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Gate Set Tomography of a Logical Qubit

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Gate Set Tomography of a Logical Qubit

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- Key takeaway: Gate set tomography shows promise as a tool for characterizing logical qubits; questions remain!

Open questions about logical qubits

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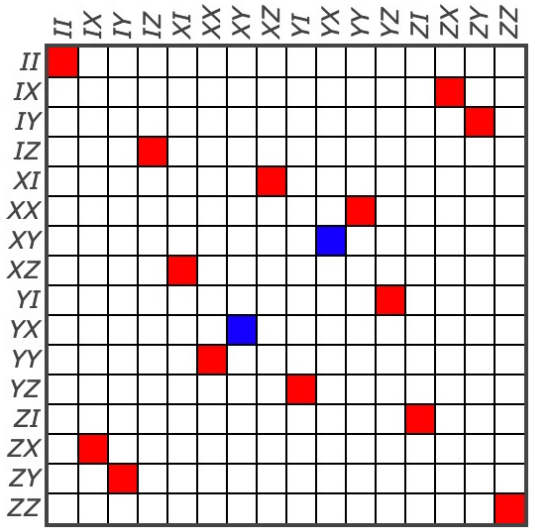
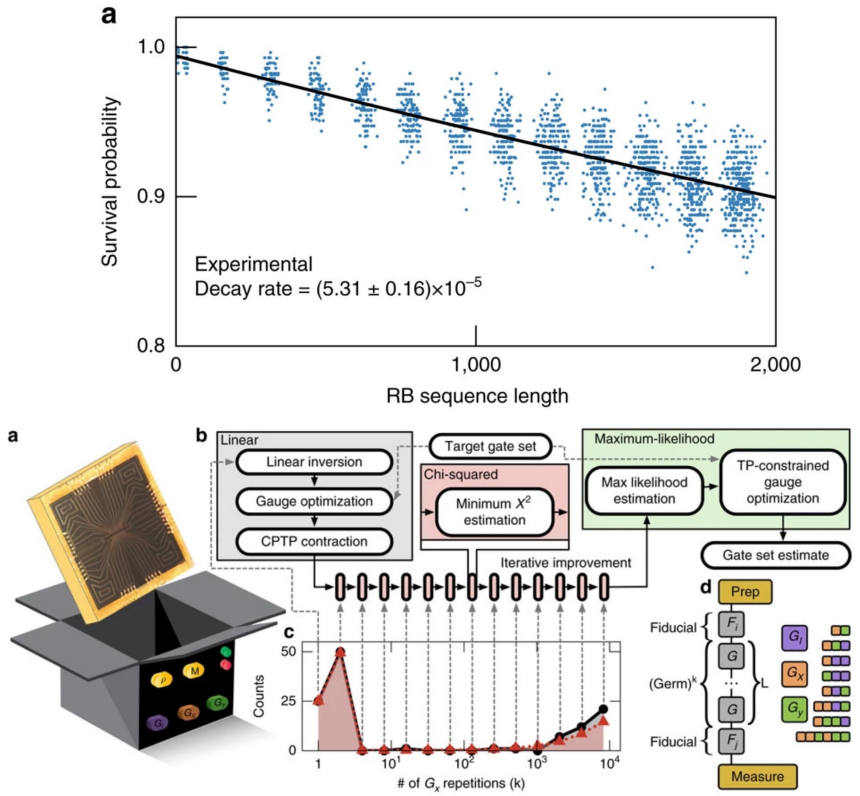
- How should we *describe* logical operations on logical qubits?



Open questions about logical qubits

- How should we *describe* logical operations on logical qubits?
- **How should we *characterize* logical operations on logical qubits?**



Open questions about logical qubits

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- How should we *describe* logical operations on logical qubits?
- How should we *characterize* logical operations on logical qubits?
- **What can we *learn* from logical qubit characterization?**

Open questions about logical qubits

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- What can we *learn* from logical qubit characterization?

- → **Can we build predictive and debugging tools for logical qubits?**

Open questions about logical qubits

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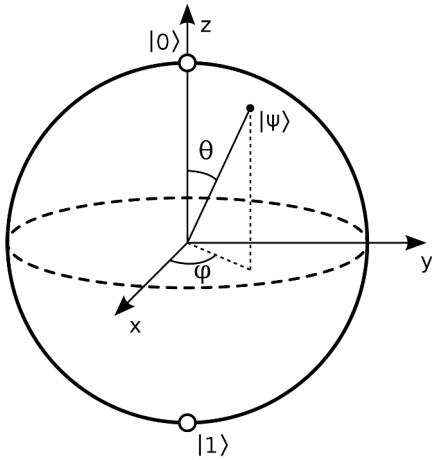
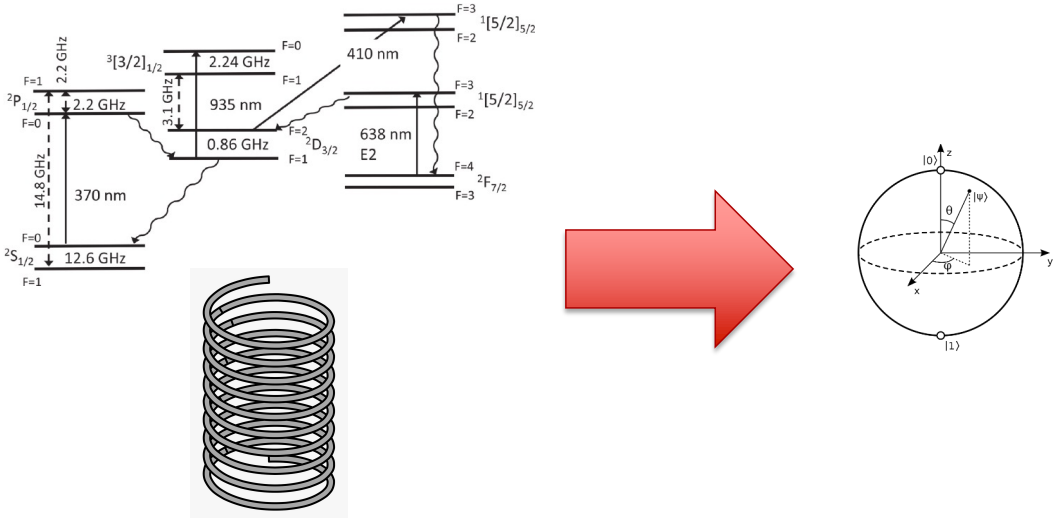
- How should we *describe* logical operations on logical qubits?
- How should we *characterize* logical operations on logical qubits?
- What can we *learn* from logical qubit characterization?

