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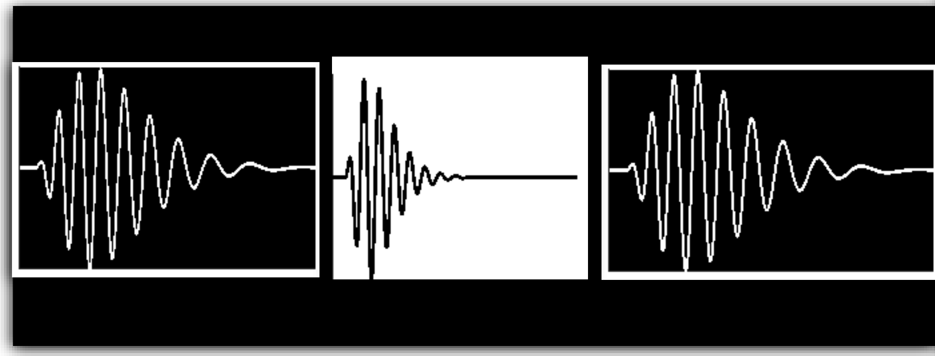

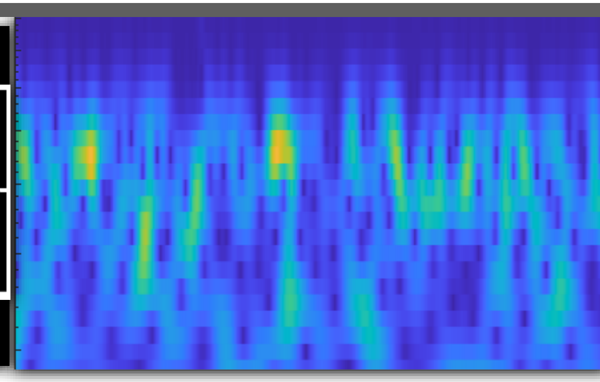

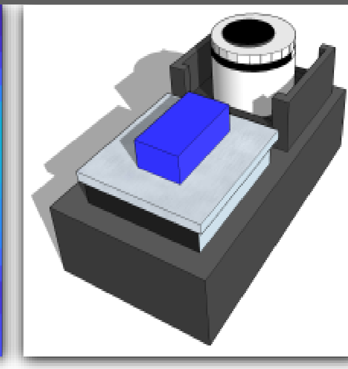

Detection of Transient Structural Response with Information Theory

Brief Overview with an Example

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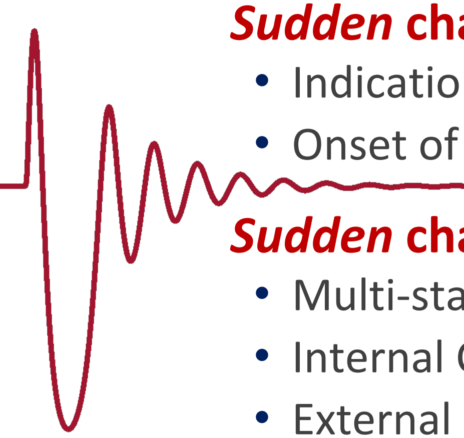
Transients in Structural Response Data

Transients: Events of engineering interest that produce a sudden change in dynamic response.

Knowing when the event occurred is the first step to further analysis



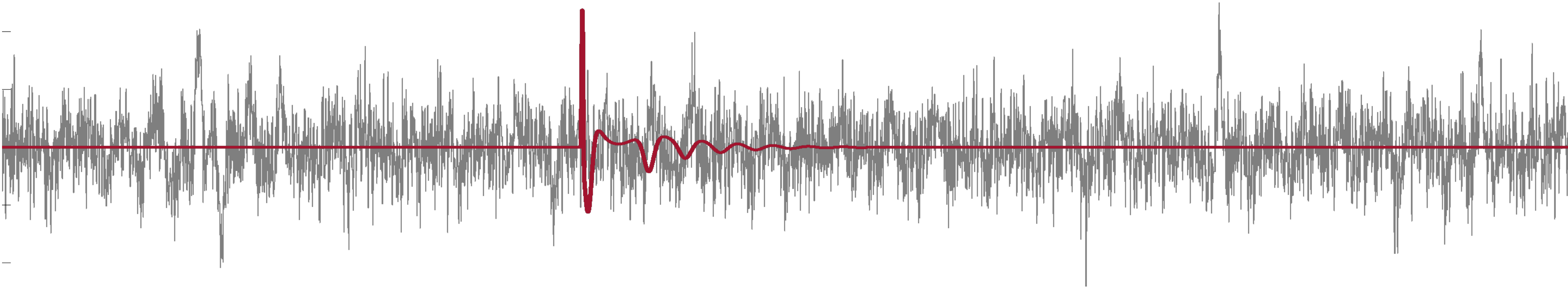
***Sudden* change in system behavior**

- Indication of instability
- Onset of damage

***Sudden* change in excitation**

- Multi-stage Rocket Separation Events
- Internal Component Impact
- External Impact
- Earthquakes

