

Practical database enrichment strategies with iterative learning: Neural Network Potentials for Phase Change Memory

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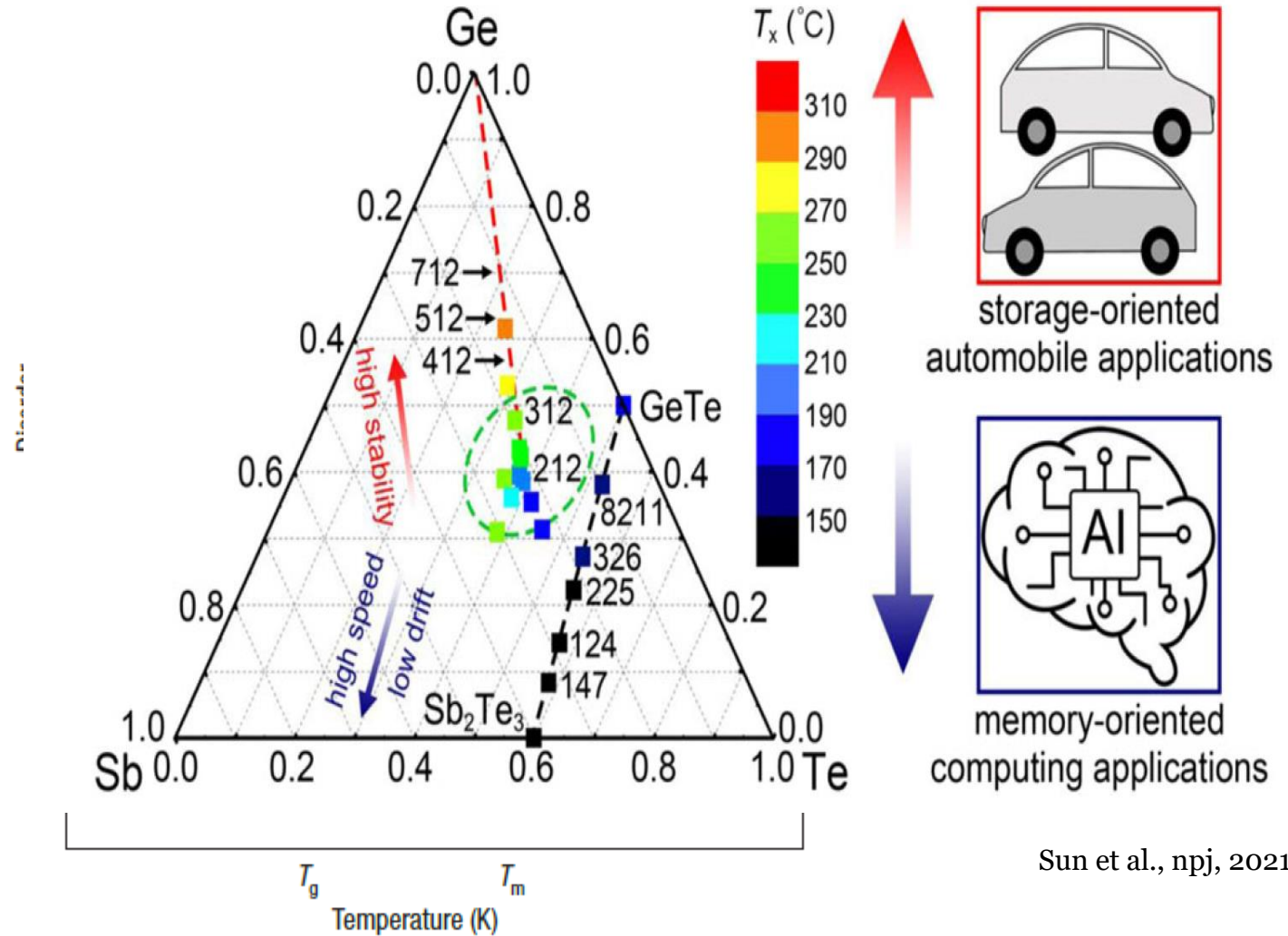
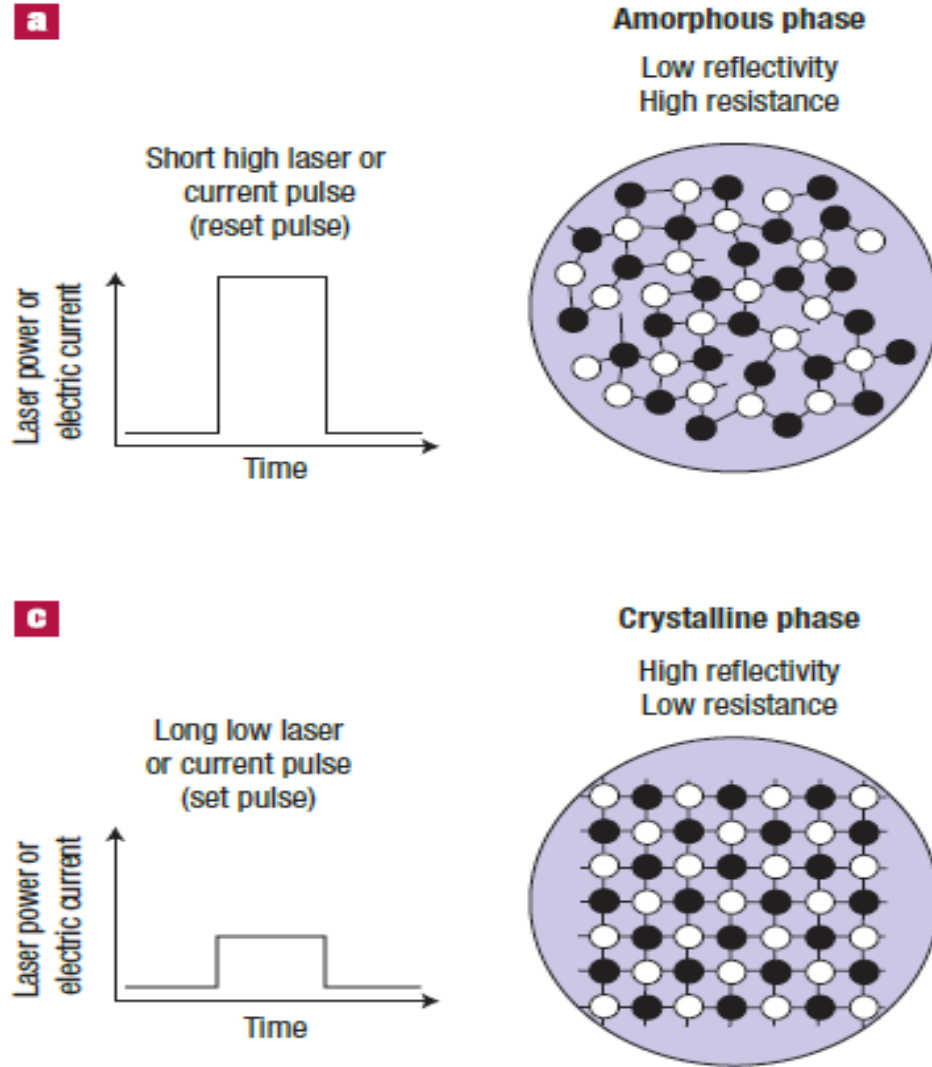


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Phase Change Memory: Atomistic precision for macroscale applications



Wuttig and Yamada, Nat Mat,
2009

Sun et al., npj, 2021

Stabilizing the amorphous phase

Sun et al., npj, 2021

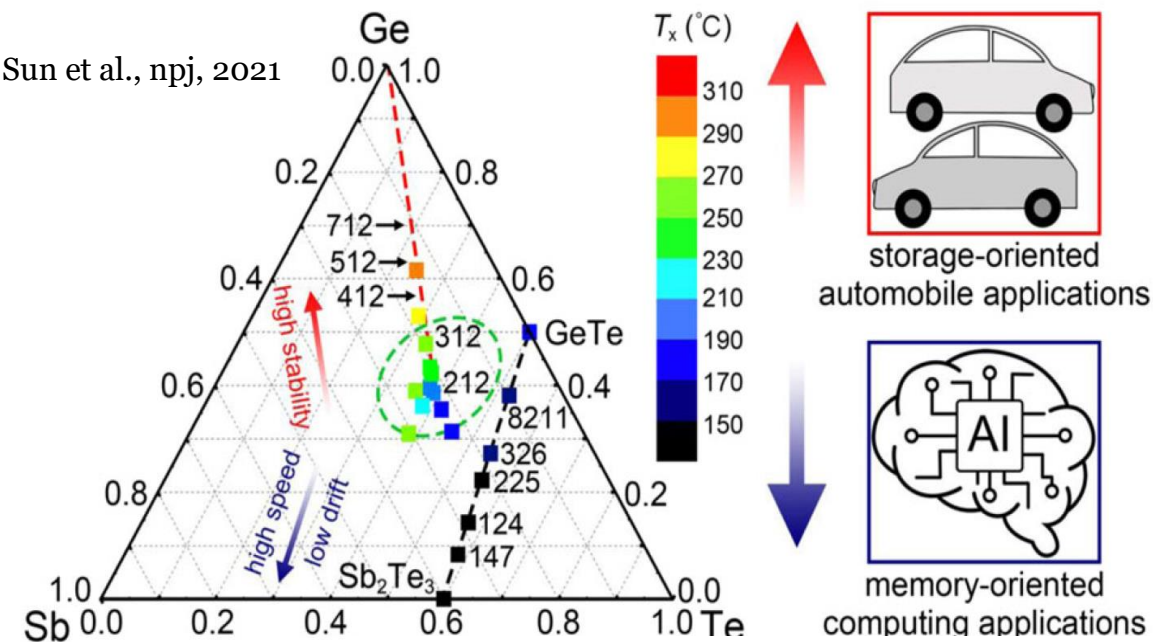
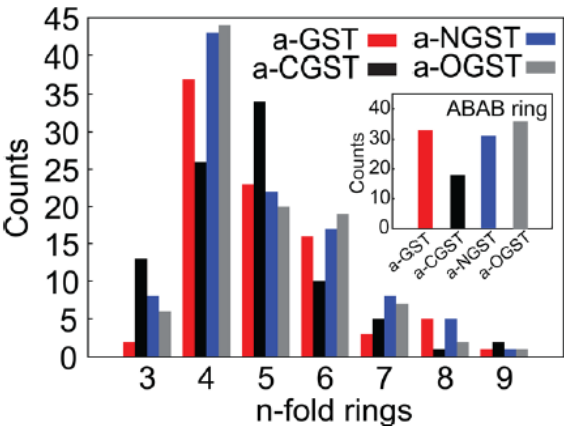
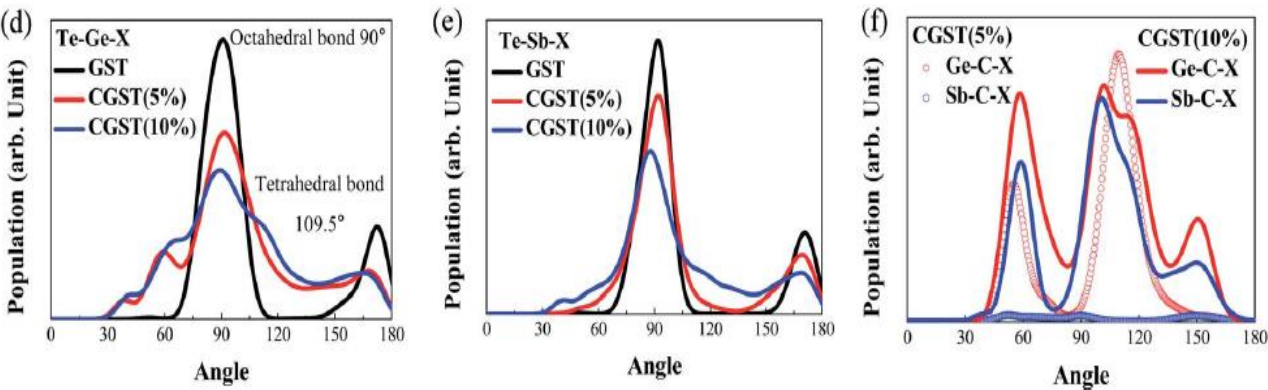


TABLE I. The coordination numbers, bond lengths, the first peak position of ADF around Ge atoms in amorphous GST structures. Ge(I) atoms are bonded to dopants, while Ge(II) atoms are not.

	Coordination numbers		Bond lengths (Å)		Averaged angle around Ge atoms (°)	
	Ge(I)	Ge(II)	Ge(I)-Te	Ge(II)-Te	-Ge(I)-	-Ge(II)-
a-GST		3.67		2.78		93
a-CGST	4.00	3.67	2.67	2.74	106	98
a-NGST	4.00	4.00	2.73	2.80	102	95
a-OGST	3.74	3.90	2.84	2.81	98	93

Han et al., RSC Advances, 2021



Cho et al., APL, 2011

FIG. 2. (Color online) Ring statistics for amorphous GST structures counted per supercell. The inset figure shows the numbers of ABAB-type squared rings.

What atomistic process can we model so far?

Ab initio methods:

- Phase change
- Density of states
- Dopant stability
- Recrystallization
- Limited to constant volume

**limited to small system sizes

ML MD Potential:

Gaussian Approximation Potential –
DFT accuracy with scaling to ~7K
atoms

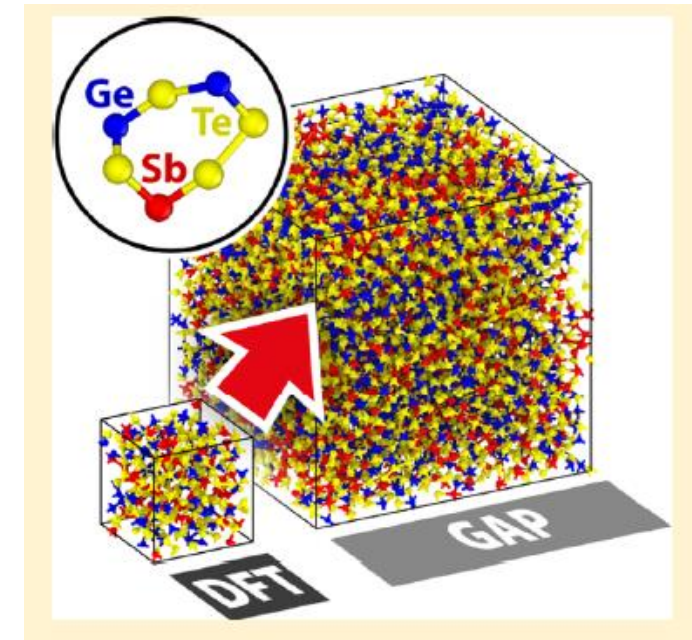
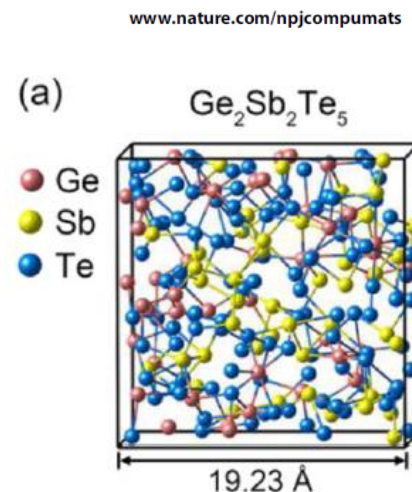
NPT possible for phase change

npj | Computational Materials

ARTICLE OPEN

Ab initio molecular dynamics and material
embedded phase-change memory

Liang Sun^{1,8}, Yu-Xing Zhou^{2,3,7,8}, Xu-Dong Wang^{2,3}, Yu-Han Chen^{2,3}, Volker L. Deringer⁴

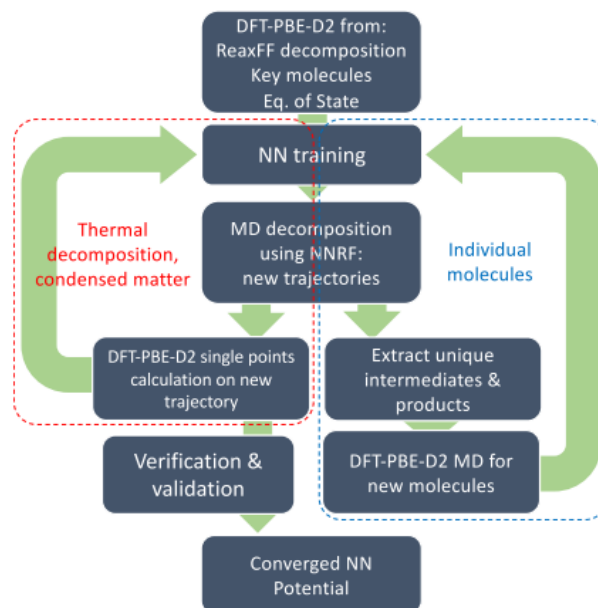


Mocanu et al., JPC B, 2018

Developing machine learning potentials

High density neural net (HDNN)

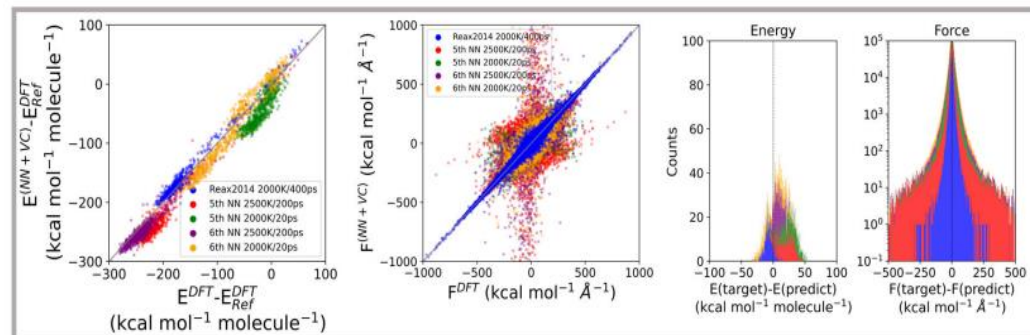
- High variability due to hyperparameters
- Requires higher variance
- High sensitivity to initialization



