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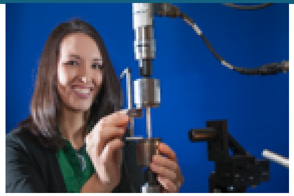
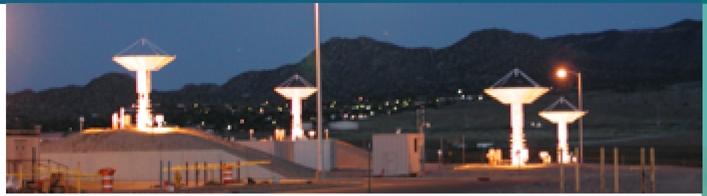
Effective Mass Computation Using a Modal Hurty-Craig-Bampton Framework





PRESENTED BY

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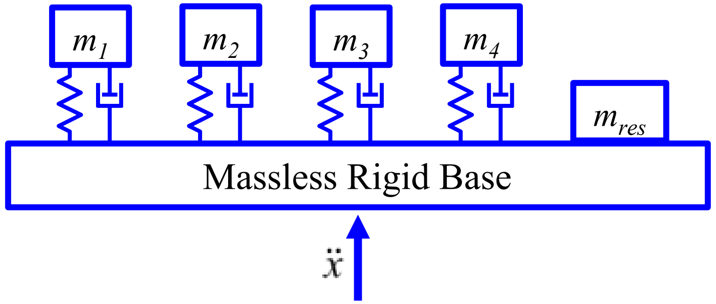
Motivation

- Quantifying the response of a structure mounted to a vibration shaker table is critical to design
  - This helps capture important measures such as failure margin

- One method to accomplish this is the **effective mass model:** a modal model that simulates the response of a component due to a base acceleration input in one direction



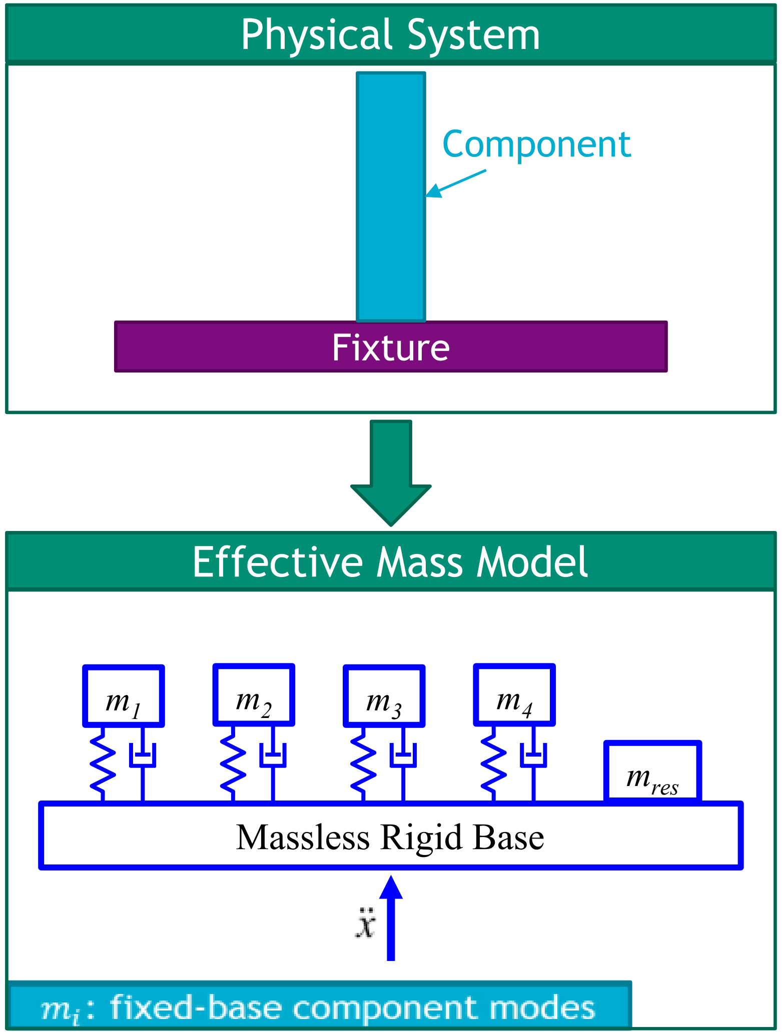
- Effective mass models can be extracted from either a finite element model (FEM) or experiment

