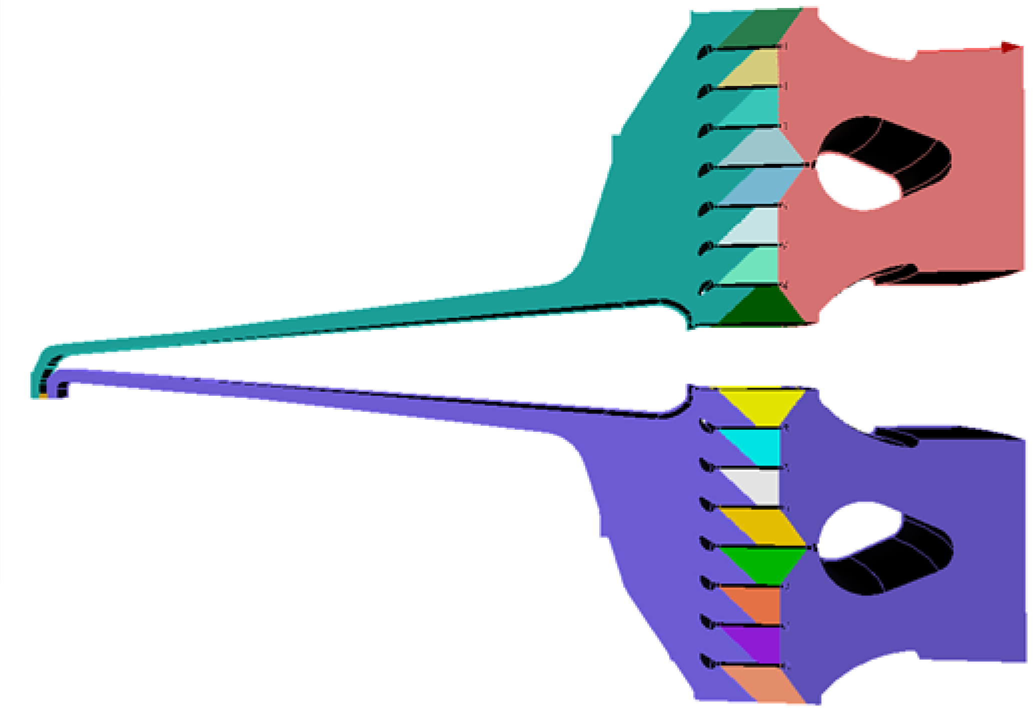
Assuming a cylindrical cross-section

$$\omega^2 - \omega_{\mathrm{TM},n,m}^2 = c^2 k^2$$

$$\omega_{\mathrm{TM},n,m} = c\sqrt{\left(\frac{2\pi n}{|AK|}\right)^2 + \left(\frac{m}{r}\right)^2}$$

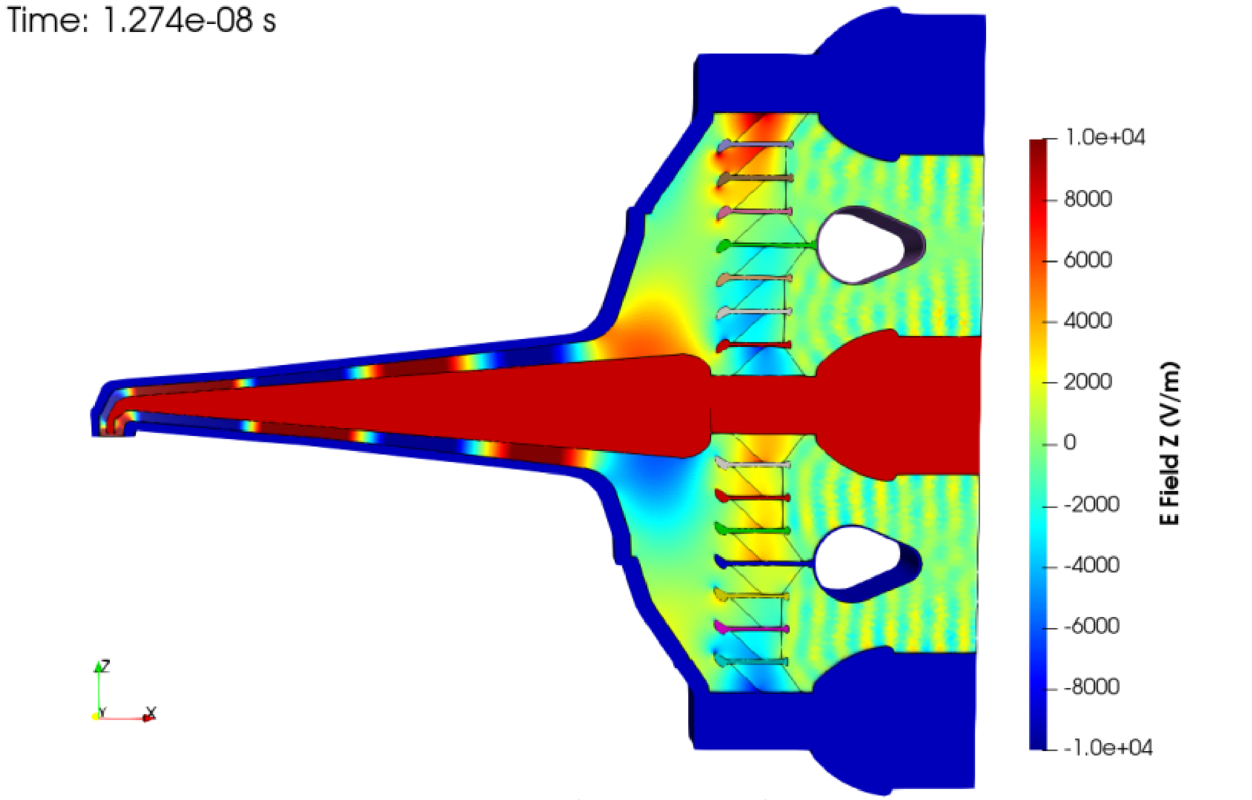
- Decrease in AK gap should regularize z directed jitter
- Radial convergence should regularize θ directed jitter
- Hypothesis: energy in TM modes should decrease as r->0



