

Practical database enrichment strategies with iterative learning: Neural Network Potentials for Phase Change Memory

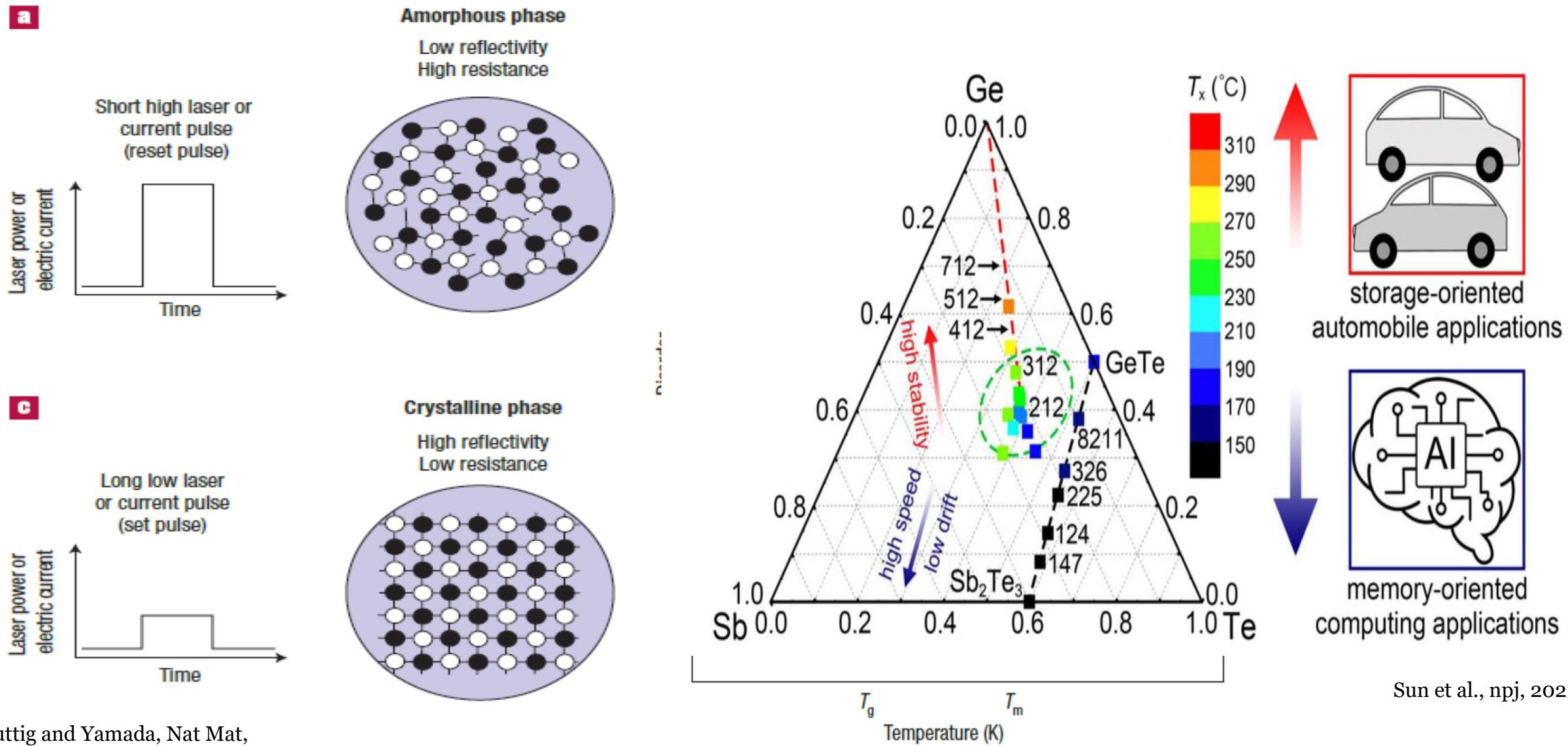
Zachary McClure*, Robert Appleton, David Adams, Alejandro Strachan
zmcclure@purdue.edu



Project Supported by:
<https://www.sandia.gov/>



Phase Change Memory: Atomistic precision for macroscale applications



Stabilizing the amorphous phase

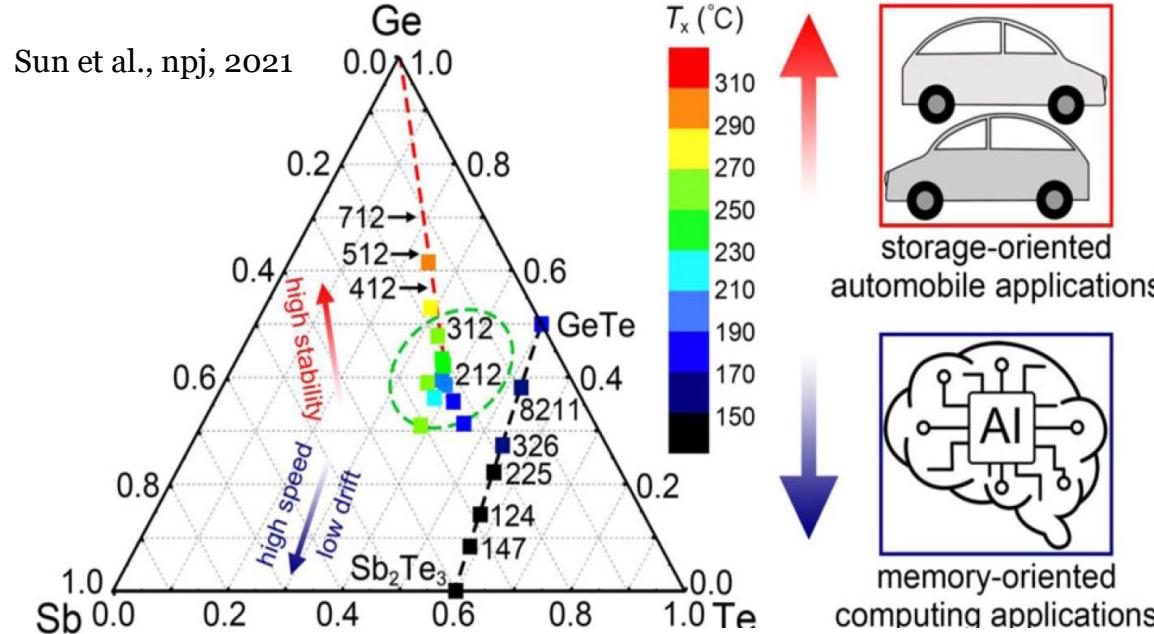
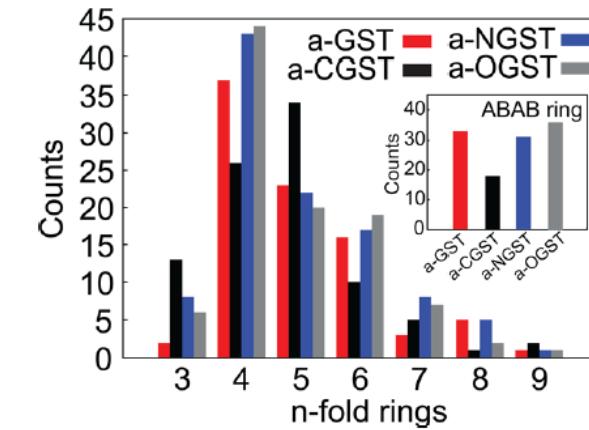
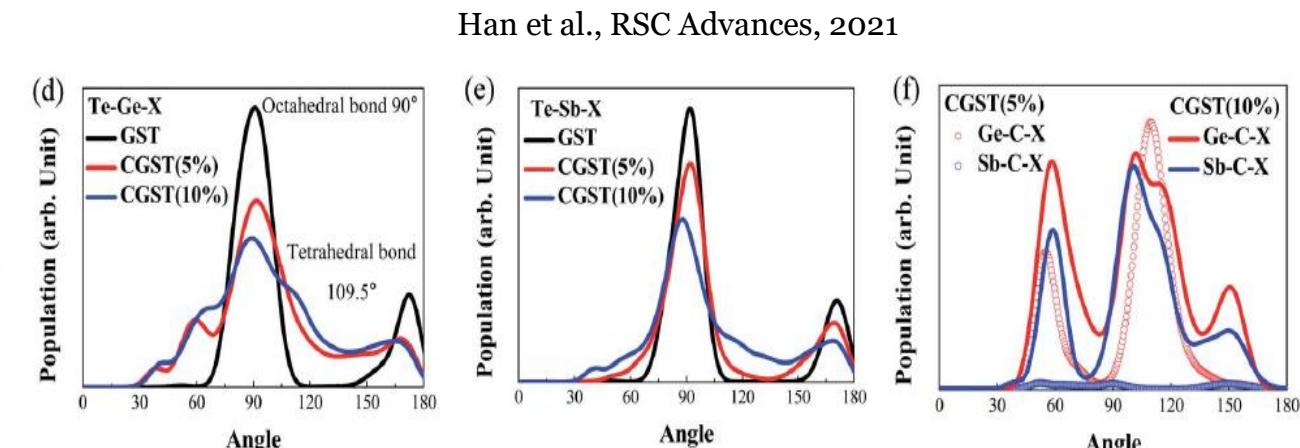


TABLE I. The coordination numbers, bond lengths, the first peak position of ADF around Ge atoms in amorphous GST structures. Ge(I) atoms are bonded to dopants, while Ge(II) atoms are not.

| | Coordination numbers | | Bond lengths (Å) | | | Averaged angle around Ge atoms (°) |
|--------|----------------------|--------|------------------|-----------|---------|------------------------------------|
| | Ge(I) | Ge(II) | Ge(I)-Te | Ge(II)-Te | -Ge(I)- | |
| | Ge(I) | Ge(II) | Ge(I)-Te | Ge(II)-Te | -Ge(I)- | |
| a-GST | 3.67 | | 2.78 | | 93 | |
| a-CGST | 4.00 | 3.67 | 2.67 | 2.74 | 106 | 98 |
| a-NGST | 4.00 | 4.00 | 2.73 | 2.80 | 102 | 95 |
| a-OGST | 3.74 | 3.90 | 2.84 | 2.81 | 98 | 93 |



Cho et al., APL, 2011

FIG. 2. (Color online) Ring statistics for amorphous GST structures counted per supercell. The inset figure shows the numbers of ABAB-type squared rings.

What atomistic process can we model so far?

Ab initio methods:

- Phase change
- Density of states
- Dopant stability
- Recrystallization
- Limited to constant volume

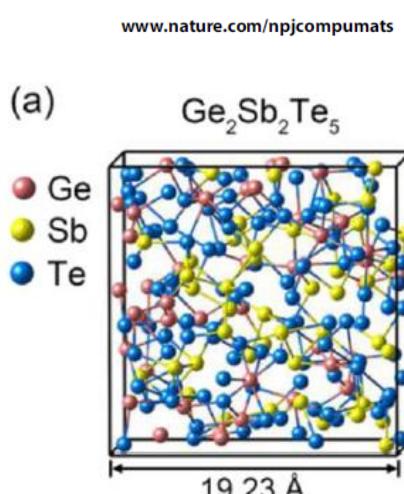
**limited to small system sizes

npj Computational Materials

ARTICLE OPEN

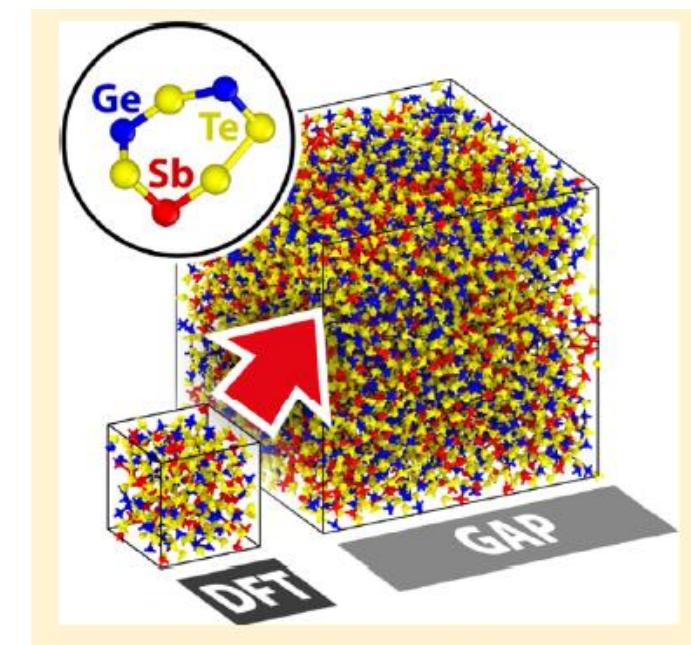
Ab initio molecular dynamics and material embedded phase-change memory

Liang Sun^{1,8}, Yu-Xing Zhou^{2,3,7,8}, Xu-Dong Wang^{2,3}, Yu-Han Chen^{2,3}, Volker L. Deringer⁴



ML MD Potential:

Gaussian Approximation Potential –
DFT accuracy with scaling to ~7K
atoms
NPT possible for phase change

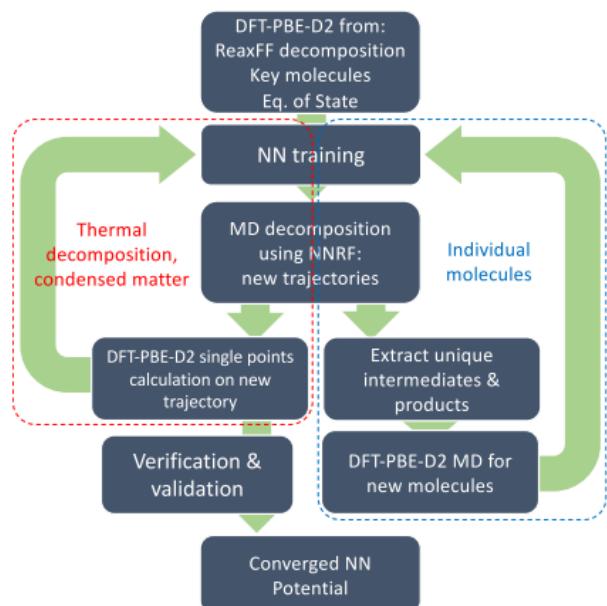


Mocanu et al., JPC B, 2018

Developing machine learning potentials

High density neural net (HDNN)

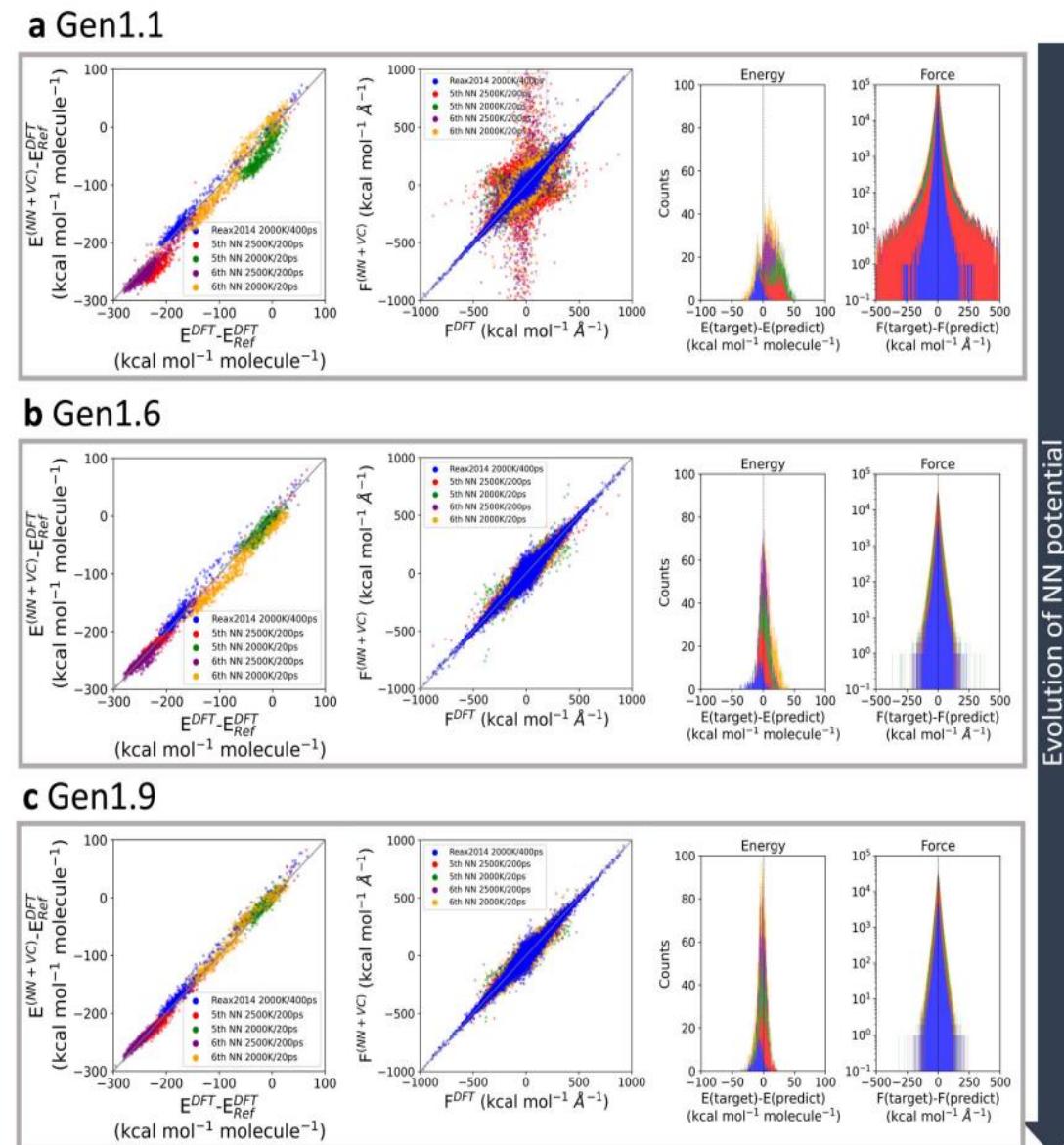
- High variability due to hyperparameters
- Requires higher VC
- High sensitivity to initialization for



HDNNP+Iteration:

