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Physics-based Reduction with Monitoring Data Assimilation for Adaptive Representations in Structural Systems

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#Applied Machine Intelligence, Sandia National Laboratories, Albuquerque, New Mexico

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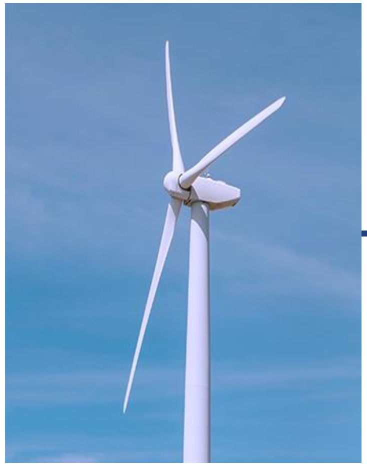
Problem Statement

Virtualization of nonlinear dynamical systems



Reference system (real-life)

High Fidelity Model
(finite element model)

HFM features:

Complex dynamics

Nonlinear behaviour

Parametric dependencies on:

- *Geometric features*
- *Material properties*
- *EOPs:*
  - *Environmental conditions*
  - *Operational conditions*
- *Excitation*

Low-order representation that:

Captures underlying dynamics

Reproduces physical behaviour

Retains parametric dependencies

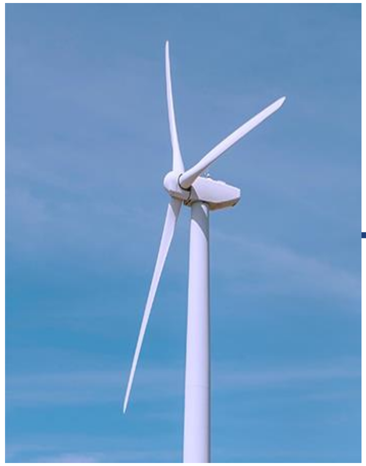
Computationally efficient

parametric Reduced Order Model
(low-order, equivalent model)

Virtual representation

Problem Statement

Condition deterioration or damage during operation



Reference system (real-life)

High **F**idelity **M**odel
(finite element model)

parametric **R**educed **O**rder **M**odel
(low-order, equivalent model)

Response information from sensors

Ground truth
representation

System 'as-is'

Uncertainties on EOPs

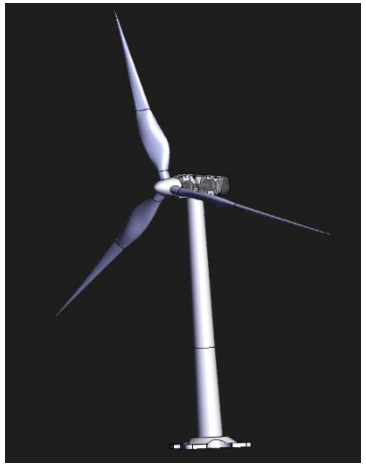
Damage and condition deterioration

Monitoring Data + ***pROM framework***

Adapt subspaces on varying dynamics

Real-time information for policy-makers

Adapt and optimize pROM performance



Virtual representation
(***Digital-Twin***)

Approach conceptualization
Framework components

Reference system (real-life)

High Fidelity Model *(finite element model)*

+

Parametric dependencies *(structural properties & excitation traits)*

Vector of parameters **p**

Sample parametric realizations => Full-order HFM evaluations

Latin Hypercube Sampling design on realizations of vector **p**

Projection-based parametric ROM as forward simulator

Projection on governing equations of motion:

$$\mathbf{M}(\mathbf{p})\ddot{\mathbf{u}}(t) + \mathbf{g}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t), \mathbf{p}\right) = \mathbf{f}(t, \mathbf{p})$$

$$\mathbf{u}(t) \in \mathbb{R}^n, \mathbf{M}(\mathbf{p}) \in \mathbb{R}^{n \times n}, \mathbf{f}(t, \mathbf{p}) \in \mathbb{R}^n, \mathbf{g}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t)\right) \in \mathbb{R}^n$$

Response time history

Mass matrix

External excitation

Internal forces (nonlinear)

Approach conceptualization

Framework components

High Fidelity Model
(finite element model)

\+

Parametric dependencies
(structural properties & excitation traits)

Vector of parameters **p**

Sample parametric realizations
=> Full-order HFM evaluations

Latin Hypercube Sampling design on realizations of vector **p**

Projection-based parametric ROM
as forward simulator

Projection on governing equations of motion:

$$\mathbf{M}_{\mathrm{r}}(\mathbf{p}_j)\ddot{\mathbf{u}}_{\mathrm{r}}(t) + \mathbf{g}_{\mathrm{r}}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t), \mathbf{p}_j\right) = \mathbf{f}_{\mathrm{r}}(t, \mathbf{p}_j)$$

$$\mathbf{M}_{\mathrm{r}}(\mathbf{p}_j) \in \mathbb{R}^{r\times r}, \mathbf{g}_{\mathrm{r}}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t), \mathbf{p}_j\right) \in \mathbb{R}^{r}, \mathbf{f}_{\mathrm{r}}(t, \mathbf{p}_j) \in \mathbb{R}^{r}$$

$$r \ll n$$

$$\mathbf{u}(t) = \mathbf{V}(\mathbf{p}_j)\mathbf{u}_{\mathrm{r}}(t) \quad \mathbf{M}_{\mathrm{r}}(\mathbf{p}_j) = \mathbf{V}(\mathbf{p}_j)^T\mathbf{M}(\mathbf{p}_j)\mathbf{V}(\mathbf{p}_j)$$

$$\mathbf{g}_r(\mathbf{p}_j) = \mathbf{V}(\mathbf{p}_j)^T\mathbf{g}\left(\mathbf{u}(t), \dot{\mathbf{u}}(t), \mathbf{p}_j\right)$$

Projection (Reduced-Order) Basis

Reference system
(real-life)

Approach conceptualization

Framework components

Reference system (real-life)

High **F**idelity **M**odel
(finite element model)

\+

Parametric dependencies
(structural properties & excitation traits)

Vector of parameters **p**

Sample parametric realizations
=> Full-order HFM evaluations

Latin Hypercube Sampling design on realizations of vector **p**

Projection-based parametric ROM as forward simulator

- *A subspace for each sample via **P**roper **O**rthogonal **D**ecomposition* on response data*
- *Form clusters on parametric domain based on local dynamics => POD subspaces similarity*
- *Validation sample uses POD basis of assigned cluster for projection & ROM integration*

*Proper Orthogonal Decomposition

$$\mathbf{U} \equiv [\mathbf{u}(t_1)...\mathbf{u}(t_N)] = \mathbf{W}\boldsymbol{\Sigma}\mathbf{R}^T$$

$$\mathbf{u}(t) = \mathbf{V}(\mathbf{p})\mathbf{u}_r(t) \quad \mathbf{V} \equiv \mathbf{W}_k = \mathbf{W}(:, 1:k)$$

POD modes

Approach conceptualization

Framework components

Reference system
(real-life)

High Fidelity Model
(finite element model)

\+

Parametric dependencies
(structural properties & excitation traits)

Sample parametric realizations
=> Full-order HFM evaluations

Projection-based parametric ROM
as forward simulator

Damage indicator

Relying on response comparison between ROM estimation and monitoring data

Data Assimilation scheme

- *Receives response data and current ROM response estimation on monitored nodes*
- ***Approximates** the correction of the ROM estimation on all system nodes*

Approach conceptualization

Framework components

High **F**idelity **M**odel
(finite element model)

+

Parametric dependencies
(structural properties & excitation traits)

Sample parametric realizations
=> Full-order HFM evaluations

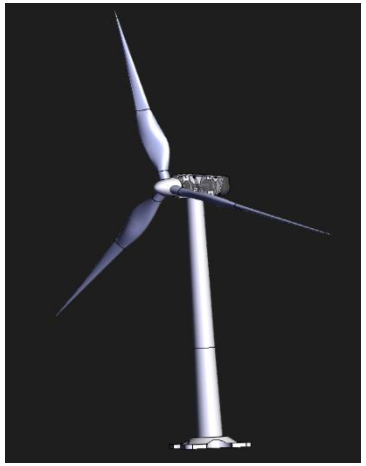
Projection-based parametric ROM
as forward simulator

