



From pulses to circuits and back again

A quantum control perspective on variational quantum algorithms

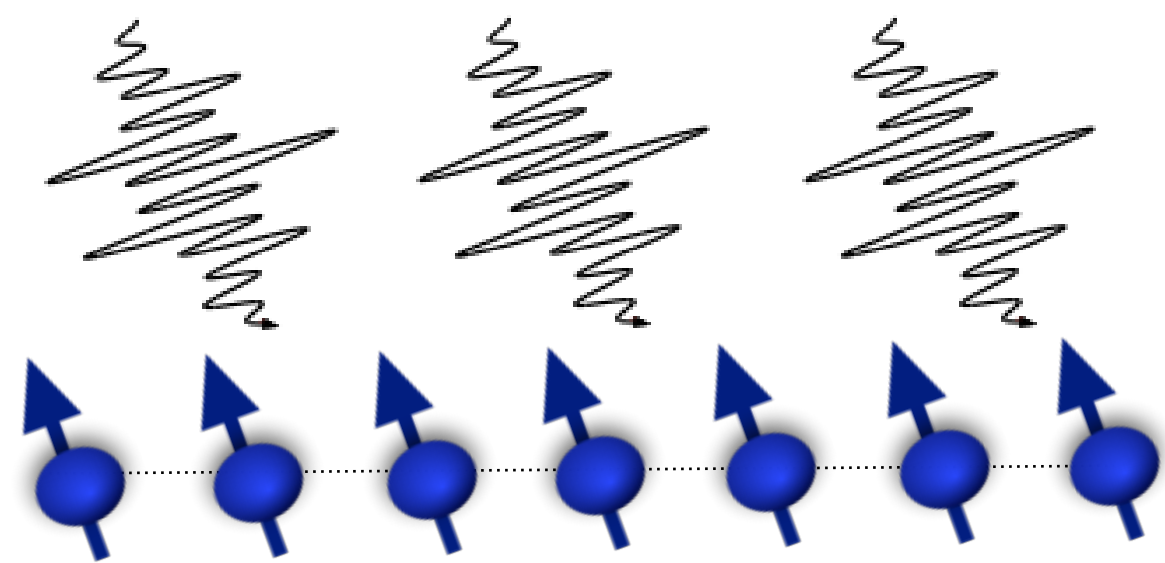
Alicia B Magann, Christian Arenz, Matthew D. Grace, Tak-San Ho, Robert L. Kosut, Jarrod R. McClean, Herschel A. Rabitz, and Mohan Sarovar



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Quantum computing

Early years: pulse-level advances



Published: 29 April 1999

Coherent control of macroscopic quantum states in a single-Cooper-pair box

[Y. Nakamura](#) ✉, [Yu. A. Pashkin](#) & [J. S. Tsai](#)

Nature **398**, 786–788 (1999) | [Cite this article](#)

Quantum control

Published: 30 October 2003

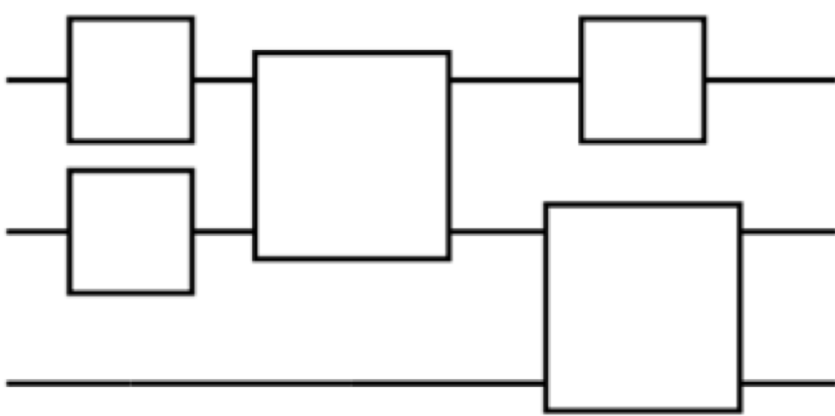
Demonstration of conditional gate operation using superconducting charge qubits

[T. Yamamoto](#) ✉, [Yu. A. Pashkin](#), [O. Astafiev](#), [Y. Nakamura](#) & [J. S. Tsai](#)

Nature **425**, 941–944 (2003) | [Cite this article](#)

AB Magann*, C Arenz*, MD Grace, TS Ho, RL Kosut, JR McClean, HA Rabitz, M Sarovar, “From pulses to circuits and back again: A quantum optimal control perspective on variational quantum algorithms”, *PRX Quantum* (2021); arXiv:2009.06702

More recently: circuit-level advances



Variational quantum algorithms

Open Access | Published: 23 July 2014

A variational eigenvalue solver on a photonic quantum processor

[Alberto Peruzzo](#) ✉, [Jarrod McClean](#), [Peter Shadbolt](#), [Man-Hong Yung](#), [Xiao-Qi Zhou](#), [Peter J. Love](#), [Alán Aspuru-Guzik](#) ✉ & [Jeremy L. O’Brien](#) ✉

Nature Communications **5**, Article number: 4213 (2014) | [Cite this article](#)

