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Discontinuous Petrov-Galerkin Methods for Transient PDEs

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Introduction / Motivation

The **Discontinuous Petrov-Galerkin (DPG)** method is a well-established method introduced by Prof. Demkowicz and Gopalakrisnan to numerically approximate the solution of Partial Differential Equations (PDEs). It is a minimum-residual method that delivers a stable solution and a built-in error representation usually employed to perform adaptivity. In the last decade, it has been applied to a wide variety of steady-state problems.

Recently, several authors in the DPG community have contributed to the extension of the method to **time-dependent problems**. In particular: (a) J. Muñoz-Matute and L. Demkowicz developed a DPG-based time-marching scheme and (b) N. V. Roberts combined the DPG method in space together with finite differences in time. Our goal is to combine the knowledge and expertise of both groups to further expand the DPG ideas and exploit the properties of this method for solving challenging nonlinear transient problems like **Vlasov equations** in plasma physics.

Approach

There exist three different approaches to apply the DPG method to time-dependent PDEs:

1. Method of discretization in time: We first apply finite differences in time (i.e., Backward Euler method) and then we discretize the sequence of variational problems in space employing the DPG method [2].
2. Space-time DPG method: We consider the time variable as an extra space dimension and we apply the DPG method in the full space-time domain. To reduce the computational cost we can solve the problem over space-time slabs.
3. Method of Lines: We first consider a variational formulation in space and discretize it employing the DPG method or the classical Bubnov-Galerkin method. After semidiscretization in space we obtain a system of ordinary differential equations. Finally, we employ the DPG-based time-marching scheme developed in [1] which is of exponential-type.

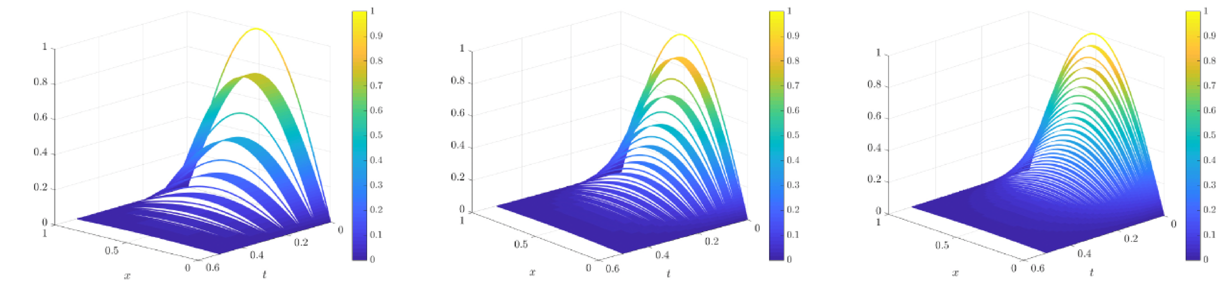

Figure 1: Solution of the 1D+time heat equation employing the DPG-based time-marching scheme.

References

[1] J. M-Matute, D. Pardo, L. Demkowicz. A DPG-based time-marching scheme for linear hyperbolic problems. CMAME, 373:113539, 2021
[2] N. V. Roberts, S. Henneking. Time-stepping DPG formulation for the heat equation. CAMWA, 95:242-255, 2021
[3] N. V. Roberts. Camellia: A rapid development framework for finite element solvers. CMAM, 19(3):581-602, 2019

Current Status / Results

- Author N. V. Roberts at Sandia has successfully applied approaches 1. and 2. for the **Vlasov-Poisson equations.** The software employed is **Camellia** [3], the Trilinos-based DPG library.