Damage indicator

Data Assimilation scheme

Adaptive parametric ROM representation
though monitoring information

Reference system
(real-life)



Virtual representation
(***Digital-Twin***)

Approach conceptualization

Adaptive pROM for robust Structural Health Monitoring

(Initial) parametric ROM framework

- ***Projection-based*** *approach relying on* ***POD subspaces***
- ***Propagates*** *dynamics forward in time in reduced coordinates*
- *Utilizes* ***local ROMs*** *through* ***clustering to retain dependencies*** *throughout domain of operation*

Earthquake induced damage / System deterioration

The pROM is no longer able to perform estimation tasks accurately

Subspaces on training set do not sufficiently capture occurring phenomena

=> Performance bottleneck

Adaptivity in a pROM context

Condition indicator to highlight failure of ROM on the fly

Update subspace / Approximate *deformation modes* anew

=> Adaptive pROM

Approach conceptualization

Adaptivity through data assimilation

Adaptivity in a pROM context

Condition indicator to highlight failure of ROM on the fly

Update subspace / Approximate *deformation modes* anew

=> Adaptive pROM

*Update pROM **"on-the-fly"** through correction on POD modes*

Noisy input signal from ***sparsely*** monitored system

Condition indicator highlights pROM performance ***failure at time*** t_k

Data-driven mapping approximates ***correction*** on ***ROM estimation*** at ***all nodes*** via ***input information*** only from ***monitored nodes***

Output approximation is employed to estimate ***"updated deformation modes"***

Updated modes are utilized to ***adjust pROM*** projection basis

Approach conceptualization

Adaptive pROM framework based on data assimilation

Offline / Training Strategy:

✓ Derive initial ***pROM as forward simulator*** :

Examples:

- ***Initial linear state*** & ***nonlinearities*** *during operation* ***to represent damage***
- ***Initial nonlinear state*** & ***deterioration effects*** during operation

✓ Assemble ***Damage Indicator*** :

- ***Deterministic nature*** based on ***response comparison*** metrics between ROM estimation and response data from monitored nodes
- Relies on ***limited nodal measurements***
- Includes input noise / exploit ***noise statistics to define activation threshold***

✓ ***Gaussian Process Regression (GPR)***

- Estimates ***correction on ROM prediction*** based on ***residual between ROM and response data*** on monitored nodes
- GPR ***trained on pool of snapshots***, without compromising efficiency

Examples:

- GPR trained on certain parametric states representing damage

The ***enrichment mode*** as defined:

✓ represents an approximation of the "true" system deformed configuration

✓ ***On the monitoring nodes*** assumes values approx. ***equal to the actual*** deformation ***(monitoring data)***

✓ ***On the rest of the nodes estimates*** the deformation via the ***GPR scheme***

Online / During Operation:

- Track ***residual on monitored nodes*** only:
  - ✓ ***Residual response*** = ***Monitoring data - pROM approximation***
- ***If indicator signals*** "ROM Performance Deteriorates/Fails":
  - ✓ Employ ***GPR*** to ***approximate*** ROM ***correction*** and reconstruct full residual state ***on all system nodes***
  - ✓ ***Enrichment mode*** = ***pROM approximation + GPR output***
  - ✓ ***Enrich pROM*** by using corrected modes in POD Basis

Implementation details

Damage indicator and GPR-scheme

Damage Indicator

- Deterministic nature based on response comparison metrics
  $\Rightarrow$ ***Mahalanobis distance (MD) measure***
  (between ROM prediction and response data on monitoring nodes)
- Relies on limited nodal measurements (5-10% nodal output measured)
- Includes input noise (3%) / exploit noise statistics to define activation threshold
  => Alert threshold from Chi-Square distribution (0.01% significance level)

Gaussian Process Regression (GPR)

- Trained based on ***residual response*** between monitoring data and pROM
- GPR ***trained on pool of snapshots***, without compromising online efficiency
- ***Input***: Response information from monitoring channels
  Output: Additive correction on full coordinate space
- Leverage ***local*** and ***physical degree-of-freedom correlations***
- ***Software***: ***gpytorch*** implementation with MultitaskGPModel and RBFKernel()

Measurement Data $\mathbf{d}_k \in \mathbb{R}^{n_d}$

Vector of random values $\mathbf{r}_k \in \mathbb{R}^{n_d}$

St. Dev. of measurement signals $\sigma_\delta \in \mathbb{R}^{n_d \times n_d}$

Noise level δ

Noisy measurement data

$$\tilde{\mathbf{d}}_k = \mathbf{d}_k + \delta \sigma_d \mathbf{r}_k$$

Damage Indicator

$Input \in \mathrm{R}^{N_{channels}*2\times 1}$

Response on monitoring channels (displacements & rotations)

Output => Performance failure alert signal

Implementation details

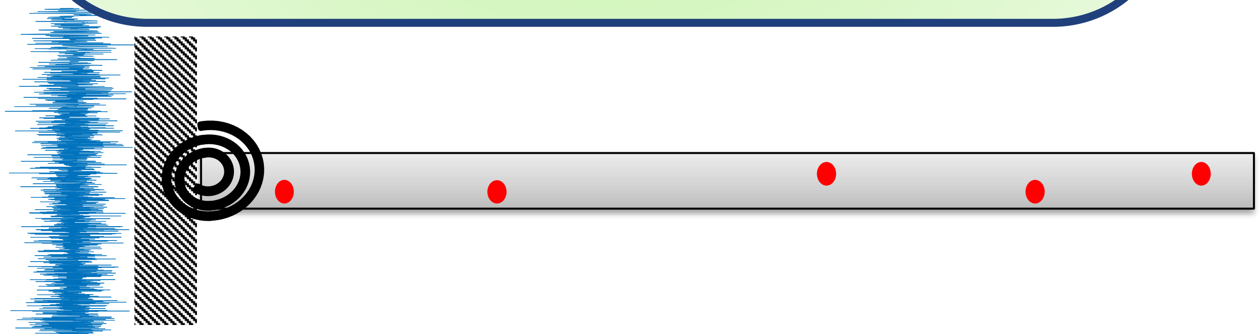
Configurations and scenarios

Cantilever Beam Case Study

- Stochastic ***ground motion excitation***
- Parametrized Boundary => ***Nonlinear rotational spring***
- ***Limited number*** of nodes monitored

Damage Scenario:

- ✓ Derive ***ROM based on "design"*** case study
- ✓ Induce **damage** by ***activating parametric boundary***
- ✓ Use ***indicator to detect*** failure
- ✓ Employ ***GPR-based scheme*** to assemble deformed modes
- ✓ ***Refine*** POD-Basis



Hysteretic spring model

- ***Total restoring force:***

$$\mathbf{R} = \mathbf{R}_{linear} + \mathbf{R}_{hysteretic} = \alpha k \mathbf{u} + (1-\alpha) k \mathbf{z}$$

- ***Bouc-Wen equation with degradation/deterioration effects:***

$$\dot{\mathbf{z}} = \frac{A\dot{\mathbf{u}} - \nu(t)(\beta|\dot{\mathbf{u}}|\mathbf{z}|\mathbf{z}|^{w-1} - \gamma\dot{\mathbf{u}}|\mathbf{z}|^{w})}{\eta(t)}$$

$$\nu(t) = 1.0 + \delta_\nu \epsilon(t), \quad \eta(t) = 1.0 + \delta_\eta \epsilon(t), \quad \epsilon(t) = \int_0^t \mathbf{z}\dot{\mathbf{u}}\delta t$$

Characteristics of the Bouc-Wen links:

β, γ, A, w : Control smoothness and shape *of hysteresis*

δ_ν, δ_η : *Degradation/Deterioration effects*

a, k : *Linear/Hysteretic contribution weighting*

Implementation details
Configurations and scenarios

Cantilever Beam Case Study

- Stochastic ***ground motion excitation***
- Parametrized Boundary => ***Nonlinear rotational spring***
- ***Limited number*** of nodes monitored

Damage Scenario:

- ✓ Derive ***ROM based on "design"*** case study
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- ✓ Use ***indicator to detect*** failure
- ✓ Employ ***GPR-based scheme*** to assemble deformed modes
- ✓ ***Refine*** POD-Basis