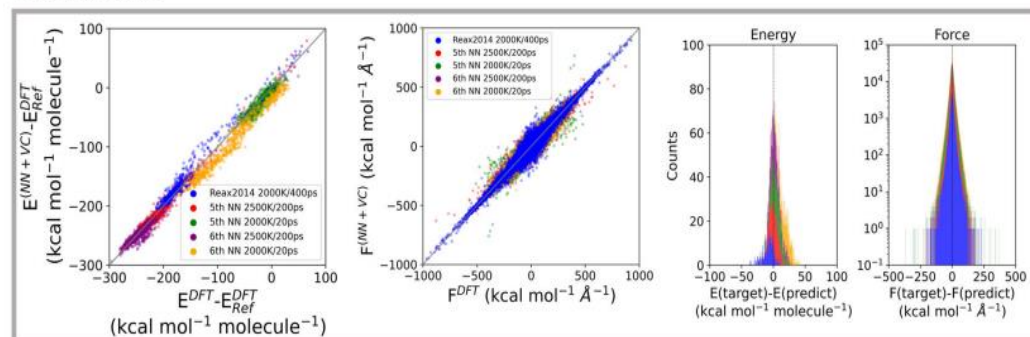
HDNNP+Iteration:

- Enrich dataset with NNMD trajectories
- Explore edge cases

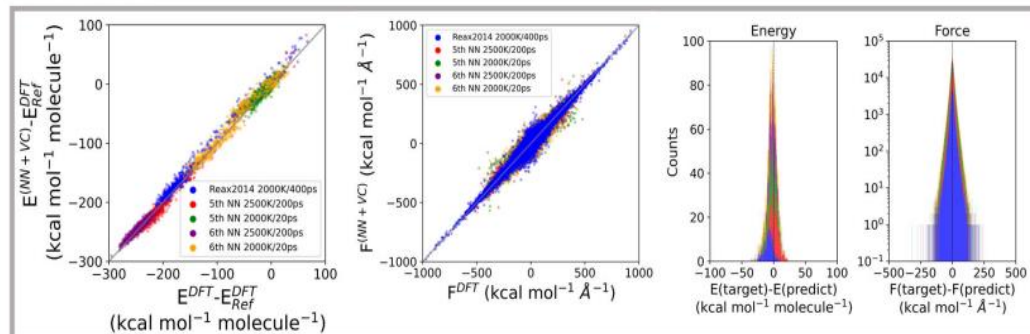
a Gen1.1



b Gen1.6



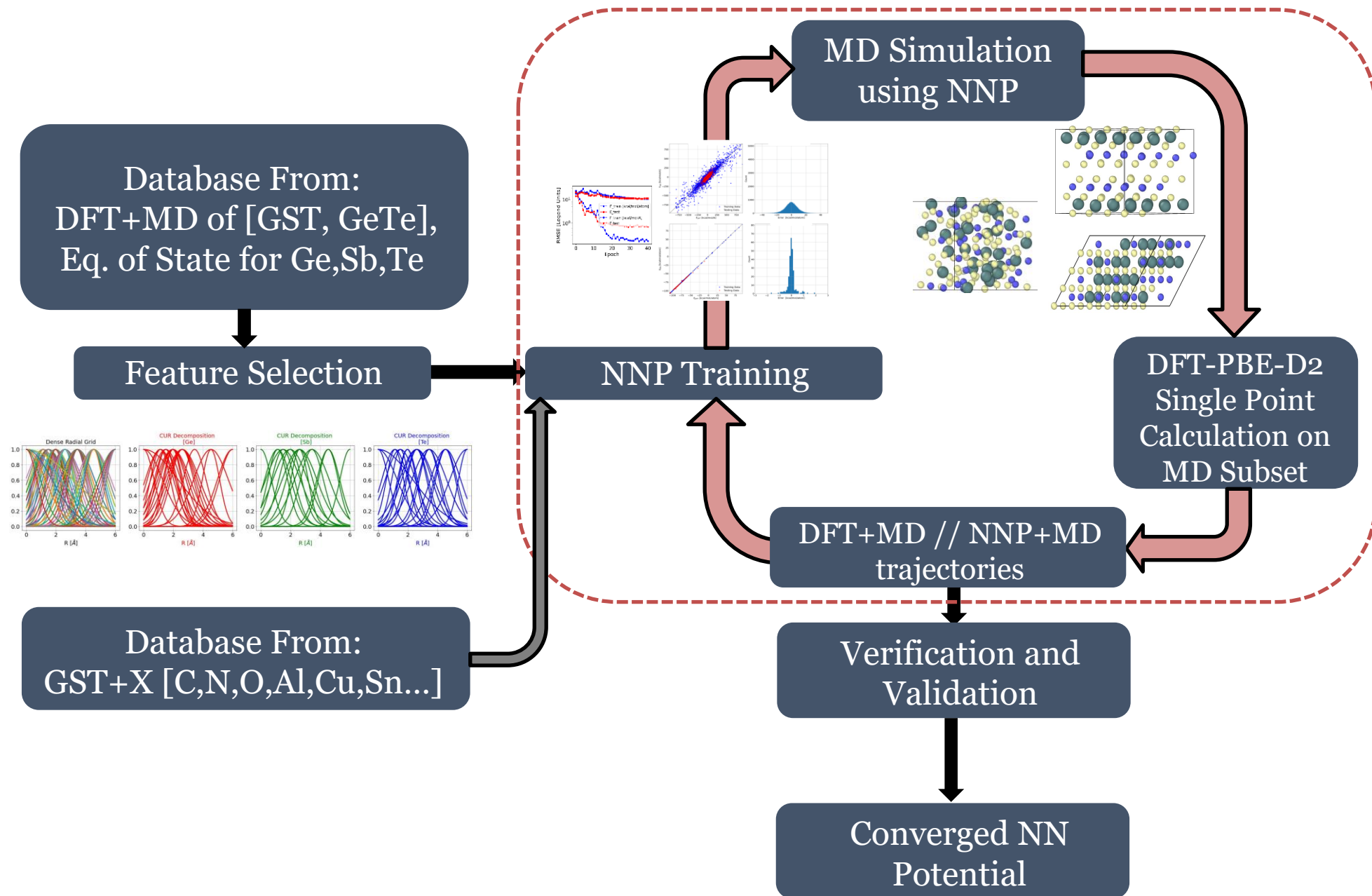
c Gen1.9



Evolution of NN potential

initialization

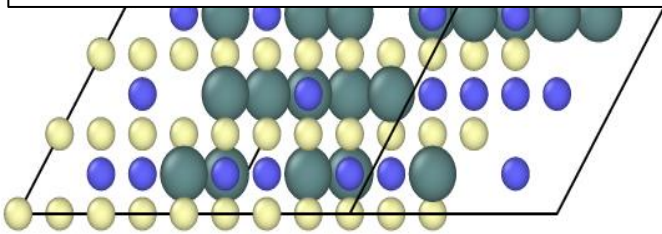
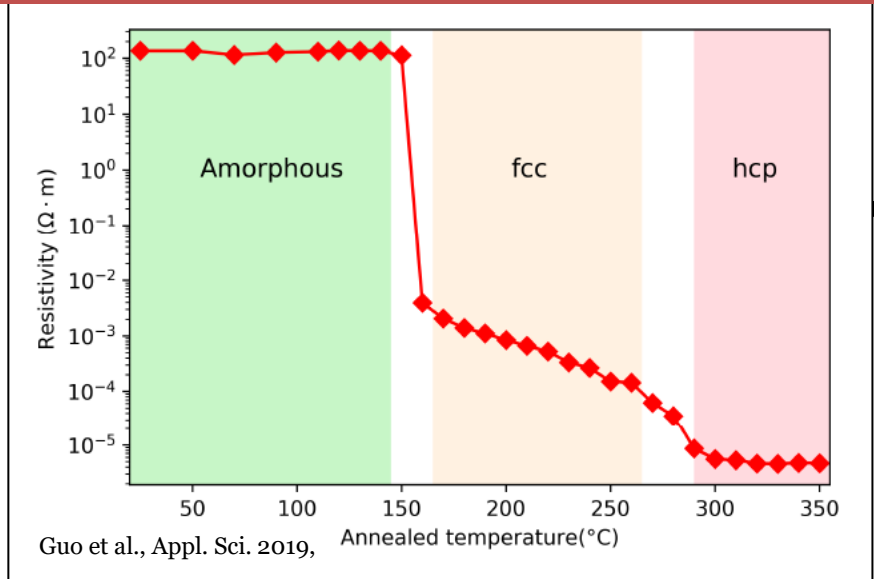
NNP Iterative Workflow



DFT Simulations – GGA – PBE+D2

Hexagonal

Three primary phases: Amorphous/Cubic
Rocksalt (FCC)/Hexagonal



$$T_m = 900 \text{ K}$$
$$T_{x\text{-tal}} = 423 \text{ K}$$

Xtal:

- $T=[400,600,800]\text{K}$
- Gather trajectories for each temp

GST+C:

- Create **cubic** structure with desired composition
- Melt & quench [see work by Robert Appleton]

Ab initio MD – NVT

Analyze and break down
free energy contributions

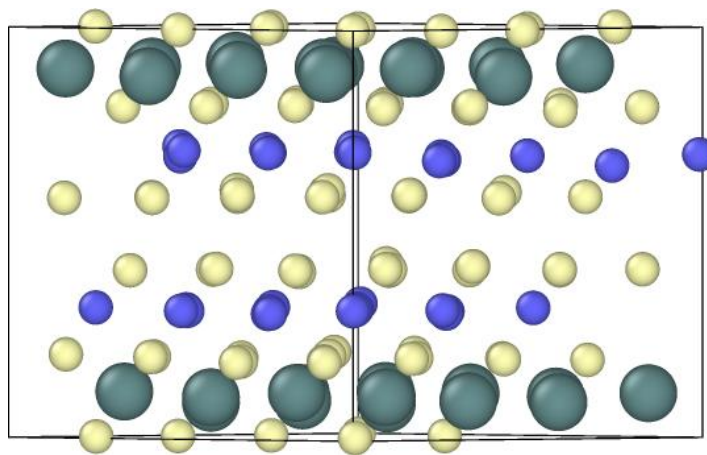
Amorph:

- Melt at 2000K
- Quench and repeat temps from Xtal

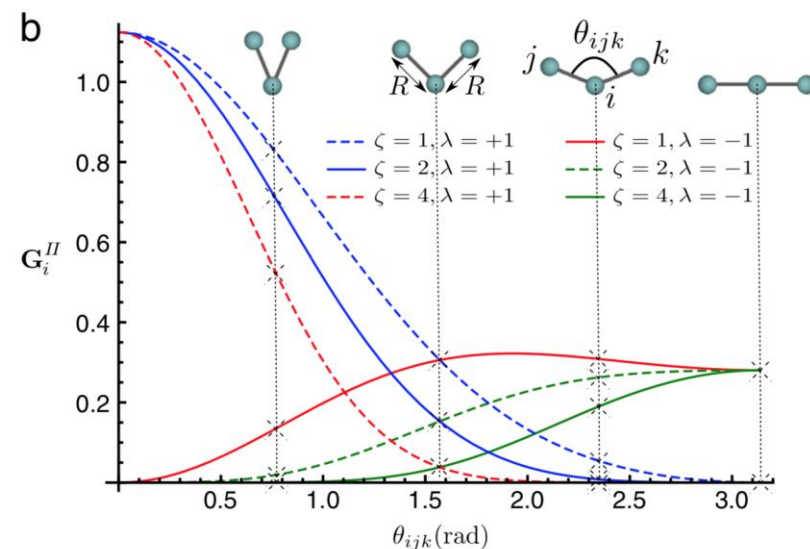
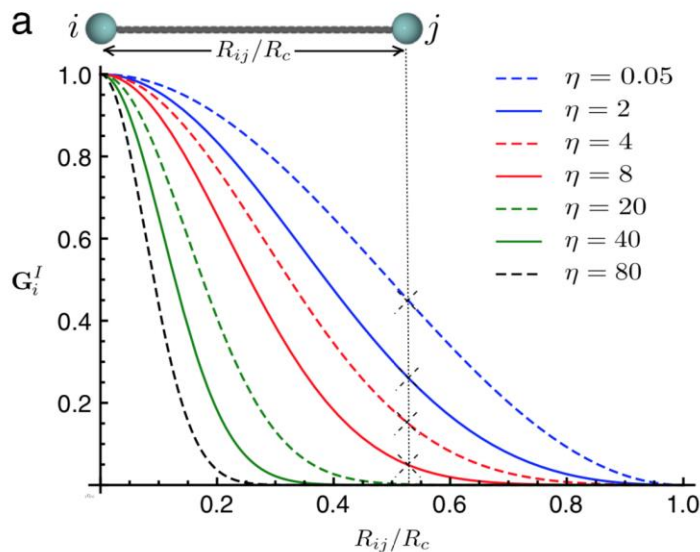
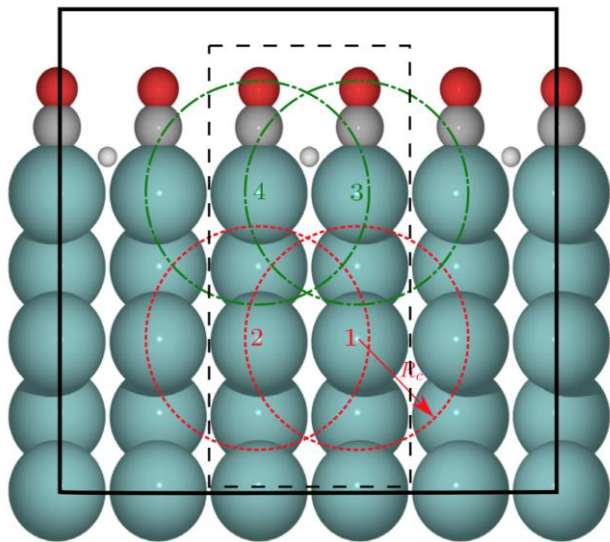
Run Params:

K-point grid: 1x1x1
Supercell: 144 atoms
KE cutoff: 300 eV

Using neural network potentials



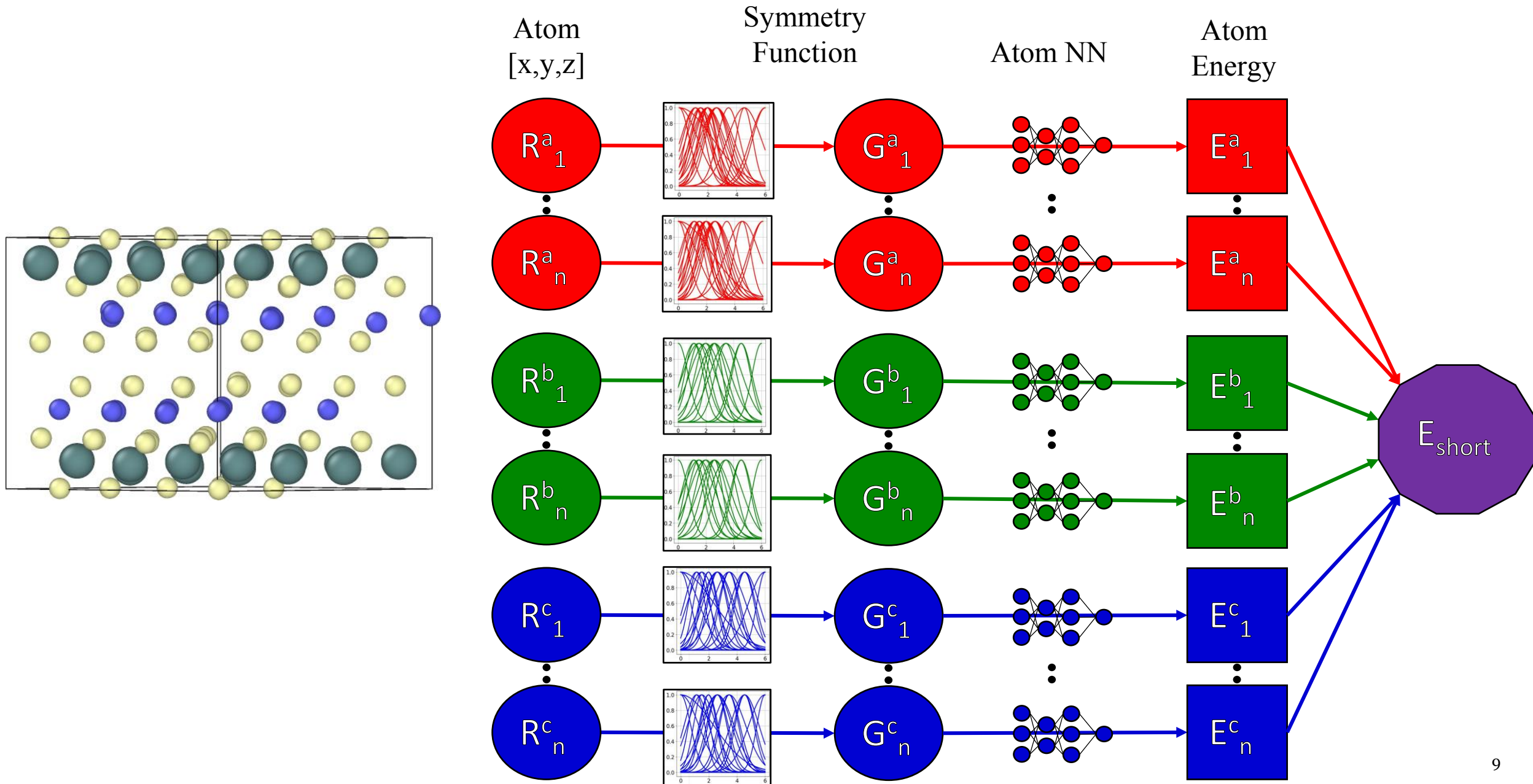
$$E_{\text{short}} = \sum_{N_i=1}^{N_{\text{elem}}} \sum_{j=1}^{N_{\text{atom}}^i} E_j^i$$



$$G_i^{w,rad} = \sum_{\substack{j \in R_c \\ j \neq i}}^{N_{\text{atom}} \in R_c} g(Z_j) e^{-\eta(R_{ij}-R_s)^2} f_c(R_{ij})$$

$$G_i^{w,ang} = 2^{1-\zeta} \sum_{\substack{j,k \neq i \\ j < k}} h(Z_j, Z_k) (1 + \lambda \cos \theta_{ijk})^\zeta e^{-\eta[(R_{ij}-R_s)^2 + (R_{ik}-R_s)^2 + (R_{jk}-R_s)^2]} \cdot f_c(R_{ij}) f_c(R_{ik}) f_c(R_{jk})$$

Using neural network potentials



Hyperparameters of features

A simple gaussian function with a cutoff radius R_c , a width tuner η , and a shift parameter R_s

$$G_i^{w,rad} = \sum_{j \neq i}^{N_{\text{atom}} \in R_c} g(Z_j) e^{-\eta(R_{ij}-R_s)^2} f_c(R_{ij})$$

A complex gaussian function with a cutoff radius R_c , a width tuner η , and a shift parameter R_s , now with a cosine function flipped by $\lambda \pm 1$, and band width tuned by ζ

$$G_i^{w,ang} = 2^{1-\zeta} \sum_{\substack{j,k \neq i \\ j < k}} h(Z_j, Z_k) (1 + \lambda \cos \theta_{ijk})^\zeta e^{-\eta[(R_{ij}-R_s)^2 + (R_{ik}-R_s)^2 + (R_{jk}-R_s)^2]} \cdot f_c(R_{ij}) f_c(R_{ik}) f_c(R_{jk})$$

How many features are we optimizing?

