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# A Matrix-Free Approach for Algebraic Multigrid

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# Introduction & Motivation

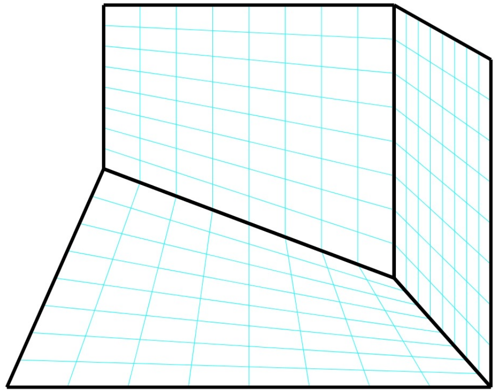
- Collaborators: Ray Tuminaro, David Noble
- Funded by ASCR

- Motivating statements:
  1. Many large-scale high-performance finite element applications are memory-bound
  2. Matrix-free methods trade lower memory usage for higher computational cost
  3. Accelerators such as GPUs may be utilized more effectively in lower-memory scenarios
  4. Without preconditioning, matrix-free performance isn't worth it

- Approaches with GMG have been studied, less with AMG

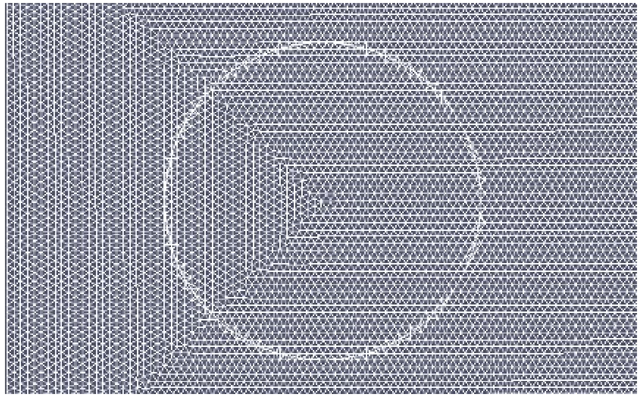
# Potential Approaches for Matrix-Free AMG

- Active subjects include p-coarsening, LOR

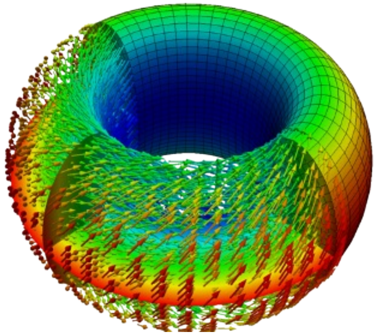


- Auxiliary operators

- AMG needs some sort of graph, explicit RAP
  - Limits benefits on simple scalar problems
  - Mesh graph (CG FEMs) or dual graph (DG FEMs)
  - Alleviate memory constraints via aggressive coarsening



- Applications of interest
  - GMG-AMG combinations
  - Hybrid-structure meshes (HHGs)
  - Collocated DOF multiphysics problems



$$\begin{bmatrix} A & B^T \\ B & -D \end{bmatrix}$$

# Potential Approaches for Matrix-Free AMG

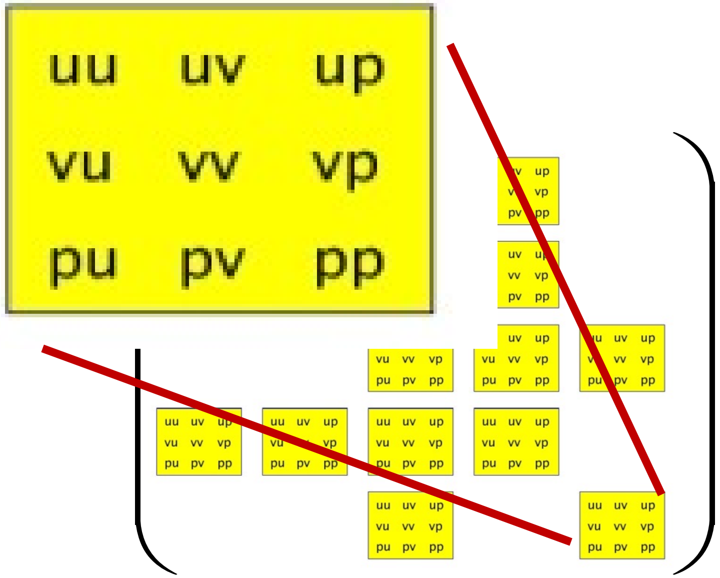
- Specialized “probing” of a matrix-free operator
  - Combine with cheap graph coloring algorithms
  - Compute all matrix entries with few matvecs