Hysteretic Bouc-Wen spring model

- ***Total restoring force:***

$$\mathbf{R} = \mathbf{R}_{linear} + \mathbf{R}_{hysteretic} = \alpha k \mathbf{u} + (1-\alpha) k \mathbf{z}$$

- ***Bouc-Wen equation with degradation/deterioration effects:***

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Characteristics of the Bouc-Wen links:

β, γ, A, w : Control smoothness and shape of *hysteresis*

δ_ν, δ_η : *Degradation/Deterioration effects*

a, k : *Linear/Hysteretic contribution weighting*

Scenario B:

- ***Initial*** "design" case study is ***nonlinear***
- ***Damage*** is represented through ***degradation / deterioration*** effects during ***operation***

Scenario A:

- ***Initial*** "design" case study is ***linear***
- ***Nonlinear spring*** is activated during ***operation***

Implementation details

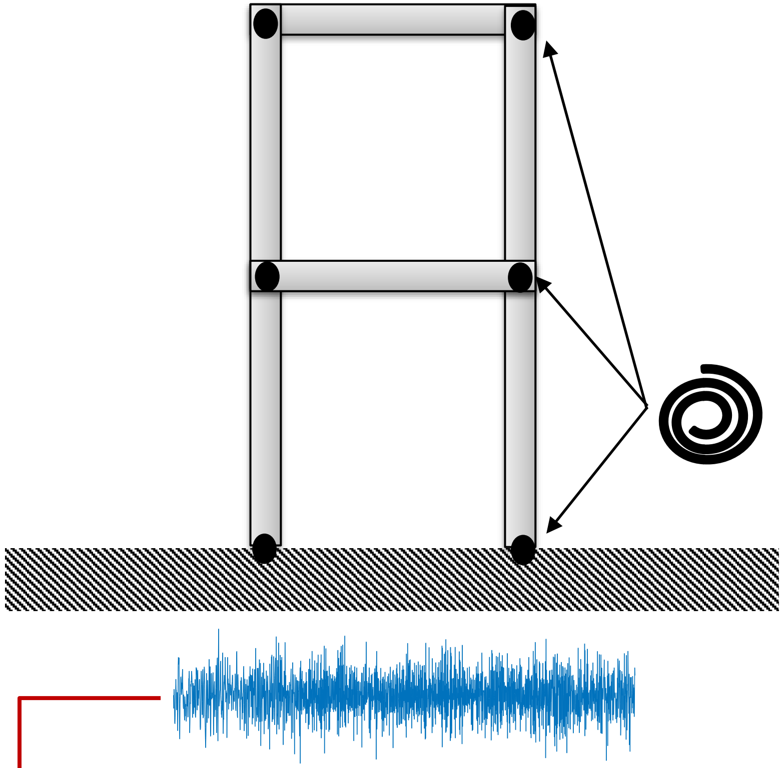
Configurations and scenarios

Plane Frame Case Study

- Stochastic ***parametrized*** ground motion ***excitation (Amplitude)***
- ***Nonlinear*** parametric rotational ***spring on all nodal connections***
- ***Limited number*** of nodes monitored

Damage Scenario:

- ✓ Derive ***ROM based on "design"*** case study
- ✓ Induce **damage** by ***activating parametric springs***
- ✓ Use ***indicator to detect*** failure
- ✓ Employ ***GPR-based scheme*** to assemble deformed modes
- ✓ ***Refine*** POD-Basis