3 elements

G^{rad}

$R_s = 3x$

$\eta = 6x$

G^{ang}

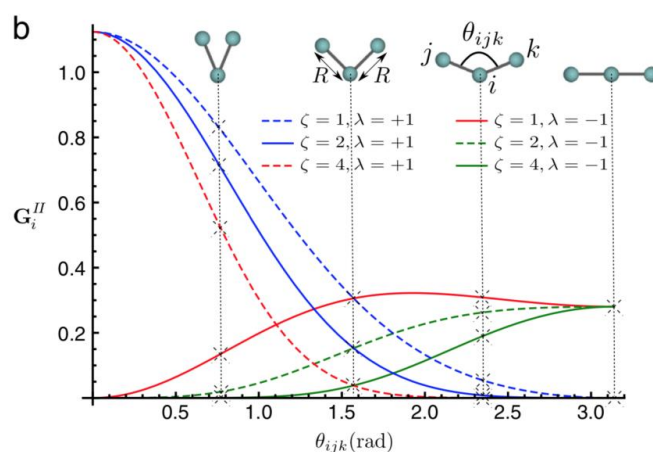
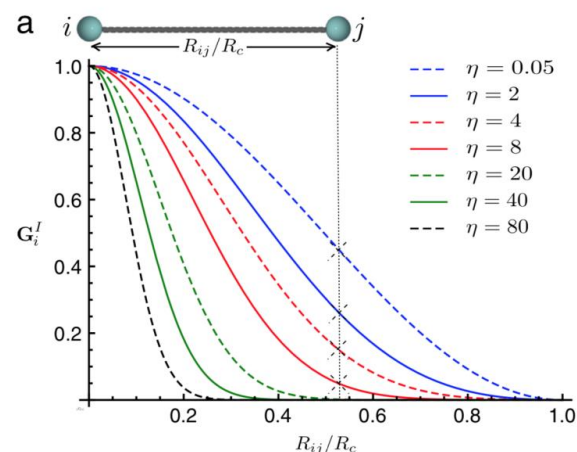
$\lambda = 2x$

$\zeta = 1x$

$R_s = 3x$

$\eta = 6x$

54 possible features /
element given initial
grid



Initial Selection Database:

Hex – GST – 225

- @ [300,400,600,800] K

Cub – GST – 225

- @ [300,400,600,800] K

Amorph – GST – 225

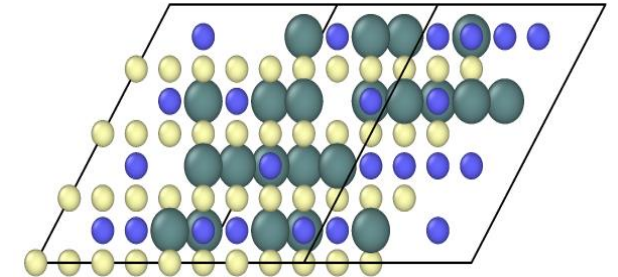
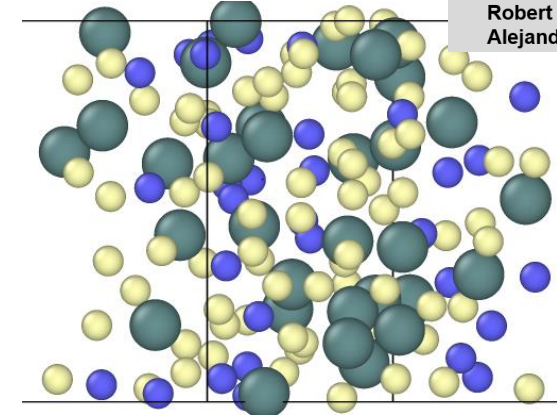
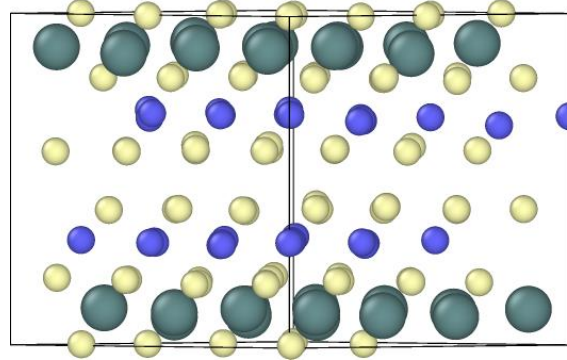
- @ [300,400,600,800] K

Liq - GST – 225

- @ [1100K, 2000] K

Liq & Crystalline GeTe

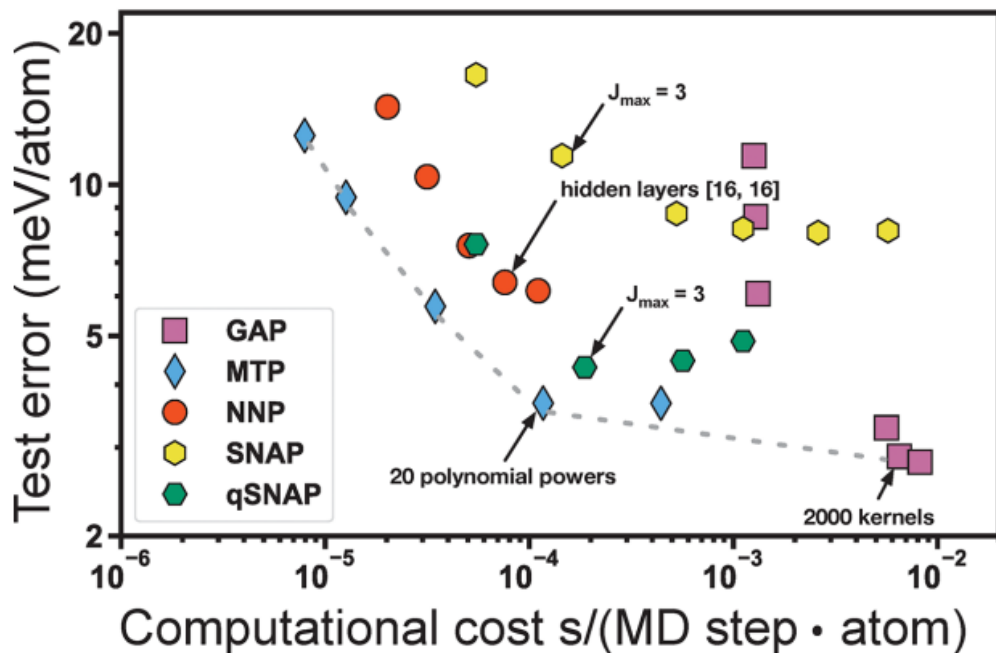
- @ varied temperatures



- Selected 1% of full DFT+MD database for initial training and calibration
- 90/10 train/test split for initial verification
- 2 Hidden Layers: [50/50]

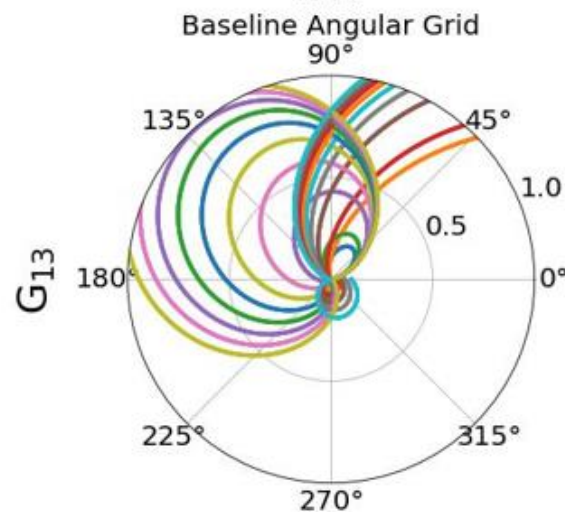
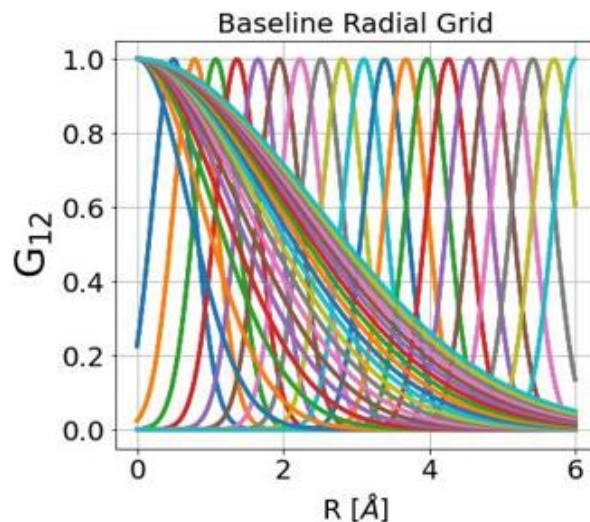
	Eq. of State [Ge,Sb,Te] S.P. DFT	GeTe MD+DFT	Hex. Ge ₂ Sb ₂ Te ₅ MD+DFT	Cub. Ge ₂ Sb ₂ Te ₅ MD+DFT	Amorph. Ge ₂ Sb ₂ Te ₅ MD+DFT	Liq. Ge ₂ Sb ₂ Te ₅ MD+DFT
Gen1.1	150	828	100	100	114	40

Limitation 1



Which symmetry functions
best describe my
environment?

What is my trade off for
description vs. computational
weight?



Limitations to overcome:

1. Feature selection
2. Architecture initialization

3 elements

G^{rad}

$R_s = 20 \times \text{constant } \eta$

$\eta = 18 \times \text{constant } R_s$

G^{ang}

$\lambda = 2 \times$

$\zeta = 1 \times$

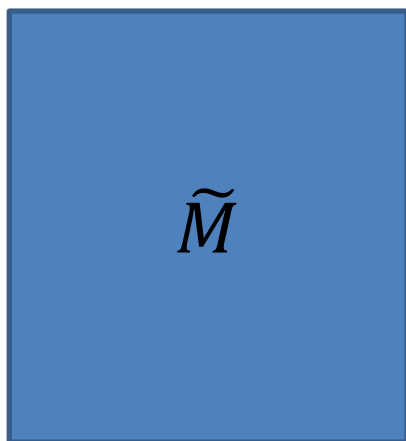
$R_s = 3 \times \text{constant } \eta$

$\eta = 3 \times \text{constant } R_s$

Solution 1: CUR Decomposition

$$\tilde{M} \approx CUR$$

Feature Matrix



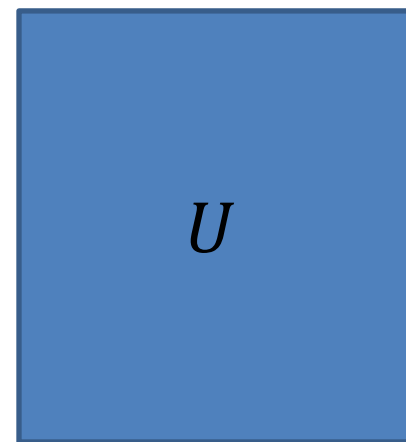
=

Elements



x

Scoring Matrix



x

Features



$$\pi_c = \sum_{j=1}^k (v_c^{(j)})^2.$$

Scaling functions from symmetry functions
used as weighting parameters in U

Highest expressivity, lowest overlap, most
unique fingerprints retained

Retain the original matrix

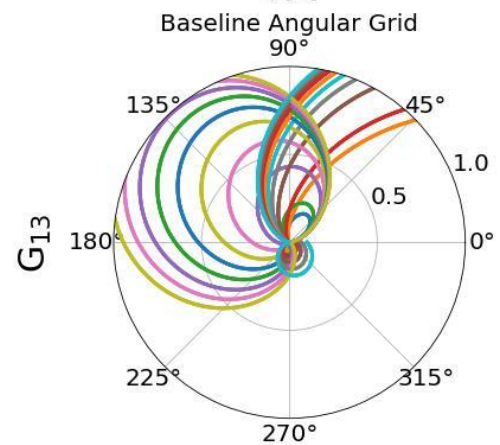
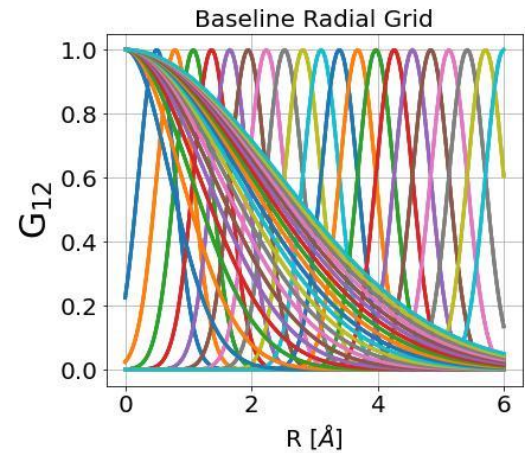
Automatic selection of atomic fingerprints and reference configurations for machine- learning potentials

Cite as: J. Chem. Phys. 148, 241730 (2018); <https://doi.org/10.1063/1.5024611>

Submitted: 02 February 2018 • Accepted: 10 April 2018 • Published Online: 30 April 2018

Giulio Imbalzano,  Andrea Anelli,  Daniele Giofré, et al.

Solution 1: CUR Decomposition

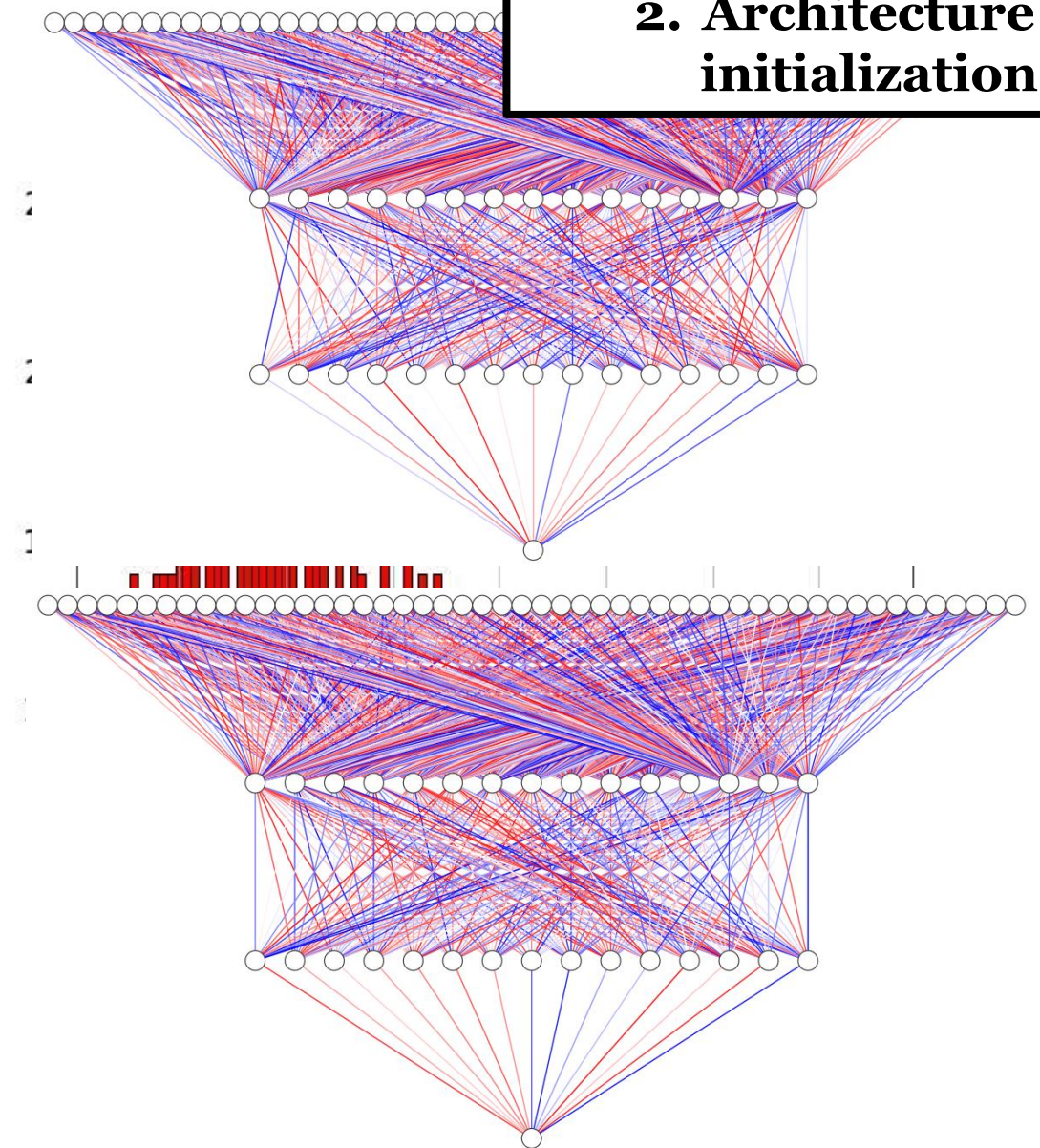
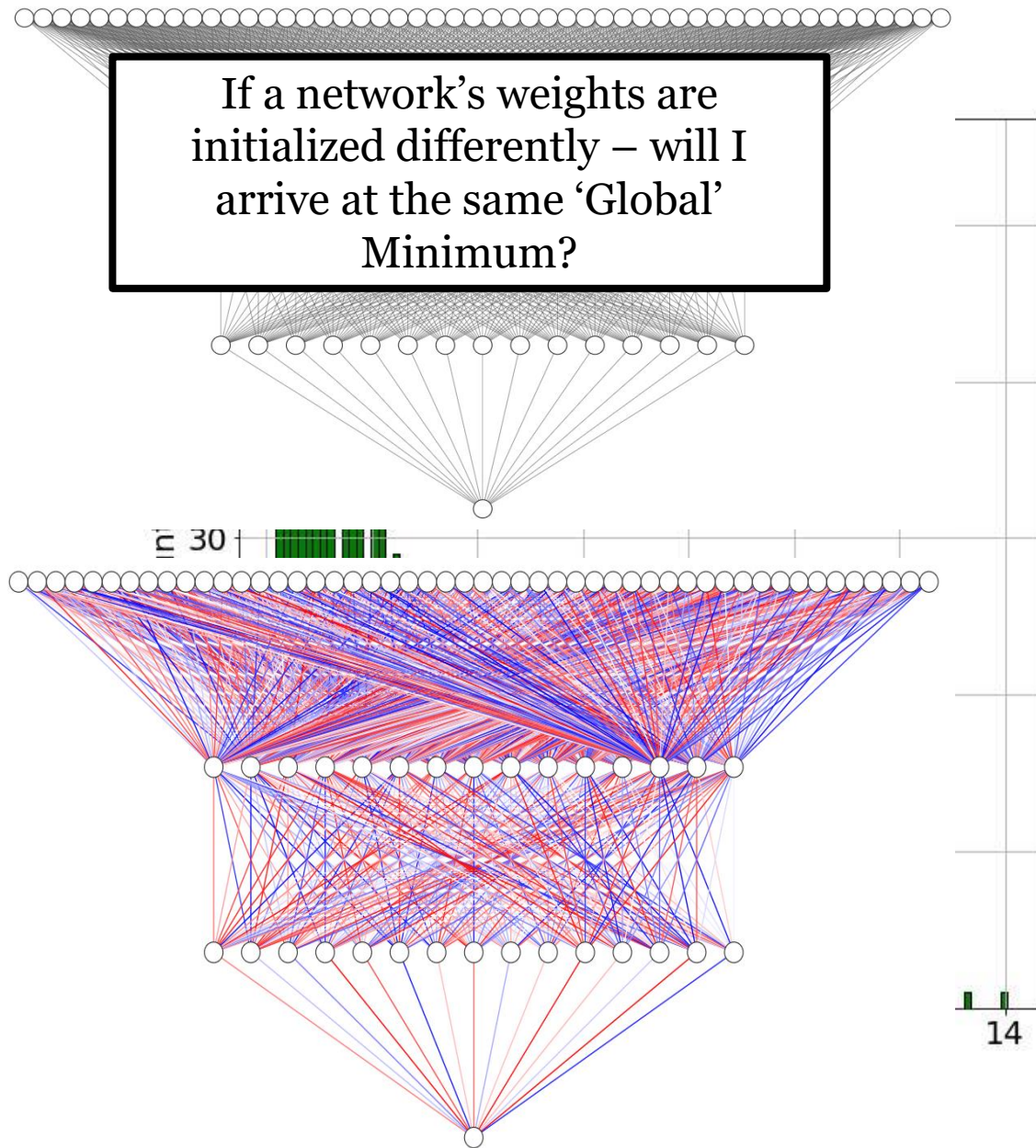


Sampling 1000 networks

Limitations to overcome:

1. Feature selection
- 2. Architecture initialization**

If a network's weights are initialized differently – will I arrive at the same 'Global' Minimum?