The Challenge

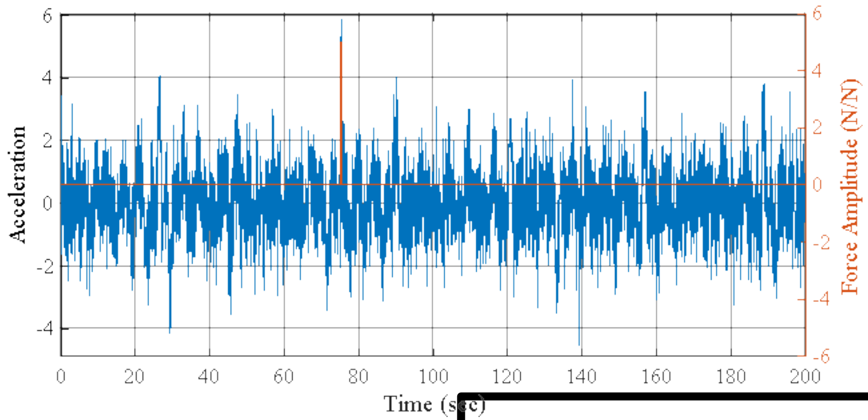


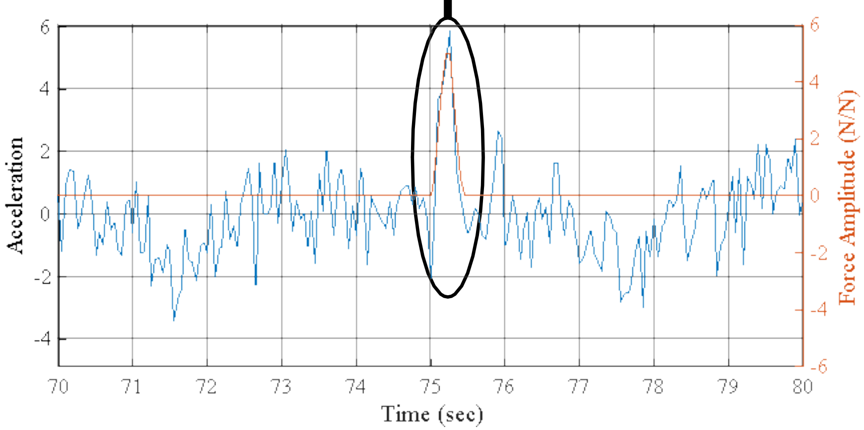
- Short duration events are easily averaged out
- Senor placement may not be optimal (lots of noise)
- Background might be difficult to characterize statistically (non-Gaussian)
- Intermittent events may or may not have lasting effects
  - In nonlinear systems, any lasting effect may eclipse the event itself – easy to mischaracterize the peak and the timing of the event

The ultimate goal is to produce a method for finding intermittent, anomalous signatures in nonlinear structural response data that can be automated.

Time-Frequency Visual Inspection

In practice, analysts use a combination of signal statistics and visual inspection with time -frequency methods to find events.



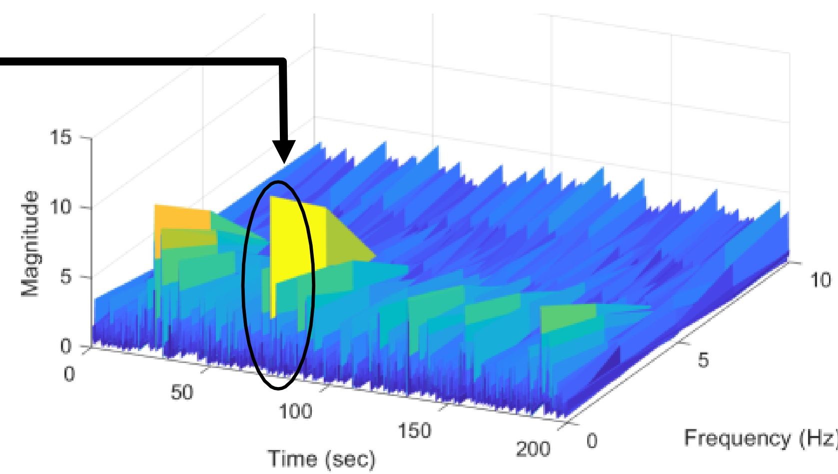


Discrete Short-Time Fourier Transform (STFT)

The STFT modulated with a periodic Hann window, W. For a signal f(τ):

$$\Upsilon(\kappa,\omega) = \sum_{-\infty}^{\infty} f(\tau)W(\tau-\kappa)e^{-i\omega\tau}$$

Υ then represents the signal in two dimensions: frequency ω and discrete time step κ.



Time-Frequency Visual Inspection

In practice, analysts use a combination of signal statistics and visual inspection with time-frequency methods to find events.

