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First-order gauge-invariant error rates in quantum processors.

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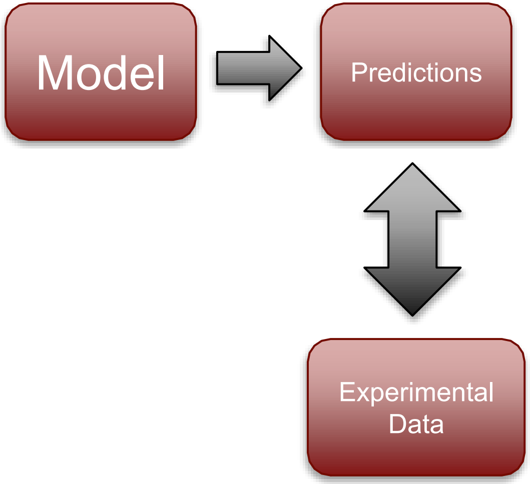
Modeling quantum processors

- We create models of quantum processors in order to:
  - Predict their behavior and performance
  - Understand how they deviate from the ideal

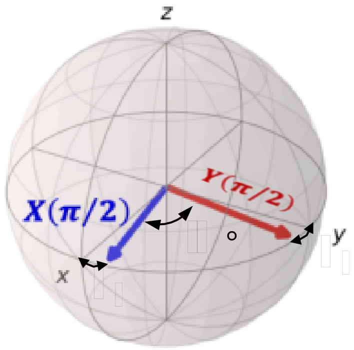


- Model = anything that makes predictions.
  - Opaque model – a black box that predicts individual/avg outcomes
  - Transparent model -- can look inside the model and gain insight from values/parameters
  - Common model = process matrix for every gate

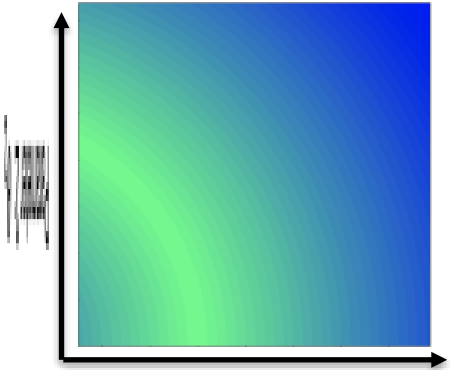
Gauge freedom in models

- **But there's a problem!** *Gauge transformations* change all the the process matrices but none of the predictions!

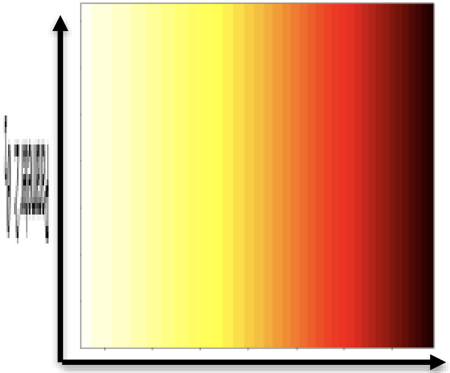
<u>Example</u>:
Gate set = $\{X(\pi/2), Y(\pi/2)\}$
Perfect except angle between rotation axes is **80° instead of 90°**



Predictions vs. data



Gauge-variant metric
(e.g. fidelity of $X(\pi/2)$)