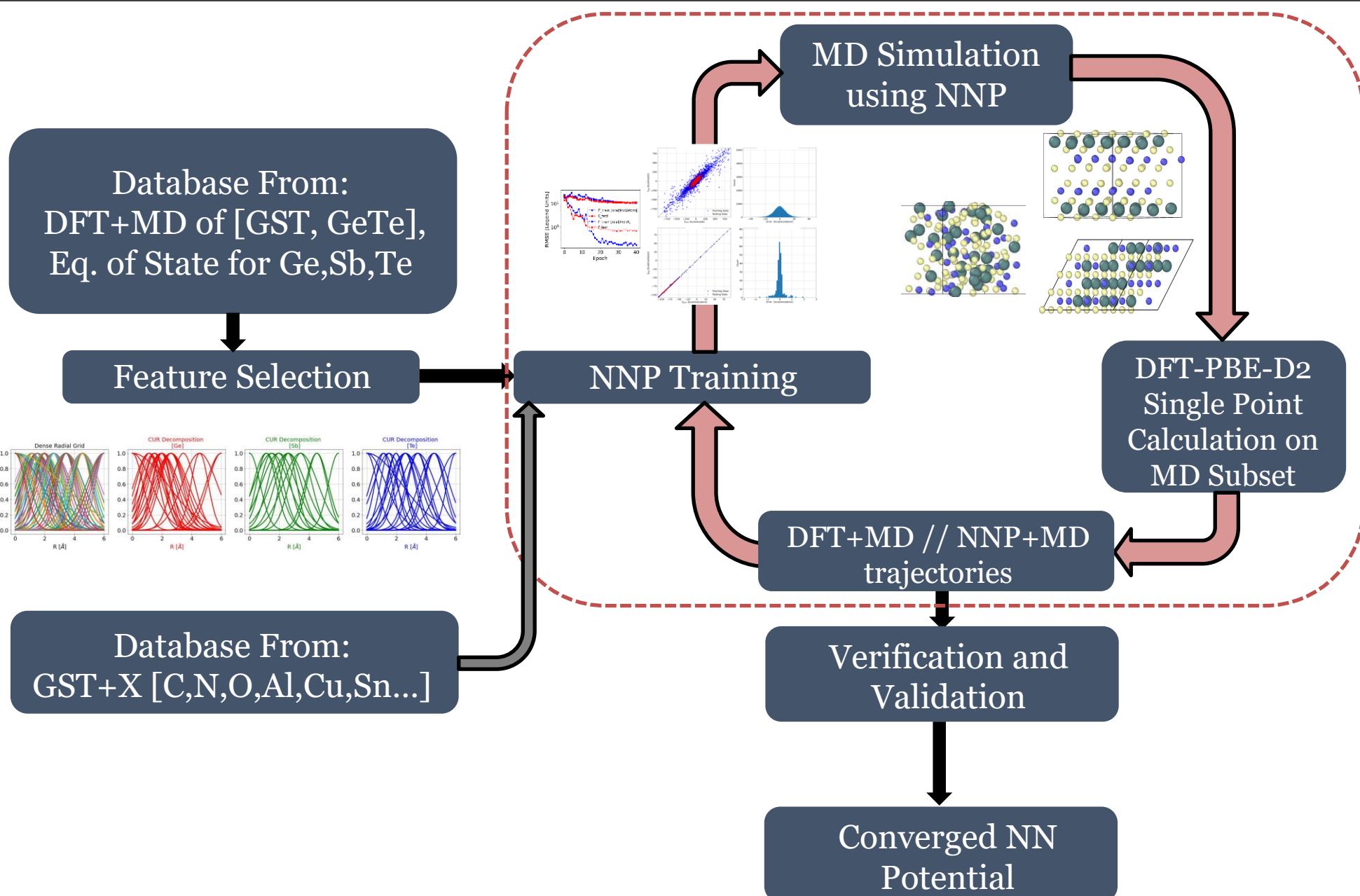
- Enrich dataset with NNMD trajectories
 - Explore edge cases

T_e
 $2 + 10^{-6}$



initialization

NNP Iterative Workflow



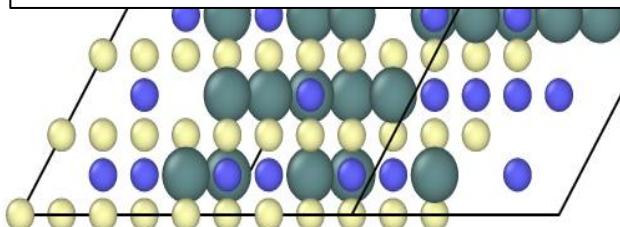
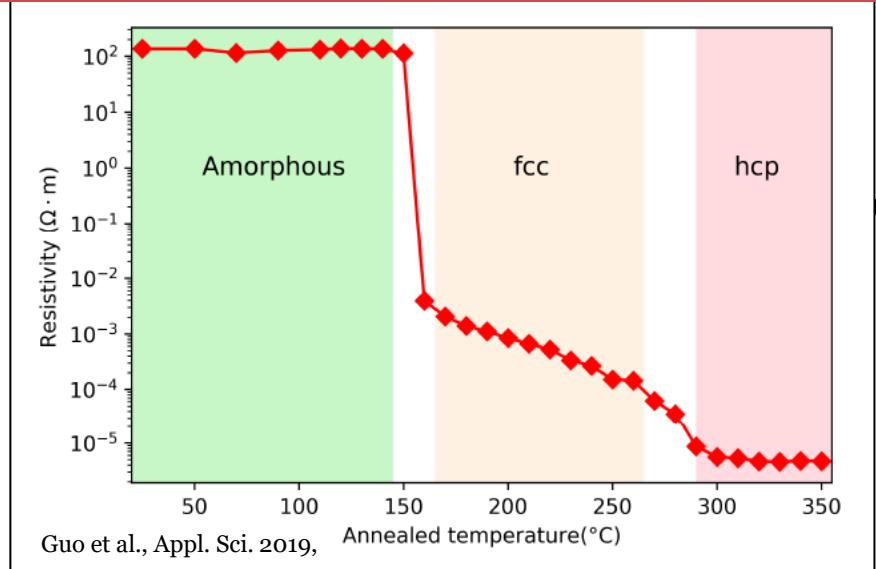
DFT Simulations – GGA – PBE+D2

Atomic Simulations of GST+C
Phase Change Materials

Robert Appleton, Zachary McClure,
Alejandro Strachan, David Adams

Hexagonal

Three primary phases: Amorphous/Cubic
Rocksalt (FCC)/Hexagonal



$$T_m = 900 \text{ K}$$
$$T_{x-tal} = 423 \text{ K}$$

Xtal:

- $T=[400,600,800]\text{K}$

Gather trajectories for each temp

GST+C:

- Create **cubic** structure with desired composition
- Melt & quench [see work by Robert Appleton]

Ab initio MD – NVT

Analyze and break down
free energy contributions

Amorph:

- Melt at 2000K
- Quench and repeat temps from Xtal

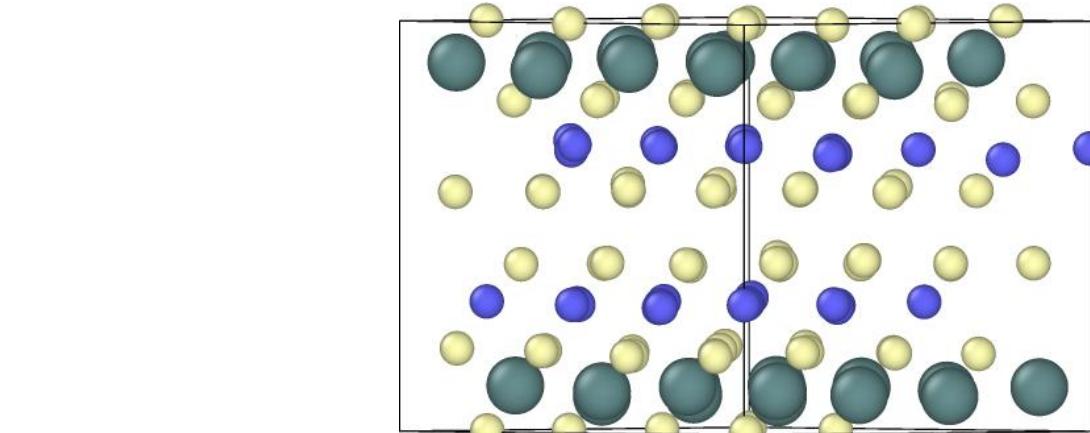
Run Params:

K-point grid: 1x1x1

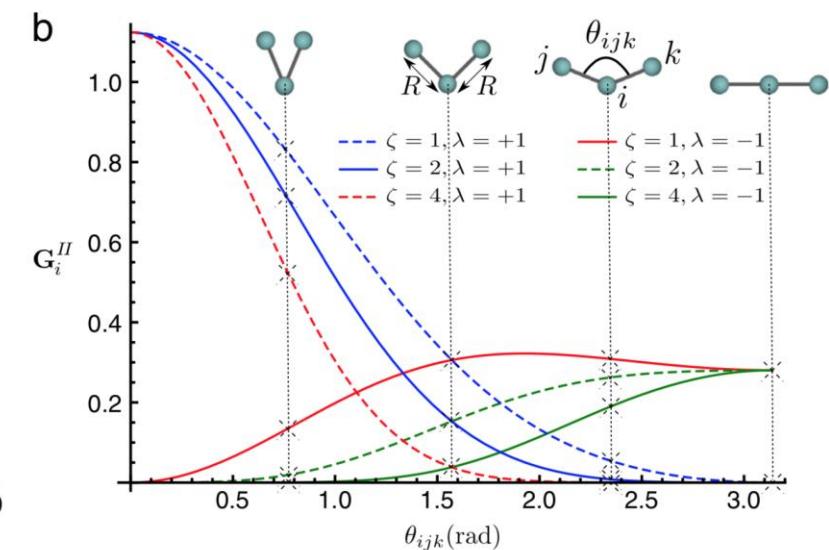
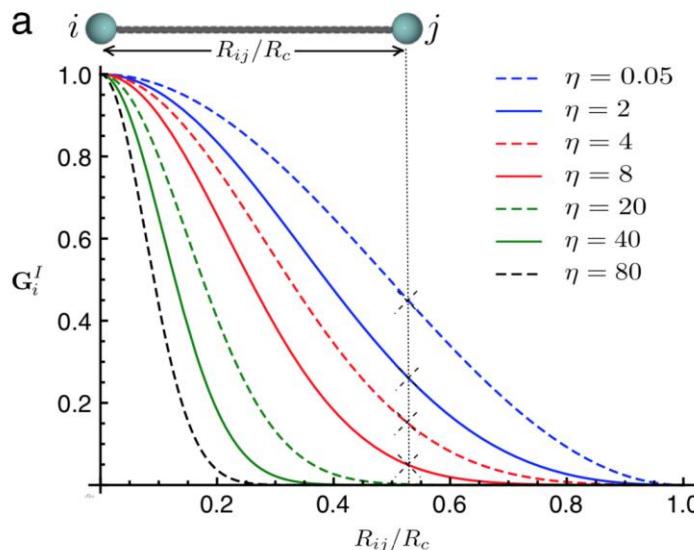
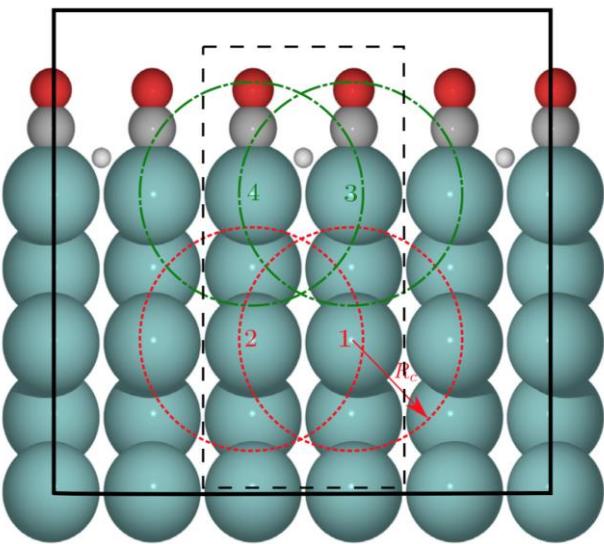
Supercell: 144 atoms

KE cutoff: 300 eV

Using neural network potentials



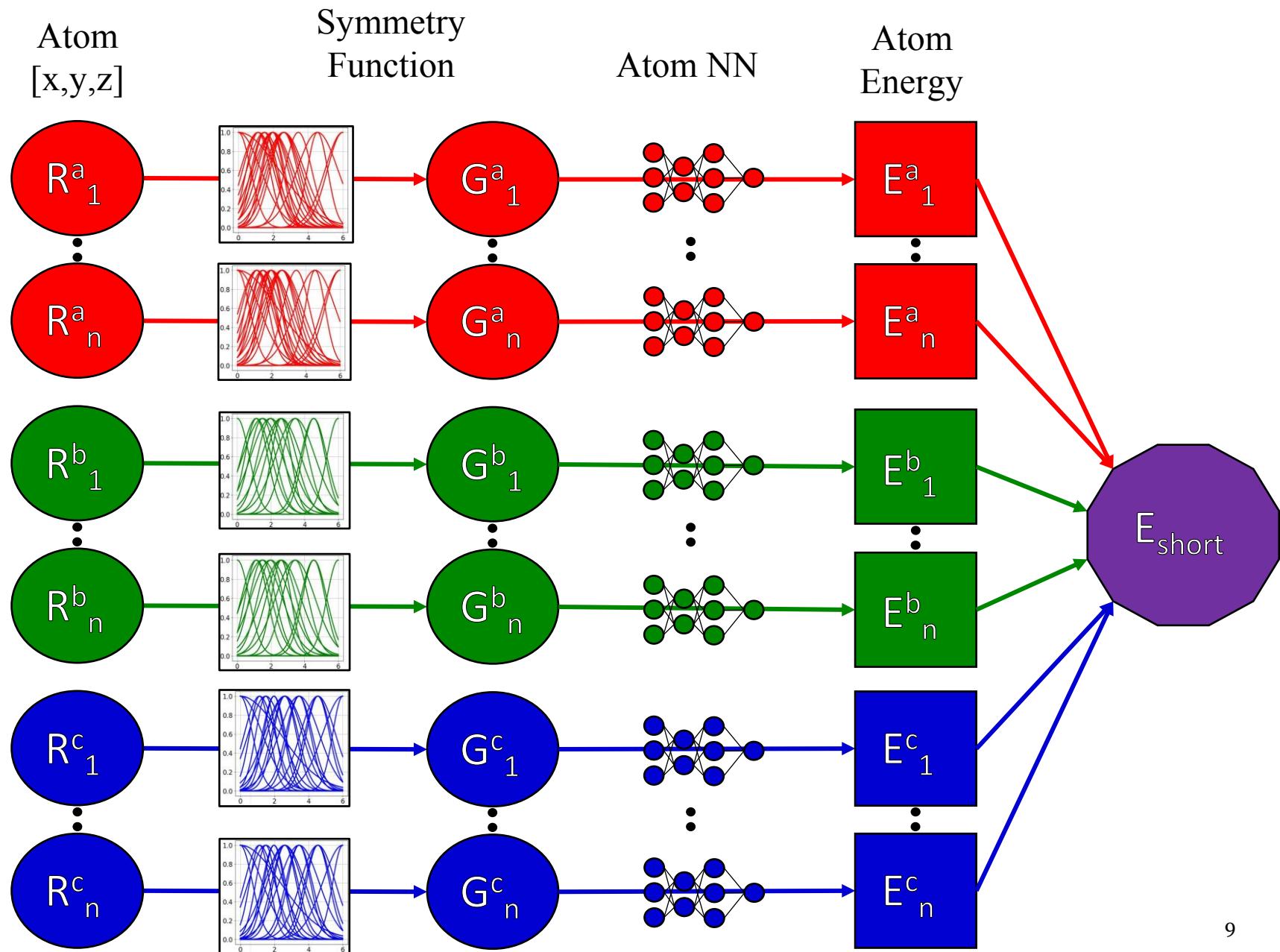
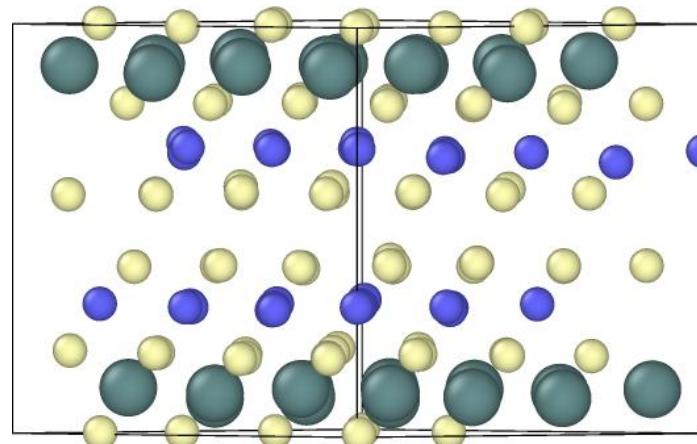
$$E_{\text{short}} = \sum_{N_{i=1}}^{N_{\text{elem}}} \sum_{j=1}^{N_{\text{atom}}^i} E_j^i$$



$$G_i^{w,rad} = \sum_{\substack{\text{Natom} \in R_c \\ j \neq i}} g(Z_j) e^{-\eta(R_{ij}-R_s)^2} f_c(R_{ij})$$

$$G_i^{w,ang} = 2^{1-\zeta} \sum_{\substack{j, k \neq i \\ j < k}} h(Z_j, Z_k) (1 + \lambda \cos \theta_{ijk})^\zeta e^{-\eta[(R_{ij}-R_s)^2 + (R_{ik}-R_s)^2 + (R_{jk}-R_s)^2]} \cdot f_c(R_{ij}) f_c(R_{ik}) f_c(R_{jk})$$

Using neural network potentials



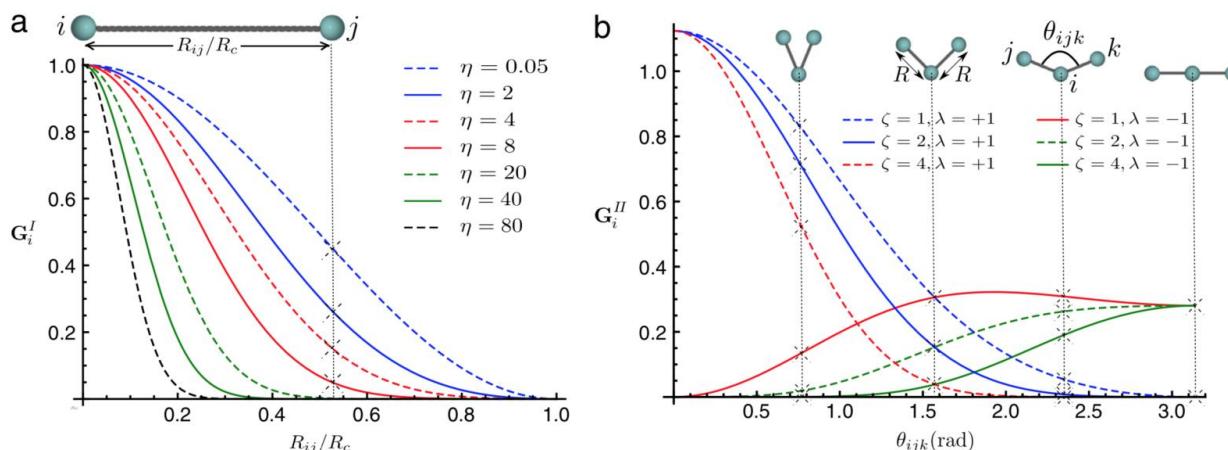
Hyperparameters of features

A simple gaussian function with a cutoff radius R_c , a width tuner η , and a shift parameter R_s

$$G_i^{w,rad} = \sum_{\substack{j \in R_c \\ j \neq i}}^{N_{\text{atom}}} g(Z_j) e^{-\eta(R_{ij} - R_s)^2} f_c(R_{ij})$$

A complex gaussian function with a cutoff radius R_c , a width tuner η , and a shift parameter R_s , now with a cosine function flipped by $\lambda \pm 1$, and band width tuned by ζ

$$G_i^{w,ang} = 2^{1-\zeta} \sum_{\substack{j, k \neq i \\ j < k}} h(Z_j, Z_k) (1 + \lambda \cos \theta_{ijk})^\zeta e^{-\eta[(R_{ij} - R_s)^2 + (R_{ik} - R_s)^2 + (R_{jk} - R_s^2)]} \cdot f_c(R_{ij}) f_c(R_{ik}) f_c(R_{jk})$$



How many features are we optimizing?

