SAND2022-2587C

Sandia National Laboratories

Physical-model gate set tomography

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Quantum Performance Laboratory &
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Sandia National Laboratories

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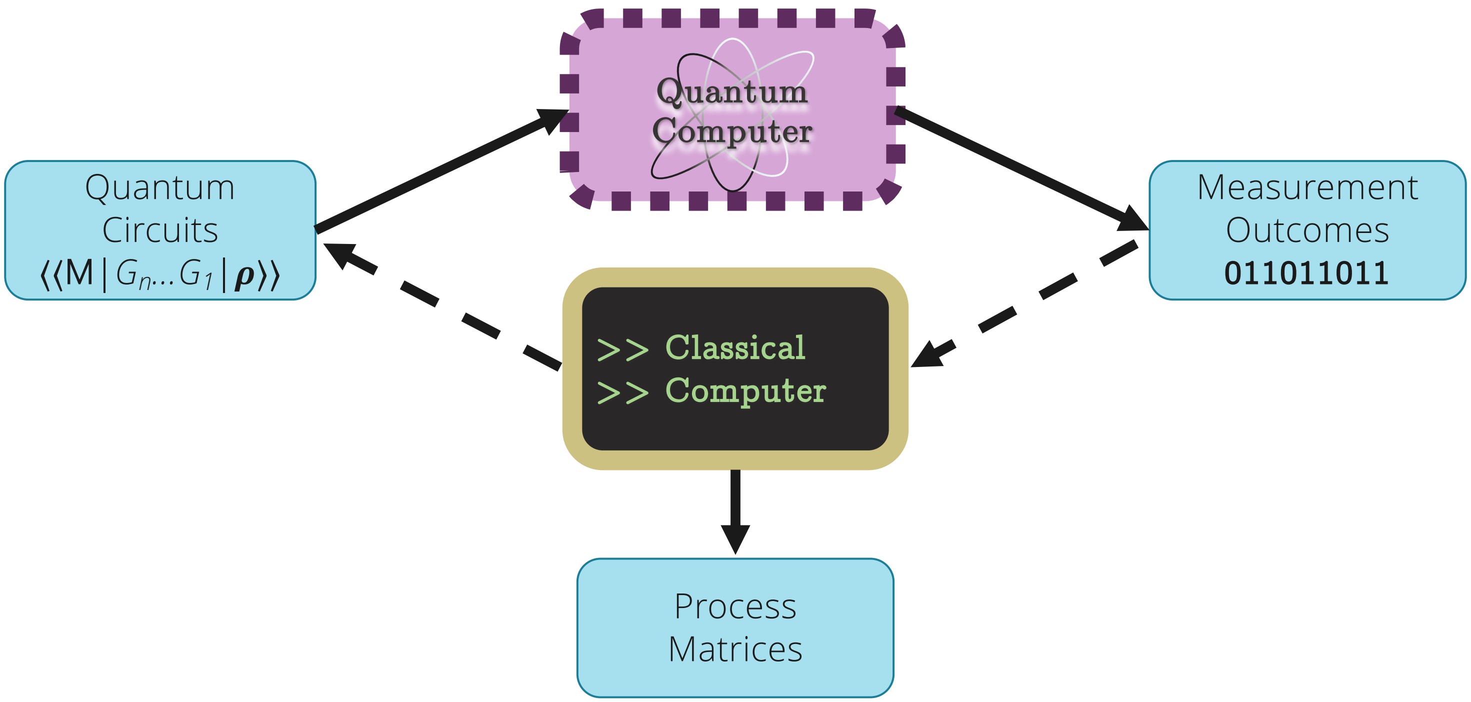
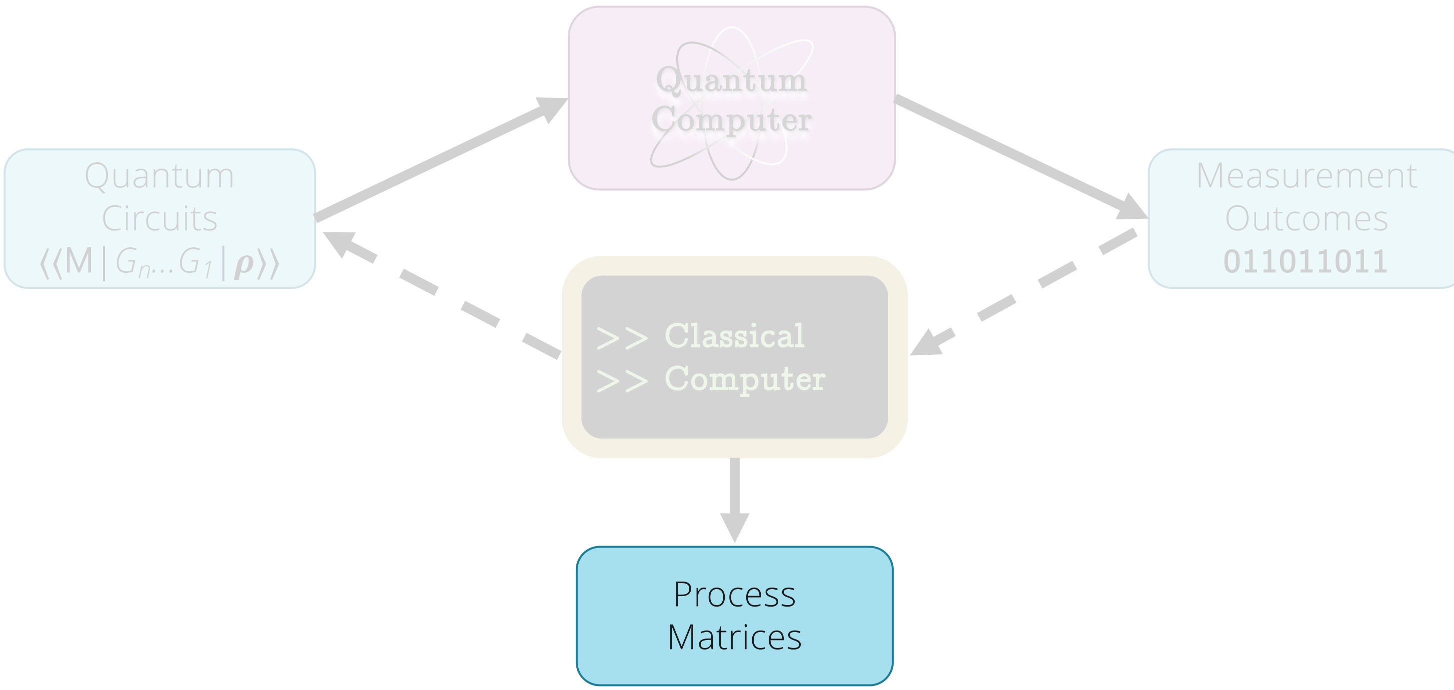
Gate Set Tomography