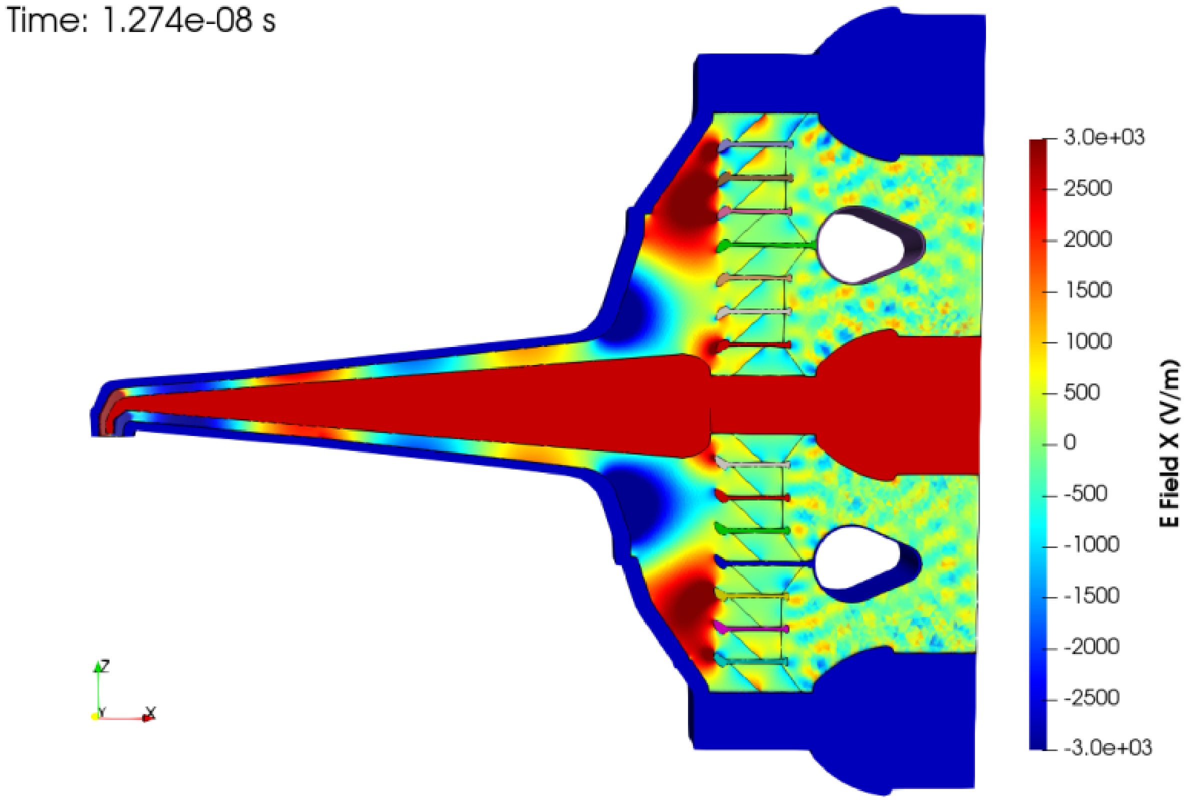


Saturn Exemplar

Reduced domain size, 5° wedge.
Drive water section with $\mathrm{TM}_{1,0}$ mode.
Drive frequency between $\omega_{\mathrm{TM},1,0}$ for the water section and vacuum section.