3 elements

G_{rad}

$R_s = 3x$

$\eta = 6x$

G_{ang}

$\lambda = 2x$

$\zeta = 1x$

$R_s = 3x$

$\eta = 6x$

54 possible features / element given initial grid

Initial Selection Database:

Hex – GST – 225

- @ [300,400,600,800] K

Cub – GST – 225

- @ [300,400,600,800] K

Amorph – GST – 225

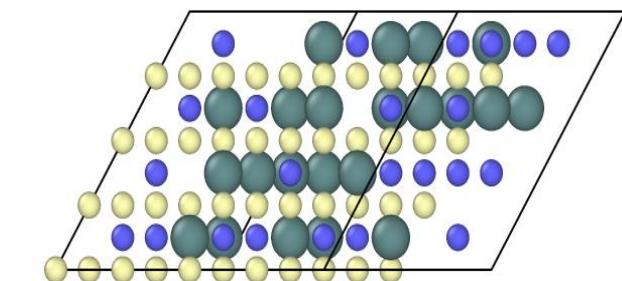
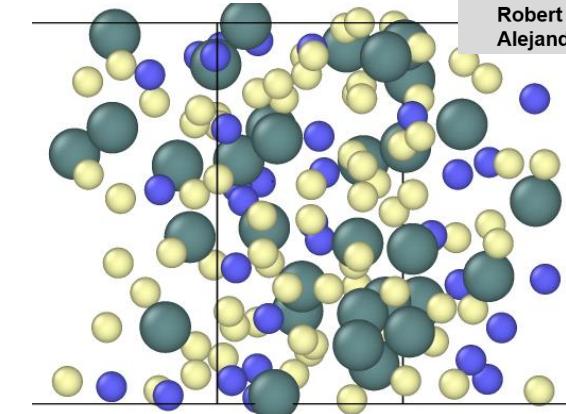
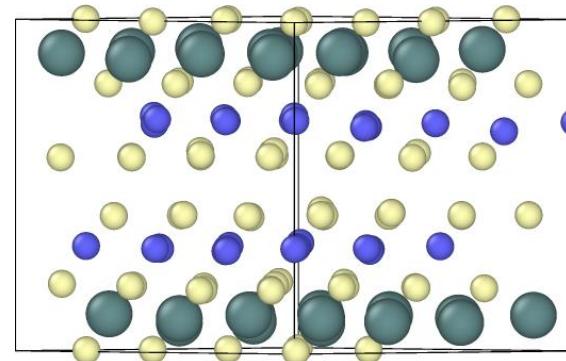
- @ [300,400,600,800] K

Liq - GST – 225

- @ [1100K, 2000] K

Liq & Crystalline GeTe

- @ varied temperatures



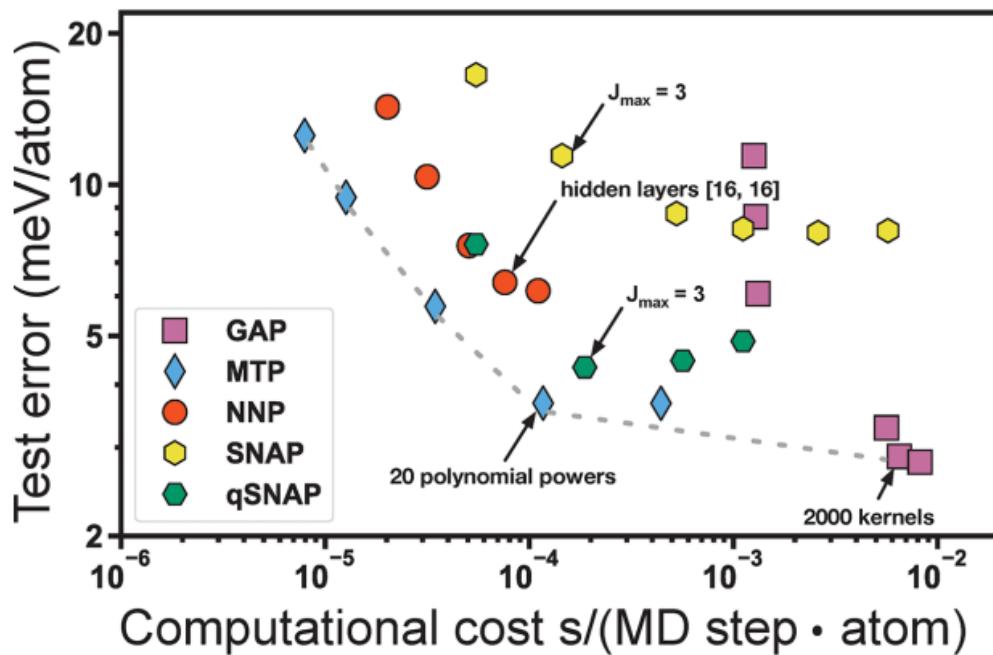
- Selected 1% of full DFT+MD database for initial training and calibration

- 90/10 train/test split for initial verification

- 2 Hidden Layers: [50/50]

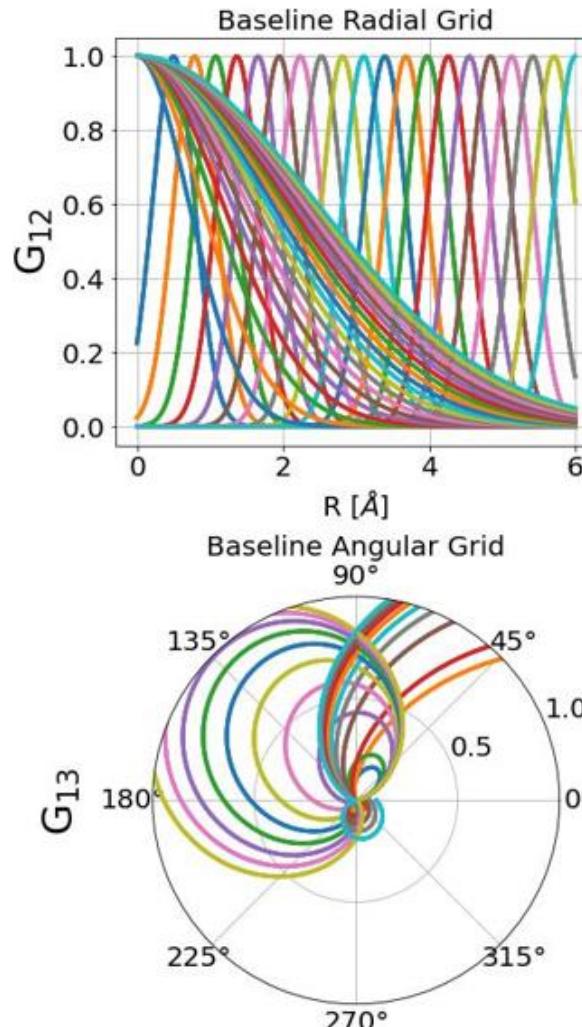
| Eq. of State [Ge,Sb,Te] S.P. DFT | GeTe MD+DFT | Hex. Ge ₂ Sb ₂ Te ₅ MD+DFT | Cub. Ge ₂ Sb ₂ Te ₅ MD+DFT | Amorph. Ge Sb ₂ Te ₅ MD+DFT | Liq. Ge ₂ Sb Te ₅ MD+DFT |
|--|----------------|---|---|---|--|
| Gen1.1 | 150 | 828 | 100 | 100 | 114 |

Limitation 1



Which symmetry functions
best describe my
environment?

What is my trade off for
description vs. computational
weight?



Limitations to overcome:

1. **Feature selection**
2. Architecture initialization

3 elements

G_{rad}

$R_s = 20x$ constant η

$\eta = 18x$ constant R_s

G_{ang}

$\lambda = 2x$

$\zeta = 1x$

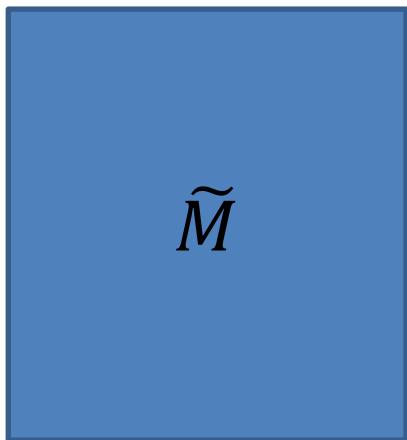
$R_s = 3x$ constant η

$\eta = 3x$ constant R_s

Solution 1: CUR Decomposition

$$\tilde{M} \approx CUR$$

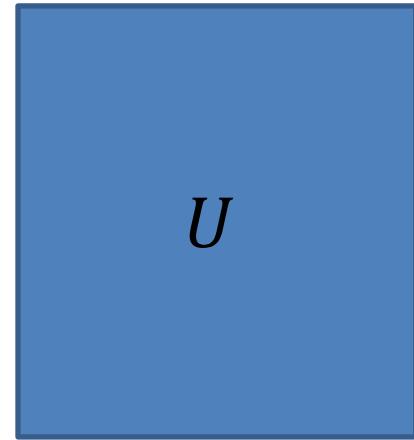
Feature Matrix



Elements



Scoring Matrix



Features



=

C

\times

U

\times

$$\pi_c = \sum_{j=1}^k (v_c^{(j)})^2.$$

Scaling functions from symmetry functions
used as weighting parameters in U

Highest expressivity, lowest overlap, most
unique fingerprints retained

Retain the original matrix

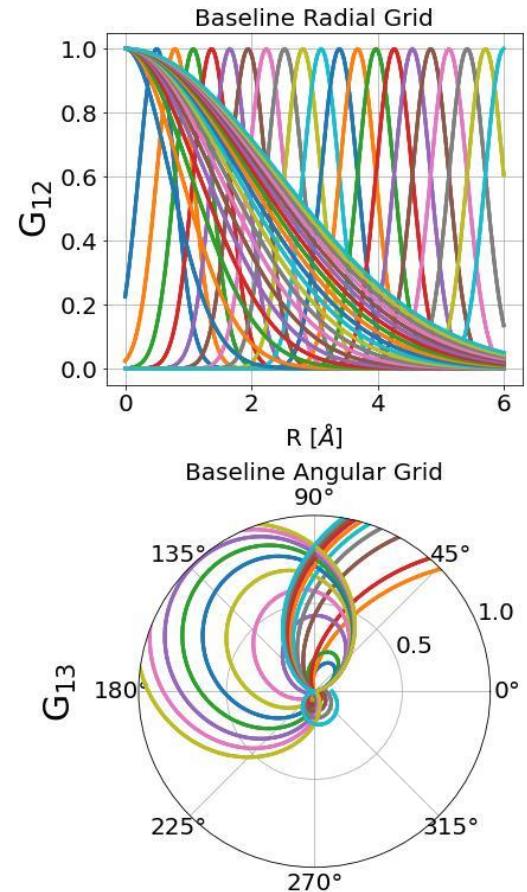
**Automatic selection of atomic fingerprints
and reference configurations for machine-
learning potentials**

Cite as: J. Chem. Phys. 148, 241730 (2018); <https://doi.org/10.1063/1.5024611>

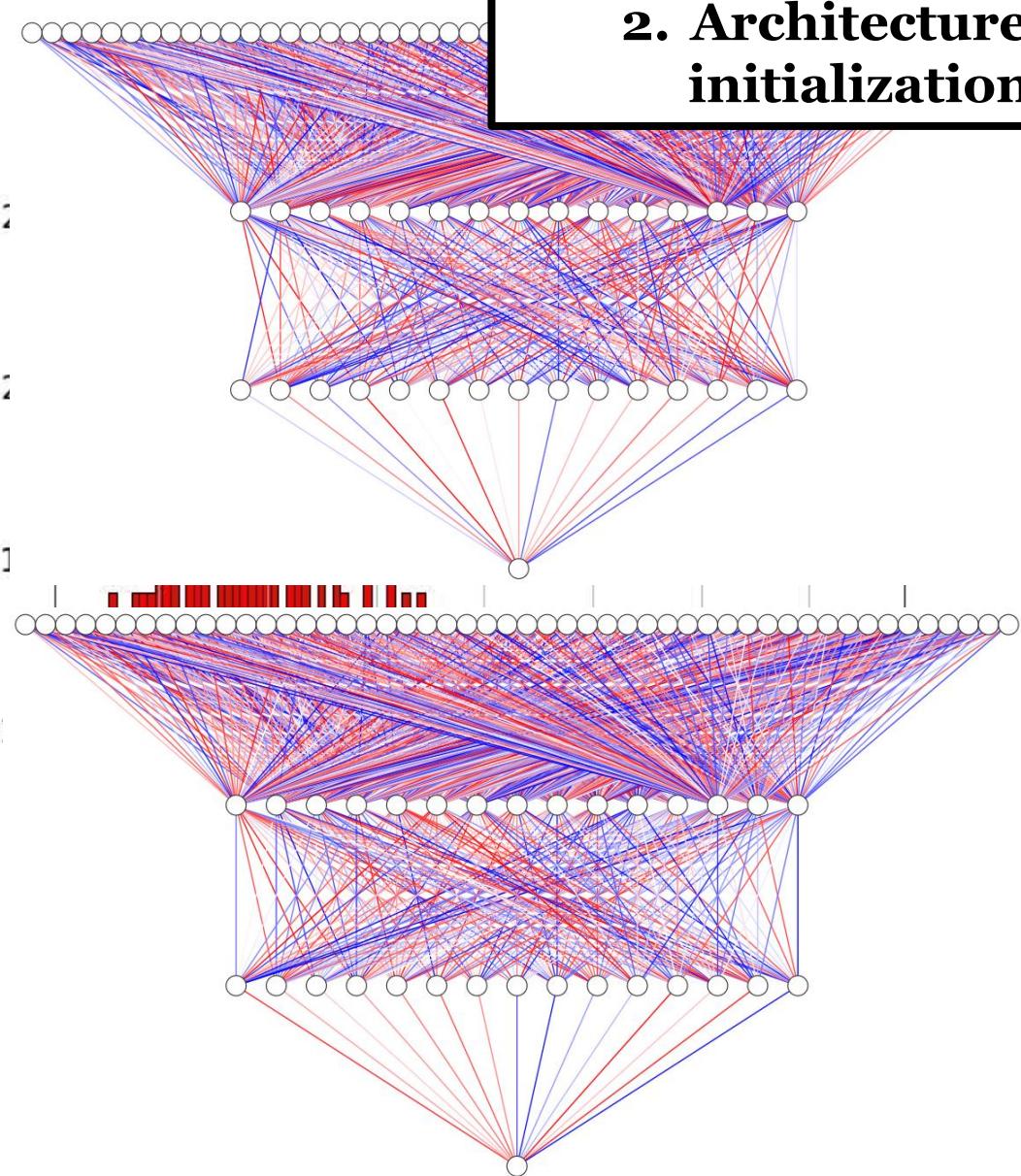
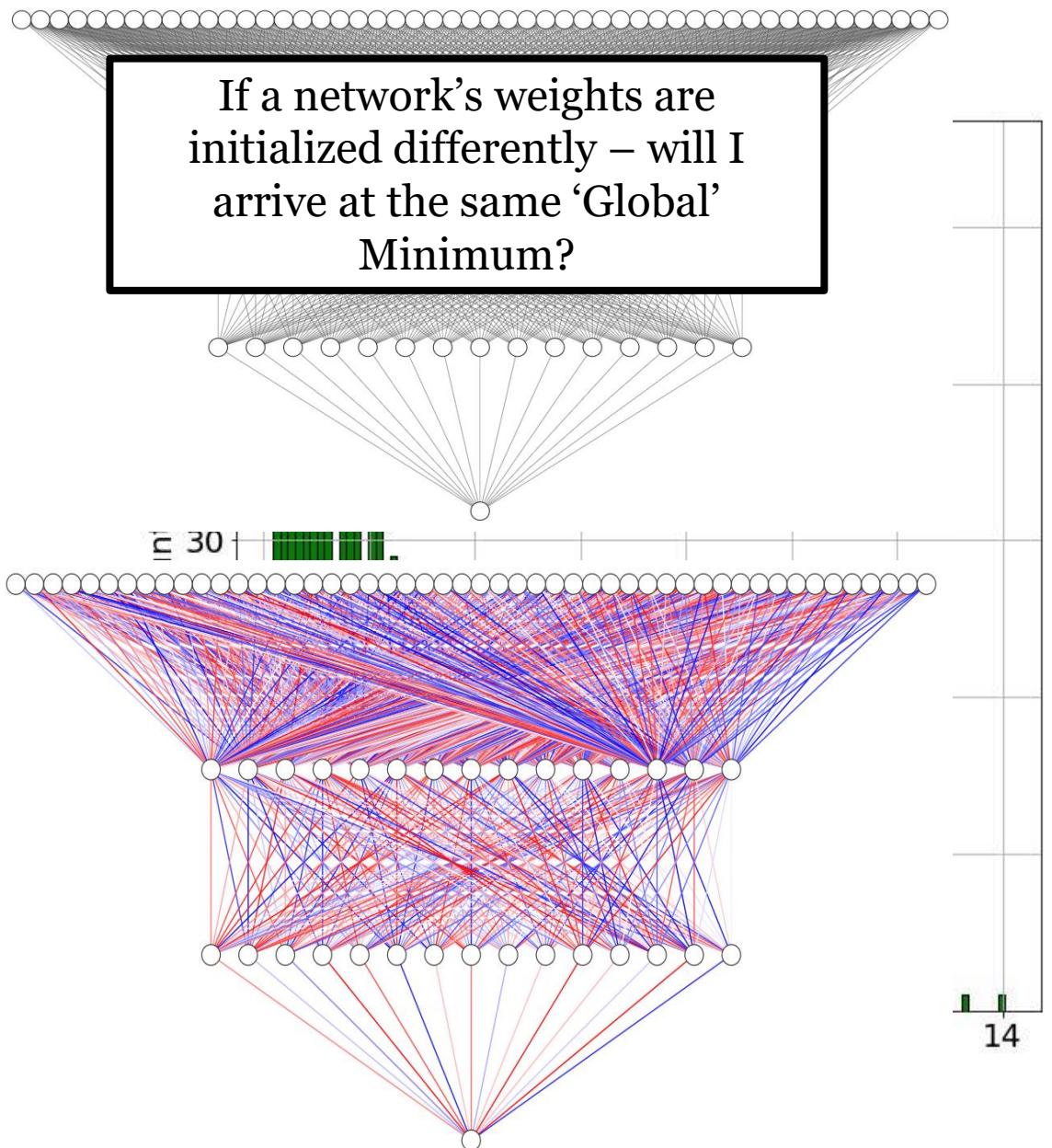
Submitted: 02 February 2018 • Accepted: 10 April 2018 • Published Online: 30 April 2018

Giulio Imbalzano,  Andrea Anelli,  Daniele Giofré, et al.

