

# MDOF Laboratory Vibration to Simulate the Field Response:

## Principles Derived from Impedance Matched Multi-Axis Testing (IMMAT)

*IMAC XL Short Course*

*Feb. 5<sup>th</sup> 2022*

*Randal Mayes*

*Dan Rohe*

*Dr. Ryan Schultz*



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# Introduction

- Instructors:
  - Randal Mayes
  - Dan Rohe
  - Ryan Schultz
- What is your background and interest?
- Notes:
  - Please ask questions!
  - Take a break if you need it
  - Room - exits

# Schedule for Today

## Objective:

***Introduce MDOF Vibration Testing Concepts, Show You How MIMO Works, and Discuss Why it is Useful***

| Section                    | Time          |
|----------------------------|---------------|
| Introduction               | 8:00 – 8:30   |
| General MDOF Overview      | 8:30 – 9:15   |
| Field vs. Lab Environments | 9:15 – 10:15  |
| Break                      | 10:15 – 10:30 |
| Example Problem            | 10:30 – 11:15 |
| Demo 1: Single-Axis Test   | 11:15 – 12:00 |
| Lunch                      | 12:00 – 1:00  |

| Section                   | Time        |
|---------------------------|-------------|
| Demo 2: Multi-Shaker Test | 1:00 – 2:00 |
| Test Design Methods       | 2:00 – 2:30 |
| Break                     | 2:30 – 2:45 |
| Rattlesnake Controller    | 2:45 – 3:30 |
| Data Quality              | 3:30 – 4:00 |
| 6DOF & 3DOF Testing       | 4:00 – 5:00 |
| Wrap-Up                   | 5:00 – 5:30 |

# **General Overview of MDOF Random Vibration Testing**

*IMAC XL Short Course*

*Feb. 5<sup>th</sup> 2022*

*Ryan Schultz*

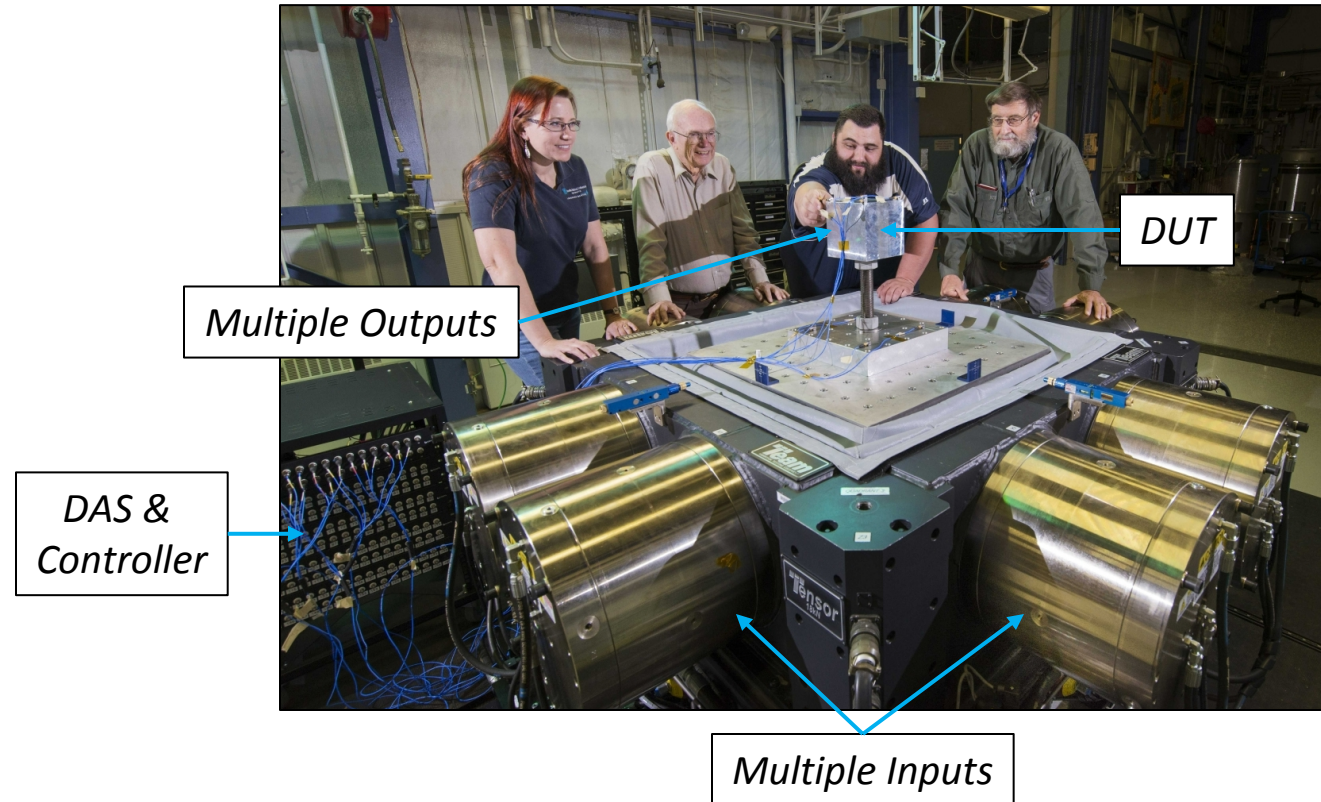


# Section Outline

- What is MDOF testing?
- Why is it good?
- Who is it for? Who is it not for?
- Terminology
- MIMO random vibration theory
- MIMO control theory
- Examining MDOF response data

# What Is MDOF (or MIMO or IMMAT or 6DOF) Testing?

*Vibration test using multiple inputs (shakers, shaker table directions) and multiple outputs (accelerometers)*



**What is MDOF testing?**

Why is it good?

Who is it for?

Who is it not for?

