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Precise, detailed error budgets for 2-qubit gates from gate set tomography

Robin Blume-Kohout,

Kenny Rudinger, Erik Nielsen, Kevin Young, Tim Proctor

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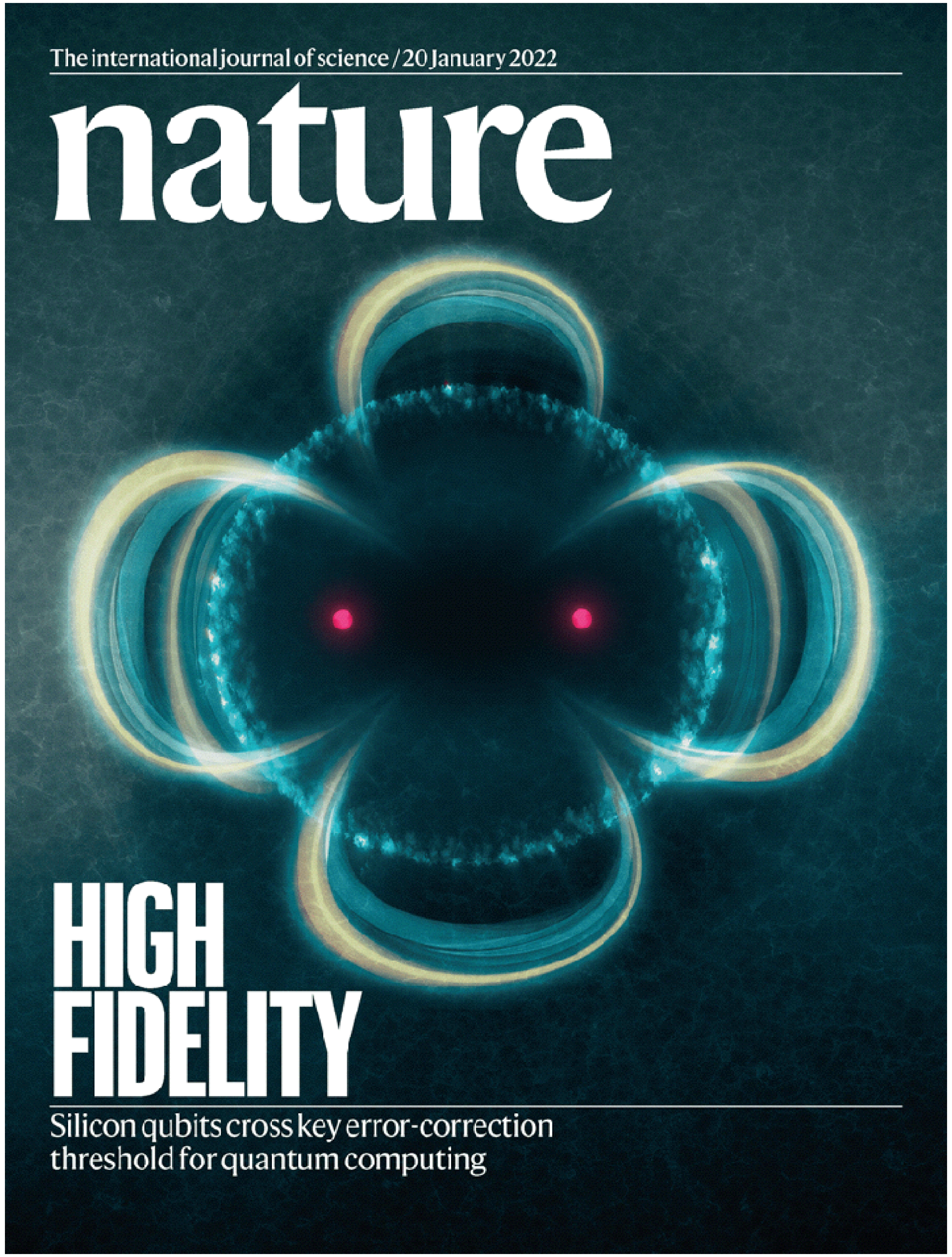

Article

Precision tomography of a three-qubit donor quantum processor in silicon

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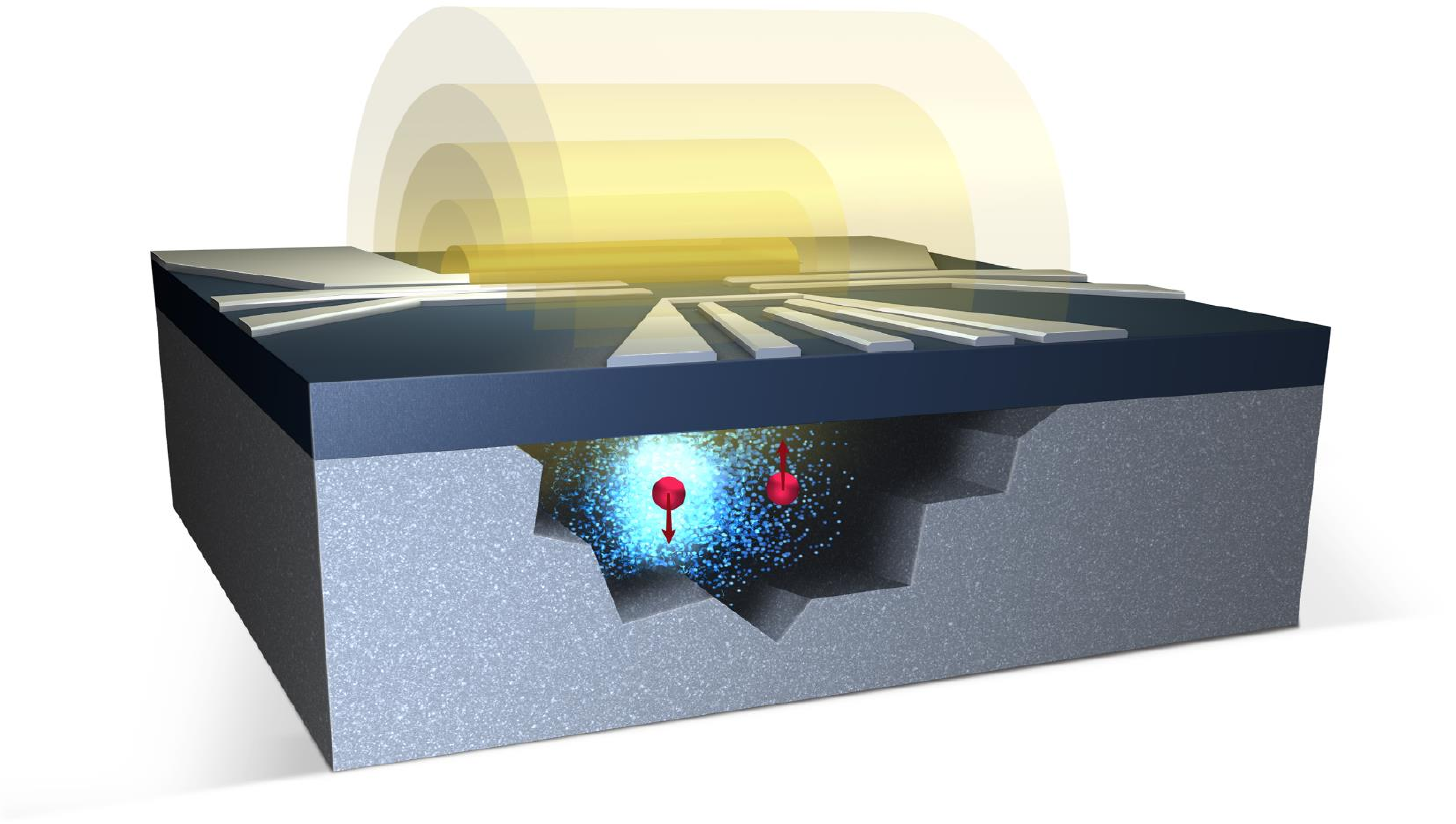
Mateusz T. Mądzik[1,9,12], Serwan Asaad[1,10,12], Akram Youssry[2,3], Benjamin Joecker[1], Kenneth M. Rudinger[4], Erik Nielsen[4], Kevin C. Young[5], Timothy J. Proctor[5], Andrew D. Baczewski[6], Arne Laucht[1,2], Vivien Schmitt[1,11], Fay E. Hudson[1], Kohei M. Itoh[7], Alexander M. Jakob[8], Brett C. Johnson[8], David N. Jamieson[8], Andrew S. Dzurak[1], Christopher Ferrie[2], Robin Blume-Kohout[4] & Andrea Morello[1]✉



M. T. Mądzik *et al*, *Precision tomography of a three-qubit electron-nuclear quantum processor in silicon*. Nature **601**, 348–353 (January 2022).