arXiv.org > quant-ph > arXiv:1411.4028

Quantum Physics

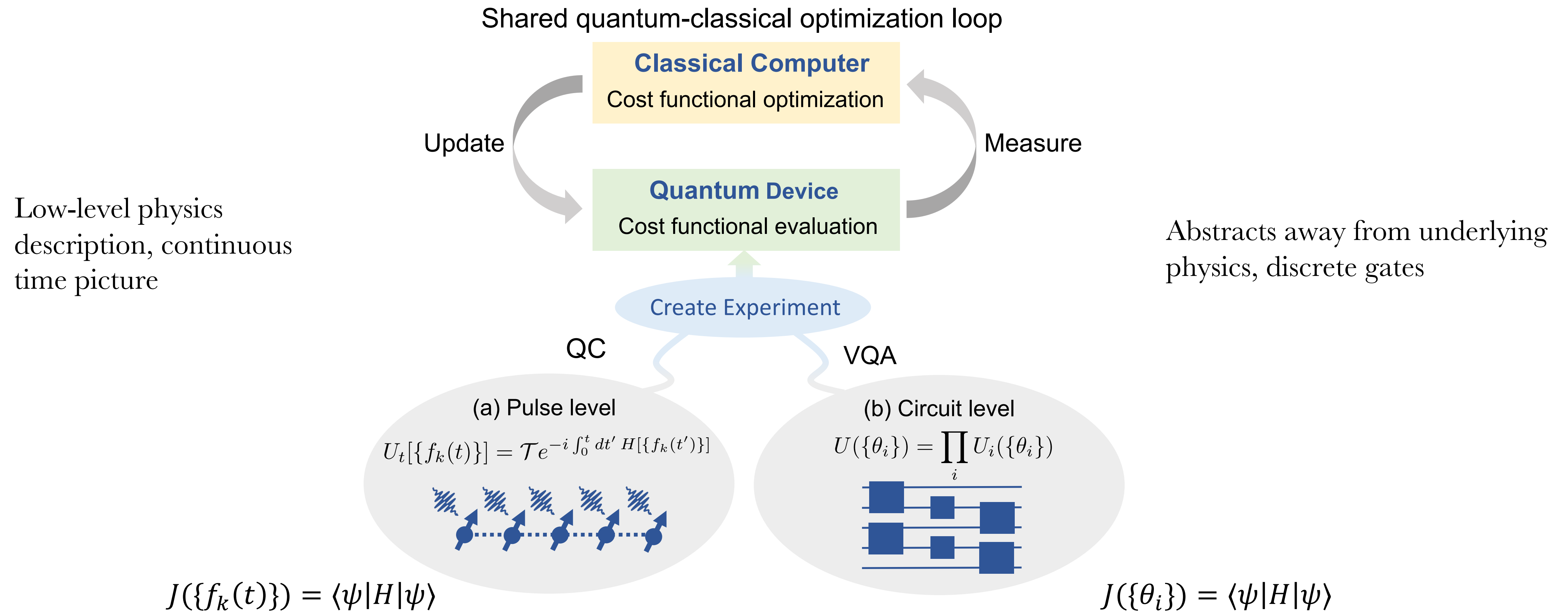
[Submitted on 14 Nov 2014]

A Quantum Approximate Optimization Algorithm

[Edward Farhi](#), [Jeffrey Goldstone](#), [Sam Gutmann](#)

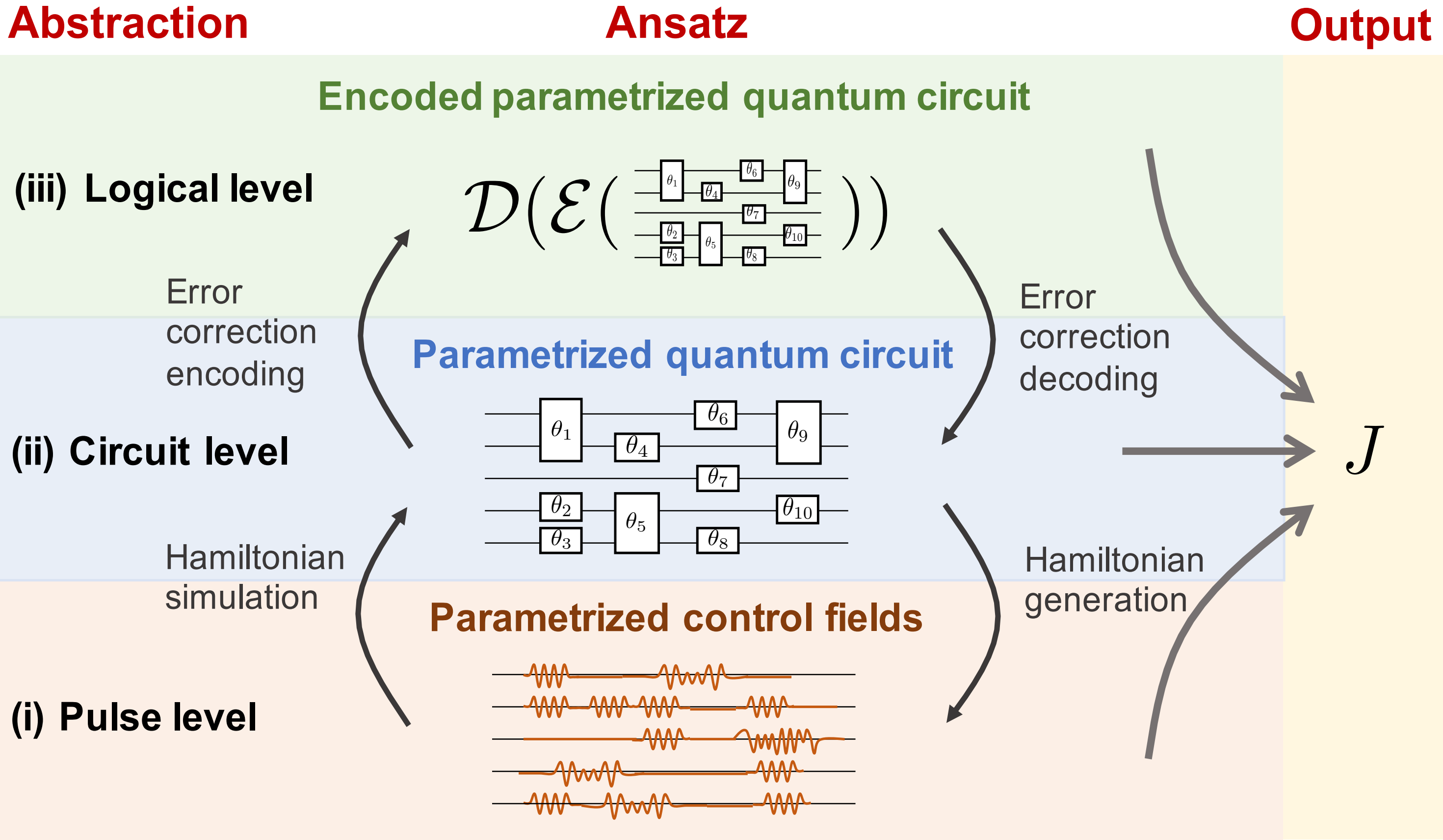
“From pulses to circuits and back again”