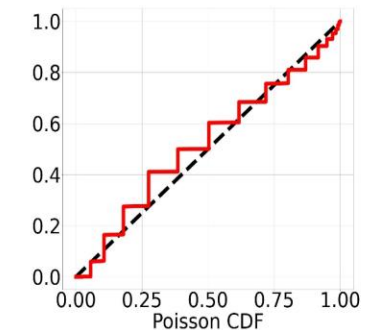
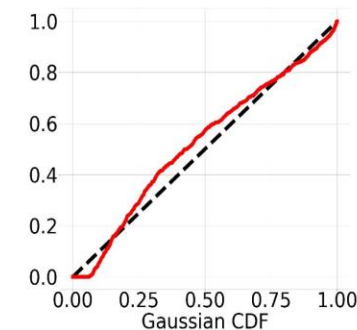
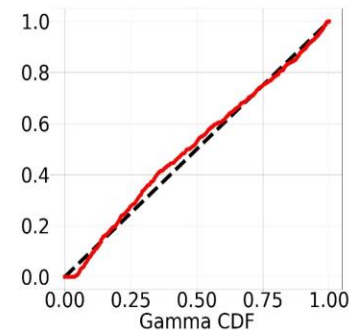
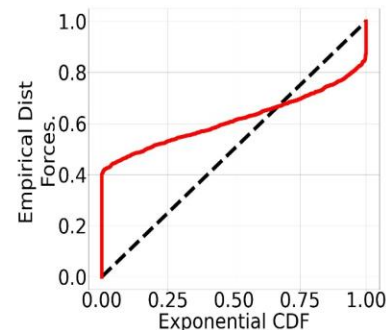
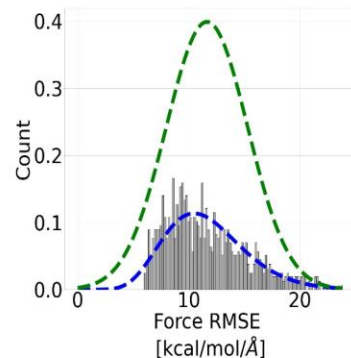
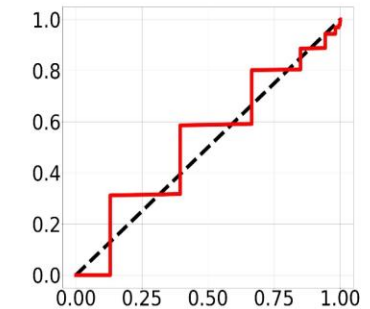
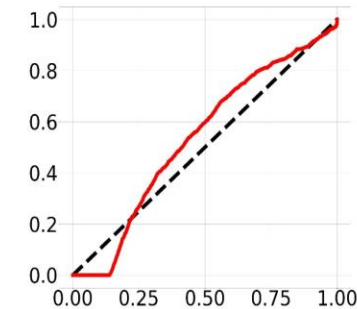
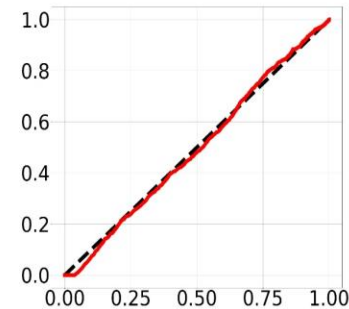
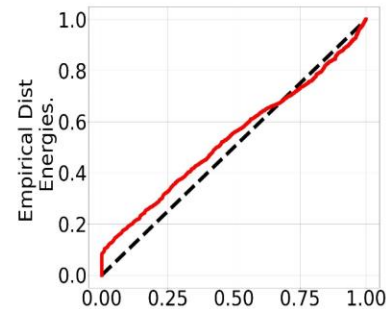
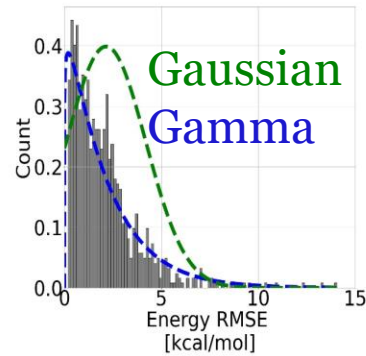
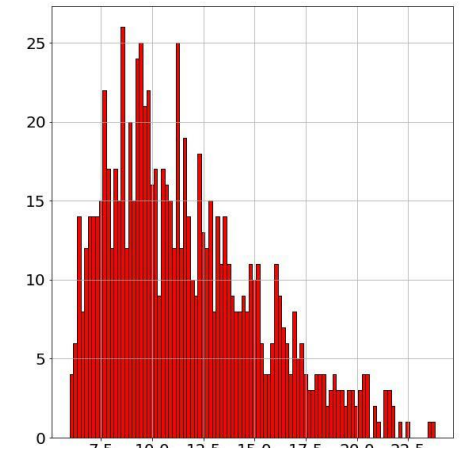
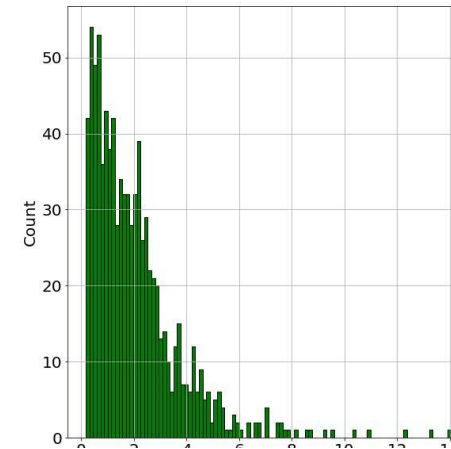
Characterizing the probability distribution

Distribution Type Hypothesis:

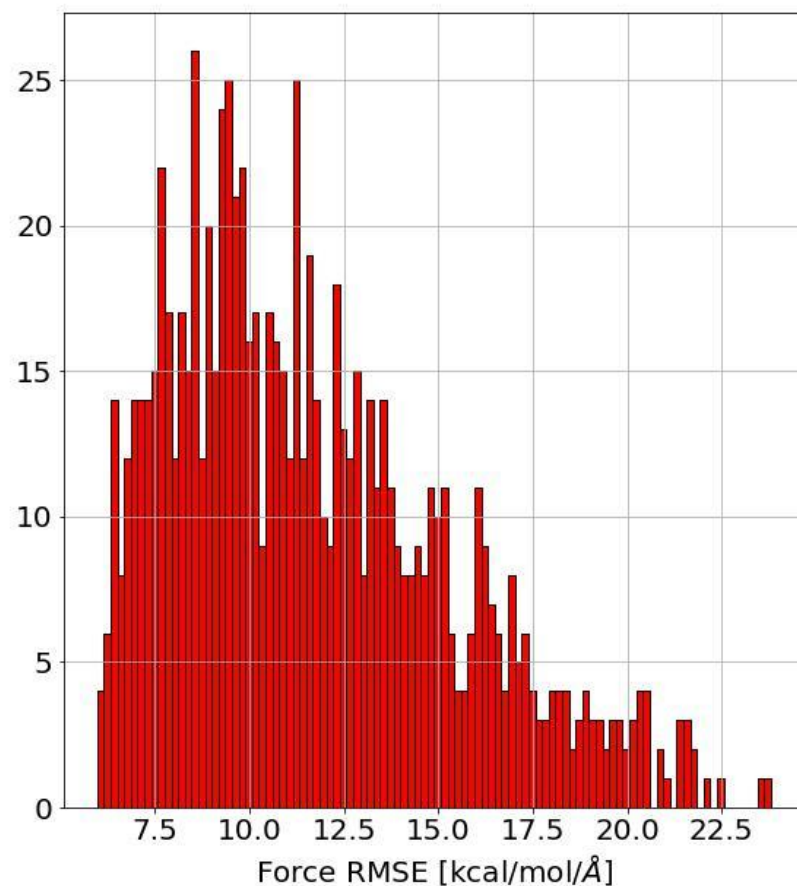
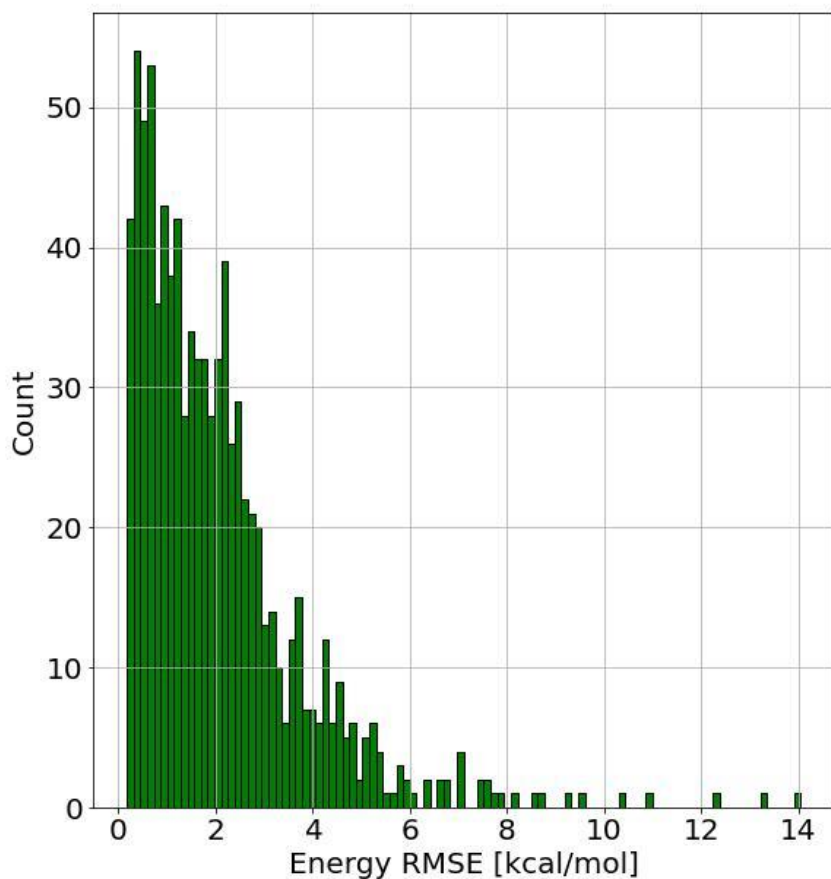
- Exponential
- Gamma
- Gaussian
- Poisson

Test with Cumulative
Distribution Function:

$$F_x(x) = P(X \leq x)$$



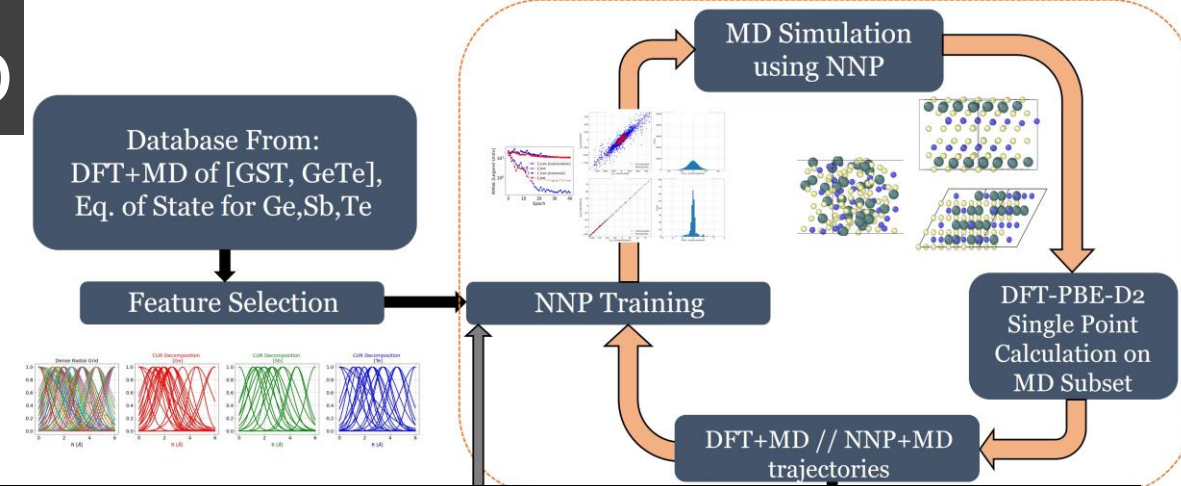
The Rule of 5



If I pick 5 different initializers, the probability of me
selecting a value below the mean is XX
[the math needs to be done before the conference]

Generational Training Roadmap

- For each NNP+MD run create isotherm
- 144 atom configurations mapped from DFT structures
- 1 ns simulation time || sample 10 trajectories



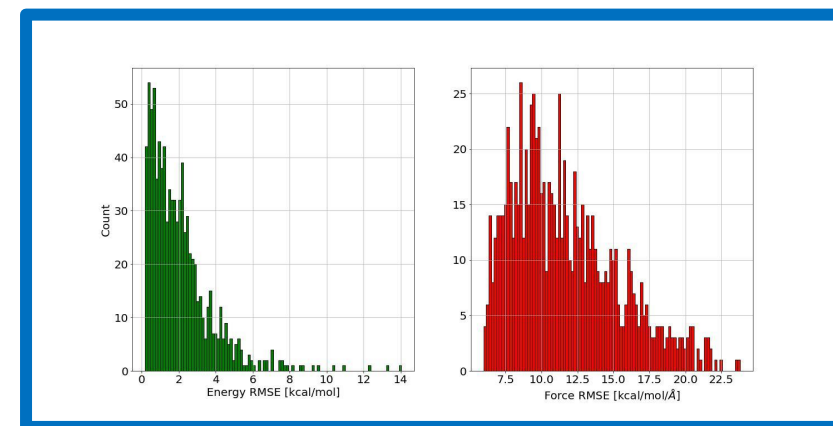
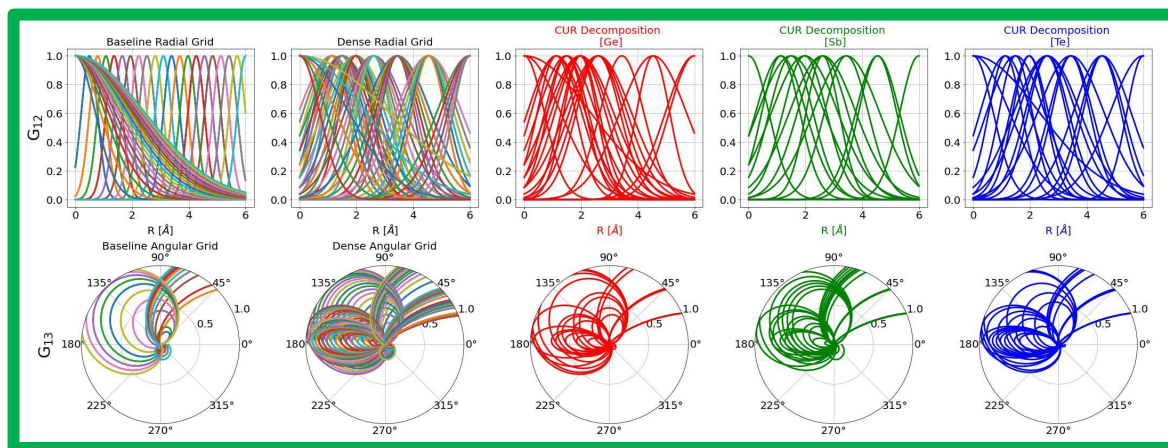
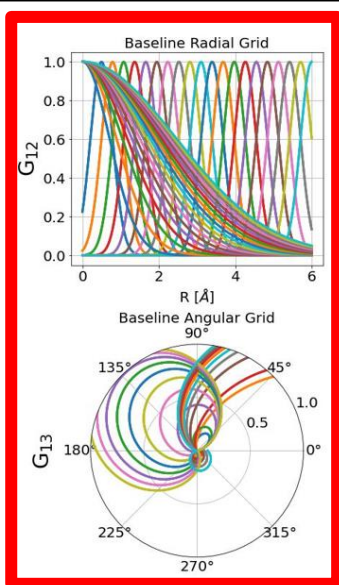
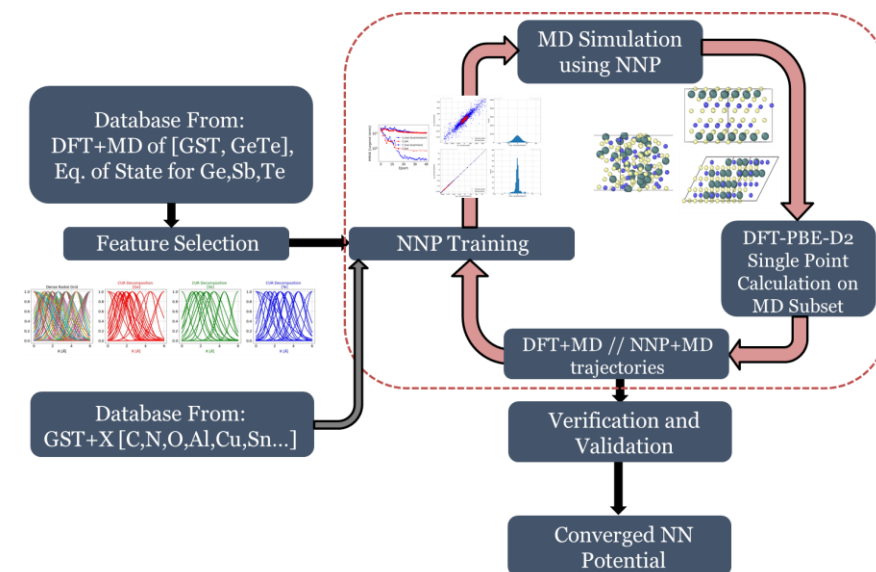
DFT+MD Database	Gen1.1	Gen1.2	Gen1.3	Gen1.4	Gen1.5	Gen1.6	Gen1.7	Gen1.8	Gen1.9	Gen1.10	Gen1.11	Gen1.12	Gen1.13	Gen1.14	Gen1.15
Ge/Sb/Te															
GeTe															
h-GST															
c-GST															
a-GST[6.2]								300K						2000K	
a-GST[6.11]															
a-GST[5.88]															
l-GST															