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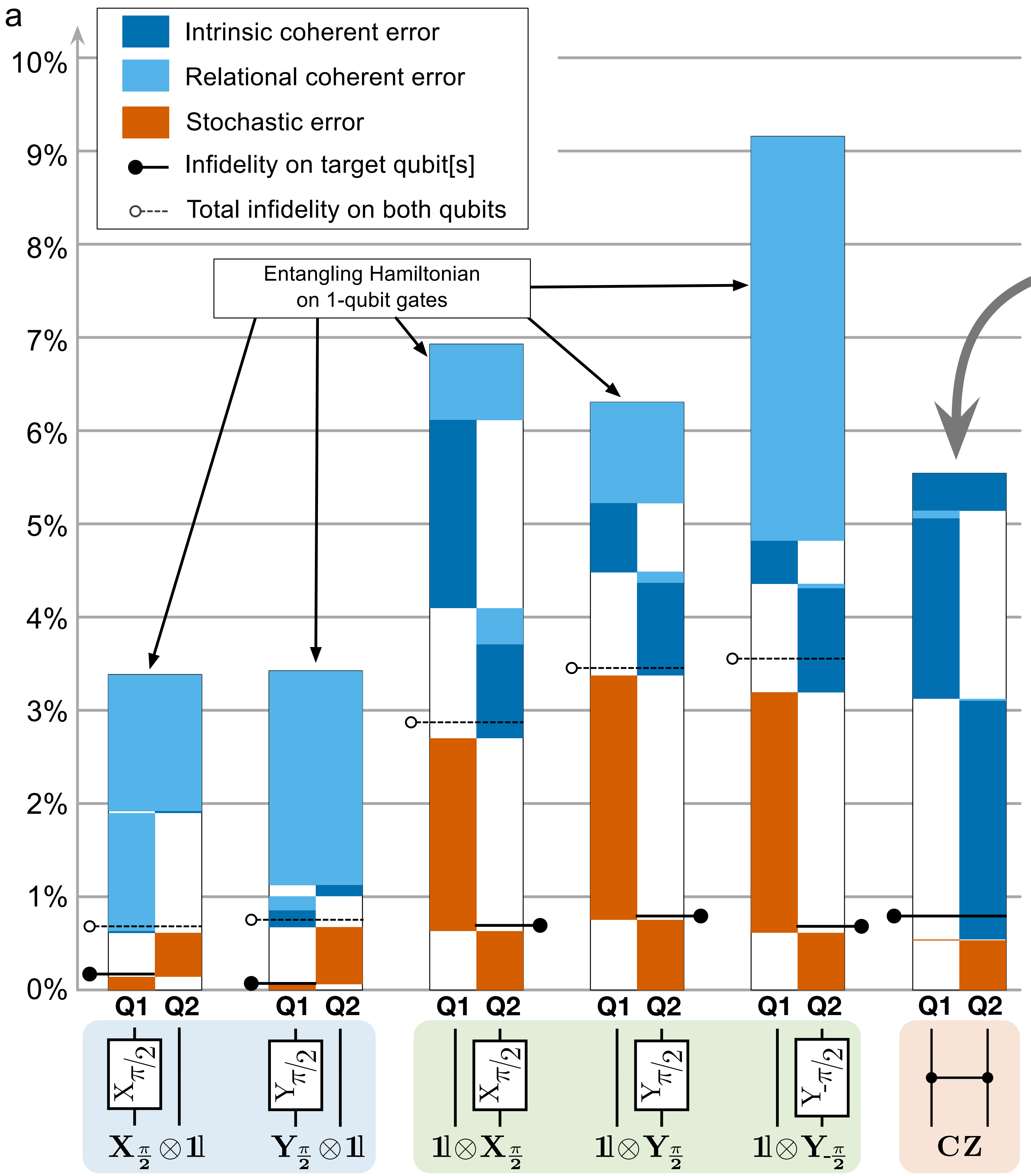

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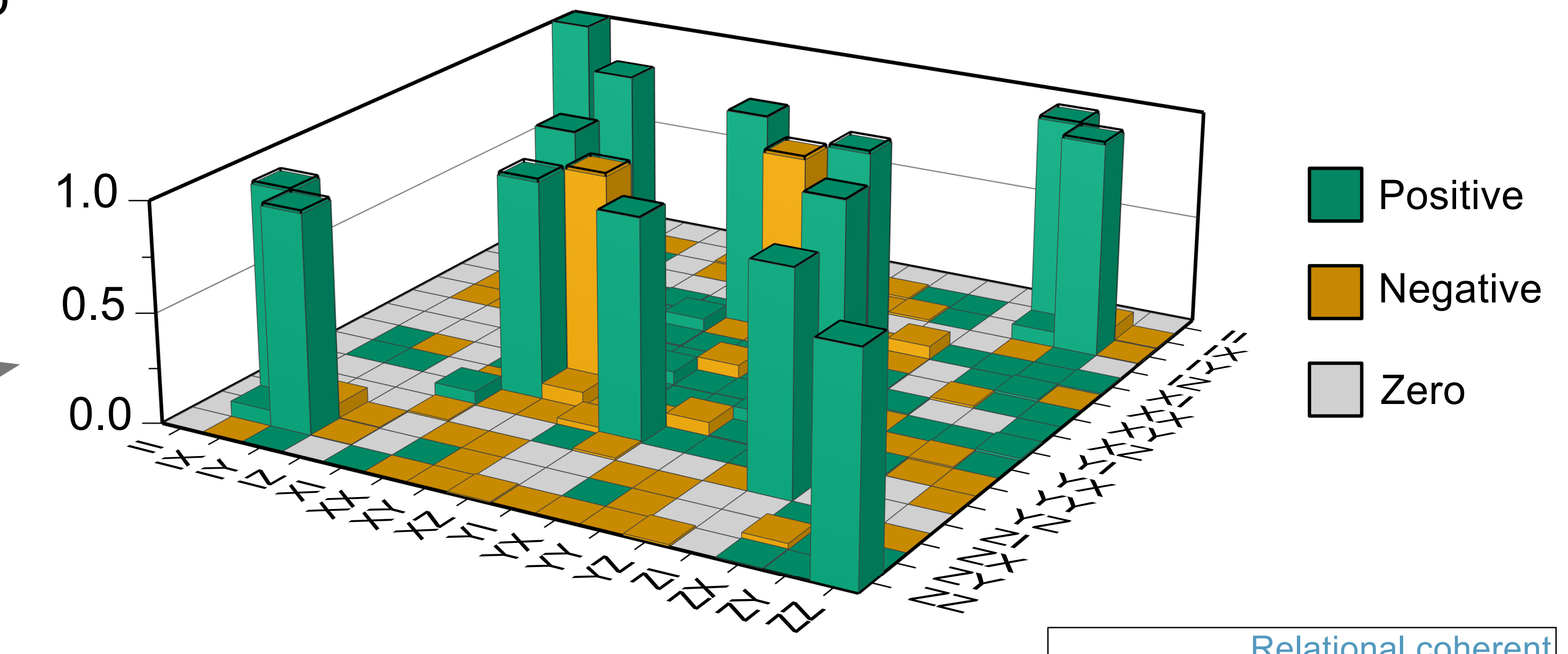