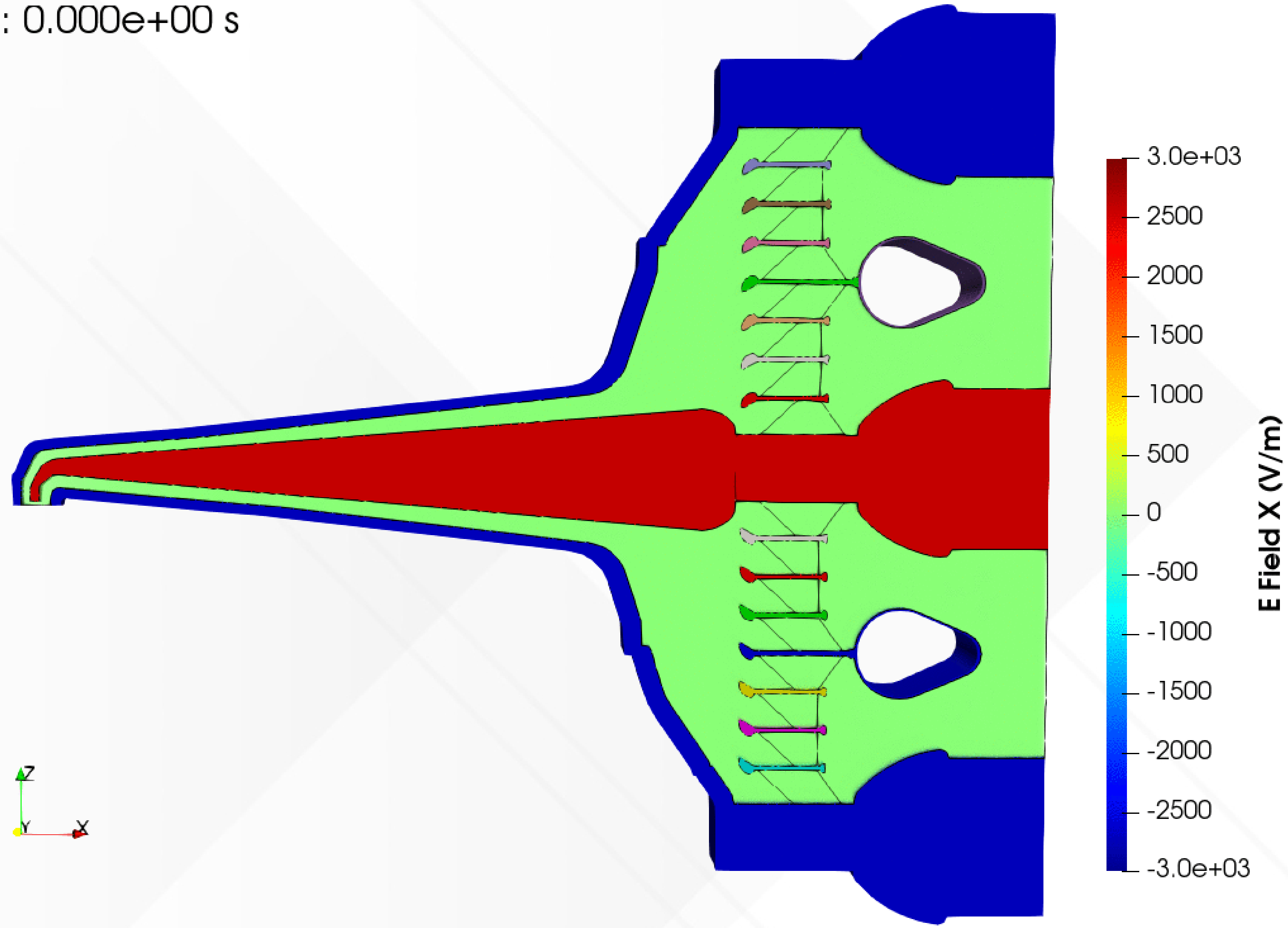
Transverse Electric Field



Normal Electric Field

Saturn Exemplar

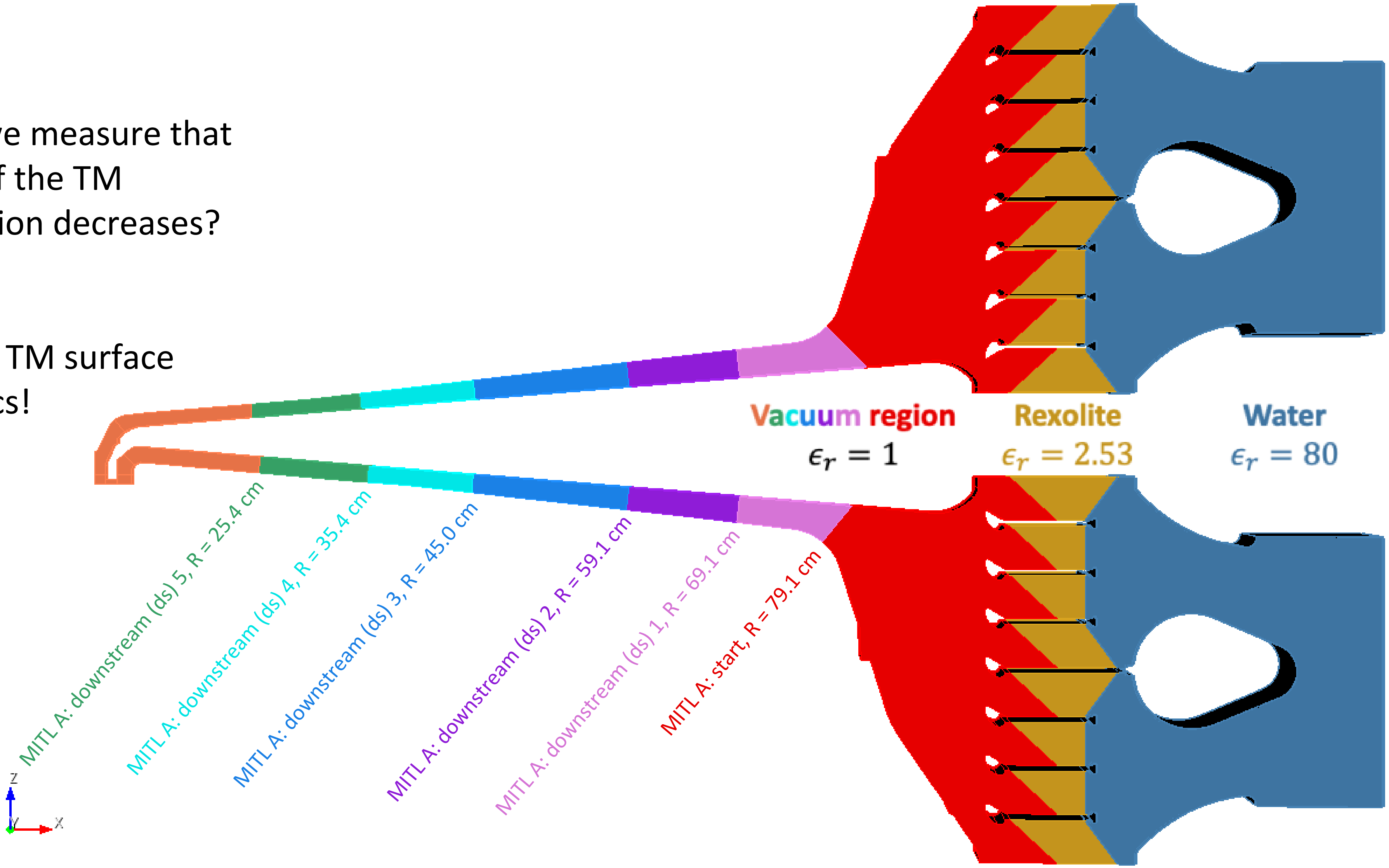