- An example multiphysics application
  - Start with mesh nodal graph
  - Perform coarsening on the graph
  - Generate graph transfers using distances
  - Create coarse matrix using a specialized multiplication
  - Coarser levels are handled more traditionally
- Reduces overall memory constraints

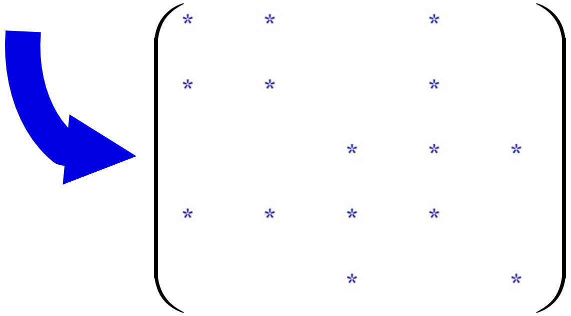
# AMG for Multiphysics

Consecutive DOFs within nodes

$$[u_1\ v_1\ p_1 \quad \cdots \quad u_n\ v_n\ p_n]$$

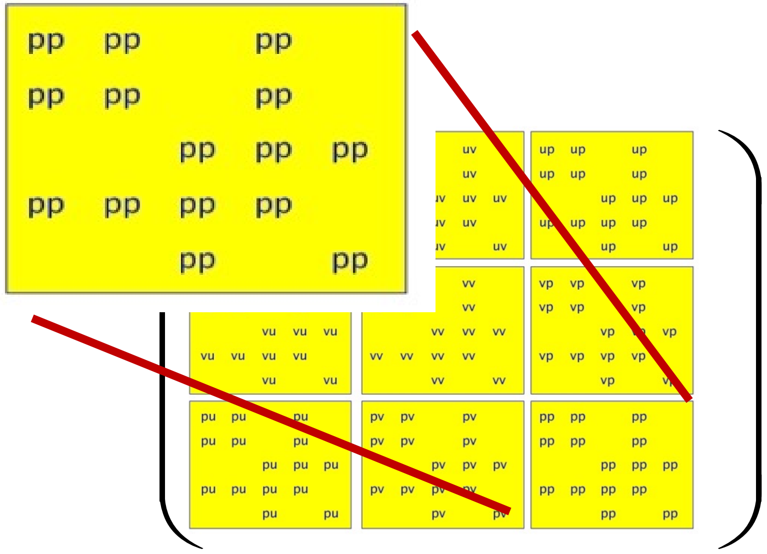



- Graph algorithms on nodes



Consecutive DOFs within fields

$$[u_1 \quad \cdots \quad u_n \quad v_1 \quad \cdots \quad v_n \quad p_1 \quad \cdots \quad p_n]$$



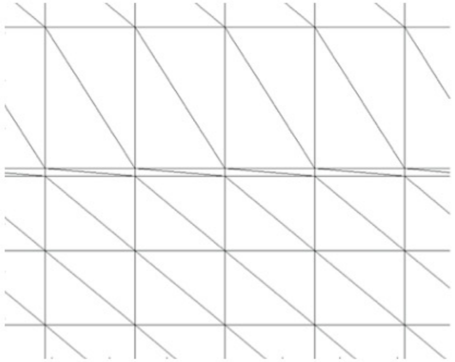
- Leads to blocked prolongator structure