Solution 1: CUR Decomposition



Sampling 1000 networks

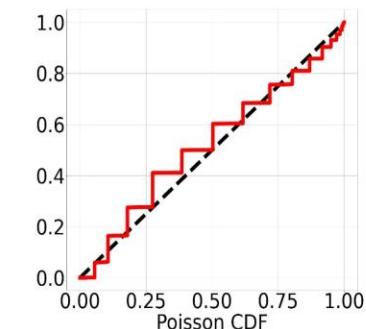
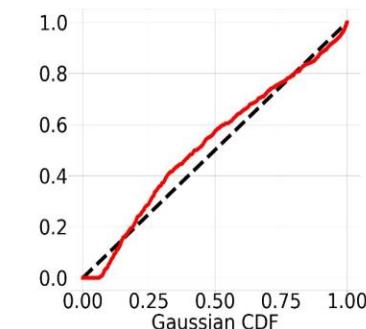
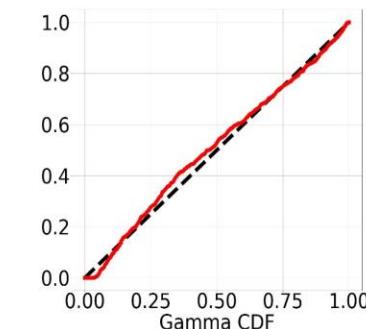
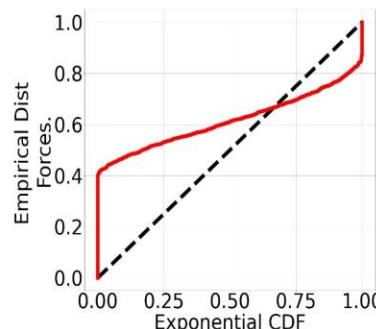
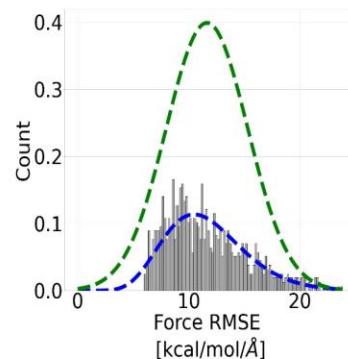
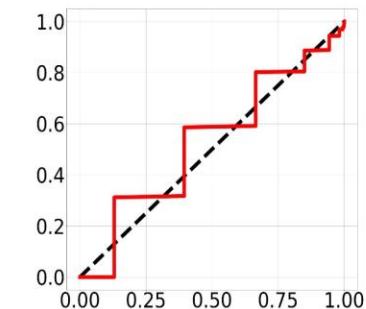
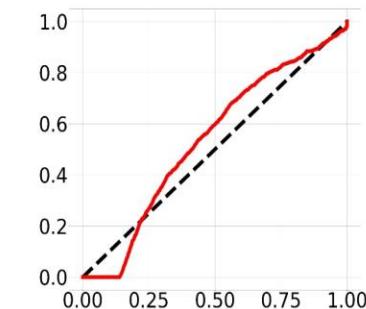
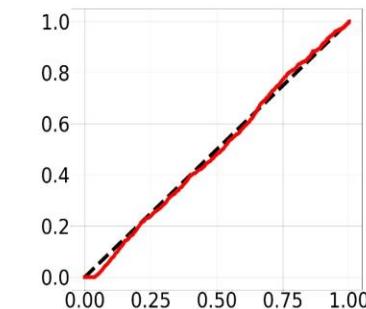
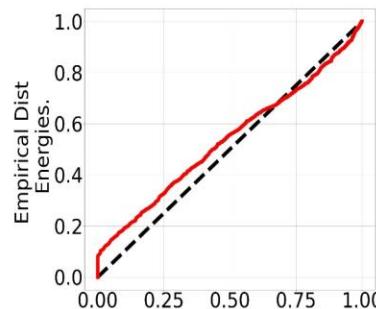
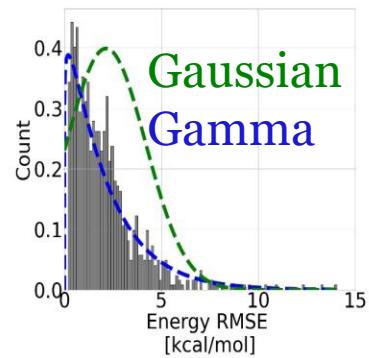
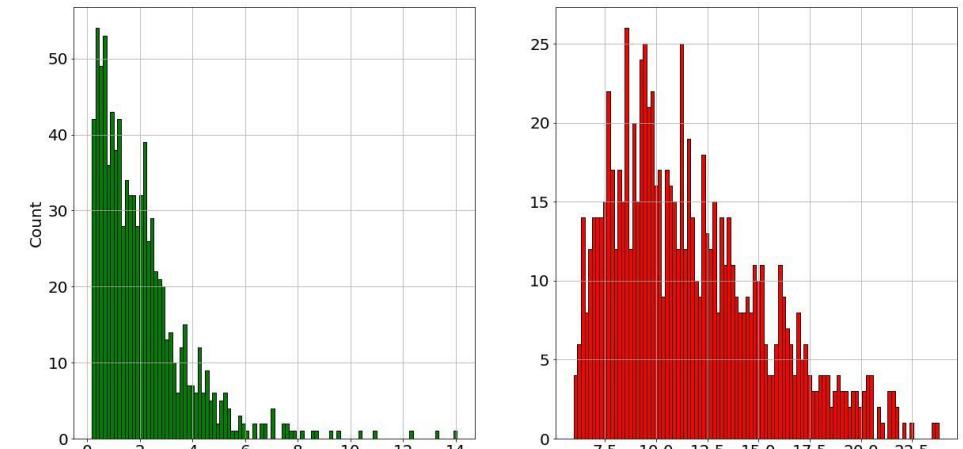


Characterizing the probability distribution

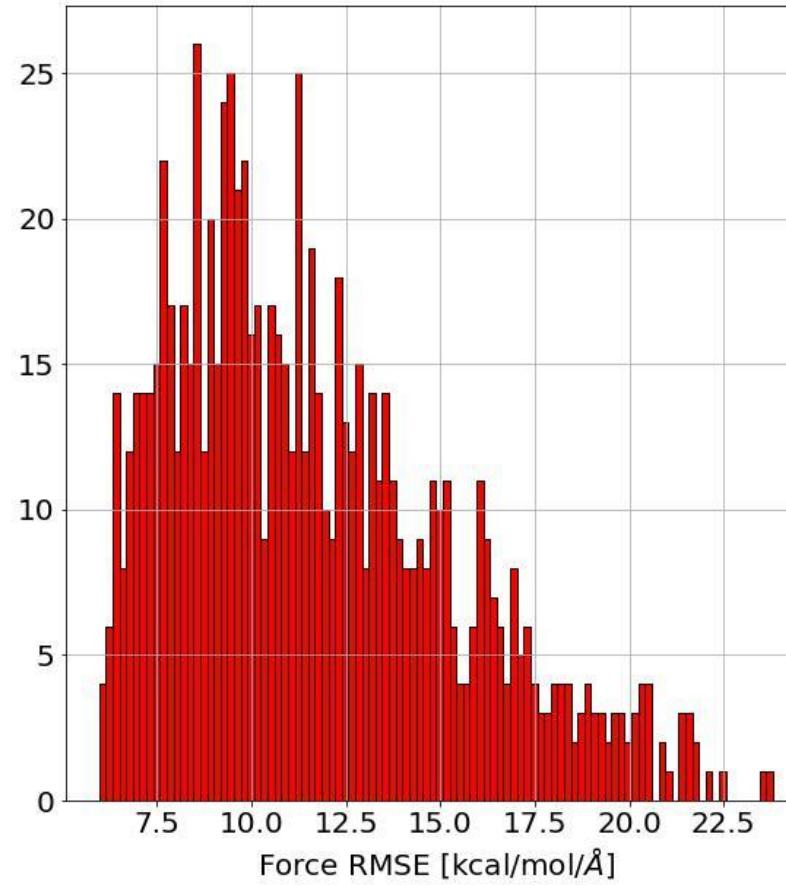
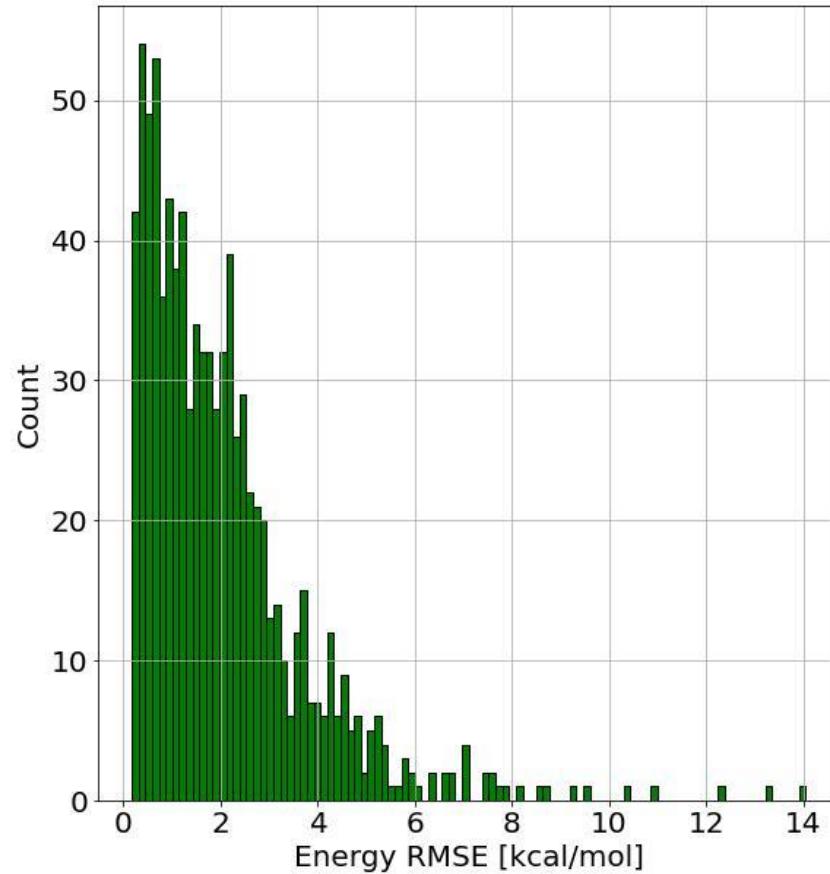
Distribution Type Hypothesis:

- Exponential
- Gamma
- Gaussian
- Poisson

Test with Cumulative Distribution Function:
 $F_x(x) = P(X \leq x)$



The Rule of 5

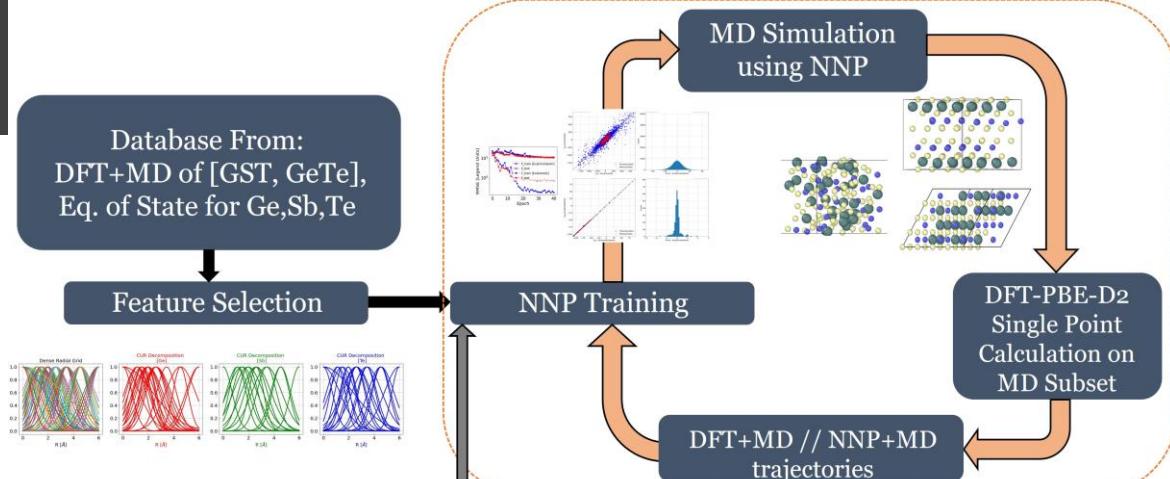


If I pick 5 different initializers, the probability of me
selecting a value below the mean is XX
[the math needs to be done before the conference]

Generational Training Roadmap

- For each NNP+MD run create isotherm
- 144 atom configurations mapped from DFT structures
- 1 ns simulation time || sample 10 trajectories

| DFT+MD Database | Gen 1.1 | Gen1.2 | Gen1.3 | Gen1.4 | Gen1.5 | Gen1.6 | Gen1.7 | Gen1.8 | Gen1.9 | Gen1.10 | Gen1.11 | Gen1.12 | Gen1.13 | Gen1.14 | Gen1.15 |
|-----------------|---------|--------|--------|--------|--------|--------|--------|--------|--------|---------|---------|---------|---------|---------|---------|
| Ge/Sb/Te | | | | | | | | | | | | | | | |
| GeTe | | | | | | | | | | | | | | | |
| h-GST | | | | | | | | | | | | | | | |
| c-GST | | | | | | | | | | | | | | | |
| a-GST[6.2] | | | | | | | | | 300K | | | | | | 2000K |
| a-GST[6.11] | | | | | | | | | | | | | | | |
| a-GST[5.88] | | | | | | | | | | | | | | | |
| l-GST | | | | | | | | | | | | | | | |

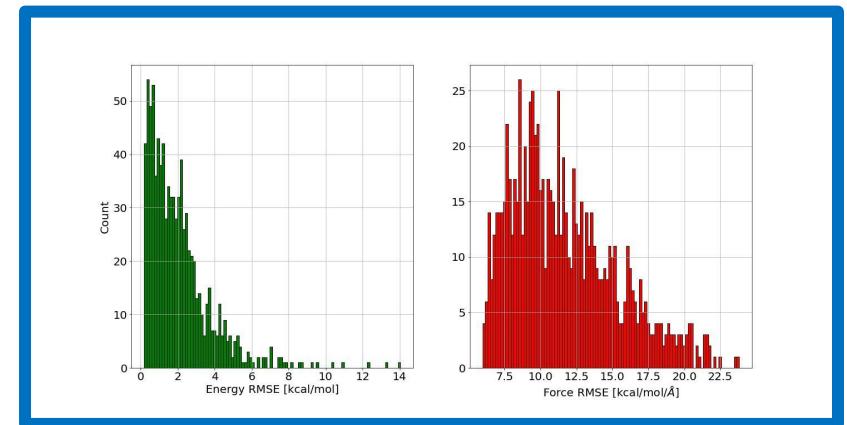
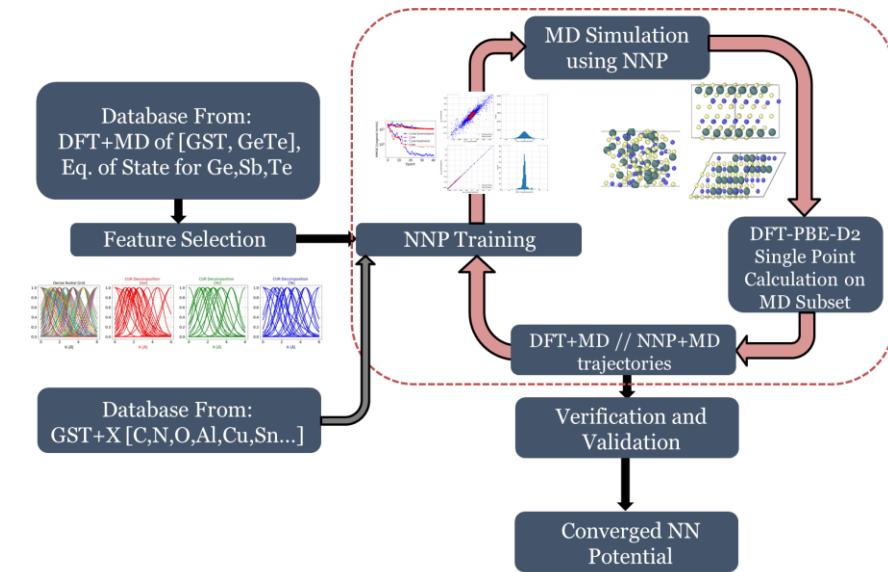
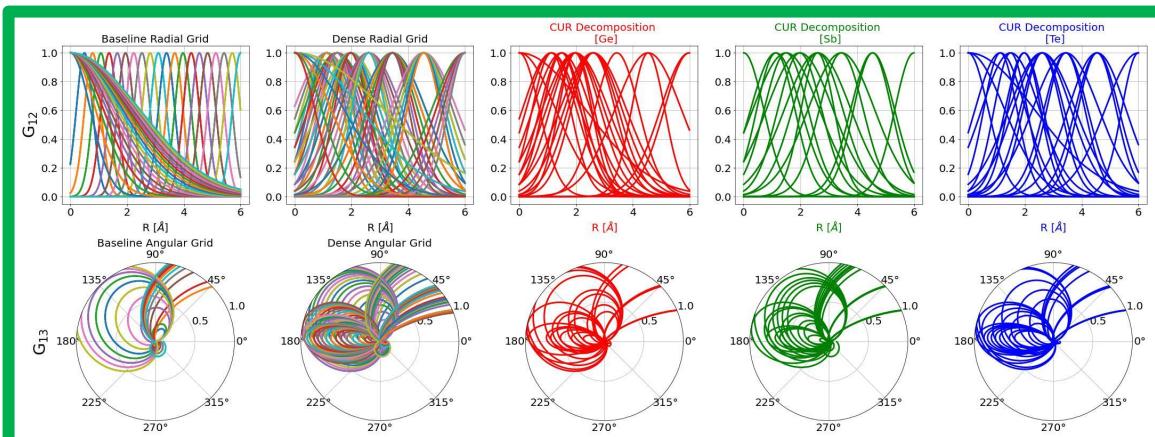
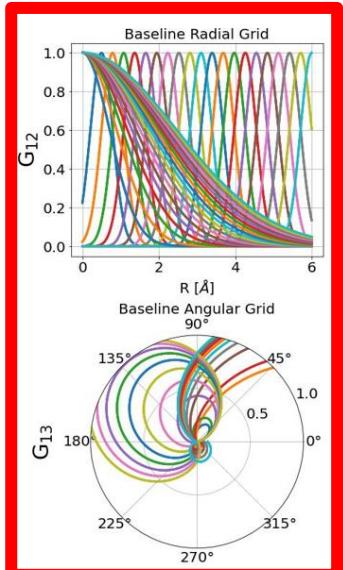