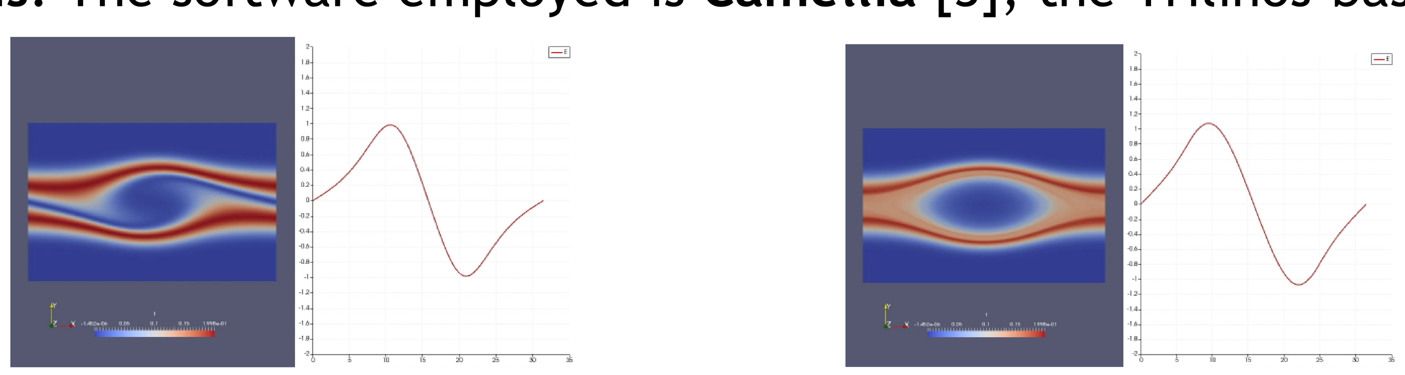

Figure 2: Two-stream instability problem at intermediate (left) and final (right) times.

- The group of Prof. Demkowicz at UT Austin have developed a **methodology** to combine the DPG method in space together with the DPG-based time marching-scheme (approach 3.) for solving linear hyperbolic transient problems like the transport equation.

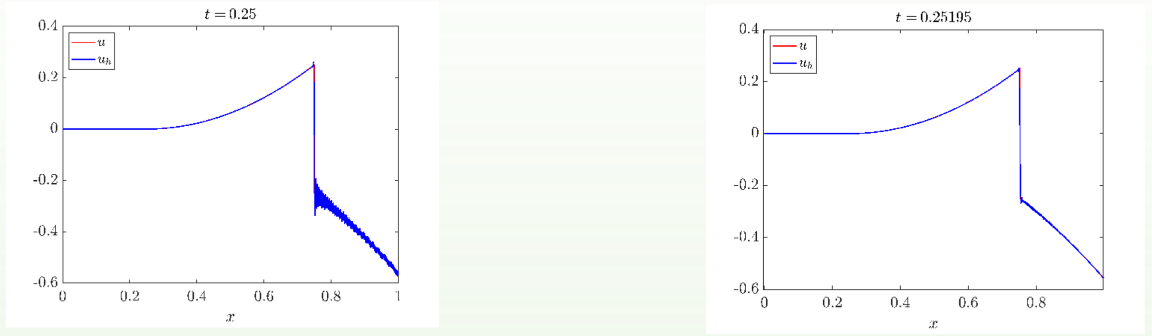

Figure 3: Solution of the 1D+time transport equation with DPG method in space together with classical Exponential Euler method (left) and the DPG method in time (right).

Challenges

The main challenges of this project are:

- The dimensionality of the Vlasov equations: we need up to 7D meshes (3 physical, 3 velocity and 1 time); planned mitigations include: Serendipity bases, smart assembly (sum factorization, etc.), and possibly matrix-free solvers (preconditioning is an open question for these).
- The DPG-based time-marching scheme requires at each time step the computation of exponential-related functions for large sparse matrices.
- The numerical analysis of the DPG method for (transient) non-linear problems is not established yet.

Future Work

Future research lines include:

- To complete the mathematical analysis of the three approaches.
- To extend the DPG time-marching scheme to transient non-linear and semilinear problems.
- To develop automatic and goal-oriented adaptive strategies both in space and time.
- To employ approach 3. to solve Vlasov-Poisson equations and other challenging problems in physics.

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