Terminology

MIMO random vibration  
theory

MIMO control theory

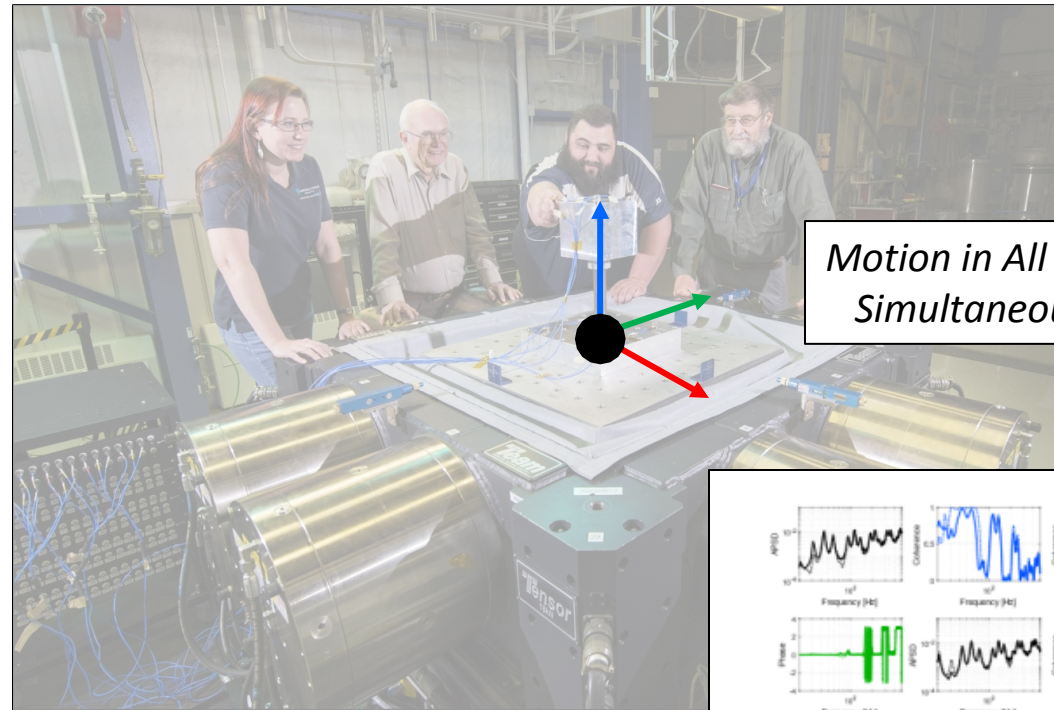
Examining MDOF response  
data

**Why is MDOF Testing Good?**

**Who is it for? Who is it not for?**

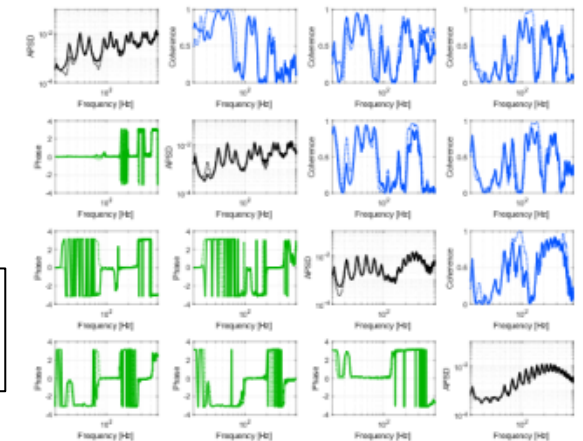
# What Is MDOF (or MIMO or IMMAT or 6DOF) Testing?

*Vibration test using multiple inputs (shakers, shaker table directions) and multiple outputs (accelerometers)*



*Motion in All Axes  
Simultaneously*

*Each DOF's Frequency Content, Levels,  
and all DOF-to-DOF Relationships*

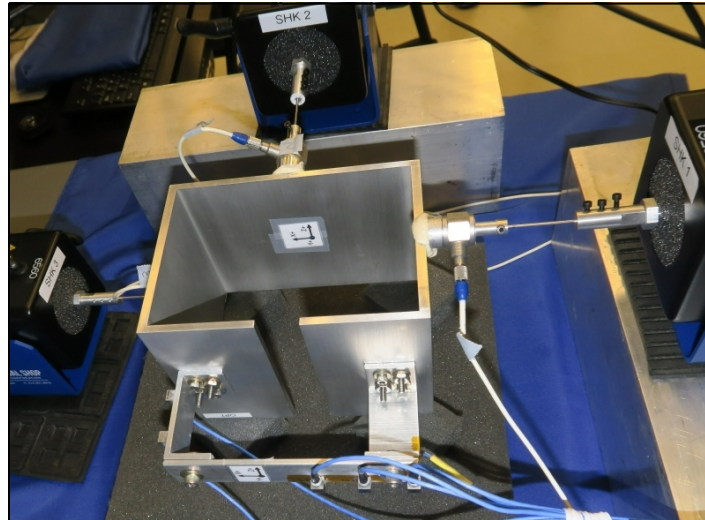


# What Is MDOF (or MIMO or IMMAT or 6DOF) Testing?

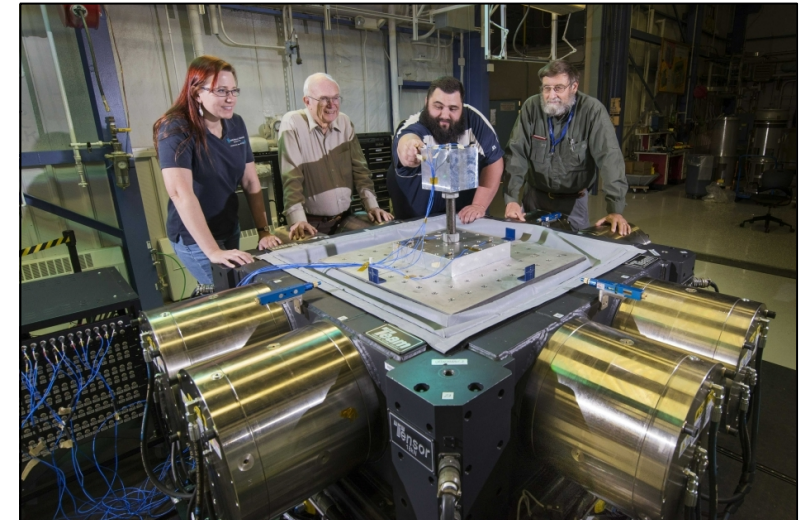
## ***MDOF vs. MIMO vs. IMMAT vs. 3- or 6-DOF***