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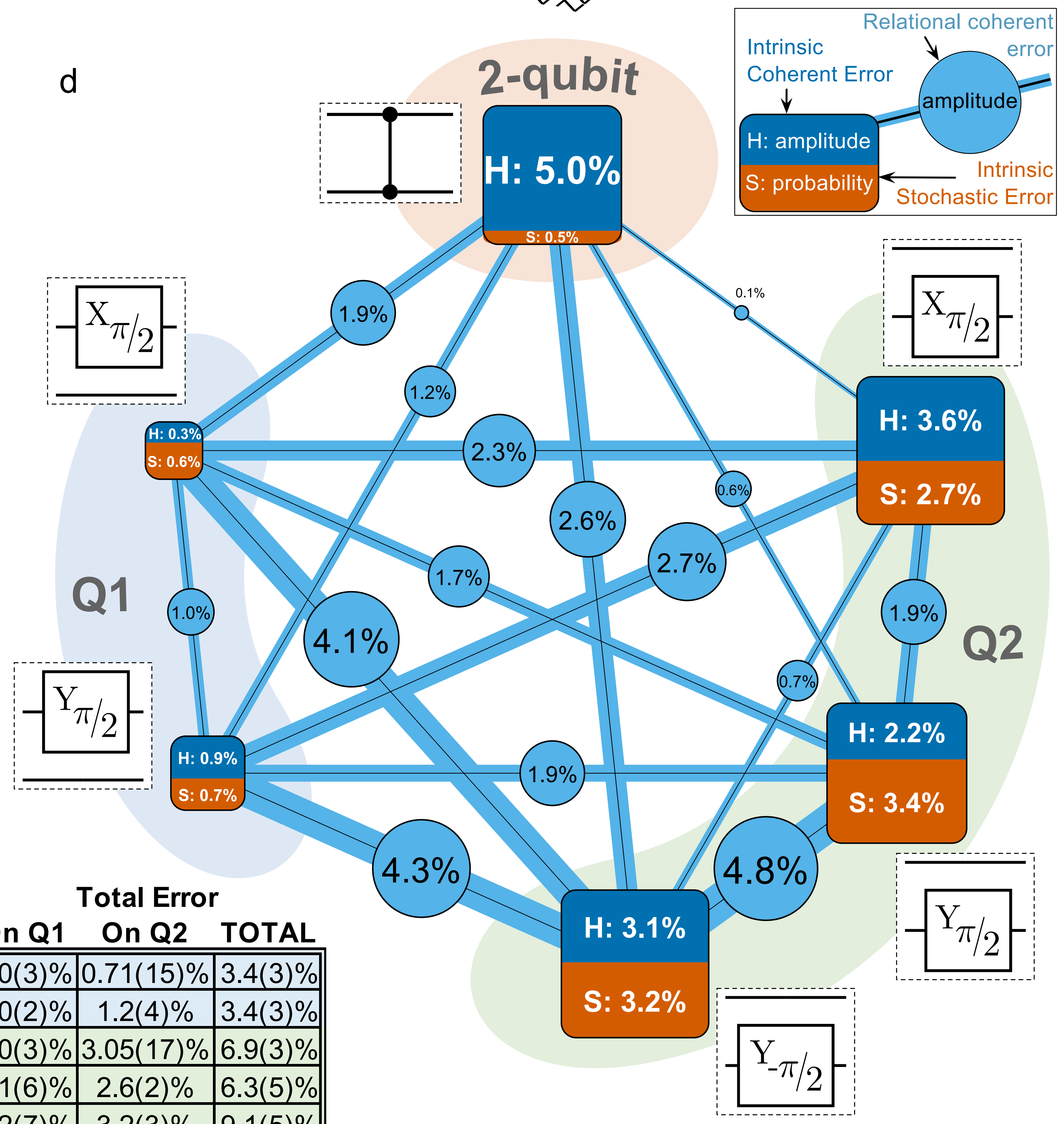
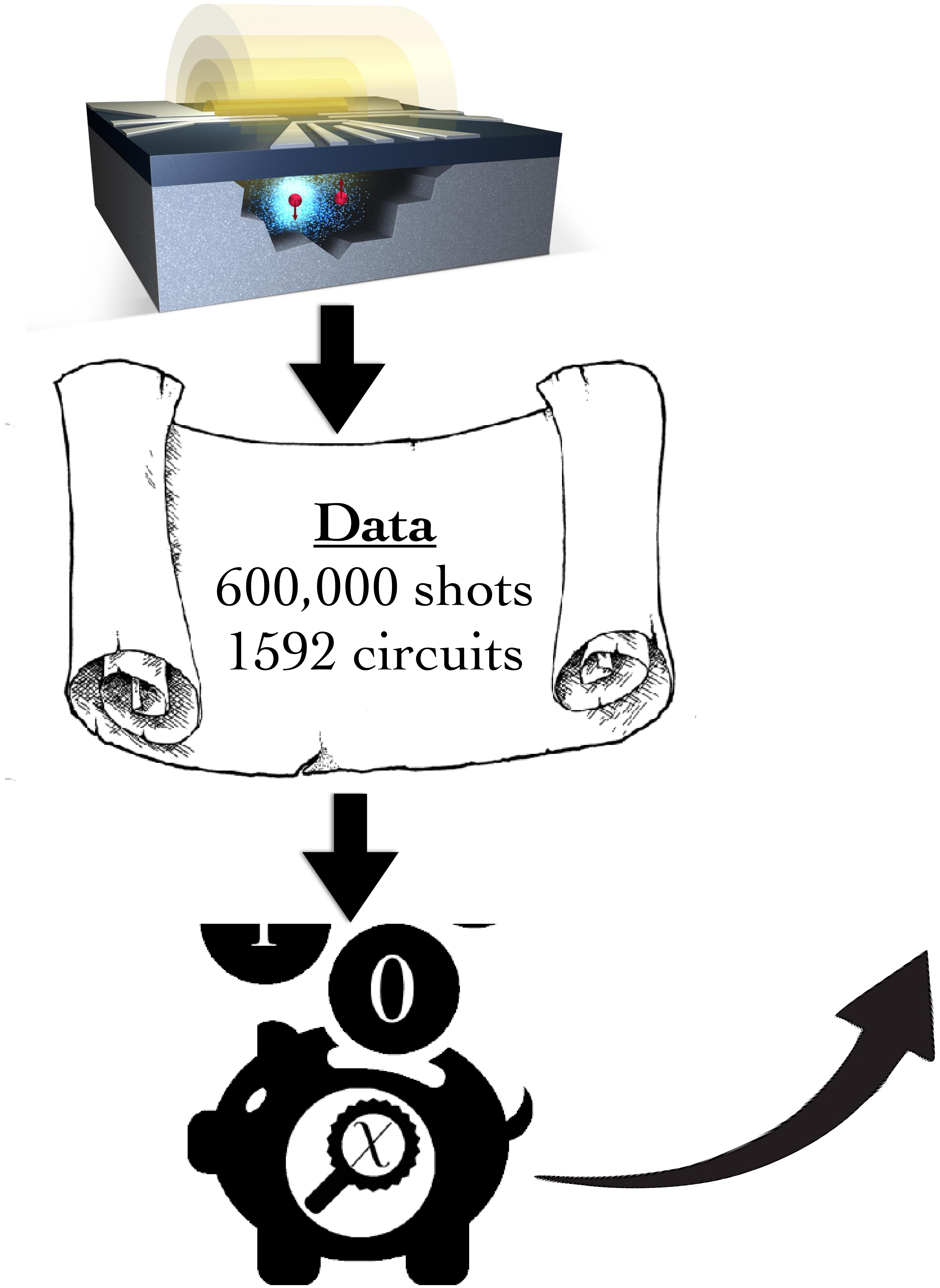
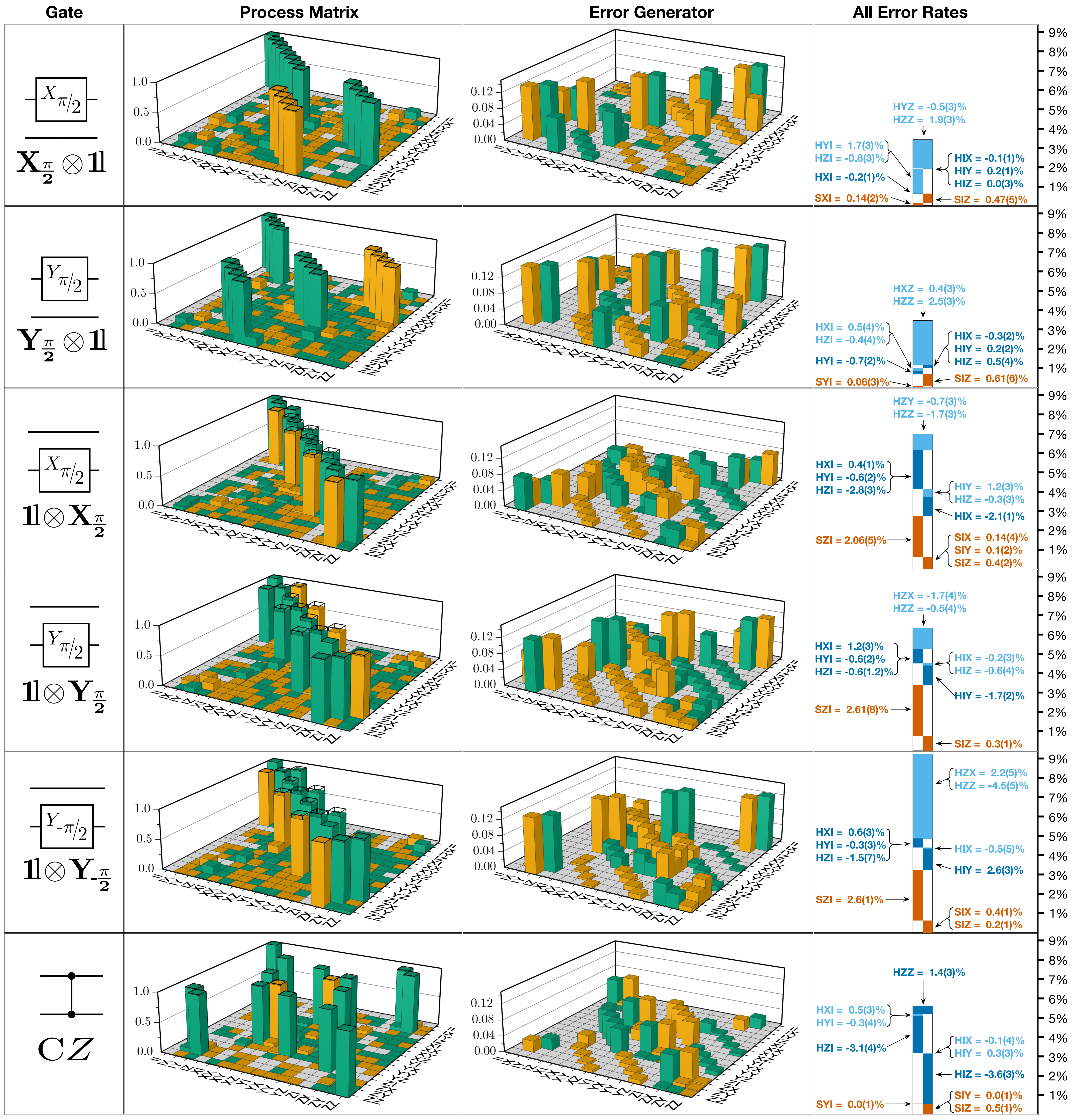