- → Can we build predictive and debugging tools for logical qubits?
- → Can we use (or adapt) existing physical qubits frameworks for understanding logical qubit behavior?



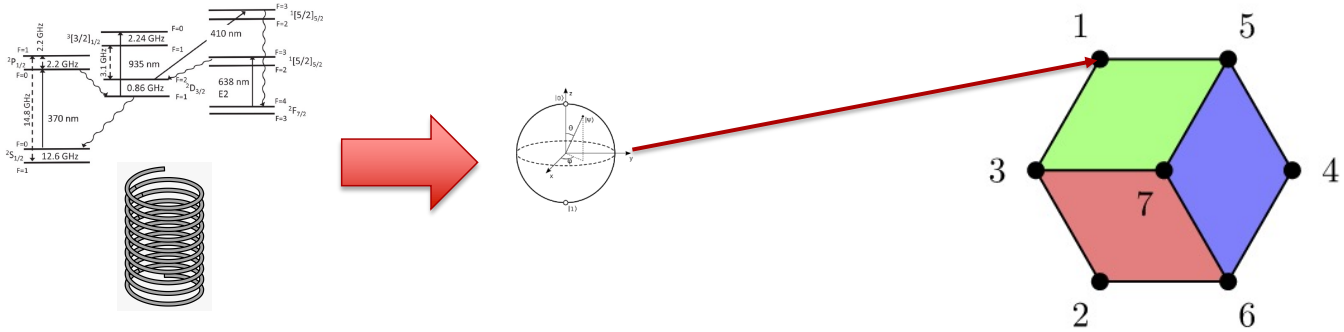
Understanding physical qubits

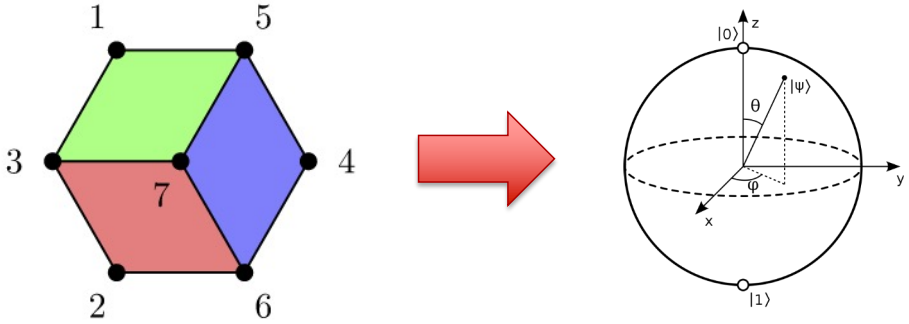


This picture is *roughly* correct!

What about logical qubits?

Why does QCVV work?





https://arxiv.org/abs/1804.06995

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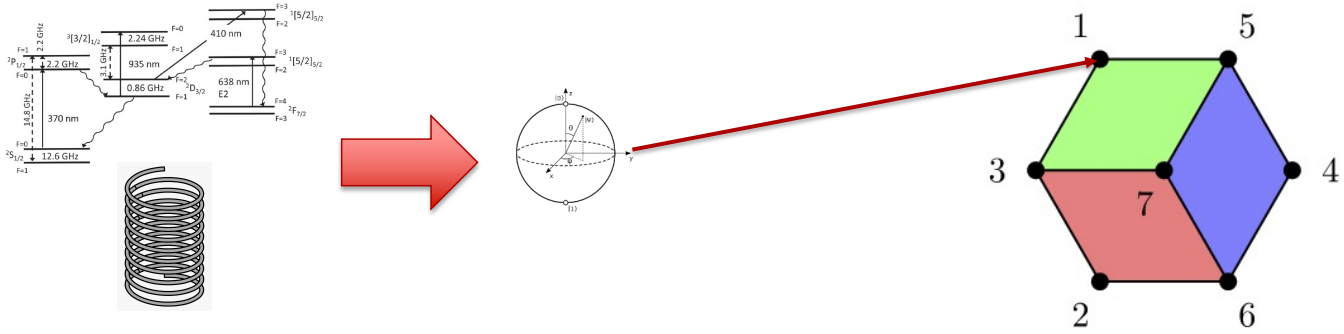