- *There is a lot of overlap in terminology!*
- *MDOF: Multiple Degree of Freedom*
- *MIMO: Multiple-Input/Multiple-Output*

*Impedance-Matched Multi  
-Axis Test (IMMAT)*



*3- or 6-DOF*

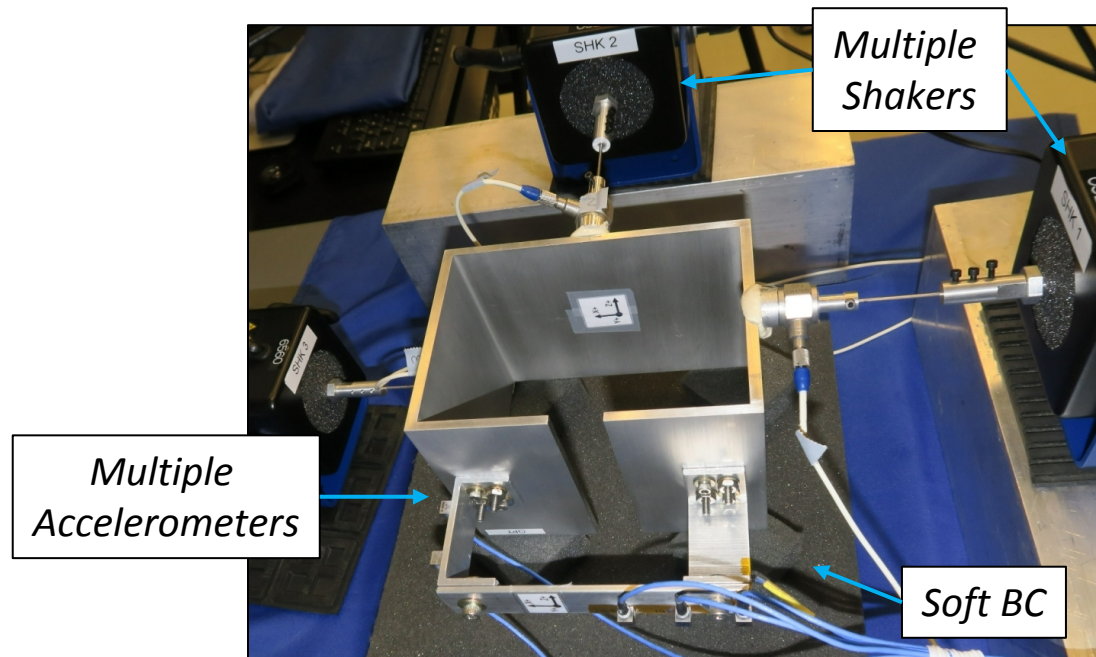




# What Is MDOF (or MIMO or IMMAT or 6DOF) Testing?

## *IMMAT or Multi-Shaker Testing*

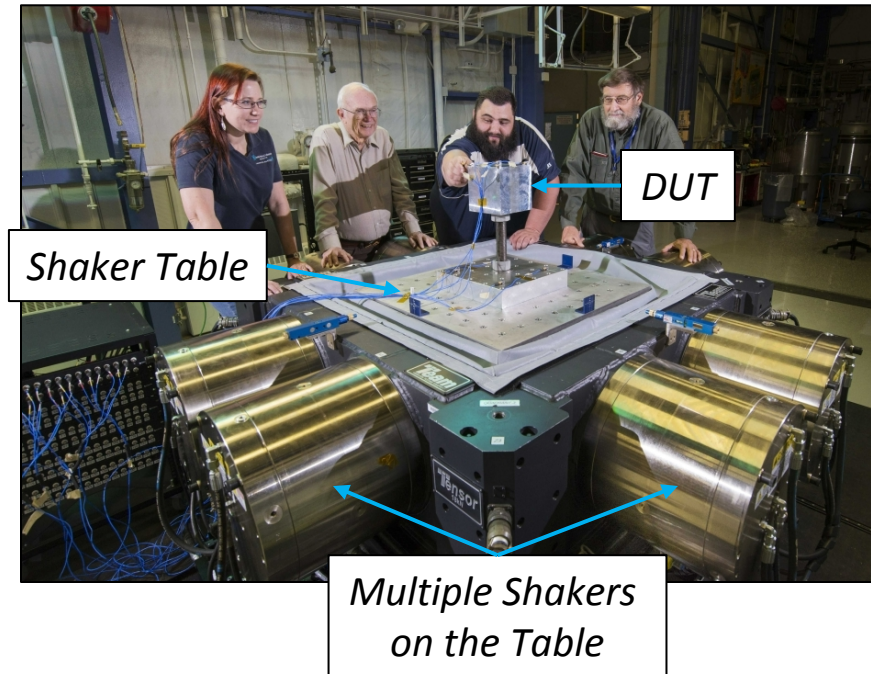
- *Individual inputs distributed on the DUT*
- *Control to multiple accelerometers (DOFs) simultaneously*
- *Approximate the service or assembly boundary conditions (impedance match)*