- Previous works have developed several methods for extracting effective mass models from experiment, but require many steps and computations

- **This work proposes a new, more efficient method to compute an effective mass model based on a modal Hurty-Craig-Bampton (HCB) framework**

Effective Mass Background

- The ratio of effective mass to total mass of a component is related to how strongly that mode will be excited in a particular vibration direction
- An effective mass model can be used to calculate the actual energy in the component during the base acceleration environment
  - Useful metric for assessing failure margin by comparing energy at failure to energy in a qualification test
- Typically there are three effective mass modal models for a component, one for each of the X, Y, and Z translational directions
  - Normally rotational directions are ignored, since standard vibration table tests are usually focused in one translational direction
- An experimental method can extract an effective mass model from a modal test of a component on a fixture



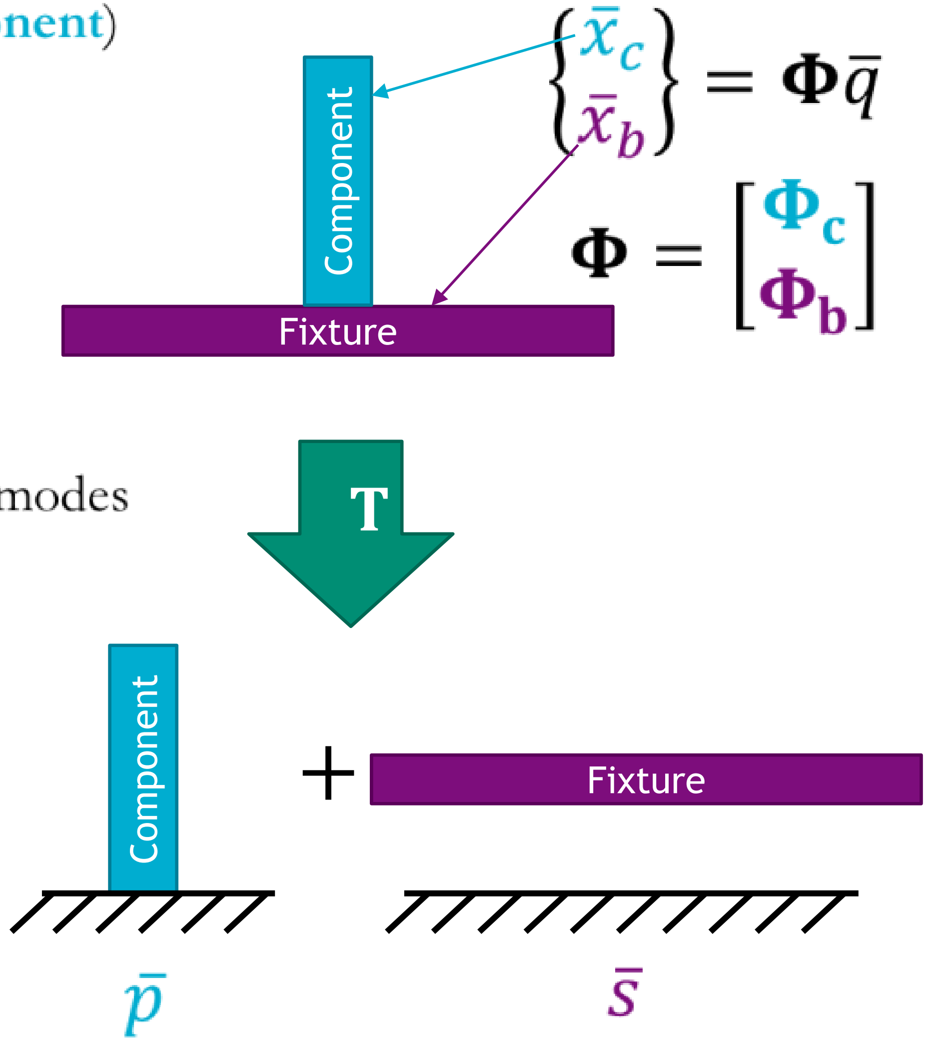
Hurty-Craig-Bampton Effective Mass Formulation

- Modal equation of motion of free-free assembly (**fixture** + **component**)
  - $[\boldsymbol{\Omega}_{n,fr} - \omega^2 \mathbf{I}]\bar{q} = 0$