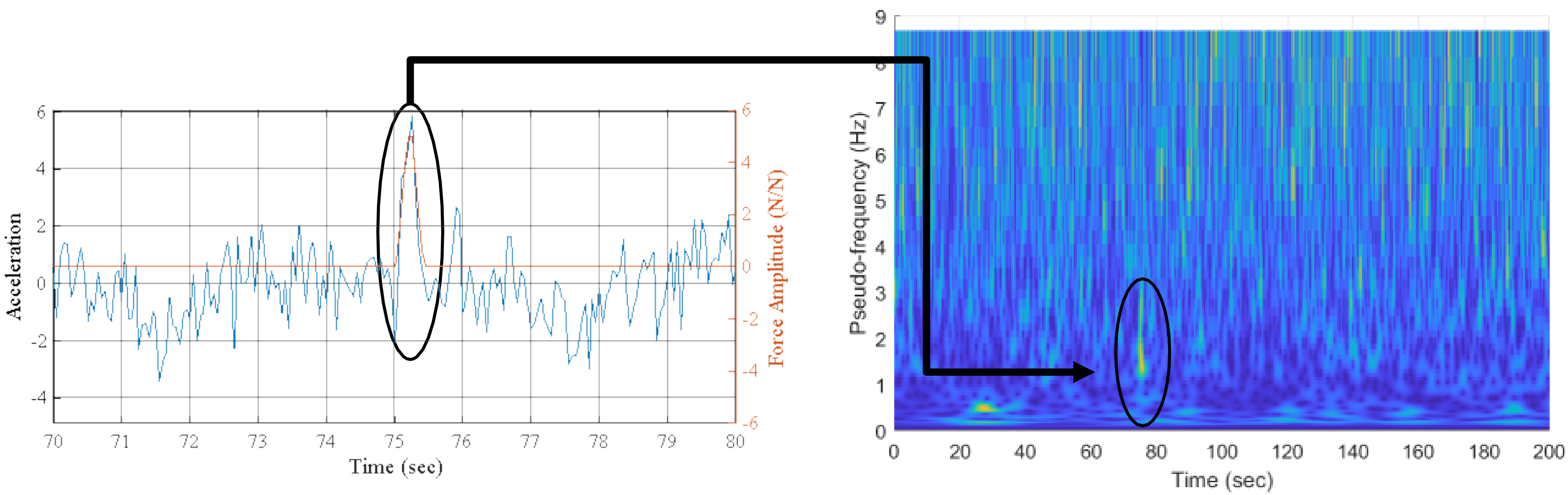
Continuous Wavelet Transform (CWT)

$$\Upsilon(s,\tau) = \int f(t)\Psi^{*}_{s,\tau}(t)dt$$

$$\Psi_{s,\tau} = \frac{1}{\sqrt{s}}\Psi\left(\frac{t-\tau}{s}\right)$$

Υ then represents the signal in two dimensions: scale factor s and time step τ. Generally, τ = t. The scale factors are converted to pseudo-frequency values.