# What Is MDOF (or MIMO or IMMAT or 6DOF) Testing?

## 3- or 6-DOF Testing

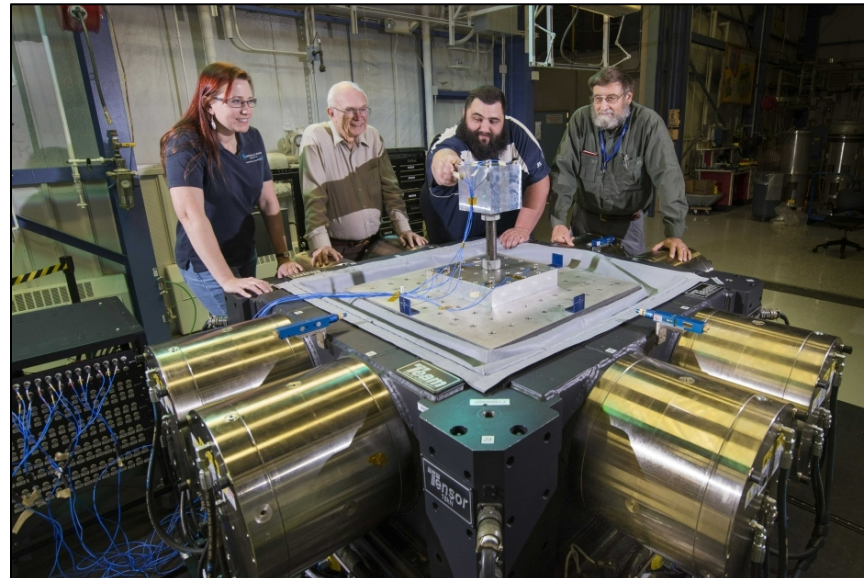
- *DUT mounted to rigid table*
- *Base excitation in multiple directions simultaneously*
- *Simultaneous control of multiple accelerometers on the DUT or the table*



# What Is MDOF (or MIMO or IMMAT or 6DOF) Testing?

## *General MIMO Testing Process*

- Setup test & check instrumentation
- Measure system output/input relationship (FRF matrix)
- Import spec into control system
- Solve MIMO control problem, get shaker drive signals
- Run the test (update drive signals to reduce error)
- Compare responses vs. spec





What is MDOF testing?

**Why is it good?**

**Who is it for?**

**Who is it not for?**

Terminology

MIMO random vibration  
theory

MIMO control theory

Examining MDOF response  
data

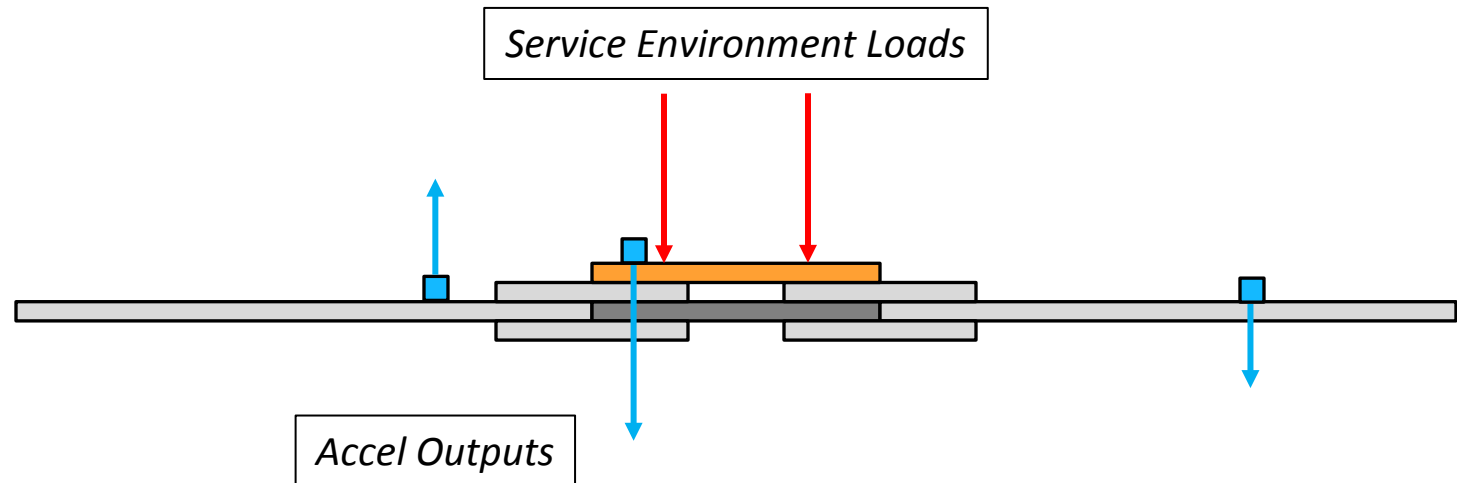
# **Why is MDOF Testing Good?**

## **Who is it for? Who is it not for?**

# MDOF Testing Enables an Accurate Match to the Response Everywhere on the DUT

***Objective: Make the lab DUT vibrate like it would in the service environment***

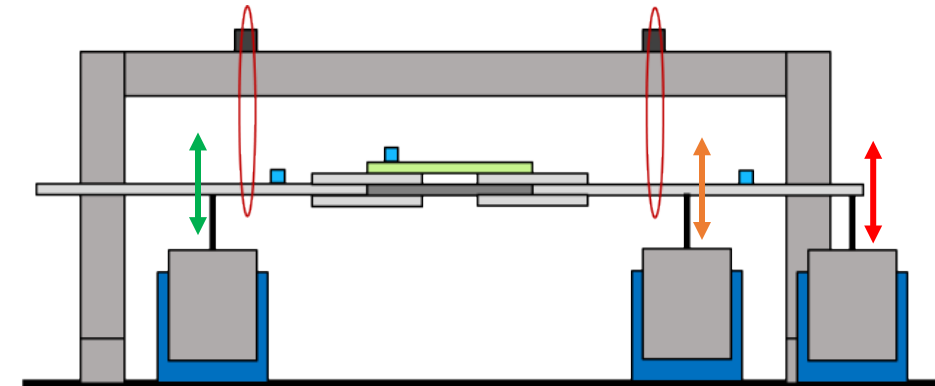
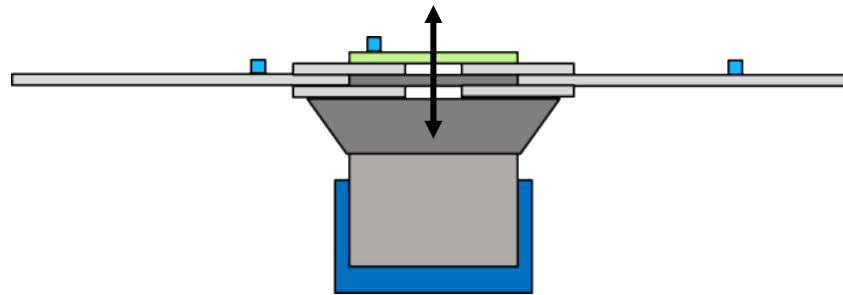
- Meaning: All locations may respond differently, and the relationship of the responses depends on the loading and the part dynamics