Modal Coordinates

Natural Frequencies

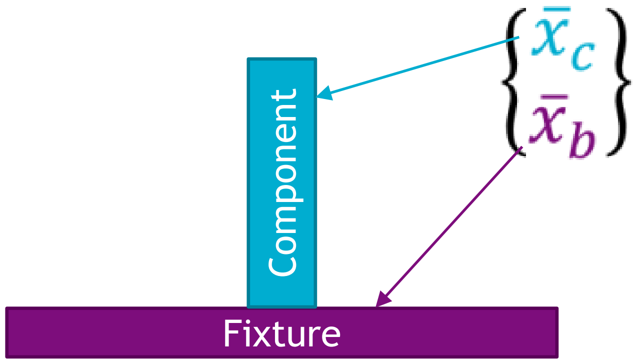
$$\boldsymbol{\Omega}_{n,fr} = \begin{bmatrix} \ddots & & \\ & \omega^2_{n,fr,i} & \\ & & \ddots \end{bmatrix}$$



- Transform free-free assembly to fixed-base component and fixture modes
  - $\bar{q} = \mathbf{T}\begin{Bmatrix} \bar{p} \\ \bar{s} \end{Bmatrix} = [\mathbf{T}_p \quad \mathbf{T}_s]\begin{Bmatrix} \bar{p} \\ \bar{s} \end{Bmatrix}$
    - $\bar{p}$: fixed-base modes of component
    - $\bar{s}$: modes of fixture
      - First six coordinates of $\bar{s}$ represent **unit displacements in each physical direction (X, Y, Z, θ_x, θ_y, θ_z)**
      - Assume for now that $\bar{s}$ only contains rigid modes

Objective: determine T

Hurty-Craig-Bampton Effective Mass Formulation, Determine $\mathbf{T_p}$



- Matrix $\mathbf{T_p}$ defines the relationship between $\bar{q}$ and $\bar{p}$
- Start with the constraints required to fix base of component
  - $x_b = \mathbf{\Phi_b}\bar{q} \approx \mathbf{\Psi_b}\bar{s}$
  - $\bar{s} = \underbrace{\mathbf{\Psi_b^+\Phi_b}}_{\mathbf{B}}\bar{q} = \mathbf{0}$

$$\mathbf{\Psi_b} = \begin{bmatrix} 1 & 0 & 0 & 0 & r_{y,1} & -r_{z,1} \\ 0 & 1 & 0 & -r_{x,2} & 0 & r_{z,2} \\ & & & \vdots & & \end{bmatrix}$$

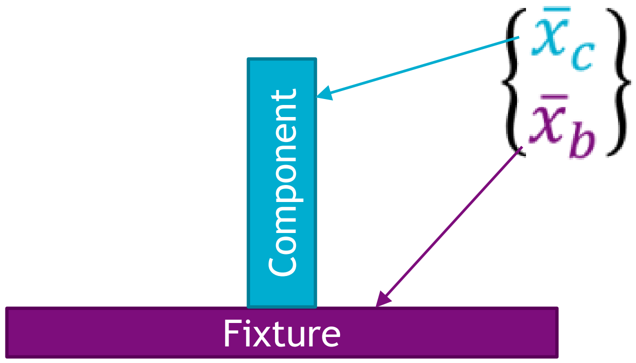
Perpendicular distance along the Y -axis to from Fixture CG to DOF 1

Constraint equation: $\bar{s} = \mathbf{B}\bar{q} = \mathbf{0}$

- Perform coordinate transform to fixed-base component modes
  - $\bar{q} = \mathbf{L}\bar{\eta}$ ← General fixed-base component coordinates
  - $\mathbf{BL}\bar{\eta} = \mathbf{0}$ ← Constraint equation in general fixed-base component coordinates
  - $\mathbf{L} = null(\mathbf{B})$
- Constrain free-free assembly using $\mathbf{L}$ to get equations of motion for fixed-base component
  - $\mathbf{L}^T\left[\mathbf{\Omega}_{n,fr} - \omega^2\mathbf{I}\right]\mathbf{L}\bar{\eta} = \mathbf{0}$
- Compute eigenvalues/vectors of this new equation of motion to get fixed-base modes
  - $\bar{\eta} = \mathbf{\Gamma}\bar{p}$ — $\mathbf{\Gamma}$ is the fixed-base component mode shapes
- Therefore, the relationship between $\bar{q}$ and $\bar{p}$ is
  - $\bar{q} = \mathbf{L\Gamma}\bar{p}$