# The Distance Laplacian

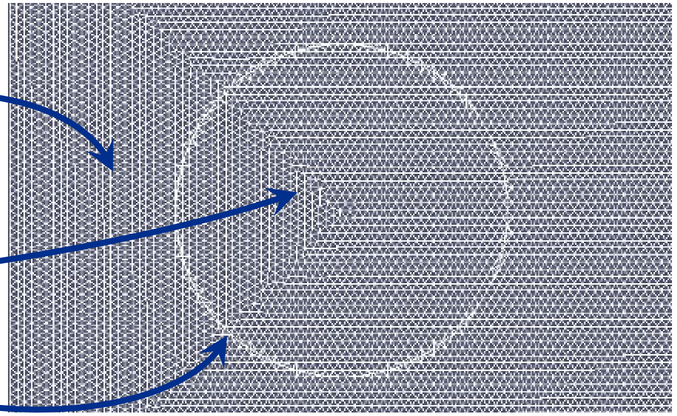
- Focus on multiphysics applications with collocated DOFs from here forward

- The distance Laplacian is defined by

$$L_{ij} = \begin{cases} -1/d(i,j), & i \neq j, A_{ij} \neq 0 \\ -\sum_{k \neq i} L_{ik}, & i = j \\ 0, & \text{otherwise} \end{cases}$$



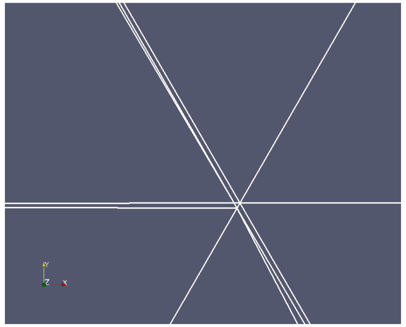
- Implemented in MueLu/ML with more features in progress
  - 3 dofs/node (velocities, water pressure)
  - 3 dofs/node (velocities, air pressure)
  - 4 dofs/node (velocities, air & water pressure)



# Matrix-Free Aspects of the Distance Laplacian

- Computing $L_{ij}$ only requires mesh, connectivity
- Reduces overall problem size

- Generally applicable (using DOF information)
  - Interface problems
  - Multiple species
  - Stabilized-equal order
  - Poor mesh quality



# Smoothed Aggregation AMG

- Smoothed aggregation = prolongator smoother * tentative prolongator

$$P_\ell = \left(I - \omega D_\ell^{-1} A_\ell\right) P_\ell^{(t)}$$

where

$$\omega = \frac{4}{3\lambda_{\ell,m}} \qquad \lambda_{\ell,m} = \rho(D_\ell^{-1} A_\ell)$$

- Distance Laplacian SA-AMG:
  - Replace $A_1$ with L
  - “Unsmoosh” resulting P
  - May resemble linear interpolation

$$\begin{array}{ccc} A & & P_A \\ \downarrow & & \uparrow \\ L & \longrightarrow & P_L \end{array}$$

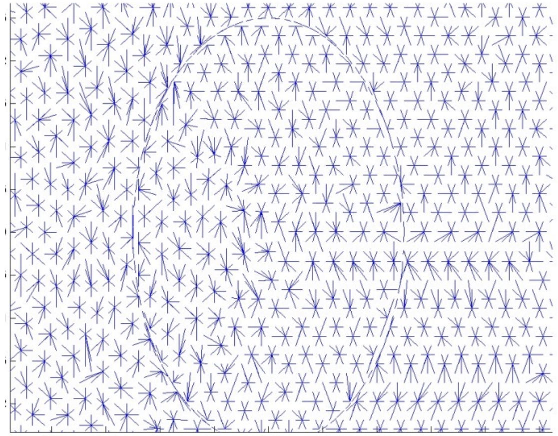
# Strategies for SA-AMG

- Matrix-free case:
  - Eigenvalue estimate and diagonal are friendly
  - Smoothers need to be carefully chosen
  - Aggregates from mesh graph