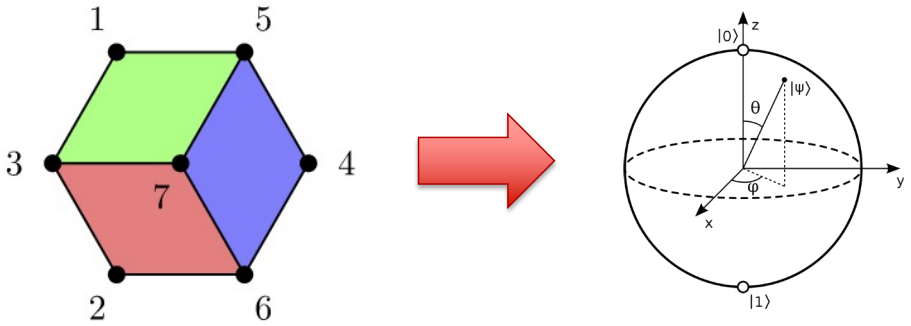

(This is the hope!)

Logical GST

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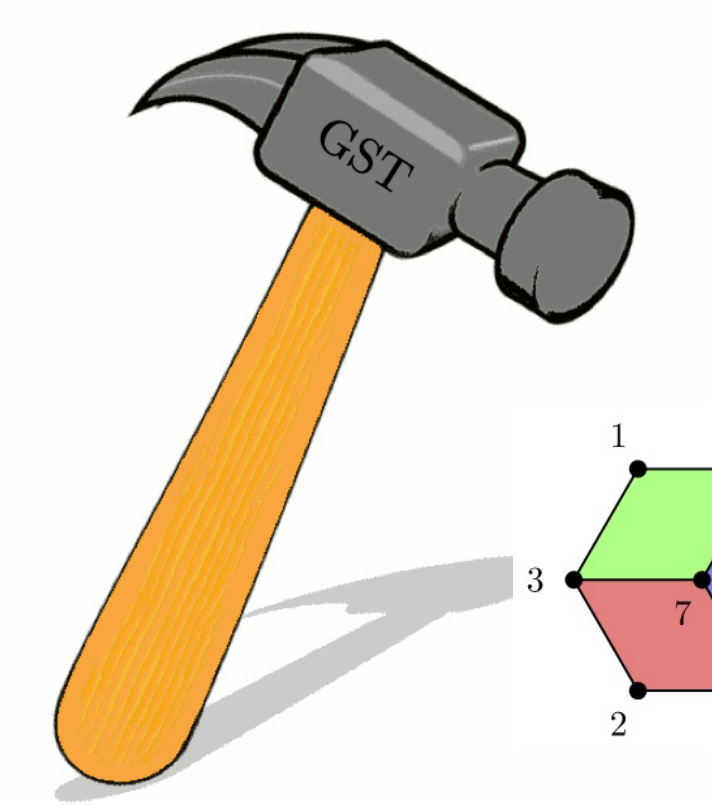

GST is a powerful tool for debugging and predicting behavior of small numbers of physical qubits. Can we use GST on logical qubits?

Why GST?

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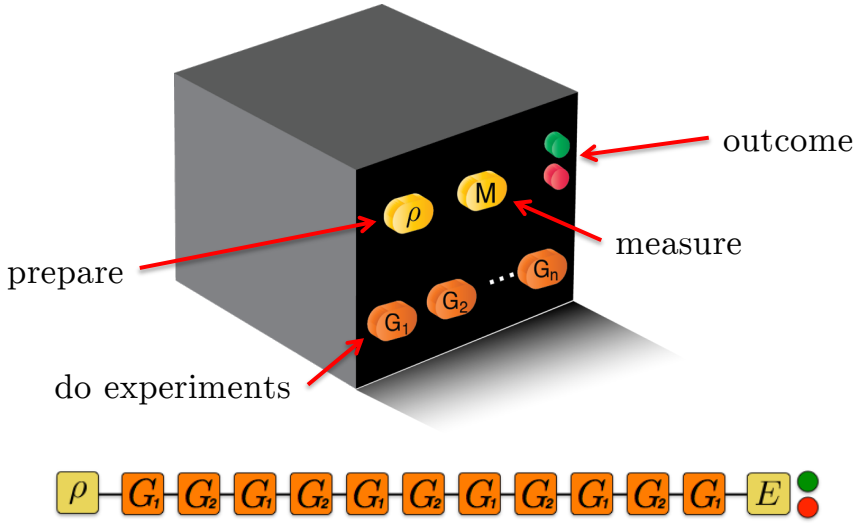
- GST works well for characterizing (physical) one- and two-qubit systems.
  - *Demonstration of qubit operations below a rigorous fault tolerance threshold with gate set tomography,* Nat. Commun. 8, 14485 (2017)
  - *Demonstration of a parametrically activated entangling gate protected from flux noise*, Phys. Rev. A 101, 012302 (2020)
  - *Machine Learning of Noise-Resilient Quantum Circuits,* Phys. Rev. X Quantum 2, 010324 (2021)
  - *Precision tomography of a three-qubit donor quantum processor in silicon,* Nature, 601, 348–353 (2022)
  - *Quantum logic with spin qubits crossing the surface code threshold,* Nature, 601, 343–347 (2022)
  - Etc.