$$\mathbf{T_p} = \mathbf{L\Gamma}$$

Hurty-Craig-Bampton Effective Mass Formulation, Determine $\mathbf{T_s}$



- Matrix $\mathbf{T_s}$ defines the relationship between $\bar{q}$ and $\bar{s}$
- Start from
  - $x_b = \mathbf{\Phi_b}\bar{q} \approx \mathbf{\Psi_b}\bar{s}$
- Since $\mathbf{\Psi_b}$ contains only rigid body modes, physical motions of the fixture can be approximated from only the rigid body modes of $\mathbf{\Phi_b}$, $\mathbf{\Phi_{b,rgd}}$
  - $\bar{q}_{rgd} = \mathbf{\Phi^+_{b,rgd}\Psi_b}\bar{s}$
  - $\bar{q}_e = \mathbf{0}\bar{s}$

$$\mathbf{T_s} = \begin{bmatrix} \mathbf{\Phi^+_{b,rgd}\Psi_b} \\ \mathbf{0} \end{bmatrix}$$

- Therefore the transformation $\mathbf{T}$ is:

$$\mathbf{T} = \left[\mathbf{L\Gamma} \middle| \begin{matrix} \mathbf{\Phi^+_{b,rgd}\Psi_b} \\ \mathbf{0} \end{matrix} \right]$$

Hurty-Craig-Bampton Effective Mass Formulation

- Transform original free-free equations of motion of the assembly to the HCB representation of the system
  - $$\left[\begin{bmatrix} \boldsymbol{\Omega}_{n,fx} & \mathbf{K}_{\mathbf{ps}} \\ \mathbf{K}_{\mathbf{ps}}^{\mathbf{T}} & \mathbf{K}_{\mathbf{ss}} \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{I} & \mathbf{M}_{\mathbf{ps}} \\ \mathbf{M}_{\mathbf{ps}}^{\mathbf{T}} & \mathbf{M}_{\mathbf{ss}} \end{bmatrix}\right] \begin{Bmatrix} \bar{p} \\ \bar{s} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \bar{F}_s \end{Bmatrix}$$

Fixed-Base Natural Frequencies

$$\boldsymbol{\Omega}_{n,fx} = \begin{bmatrix} \ddots & & \\ & \omega_{n,fx,i}^2 & \\ & & \ddots \end{bmatrix}$$

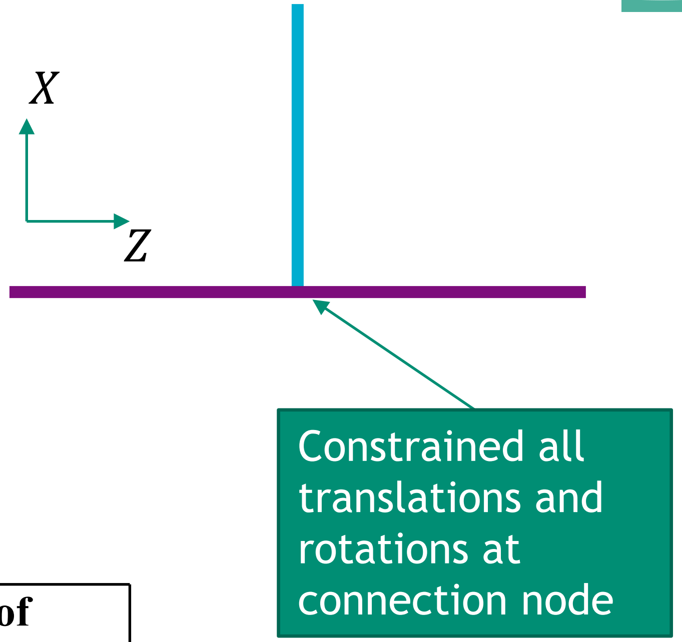
- The (i,j) terms of $\mathbf{M}_{\mathbf{ps}}$ are the modal participation factors for component mode p_i to base input direction s_j
- Since mass-normalized mode shapes of the assembly were used, the corresponding effective mass is:

Effective Mass of Mode i to input direction j

$$m_{eff,i,j} = \mathbf{M}_{\mathbf{ps}}(i,j)^2$$

- Key takeaways:
  - Effective mass models for all three directions are obtained simultaneously
  - Only requires shape information of the fixture, both in the assembly and in isolation
  - With a slightly modified $\mathbf{T}_{\mathbf{s}}$, this process also works if elastic fixture modes are included in $\boldsymbol{\Psi}_{\mathbf{b}}$. This is required when the fixture exhibits elastic motion in the free-free assembly modal test.