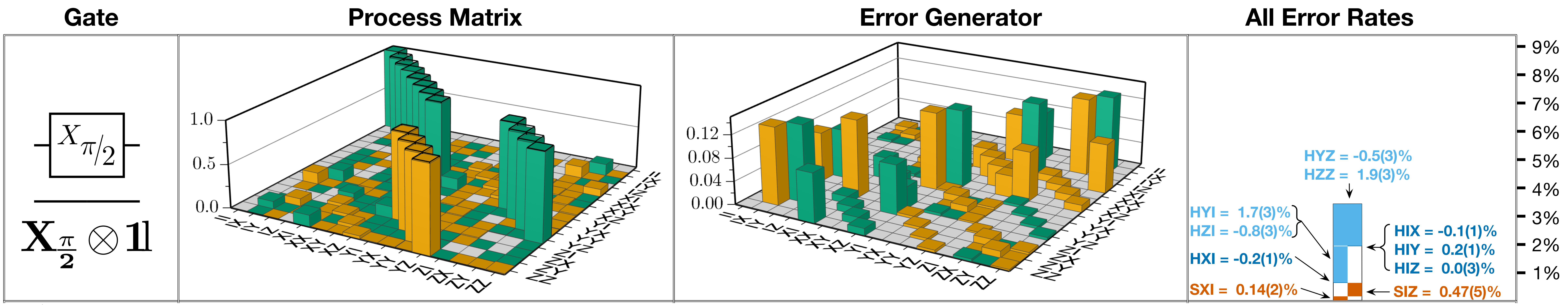
c

Gate	On-target Average Gate Fidelity	Generator Infidelity On Q1	Generator Infidelity On Q2	Generator Infidelity TOTAL	Stochastic Error	*Intrinsic* Coherent Error	*Relational* Coherent Error	Total Error On Q1	Total Error On Q2	Total Error TOTAL
$X_{\frac{\pi}{2}}\otimes 1\!\!1$	**99.87(2)%**	**0.17(3)%**	0.47(5)%	0.68(6)%	0.60(6)%	0.34(14)%	2.8(3)%	2.0(3)%	0.71(15)%	3.4(3)%
$Y_{\frac{\pi}{2}}\otimes 1\!\!1$	**99.95(2)%**	**0.07(3)%**	0.61(6)%	0.75(6)%	0.67(6)%	0.9(3)%	2.6(3)%	1.0(2)%	1.2(4)%	3.4(3)%
$1\!\!1\otimes X_{\frac{\pi}{2}}$	**99.48(3)%**	2.14(5)%	**0.69(4)%**	2.86(7)%	2.69(6)%	3.6(3)%	2.2(3)%	5.0(3)%	3.05(17)%	6.9(3)%
$1\!\!1\otimes Y_{\frac{\pi}{2}}$	**99.41(5)%**	2.63(8)%	**0.79(6)%**	3.44(10)%	3.36(10)%	2.3(4)%	1.9(4)%	4.1(6)%	2.6(2)%	6.3(5)%
$1\!\!1\otimes Y_{-\frac{\pi}{2}}$	**99.49(5)%**	2.60(13)%	**0.68(7)%**	3.54(16)%	3.18(15)%	3.1(4)%	5.1(5)%	4.2(7)%	3.2(3)%	9.1(5)%
CZ	**99.37(11)%**	0.11(10)%	0.66(11)%	**0.79(14)%**	0.54(13)%	4.9(3)%	0.7(3)%	3.2(4)%	4.1(3)%	5.5(4)%