Time: 0.000e+00 s



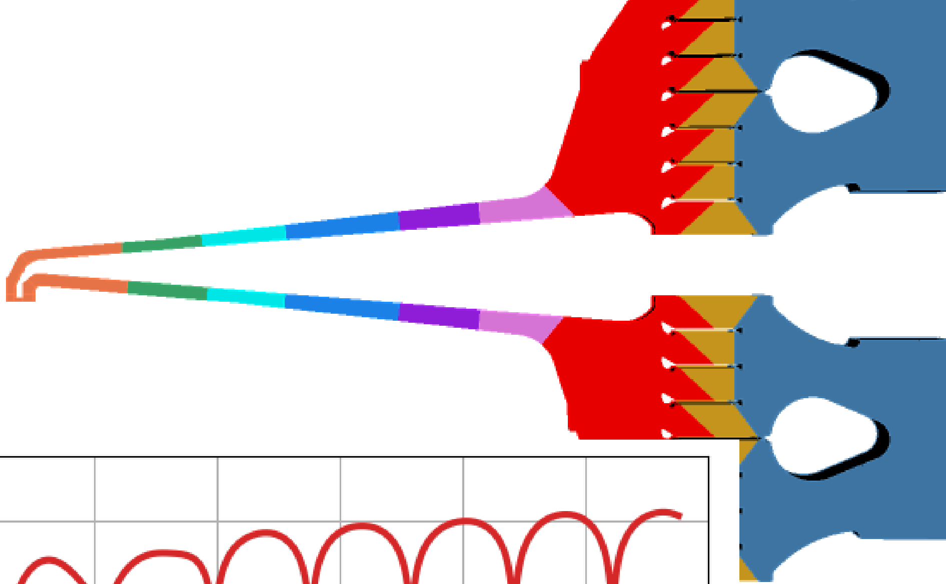
Saturn Exemplar

How do we measure that the size of the TM perturbation decreases?