Demonstration, Numerical Example



- The proposed method was used to extract the effective mass model of a planar beam assembly in two configurations:
  - Component + stiff fixture
  - Component + soft fixture
- The assembly with the soft fixture has two elastic fixture modes in the bandwidth of interest (2000 Hz)
  - Stiff fixture only exhibits rigid motion

Beam	Modulus (lbf/in^2)	Density (lbm/in^3)	Length (in)	Base (in)	Height (in)	Number of Elements	Number of Measurement DOFs
Component	1×10^6	0.098	10	1	1	51	10
Fixture, Soft	1×10^6	0.098	20	6	6	101	12
Fixture, Stiff	30×10^7	0.284	10	6	6	51	12

- The effective masses and fixed-base frequencies predicted for each assembly is compared to the truth result for the component beam
  - Effective masses are presented as percentages of the component mass

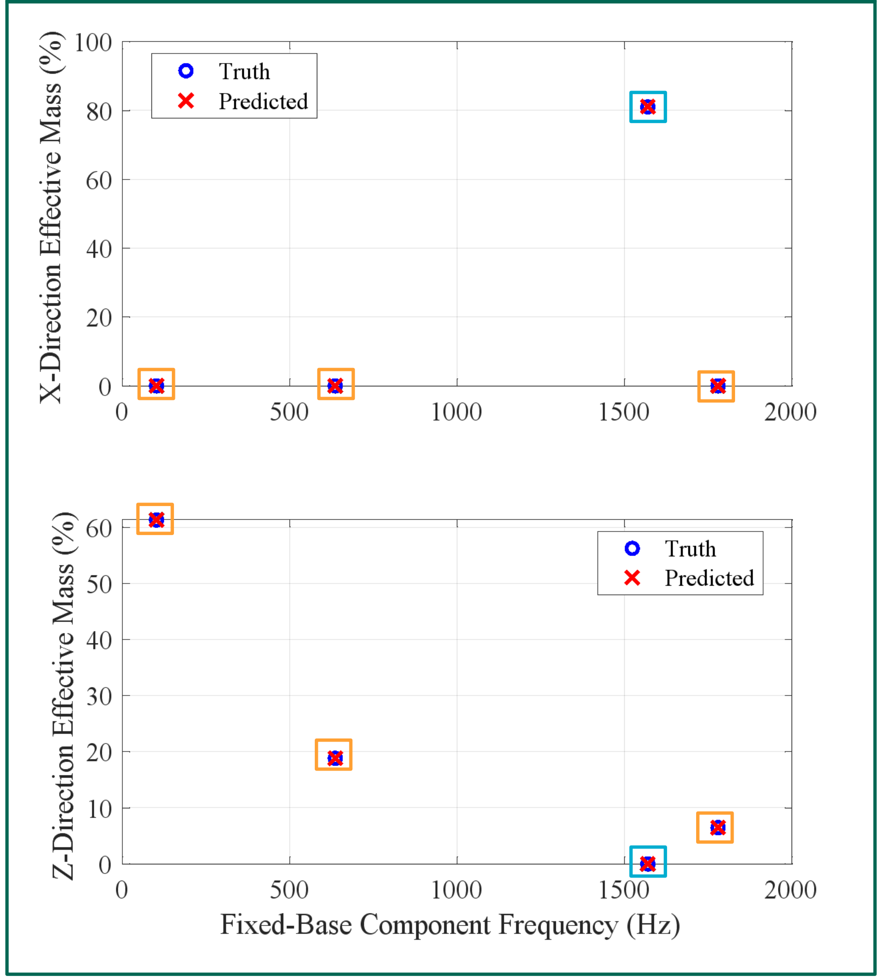
Mode	Description	Frequency (Hz)	Effective Mass	
			X	Z
1	1st bending of component	101.38	0.00	61.31
2	2nd bending of component	635.35	0.00	18.83
3	1st axial mode of component	1569.06	81.06	0.00
4	Higher bending of component	1779.00	0.00	6.47