Generational Training Roadmap

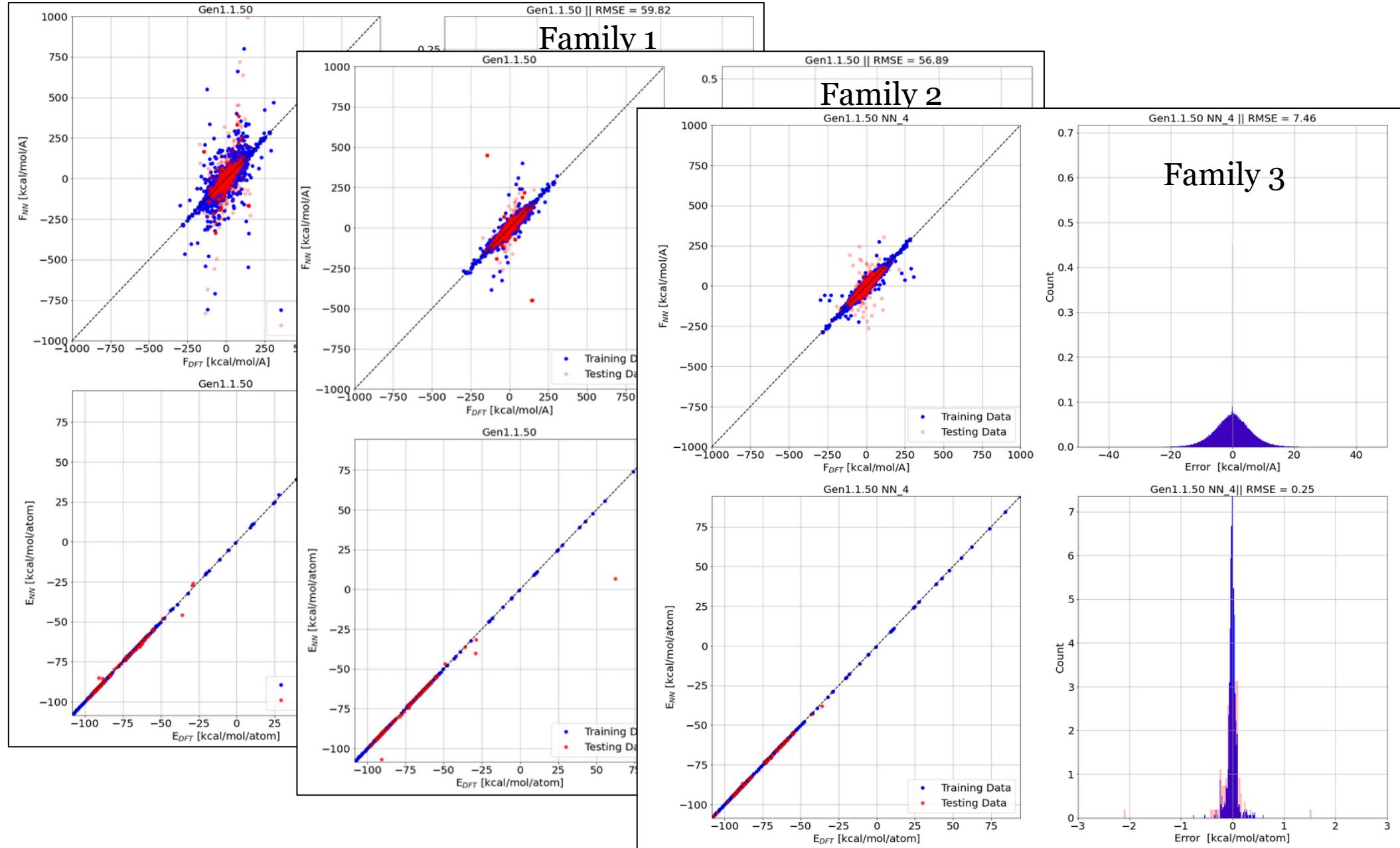
Family 1: Grid of features, use one network per iteration

Family 2: CUR selected features, use one network per iteration

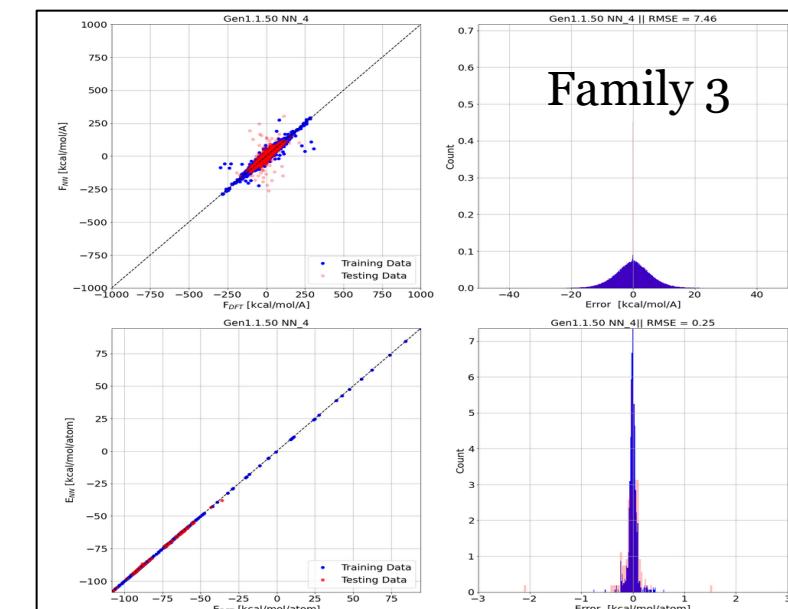
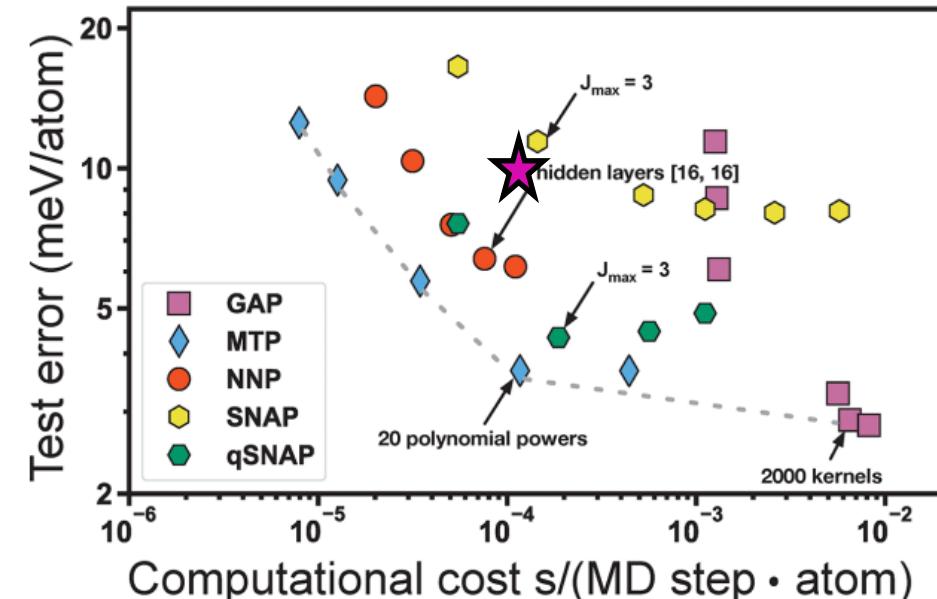
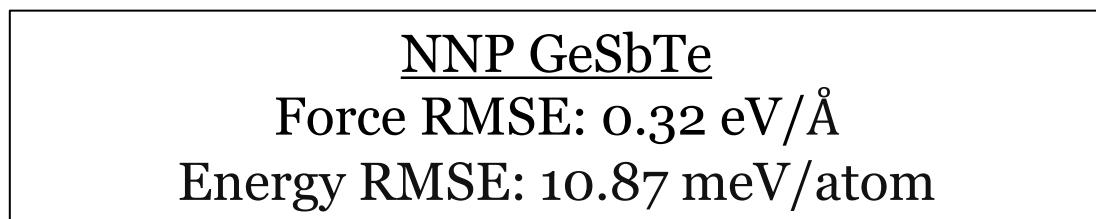
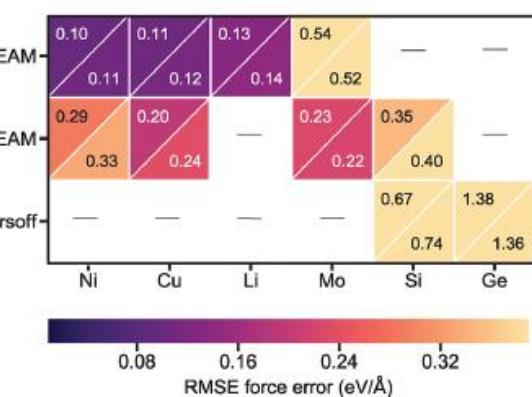
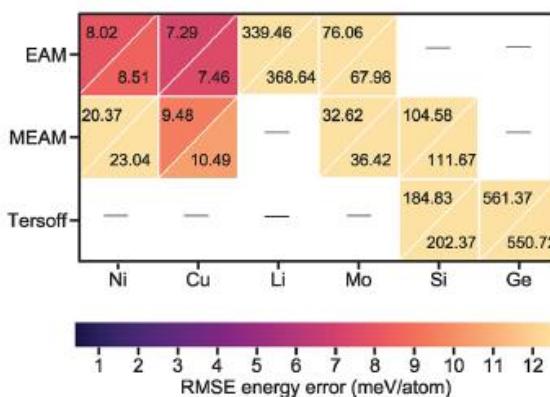
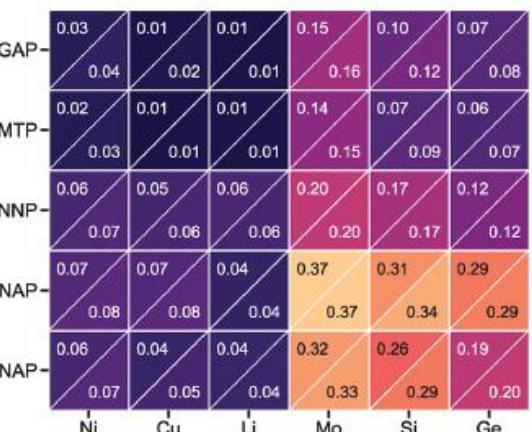
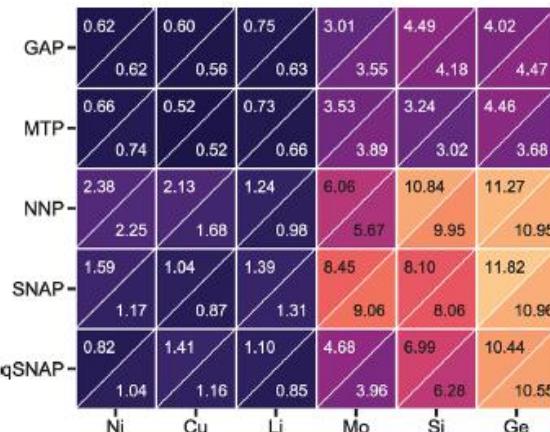
Family 3: CUR selected features, use five networks per iteration



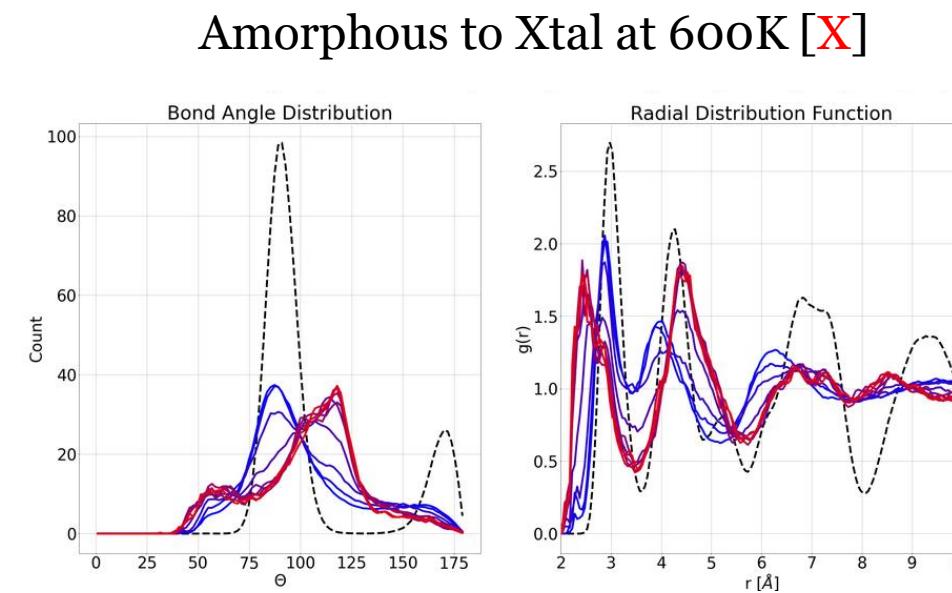
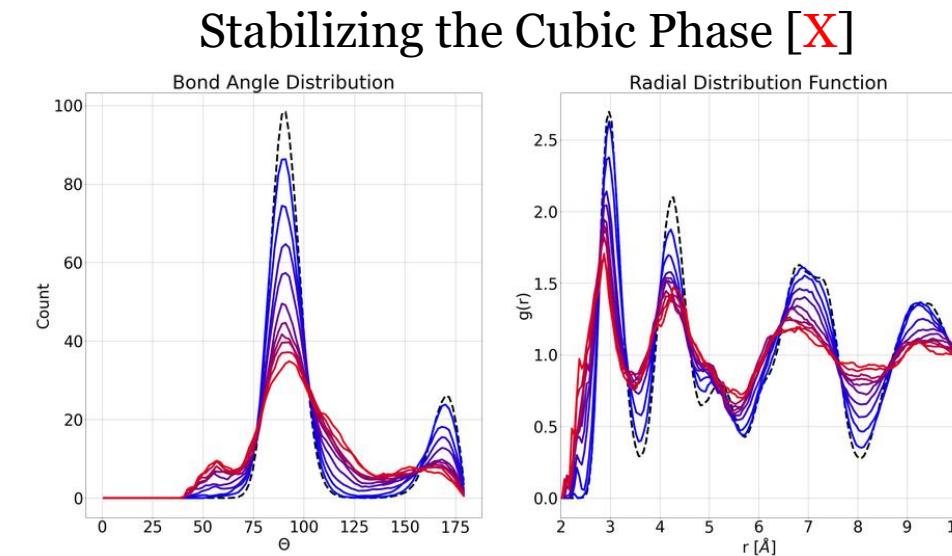
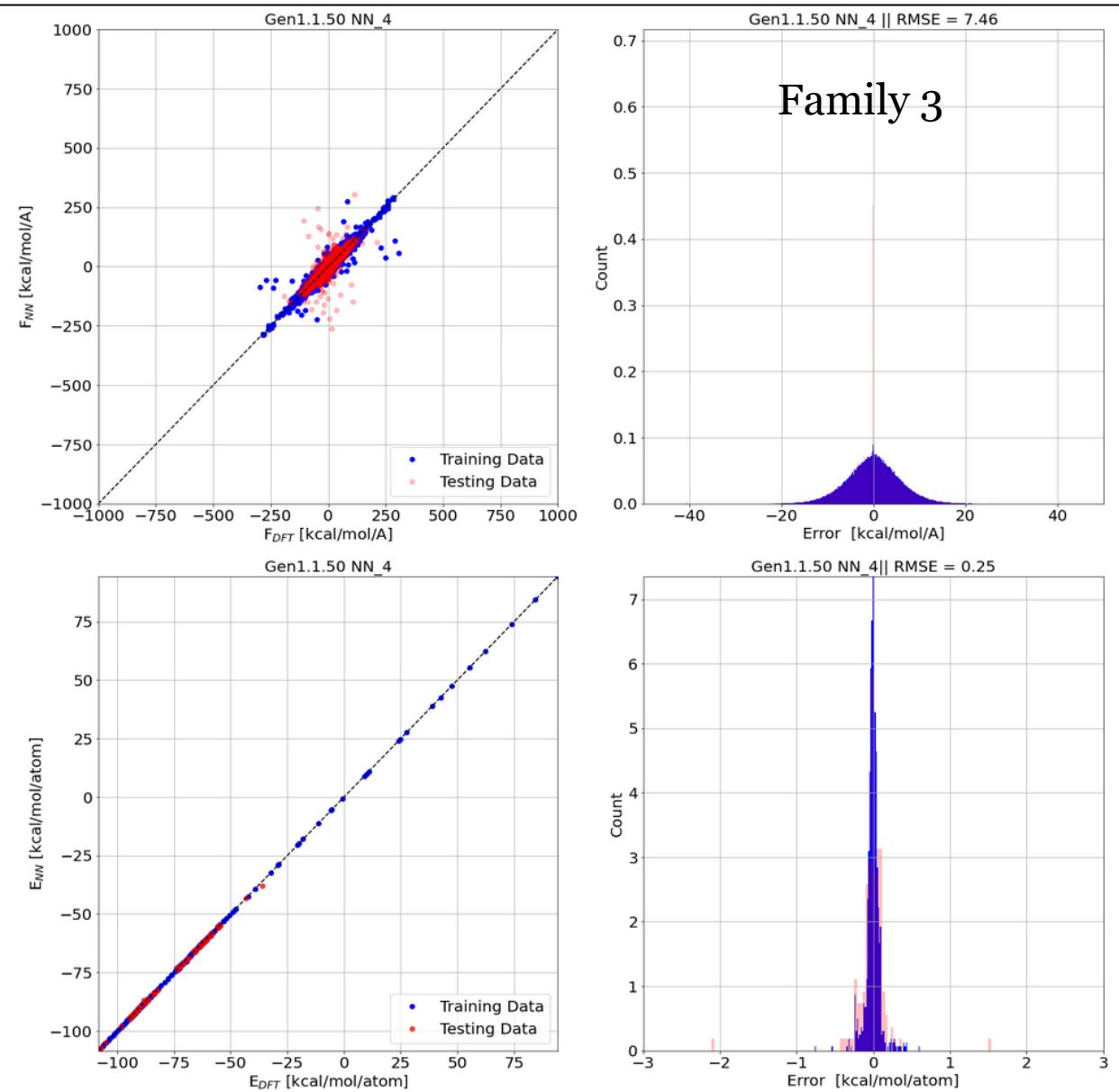
Baseline Training