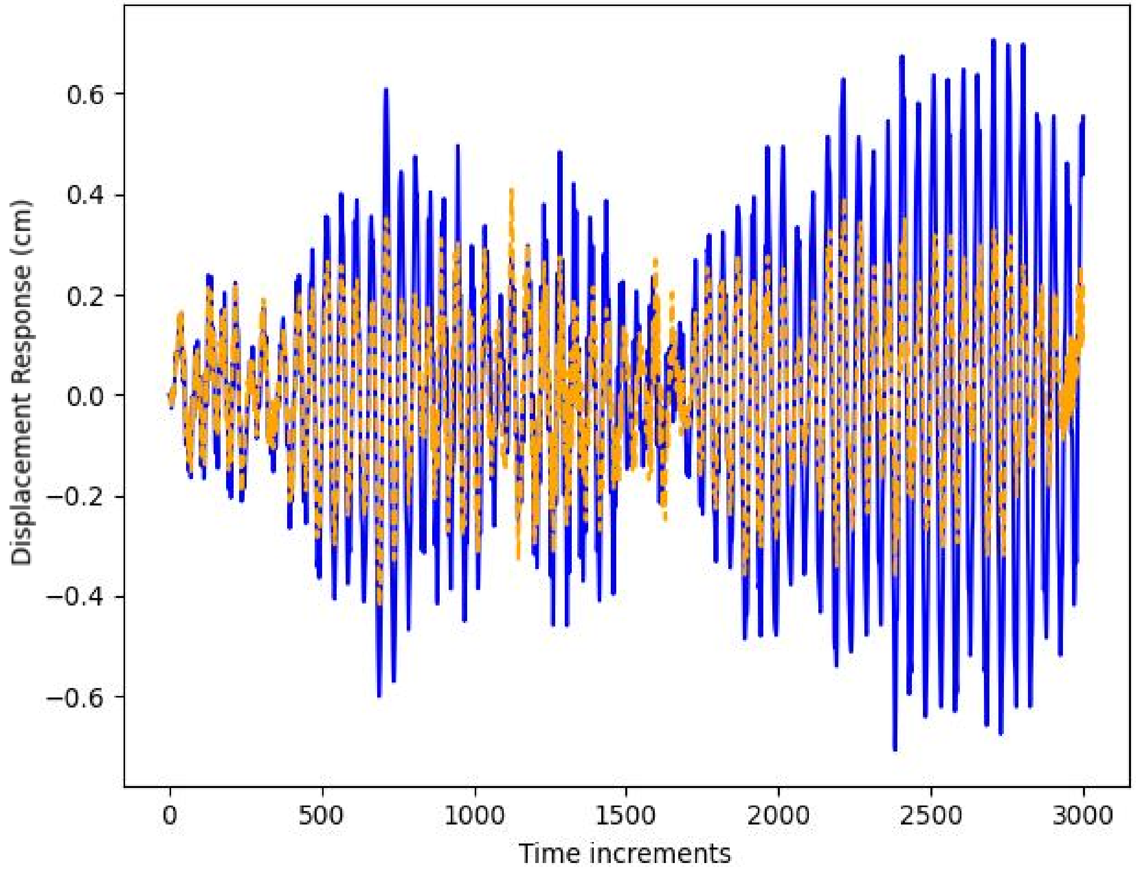


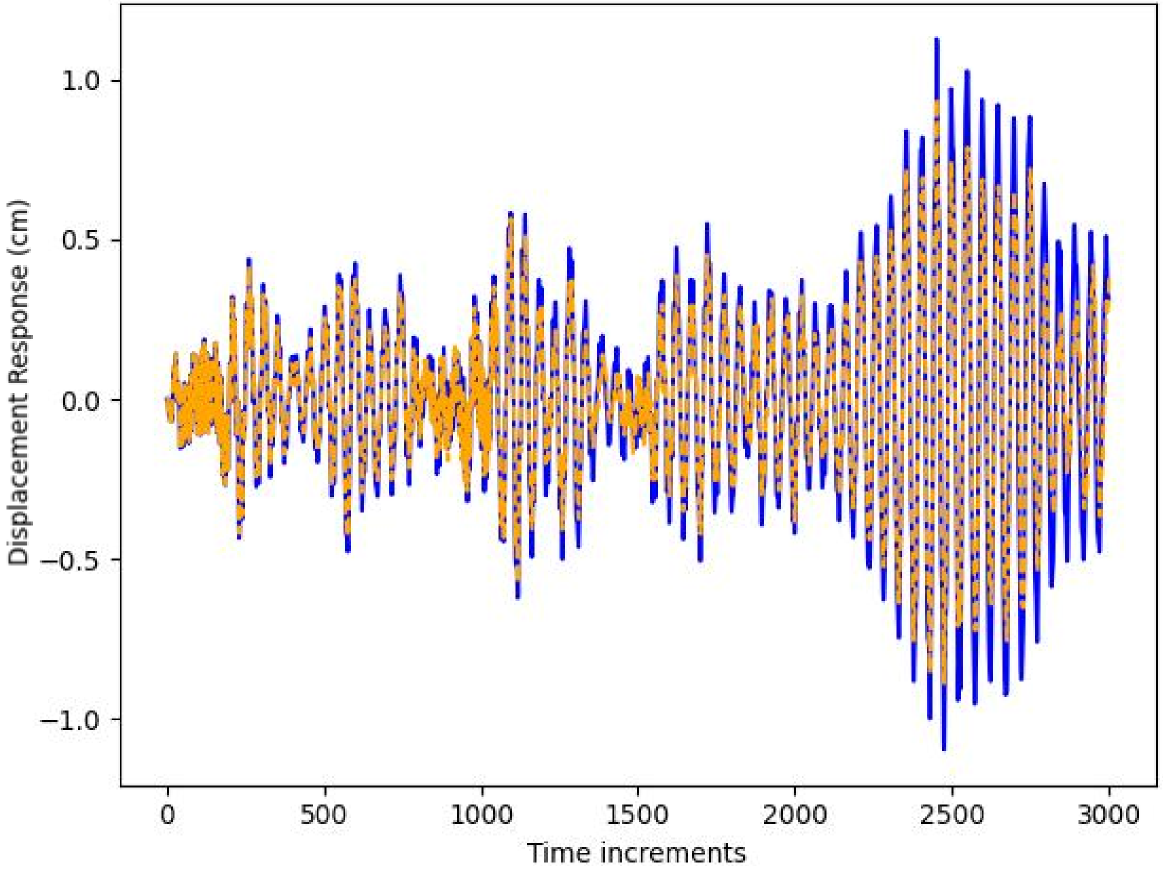
Scenario C*:*

- ***Initial*** "design" case study is ***linear***
- ***Nonlinear spring*** is activated during ***operation***
- Evaluation earthquake not included in training set

Implementation details
Configurations and scenarios

Linear vs Nonlinear response examples for ***different Bouc-Wen activation parameters***