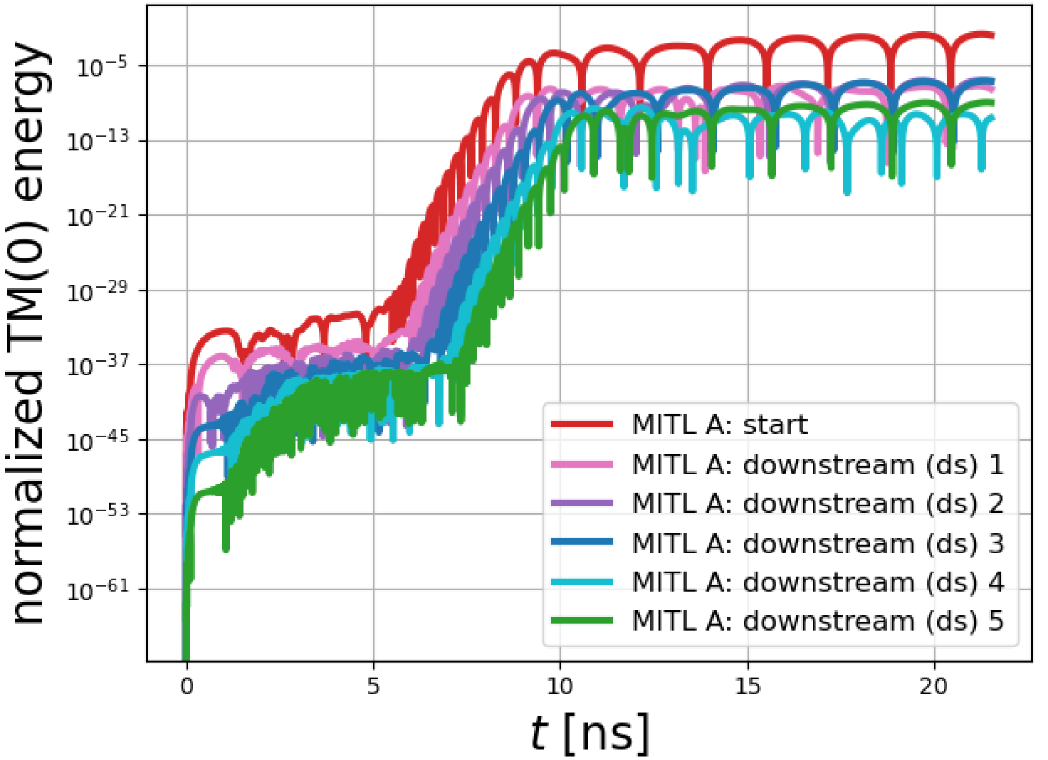
Approach TM surface diagnostics!