Connections between quantum control & variational quantum algorithms



Potential crossover areas

Different levels in a hierarchy of abstractions



➤ Algorithm design

➤ Noise mitigation

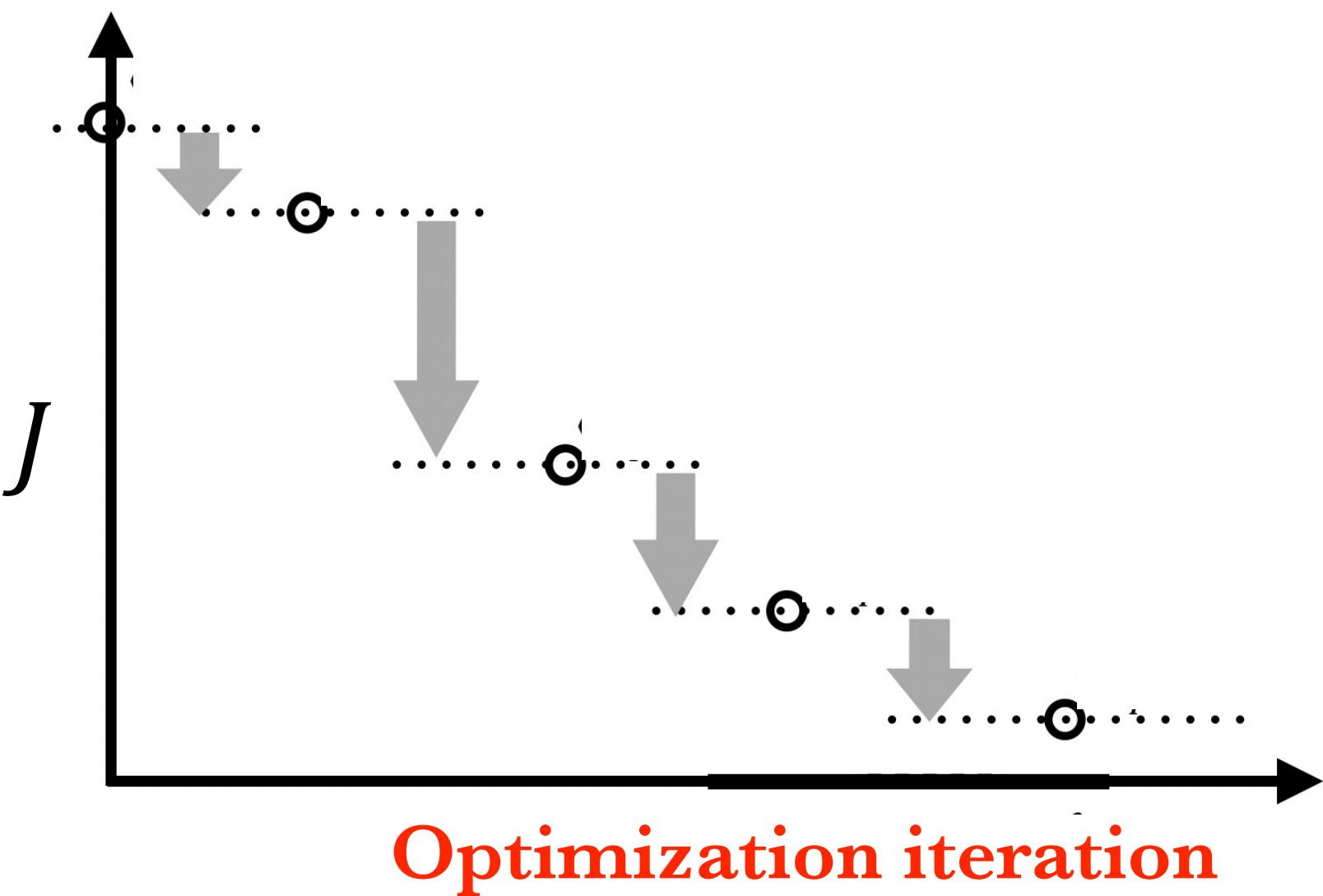
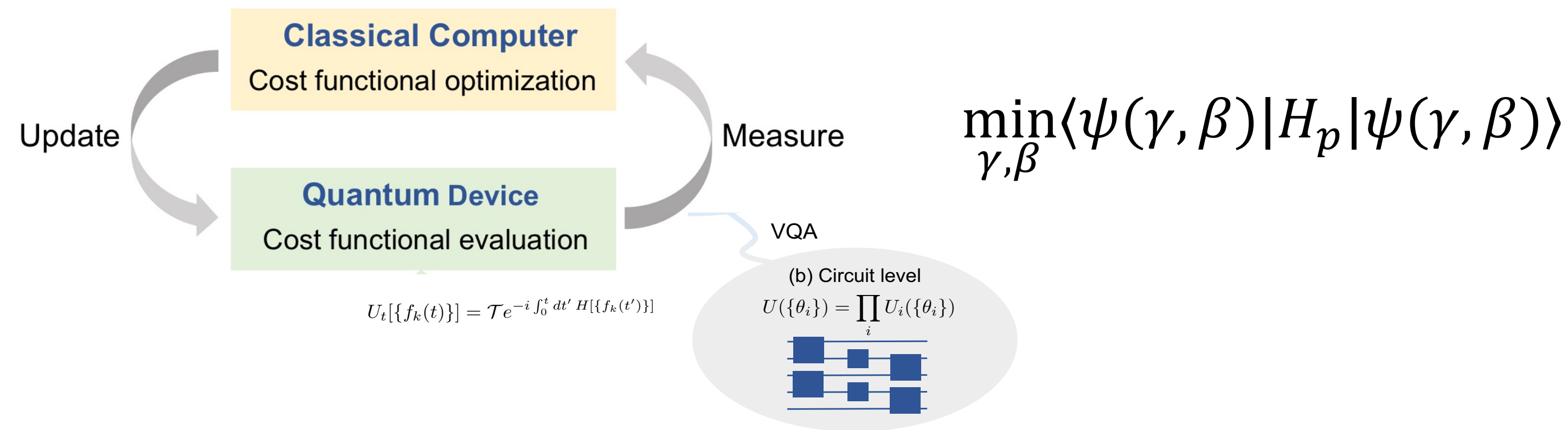
➤ Optimization landscape analysis