E. Nielsen *et al*, *Gate set tomography*. Quantum **5**, 557 (October 2021).



Target Gate

$\boldsymbol{G_0}$

Unitary

Acts only on Q1

Estimated Process

$\boldsymbol{G}$

Acts on both qubits

16x16 matrix

240 free parameters

$$G = e^L G_0$$

$$\Downarrow$$

$$L = \log\left(G G_0^{-1}\right)$$

Error Generator

$\boldsymbol{L}$

Isolates *errors* from *target*

Close to $\mathbb{1}$

Error Rates

Components of $\boldsymbol{L}$…

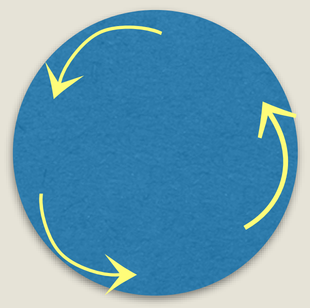
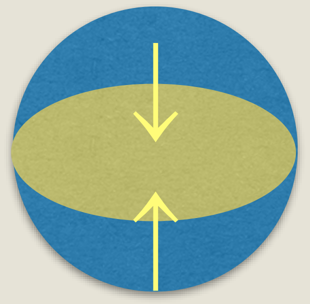
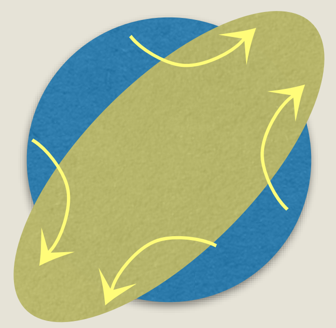
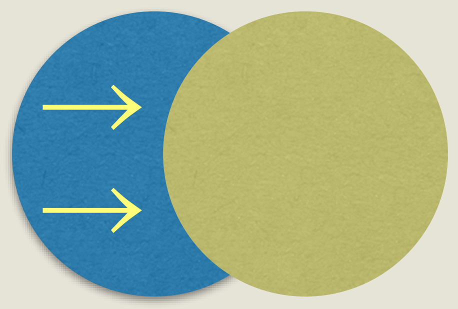
…in a useful basis.

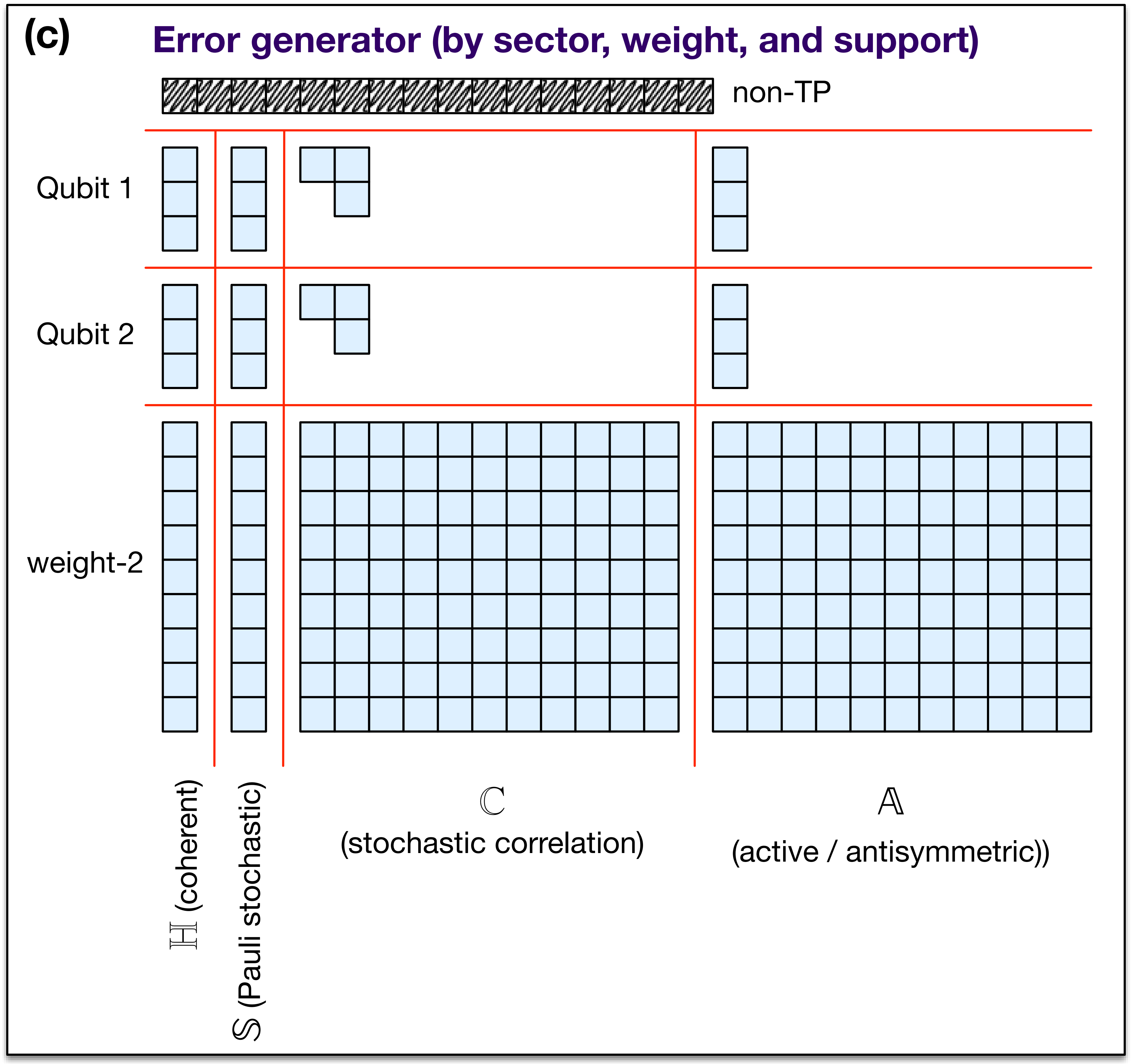
Only 10 of 240 significant!

R. Blume-Kohout *et al*, *A taxonomy of small Markovian errors*. arXiv preprint:2103.01928 (March 2021)

Elementary errors are a useful basis for (arbitrary) error generators

Classify 2Q elementary errors by their *sector* and *support*

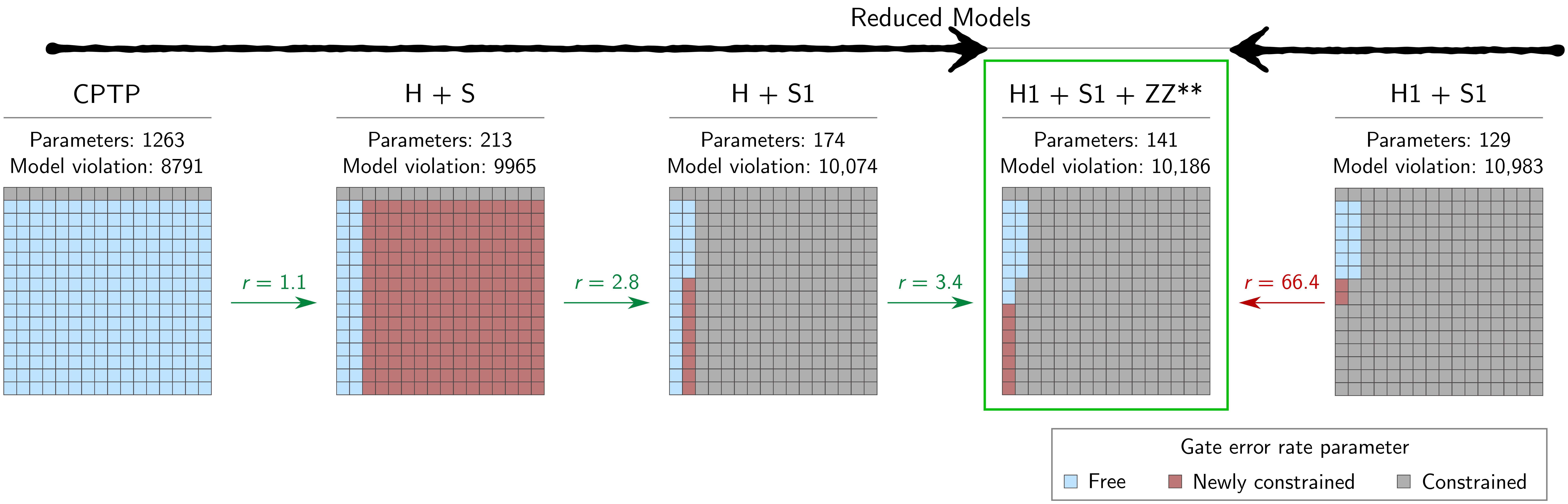
Sector		Dimension	Action	Example effect (Bloch sphere)
Hamiltonian $\mathbb{H}$		$d^2 - 1$	$H_P[\rho] = -i[P, \rho]$	
Stochastic (Pauli) $\mathbb{S}$		$d^2 - 1$	$S_P[\rho] = P\rho P - 1\!\!1\rho 1\!\!1$	
Stochastic (Pauli-correlation) $\mathbb{C}$	{P,Q}=0 [P,Q]=0	$\binom{d^2-1}{2}$	$C_{P,Q}[\rho] = P\rho Q + Q\rho P - \frac{1}{2}\{\{P,Q\},\rho\}$	
Active $\mathbb{A}$	{P,Q}=0 [P,Q]=0	$\binom{d^2-1}{2}$	$A_{P,Q}[\rho] = i\left(P\rho Q - Q\rho P + \frac{1}{2}\{[P,Q],\rho\}\right)$	