Not practical for large amounts of data



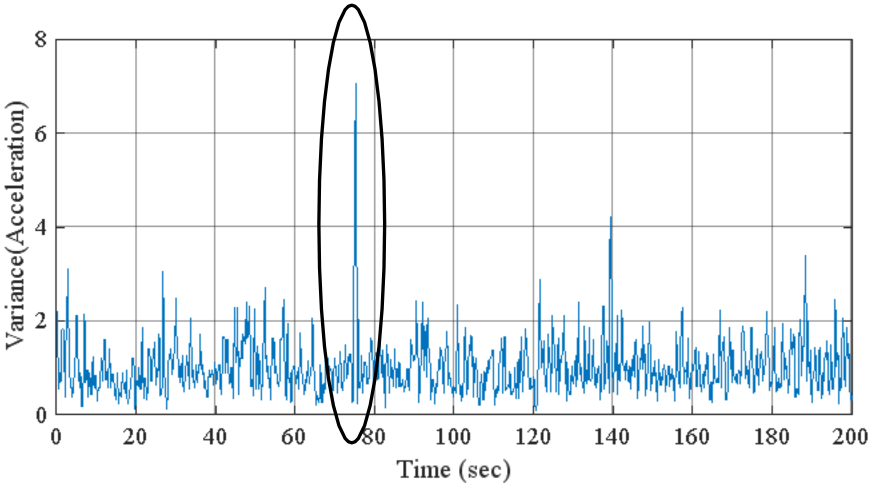
Statistical Time History Inspection

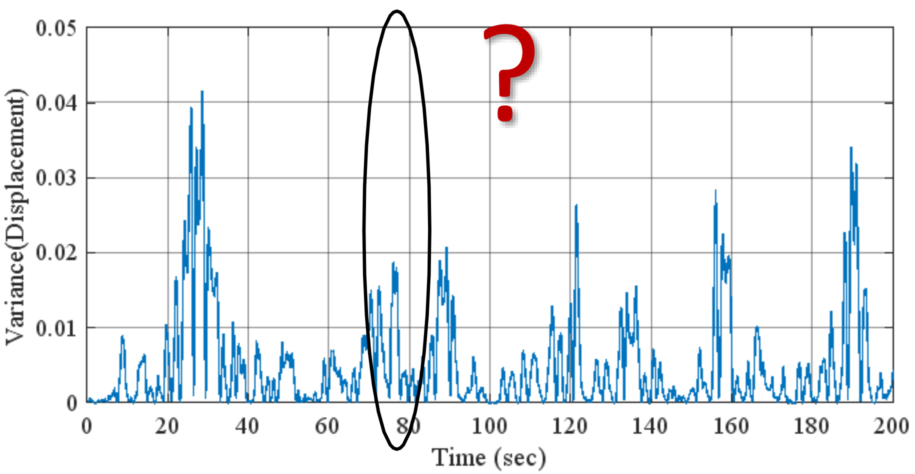
Variance

The variance is the second statistical moment. It is computed for a sequence that is N points long:

$$Var = \frac{\sum_{i}^{N} |x_i - \bar{x}_i|^2}{(N-1)}$$

Variance computed here is on a moving segment 8 points (N) with an 4 point overlap.





Variance is really good when the pulse amplitude is bigger than the background and the background has uniform variation (*random*).

Alternative Methods of Time History Inspection

Hölder Exponent

The exponent that satisfies the Hölder Condition for a continuously differentiable function

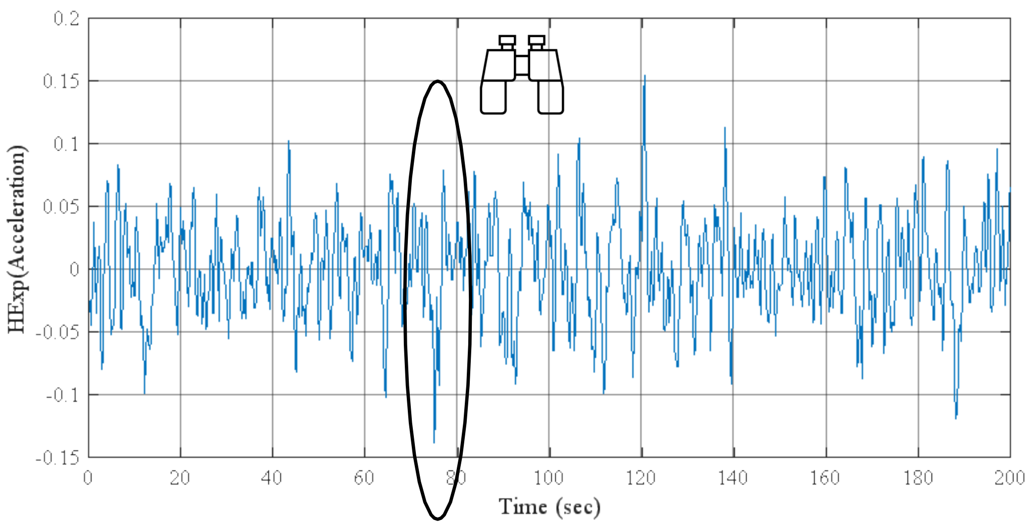
$$|f(x) - f(y)| \leq C\|x - y\|^m$$

m goes low or negative when the function is 'less differentiable'.

The Holder exponent is extracted by calculating the negative linear slope resulting from the log of the wavelet modulus over the log of the pseudo-frequency values.

$$HExp(t) = -\frac{\log(W(t,f))}{\log(f)}$$

Hexp is found by linear regression at each t between Log(W) and Log(f) ***per time t***. The intercept is discarded.



HExp is effective but requires some filtering and smoothing.

- Examples are high pass filtered with a cut-off frequency of 0.05 Hz
- Then smoothed with a 8-point running average with 4 point overlap so that the time resolution matches the IIF and the other two metrics.

Information Theory

C.E. Shannon borrowed the concept to describe information

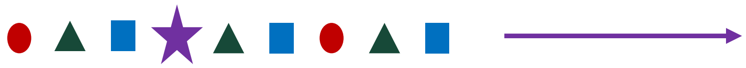
Entropy H for a macrostate with N microstates, each with probability p_i. Note, Entropy is max for homogenous data

$$H = -\frac{1}{\log_2(N)}\sum_{i=1}^{N} p_i \log_2 p_i$$

This mathematical relationship is known as the ***Boltzmann distribution***

It describes a macrostate property some call ***complexity***

$$1 = -\frac{1}{\log_2(3)}\sum_{i=1}^{3} \frac{1}{3}\log_2\frac{1}{3}$$



$$0.9455 = -\frac{1}{\log_2(4)}\left(\frac{1}{9}\log_2\frac{1}{9} + 2\,\frac{3}{9}\log_2\frac{3}{9} + \frac{2}{9}\log_2\frac{2}{9}\right)$$

- Surprise will result in a drop in entropy
- Surprise will result in an increase in the *length of the description*

A New Method Using Information

Three Observations:

1. Transient events result in a change in behavior not just amplitude
2. Transient events add to the information content of a signal
3. Higher information requires **more effort** to transmit, describe, or compress.