Process Matrices



Process Matrices

$$\rho \longmapsto \mathcal{E}(\rho)$$

$$|\rho\rangle\rangle \longmapsto E|\rho\rangle\rangle$$

- The Good:
  Completely-positive, trace-preserving maps on density matrices.
  Can predict the outcomes of any Markovian quantum circuit.

- The Bad:
  Dense $4^N \times 4^N$ matrices of real numbers are difficult to interpret.
  Learning these parameters requires many experiments.

- The Ugly:
  Gauge freedom.

  Process Matrices

  No obvious connection between parameters and experiments.

$$E \longmapsto GEG^{-1}$$

$$|\rho\rangle\rangle \longmapsto G|\rho\rangle\rangle$$

$$\langle\langle M| \longmapsto \langle\langle M|G^{-1}$$

Two methods to better characterization

1

Physics-inspired reduced error models

2

Detailed physical simulation

Reforming process matrices with error generators

Error Generators

$$\rho \longmapsto \mathcal{E}(\rho)$$

$$|\rho\rangle\rangle \longmapsto E\,|\rho\rangle\rangle$$

$$|\rho\rangle\rangle \longmapsto e^{\Delta} U\,|\rho\rangle\rangle$$

Physicists often care more about Hamiltonians than the unitary operator they induce

Similarly, *error generators* can be more informative than the process matrices they generate.

Reforming process matrices with error generators

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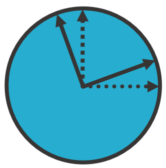
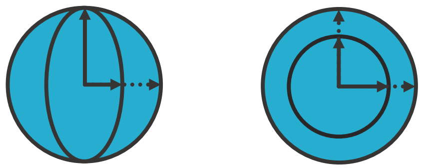


coherent (unitary) processes

$\mathbb{H}$



Pauli stochastic errors
(eg., dephasing or depolarization)

$\mathbb{S}$

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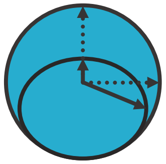
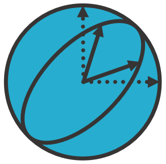


affine errors
(eg., amplitude damping)
$\mathbb{A}$



correlated Pauli stochastic errors
(eg., dephasing in the X+Z direction)
$\mathbb{C}$

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Physicists often care more about Hamiltonians than the unitary operator they induce.

Similarly, *error generators* can be more informative than the process matrices they generate.

$$E = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & .95 & .1 & -.01 \\ 0 & .1 & .95 & .01 \\ .1 & -0.05 & -.02 & .99 \end{pmatrix}$$

Reforming process matrices with error generators

Error Generators

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Physicists often care more about Hamiltonians than the unitary operator they induce.

Similarly, *error generators* can be more informative than the process matrices they generate.

$$\Delta = \begin{cases} 0.01\mathbb{H}_Y + 0.005\mathbb{H}_X \\ +0.005\mathbb{S}_X + 0.005\mathbb{S}_Y + 0.04\mathbb{S}_Z \\ +0.01\mathbb{C}_{XY} + 0.005\mathbb{C}_{YZ} + 0.1\mathbb{A}_{XX} \end{cases}$$

For reasonably good quantum computers, error generators are **small**, and many generators may be indistinguishable from ZERO.

Reduced models with error generators

Error Generators

$$\rho \longmapsto \mathcal{E}(\rho)$$

$$|\rho\rangle\rangle \longmapsto E\,|\rho\rangle\rangle$$

$$|\rho\rangle\rangle \longmapsto e^{\Delta}U\,|\rho\rangle\rangle$$

Error generators enable construction of ***reduced models.***

- Ask your theorist which error generators to expect, and only include those in your model.

Reduced models have:

- Fewer parameters
- Require fewer experiments to fit
- Display MUCH less gauge freedom
- Have more direct ties to experiments

Reduced model for silicon qubits – ***learned from data***

UNSW Silicon Qubits (with Morello group)
Experiments for full 2Q GST
Data explained using a reduced model (H1 + S1 + ZZ*)

$ZZ^* = H_{ZZ} + H_{G \cdot ZZ}$

	Q_1		Q_2			2Q
Gate	XI	YI	IX	IY	-IY	CZ
Hamiltonian (H1+ZZ*)	IX, IY, IZ, XI, YI, ZI, ZZ, YZ	IX, IY, IZ, XI, YI, ZI, ZZ, XZ	IX, IY, IZ, XI, YI, ZI, ZZ, ZY	IX, IY, IZ, XI, YI, ZI, ZZ, ZX	IX, IY, IZ, XI, YI, ZI, ZZ, ZX	IX, IY, IZ, XI, YI, ZI, ZZ
Stochastic (S1)	IX, IY, IZ, XI, YI, ZI,	IX, IY, IZ, XI, YI, ZI,	IX, IY, IZ, XI, YI, ZI,	IX, IY, IZ, XI, YI, ZI,	IX, IY, IZ, XI, YI, ZI,	IX, IY, IZ, XI, YI, ZI,

