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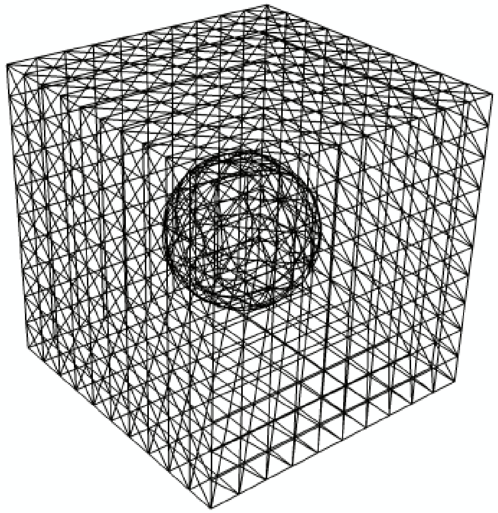
A Data-driven Approach for Improving the Existing Gurson Material Damage Model Using Genetic Programming for Symbolic Regression



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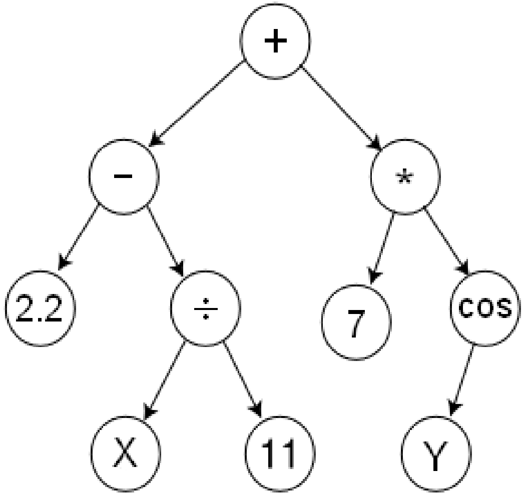

$$\left(2.2-\left(\frac{X}{11}\right)\right)+\left(7*\cos(Y)\right)$$

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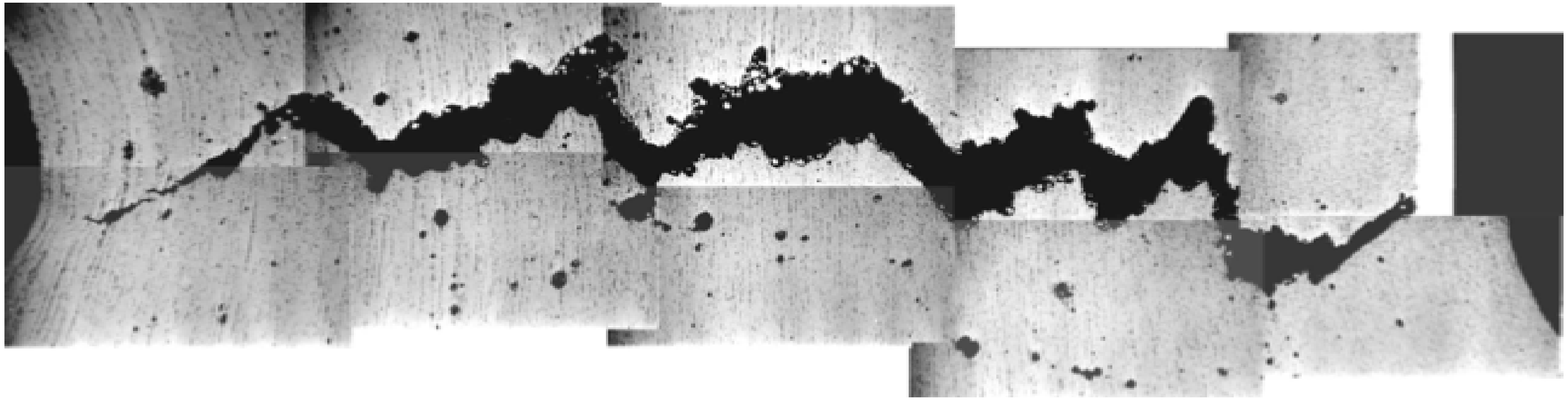
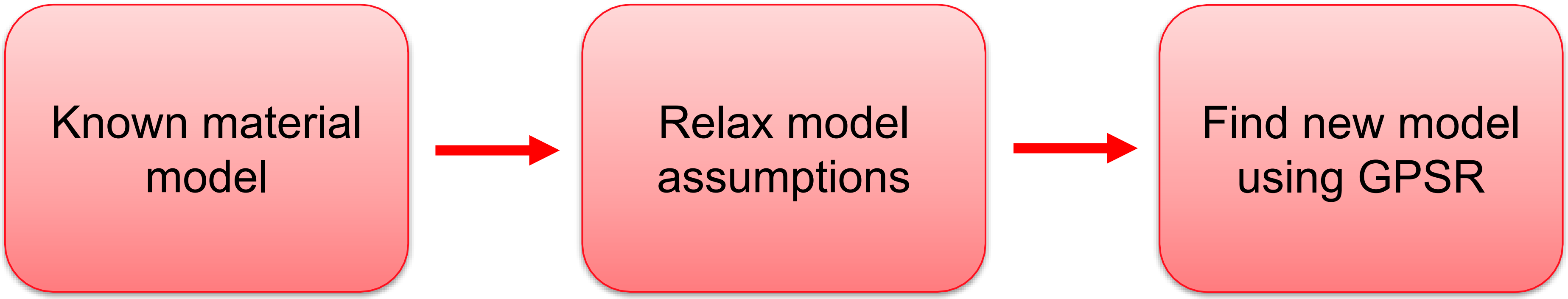
Background