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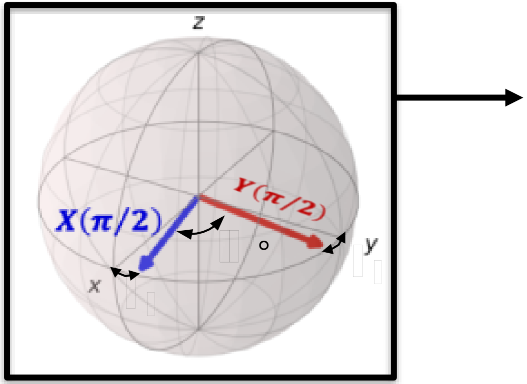
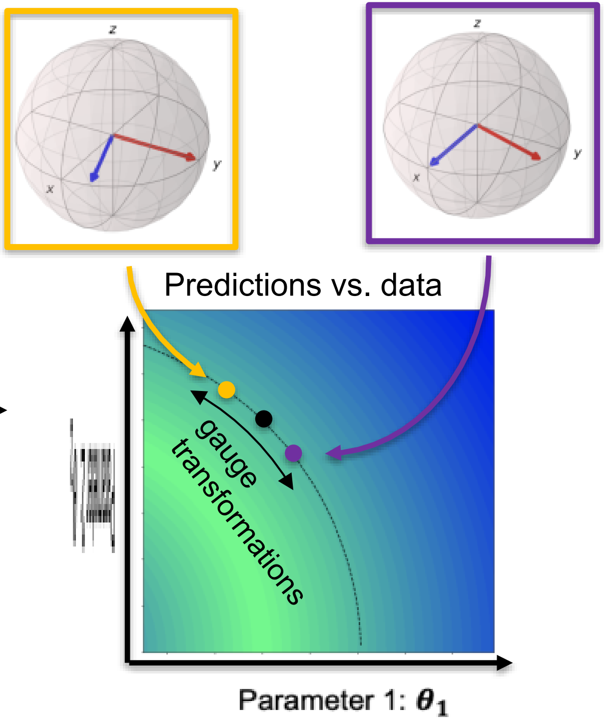
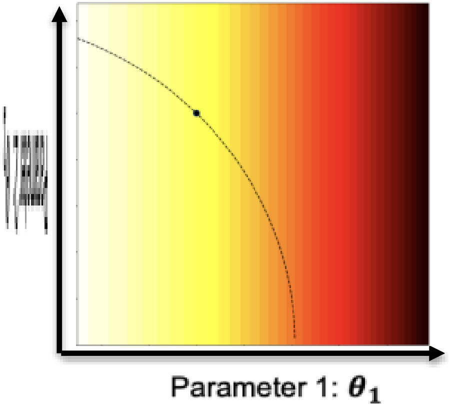




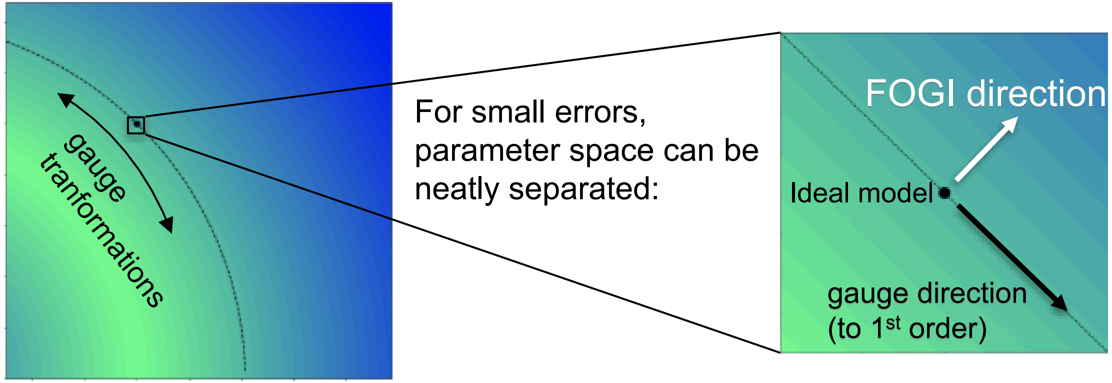
Insight based on gauge-variant quantities is flawed!

Gauge-variant metric (e.g. fidelity of $X(\pi/2)$)



FOGI directions

- **First order gauge invariant (FOGI)** directions = a first step toward gauge invariant models & metrics.
- Linearize about a the ideal (error free) model in parameter space:



- **Goal of this work: how do we find these FOGI directions?**
- First step: choose coordinates to use

Error generators instead of process matrices

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Model

$\mathbb{G}_1$ $\mathbb{G}_2$ …

Gates are parameterized by error rates:

Gate $\mathbb{G}$ superoperator $= e^{L_{\mathbb{G}}}$ Ideal gate superoperator

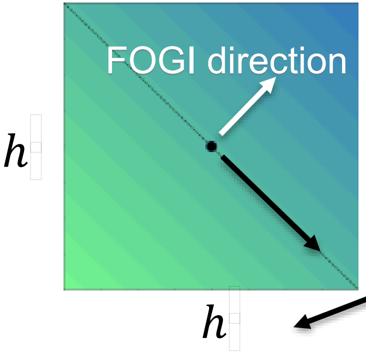
error generator

$$L_{\mathbb{G}} = \sum_P h_P^{\mathbb{G}} H_P + \sum_P s_P^{\mathbb{G}} S_P + \sum_P c_{P,Q}^{\mathbb{G}} C_{P,Q} + \sum_P a_{P,Q}^{\mathbb{G}} A_{P,Q}$$