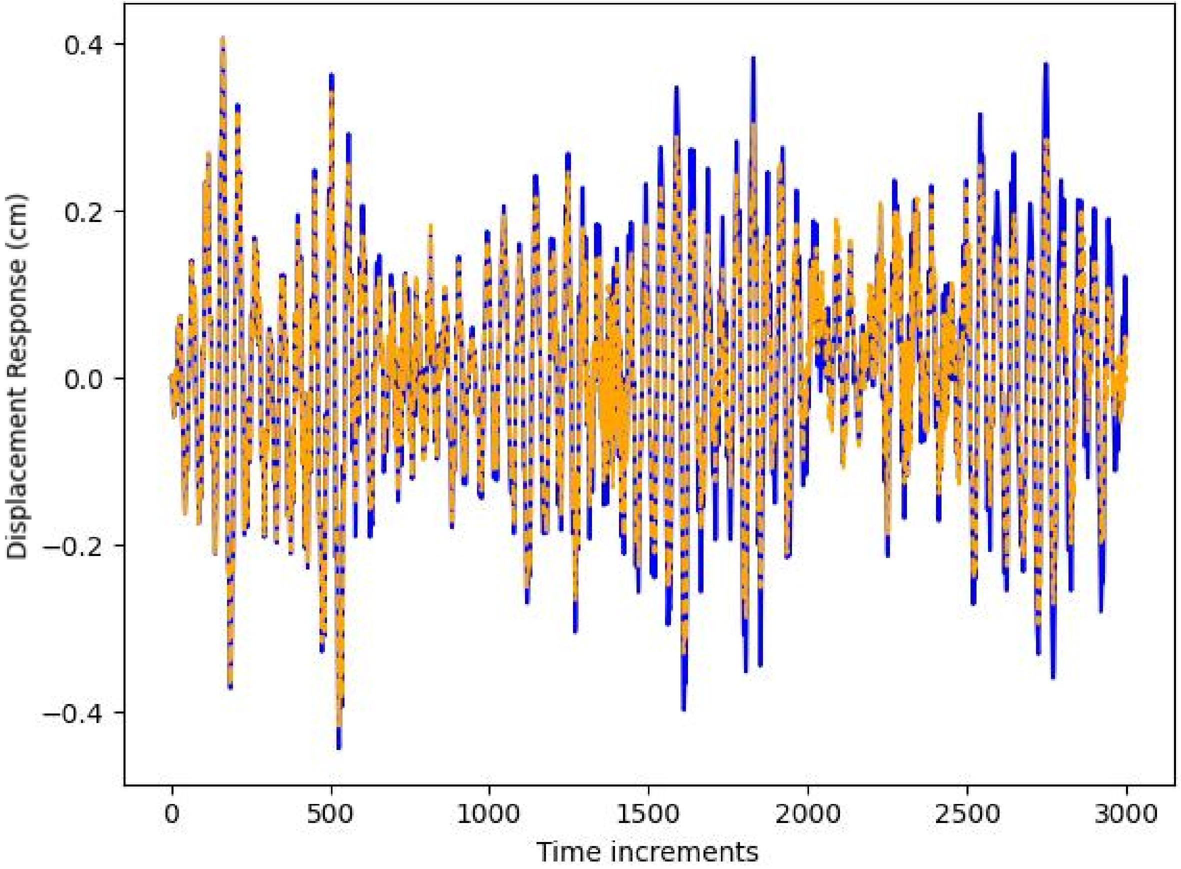


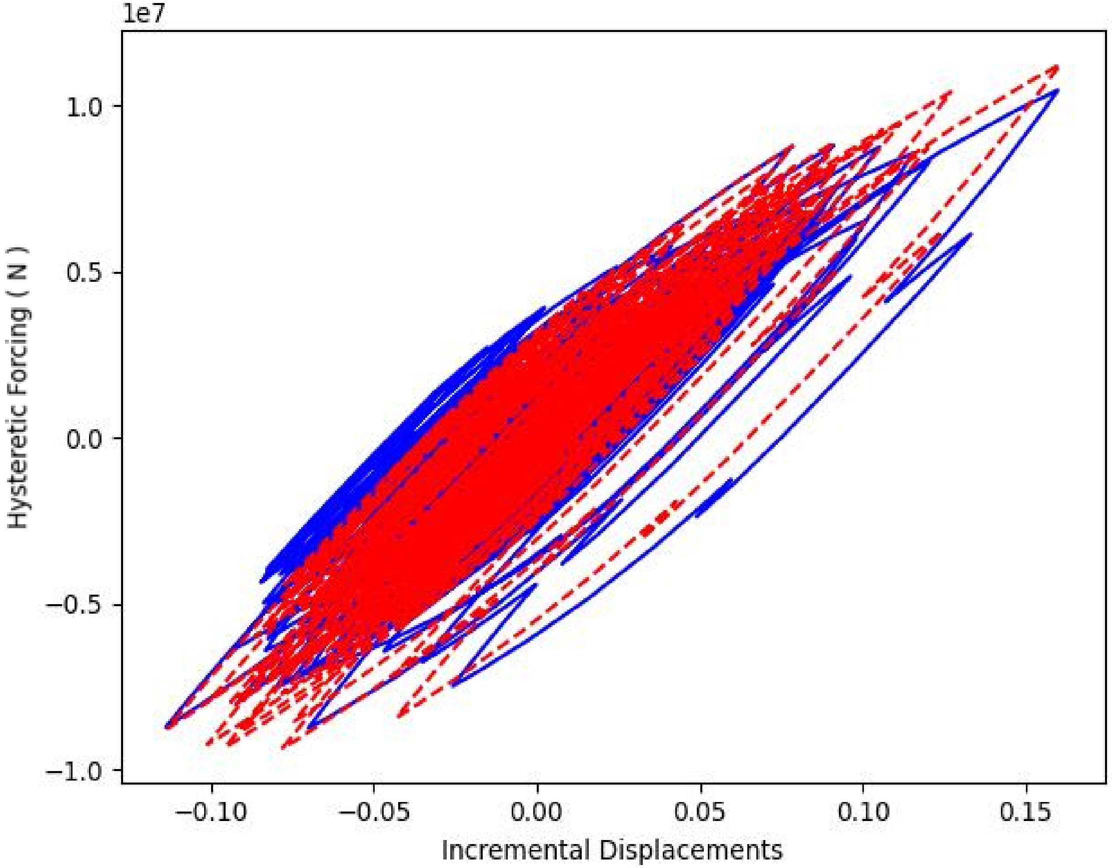


Implementation details

Configurations and scenarios

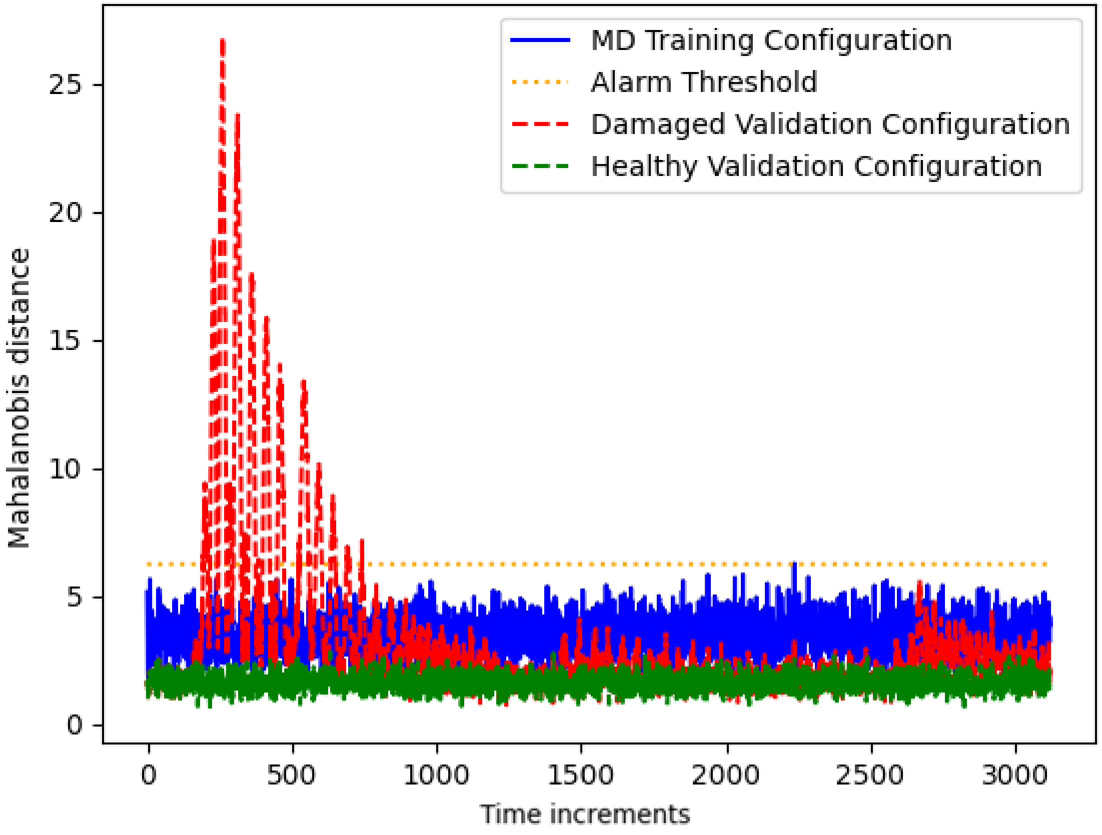
Response examples with ***Bouc-Wen degradation phenomena*** during operation



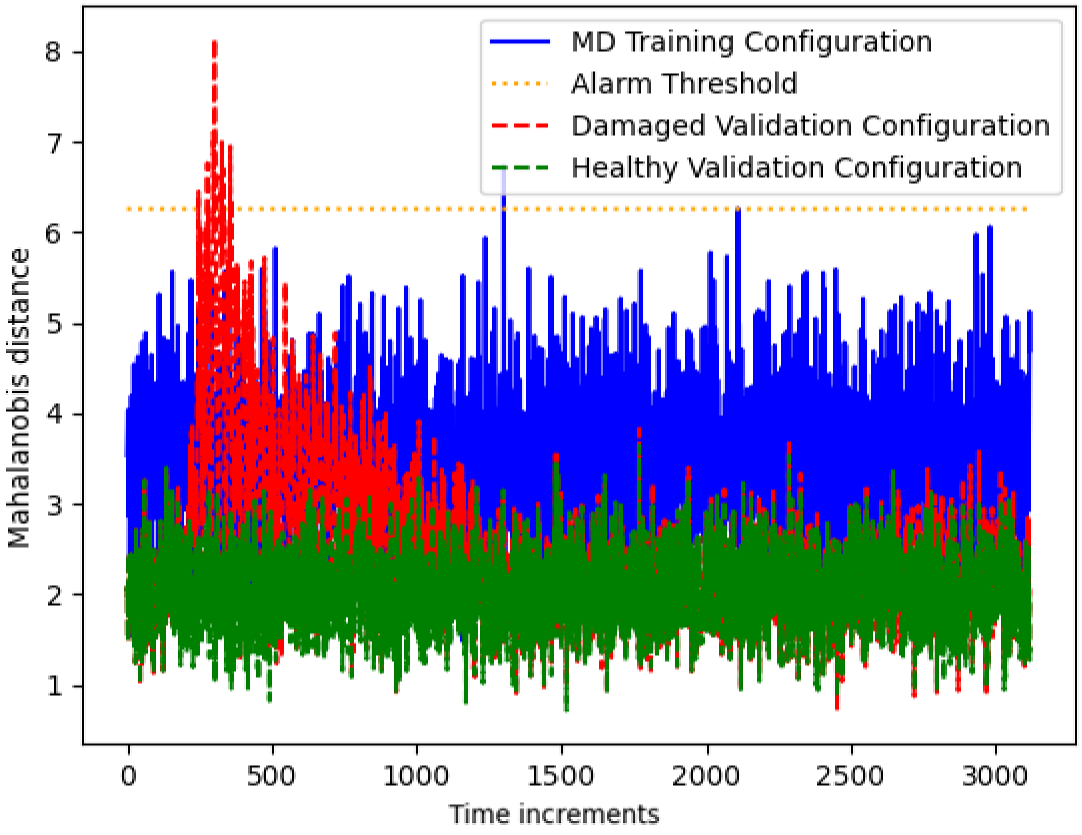


Implementation details

Damage indicator and GPR-scheme



Linear vs Nonlinear
response example (***Scenario A***)



Bouc-Wen degradation phenomena
during operation (***Scenario B***)

Implementation details

Damage indicator and GPR-scheme

Gaussian Process Regression (GPR)

- ***Input*** : Response information in monitoring channels (displacements & rotations)

  Input = True response - pROM estimation (monitored nodes)

- ***Output***: Response approximation through additive correction on full coordinate space

  Output = True response - pROM estimation (all coordinates)

  ⇒ pROM **Basis Enrichment** mode = ***pROM approximation*** **+** ***GPR output***

- Leverage ***local*** and ***physical degree-of-freedom correlations***

✓ Assemble matrix capturing ***correlations*** between the time history ***response on physical coordinate*** (coordinate = degree-of-freedom)

✓ Leverage correlation coefficients to ***define an output window*** for each monitored input channel