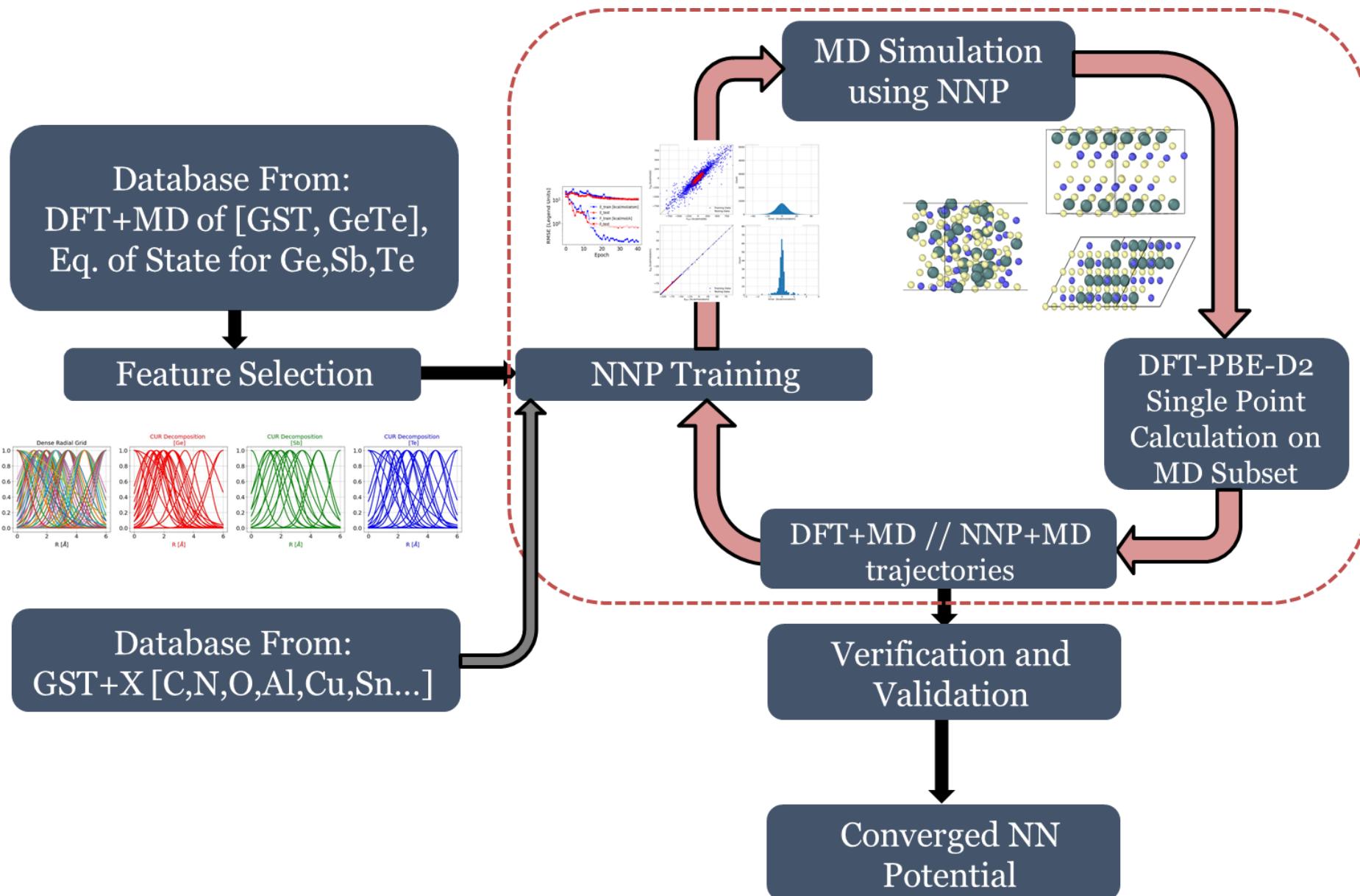
How do we compare?



Training Gen1.1 – Molecular Dynamics

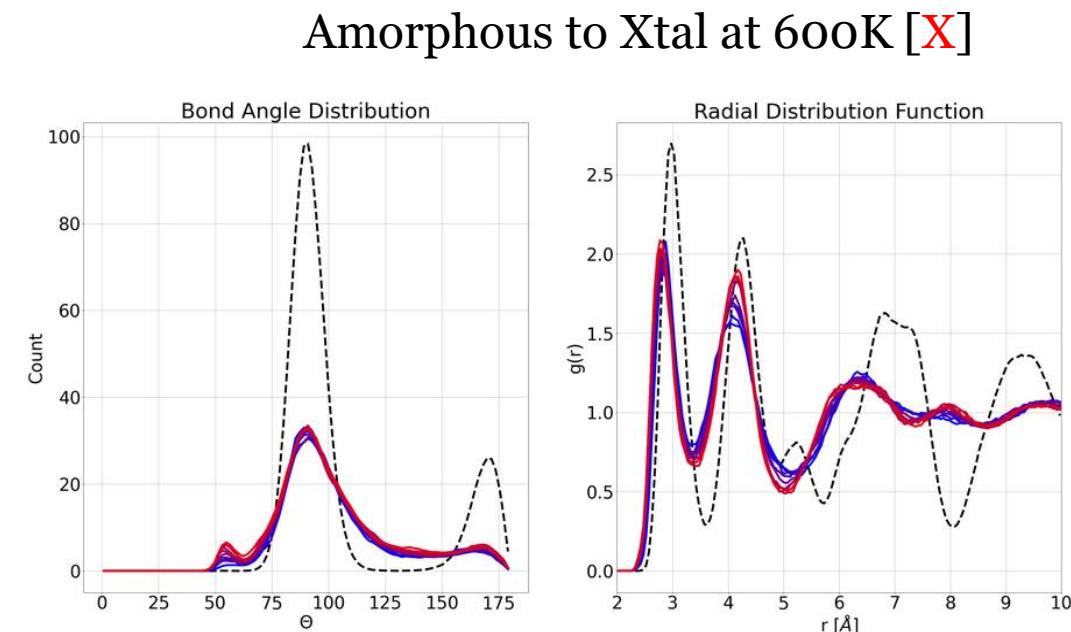
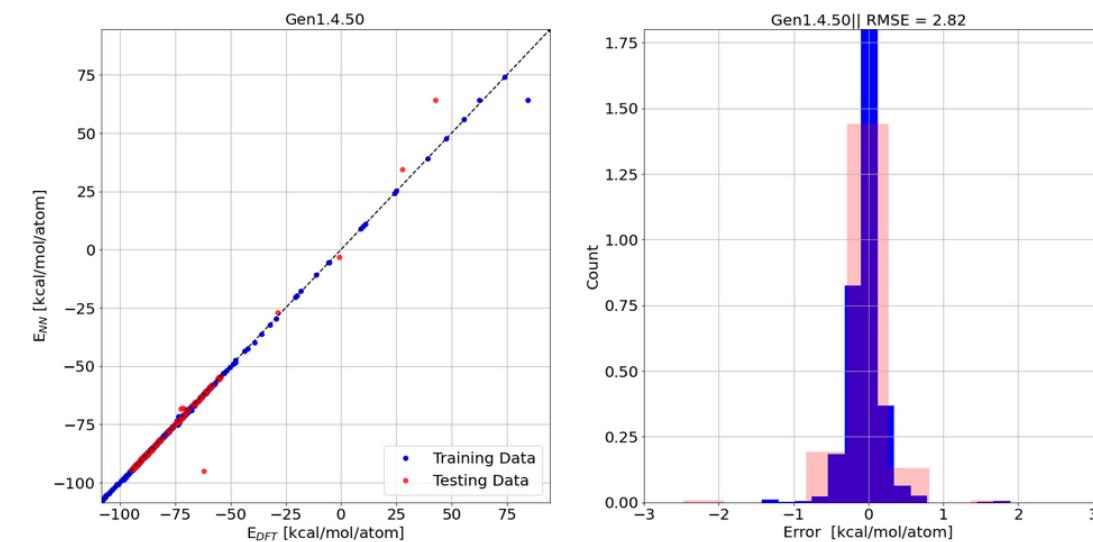
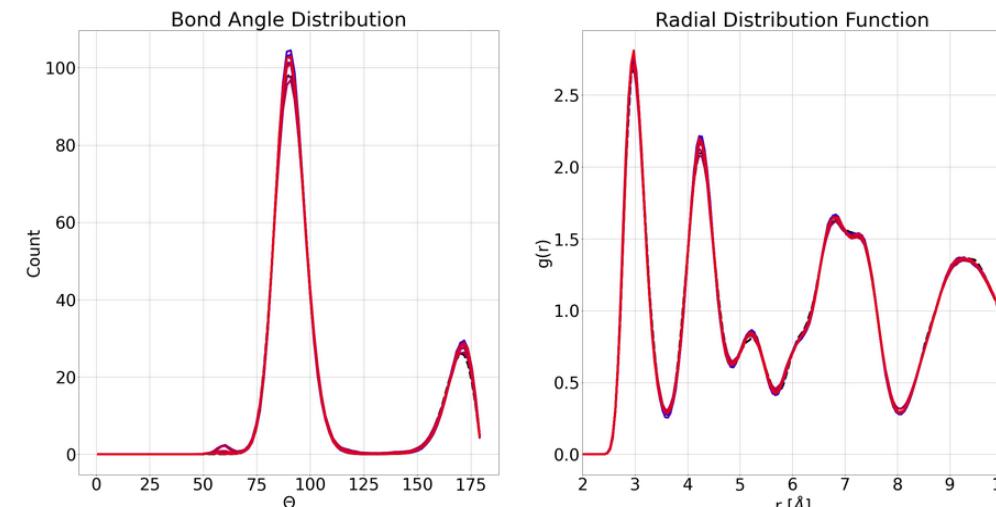
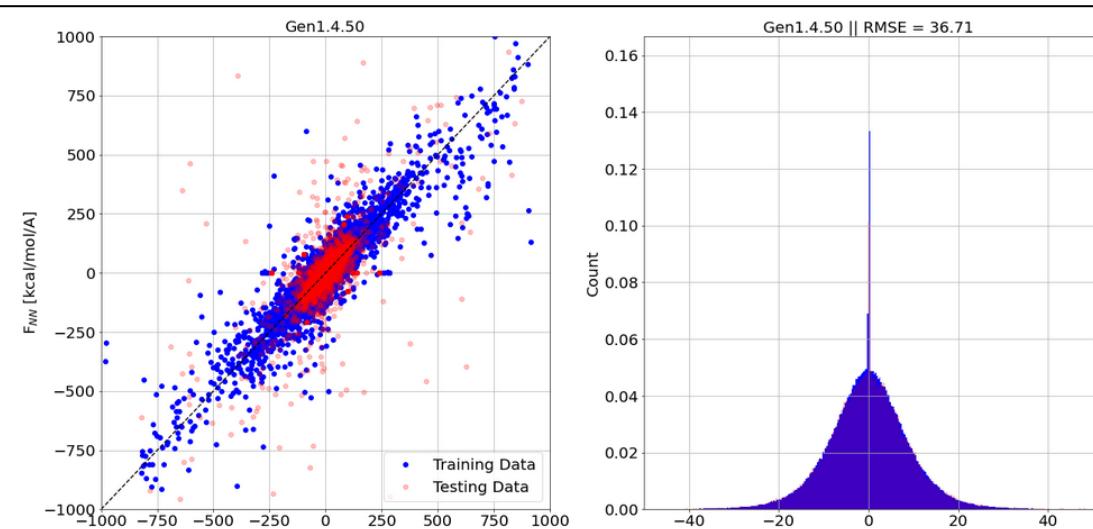


Begin the loop!

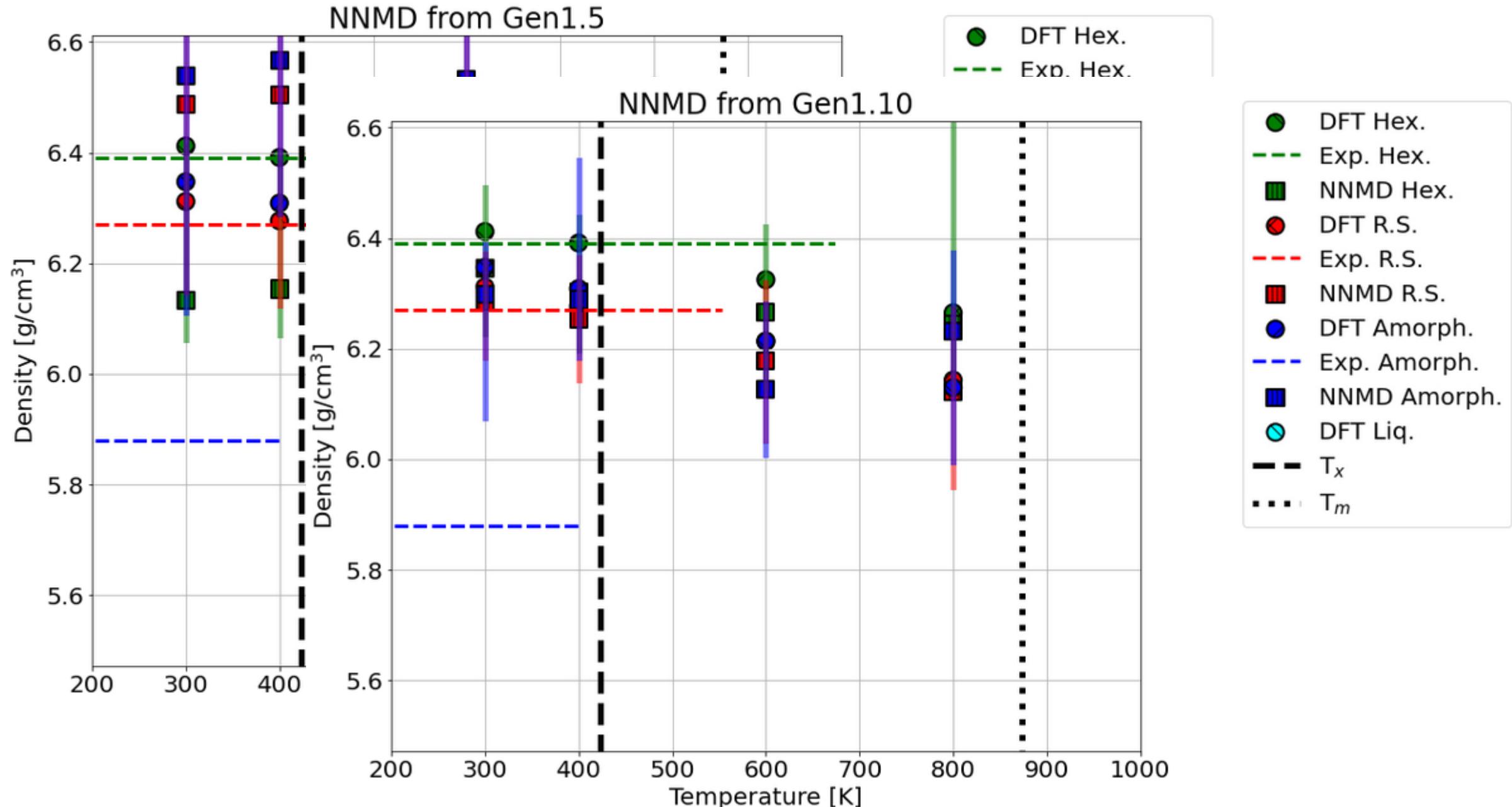


Training Gen1.4

Stabilizing the Cubic Phase [✓]



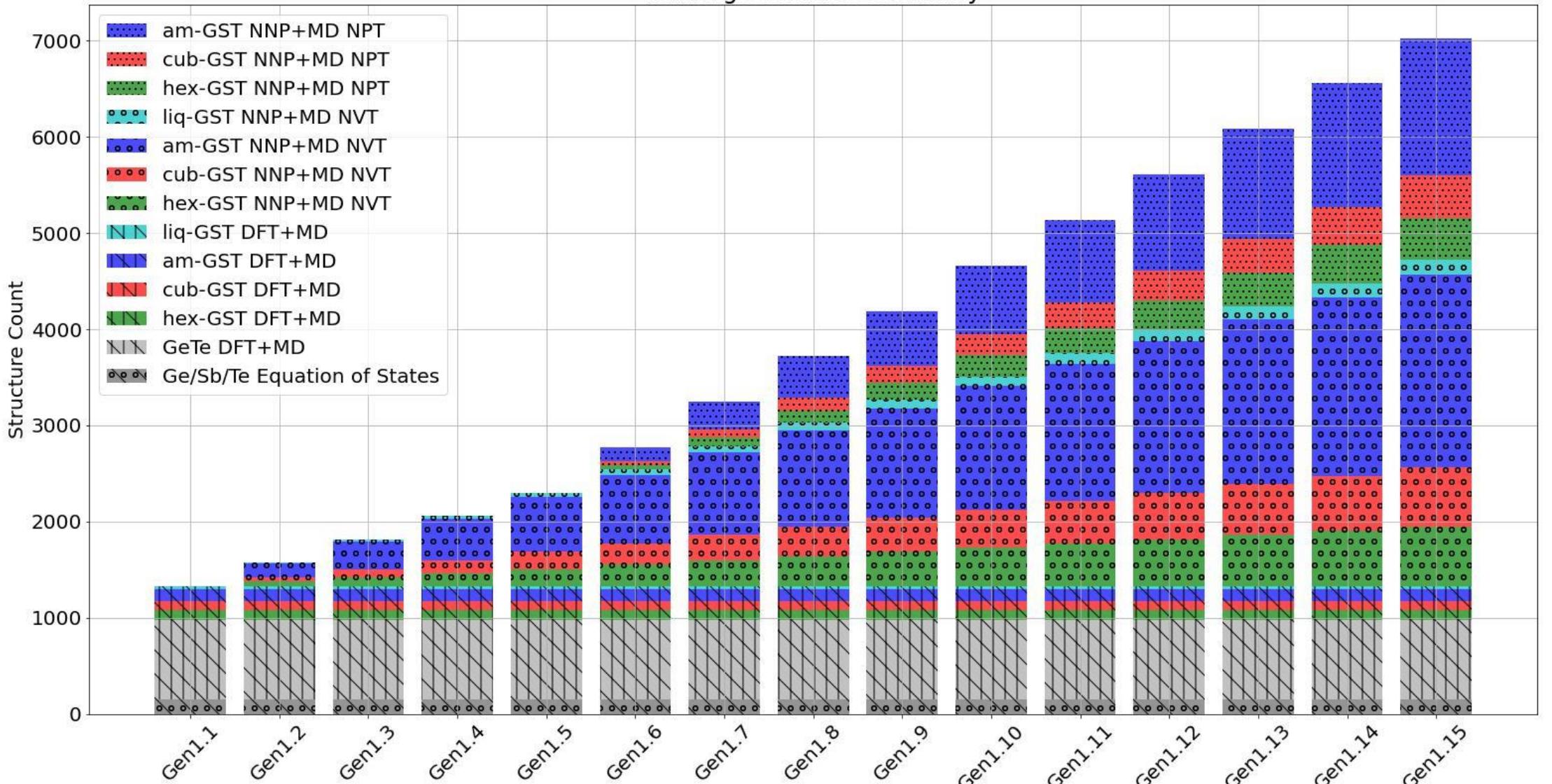
Switching to NPT and Evaluating Stability