- Engineering constitutive models that capture plasticity and damage have the following advantages:
  1. Flexibility
  2. Interpretability
  3. Computational tractability
- However, they are limited in their accuracy due to assumptions required for closed form solutions
- Examples: Gurson [1] and Cocks-Ashby [2]



Proposed Solution

- Utilize a machine learning approach known as genetic programming for symbolic regression (GPSR) to generate more accurate, data-driven, interpretable material models



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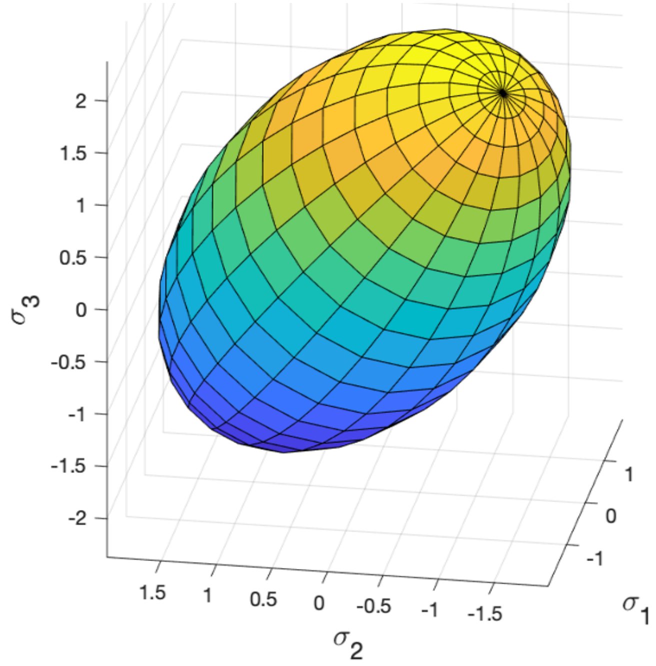
Known Material Model

- Gurson model for porous metal plasticity
- Assumptions:
  - Ignores void interaction
  - Assumes voids grow self-similarly
  - Assumes voids are arranged in a symmetric array
  - Assumes a perfectly plastic von Mises matrix material
  - Does not model void coalescence