# MDOF Testing Enables an Accurate Match to the Response Everywhere on the DUT

Matching the response everywhere on the DUT:

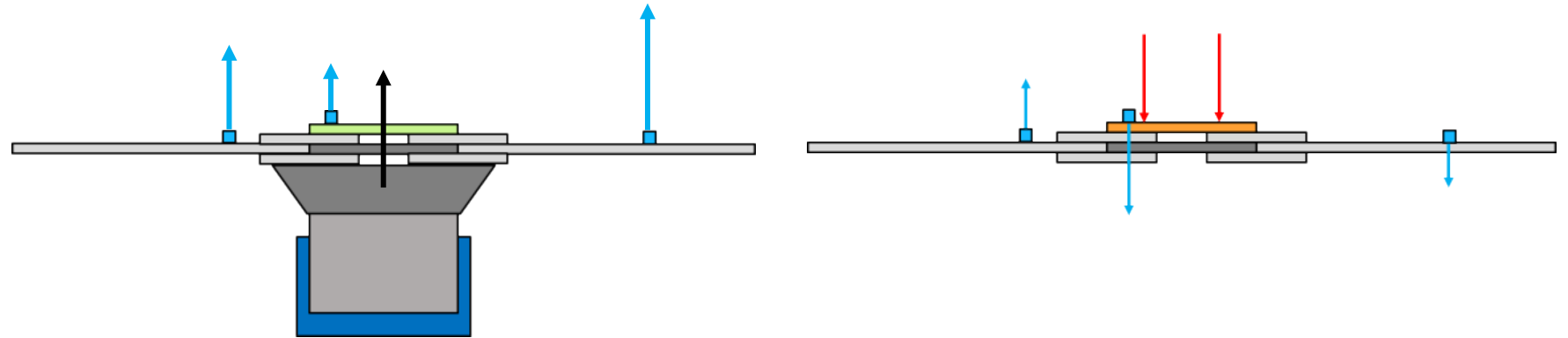
- Test inputs + DUT dynamics & BCs combine and result in some DUT response pattern
- MIMO control can tailor the inputs to change that DUT response pattern to match a spec (from the service environment response)



# MDOF Testing Enables an Accurate Match to the Response Everywhere on the DUT

With only one input (single-axis):

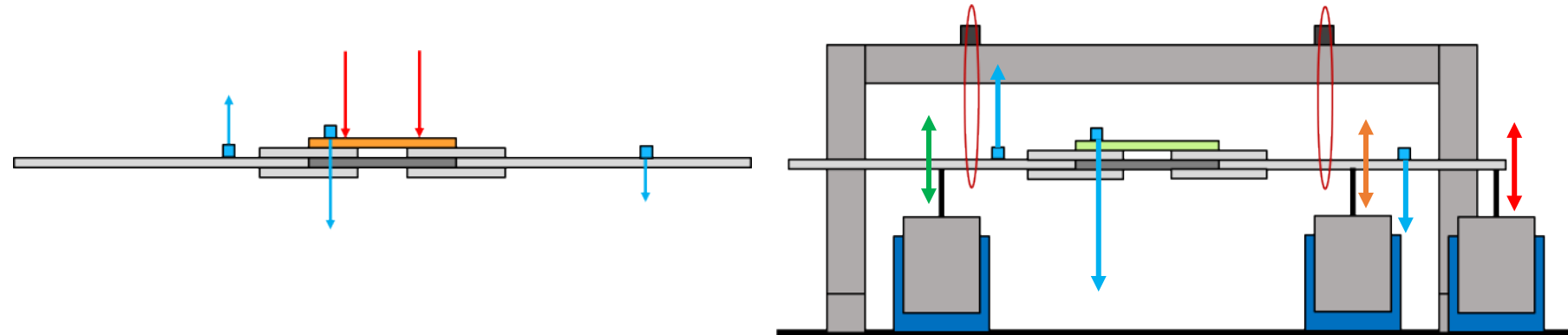
- Shaker input determines the level vs. frequency
- DUT dynamics determines the distribution of level at each output
- All outputs are perfectly correlated
- No way to change the output correlation or phase to better match the service environment response (spec)



# MDOF Testing Enables an Accurate Match to the Response Everywhere on the DUT

With multiple inputs (MIMO/MDOF/IMMAT/6DOF):

- Shaker inputs determines the level vs. frequency and the coherence and phase between all inputs
- DUT dynamics determines the distribution of level at each output
- All outputs can have desired coherence and phase
- Controller changes the inputs to make the output levels, coherence and phase to better match the service environment response (spec)