Compressibility and description length relates to A.N. Kolmogorov's idea of ***algorithmic complexity.*** *He showed that his concept is independent of probability distribution*

How much work is needed to transmit/compress signal?

Singular Value Decomposition

Rank: 100

Rank: 10

Rank reduction preserves the image by most *dominate behavior* first

Singular Value Decomposition

The **Short Time Fourier Transform** is represented by $\Upsilon(\kappa, \omega)$.

$$\Upsilon(\kappa, \omega) \cong USV^T$$

≤ ≤ , ≤ ≤

≅ U S V^T

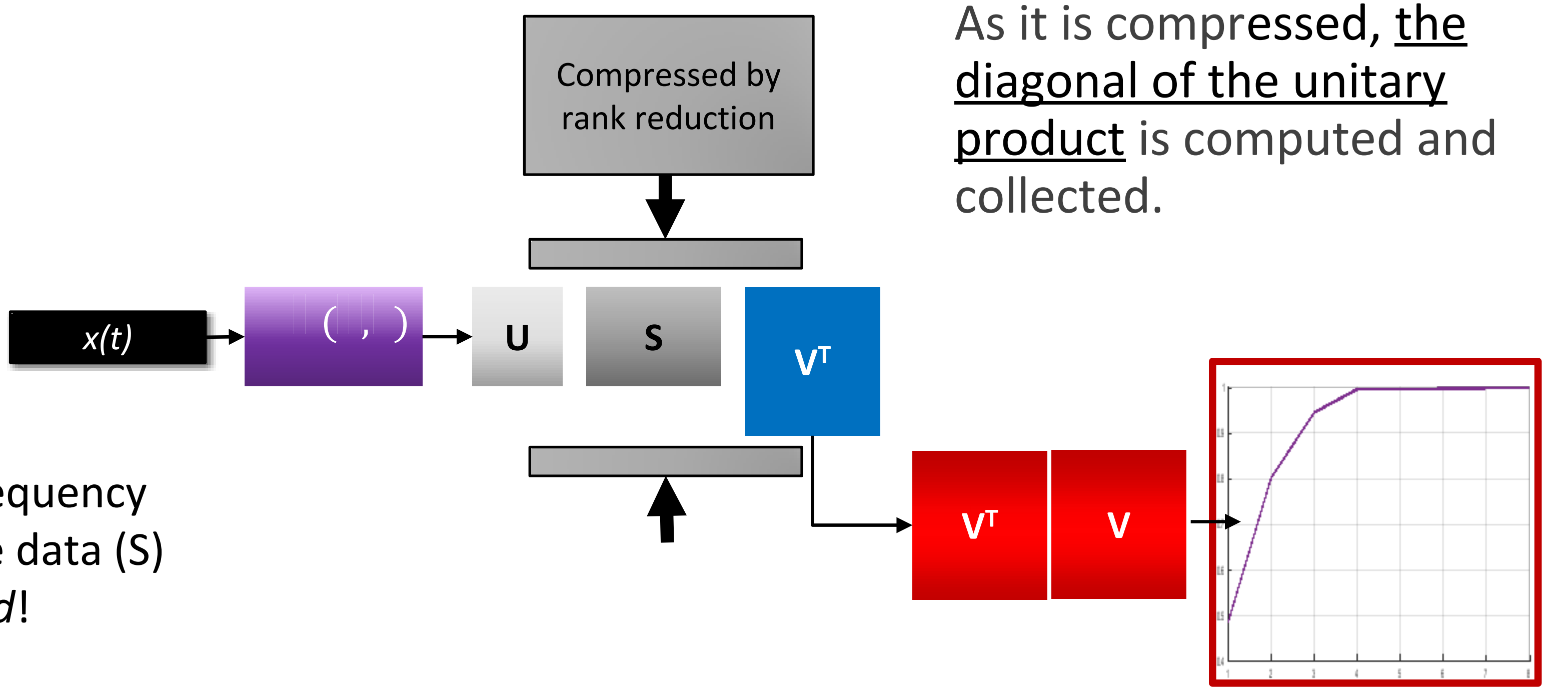
n x n

The columns of U contain the **left singular vectors** of A ($\Upsilon\, \Upsilon^T$)
The columns of V contain the **right singular vectors** of A^T ($\Upsilon^T\, \Upsilon$)
S^2 are the **singular values**, in rank-order

An approximation for Υ can be made by only retaining singular values up to a desired rank r (for $r < m$).

Information Potential Function

As it is compressed, the diagonal of the unitary product is computed and collected.