AB Magann*, C Arenz*, MD Grace, TS Ho, RL Kosut, JR McClean, HA Rabitz, M Sarovar, “From pulses to circuits and back again: A quantum optimal control perspective on variational quantum algorithms”, *PRX Quantum* (2021); arXiv:2009.06702

Case study:

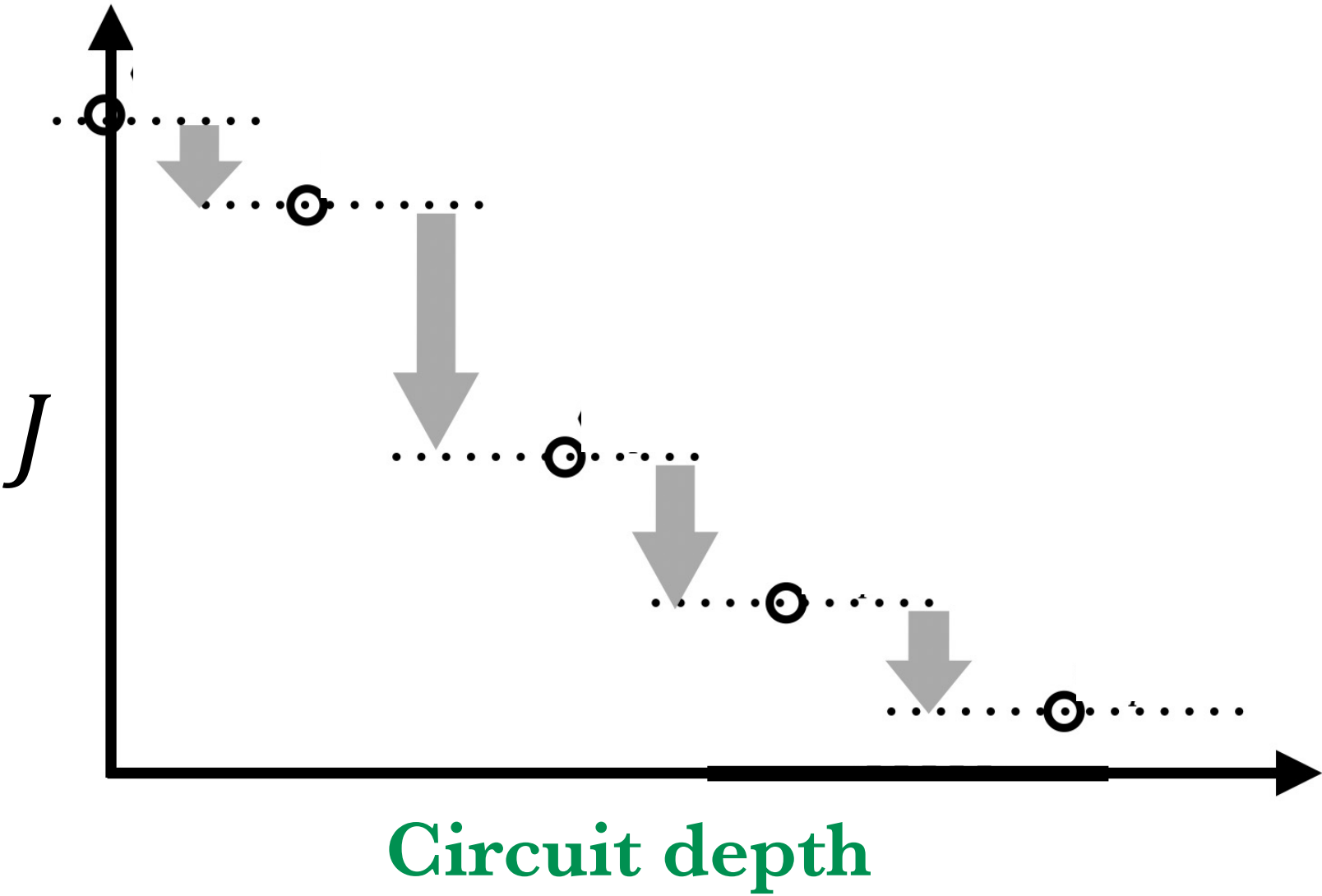
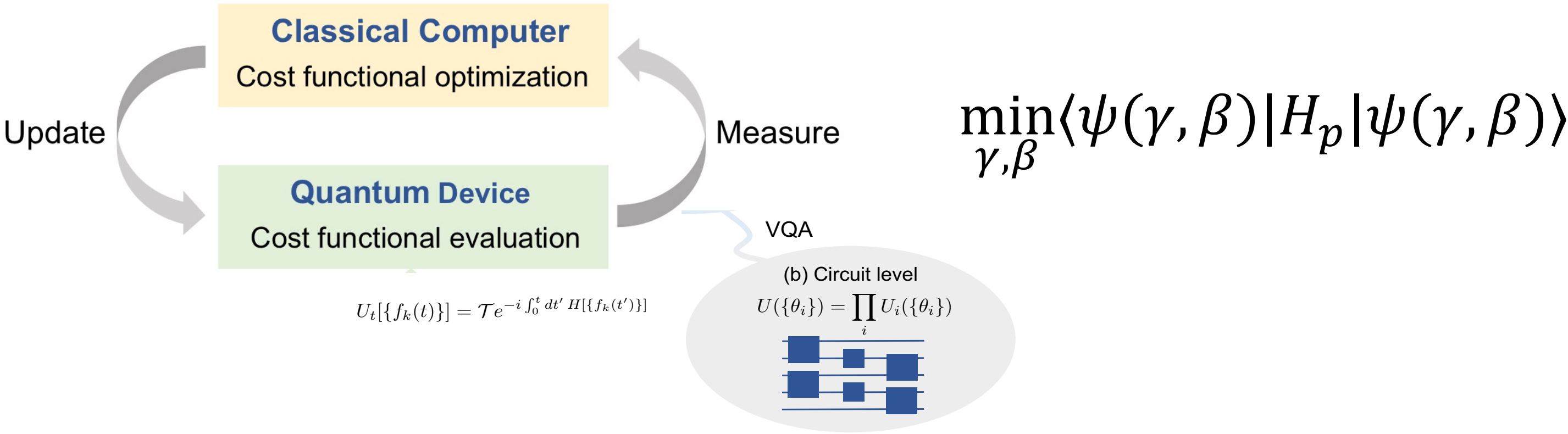
A quantum Lyapunov control-inspired quantum
algorithm for combinatorial optimization

