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Identification of Noise Covariances for Voltage Dynamics Estimation in Microgrids

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***Abstract*—For the model-based control of low-voltage microgrids, state and parameter information are required. Different optimal estimation techniques can be employed for this purpose. However, these estimation techniques require knowledge of noise covariances (process and measurement noise). Incorrect values of noise covariances can deteriorate the estimator performance, which in turn can reduce the overall controller performance. This paper presents a method to identify noise covariances for voltage dynamics estimation in a microgrid. The method is based on the autocovariance least squares technique. A simulation study of a simplified 100 kVA, 208 V microgrid system in MATLAB/Simulink validates the method. Results show that estimation accuracy is close to the actual value for Gaussian noise, and non-Gaussian noise has a slightly larger error.**

***Index Terms*—Voltage dynamics, noise covariance, noise identification, process noise, measurement noise**

I. Introduction

Microgrids generally operate at low-to-medium voltage ranges, and, as a result, the R/X ratio is relatively high [1]. This results in voltage being more sensitive to active power. Different optimal controllers have been proposed to overcome these challenges [2]–[4], but these require full states and parameters information. States and parameters can be extracted from a model of the system and time-series measurements, but each of these have unknown error — termed as process and measurement noise for the modeling and measurement error, respectively. For optimal estimation, both information should be combined in the right proportion based on the error covariance. Different optimal filters/estimators exist, e.g., Kalman filter (KF), extended Kalman filter, unscented Kalman filter, particle filter, moving horizon estimator, but each filter requires knowledge of the covariance of the process and measurement noise. Incorrect noise covariances can combine the information in the wrong proportion and deteriorate the estimator performance [5], further reducing controller performance. As the error is unknown, the value of error covariances cannot be computed directly. Thus, it is necessary to identify process and measurement noise covariance for optimal state estimation.

Noise covariances identification techniques are divided into four classes: Bayesian [6], maximum likelihood [7], covariance matching [8], and correlation-based [9], [10]. In some cases, Bayesian and maximum likelihood-based methods require higher computational costs. The covariance matching technique requires less computational power, but it gives a biased estimate of the true covariance. The correlation-based technique presented in [9] and [10] gives an estimate with higher variance. The improved autocovariance least squares (ALS) method is presented in [11]. This method identifies noise covariances with lower uncertainty and requires less computational cost compared to other methods. We presented state and parameter estimation for voltage dynamics of microgrids in [12] where we assumed known process and measurement noise covariance. In this paper, we apply the ALS method from [11] to identify noise covariances for voltage dynamics of a microgrid.

The paper is organized as follows: Section II presents the noise covariances identification technique. In Section III, simplified voltage dynamics equations are presented. The simulation setup is presented in Section IV, with results and findings summarized in Section V. Section VI concludes the paper.

II. Autocovariance Least Squares Technique

In this section, the noise covariances identification technique is explained. The technique requires design of a stable observer, and the estimate provided by the observer will be used to identify the noise covariances. We consider the state and output equations of following form:

$$x_k = Ax_{k-1} + Bu_{k-1} + Gw_{k-1} \tag{1a}$$

$$y_k = Cx_k + v_k \tag{1b}$$

where x_k is the state, u_k is the input, y_k is the measurement, A is state matrix, B is input matrix, G is input matrix for process noise w_k, and v_k is the measurement noise.

A. Observer Design

Let the covariance of measurement noise be R_v and that of the process noise be Q_w; the error dynamics of the observer

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as given in [11] is

$$\varepsilon_k = \bar{A}\varepsilon_{k-1} + \bar{G}\bar{w}_{k-1} \quad (2a)$$

$$\mathscr{Y}_k = y_k - C\hat{x}_k^- \quad (2b)$$

where $\hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1}$, $\bar{A} = A - ALC$, $\bar{G} = [G,\ -AL]$, $\bar{w}_k = [w_k^\top,\ v_k^\top]^\top$, and L is the observer gain. The observer gain is calculated using a guessed value of Q_w and R_v, thus the designed observer is non-optimal as the guessed values are most likely to be far from correct values. However, this does not affect the noise identification process as long as the observer is stable as shown in [11]. Solution to following equation gives state covariance P in steady state

$$P = APA^\top + GQ_wG^\top - APC^\top(CPC^\top + R_v)^{-1}CPA^\top. \quad (3)$$

From the value of P, the observer gain is calculated as

$$L = PC^\top(CPC^\top + R_v)^{-1}. \quad (4)$$

B. Noise Covariance Identification

The noise covariance identification technique is based on the method given in [11]. From the estimate provided by the observer, $\mathscr{Y}$ can be calculated with autocovariance given by