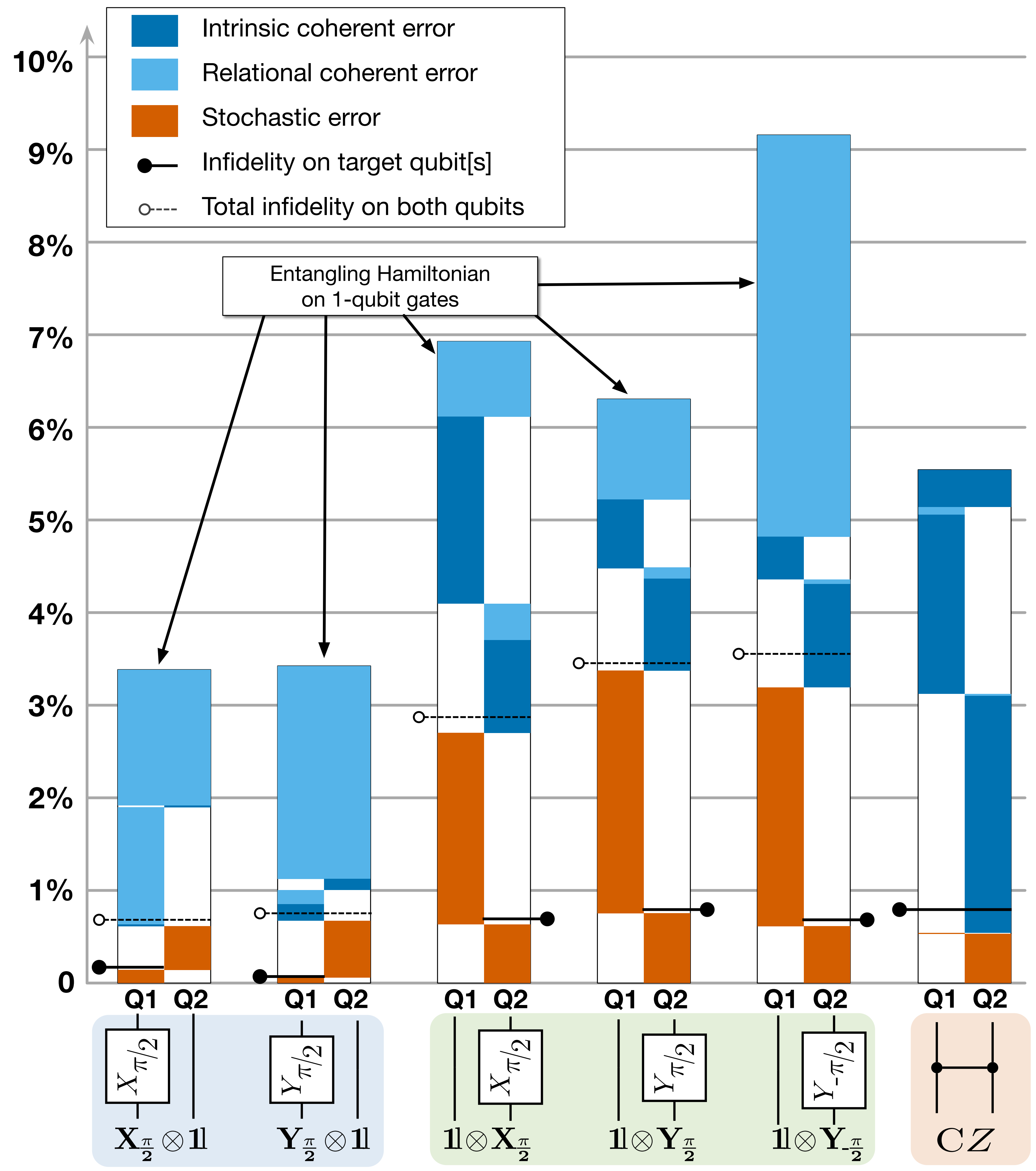
R. Blume-Kohout *et al*, *A taxonomy of small Markovian errors*. arXiv preprint:2103.01928 (March 2021)

A subset of elementary error generators defines a *reduced model* for noisy gates.

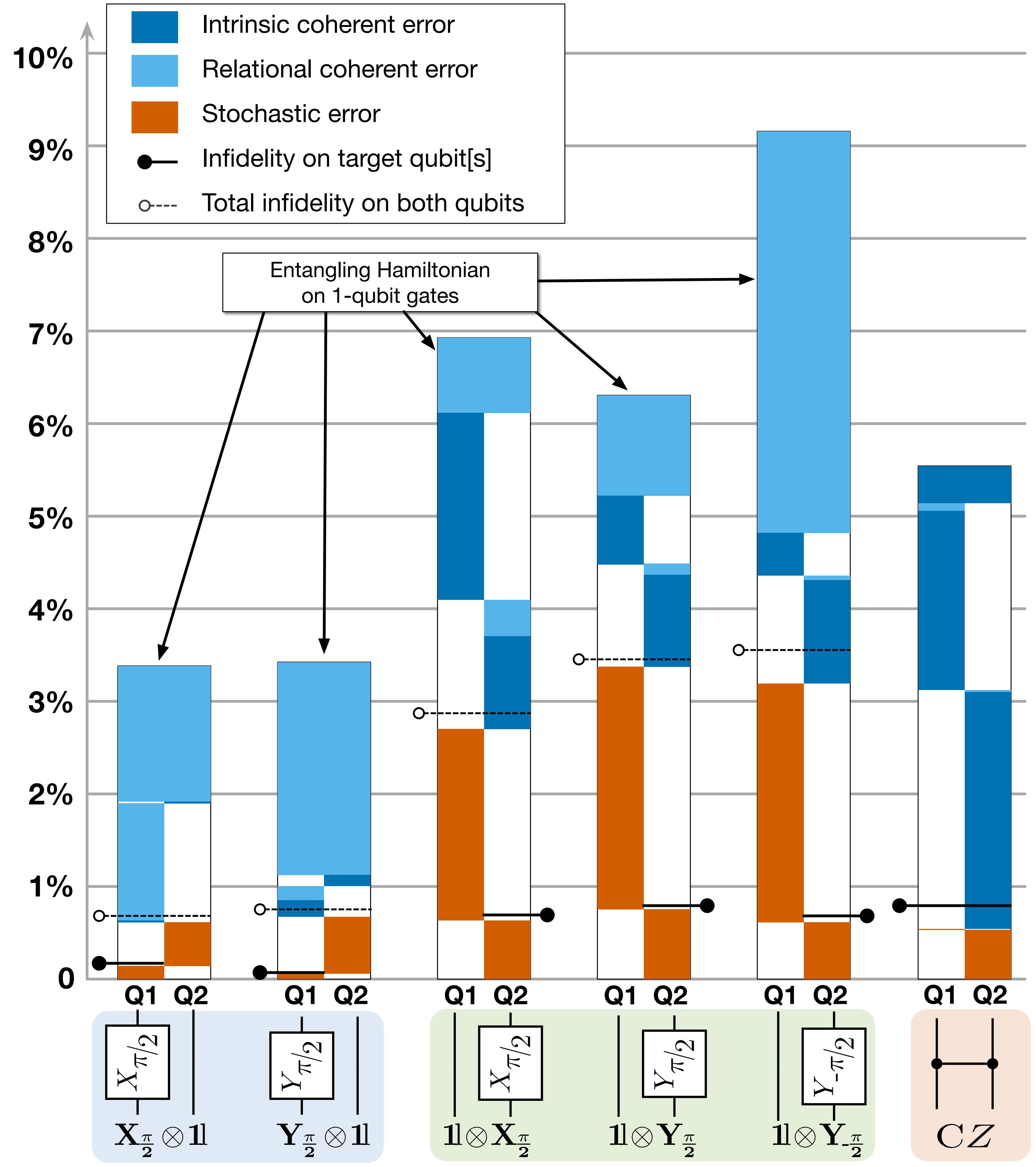
We used *model selection* to identify the smallest model that fit the data well. It only contained (1) local stochastic and (2) local *and* ZZ Hamiltonian errors.