Background: Quantum approximate optimization algorithm (QAOA)



Bottleneck: searching for the optimal (γ, β) parameters

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Bottleneck: searching for the optimal (γ, β) parameters

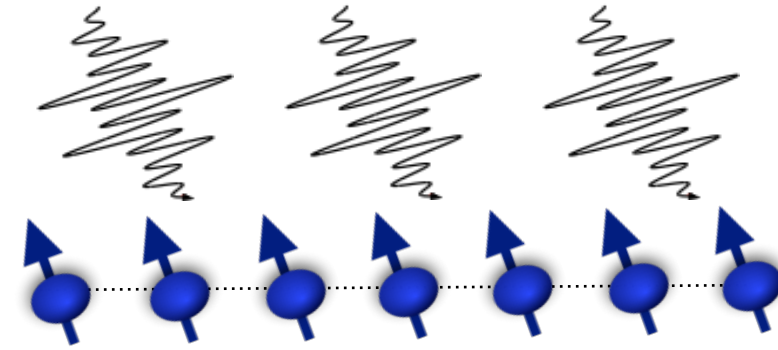
This case study: a constructive parameter setting protocol that does not require classical optimization & is inspired by quantum control theory

AB Magann, KM Rudinger, MD Grace, M Sarovar, “Feedback-based quantum optimization”: arXiv:2103.08619 (2021)

AB Magann, KM Rudinger, MD Grace, M Sarovar, “Lyapunov control-inspired strategies for quantum combinatorial optimization”: arXiv:2108.05945 (2021)

Lyapunov control-inspired algorithm for quantum optimization

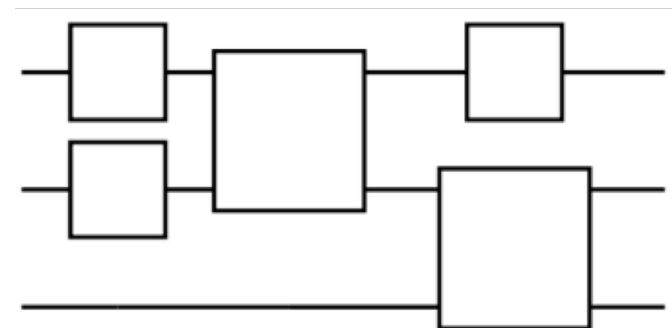
Pulse-level control perspective



The Lyapunov control principle is to design a control function $\beta(t)$ to ensure the cost function is strictly decreasing over time by enforcing

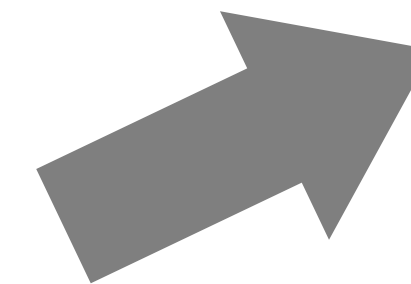
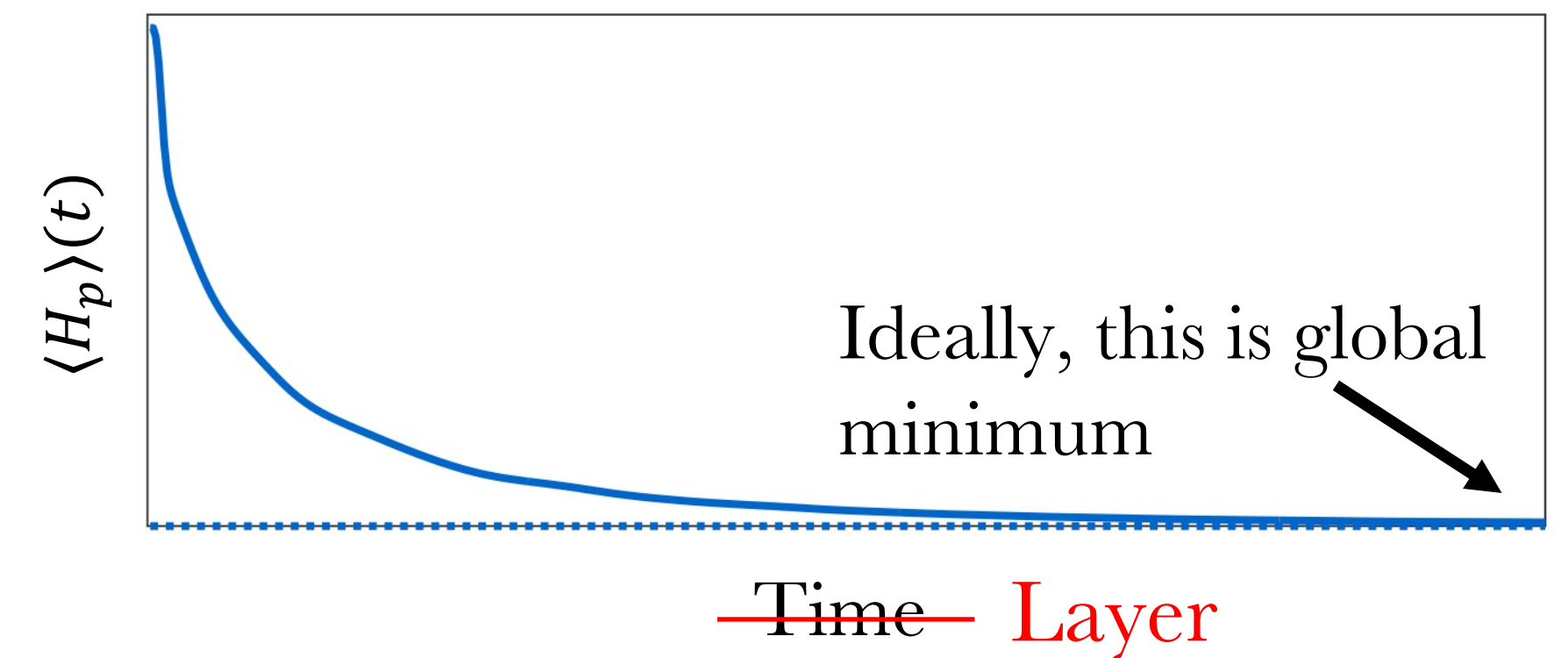
$$\frac{d}{dt} \langle H_p \rangle(t) \leq 0, \forall t$$