$$\widehat{\mathscr{C}}_j = \frac{1}{N_d - j}\sum_{i=1}^{N_d-1} \mathscr{Y}_i \mathscr{Y}_{i+j}^\top \quad (5)$$

where a non-negative integer j represents the lag and N_d represents the number of data points. The autocovariance can be calculated for different values of lag. For stationary data, the autocovariance becomes non-significant after a certain lag [13]. Let the maximum lag for significant autocovariance be $N-1$, then an autocovariance matrix (ACM) is created as

$$\widehat{\mathscr{R}}(N) = \begin{bmatrix} \widehat{\mathscr{C}}_0 & \dots & \widehat{\mathscr{C}}_{N-1} \\ \vdots & \ddots & \vdots \\ \widehat{\mathscr{C}}_{N-1}^\top & \dots & \widehat{\mathscr{C}}_0 \end{bmatrix}. \quad (6)$$

The mathematical expression for autocovariance presented in [11] can be equated to the calculated ACM, and the equations can be solved for Q_w and R_v. However, the number of equations depends upon the maximum lag chosen, which is generally greater than the number of unknowns. Thus, we seek to solve the least-squares optimization problem such that the sum of the square error in each equation is minimized instead of solving the equations. As given in [11], the least-squares optimization problem can be written as

$$\min_{\mathscr{X}} \quad \left|\left|\mathscr{A}\mathscr{X} - \hat{b}\right|\right|_2^2 \quad (7)$$

where

$$\mathscr{A} = \left[D(G\otimes G) | D(AL\otimes L) + [\Psi \oplus \Psi + I_{n_y^2 N^2}]\mathscr{I}_{n_y,N}\right],$$
$$D = [(\mathscr{O}\otimes\mathscr{O})(I_{n_x^2} - \bar{A}\otimes\bar{A})^{-1} + (\Gamma\otimes\Gamma)\mathscr{I}_{n_x,N}],$$
$$\mathscr{O} = \begin{bmatrix} C \\ C\bar{A} \\ \vdots \\ C\bar{A}^{N-1}\end{bmatrix}, \Gamma = \begin{bmatrix} 0 & 0 & 0 & 0 \\ C & 0 & 0 & 0 \\ \vdots & \ddots & & \vdots \\ C\bar{A}^{N-2} & \dots & C & 0\end{bmatrix},$$
$$\hat{b} = \text{vec}\left(\widehat{\mathscr{R}}(N)\right), \Psi = \Gamma\left[\bigoplus_{j=1}^{N}(-AL)\right],$$
$$\text{and } \mathscr{X} = \begin{bmatrix}\text{vec}(Q_w) \\ \text{vec}(R_v)\end{bmatrix}.$$

Here $\otimes$ represents the Kronecker product, $\oplus$ the Kronecker sum, $\bigoplus$ the direct sum, $\text{vec}(\cdot)$ the columnwise stacking of matrix into a vector, and $\mathscr{I}_{n,N}$ the permutation matrix of size $(nN)^2 \times n^2$ whose elements a_{ij} are either 0 or 1. The elements must follow the condition:

$$\text{vec}\left(\bigoplus_{i=1}^{N} X\right) = \mathscr{I}_{n,N}\ \text{vec}(X) \quad \forall\ X \in \mathbb{R}^{n\times n}. \quad (8)$$

The solution to (7) is given as

$$\widehat{\mathscr{X}} = (\mathscr{A}^\top\mathscr{A})^{-1}\mathscr{A}^\top\hat{b}, \quad (9)$$

which gives the estimate of Q_w and R_v that are denoted by $\widehat{Q}_w$ and $\widehat{R}_v$, respectively.

III. Simplified Voltage Dynamics

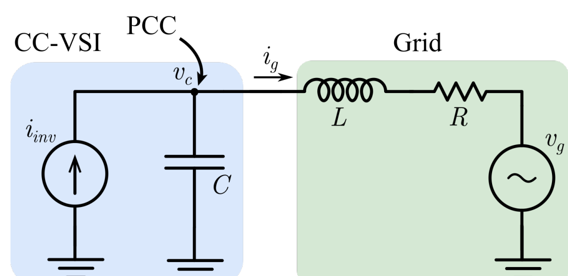

Fig. 1: Schematic representing an ESS connected to a microgrid. The ESS is represented as CC-VSI for modeling purposes.