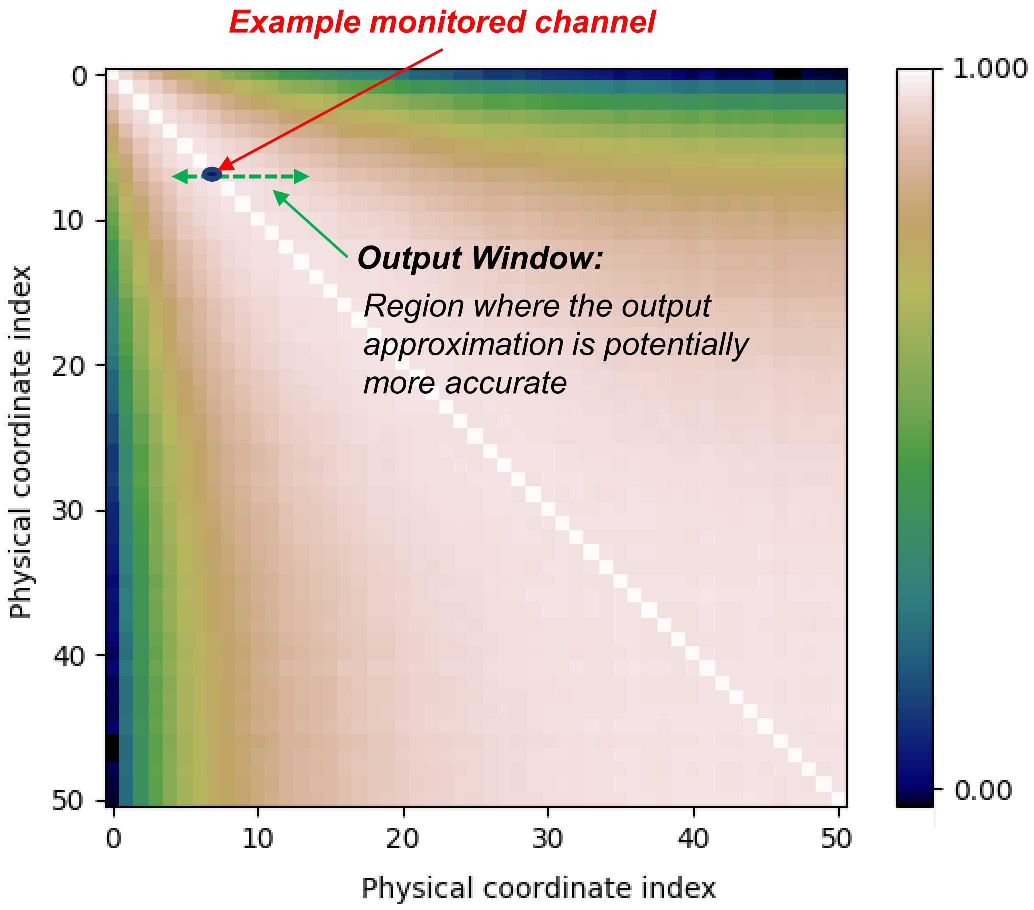
Thus:

❖ ***Not all*** monitoring ***channels*** are used as ***input at once***

❖ Instead, the *data from each channel* are employed *as input to approximate the response only on "correlated" coordinates* based on output window

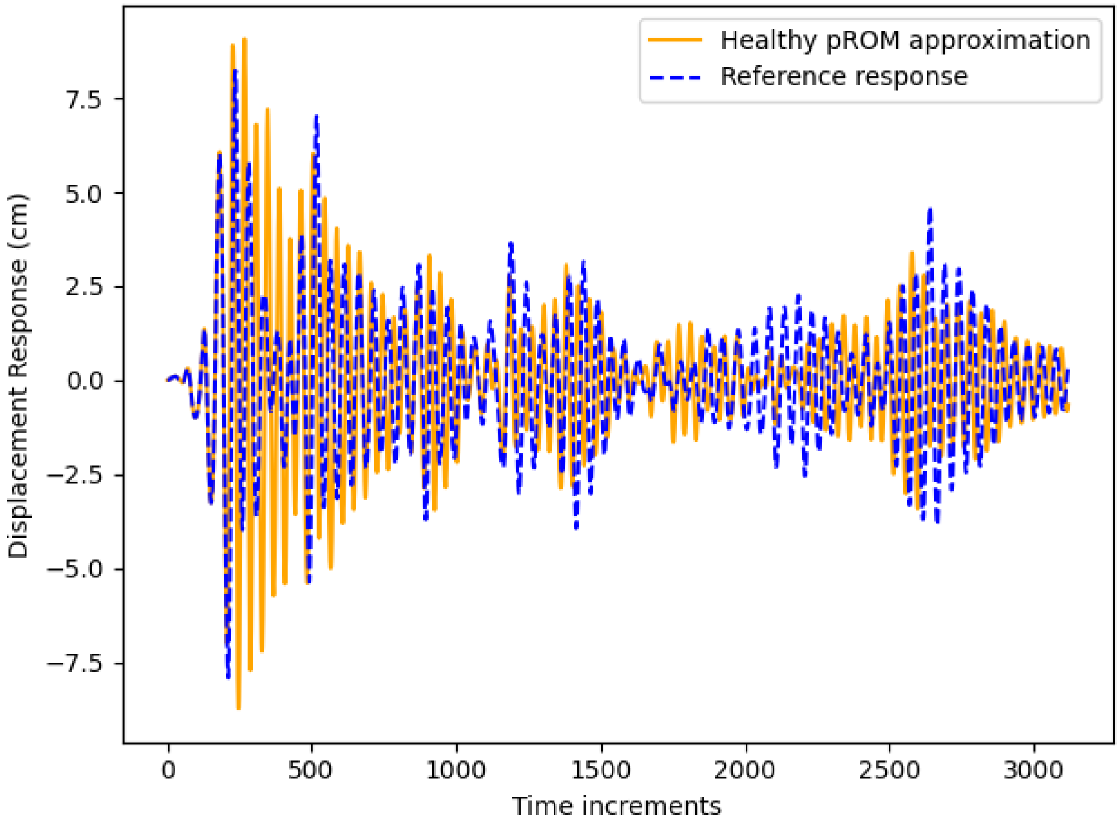
❖ ***Overlapping*** to ensure quality of approximation

Matrix of correlation coefficients (Scenario A)