Circuit-level perspective

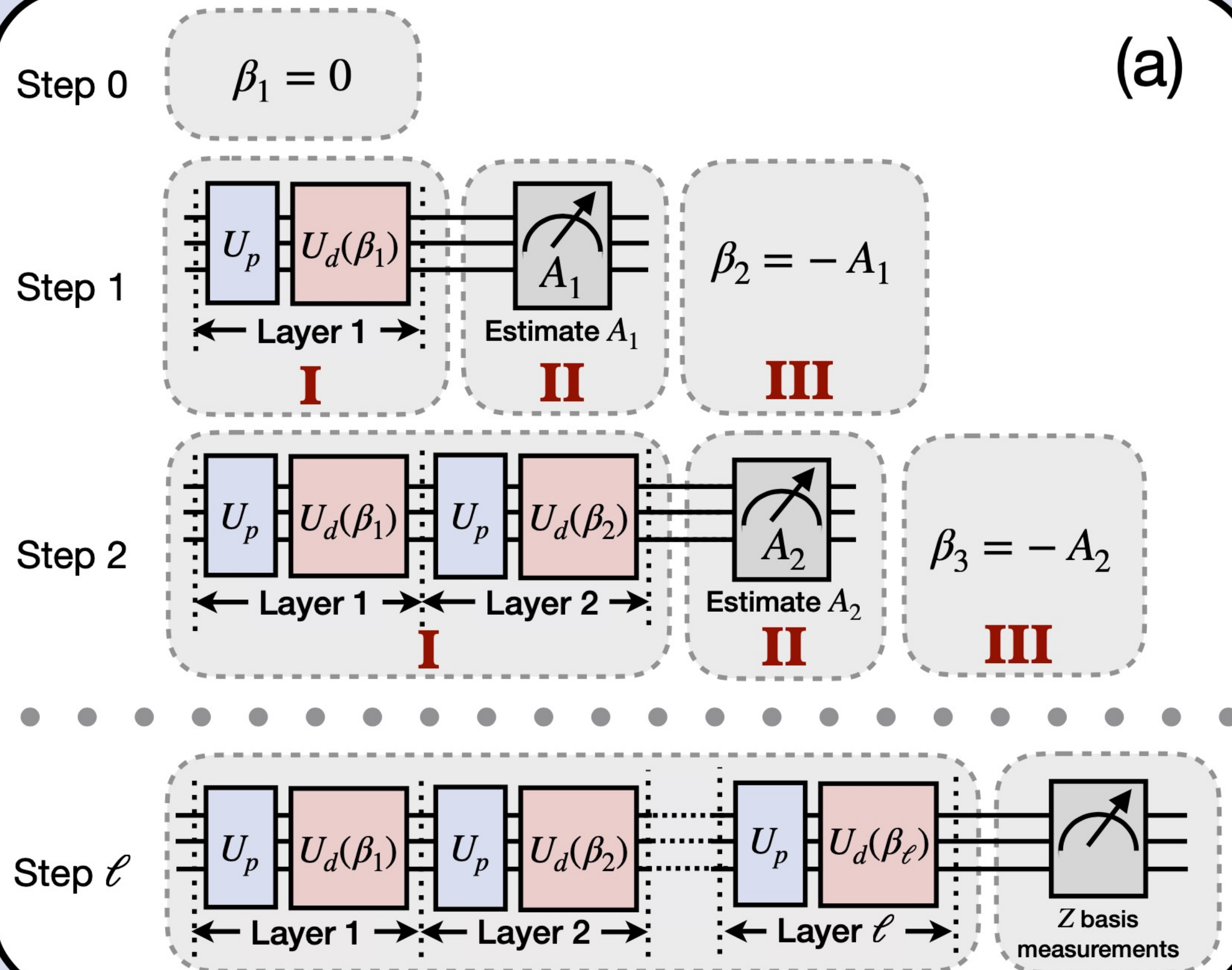


We define control law for $\beta(t)$, and then develop Trotterization-inspired quantum algorithm, with circuits akin to QAOA