Generational Training Roadmap

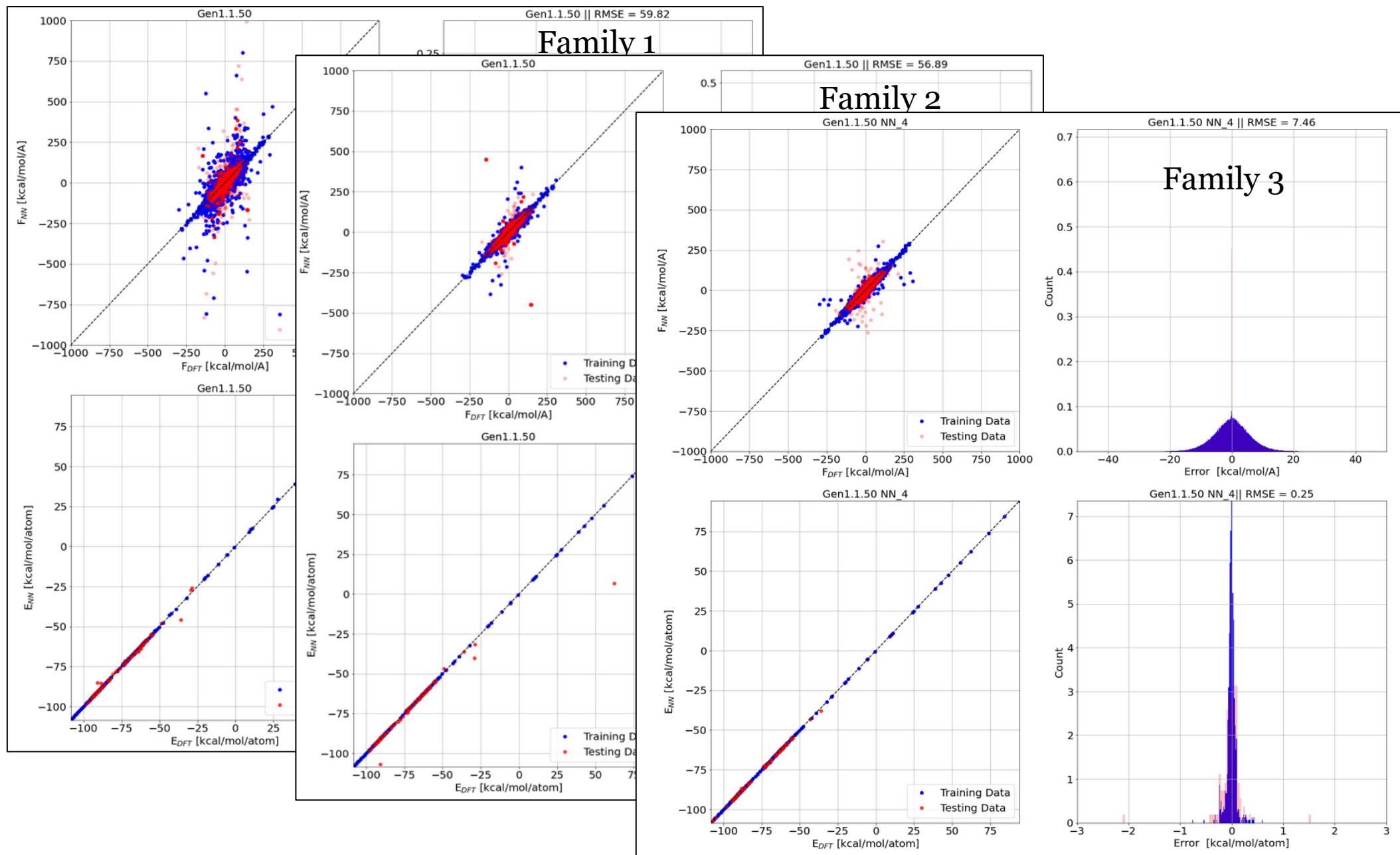
Family 1: Grid of features, use one network per iteration

Family 2: CUR selected features, use one network per iteration

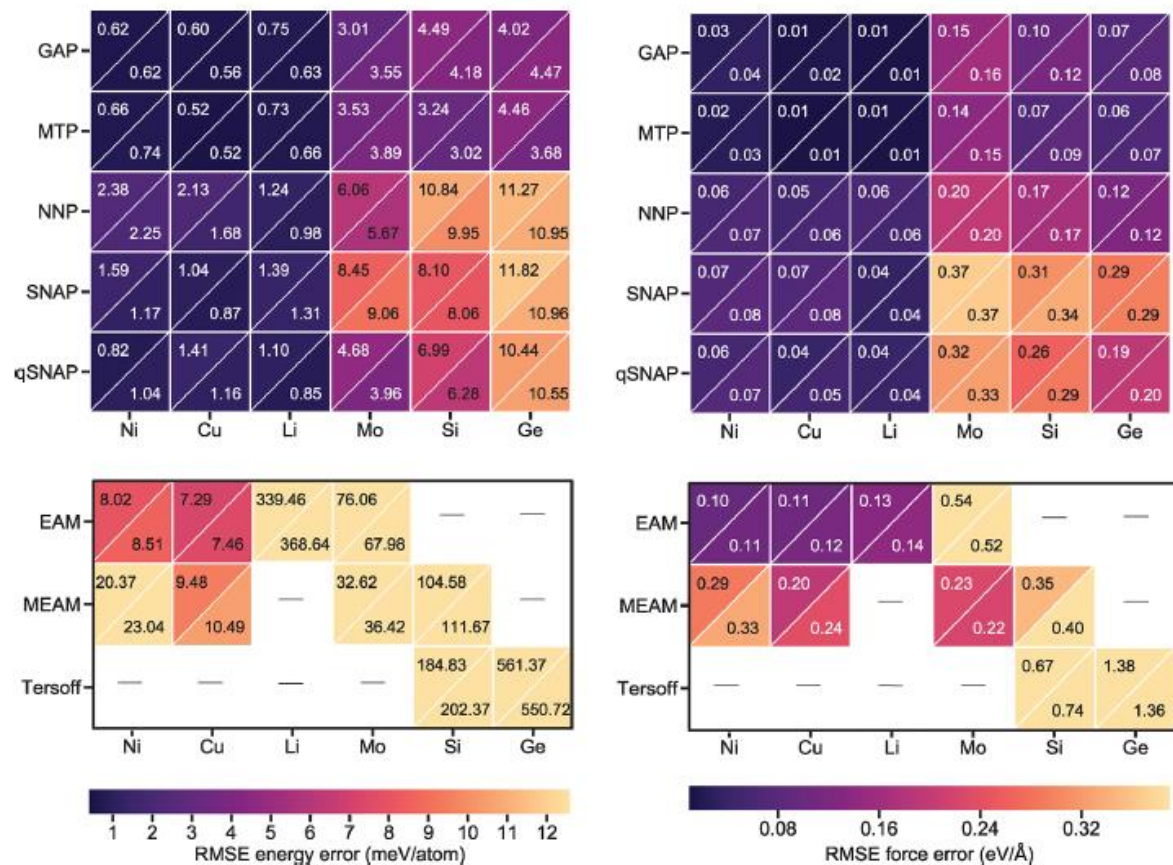
Family 3: CUR selected features, use five networks per iteration



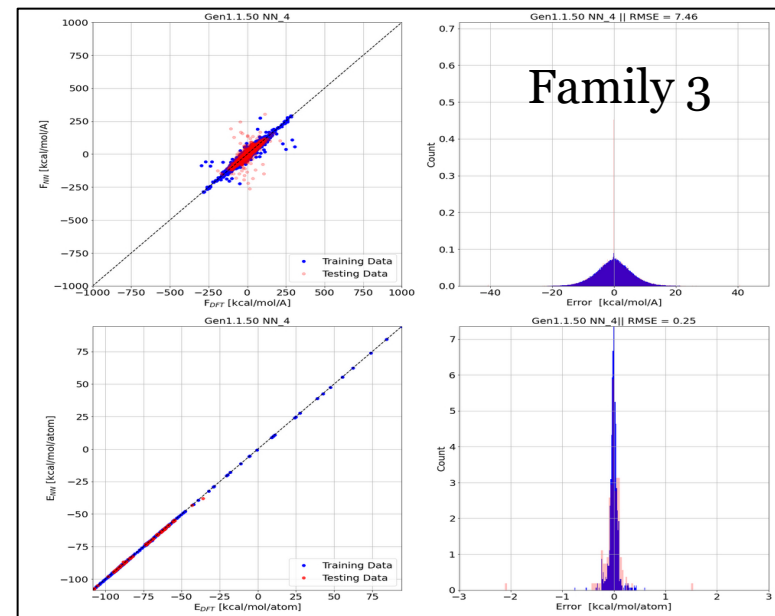
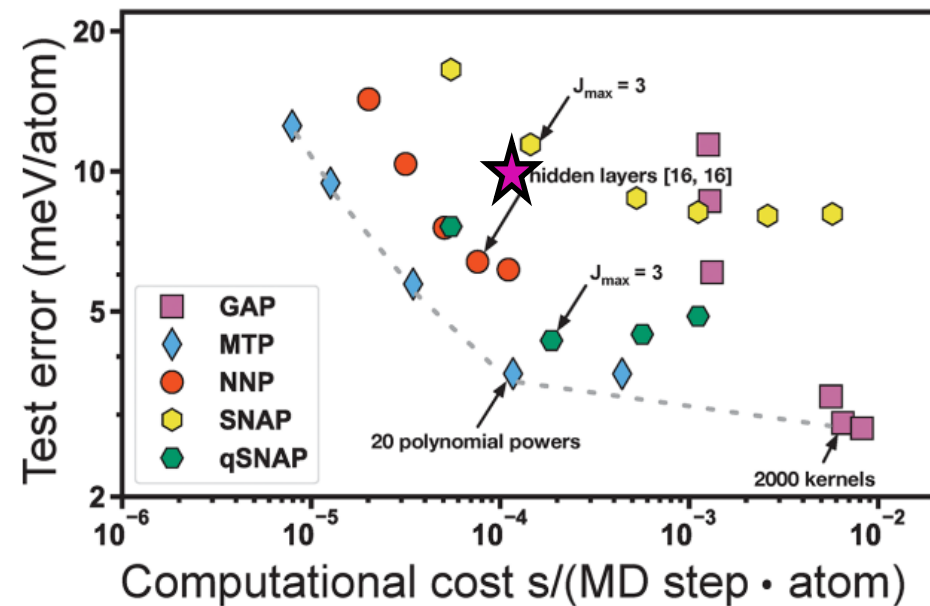
Baseline Training



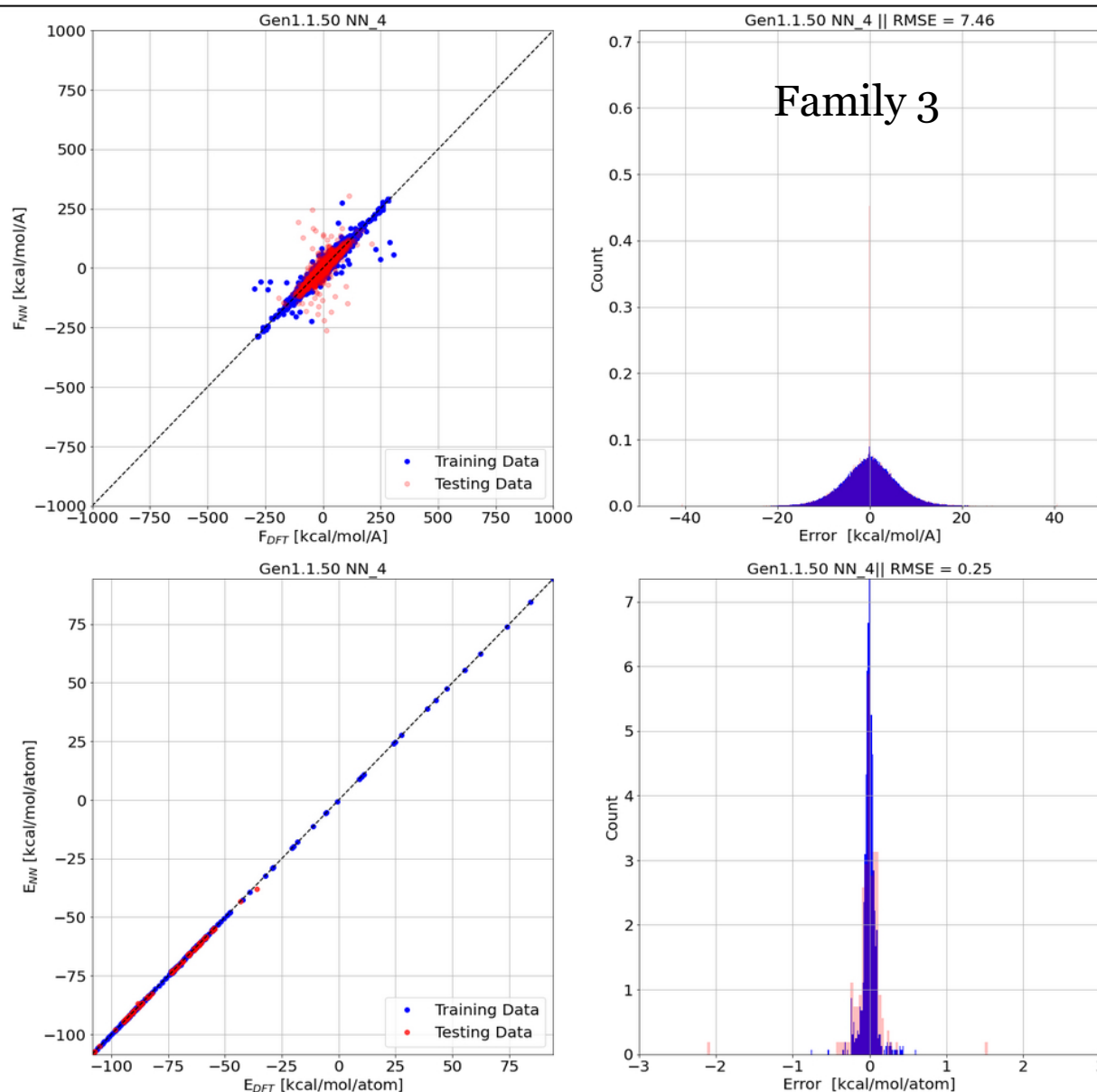
How do we compare?