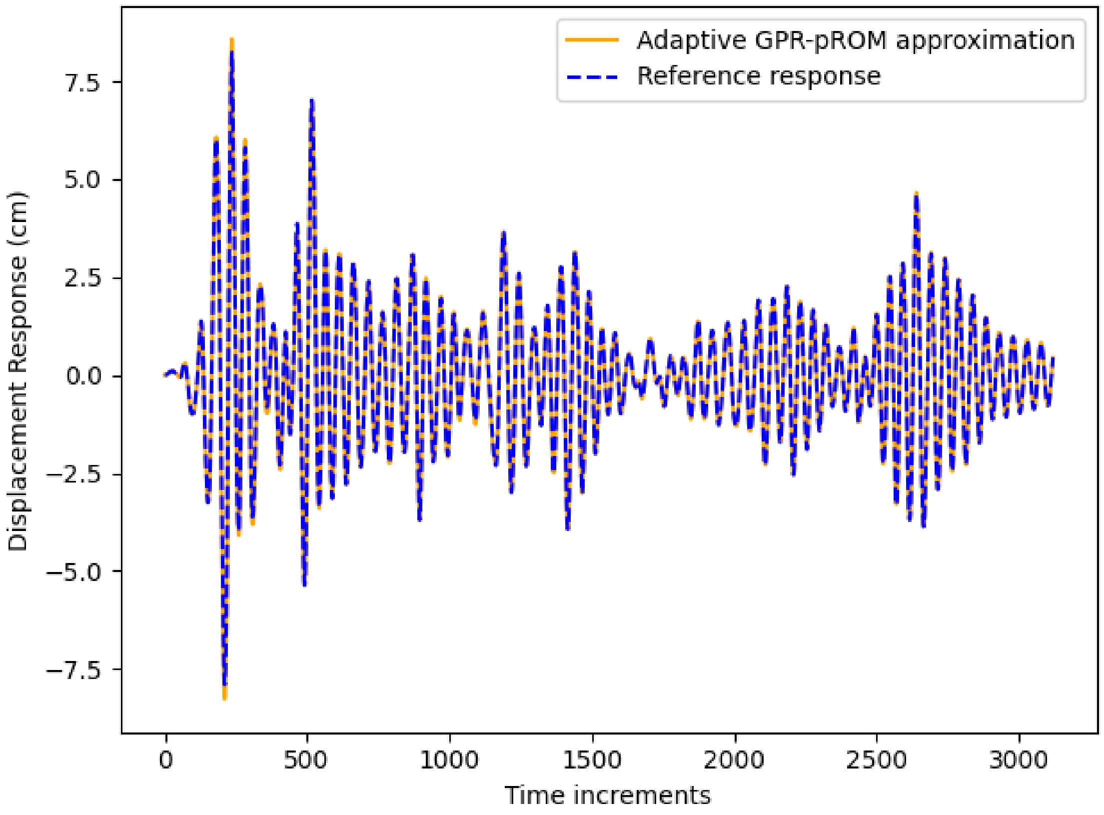


Case studies

Accuracy performance of the framework



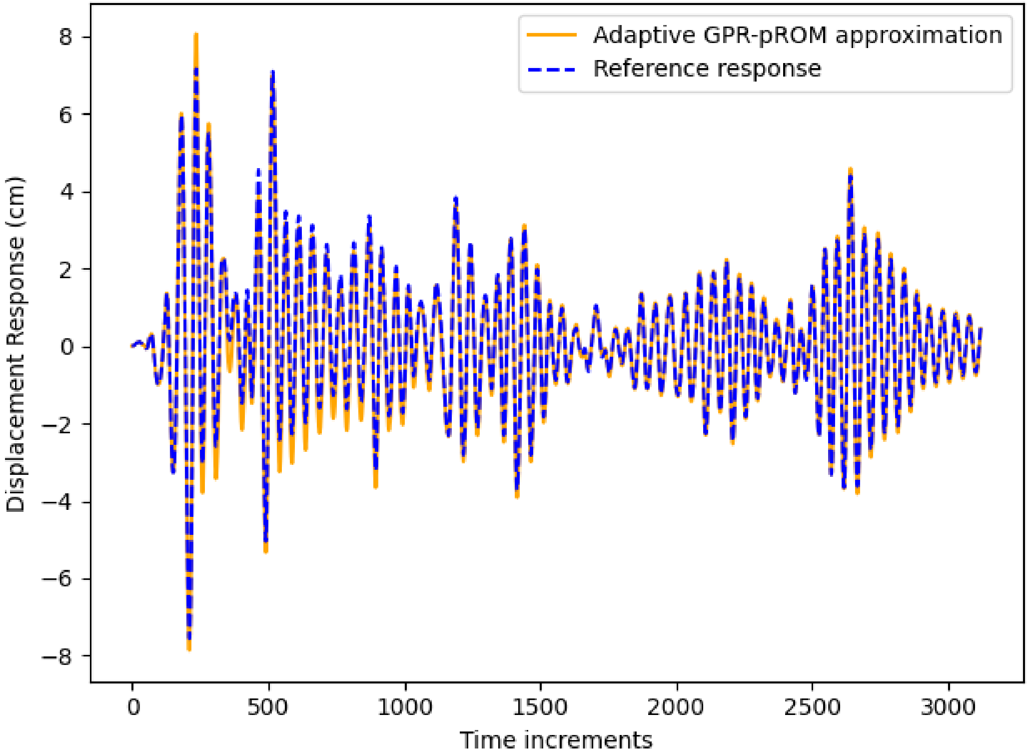
Healthy pROM uses
initial linear Basis (***Scenario A***)

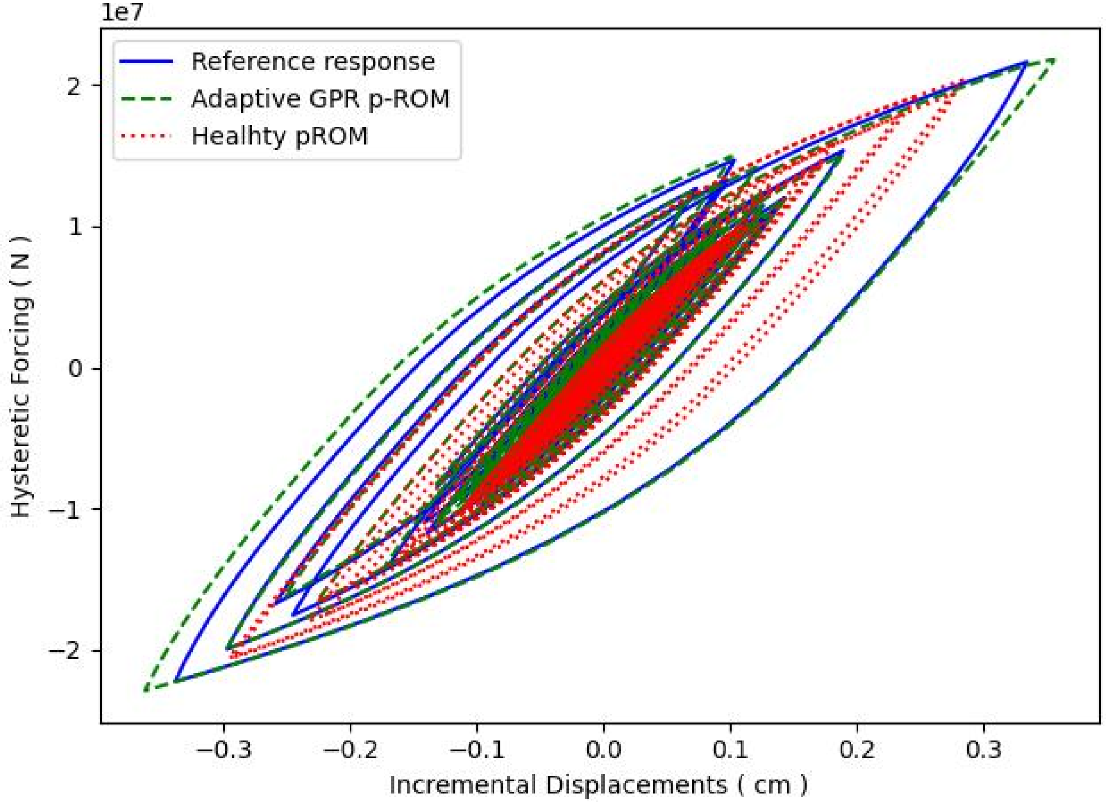


GPR-pROM adapts
projection Basis (***Scenario A***)