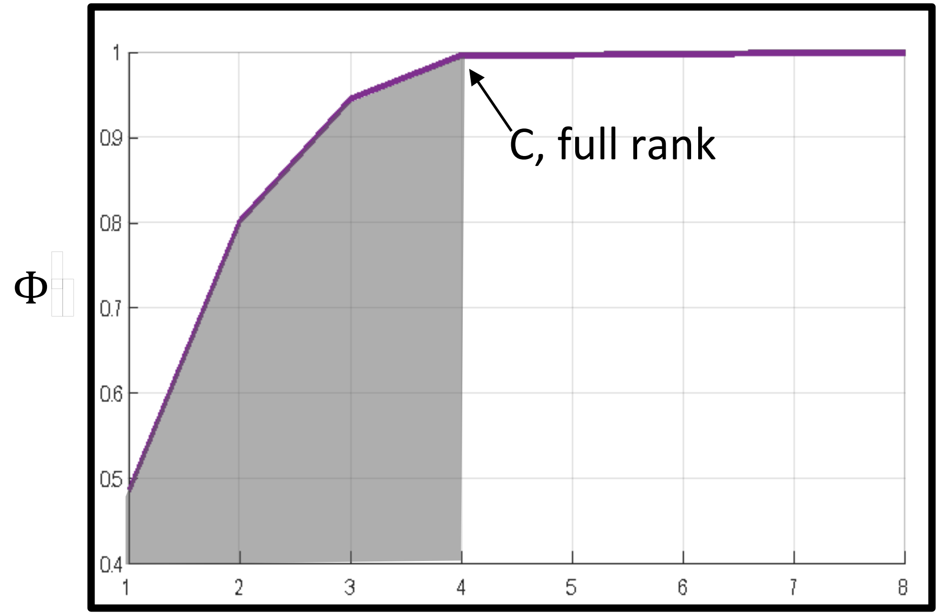
Note- the frequency (U) and scale data (S) are *discarded*!

This results in a potential function corresponding to information transferred per level of compression

Information Impulse Function (IIF)

$$\Phi_{ij}^{R} = \sum_{q=1}^{j} |V_{iq}|^2 \quad \begin{matrix} i = 1, \dots, n \\ j = 1, \dots, n \end{matrix}$$

Curve for one value of i



Rank (j)

Certain indexes (i) will have a greater 'information requirement' -> steeper curve

IIF = Area Under Curve

$$IIF^{R}(i) = \frac{2n}{C(C+1)} \sum_{j=1}^{C} \Phi_{ij}^{R} \quad \begin{matrix} i = 1, \dots, n \\ j = 1, \dots, C \end{matrix} \quad C \leq m$$