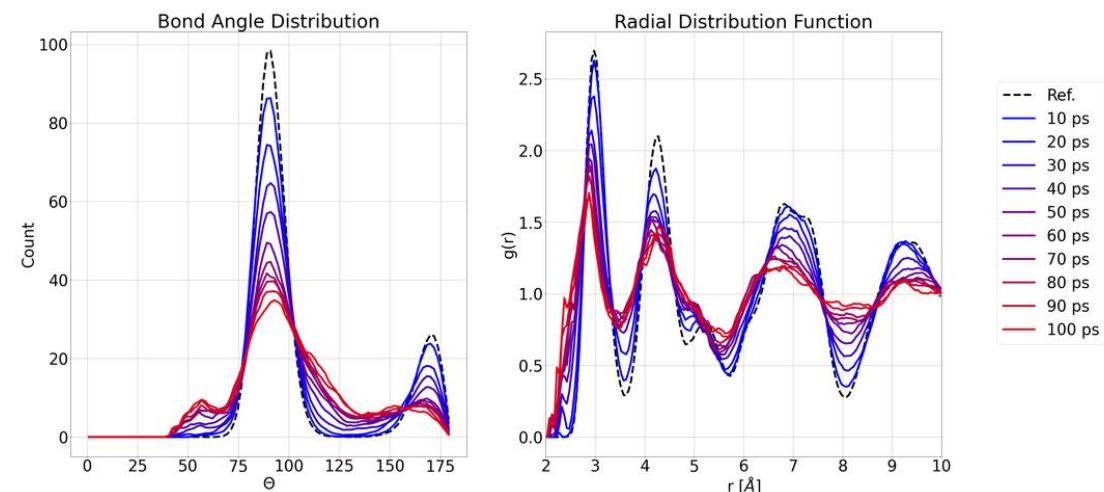
NNP GeSbTe
 Force RMSE: 0.32 eV/Å
 Energy RMSE: 10.87 meV/atom



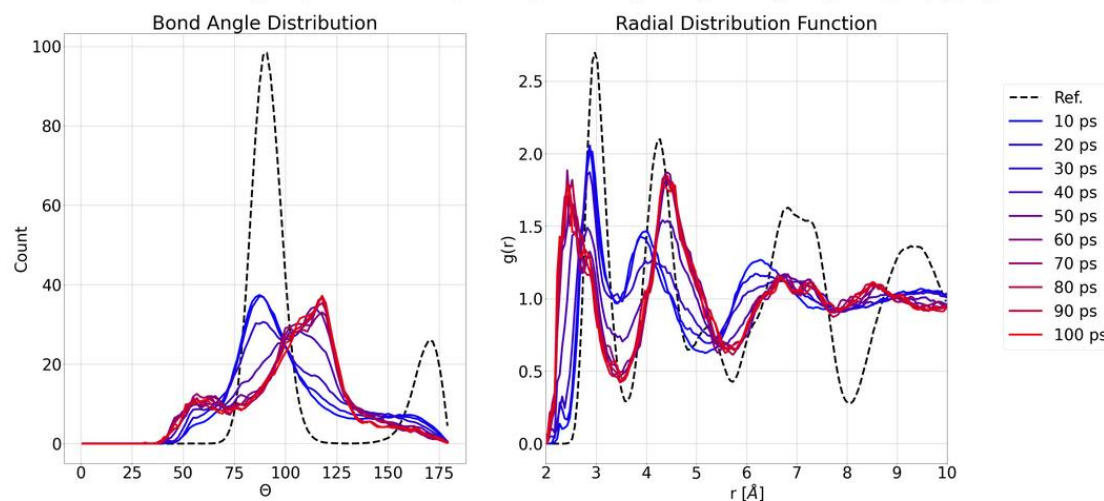
Training Gen1.1 – Molecular Dynamics



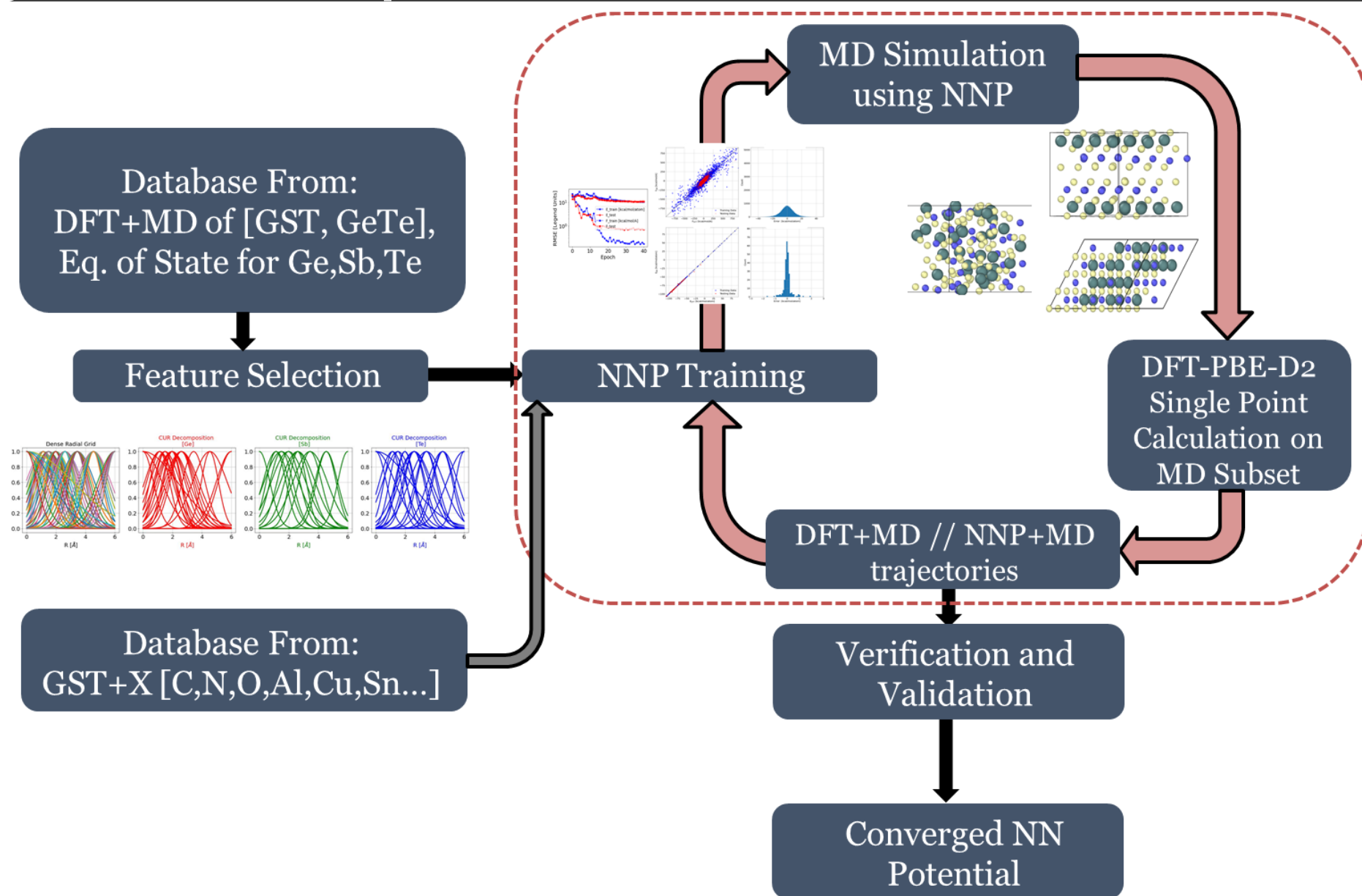
Stabilizing the Cubic Phase [X]



Amorphous to Xtal at 600K [X]

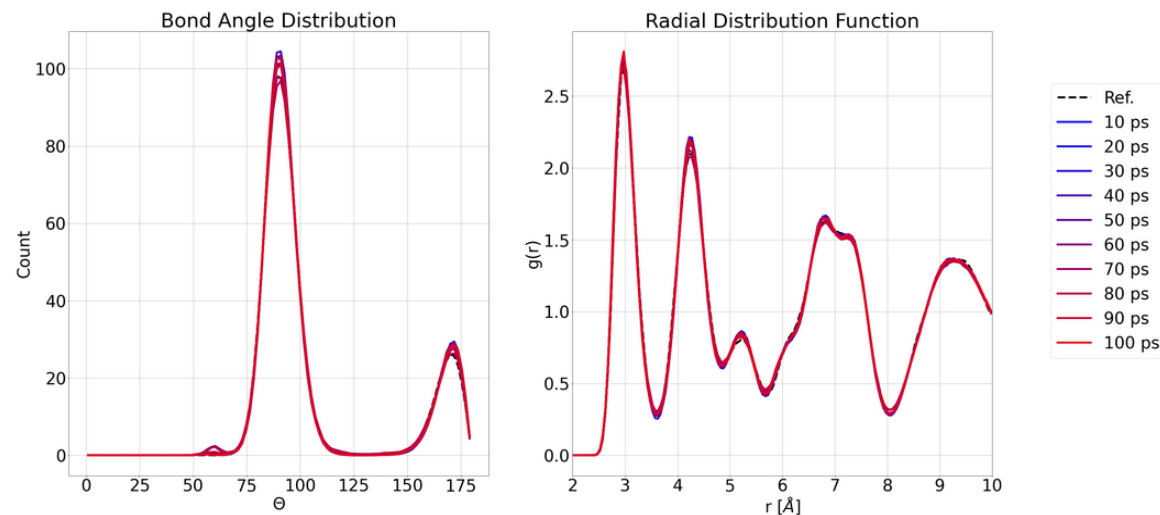
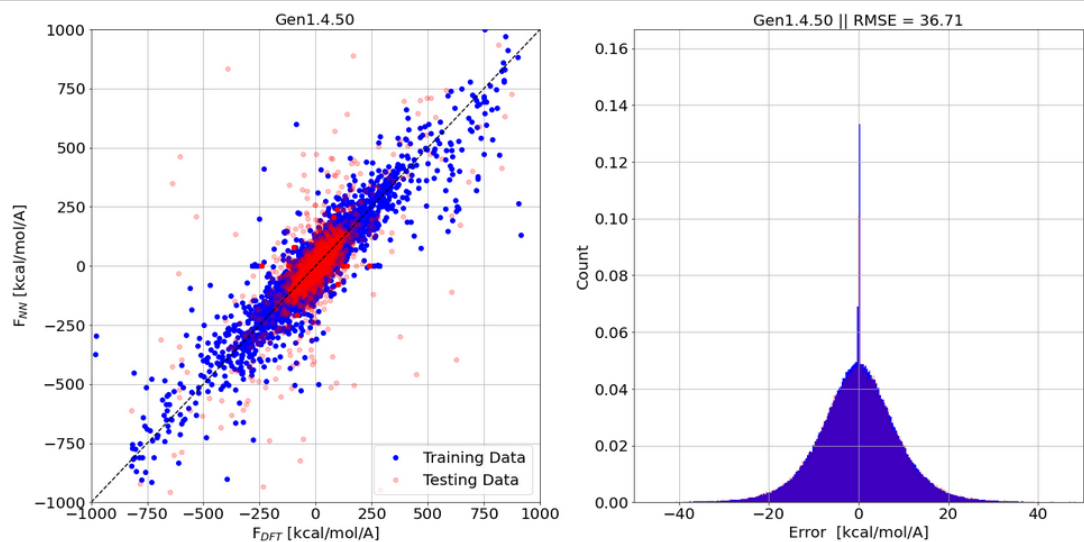


Begin the loop!

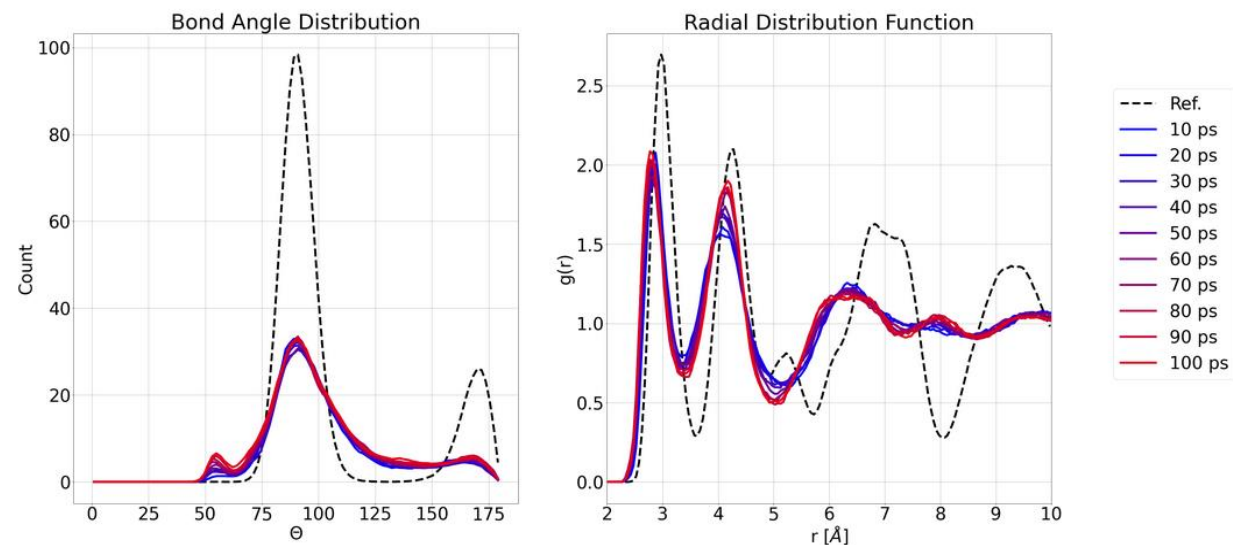
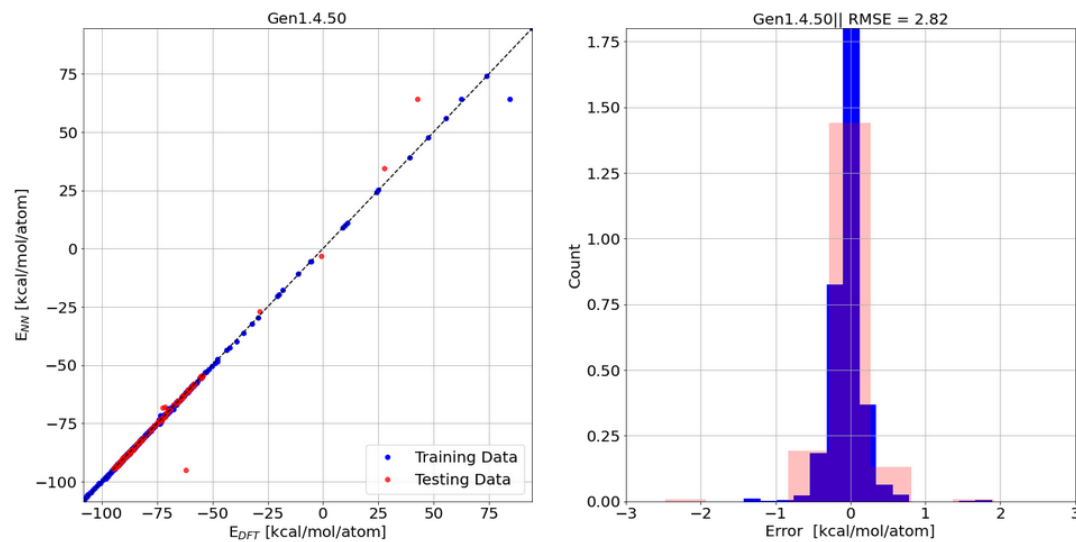


Training Gen1.4

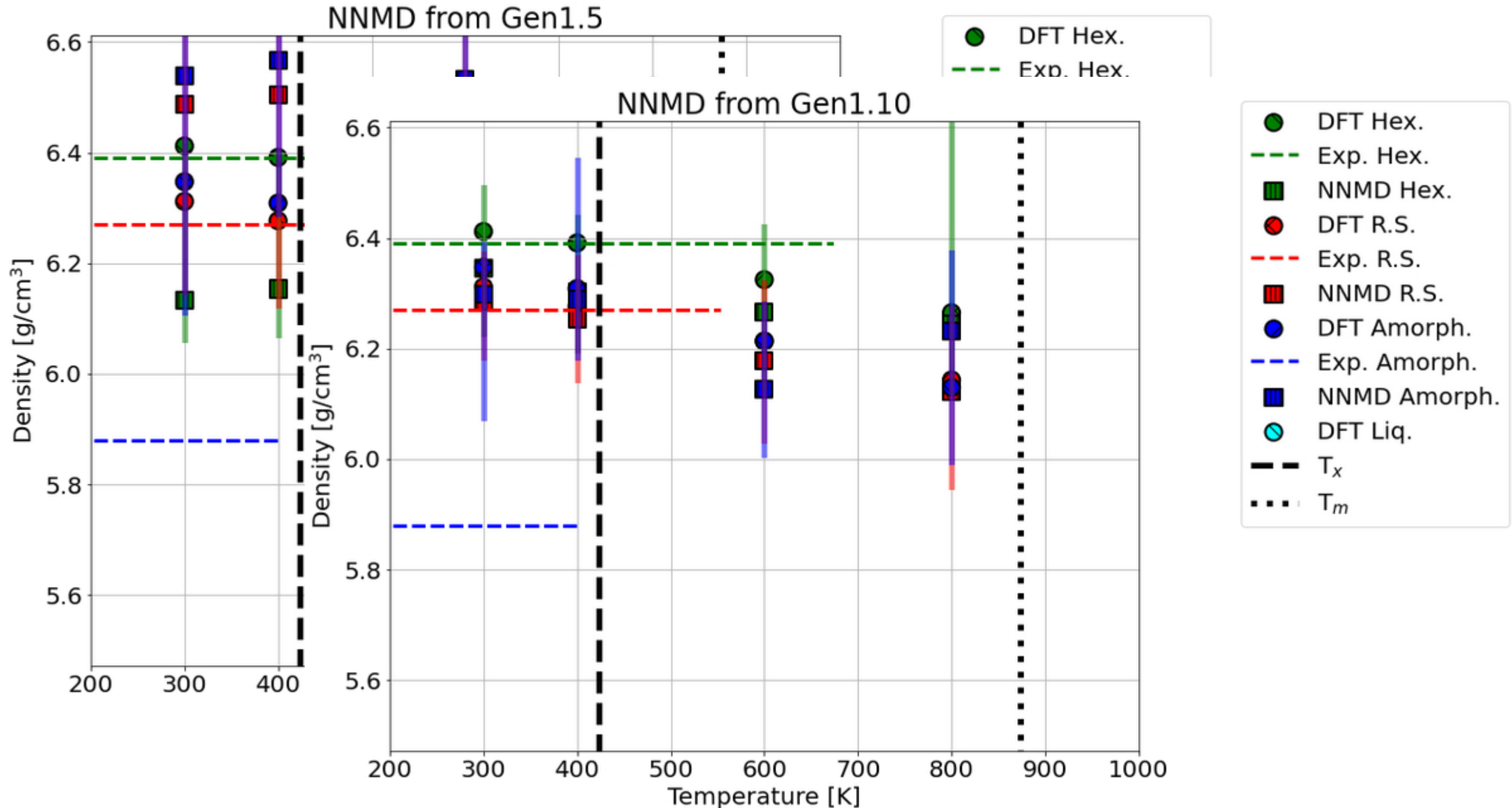
Stabilizing the Cubic Phase [✓]



Amorphous to Xtal at 600K [X]