Illustration of the principle



Lyapunov control-inspired algorithm for quantum optimization

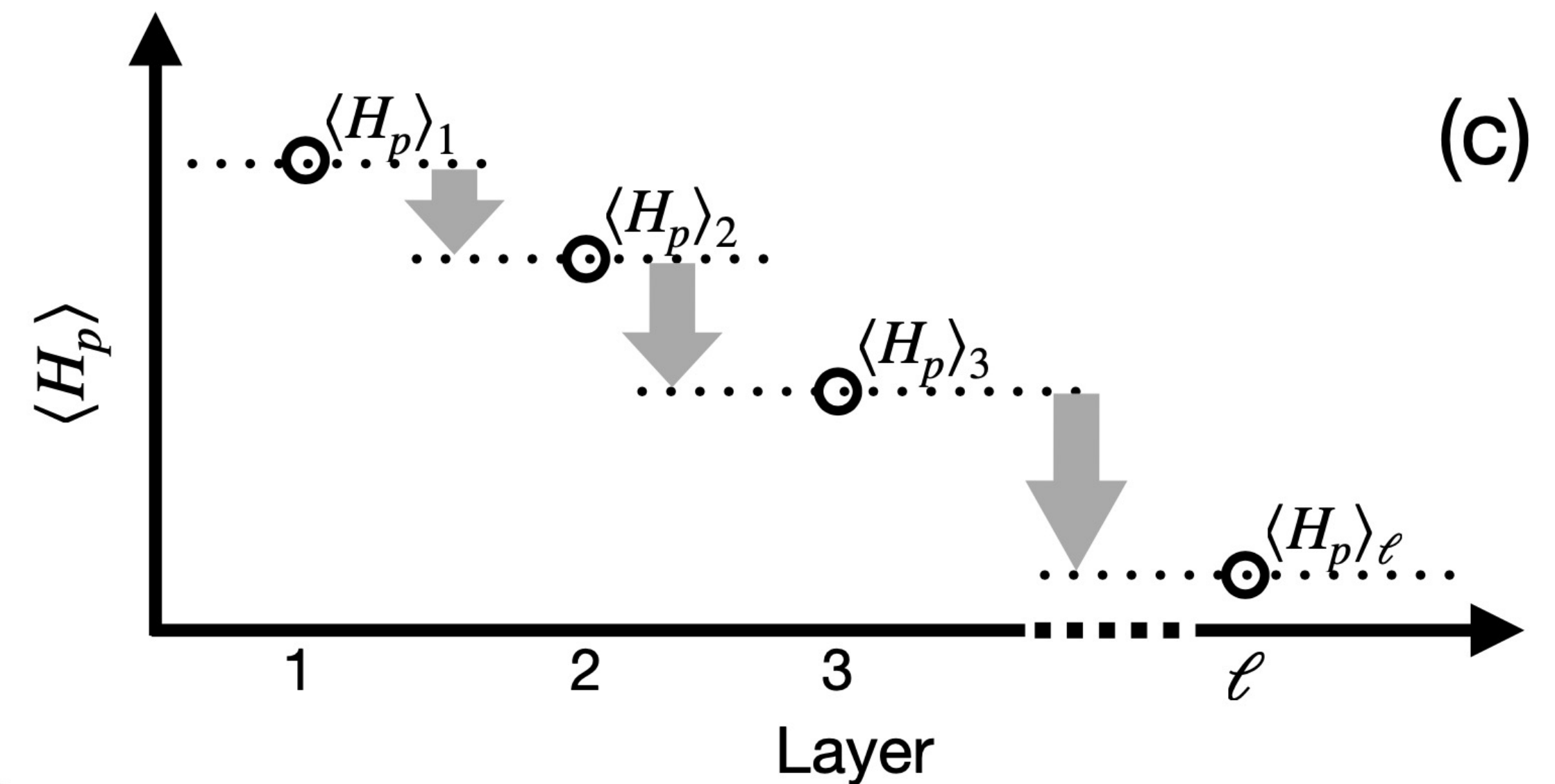


Step k

I Prepare qubits in state $|\psi_k\rangle = U_d(\beta_k)U_p \cdots U_d(\beta_1)U_p |\psi_0\rangle$

II Measure qubits to estimate $A_k = \langle \psi_k | i[H_d, H_p] | \psi_k \rangle$