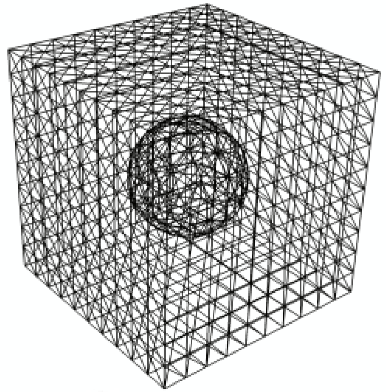


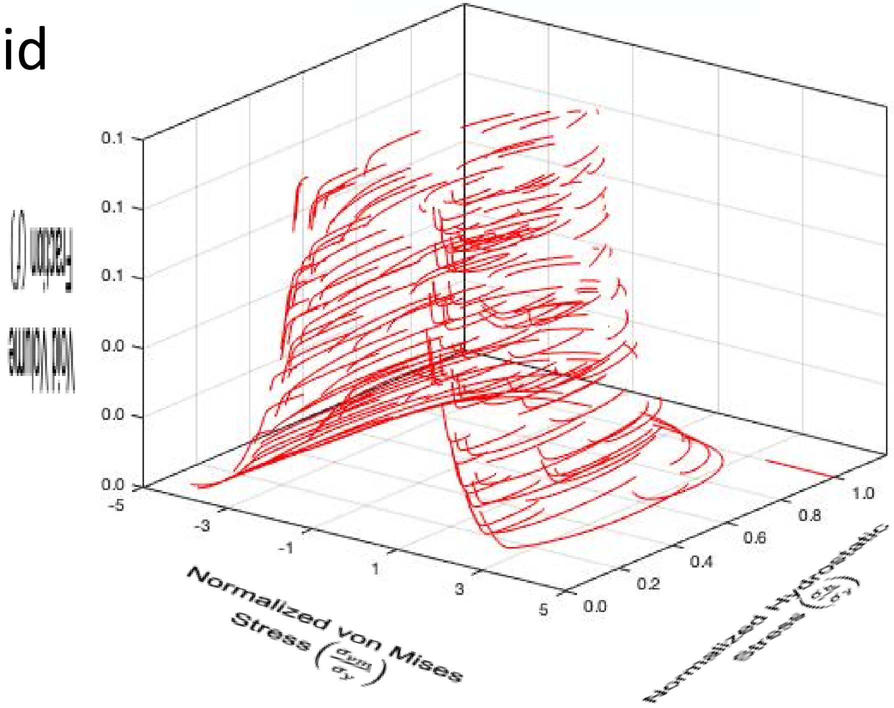
$$\Phi = \left(\frac{\sigma_{vm}}{\sigma_y}\right)^2 + 2\, f \cosh\left(\frac{3\sigma_h}{2\sigma_y}\right) - 1 - f^2 = 0$$

σ_h - Hydrostatic Stress
σ_{vm} - von Mises Stress
σ_y - Matrix Yield Stress
f – Void Volume Fraction

Relax Model Assumptions

- Generate training data via finite element (FE) simulations
- Model is consistent with all assumptions Gurson makes except void interaction
  - Representative volume element (RVE) with single void
  - Nodes surrounding void are limited to spherical deformation
  - Periodic boundary conditions (PBCs)
  - Varying size of void
  - 200 triaxial load cases

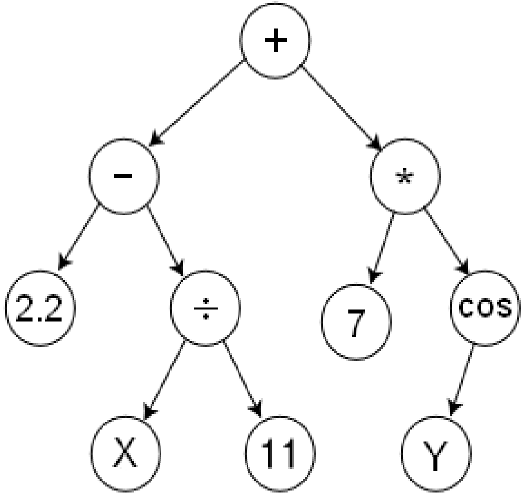




Find New Model Using GPSR

- Symbolic regression searches space of known equations via combinations of variables and weights
- Genetic programming evolves equations based on fitness with data
  - Fitness of implicit equations compares partial derivatives of proposed model with data
- Previous studies show GPSR can find known material models such as von Mises [4] and Gurson (unpublished)

[3]