In this section, a simplified model for voltage dynamics of a microgrid system is derived and then transformed into a form suitable to apply the noise covariances identification technique. The simplified model is used as a prediction model for noise covariances identification. The single line diagram of an inverter-based energy storage system (ESS) connected to a microgrid is shown in Fig. 1. The inverter is modeled as an average controlled current source (neglecting the current control loop of the inverter), while the grid is represented by the Thevenin equivalent voltage source with voltage v_g and equivalent resistance and inductance represented by R and L, respectively. At the point of common coupling (PCC), a capacitor C is added that is a part of the inverter filter. The voltage across the capacitor is represented by v_c, the current to the grid by i_g, and the inverter current by i_{inv}.

Grid current i_g and capacitor voltage v_c can be taken as state variables, and the dynamics of the system can be represented as:

$$\frac{di_{g,abc}}{dt} = \frac{v_{c,abc} - v_{g,abc} - i_{g,abc}R}{L} \tag{10a}$$

$$\frac{dv_{c,abc}}{dt} = \frac{i_{inv,abc} - i_{g,abc}}{C} \tag{10b}$$

With the application of Park's transformation and neglecting the zero component, we get the following state-space representation of the system in $dq0$ frame:

$$\frac{di_{gd}}{dt} = -\frac{R}{L}i_{gd} + \omega i_{gq} + \frac{v_{cd}}{L} - \frac{v_{gd}}{L} \tag{11a}$$

$$\frac{di_{gq}}{dt} = -\omega i_{gd} - \frac{R}{L}i_{gq} + \frac{v_{cq}}{L} - \frac{v_{gq}}{L} \tag{11b}$$

$$\frac{dv_{cd}}{dt} = -\frac{i_{gd}}{C} + \omega v_{cq} + \frac{i_{invd}}{C} \tag{11c}$$

$$\frac{dv_{cq}}{dt} = -\frac{i_{gq}}{C} - \omega v_{cd} + \frac{i_{invq}}{C} \tag{11d}$$

In the above equations, state variables are i_{gd}, i_{gq}, v_{cd}, and v_{cq}, and the input variables are i_{invd} and i_{invq}.

Equation (11) has four state variables, however v_{gd} and v_{gq} are also time-varying so they should also be estimated. Thus we can also incorporate v_{gd} and v_{gq} as state variables. Because the dynamics of v_{gd} and v_{gq} are governed by a load change in the microgrid which is stochastic and variation in their value is relatively small, we assume a constant process model for v_{gd} and v_{gq}. The state equations can then be written as

$$\frac{d}{dt}\begin{bmatrix} i_{gd} \\ i_{gq} \\ v_{cd} \\ v_{cq} \\ v_{gd} \\ v_{gq} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega & \frac{1}{L} & 0 & -\frac{1}{L} & 0 \\ -\omega & -\frac{R}{L} & 0 & \frac{1}{L} & 0 & -\frac{1}{L} \\ -\frac{1}{C} & 0 & 0 & \omega & 0 & 0 \\ 0 & -\frac{1}{C} & -\omega & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{gd} \\ i_{gq} \\ v_{cd} \\ v_{cq} \\ v_{gd} \\ v_{gq} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{C} & 0 \\ 0 & \frac{1}{C} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_{invd} \\ i_{invq} \end{bmatrix}. \tag{12}$$

The above equation can be discretized, and process and measurement noise can be incorporated. The final discrete-time equations take the form of equation (1), where w_k represents the process noise, v_k the measurement noise, $x_k = [i_{gdk}\ i_{gqk}\ v_{cdk}\ v_{cqk}\ v_{gdk}\ v_{gqk}]^\top$ the state vector, and $u_k = [i_{invdk}\ i_{invqk}]^\top$ the input vector. Because each element of w_k appears in each equation, we choose $G = I_{4\times 6}$, where $I_{m\times n}$ represents an identity matrix of size $m \times n$. Similarly, i_{gd}, i_{gq}, v_{cd}, and v_{cq} are measurable; hence we choose $C = I_{4\times 6}$.