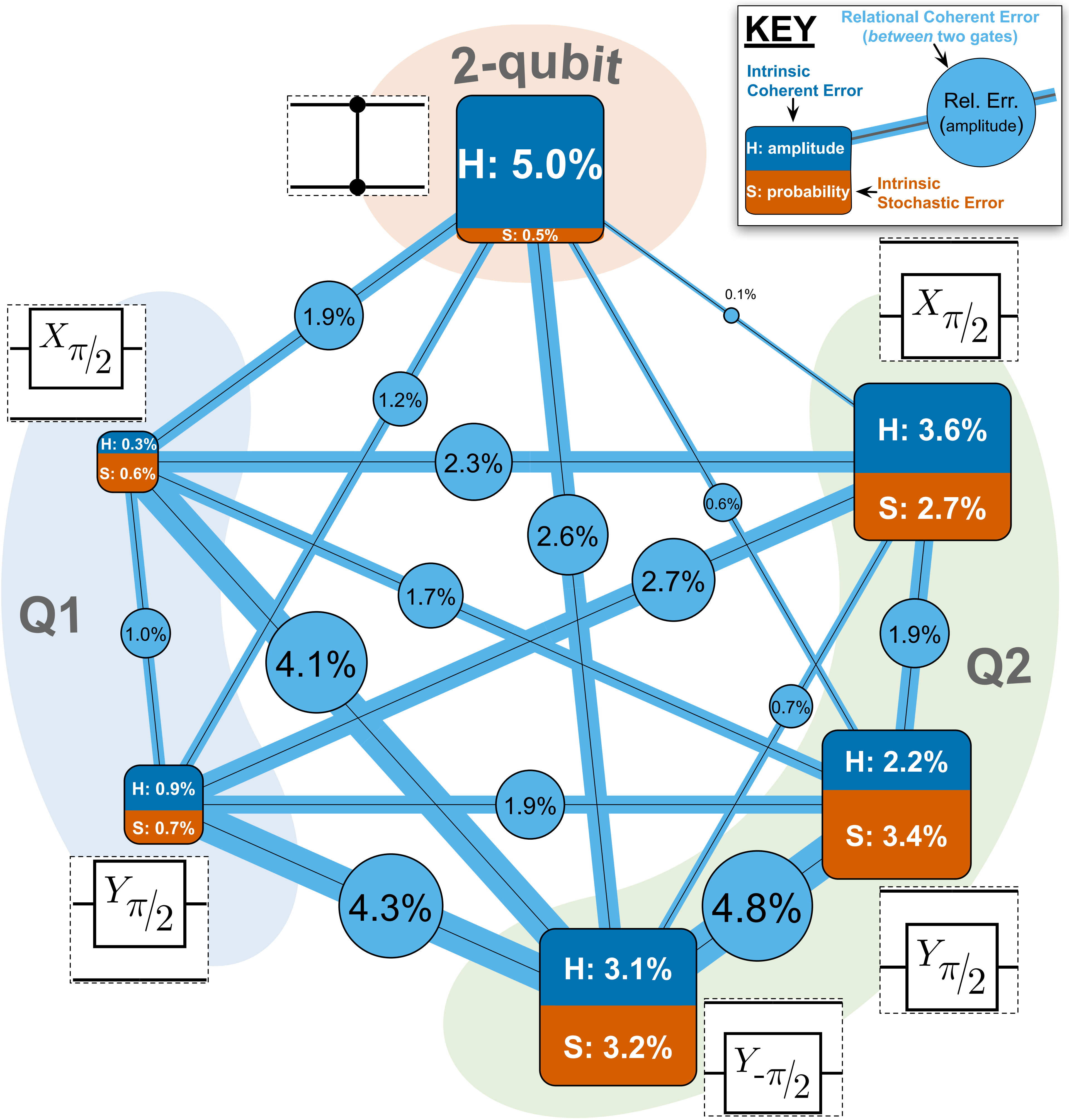
Coefficients of that model yield error rates for each of the 6 gates.