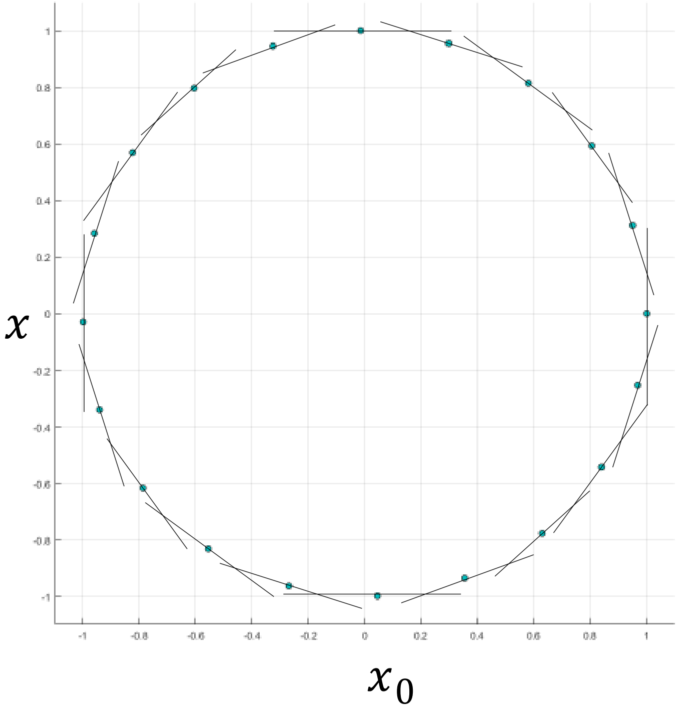


$$\left(2.2 - \left(\frac{X}{11}\right)\right) + \left(7 * \cos(Y)\right)$$

Find New Model Using GPSR

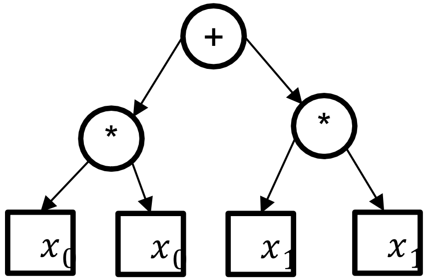
- Defining fitness for implicit equations

Given data: $X(x_0, x_1)$



Calculate $\frac{\Delta x_0}{\Delta t}$ and $\frac{\Delta x_1}{\Delta t}$

Propose model: $f(x_0, x_1)$



Calculate $\frac{\partial f}{\partial x_0}$ and $\frac{\partial f}{\partial x_1}$

Calculate $\frac{\partial f}{\partial t}$ via chain rule $\frac{\partial f}{\partial x_0}\frac{\Delta x_0}{\Delta t}$ and $\frac{\partial f}{\partial x_1}\frac{\Delta x_1}{\Delta t}$

$$E(f, X) = \frac{1}{N}\sum_{i=1}^{N}\left|\sum_{j=1}^{P}\frac{\frac{\partial f}{\partial x_j}\frac{\Delta x_j}{\Delta t}}{\left|\frac{\partial f}{\partial x_j}\frac{\Delta x_j}{\Delta t}\right|}\right|^{(i)}$$

Find New Model Using GPSR

- Leverage prior knowledge to generate models that are interpretable and abide by known physics:
- Boosting
- Seeding

Find New Model Using GPSR

- Leverage prior knowledge to generate models that are interpretable and abide by known physics:
- Boosting – a machine learning strategy where many weak learners are combined to form one strong learner
    - For each assumption relaxation:

Find New Models Using GPSR

- Boosting implementation:

  1. Given input data X for assumption A_i
  2. Calculate weights
     a. Retrieve model from previous boost stage f_{i-1}
     b. $W_i = E(f_{i-1}, X)$
  3. Minimize $Fitness$
     Propose new model $f_i(X)$
     Calculate new model fitness $Fitness = W_i \times E(f_i, X)$

  Iterate

- Datapoints with worse fit are weighted higher in next boosting stage

Find New Models Using GPSR

- Boosting implementation:

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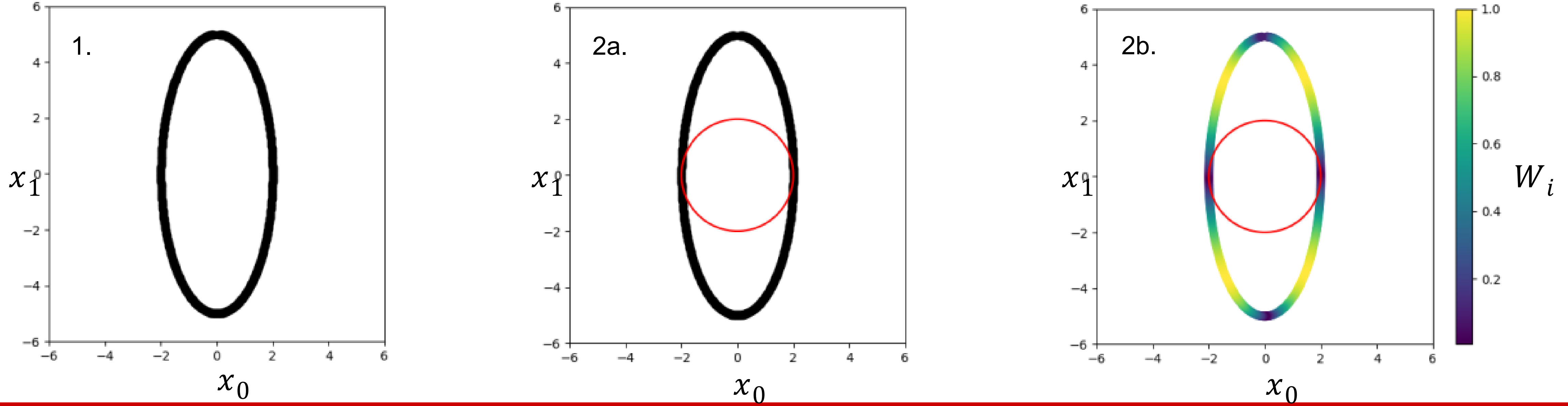
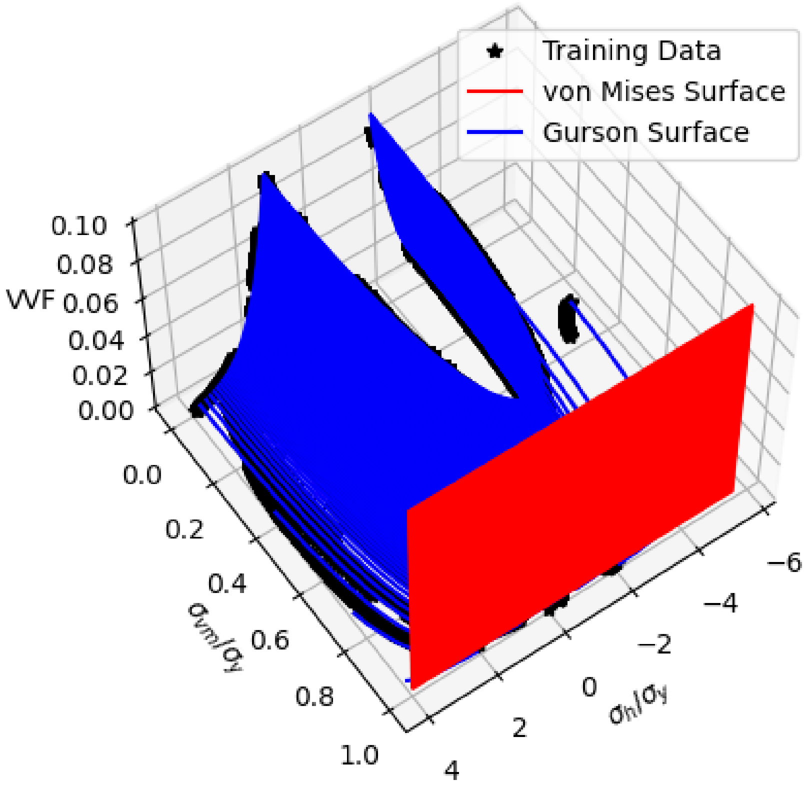
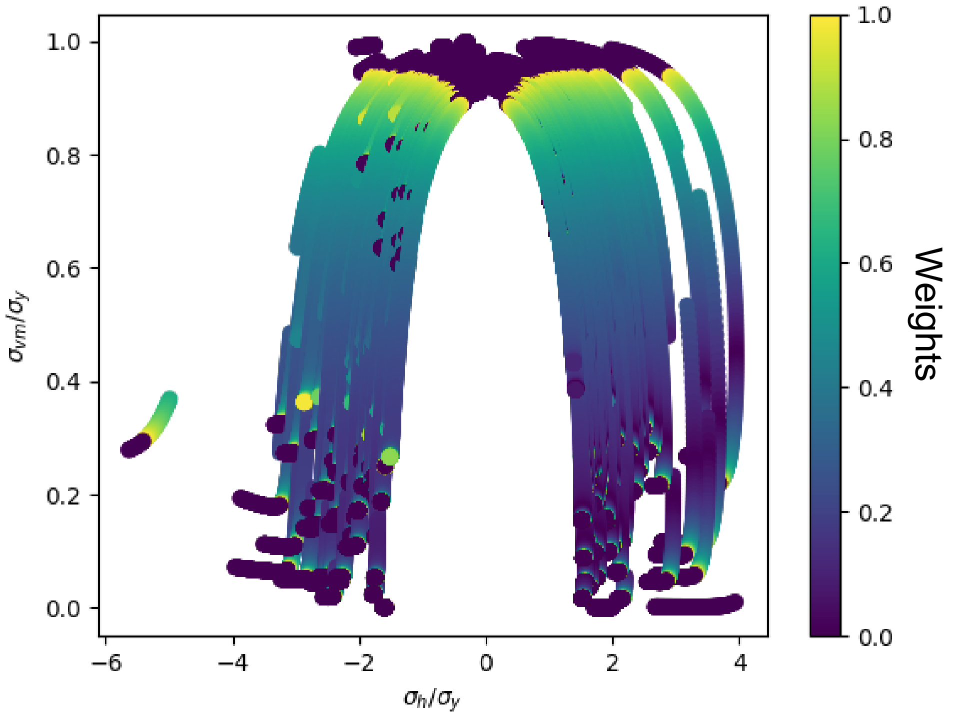