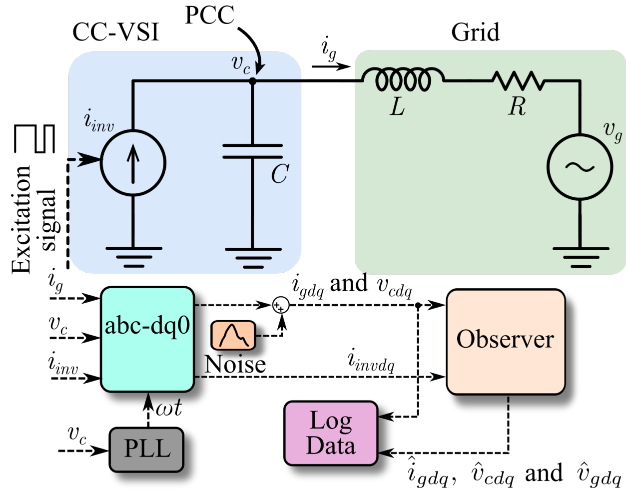

Fig. 2: Simulation setup of the test system to extract the data to identify noise covariances.

IV. Simulation Setup

A simulation study was carried out to validate the approach by comparing the identified noise covariances with the known applied noise. The setup is illustrated in Fig. 2 and was modeled in MATLAB/Simulink. The microgrid system considered is 100 kVA, 208 V with a 40 kVA inverter. The system parameters are summarized in Table I. Continuous-time state equations were discretized using *Runge-Kutta* method of order 4. The sample time for the observer was selected based on the time constant of the system [14]. A standard phase-locked loop (PLL) from Simulink was used to extract the instantaneous angle of v_c to transform variables in abc to $dq0$ frame. Gaussian noise of a known covariance is added to these transformed values and used as measurements which are then fed to the designed observer. Because process noise corresponds to modeling error, we used incorrect parameter values to induce modeling error/process noise. Simulation parameters along with noise covariances are summarized in Table I. The guessed value of noise covariances in Table I were used to calculate the observer gain. The estimates provided by the observer and the measurements were logged. The ALS technique was applied offline to the logged data to get $\widehat{Q}_w$ and $\widehat{R}_v$. Python's numpy library was used to perform matrix operations on the data.

TABLE I: Summary of parameters

Parameters	Values	Parameters	Values
R	0.08 Ω	Incorrect R	0.09 Ω
L	0.22 mH	Incorrect L	0.33 mH
C	220 µF	Incorrect C	120 µF
True R_v	$\text{diag}(2.6, 2.6, 17.3, 17.3) \times 10^{-4}$		
Guessed Q_w	$\text{diag}(0.65, 2.1, 6.92, 6.92) \times 10^{-5}$		
Guessed R_v	$\text{diag}(0.65, 0.65, 24.2, 24.2) \times 10^{-5}$		

V. Results and Analysis

A. Noise Covariance Identification

The simulation was run 500 times, each time with a different random seed value with a runtime of 0.2 seconds. The data obtained from each simulation were passed to the observer separately. From the estimates provided by the observer and