- GST does an excellent job of characterizing qubits that aren't perfectly well-behaved (exhibit leakage, drift, other non-Markovian errors) *without* necessarily understanding or fully capturing those dynamics.
- Logical qubits *should* behave like nice two-level systems. If so, characterize with GST! If not, capture violation of that model with GST.

Gate Set Tomography - Review

- Circuits: A collection of structured, "process-tomography-like" circuits.
- Analysis: Maximum likelihood estimation on observed outputs
- Output: Process matrix estimates for quantum logic operations (gates, state preparations/measurements).
- Number of circuits <100 – 1000s
- Estimates are self-consistent and Heisenberg-limited in accuracy; provide built-in Markovianity check (how well does the estimate fit the data?)



Logical GST

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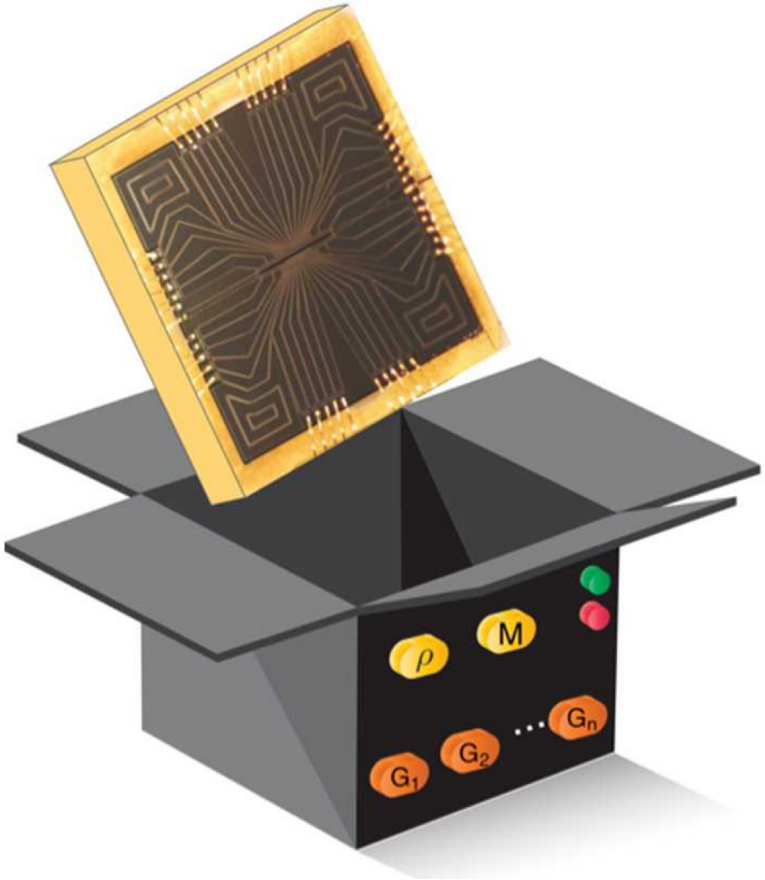
- What does it even mean to "do logical GST"?
- Simple "naïve" approach (don't criticize me yet!)
  - Take GST circuits and run them on a logical qubit (simulation or experiment), promoting physical gates to logical gates. (Button-pushes are *logical operations not physical.* E.g., $H \rightarrow H^{\otimes 7}$)



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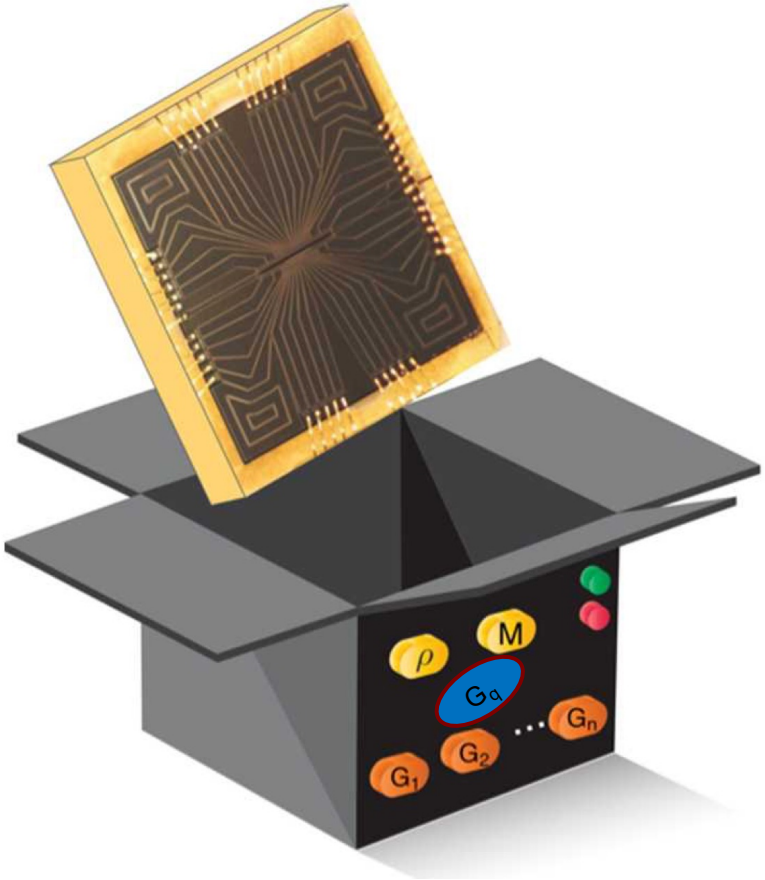
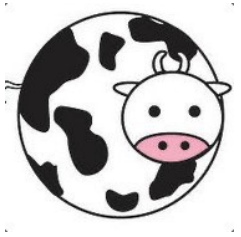
  - Take GST circuits and run them on a logical qubit (simulation or experiment), promoting physical gates to logical gates. (Button-pushes are *logical operations not physical.* E.g., $H \rightarrow H^{\otimes 7}$)
  - Optional: Have a new button for QEC.