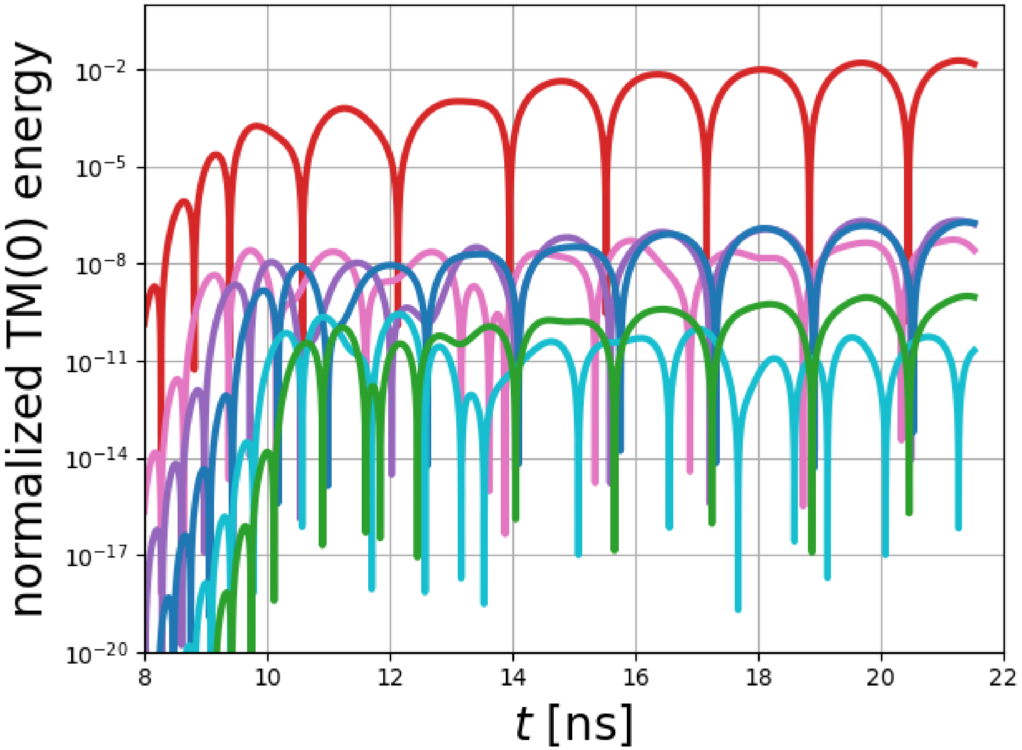


Saturn Exemplar



Electric Energy per unit length normalized by drive energy for the mode





We observe energy content of the TM wave perturbation decreases along the length of the vacuum mitls

Life after LDRD

Productionization in EMPIRE
- Harden capability as its used
- Implement TE modes
- Refactor EMTL Coupling
- Implement EM-TE coupling

EMPIRE-Cable
- EMPIRE's TL simulation capability has been spun off into a standalone capability (BERTHA-like)
- Sceptre Coupling
- Xyce Coupling

Enable new Capabilities/Directions
- HPM applications
- Near-field/Far-field coupling
- High order outflow conditions

Design and Assessment
- Capabilities developed in this LDRD can be used to design new powerflow devices or assess existing in terms of "power flow efficiency"

Regrets

Extended Transmission line models
- Still desire for capability
- Added to EMPIRE-Cable Roadmap FY25

Target Coupling
- No funding stream identified
- Raises important programmatic question
  "Who owns a code coupling capability"
  Examples: ITS-EMPIRE, FORTE, Charon-Xyce