Different types of **elementary error generators**

Sector		Dimension	Action	Example effect (Bloch sphere)
Hamiltonian	$\mathbb{H}$	d^2-1	$H_P[\rho] = -i[P,\rho]$	
Stochastic (Pauli)	$\mathbb{S}$	d^2-1	$S_P[\rho] = P\rho P - \mathbb{1}\rho\mathbb{1}$	
Stochastic (Pauli-correlation) $\mathbb{C}$	{P,Q}=0 [P,Q]=0	$\binom{d^2-1}{2}$	$C_{P,Q}[\rho] = P\rho Q + Q\rho P - \frac{1}{2}\{\{P,Q\},\rho\}$	
Active $\mathbb{A}$	{P,Q}=0 [P,Q]=0	$\binom{d^2-1}{2}$	$A_{P,Q}[\rho] = i\left(P\rho Q - Q\rho P + \frac{1}{2}\{[P,Q],\rho\}\right)$	

(arXiv:2103.01928)

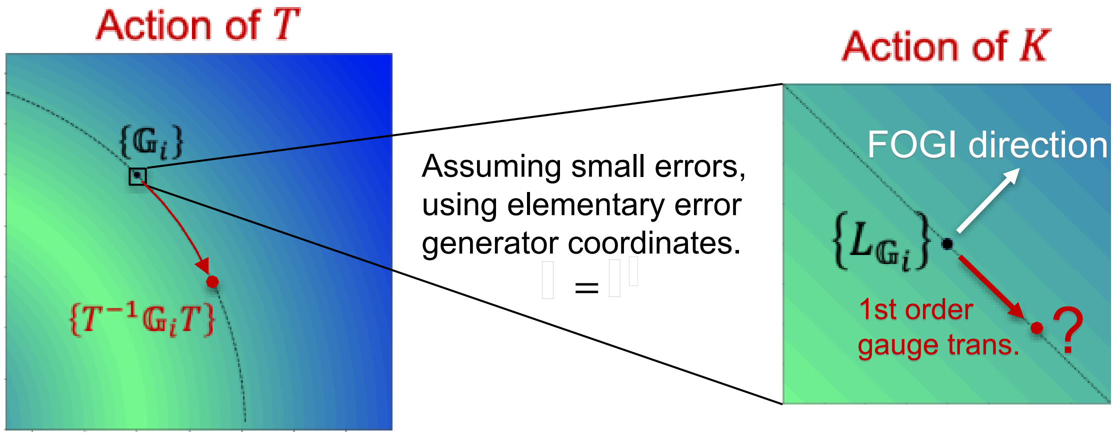


Coefficients = ***coordinates*** we work in.

Gauge transformations

- **Step 1:** coordinates = elementary error generator coefficients ✓
- **Step 2**: figure out what gauge transformations do within this space.
  - We know what they do to process matrices:

    gauge transformation T takes process matrices $\mathbb{G} \rightarrow T^{-1}\mathbb{G}T$



Gauge transformations at 1st order

Gauge transformation on process matrices (**nonlinear**)

$$\mathbb{G} \rightarrow T^{-1}\mathbb{G}T$$

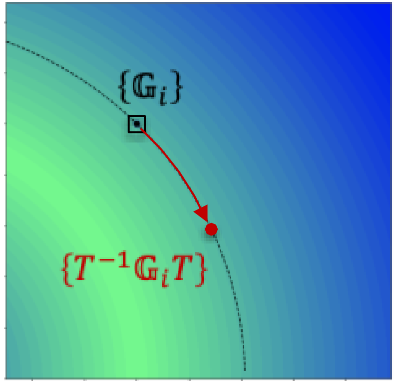
Write using error & gauge generators

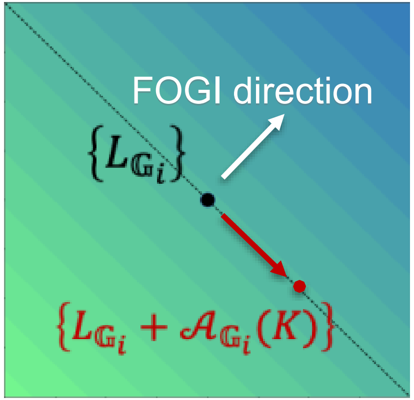
$$T = e^{K}$$
$$\mathbb{G} = e^{L_{\mathbb{G}}}\mathbb{U}$$

linearize (assume K and $L_{\mathbb{G}}$ are small)