Logical GST- Let's do it!



- Let's do this on a simple logical gate set:
  - Steane [[7,1,3]] code
  - All-to-all CNOT connectivity
  - {H, S, QEC} logical operations
  - Simple error model:
    - *Only error* is 10% depolarization on physical S, $S^\dagger$ ($S_{logical}$=S⊗$S^\dagger$⊗S⊗$S^\dagger$...)
  - Maximum germ depth is 16; 774 circuits.
  - 1000 shots per circuit.
  - Simulation run using Logical Qubit Simulator (LoQS)- a hybrid density matrix/Monte Carlo simulation tool built on pyGSTi and Quantumsim (not yet publicly available, but soon we hope!)
  - 5.5 wall-clock-hours; 790 compute-hours. (Most of the time spent on circuits with rounds of QEC).

Logical GST

- What's it look like?!
- Look at *physical* GST first:

Figure 1. Model violation summary.

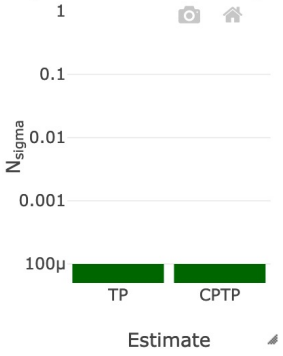


Figure 2. Model violation per iteration.

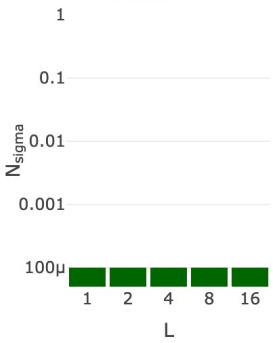


Figure 3. Histogram of per-circuit model violation.

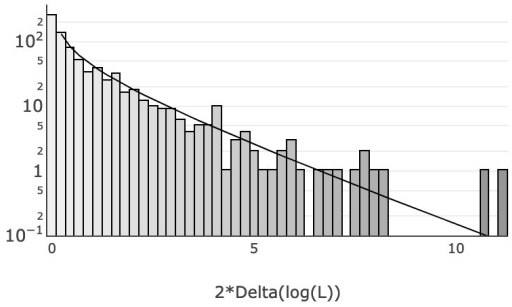


Table 1. Comparison of estimated gates to targets.

Gate	Entanglement Infidelity	1/2 Trace Distance	1/2 Diamond-Dist	Eigenvalue Ent. Infidelity	Eigenvalue 1/2 Diamond-Dist
Gh:0	5×10^{-7}	0.000698	0.000698	0	0.000043
Gp:0	0.073964	0.073991	0.074478	0.07459	0.075084
[]	2×10^{-7}	0.000435	0.000435	2×10^{-7}	0.000653

Physical GST

Physical GST

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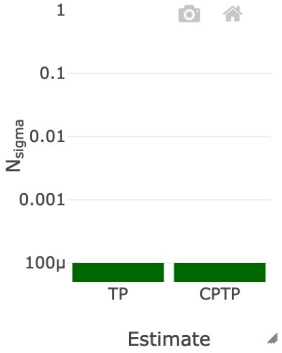

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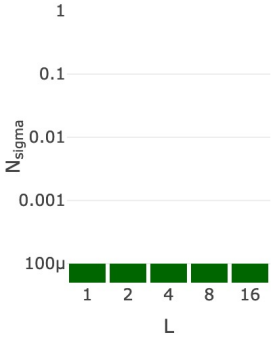

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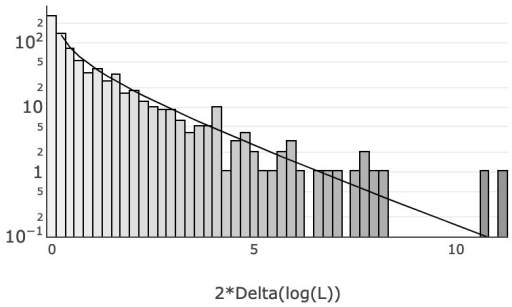

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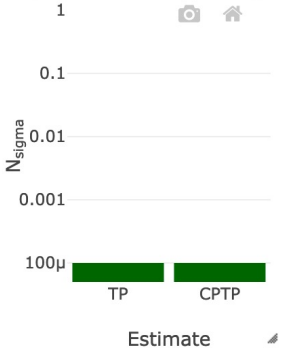

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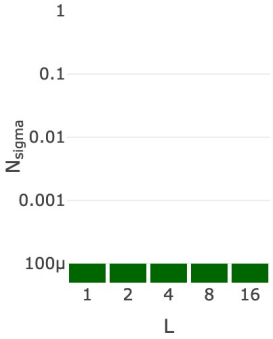