Boosting Test Results

- Train from the von Mises surface to the Gurson surface
- After 50,000 generations, the correct Gurson equation is found





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Find New Model Using GPSR

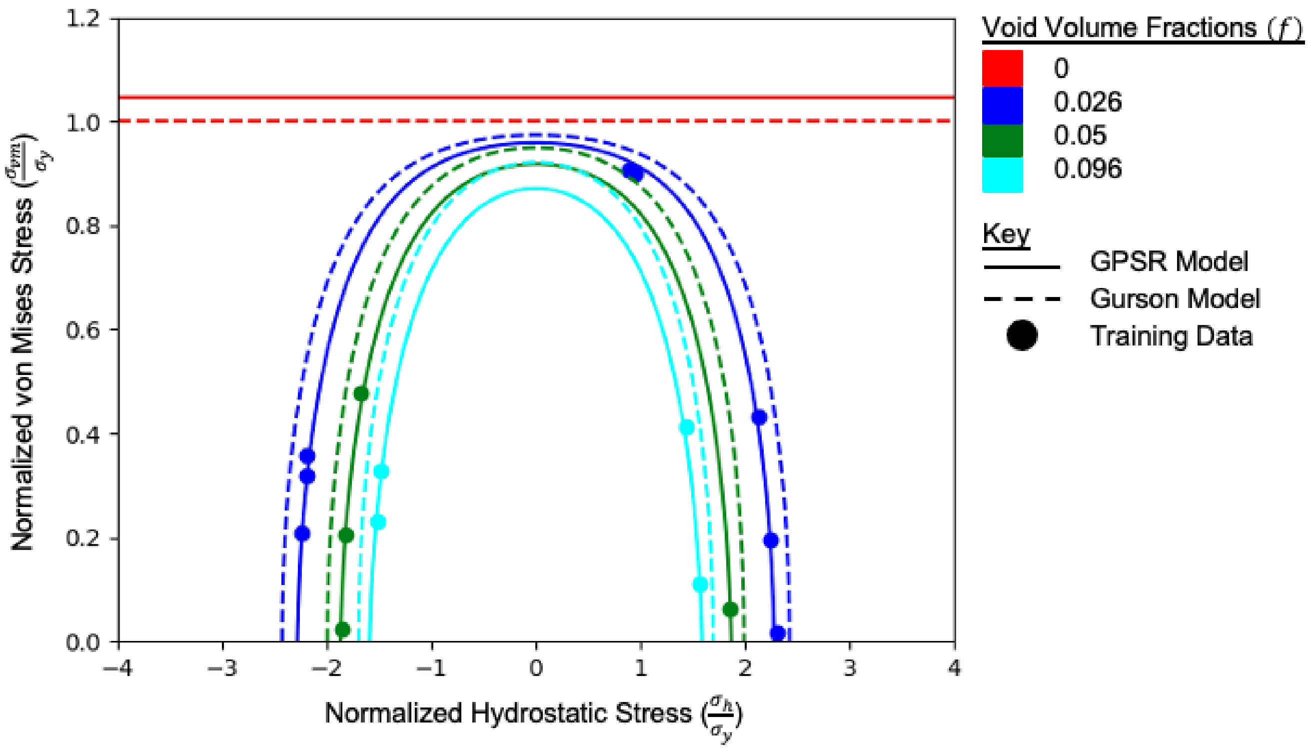
- Leverage prior knowledge to generate models that are interpretable and abide by known physics:
- Boosting
- Seeding – replacing a portion of the initial genetic population with ”seeds” that will likely be in the final solution [5]
- Genetic population is typically randomly generated
  - Seeding helps narrow in on search space
  - Decreases training times, increases interpretability

Seed 1 Seed 2

$$\Phi = \left(\frac{\sigma_{vm}}{\sigma_y}\right)^2 + 2\, f \cosh\left(\frac{3\sigma_h}{2\sigma_y}\right) - 1 - f^2 = 0$$

Seeding Results

- Relaxing assumption 1: void interaction
- Replaced 10 percent of GPSR population with parts of Gurson equation
- Trained for 200,000 generations