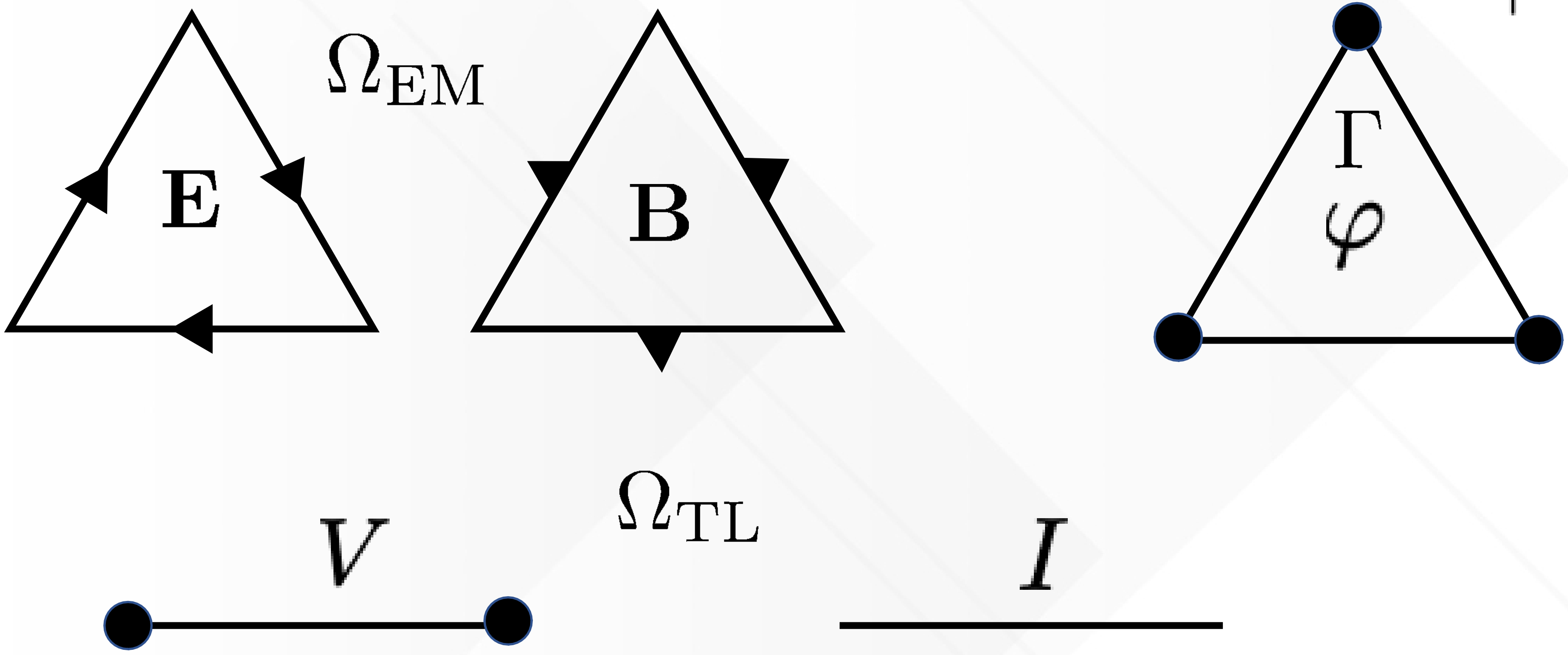
Back up

Discretization

RK Time Integration

0	0	0
1	0	1
	1/2	1/2

Compatible Finite Elements



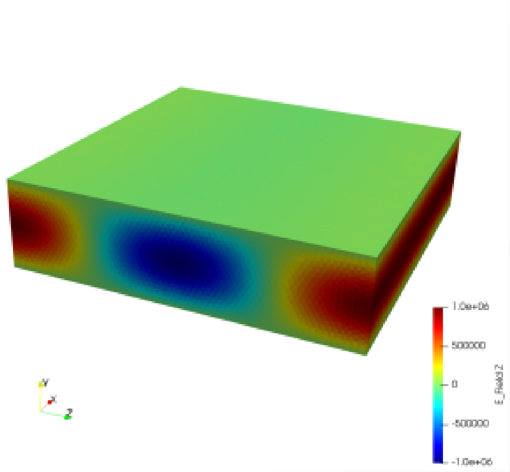
Code Verification

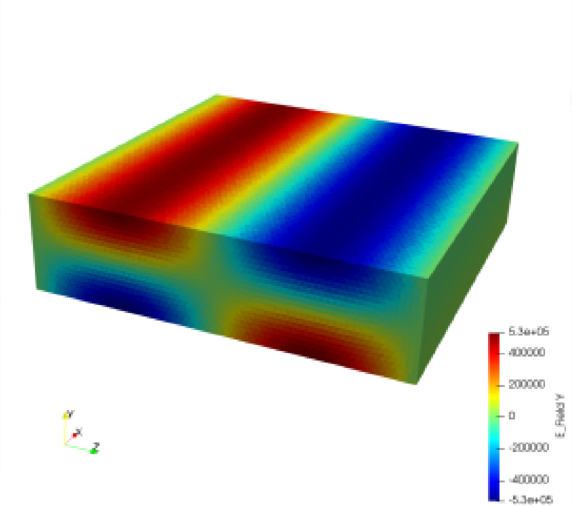
TL DOMAIN

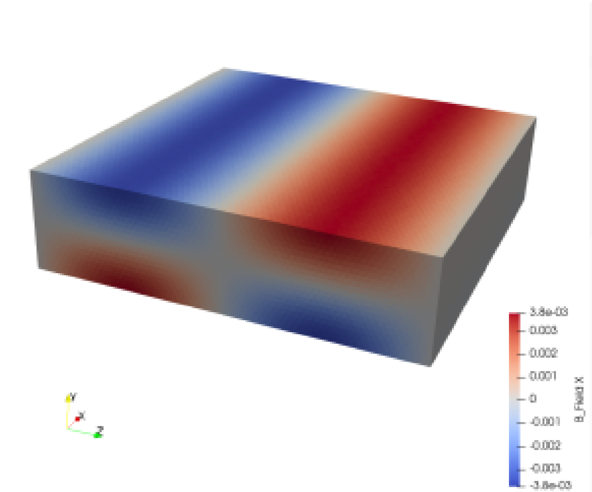
EM DOMAIN

3D TM Wave on a TL-EM domain

$z = -L_z$ $\qquad z = 0$ $\qquad z = L_z$







$$E_x = 0$$

$$E_y = \left(\frac{k_z^{[m]}}{k_c^{[m]}} E_{0,z}^{[m]} \right) \cos\left(k_c^{[m]} y \right) \sin(\omega t - k_z^{[m]} z)$$

$$E_z = E_{0,z}^{[m]} \sin\left(k_c^{[m]} y \right) \cos(\omega t - k_z^{[m]} z)$$

$$H_x = -\left(\frac{\omega \epsilon}{k_c^{[m]}} E_{0,z}^{[m]} \right) \cos\left(k_c^{[m]} y \right) \sin(\omega t - k_z^{[m]} z)$$

$$H_y = 0$$

$$H_z = 0$$

$$k_z^{[m]} \approx 2 k_c^{[m]}$$

Numerical Method (transverse profiles)

To solve the for wavenumber and eigenmode we will solve the eigenproblem on the coupling surface.
We use nodal finite elements on the surface mesh for this.