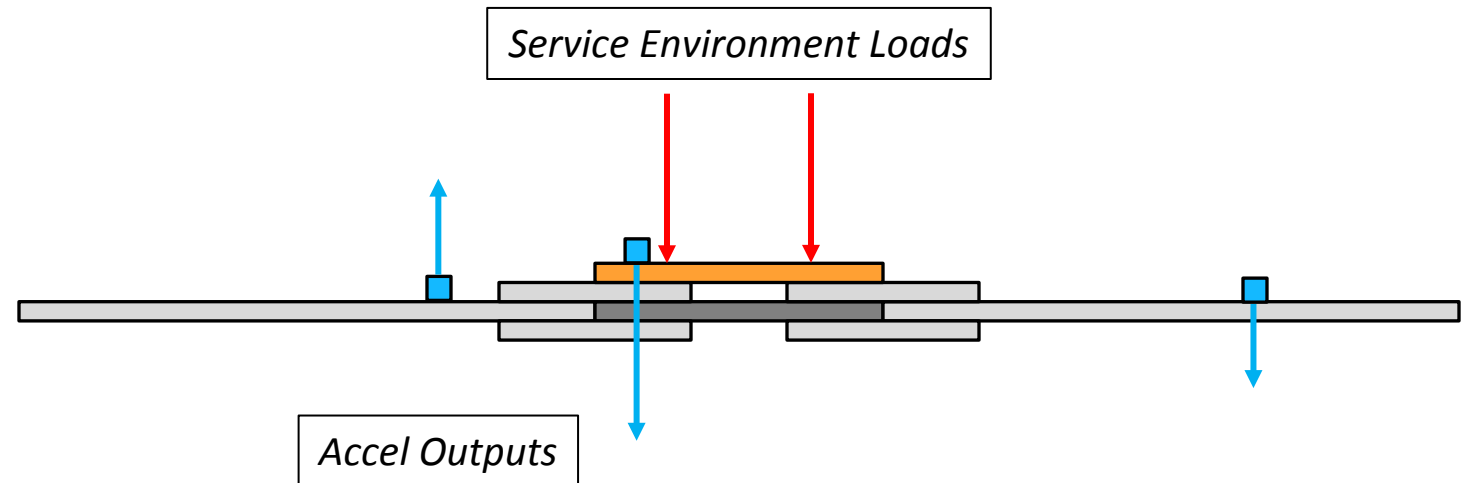
# Why Does Matching the Response Everywhere Matter?

Important components at various locations

- Don't want to over- or under-test

Response pattern determines stress on the system

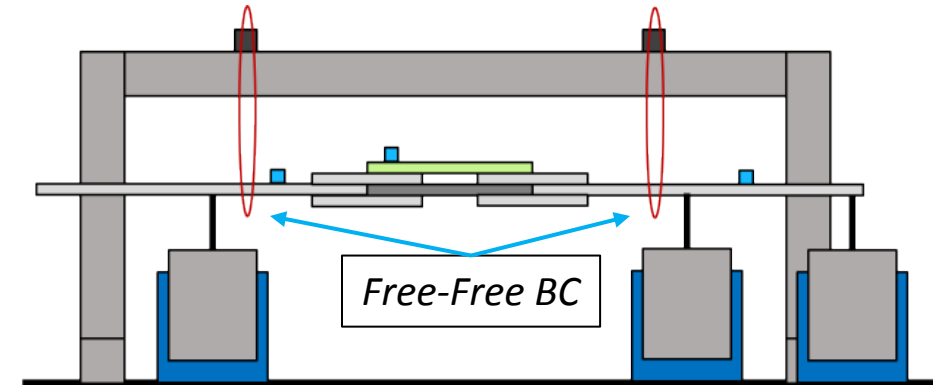
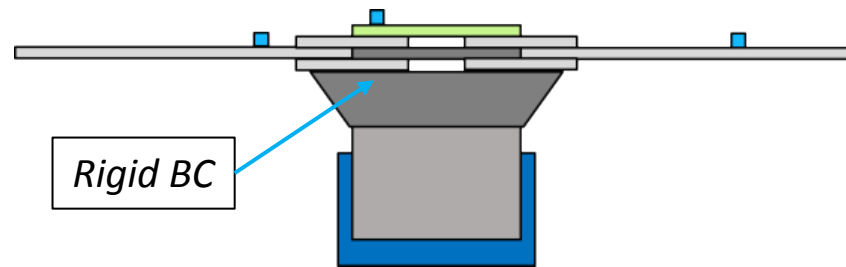
- Don't want to over- or under-stress structure



# MDOF Testing Allows Closer Matching of Service Boundary Conditions

Not constrained to only mounting the DUT to a shaker table using a fixture

- Often this overly stiffens the BCs
- The fixture and shaker table change the DUT modes (shapes and frequencies)
- IMMAT or multi-shaker testing can be done with various BCs: free-free, using impedance-approximating fixtures, etc.



# MIMO Testing:

Who is it for?

Who should  
avoid it?

MIMO testing is (generally) more complicated than single-axis vibration testing and requires a few extra things to work well

- Multiple shakers or a 6-DOF machine
- MIMO control system
- Multiple accelerometers in good locations
- MIMO-compatible specification



# MIMO Testing:

## Who is it for?

## Who should avoid it?

MIMO testing is (generally) more complicated than single-axis vibration testing and requires a few extra things to work well

- Who should utilize MIMO testing?
  - If you need an accurate response at multiple points on your DUT
  - If you need a more realistic ground test and a more realistic assessment of your DUT's margins and functional performance
  - If your part is unnecessarily failing during single-axis testing
- Who should avoid it?
  - If you don't have a MIMO specification for your system in your desired service environment, it may not be worth the extra work (a bad spec but a good setup won't make for a good test)
  - If you test lots of different types of parts the setup time and need for different shaker configurations for each part may be a problem

What is MDOF testing?

Why is it good?

Who is it for?

Who is it not for?