Equation	Fitness
Gurson	
$\Phi(\sigma, f) = \left(\frac{\sigma_{vm}}{\sigma_y}\right)^2 + 2\, f \cosh\left(\frac{3\sigma_h}{2\sigma_y}\right) - 1 - f^2 = 0$	0.094
GPSR	
$\Phi(\sigma, f) = \left(\frac{\sigma_{vm}}{\sigma_y}\right)^2 + f \cosh\left(\frac{\sigma_h}{\sigma_y}\right)\left(\left(\frac{\sigma_h}{\sigma_y}\right)^2 + \frac{\sigma_{vm}}{\sigma_y} + 2.603\right) - 1.03 = 0$	0.069

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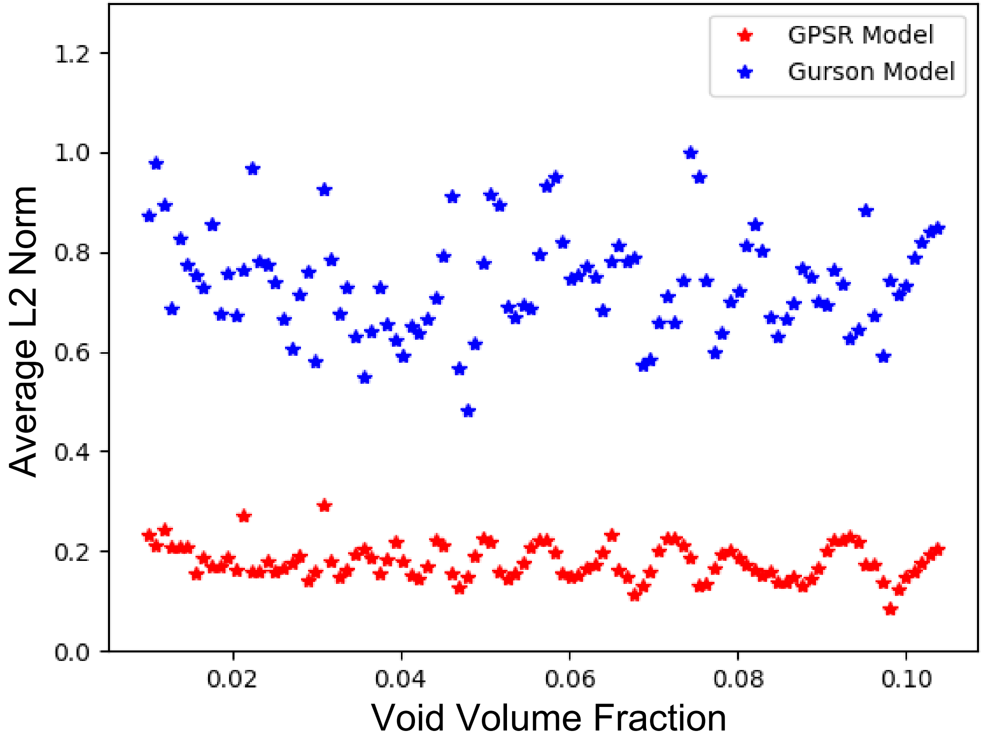

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GPSR	
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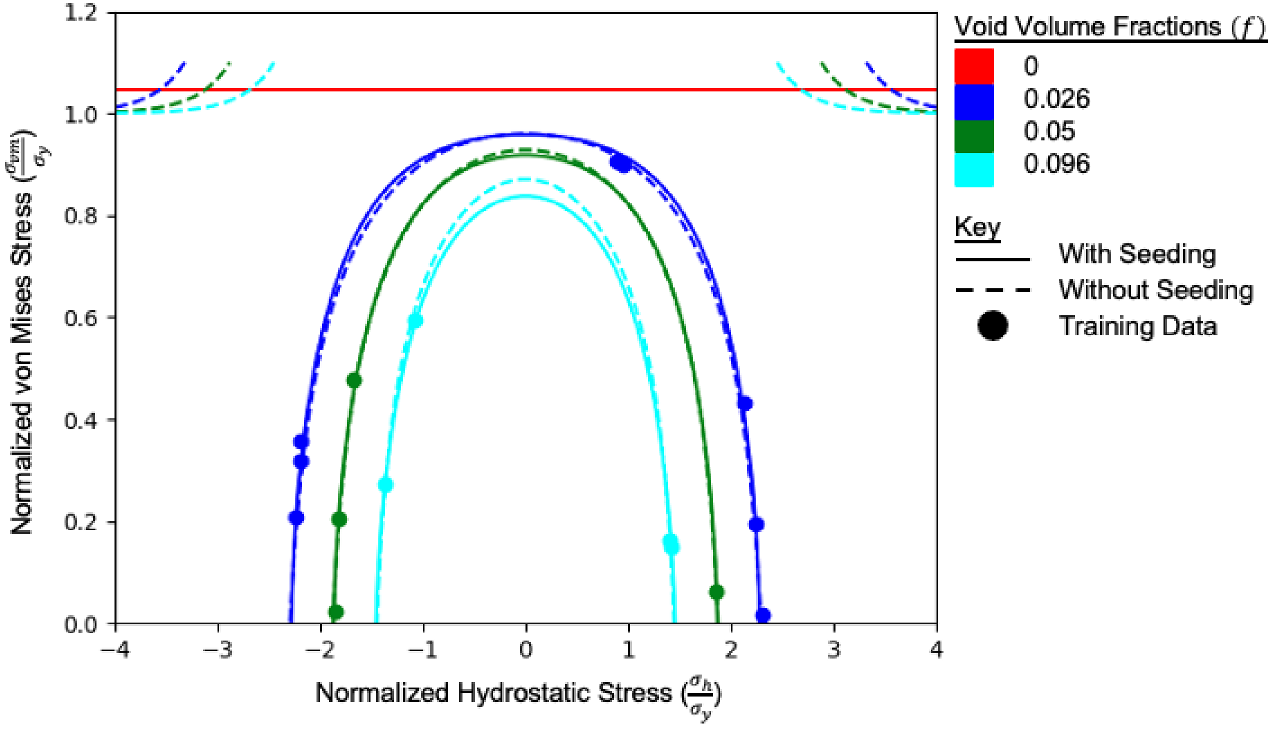
Seeding Results