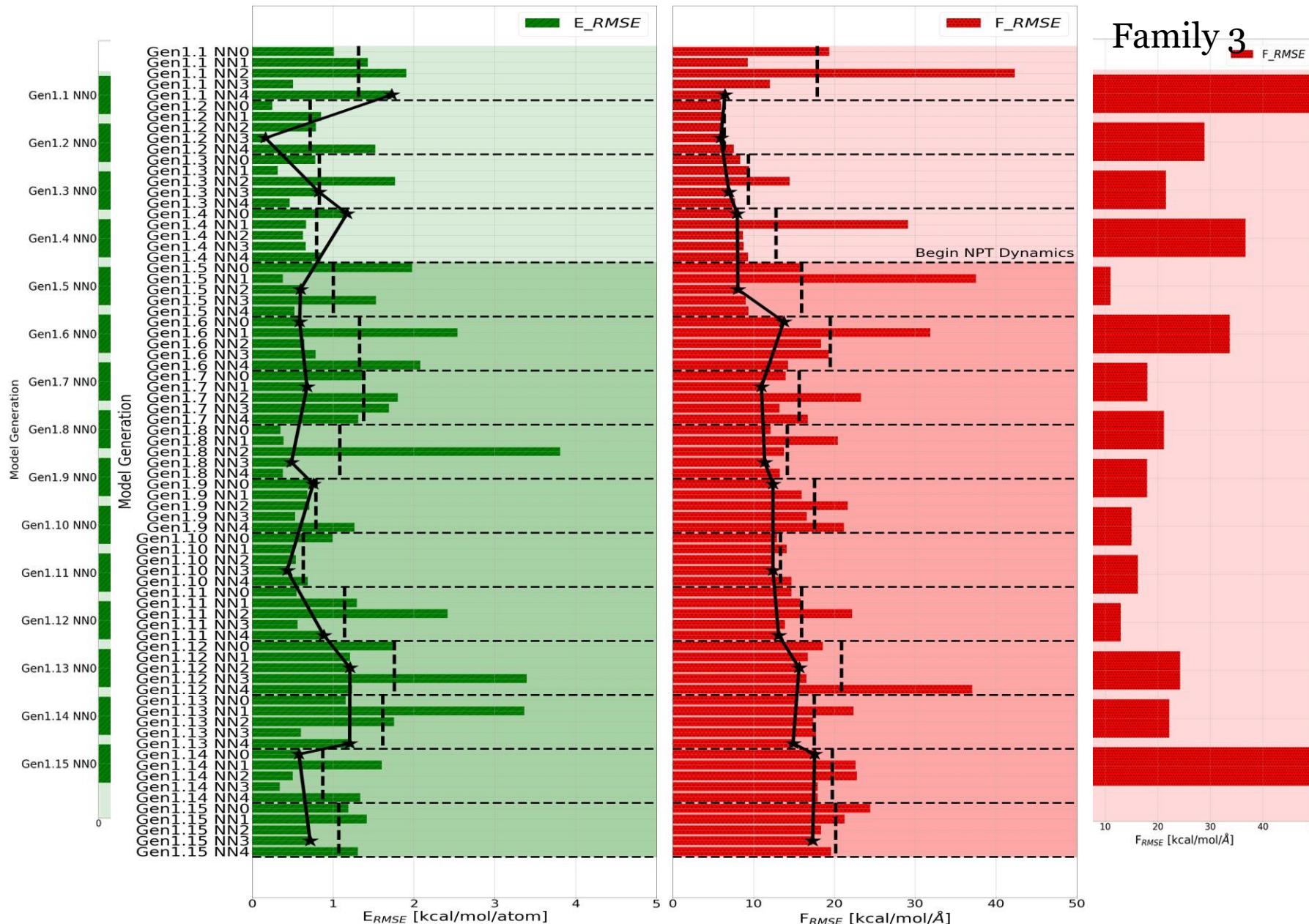
Database Composition

| | Eq. of State [Ge,Sb,Te] S.P. DFT | GeTe MD+DFT | Hex. Ge ₂ Sb ₂ Te ₅ MD+DFT | Cub. Ge ₂ Sb ₂ Te ₅ MD+DFT | Amorph. Ge ₂ Sb ₂ Te ₅ MD+DFT | Liq. Ge ₂ Sb ₂ Te ₅ MD+DFT | Hex. Ge ₂ Sb ₂ Te ₅ NVT | Cub. Ge ₂ Sb ₂ Te ₅ NVT | Amorph. Ge ₂ Sb ₂ Te ₅ NVT | Liq. Ge 2 Sb ₂ Te ₅ NPT | Hex. Ge ₂ Sb ₂ Te ₅ NPT | Cub. Ge ₂ Sb ₂ Te ₅ NPT | Amorph. Ge ₂ Sb ₂ Te ₅ NPT | |
|--------|--|----------------|---|---|--|---|---|---|--|--|---|---|--|---|
| Gen1.1 | 150 | 828 | 100 | 100 | 114 | 40 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Gen1.2 | 150 | 828 | 100 | 100 | 114 | 40 | 44 | 44 | 143 | 11 | 0 | 0 | 0 | 0 |
| Gen1.3 | 150 | 828 | 100 | 100 | 114 | 40 | 88 | 88 | 286 | 22 | 0 | 0 | 0 | 0 |
| | | | | | | | | | | 33 | 0 | 0 | 0 | 0 |
| | | | | | | | | | | 44 | 0 | 0 | 0 | 0 |
| | | | | | | | | | | 55 | 44 | 44 | 143 | |
| | | | | | | | | | | 66 | 88 | 88 | 284 | |
| | | | | | | | | | | 77 | 132 | 132 | 427 | |
| | | | | | | | | | | 88 | 176 | 176 | 570 | |
| | | | | | | | | | | 99 | 220 | 220 | 713 | |
| | | | | | | | | | | 110 | 264 | 264 | 856 | |
| | | | | | | | | | | 121 | 308 | 308 | 999 | |
| | | | | | | | | | | 132 | 352 | 352 | 1142 | |
| | | | | | | | | | | 143 | 396 | 396 | 1285 | |
| | | | | | | | | | | 154 | 440 | 440 | 1428 | |

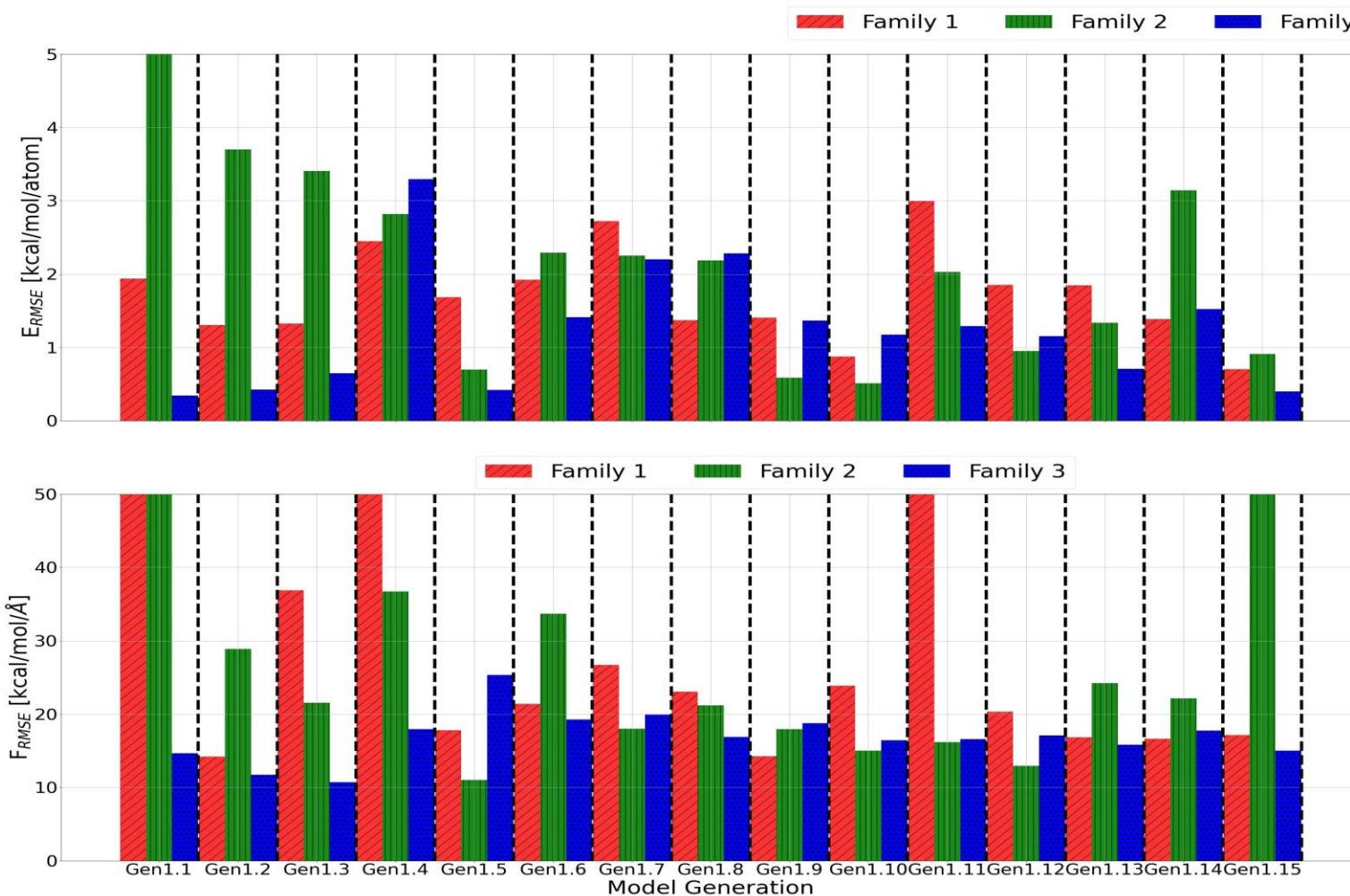
Training Database Summary



Comparing Three Families



Comparing Three Families

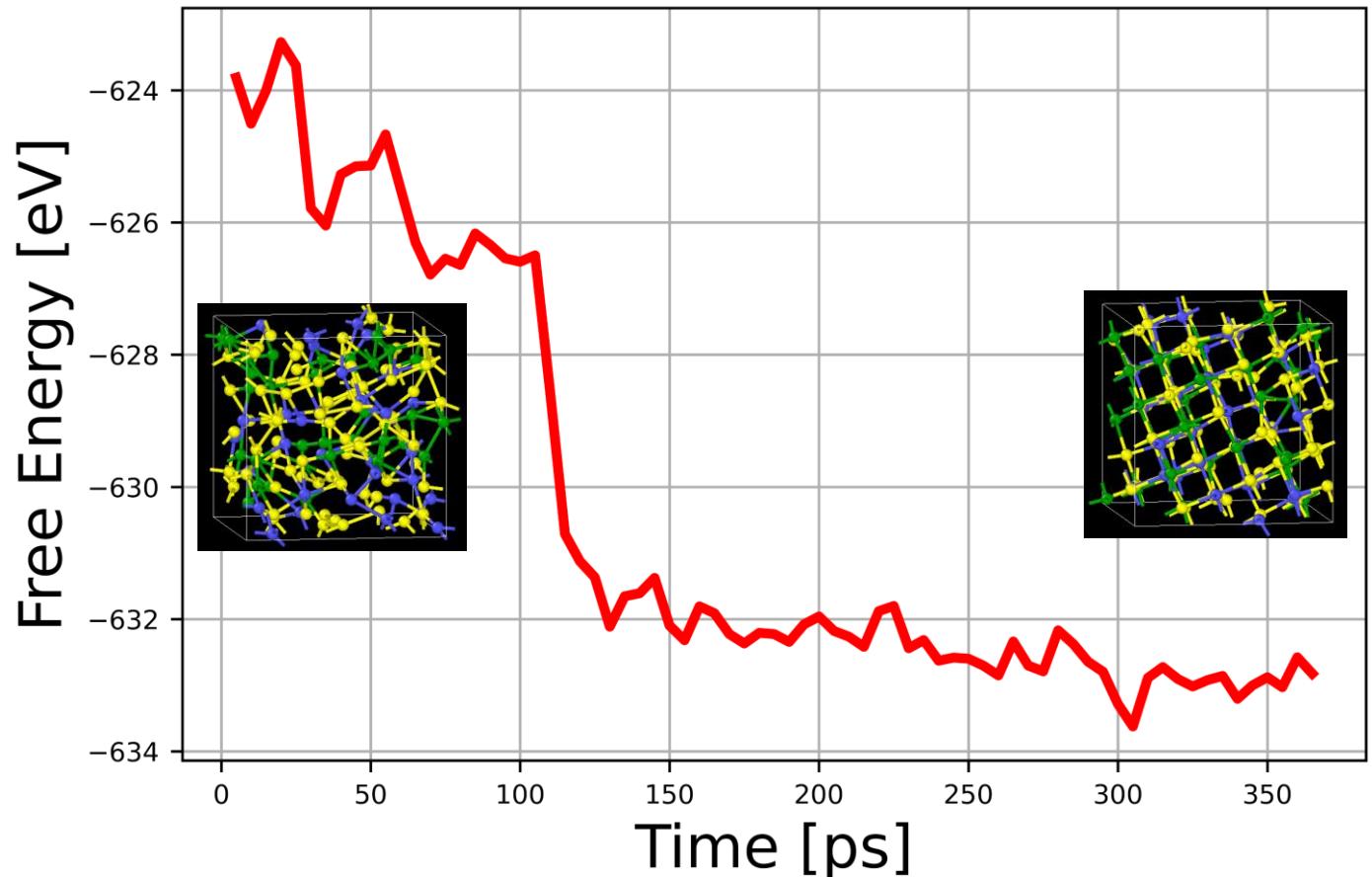
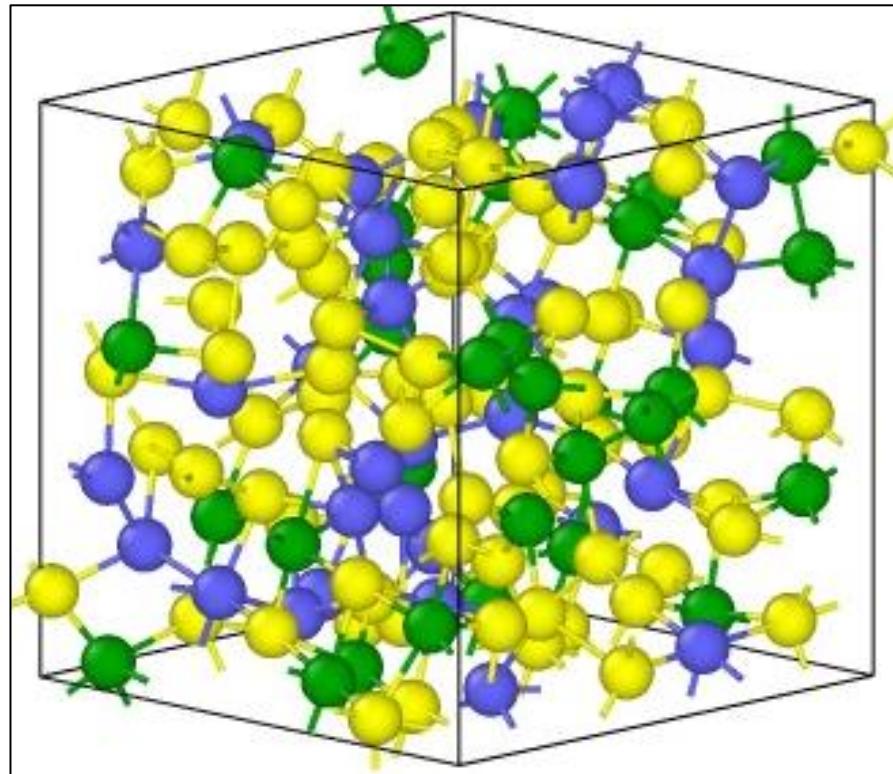


- CUR selected features [Family 2/3] outperform grid of features [Family 1]
- [Family 3] allows for well controlled dynamics and database enrichment