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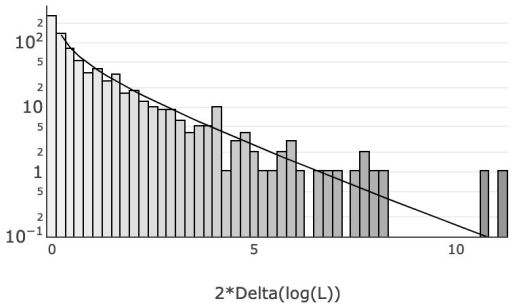

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Logical GST- same physical error model

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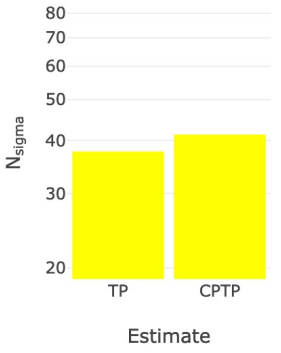

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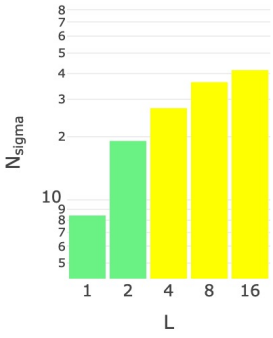

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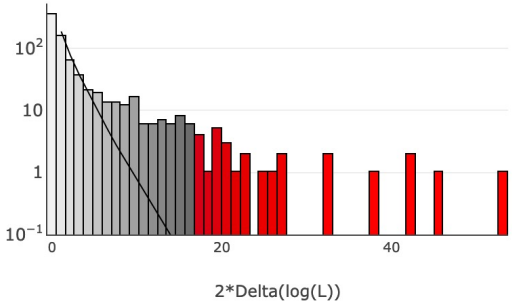

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Logical GST

- What happened?! Markovian physical qubits but non-Markovian logical qubit?!

Logical GST- same physical error model

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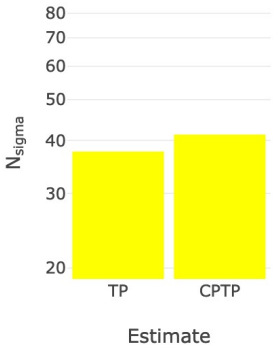

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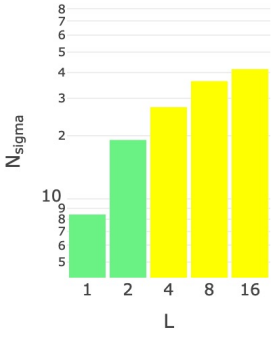

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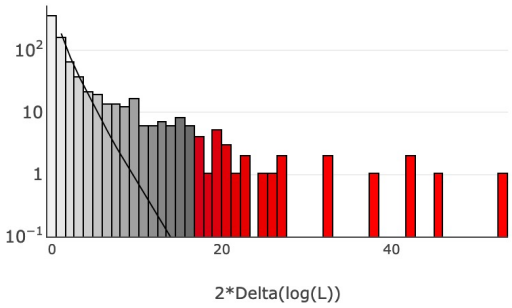

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Logical GST

- What happened?! Markovian physical qubits but non-Markovian logical qubit?!
- QEC makes previous gates *better*. *This is explicitly not Markovian.*

Logical GST- same physical error model

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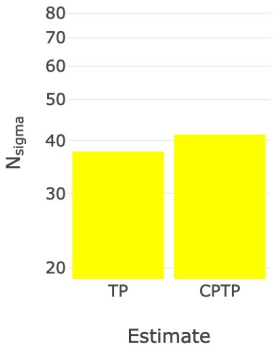

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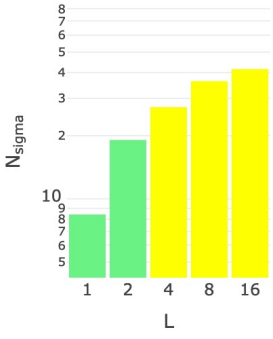

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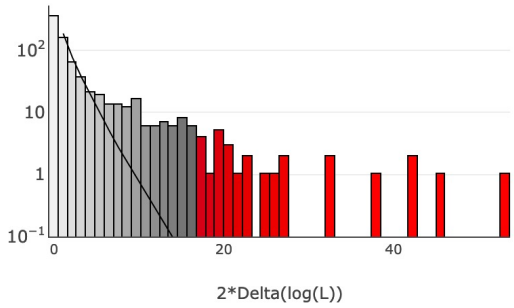