Fully general process matrix model
- 1503 parameters
- Used 1592 circuits

Reduced error generator model
- 83 gate + 63 SPAM = 146
- *Could have been fit with far fewer circuits!*
- *All of these parameters make physical sense!*

- Mądzik et al., Nature 601, 348–353 (2022)

Reduced model for ^{171}Yb qubits

Laser phase, amplitude, timing, 1-photon detuning, 2-photon detuning, spontaneous scattering, clock errors,

	Q_1		Q_2		2Q		
Gate	G_{IX}	G_{IY}	G_{XI}	G_{YI}	G_{II}	G_{XX}	G_{MS}
Hamiltonian (H)	IX, IY, IZ, XI, YI, ZI	IX, IY, IZ, XI, YI, ZI	IX, IY, IZ, XI, YI, ZI	IX, IY, IZ, XI, YI, ZI	IX, IY, IZ, XI, YI, ZI	IX, IY, IZ, XI, YI, ZI	IX, IY, IZ, XI, YI, ZI XX, XY, YX, YY
Stochastic (S)	IX, IY, IZ, XI, YI, ZI	IX, IY, IZ, XI, YI, ZI	IX, IY, IZ, XI, YI, ZI	IX, IY, IZ, XI, YI, ZI	IX, IY, IZ, XI, YI, ZI	IX, IY, IZ, XI, YI, ZI	IX, IY, IZ, XI, YI, ZI XX, XY, YX, YY
Correlation (C)	ZI:IZ	ZI:IZ	ZI:IZ	ZI:IZ	ZI:IZ	ZI:IZ XI:IX	ZI:IZ
Active (A)	$I\sigma_-$	$I\sigma_-$	$\sigma_- I$	$\sigma_- I$		$I\sigma_-$, $\sigma_- I$	$I\sigma_-$, $\sigma_- I$

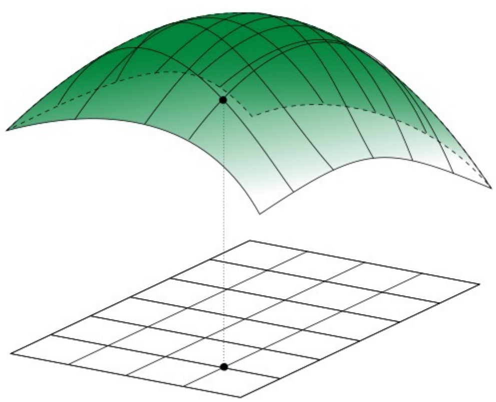
Fully general process matrix model
- 240*7 Gate + 63 SPAM = 1743

Reduced error generator model
- 108 gate + 63 SPAM = 171

pyGSTi.extras.interpygate

$$|\rho\rangle\rangle \longmapsto e^{\Delta} U \, |\rho\rangle\rangle$$

$$\Delta = \Delta(\vec{a})$$



Physics models functions:

experimental parameters → process matrices

Generally **too expensive** for iterative optimizers that fit models to data.

`pyGSTi.extras.interpygate` approximates this function by:

1. Evaluate on hypergrid of the input parameter space
2. Computing an error generator at each point
3. Interpolate with a multivariate cubic spline
4. Save as a file, and share

This procedure is embarrassingly parallel, so is compatible with HPCs.

Now you can do GST with far fewer circuits, no gauge problem, and parameters that mean something to experimentalists!

Acknowledgements

We'd love to deploy these tools on your hardware! If you have a simulator and would like to use to to do tomography, let us know!

This material was funded in part by the U.S. Department of Energy, Office of Science, Office of Advanced Scientific Computing Research's Early Career Research Program and Testbed Pathfinder Program.

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