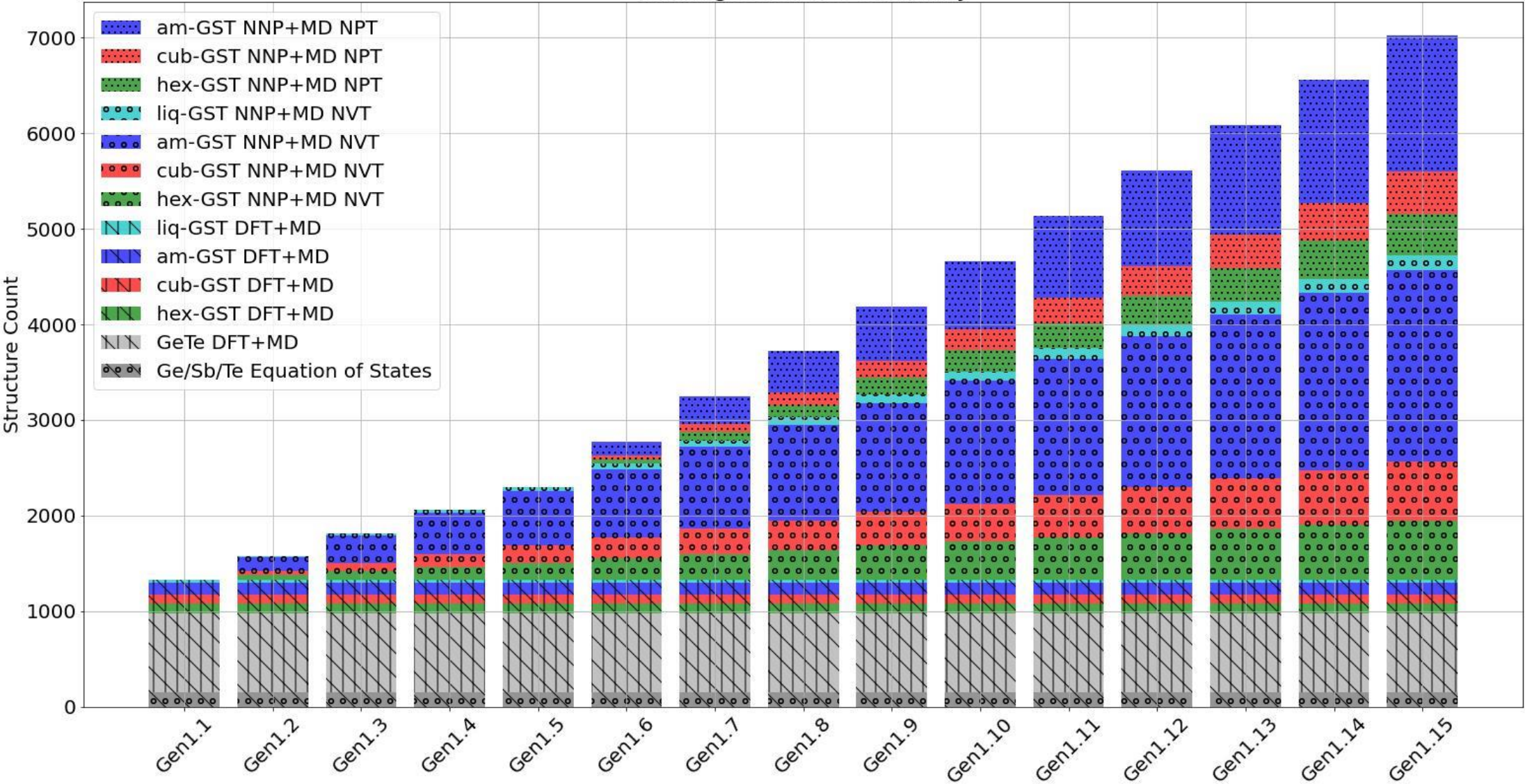
Switching to NPT and Evaluating Stability



Database Composition

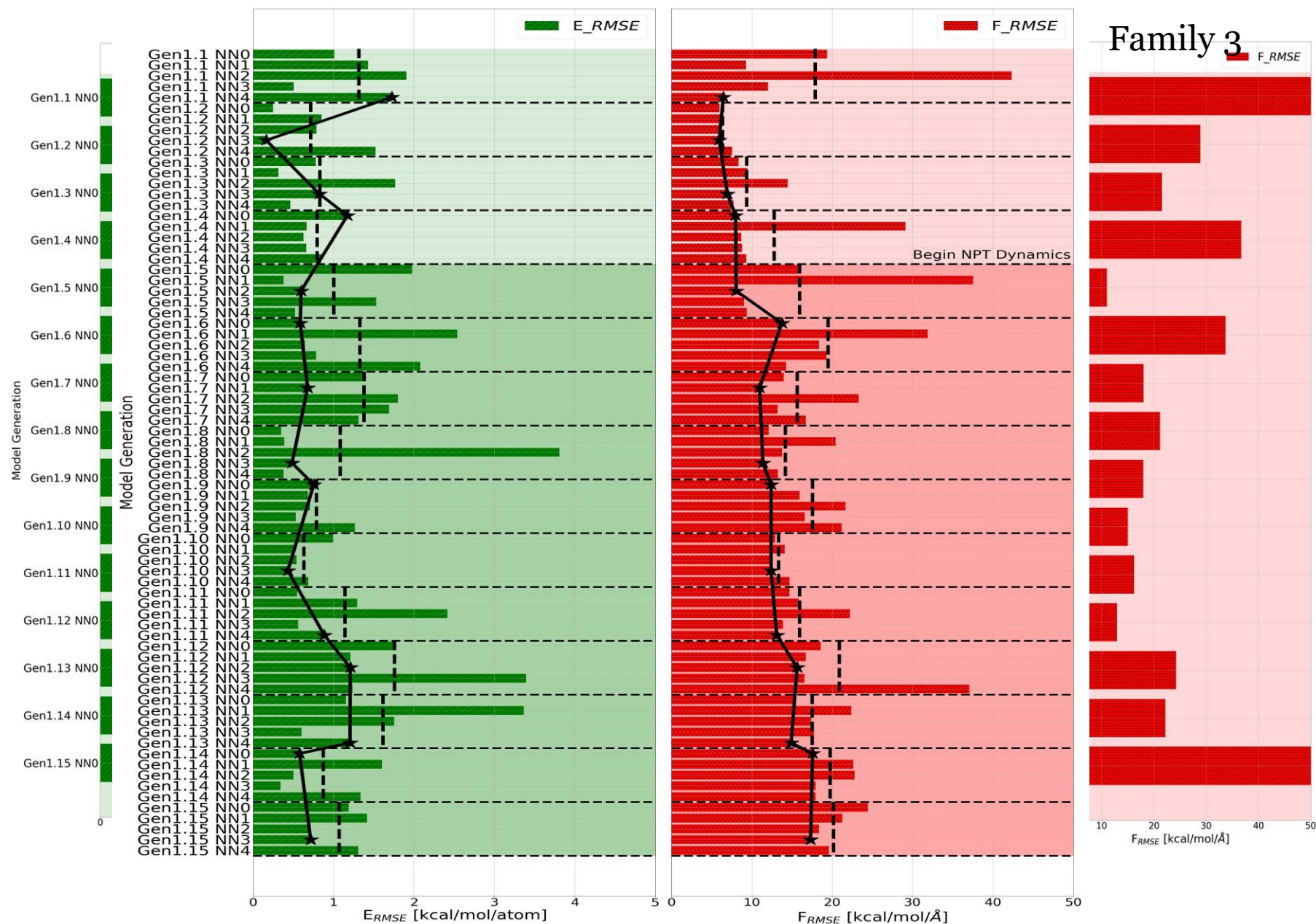
	Eq. of State [Ge,Sb,Te] S.P. DFT	GeTe MD+DFT	Hex. Ge ₂ Sb Te ₅ MD+DFT	Cub. Ge ₂ Sb Te ₅ MD+DFT	Amorph. Ge ₂ Sb ₂ Te ₅ MD+DFT	Liq. Ge ₂ Sb Te ₅ MD+DFT	Hex. Ge ₂ Sb Te ₅ NVT	Cub. Ge ₂ Sb Te ₅ NVT	Amorph. Ge ₂ Sb ₂ Te ₅ NVT	Liq. Ge ₂ Sb Te ₅ NVT	Hex. Ge ₂ Sb Te ₅ NPT	Cub. Ge ₂ Sb Te ₅ NPT	Amorph. Ge ₂ Sb ₂ Te ₅ NPT
Gen1.1	150	828	100	100	114	40	0	0	0	0	0	0	0
Gen1.2	150	828	100	100	114	40	44	44	143	11	0	0	0
Gen1.3	150	828	100	100	114	40	88	88	286	22	0	0	0

Training Database Summary

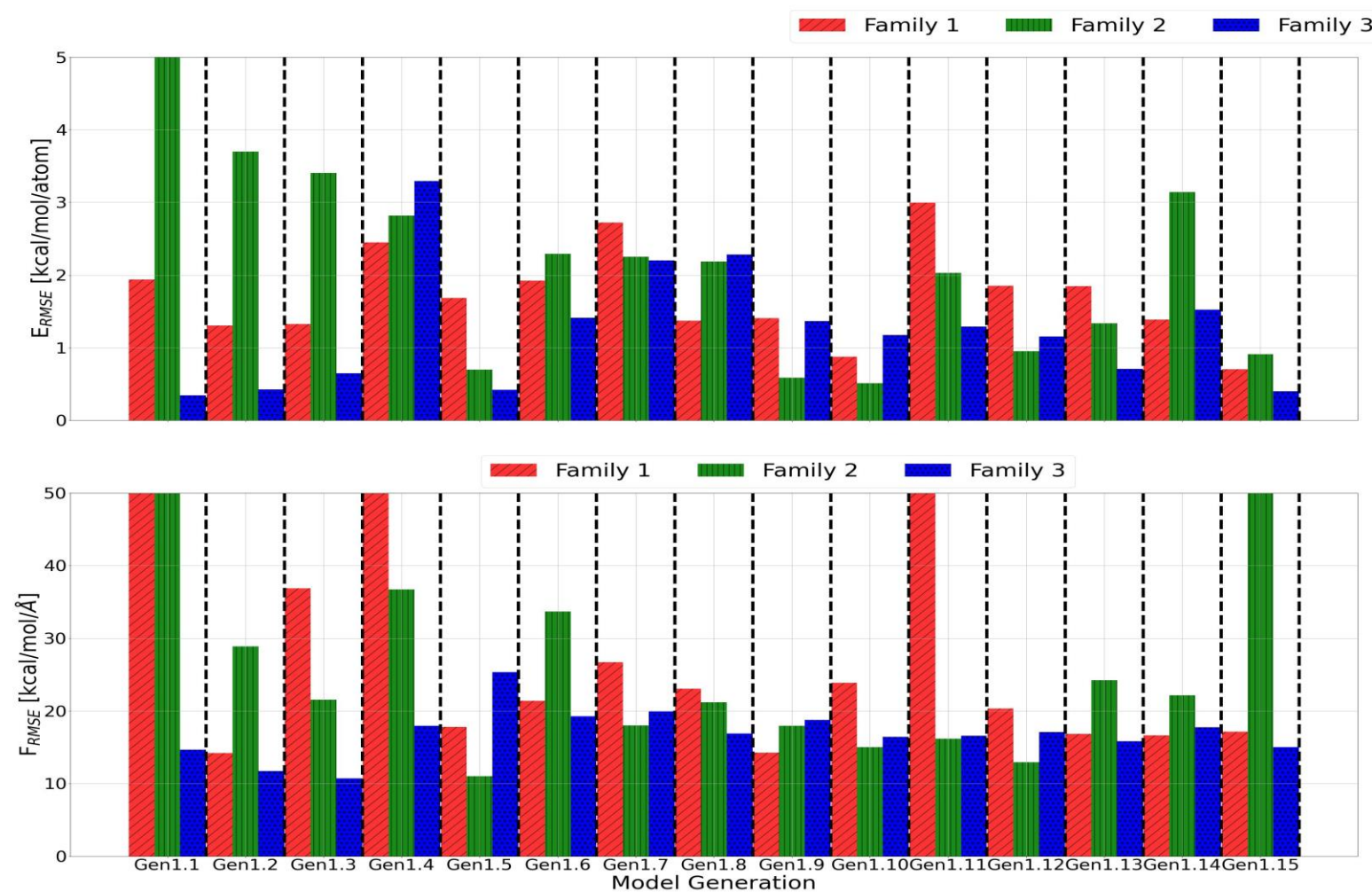


33	0	0	0
44	0	0	0
55	44	44	143
66	88	88	284
77	132	132	427
88	176	176	570
99	220	220	713
110	264	264	856
121	308	308	999
132	352	352	1142
143	396	396	1285
154	440	440	1428

Comparing Three Families



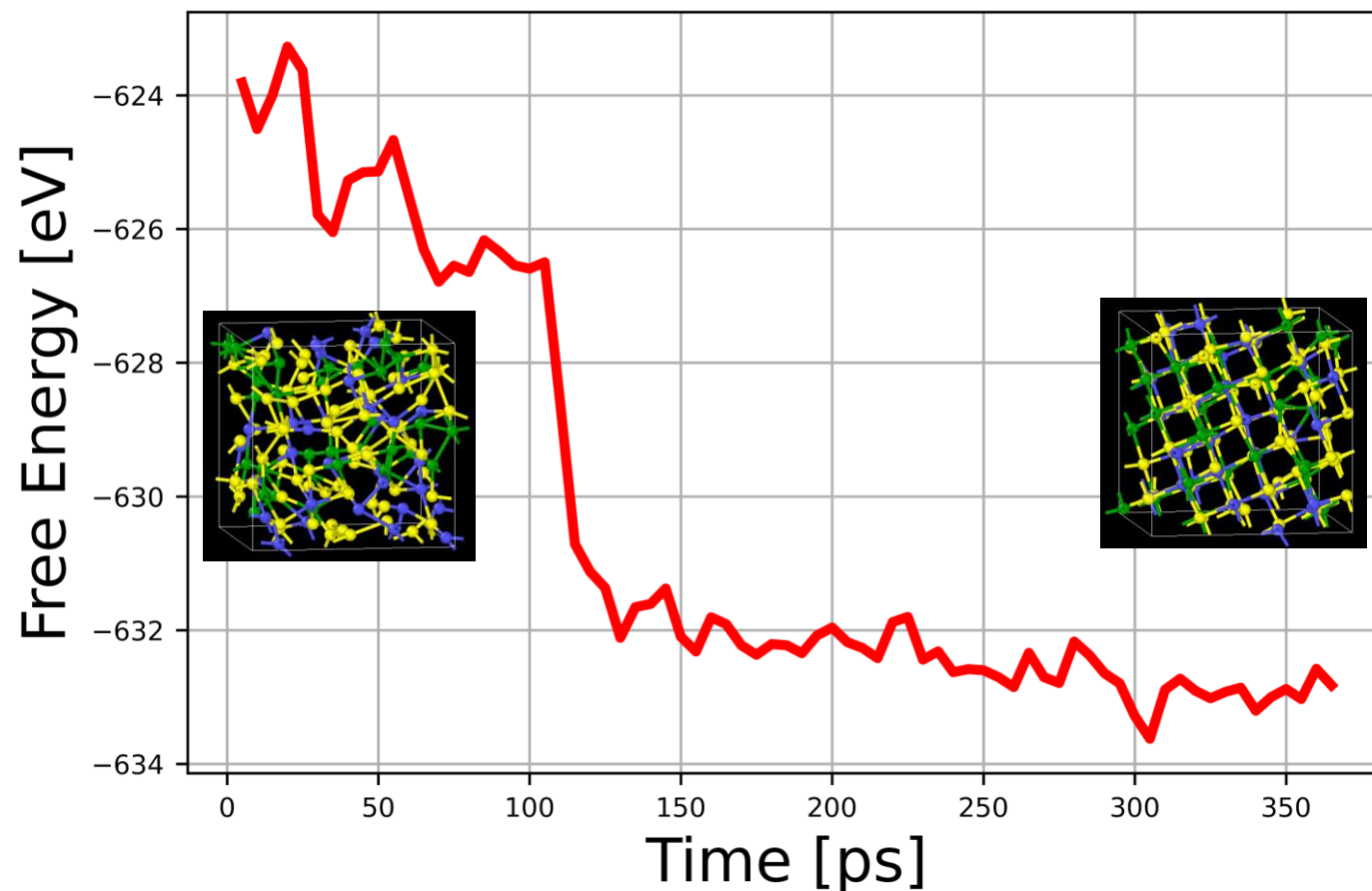
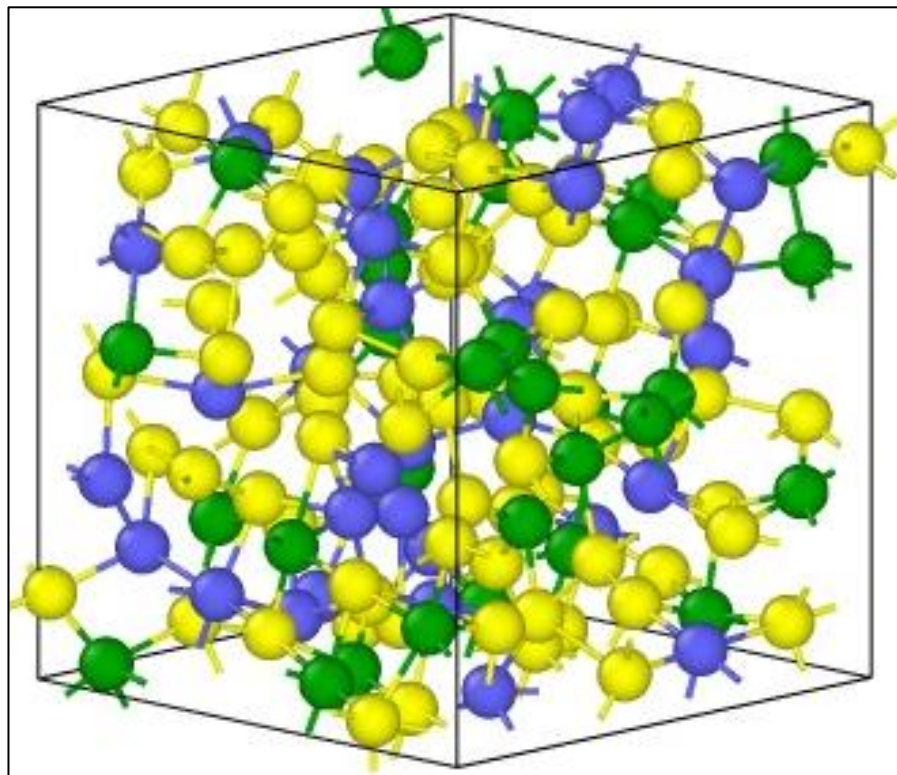
Comparing Three Families



- CUR selected features [Family 2/3] outperform grid of features [Family 1]
- [Family 3] allows for well controlled dynamics and database enrichment