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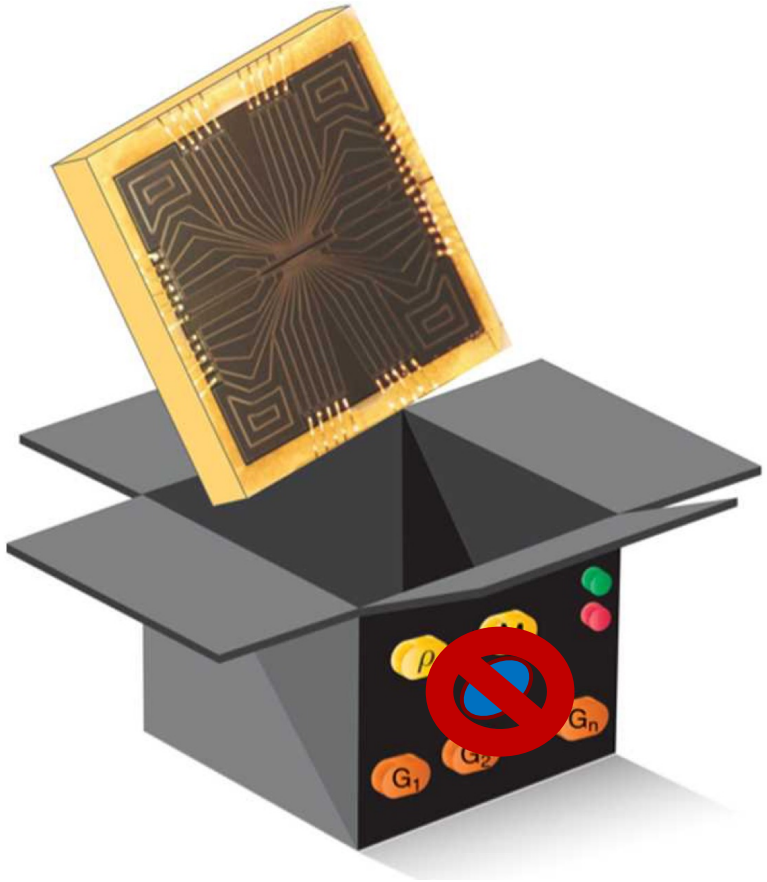
Logical GST

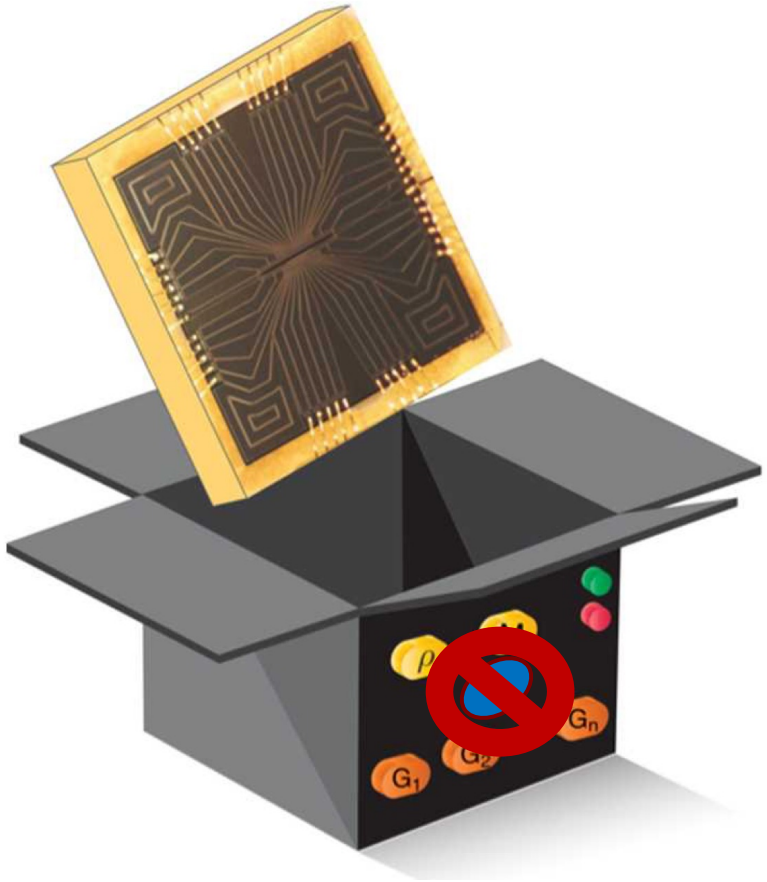
- *What should the "API" for logical GST be?*
- As you may have expected, we do *not* want a "QEC button". (Or work out valid way to model QEC operations within GST framework.)
- Follow every logical gate with a round (or more) of QEC.

  E.g., $G_x \rightarrow G_x G_{QEC}$



Logical GST – Take 2

- Each button implements transversal gate followed by one round of QEC.
- H: 10^{-2} depolarization
- S, $S^{\dagger}$: $1.7*10^{-1}$ over-rotation (same infidelity as H)
- CNOT: $2*10^{-2}$ depolarization



Logical GST – Take 2

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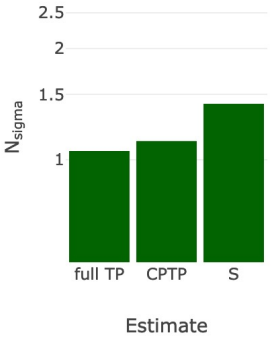

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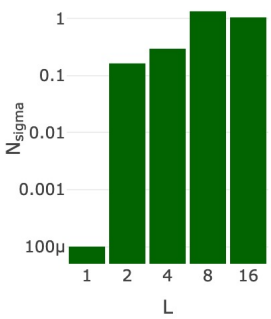

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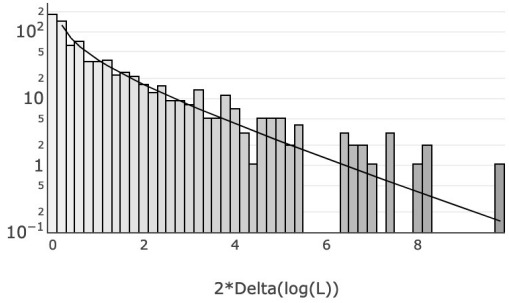

Table 1. Comparison of estimated gates to targets.