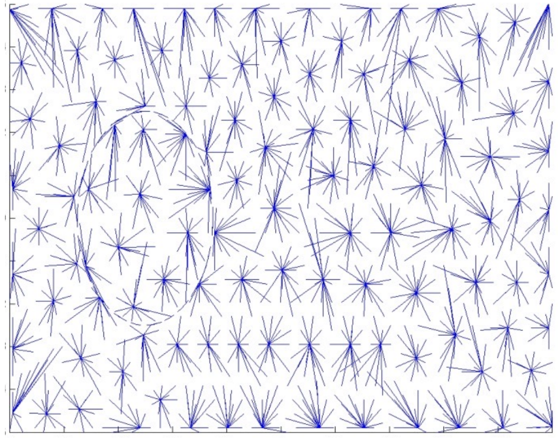
- Aggressive coarsening:
  - Multiple prolongator smooths increases stencil
  - Dropping to fix any poor qualities
    - e.g. locate material jumps with $A_{ii}/L_{ii}$

# Results – Distance Laplacian Only

- Rising bubble problem in Aria:



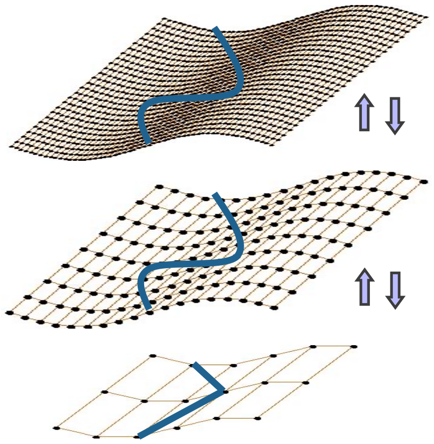
1st level aggregates

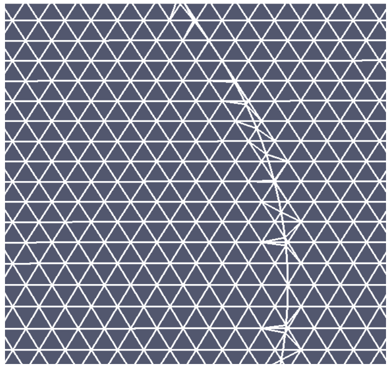


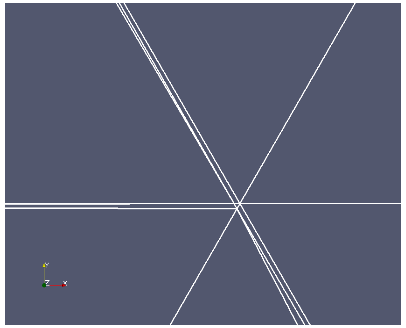
2nd level aggregates

- ILU relaxation to address smoothing concerns
  - Associated with incompressibility constraint
  - Tiny mesh spacing @ interface

| method | iterations |
|---|---|
| ILU only | 180 |
| Unsmoothed/plain aggregation | 25 |
| Smoothed aggregation | 19 |







# Performance Portability for Matrix-Free Solvers

- Performance portability is important as new architectures arise
- Trilinos packages leverage Kokkos for performance portability
- We utilize
  - Intrepid2
  - Panzer
  - Tpetra
  - Belos
  - MueLu

```
apply(const MV& X,
      MV& Y,
      Teuchos::ETransp mode = Teuchos::NO_TRANS,
      scalar_type alpha = Teuchos::ScalarTraits<scalar_type>::one(),
      scalar_type beta = Teuchos::ScalarTraits<scalar_type>::zero()) const
```

```
Y.scale(beta);
```

```
Kokkos::DynRankView<double,PHX::Device>
    physbasisvals("phys basis vals", num_elems, num_basis, num_ip);
Intrepid2::FunctionSpaceTools<PHX::Device::execution_space>
    ::HGRADtransformVALUE(phys_basis_vals, basis_vals);
```

```
kokkos_view_Y(LIDs(e,i),c) += alpha*local_mass(i,j)*kokkos_view_X(LIDs(e,j),c);
```

- Best value for programmatic effort

# Conclusion

- Distance Laplacian
  - Compressed representation of problem
  - Handles poor mesh quality well
- Smoothed aggregation
  - Versatile
- Matrix-Free
  - Plays well with each
  - Hybrid hierarchy approaches
- Plenty of combinations to explore
- Future work
  - Performance tests/improvement
  - Integration with MueLu

# Thank you!

- Questions?