Validation with DFT

Atomic Simulations of GST+C
Phase Change Materials

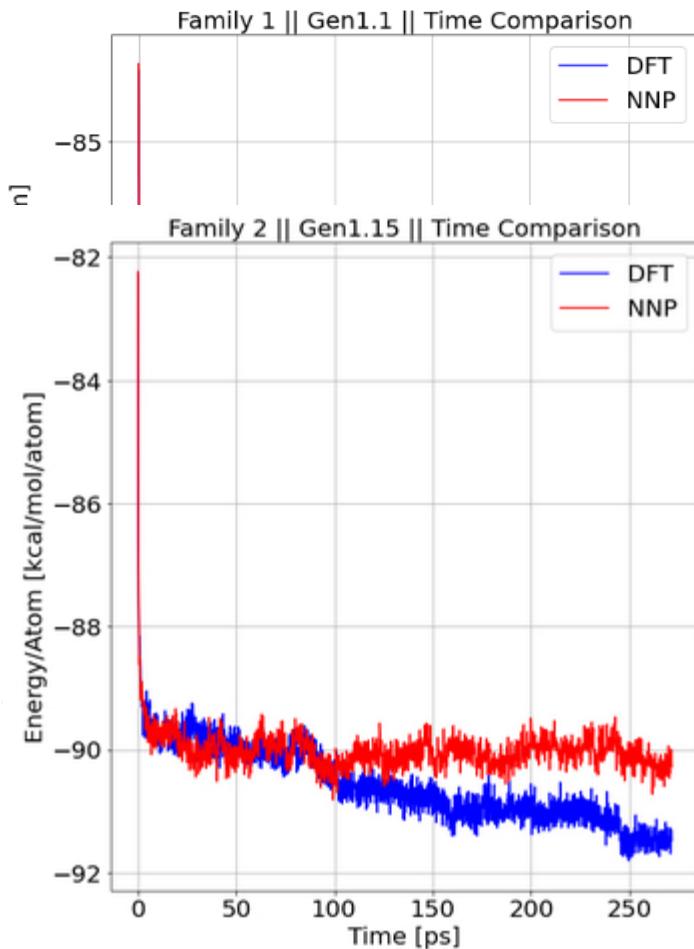
DFT+MD Simulation @ 600K ||
NVT ensemble



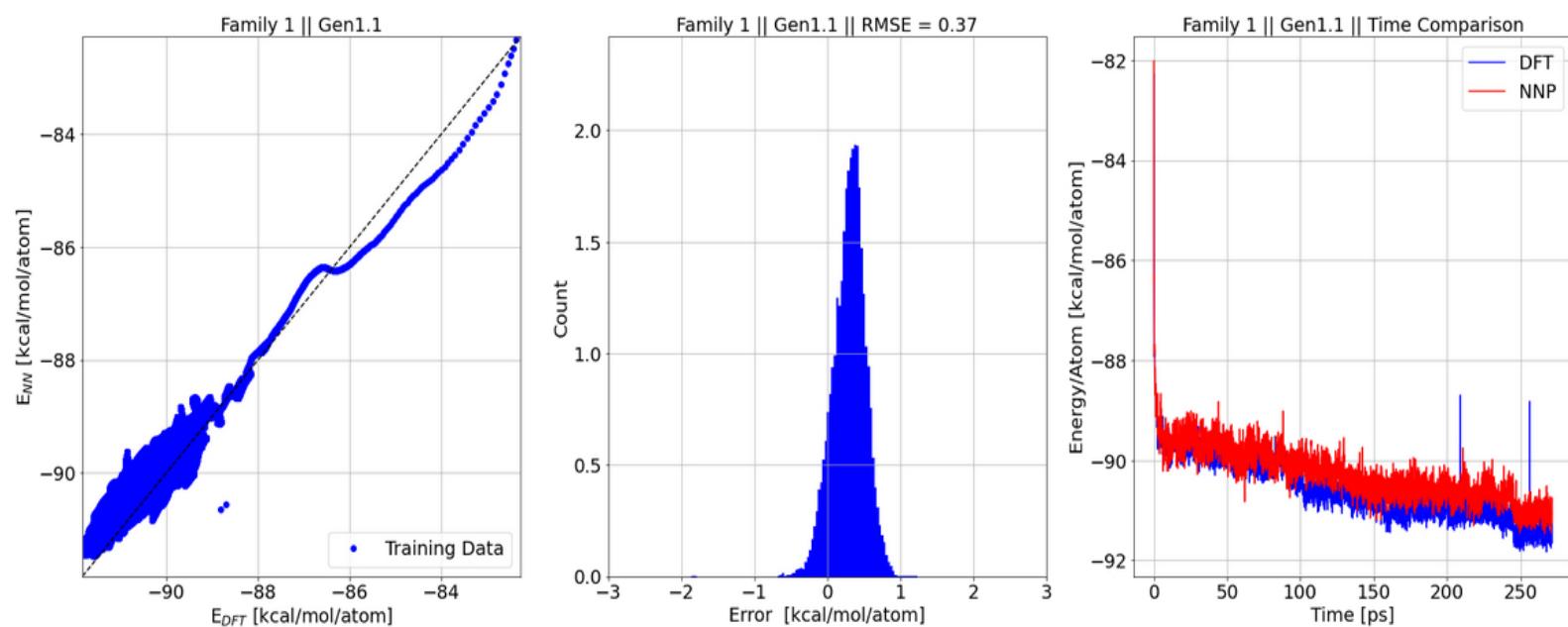
Crystallization via molecular dynamics

2 Flavors of Validation

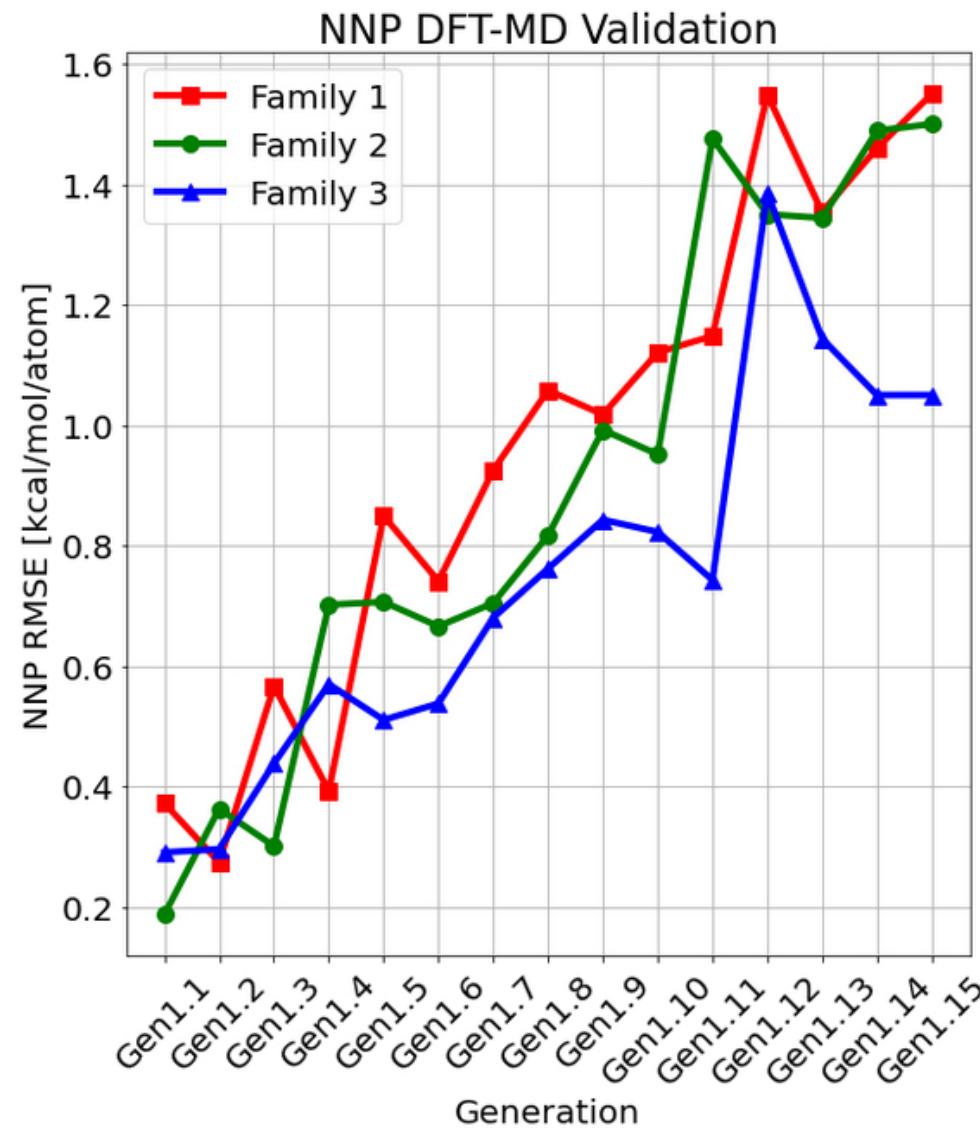
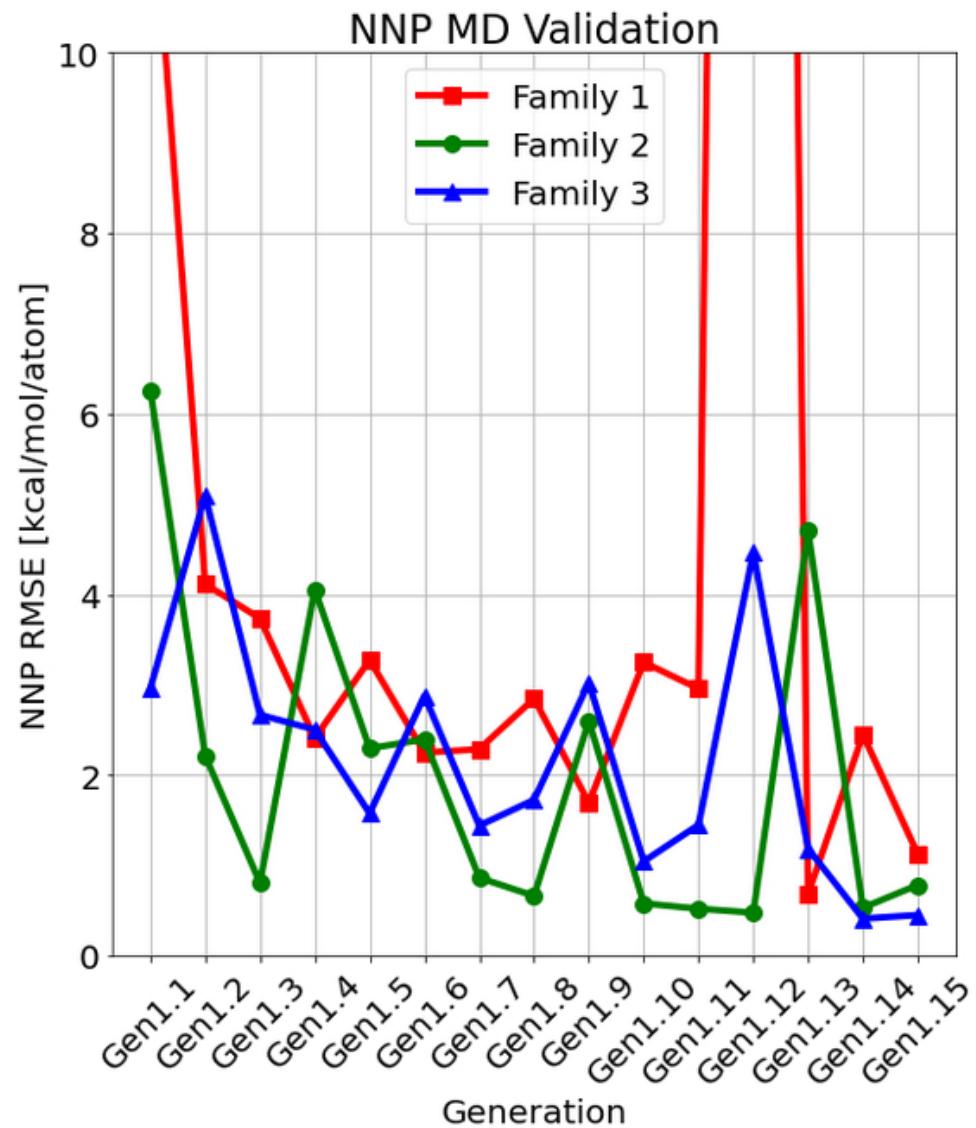
1.) How well can my trained weights reproduce an DFT+MD simulation of amorphous recrystallization with NNP+MD?



2.) How well can my weights reproduce energies of the DFT+MD trajectory

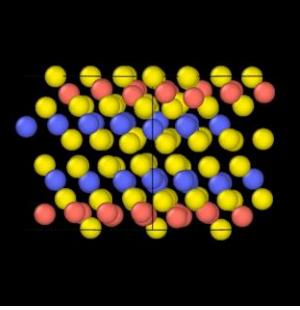
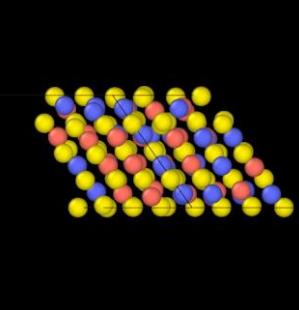
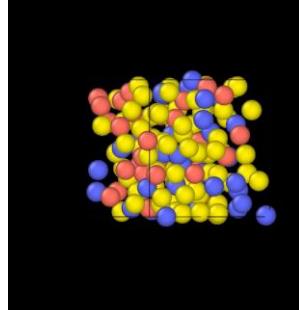
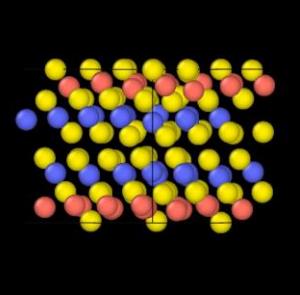
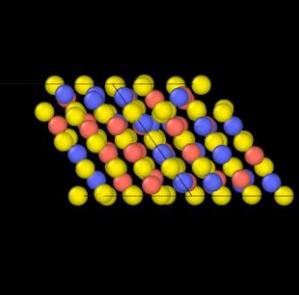
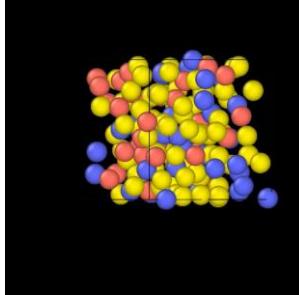
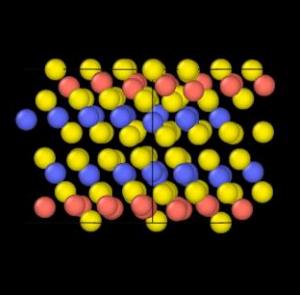
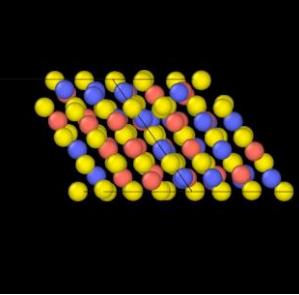
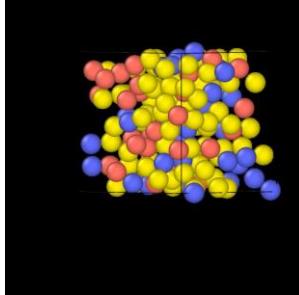


Generation Validation



Three Families – Three Phases

Ge Sb Te

| | Hexagonal | Cubic | Amorphous |
|---|---|---|---|
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |

Current State:

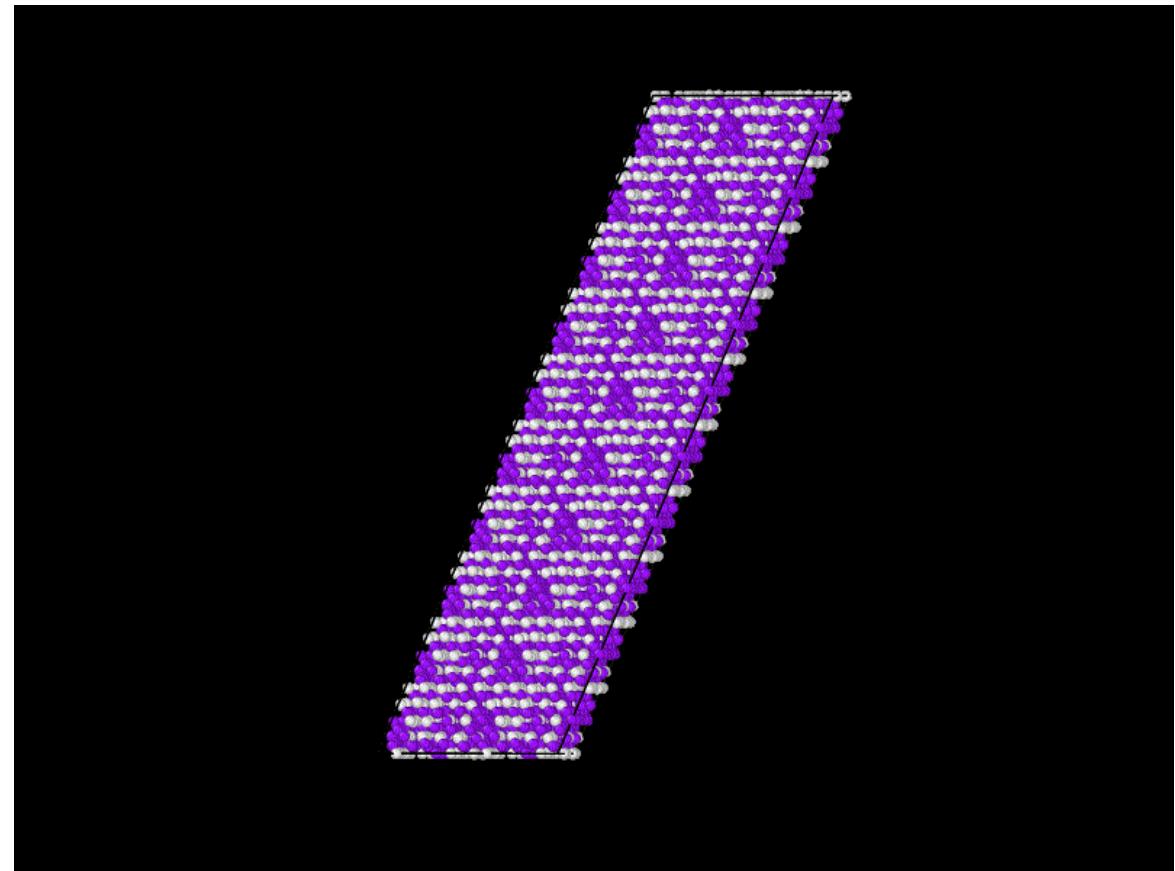
Successes:

- Stability of all three phases in NVT/NPT
- Recrystallization of amorphous into cubic phase shown
 - Good agreement with DFT structural representations for xtal/amorphous

Limitations:

- Large scale recrystallization needs a heterogeneous seed
- Liquid temperatures above 1200K become unstable in NPT

~7000 atoms – recrystallizing anneal 600K – 2 ns run



Soon to come:
GST+C iterative training

Acknowledgements

Strachan Research Group – Purdue University

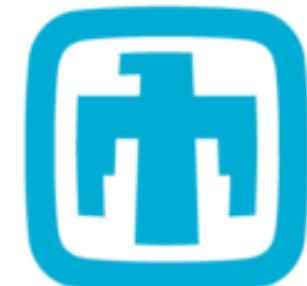


Project Colleague – Robert Appleton

PI Alejandro Strachan – Purdue University

Co PI David Adams – Sandia National Laboratory

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