Demonstration, Numerical Example

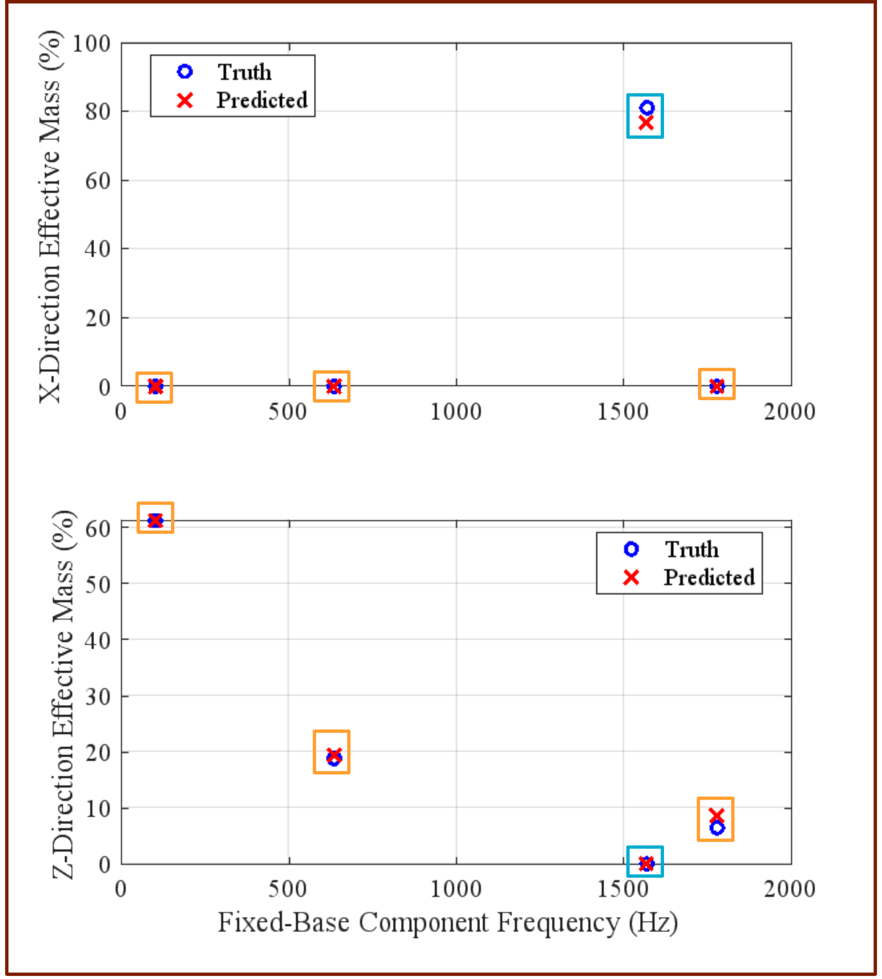
Axial mode

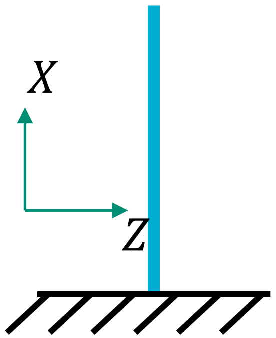
Bending mode

Stiff Fixture



Soft Fixture

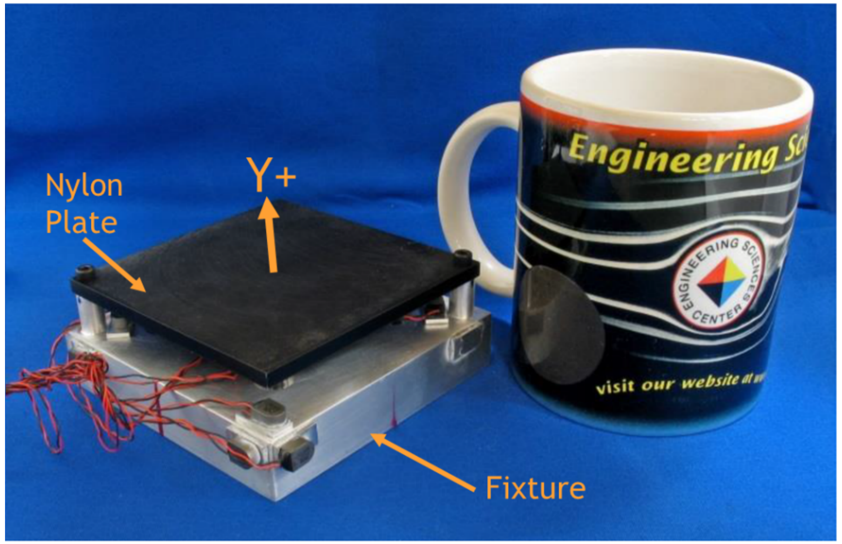




The predicted results match the truth data for both configurations, but the soft fixture has larger errors