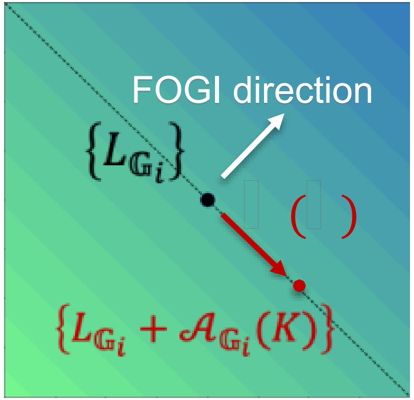
Gauge transformation on error generators (**linear!**)

$$L_{\mathbb{G}} \rightarrow L_{\mathbb{G}} + \underbrace{(\mathbb{U}K\mathbb{U}^{-1} - K)}_{\mathcal{A}_{\mathbb{G}_i}(K)}$$





FOGI directions



$$\begin{bmatrix} L_{\mathbb{G}_1} \\ L_{\mathbb{G}_2} \\ \vdots \end{bmatrix} \rightarrow \begin{bmatrix} L_{\mathbb{G}_1} \\ L_{\mathbb{G}_2} \\ \vdots \end{bmatrix} + \begin{bmatrix} \mathcal{A}_{\mathbb{G}_1} \\ \mathcal{A}_{\mathbb{G}_2} \\ \vdots \end{bmatrix} K$$

- 1st order gauge direction for gauge generator K = $\mathcal{A}(K)$
- All 1st order gauge directions = range($\mathcal{A}$)
- All FOGI directions = $\overline{\text{range}(\mathcal{A})}$ (complement of range)

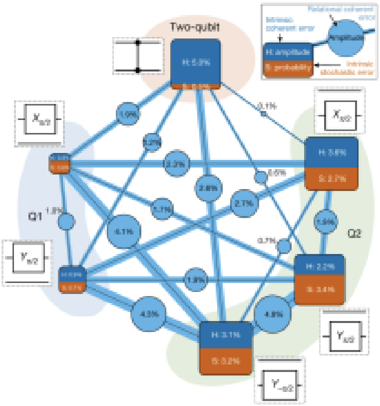
We're done!
We've found a basis for the FOGI space!

Constructing a *nice* FOGI space basis

- We'd like to find a *nice* basis, where elements have minimal support.

- Additional work (see arxiv paper) shows how to construct a basis composed of:
  - *intrinsic* FOGI directions: support on a single gate
  - *relational* FOGI directions: support on a pair or limited subset of gates.

- Presents gate set error as a graph that attributes error to one or more nodes (gates).



[Nature **601**, 348 (2022)]

1-qubit example

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- Gate set = $\{X(\pi/2), Y(\pi/2), \rho_0, M\}$
- Each gate has 6 parameters; model has **12 parameters total.**
  - 3 Hamiltonian: $h_X^{\mathbb{G}}, h_Y^{\mathbb{G}}, h_Z^{\mathbb{G}}$ 3 Pauli Stochastic: $s_X^{\mathbb{G}}, s_Y^{\mathbb{G}}, s_Z^{\mathbb{G}}$

FOGI construction: 12 = 7 FOGI + 5 gauge parameters

$X(\pi/2)$

$Y(\pi/2)$

Intrinsic

FOGI directions

1. $h_X^{X(\pi/2)}$
2. $s_X^{X(\pi/2)}$
3. $\left(s_Y^{X(\pi/2)} + s_Z^{X(\pi/2)}\right)$

Over-rotation angle

On-axis stochastic error

Total off-axis stochastic error

4. $h_Y^{Y(\pi/2)}$
5. $s_Y^{Y(\pi/2)}$
6. $\left(s_X^{Y(\pi/2)} + s_Z^{Y(\pi/2)}\right)$

Relational

FOGI directions

7. $\left(h_Y^{X(\pi/2)} - h_Z^{X(\pi/2)}\right) - \left(h_X^{Y(\pi/2)} - h_Z^{Y(\pi/2)}\right)$

Summary

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- Gauge degrees of freedom make model-based characterization difficult.

- First-order gauge-invariant (FOGI) directions partially solve this problem:
  - Assumes small errors and gauge transformations
  - Applies linear algebra to separate (first-order) gauge from gauge-invariant directions.

- Procedure exists for constructing a nice basis for FOGI space.
  - View a gate set's errors as a mix of **intrinsic** (attributed to a single gate) and **relational** (quantifies a relationship between gates) errors that are 1st-order insensitive to gauge transforms.