Gate	Entanglement Infidelity	1/2 Trace Distance	1/2 Diamond-Dist	Eigenvalue Ent. Infidelity	Eigenvalue 1/2 Diamond-Dist
Gh:0	0.222543 ± 0.088974	0.225698 ± 0.092284	0.226201 ± 0.091305	0.22176 ± 0.086558	0.299398 ± 0.089373
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[]	0.224359 ± 0.108209	0.297149 ± 0.202904	0.313971 ± 0.212221	0.224359 ± 0.108209	0.430875 ± 0.29547

- It works!
- Even a stochastic-only model fits the data!

Logical GST – Take 2

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Figure 1. Model violation summary.

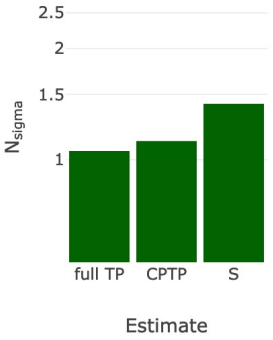


Figure 2. Model violation per iteration.

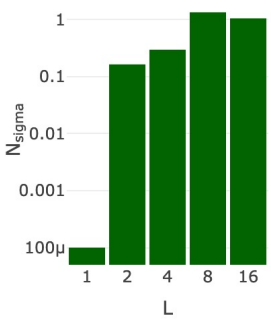
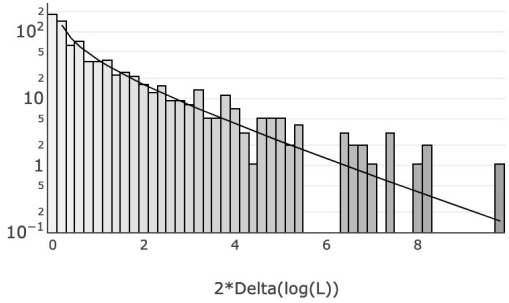


Figure 3. Histogram of per-circuit model violation.



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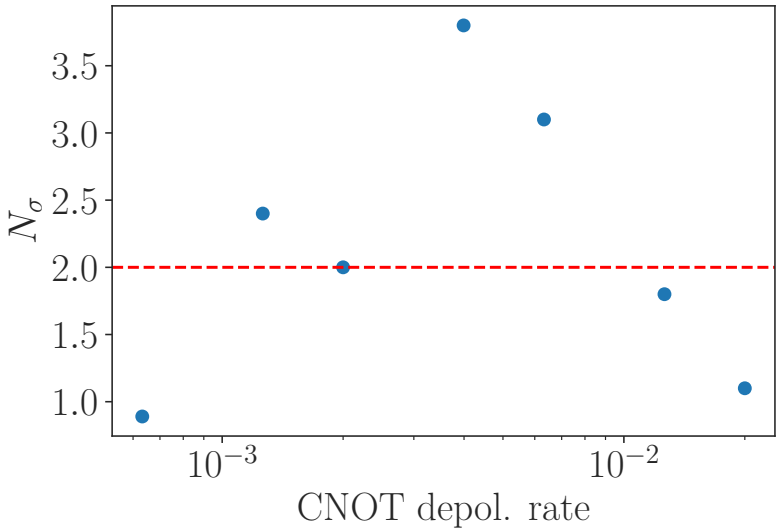
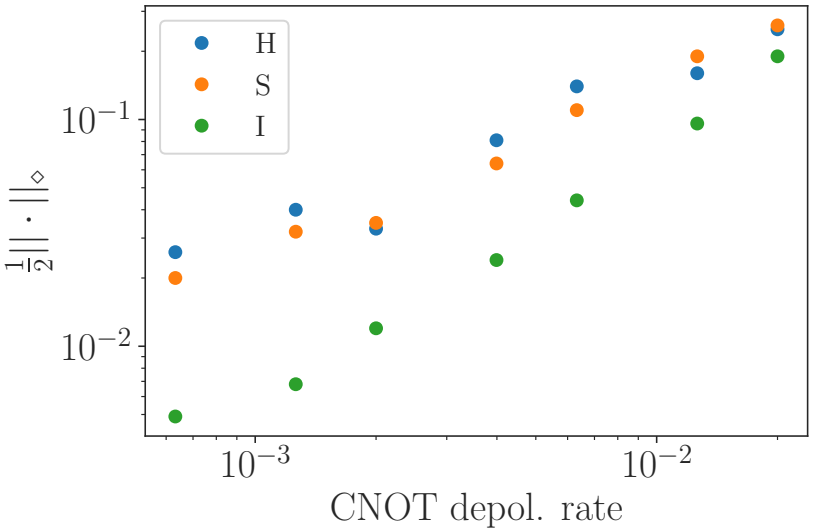
- It works!
- Even a stochastic-only model fits the data!
- What about a range of CNOT depolarizations?

Logical GST – Take 2





- Behavior is *mostly* as expected, though somewhat high N_σ values. ...some subtle non-Markovian effect?

Conclusions

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- There exist regimes in which logical qubit operators are well-modeled by (Markovian) single-qubit process matrices.
- GST can be extended to logical qubits to learn those process matrices.
- The full scope of logical GST's applicability and utility remains an open question.
  - What about other codes? What about non-fault-tolerant operations? Can we extract useful information from syndrome data? Does pseudothreshold matter? Etc.

- www.pygsti.info
- Quantum Performance Laboratory looking to hire: qpl.sandia.gov
- kmrudin@sandia.gov
- Thanks to co-authors, Tom O'Brien, Marcello Caio, and you!