Simulation Method

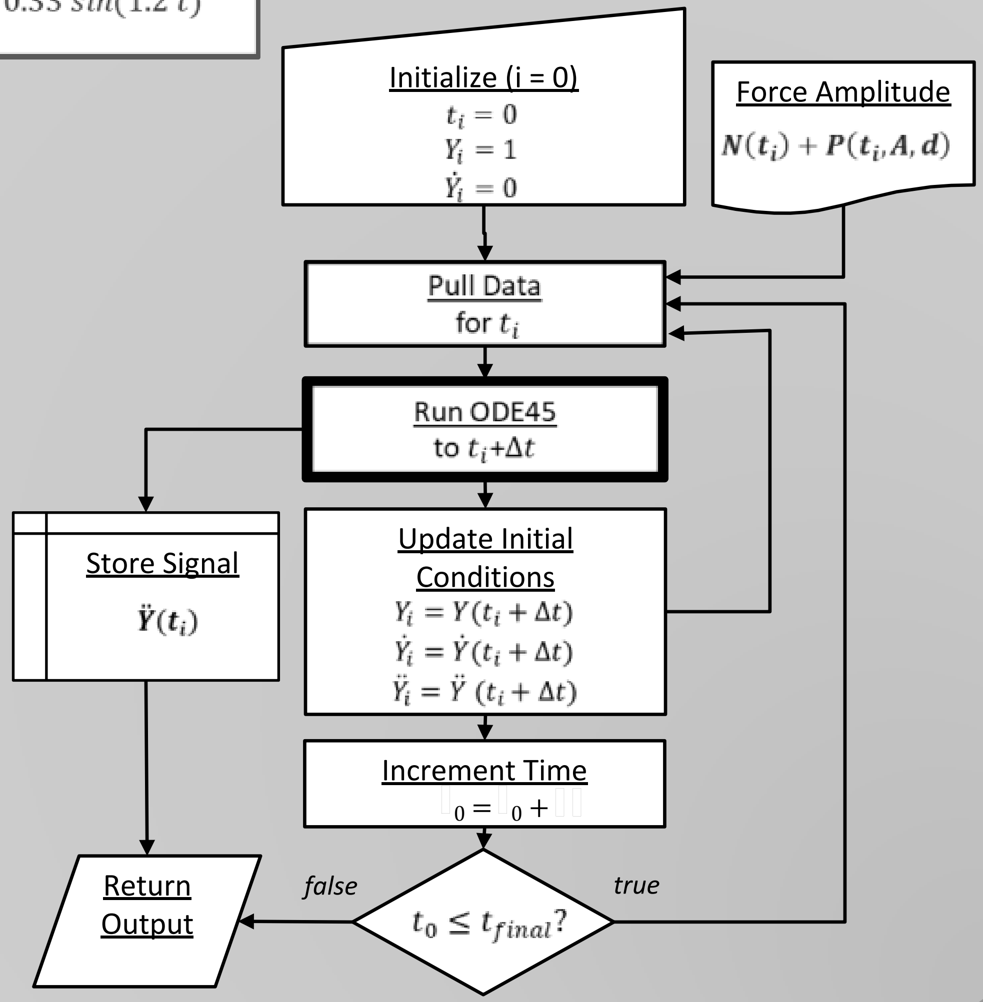
Equation of motion

$$\ddot{Y}(t) + 0.3\dot{Y}(t) + \left(Y(t)^3 - Y(t)\right) = \boldsymbol{R(t)} + \boldsymbol{P(t, A, d)} + 0.33\, sin(1.2\, t)$$

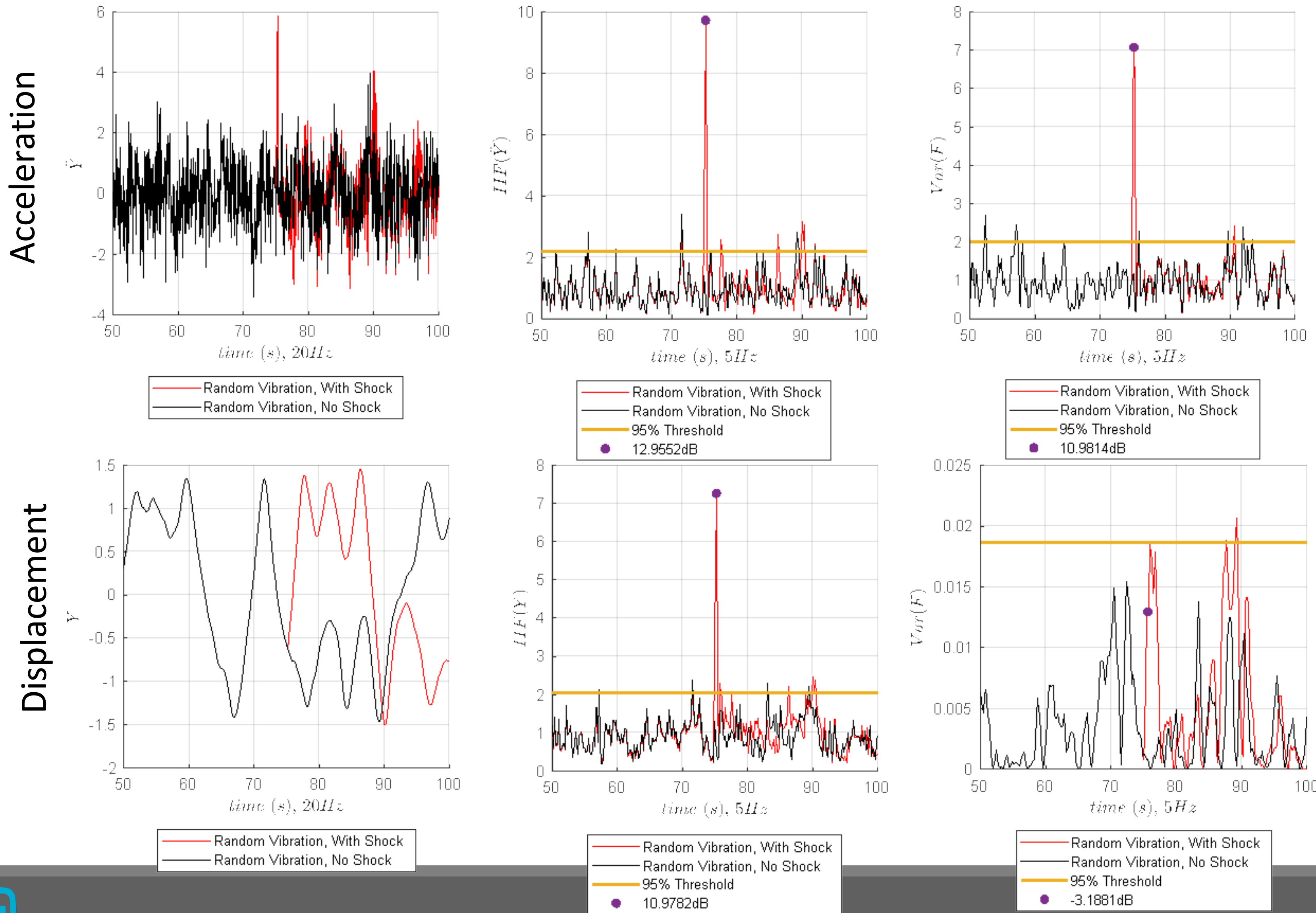
The solver is set up similar to finite element solvers – solving parameterized equation of motion and updating the initial conditions at each Δt.