Validation with DFT

DFT+MD Simulation @ 600K ||
NVT ensemble

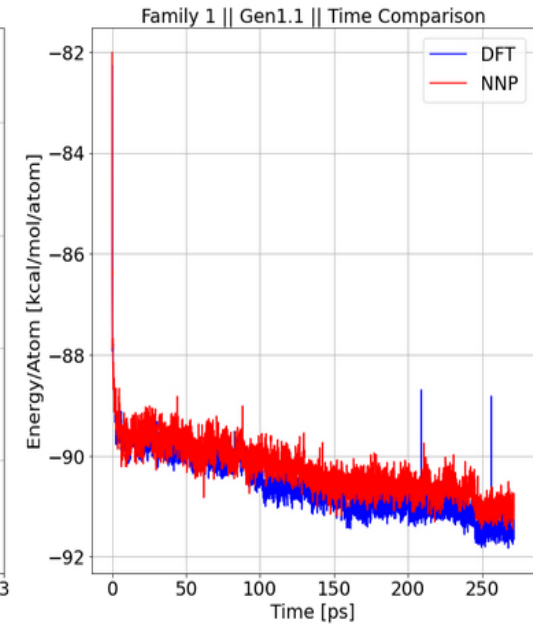
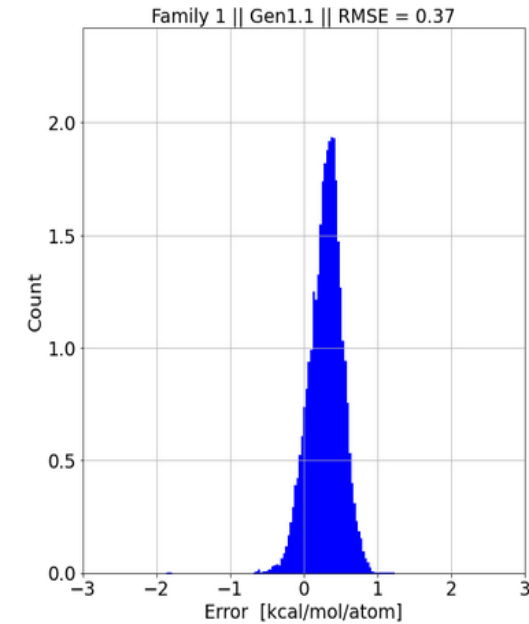
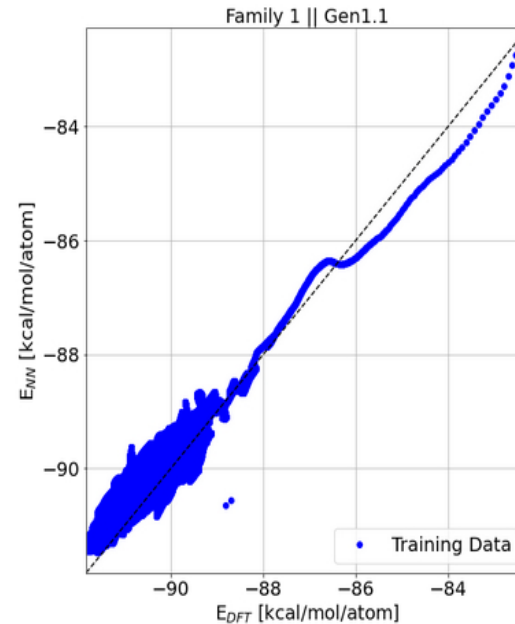
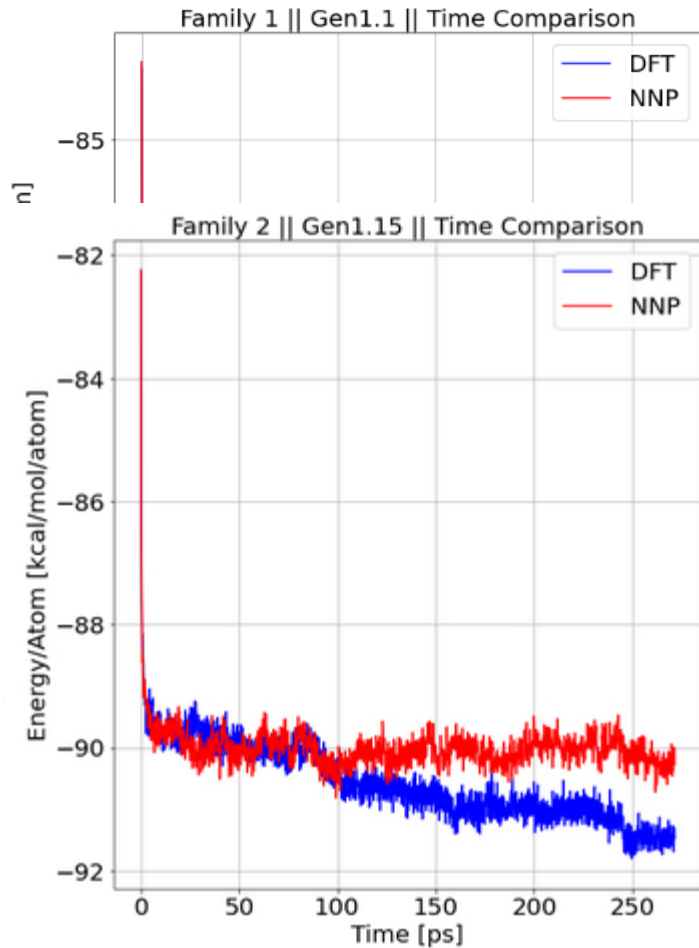


Crystallization via molecular dynamics

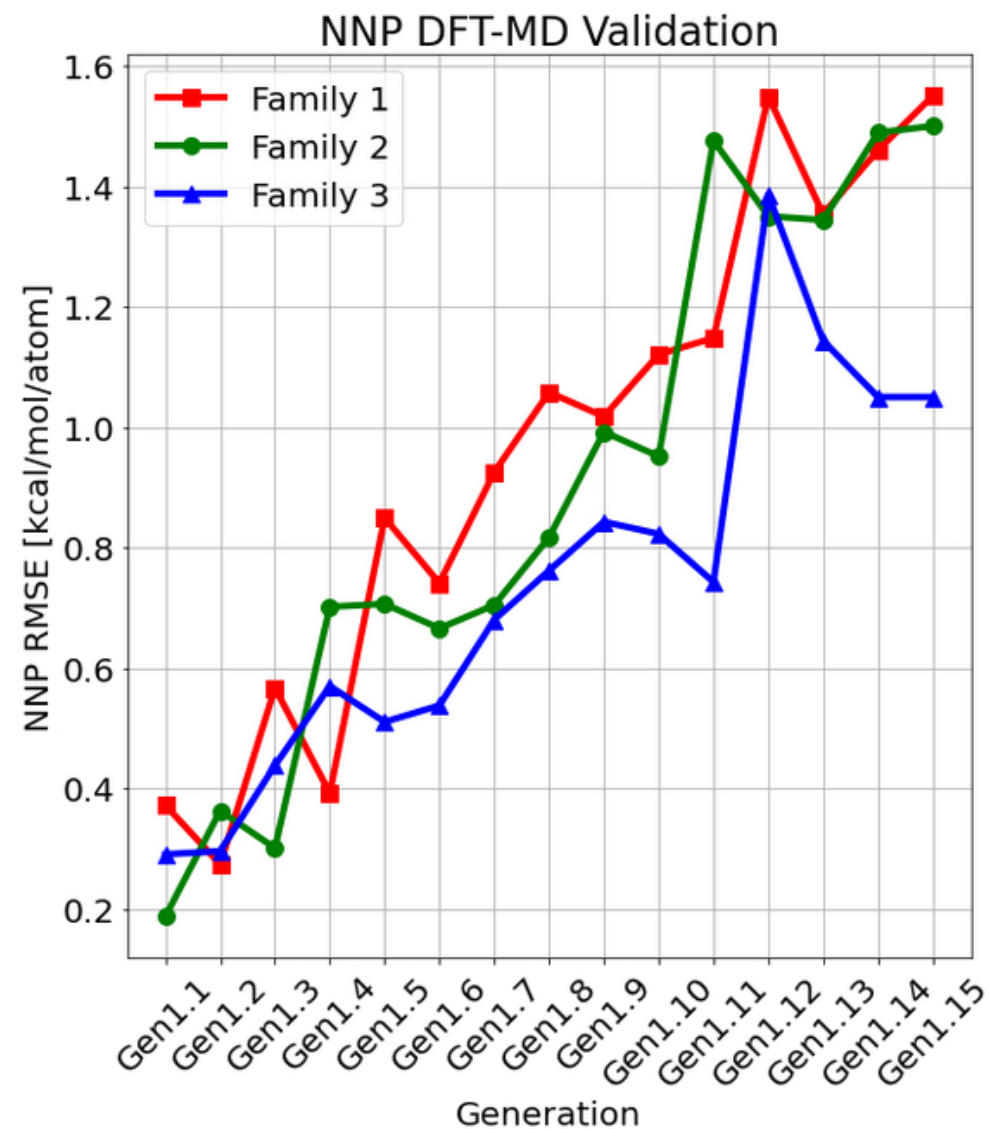
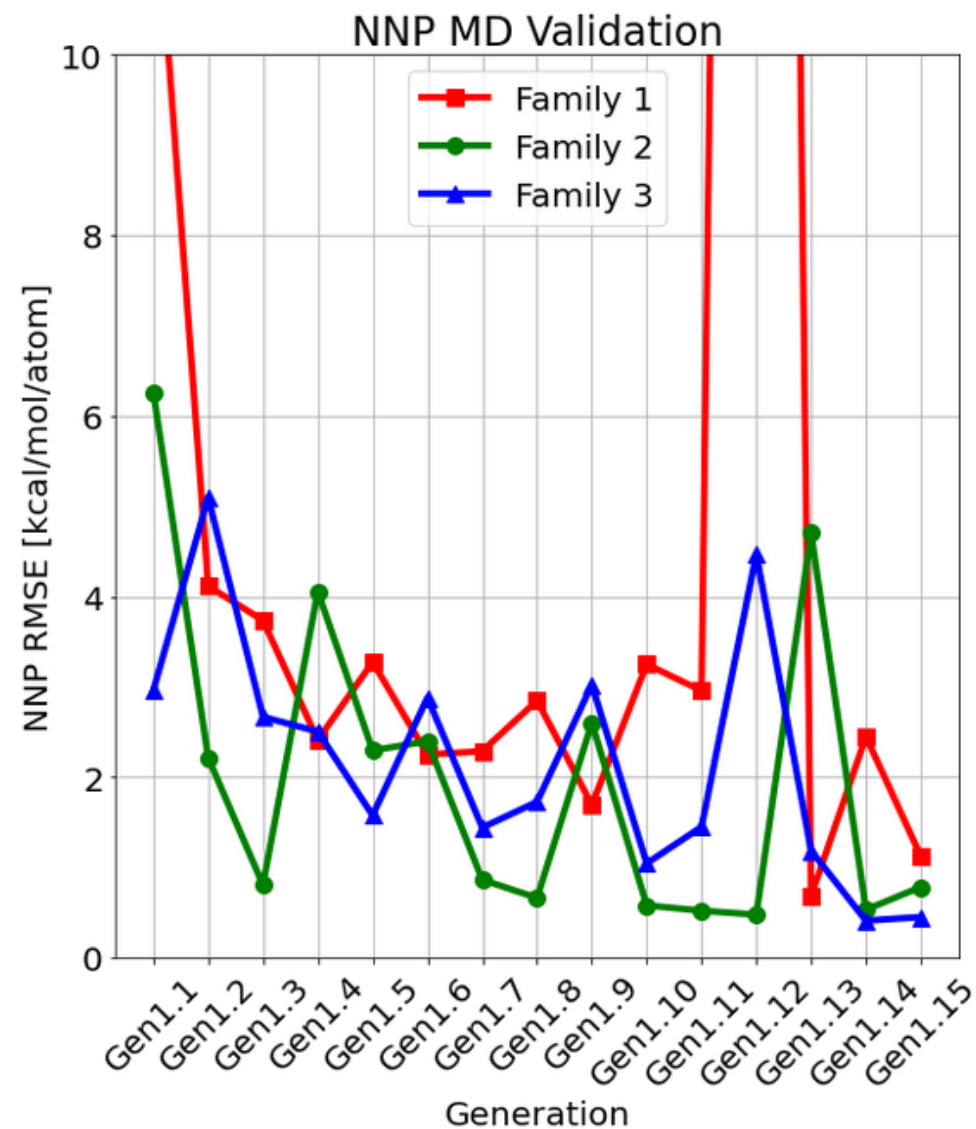
2 Flavors of Validation

1.) How well can my trained weights reproduce an DFT+MD simulation of amorphous recrystallization with NNP+MD?

2.) How well can my weights reproduce energies of the DFT+MD trajectory

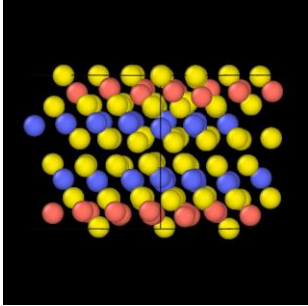

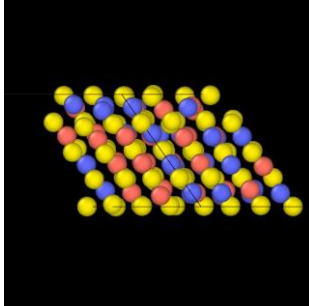

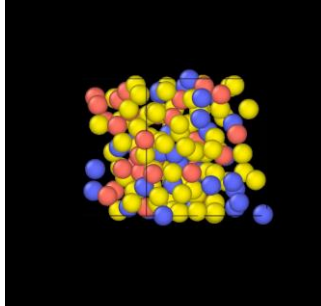

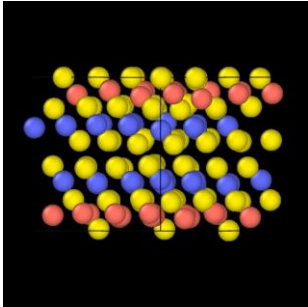

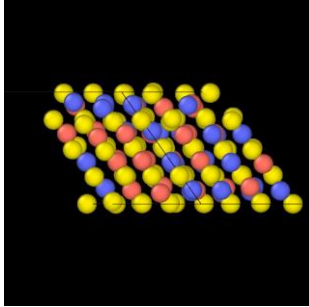

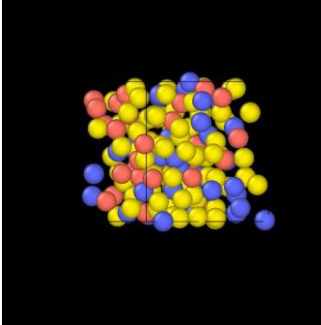

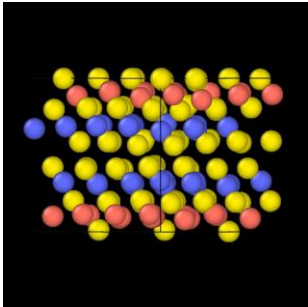

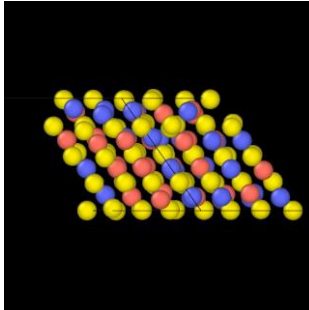

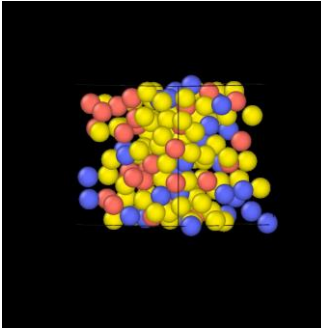



Generation Validation



Three Families – Three Phases



	Hexagonal	Cubic	Amorphous
1	 	 	 
2	 	 	 
3	 	 	 

Current State:

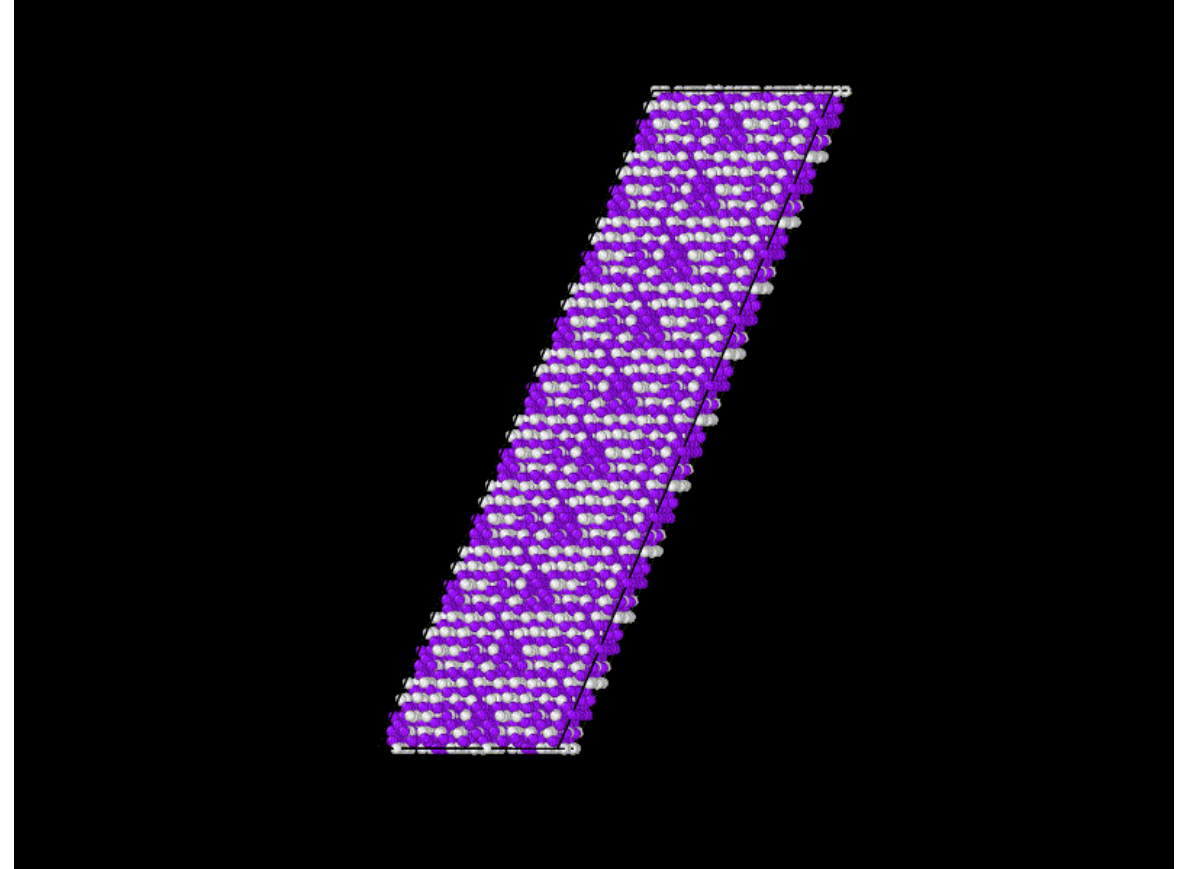
Successes:

- Stability of all three phases in NVT/NPT
- Recrystallization of amorphous into cubic phase shown
- Good agreement with DFT structural representations for xtal/amorphous

Limitations:

- Large scale recrystallization needs a heterogeneous seed
- Liquid temperatures above 1200K become unstable in NPT

~7000 atoms – recrystallizing anneal 600K – 2 ns run



Soon to come:
GST+C iterative training

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