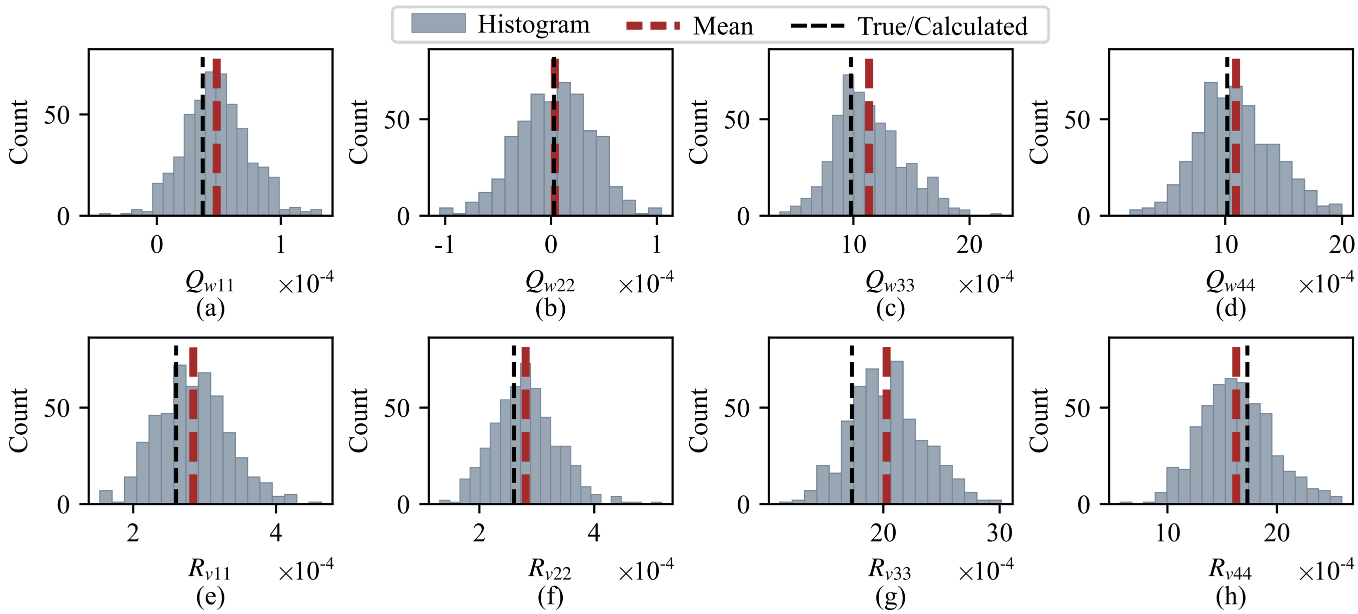

Fig. 3: Histogram of the identified noise covariances. First row shows histogram of process noise covariances and second row shows histogram of measurement noise covariances.

the measurements, autocovariance is calculated. For most of the scenarios, it was found that autocovariance after a lag of 6 was very low, so $N = 6$ was chosen. From the identification of noise covariances we get from multiple simulation data, a histogram is created as shown in Fig. 3. Note that the figure shows the histogram only for diagonal elements of noise covariances. To validate the process noise covariance, we calculate the modeling error by solving for w_k from (1) with the true value of x_k. The distribution of calculated process noise is shown in Fig. 4.

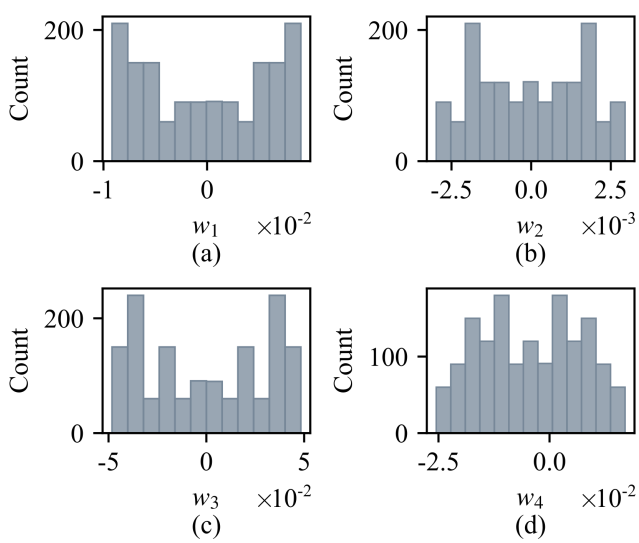

Fig. 4: Distribution of process noise calculated from true value of states (to validate the identified noise covariances).

The estimated and true/calculated mean are summarized in Table II. The results show that the variance of the first three process noise has a large error with respect to the calculated value. This is because process noise was induced by using the incorrect value of parameters that does not necessarily follow a Gaussian distribution as shown in Figs. 4(a)–(c). As the ALS method assumes that all noise are Gaussian, a large discrepancy in process noise is observed. The variance of the last term of process noise is lower because the corresponding distribution is closer to Gaussian as observed in Fig. 4(d). Gaussian measurement noise was introduced for this study, and hence the error is smaller for their variance identification.

TABLE II: Comparison of true and identified noise covariances components

Noise	True/Calculated	Identified mean	Error (%)
Q_{w11}	0.37×10^{-4}	0.48×10^{-4}	29.7
Q_{w22}	0.027×10^{-4}	0.0350×10^{-4}	22.9
Q_{w33}	9.79×10^{-4}	11.3×10^{-4}	13.4
Q_{w44}	10.2×10^{-4}	10.9×10^{-4}	6.42
R_{v11}	2.6×10^{-4}	2.84×10^{-4}	9.23
R_{v22}	2.6×10^{-4}	2.80×10^{-4}	7.69
R_{v33}	17.3×10^{-4}	20.2×10^{-4}	16.8
R_{v44}	17.3×10^{-4}	16.3×10^{-4}	5.78

B. *Estimation with Identified Noise Covariances*

After the noise covarianced are identified, state estimation with KF was performed on two cases. In the first case, the guessed value of noise covariances are taken (which are far from true values). In the second case, the identified noise covariances are used. The first case is named "incorrect KF", and the second case "correct KF". While implementing KF, we assumed that there is uncertainty in the parameters so incorrect values of R, L, and C were used (i.e., the values in Table I). The performance of KF under these two conditions are shown in Fig. 5. Normalized root mean square error (NRMSE) was used to characterize the error of the estimates with respect to true values, shown in Table III. The results show that KF with identified noise estimates the states closer to the true value; KF using identified noise covariance performs better than with the incorrect values of covariance.