For R(t) a standard normal distribution was used.

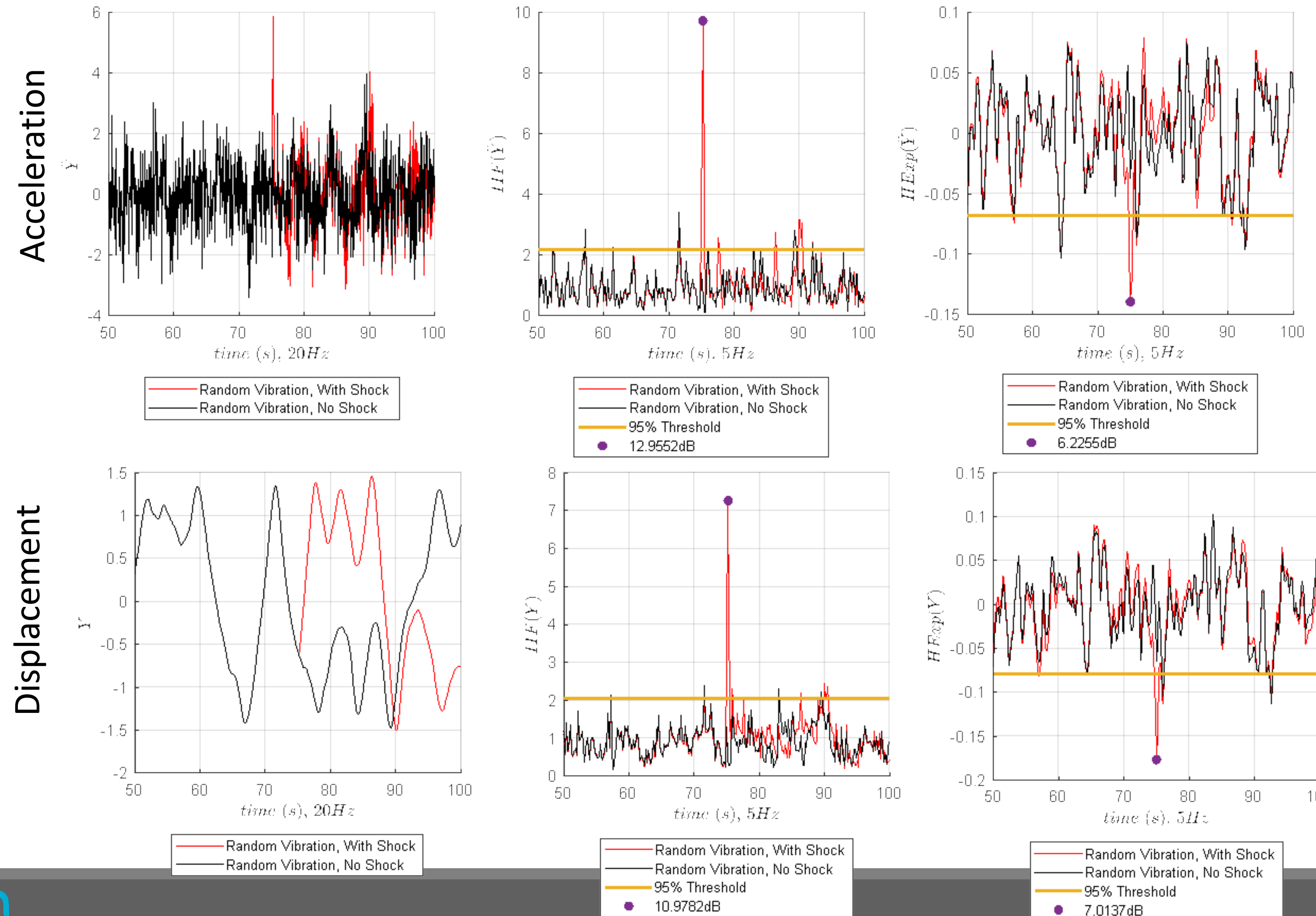
The pulse was inserted at 75 seconds, at an amplitude 5 and lasted for 0.5 seconds



Comparison with Variance



Comparison with the Holder Exponent



Discussion

- The IIF provides the analyst with a qualitative assessment of the relative information content in the signal
- It is specific to time-dependent behavior
  - It is less sensitive to changes in amplitude that part of system dynamics
- It works as well or better as running variance when the data are dominated by randomness
- It works as well or better as the Holder Exponent when the data are smooth, but does not require any filtering

The IIF is a very simple algorithm that can be easily evaluated with a simple threshold. It is a good candidate for automation.

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This presentation describes objective technical results and analysis. Any subjective views or opinions that might be expressed in the paper do not necessarily represent the views of the U.S. Department of Energy or the United States Government.

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