- Main difference with/without seeding/boosting is generation of physically reasonable and interpretable results



Equation	Complexity
With Seeding	
$\Phi(\sigma, f) = \left(\frac{\sigma_{vm}}{\sigma_y}\right)^2 + f\cosh\left(\frac{\sigma_h}{\sigma_y}\right)\left(\left(\frac{\sigma_h}{\sigma_y}\right)^2 + \frac{\sigma_{vm}}{\sigma_y} + 2.603\right) - 1.03 = 0$	12
Without Seeding (truncated)	
$\Phi(\sigma,f) = -\frac{-\frac{\sigma_{vm}}{\sigma_y}\left(f^3\left(\frac{\sigma_{vm}}{\sigma_y} - \cosh\left(1.53\frac{\sigma_h}{\sigma_y}\right)\right)^2\left(f\left(\frac{\sigma_{vm}}{\sigma_y} - \cosh\left(1.53\frac{\sigma_h}{\sigma_y}\right)\right) - \cosh\left(1.53\frac{\sigma_h}{\sigma_y}\right)\right) + 0.27\right) + f^2\left(f\left(\frac{\sigma_{vm}}{\sigma_y} - \cosh\left(1.53\frac{\sigma_h}{\sigma_y}\right)\right) -}{f\left(f\left(\frac{\sigma_{vm}}{\sigma_y} - \cosh\left(1.53\frac{\sigma_h}{\sigma_y}\right)\right) - \cosh\left(1.53\frac{\sigma_h}{\sigma_y}\right)\right)}$	31

Conclusions

- We can leverage prior knowledge to improve GPSR training on finite element model data
- Boosting can tie directly with relaxing assumptions to improve GPSR performance and improve interpretability of output equations
- Seeding the initial GPSR population reduces search space and improves final model fitness and interpretability
- **Future Work:** Apply these methods to further relaxed assumptions to generate more accurate models for real-world materials

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Questions?

Lab

References

[1] Gurson AL. Continuum theory of ductile rupture by void nucleation and growth. *J Eng Mater Technol* 1977; 99: 2–15.

[2] F. Cocks and M. F. Ashby, "Creep fracture by coupled power-law creep and diffusion under multiaxial stress," no. September, pp. 395–402, 1980.

[3] G. Bomarito, T. Townshend, J. Hochhalter, E. Adams, K. Esham, and D. Vera, "Bingo," 2018. [Online]. Available: https://github.com/nasa/bingo.

[4] G. F. Bomarito, T. S. Townsend, K. M. Stewart, K. V. Esham, J. M. Emery, and J. D. Hochhalter, "Development of interpretable, data-driven plasticity models with symbolic regression," *Comput. Struct.*, vol. 252, p. 106557, 2021.

[5] Schmidt, Michael D., and Hod Lipson. 2009. "Incorporating Expert Knowledge in Evolutionary Search: A Study of Seeding Methods." *Proceedings of the 11th Annual Genetic and Evolutionary Computation Conference, GECCO-2009*, 1091–97. https://doi.org/10.1145/1569901.1570048.