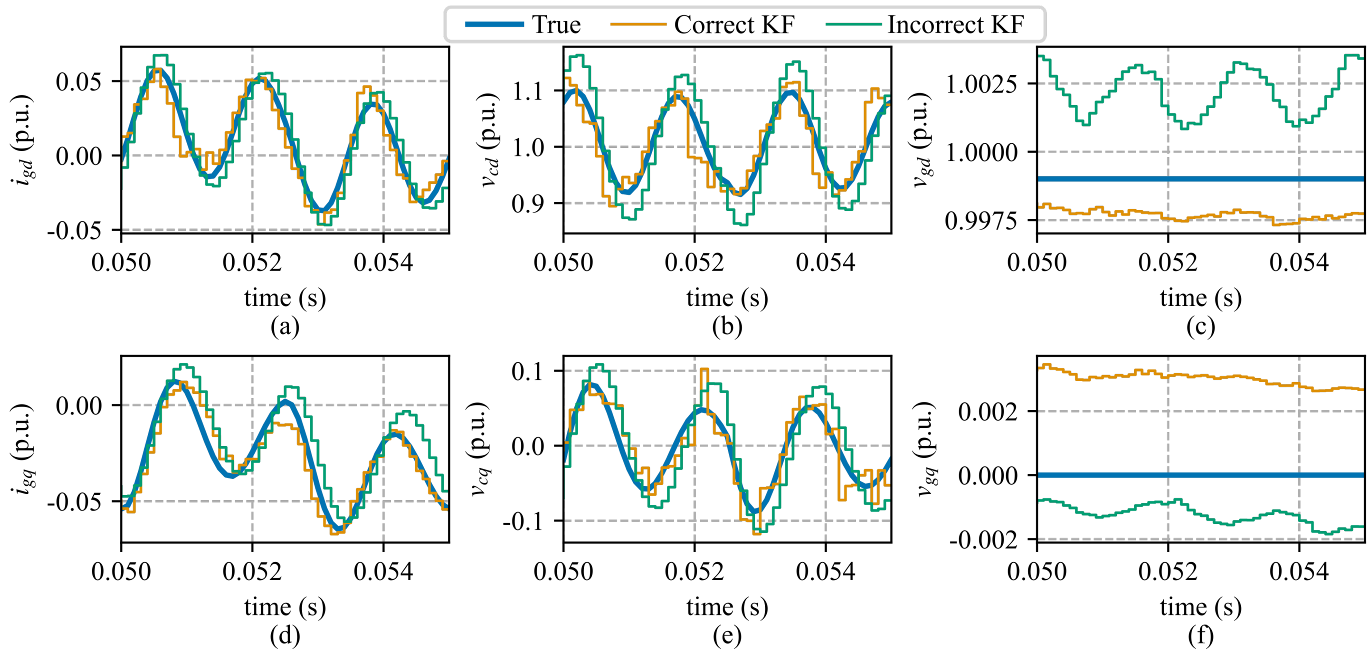

Fig. 5: Comparison of state estimates provided by correct and incorrect KF; KF with identified noise tracks true states more closely.

TABLE III: Comparison of NRMSE of state estimates provided by correct and incorrect KF

States	NRMSE (%)		States	NRMSE (%)	
	Correct KF	Incorrect KF		Correct KF	Incorrect KF
i_{gd}	12.5	18.3	v_{cd}	16.3	31.3
i_{gq}	14.5	22.0	v_{cq}	18.9	27.8

VI. Conclusions

The paper presented a process and measurement noise identification technique for voltage dynamics estimation of microgrids. A simplified model was used to model the microgrid and was employed to identify noise covariance. The method was able to accurately identify Gaussian noise, as well as identify the non-Gaussian process noise. The identified noise covariance are of same order as those calculated. The larger discrepancy in calculated and identified covariance is due to the non-Gaussian distribution of process noise. Further, KF was implemented using incorrect ("guessed") and identified values of noise covariances, and it was found that KF using the identified noise covariances closely tracks the true states.

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