Demonstration, Experimental Example

- The proposed method was applied to the nylon plate structure from [1]



- In [1], the effective mass model of the plate was extracted from a FEM and an experiment. Their results were then compared.
  - The FEM result was used as the truth data
  - The experiment was a free-free modal test of the assembly comprised the fixture and nylon plate
    - Note: one elastic fixture mode was within the bandwidth of interest
- The proposed HCB method utilized the data from this experiment to compute the effective mass model
- All three results are compared

[1] R. L. Mayes and D. W. Linehan, "Measuring Effective Mass of a Circuit Board," *Topics in Modal Analysis II,* vol. 8, pp. 207-217, 2014.

Demonstration, Experimental Example

Effective Mass Results

Predicted Fixed-Base Mode	Frequency (Hz)			Effective Mass (% of Nylon Plate Mass)		
	FEM	Method from [1]	Modal HCB	FEM	Method from [1]	Modal HCB
1	344-357	339.4	339.5	81.6-83.1	81.5	81.5
2	1000-1012	1081.4	1082	5.8-6.2	6.9	6.9
3	2590-2654	2705	2710	0.41-0.43	4	4

Errors

Predicted Fixed-Base Mode	Frequency Error (% of FEM)		Effective Mass Error (% of Nylon Plate Mass)	
	Method from [1]	Modal HCB	Method from [1]	Modal HCB
1	-4.9 to -1.3	-4.9 to -1.3	-1.6 to -0.1	-1.6 to -0.1
2	6.9 to 8.1	6.9 to 8.2	0.7 to 1.1	0.7 to 1.1
3	1.9 to 4.4	2.1 to 4.6	3.57 to 3.59	3.57 to 3.59

In [1], the FEM utilized two different methods to attach the plate to the fixture, so a range of frequencies and effective masses are given

Both methods achieved **frequency errors within 10%**

Both methods achieved **effective mass errors within 4%**

The proposed method yielded similar results as that used in [1]

Summary and Conclusions

- A new method for extracting an effective mass model from an experiment was presented which utilized a modal Hurty-Craig-Bampton framework
- This new method is simpler and more efficient than previous techniques while still offering the same prediction accuracy
  - There are fewer steps
  - Effective mass models for all three translational input directions are obtained simultaneously
- The proposed method relies on substructuring techniques, so corresponding standard best-practices and heuristics apply
  - Example: well-conditioned shape matrices, $\mathbf{\Phi_b}$ and $\mathbf{\Psi_b}$ (i.e. sufficient instrumentation to acquire independent modes of the fixture)

Thank You!

Back Ups

Craig-Bampton Effective Mass Formulation

- The derivation is slightly different if there are elastic modes in Ψ_b in addition to rigid body modes.
- Here we assume that the rigid modes of the fixture in the free-free assembly at the fixture DOFs can be approximated by the rigid modes of the fixture by itself. This is also assumed to hold true for the elastic modes. Therefore
  - $\Phi_{b,rgd}\bar{q}_{rgd} = \Psi_{b,rgd}\bar{s}_{rgd}$
  - $\Phi_{b,el}\bar{q}_{el} = \Psi_{b,el}\bar{s}_{el}$
  - $\bar{q} = \begin{Bmatrix} \bar{q}_{rgd} \\ \bar{q}_{el} \end{Bmatrix}$
  - $\bar{s} = \begin{Bmatrix} \bar{s}_{rgd} \\ \bar{s}_{el} \end{Bmatrix}$
- Solving for the $\bar{q}$:
  - $\bar{q}_{rgd} = \Phi_{b,rgd}^{+}\Psi_{b,rgd}\bar{s}_{rgd}$
  - $\bar{q}_{el} = \Phi_{b,el}^{+}\Psi_{b,el}\bar{s}_{el}$
- Thus the transformation becomes
  - $T = \begin{bmatrix} L\Gamma & \Phi_{b,rgd}^{+}\Psi_{b,rgd} & 0 \\ & 0 & \Phi_{b,el}^{+}\Psi_{b,el} \end{bmatrix}$
- The computation of modal participation factors and effective mass proceeds as before using this new transformation matrix