We perform a solve for the M smallest eigenvalues/eigenfunctions of the Laplace operator

$$\varphi_{\mathrm{TM},j,h} \in \mathcal{N}_{\Gamma,\mathrm{TM},h} : \int_\Gamma \varphi_{\mathrm{TM},j,h} \widetilde{\varphi}_h \; dA = \frac{1}{k_j^2} \int_\Gamma \nabla \varphi_{\mathrm{TM},j,h} \cdot \nabla \widetilde{\varphi}_h \; dA, \quad \forall \widetilde{\varphi}_h \in \mathcal{N}_{\Gamma,\mathrm{TM},h}$$

We solve the problem using Block Davidson algorithm found in Anasazi package

Once the solve is complete we normalize and orient (with a reference function) the eigenfunction

Our coupling algorithm only requires the tangent E field so we compute

$$\mathbf{E}_{\mathrm{TM},\tau,j,h} \in \mathcal{E}_{\Gamma,h} : \mathbf{E}_{\mathrm{TM},\tau,j,h} = -\nabla \varphi_{\mathrm{TM},j,h}$$

Numerical Method (for TL domain)

We then evolve the decoupled 1D telegrapher equations using your favorite method

We use Crank-Nicolson time integration in time and P1/P0 elements in space

$$\begin{cases} C\dfrac{\partial V_\tau}{\partial t} + \dfrac{\partial I_\tau}{\partial n} = 0 \\ C\dfrac{\partial V_n}{\partial t} - k_n I_\tau = 0 \\ L\dfrac{\partial I_n}{\partial t} + k_n V_n + \dfrac{\partial V_\tau}{\partial n} = 0 \end{cases}$$

$$(V_\tau, V_n, I_\tau) \in \mathcal{N}_h \times \mathcal{I}_h \times \mathcal{I}_h$$

$$\begin{cases} \mathbb{M}_\mathcal{N}(C)\dfrac{V_\tau^{n+1} - V_\tau^n}{\Delta t} - D_h^T \mathbb{M}_\mathcal{I} \dfrac{I_\tau^{n+1} + I_\tau^n}{2} = 0 \\ \mathbb{M}_\mathcal{I}(C)\dfrac{V_n^{n+1} - V_n^n}{\Delta t} - \mathbb{M}_\mathcal{I}(k)\dfrac{I_\tau^{n+1} + I_\tau^n}{2} = 0 \\ \mathbb{M}_\mathcal{I}(L)\dfrac{I_\tau^{n+1} - I_\tau^n}{\Delta t} + \mathbb{M}_\mathcal{I}(k)\dfrac{V_n^{n+1} + V_n^n}{2} + \mathbb{M}_\mathcal{I} D_h \dfrac{V_\tau^{n+1} + V_\tau^n}{2} = 0 \end{cases}$$

We reduce the TM solve to a TEM solve by discretely eliminating Vn in every time step

Data: Residual vectors R_{V_τ}, R_{V_n}, R_{I_τ}

Result: Increments $\Delta V_\tau, \Delta V_n, \Delta I_\tau$

Compute $\tilde{R}_{I_\tau} = R_{I_\tau} - \frac{\beta}{\alpha}\mathbb{I}_\mathcal{I}^{k/C} R_{V_n}$;

Compute $\tilde{R}_{V_\tau} = R_{V_\tau} - \frac{\beta}{\alpha} D_h^T (\mathbb{S}_\mathcal{I})^{-1} \tilde{R}_{I_\tau}$;

Solve $\mathbb{S}_\mathcal{N} \Delta V_\tau = \tilde{R}_{V_\tau}$;

Update $\Delta I_\tau = \frac{1}{\alpha}(\mathbb{S}_\mathcal{I})^{-1}(\tilde{R}_{I_\tau} - \beta D_h \Delta V_\tau)$;

Update $\Delta V_n = \frac{1}{\alpha}(\mathbb{Q}_\mathcal{I}^C)^{-1} R_{V_n} + \frac{\beta}{\alpha}\mathbb{I}_\mathcal{I}^{k/C} \Delta I_\tau$;

TEM Solve

$$\mathbb{S}_\mathcal{I} = \mathbb{M}_\mathcal{I}\left(L + \frac{\alpha^2}{\beta^2}\frac{k^2}{C}\right)$$

$$\mathbb{S}_\mathcal{N} = \alpha \mathbb{M}_\mathcal{N}(C) - \frac{\beta^2}{\alpha} D_h^T (\mathbb{S}_\mathcal{I})^{-1} D_h$$