Case studies

Accuracy performance of the framework

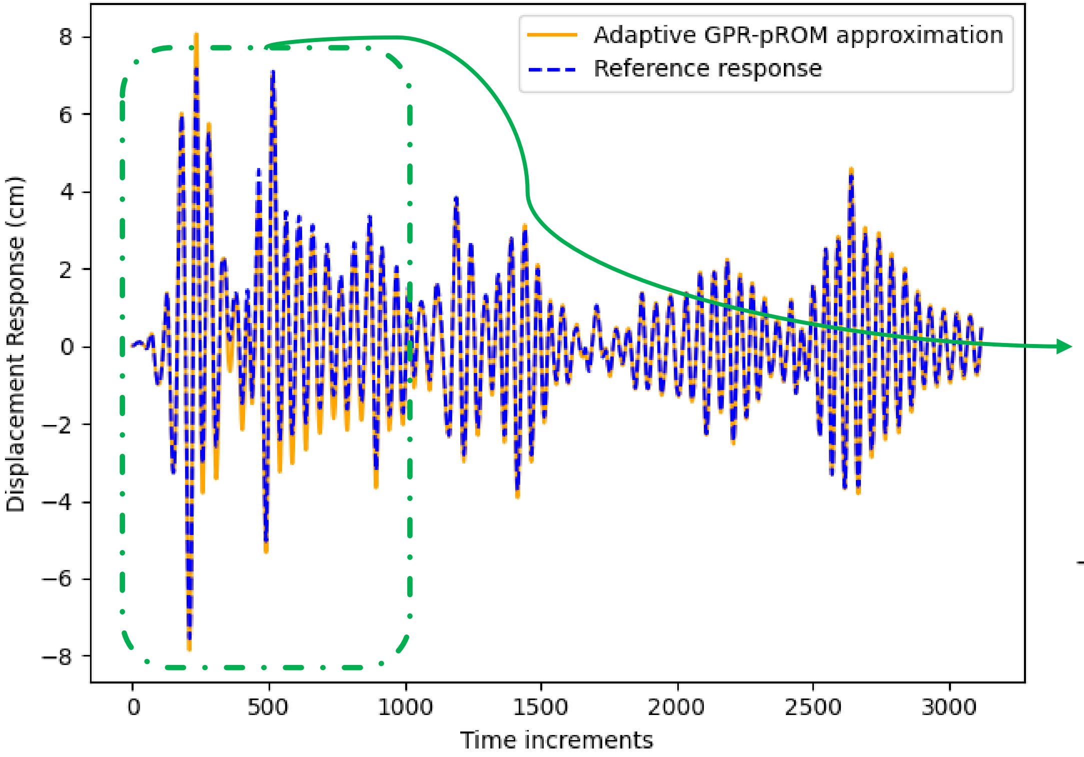




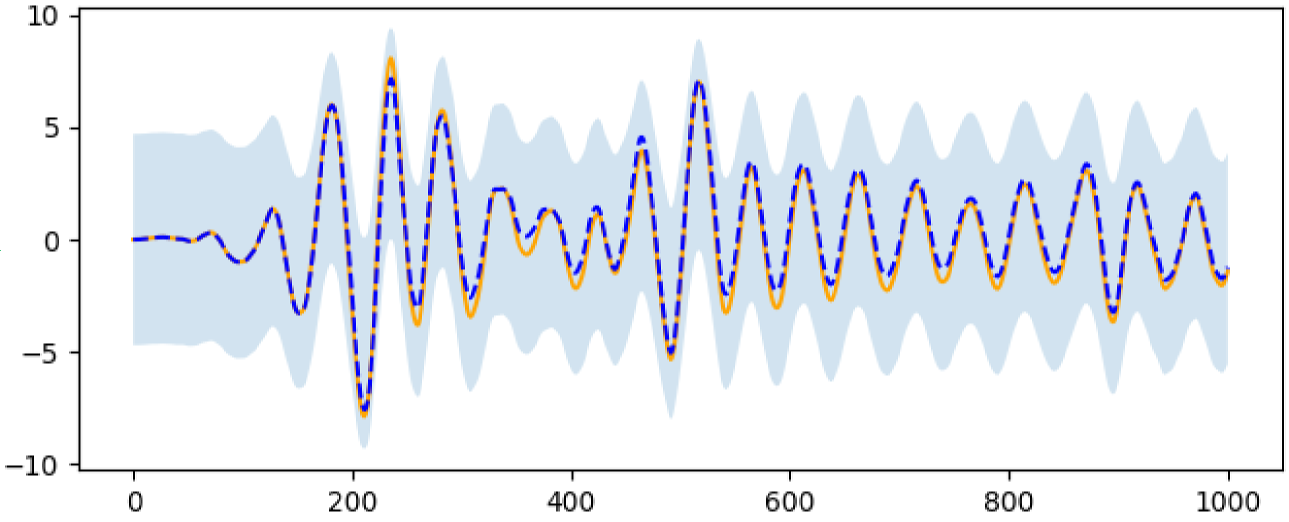
GPR-pROM adapts
projection Basis (***Scenario C***)

Case studies

Accuracy performance of the framework



GPR-pROM adapts
projection Basis (***Scenario C***)

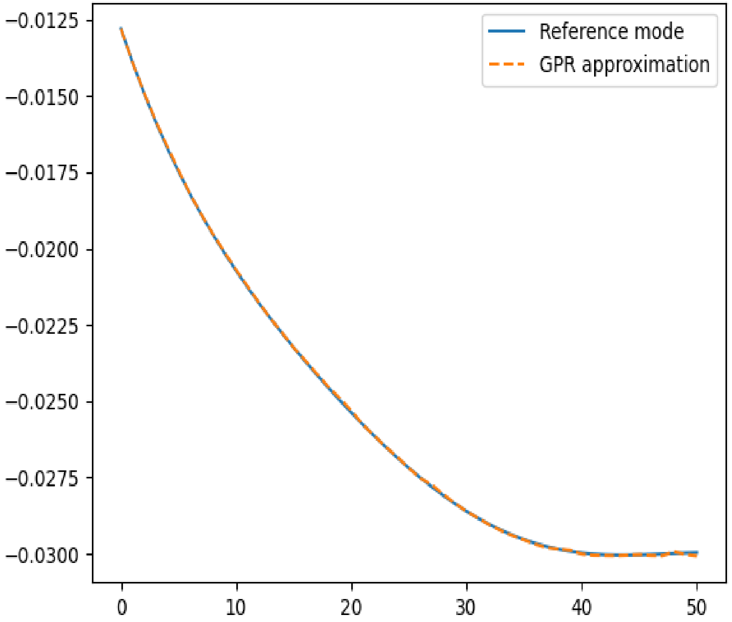


Confidence Bounds of
GPR-pROM prediction (***Scenario C***)

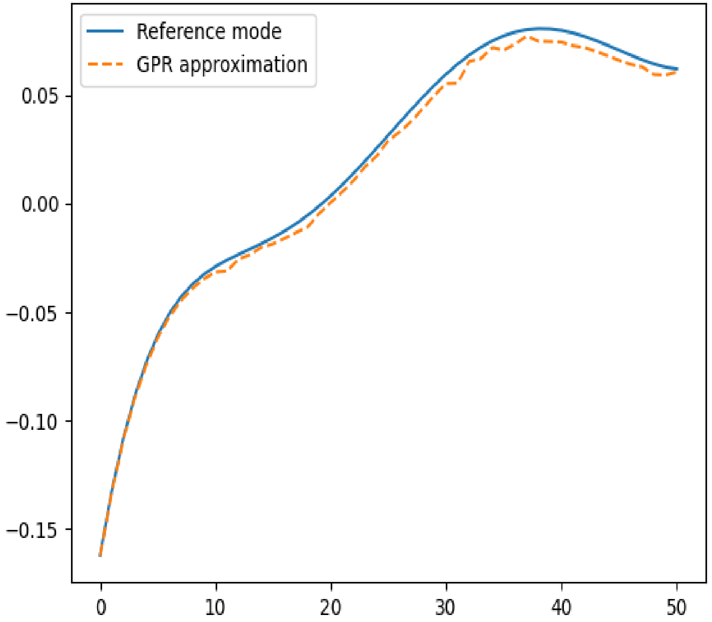
Case studies

Accuracy performance of the framework

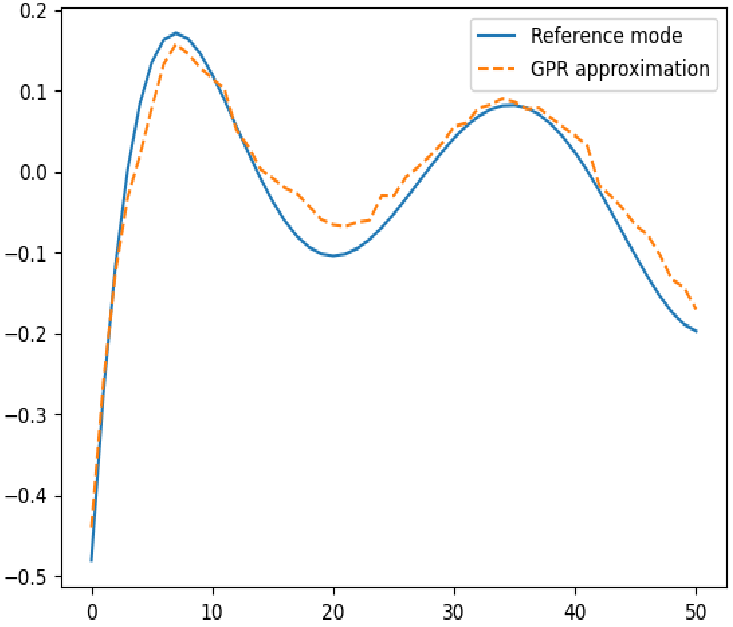
Reduced-order of pROM : 4 modes



GPR approximation
on ***first mode*** (***Scenario C***)



GPR approximation
on ***fourth mode*** (***Scenario C***)



GPR approximation
on ***sixth mode*** (***Scenario C***)

Concluding remarks
Limitations and outlook

The proposed adaptive GPR-pROM framework

- ✓ *Extends performance range* of traditional projection-based pROMs
- ✓ *Captures underlying dynamics* and dependencies *during damage or condition deterioration scenarios*
- ✓ Achieves ***on the fly correction*** of the pROM ***based on sparse measurements***
- ✓ Provides confidence bounds for response estimation
- ✓ May be adapted as an ***approximative, online low-cost surrogate*** for *Structural Health Monitoring* applications

- ➢ Hyper-Reduction implications for additional efficiency need further investigation
- ➢ GPR approximation scheme fails to capture higher order modes
- ➢ GPR approximation performance is strongly dependent on noise level
- ➢ GPR input-output channels discretization needs to be automated and optimized

Next short-term steps:

- ❖ Generalize implementation – adjust overall scope:
  Train pROM on earthquake database => Estimate damage in real-case scenarios
- ❖ Couple with filtering scheme to demonstrate potential on parameter/state/input estimation

Question session