### **Terminology**

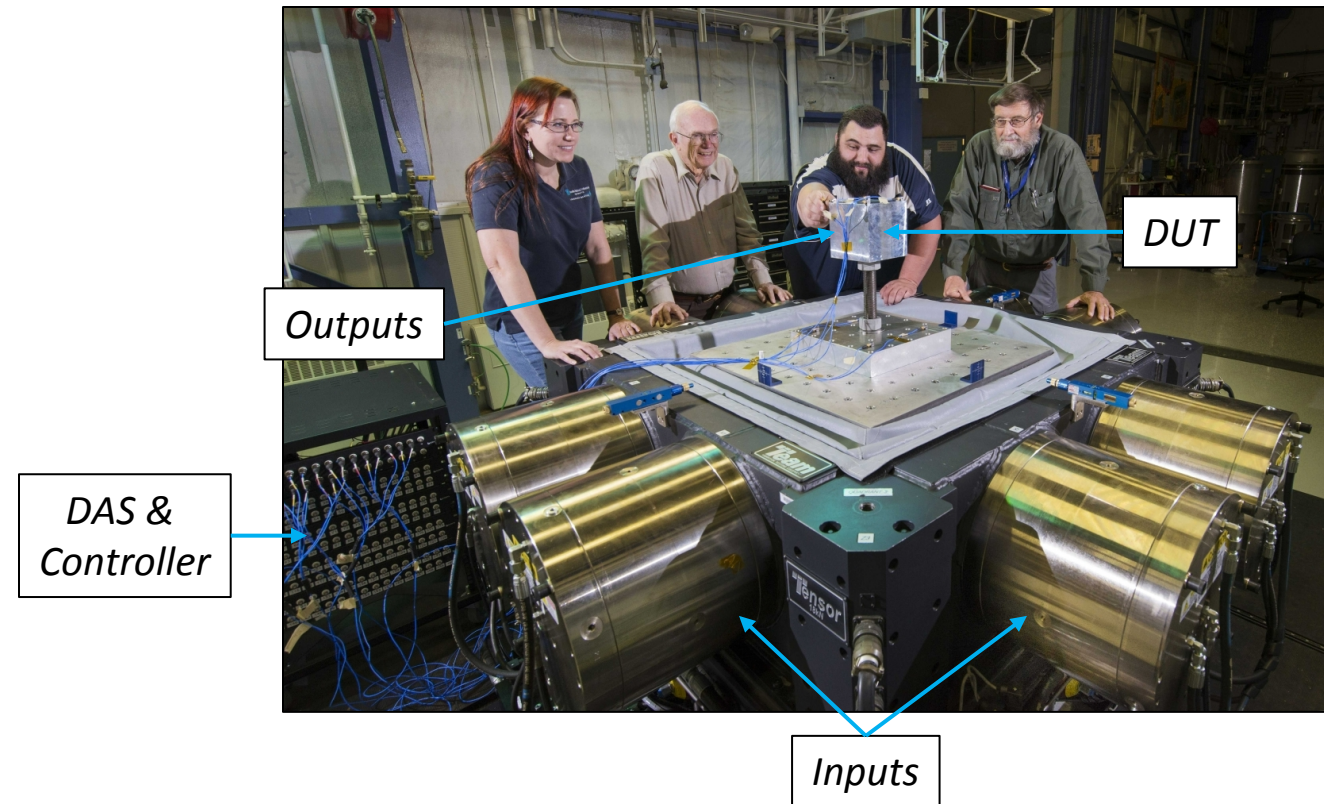
MIMO random vibration  
theory

MIMO control theory

Examining MDOF response  
data

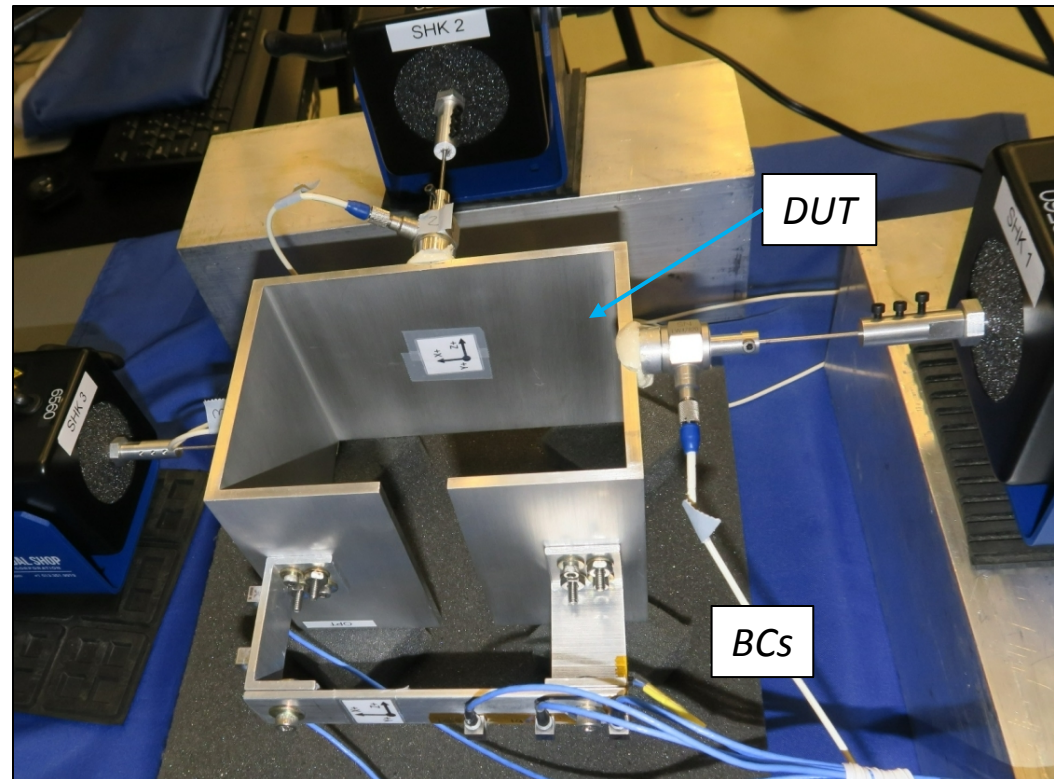
# **MDOF Testing Terminology**

# MDOF Testing Terminology



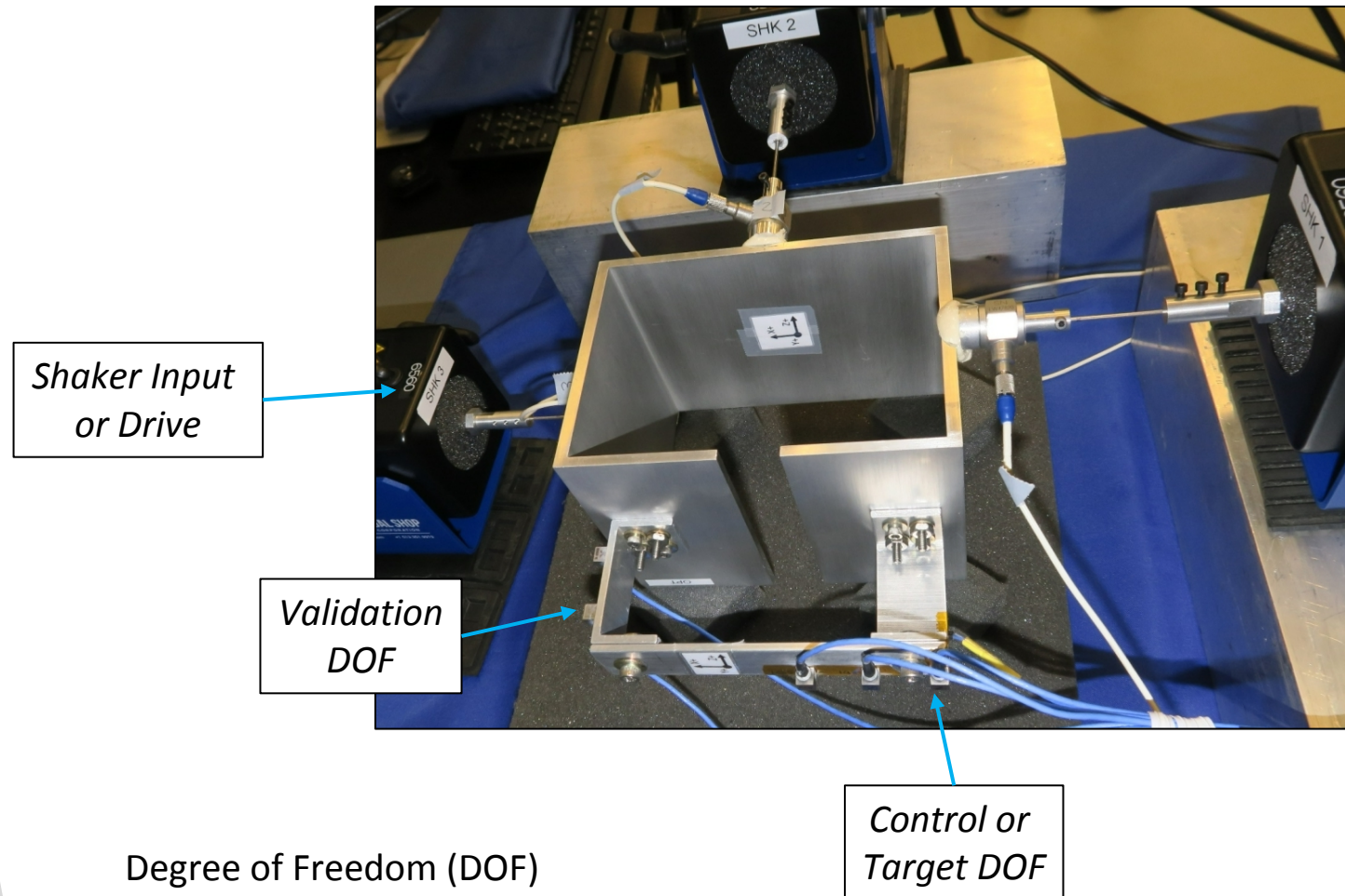
Data Acquisition System (DAS)  