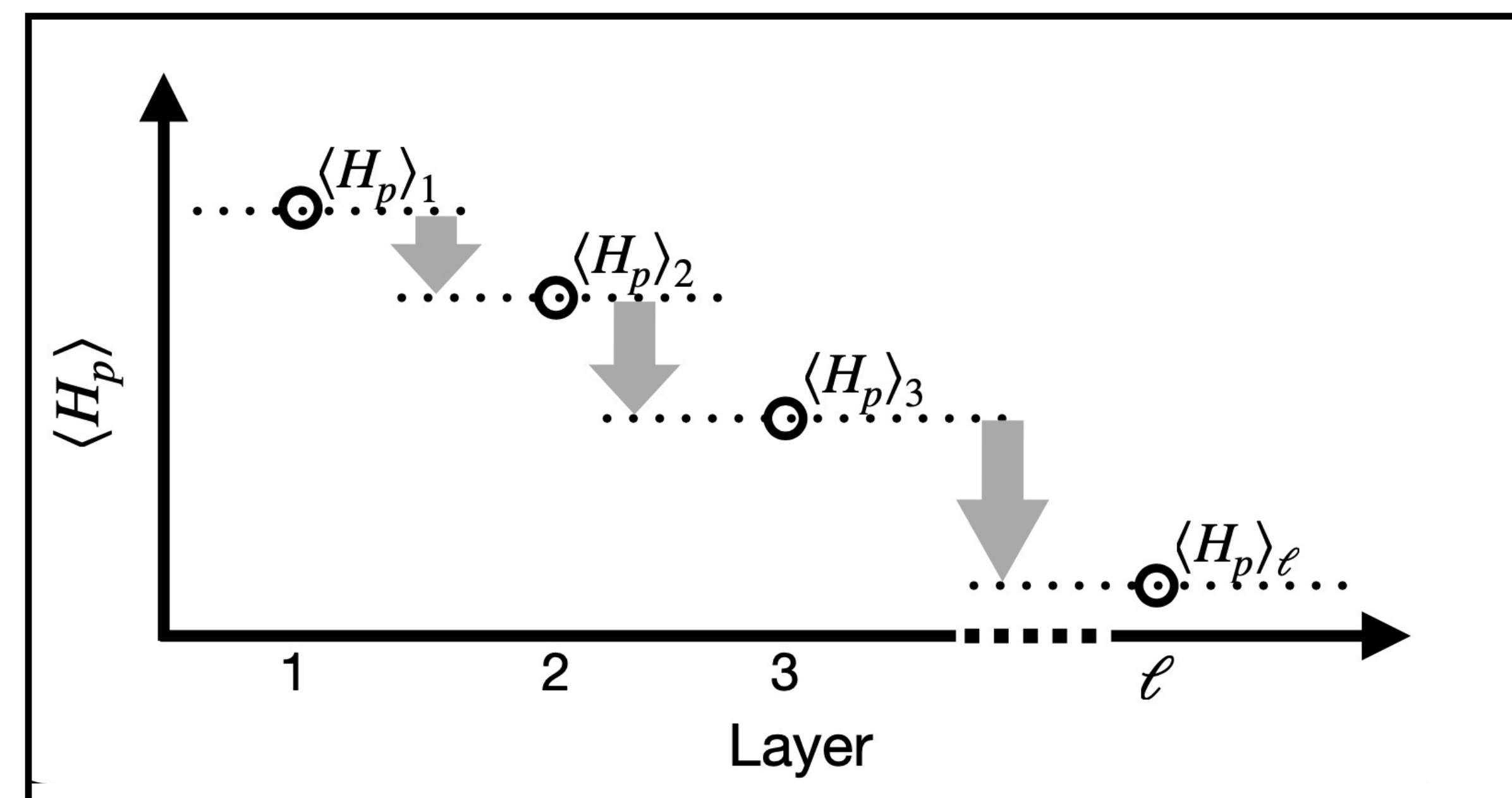
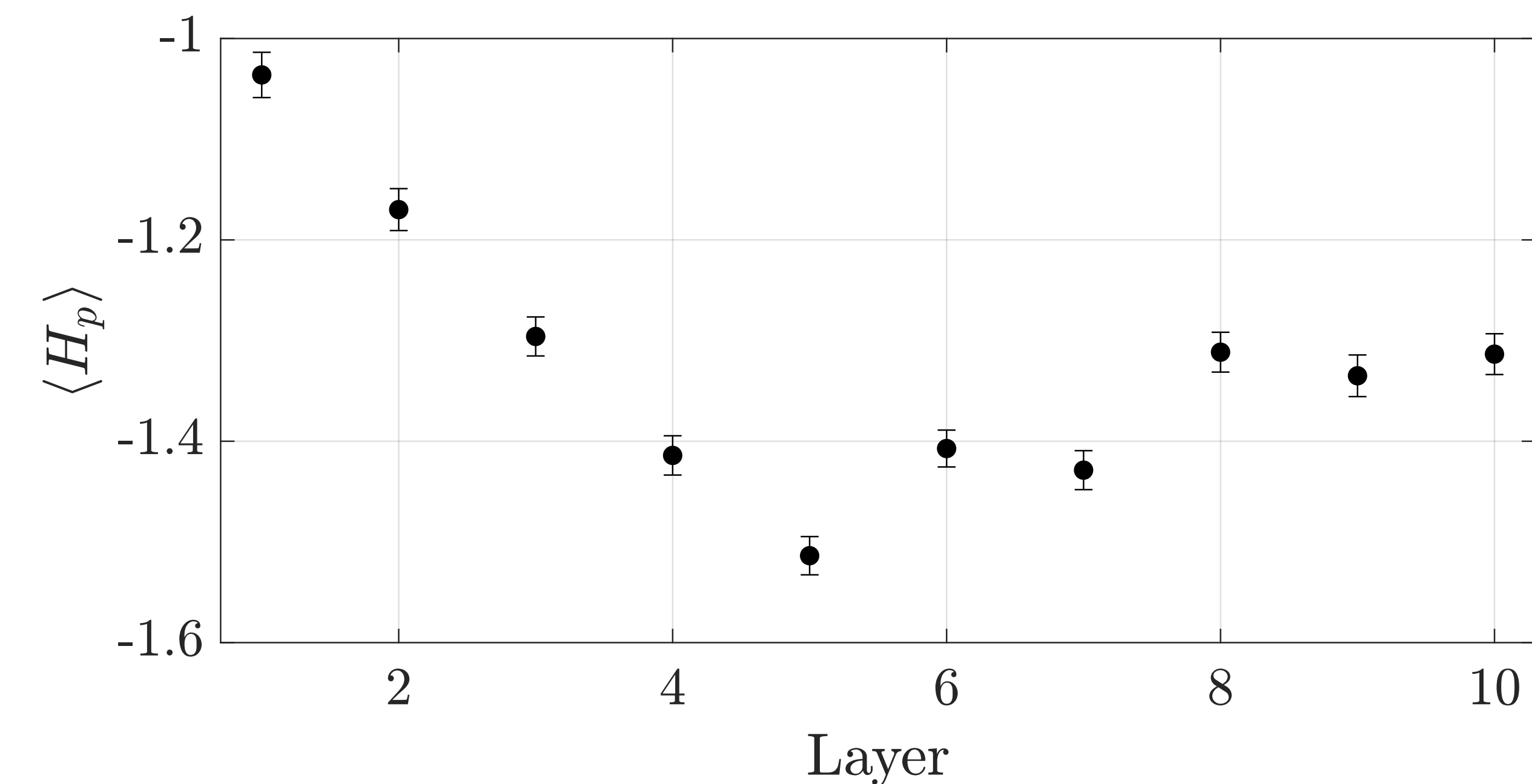
III Set $\beta_{k+1} = -A_k$



Lyapunov control-inspired algorithm for quantum optimization

Hardware demonstration

Implementation of our protocol to solve a 3 qubit instance of MaxCut on the ibmq_manila processor



Summary

- Discussed connections and analogies between (pulse-level) quantum control and (circuit-level) variational quantum algorithms
- Outlined potential crossover areas: noise, new ansätze, and optimization landscapes
- Described a Lyapunov control-based algorithm for quantum optimization as an example of quantum control/quantum algorithm crossover