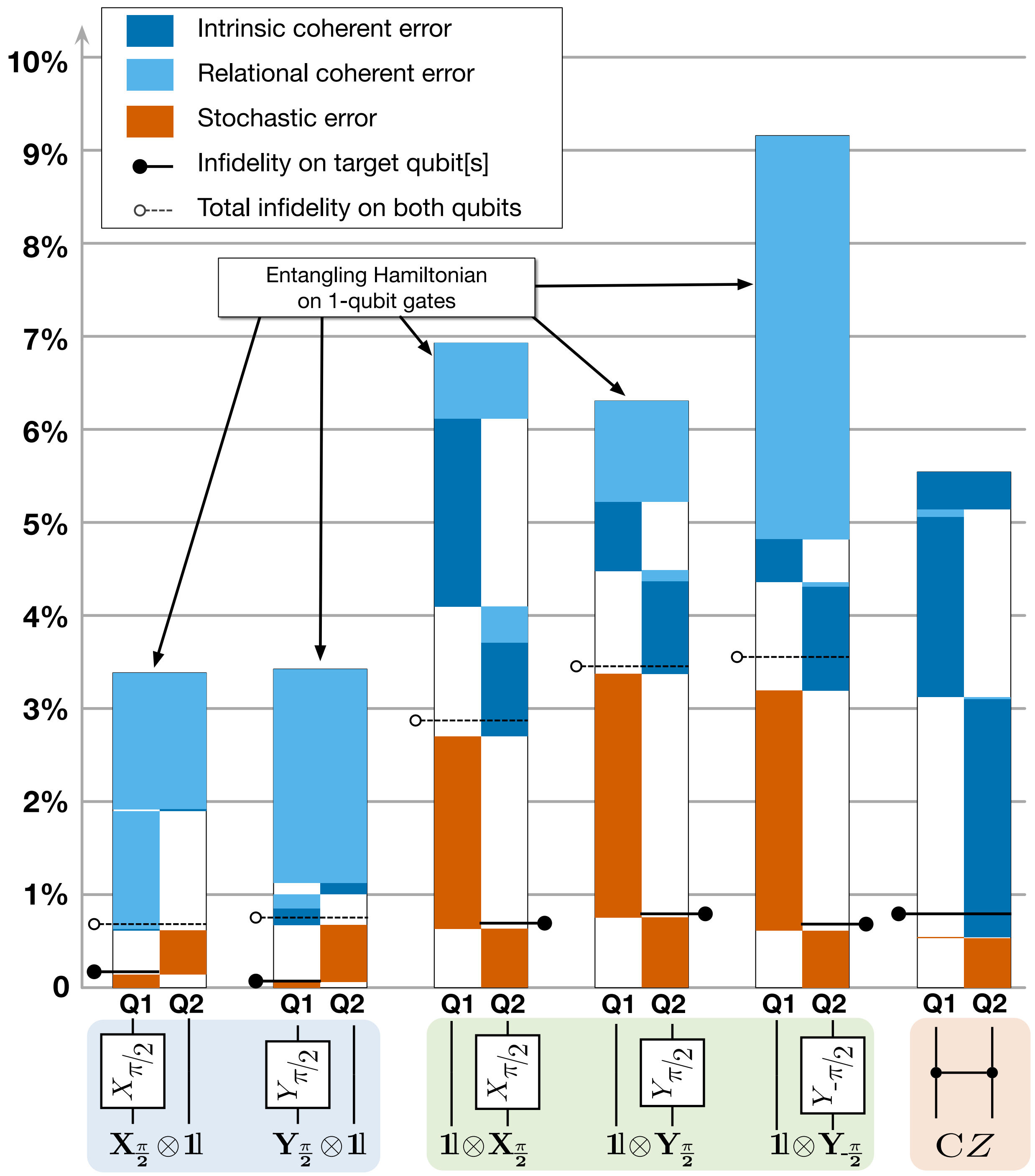


Coefficients of that model yield error rates for each of the 6 gates.



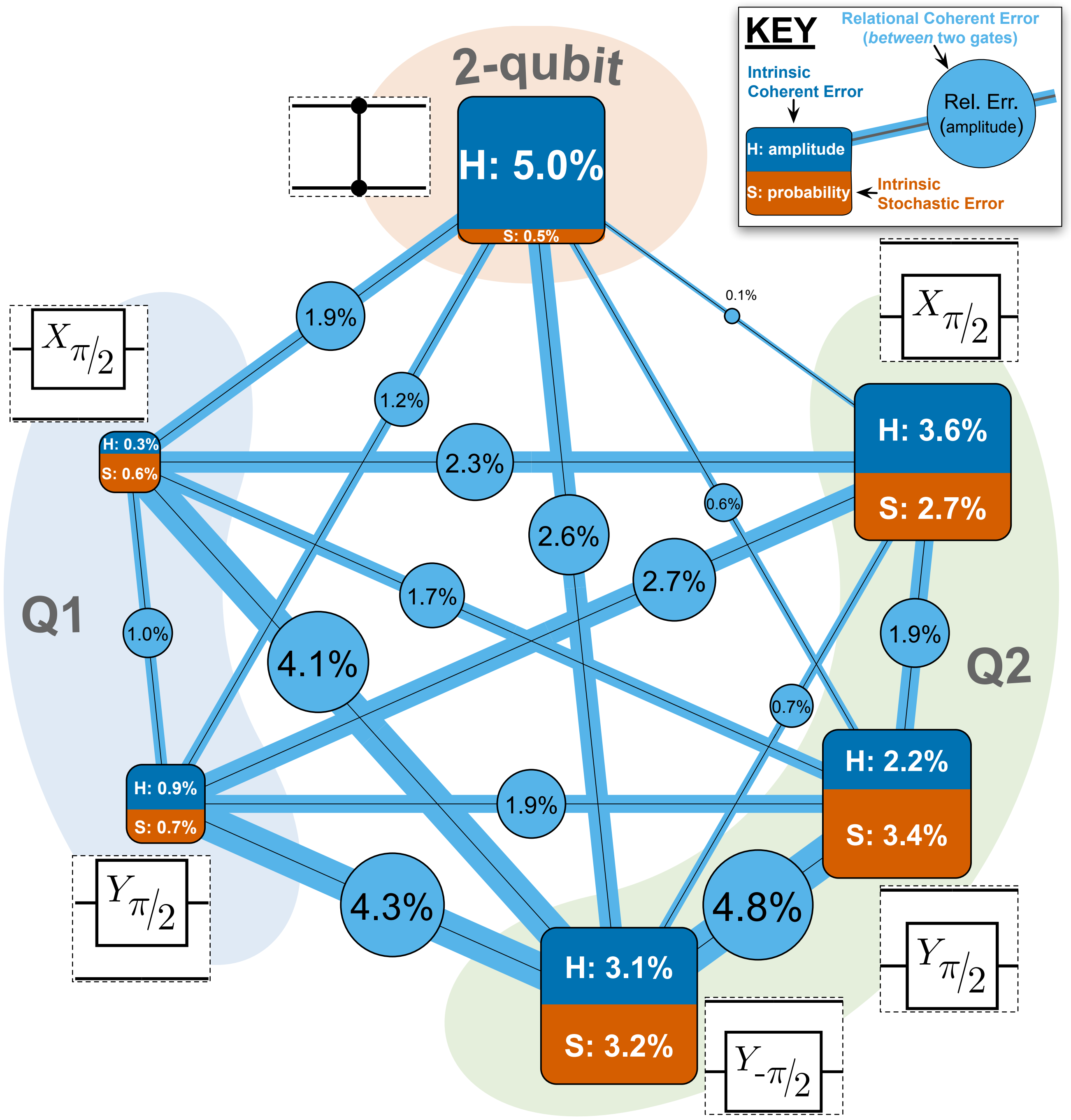
But some errors are actually *relational* between two gates. We separated relational errors from *intrinsic* errors associated with a unique gate.





Our final error budget identifies:

- *every* error's rate (w/error bars)
- each error's contribution to:
  - ➢ infidelity
  - ➢ total (worst-case) error
- contributions from each *sector*
- contributions on each *qubit*



Gate	Infidelity (Q1)	Infidelity (Q2)	Infidelity TOTAL	Stochastic Error	*Intrinsic* Coherent Error	*Relational* Coherent Error	Total Error (Q1)	Total Error (Q2)	Total Error TOTAL
Gxi	**0.17(3)%**	0.47(5)%	0.68(6)%	0.60(6)%	0.34(14)%	2.8(3)%	2.0(3)%	0.71(15)%	3.4(3)%
Gyi	**0.07(3)%**	0.61(6)%	0.75(6)%	0.67(6)%	0.9(3)%	2.6(3)%	1.0(2)%	1.2(4)%	3.4(3)%
Gix	2.14(5)%	**0.69(4)%**	2.86(7)%	2.69(6)%	3.6(3)%	2.3(3)%	5.0(3)%	3.05(17)%	6.9(3)%
Giy	2.63(8)%	**0.79(6)%**	3.44(10)%	3.36(10)%	2.3(4)%	1.9(4)%	4.1(6)%	2.6(2)%	6.3(5)%
Gimy	2.60(13)%	**0.68(7)%**	3.54(16)%	3.18(15)%	3.1(4)%	5.1(5)%	4.2(7)%	3.2(3)%	9.1(5)%
Gcz	0.11(10)%	0.66(11)%	**0.79(14)%**	0.54(13)%	4.9(3)%	0.7(3)%	3.2(4)%	4.1(3)%	5.5(4)%

Conclusions

A custom 2-qubit GST experiment (just 1592 circuits & 600K shots),

combined with a cutting-edge analysis using error generators,

- a complete predictive model (process matrices) with sub-10^{-3} error bars,
- and simple measures of infidelity and worst-case error for each gate…
- …that can be tracked and divided all the way down to individual errors.

This is what complete understanding of a system's <u>Markovian</u> errors looks like.

Outstanding challenges: <u>non-Markovian errors</u>, and <u>systems of 3+ qubits</u>.