Device Under Test (DUT)

# MDOF Testing Terminology



Boundary Conditions (BCs)  
Device Under Test (DUT)

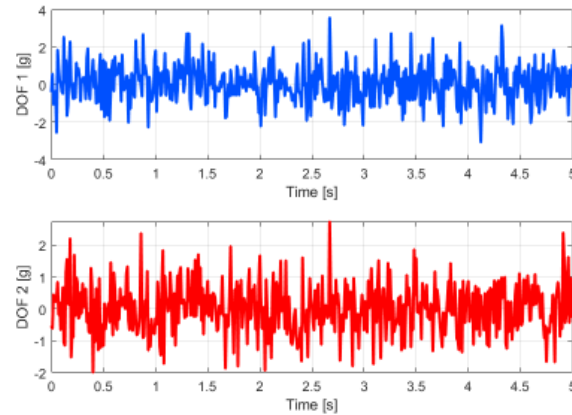
# MDOF Testing Terminology



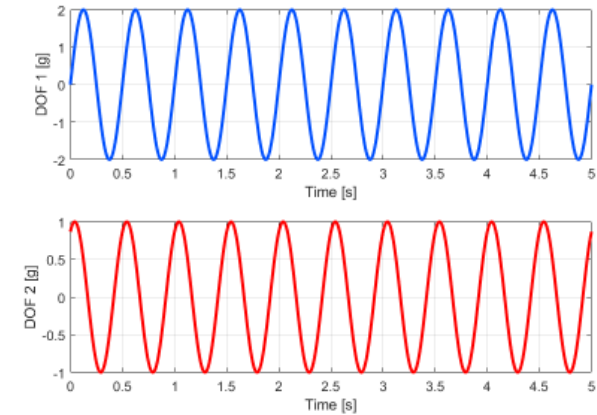


# MDOF Testing Terminology

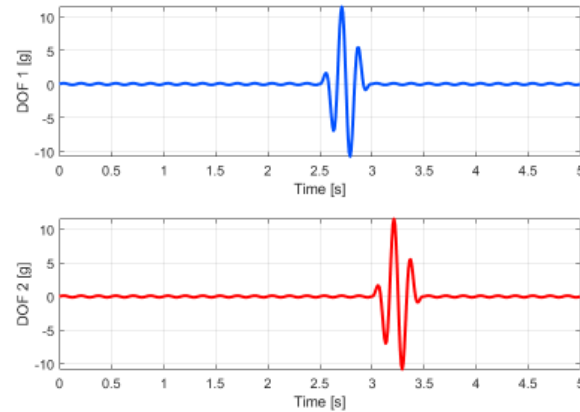
*MIMO Random*



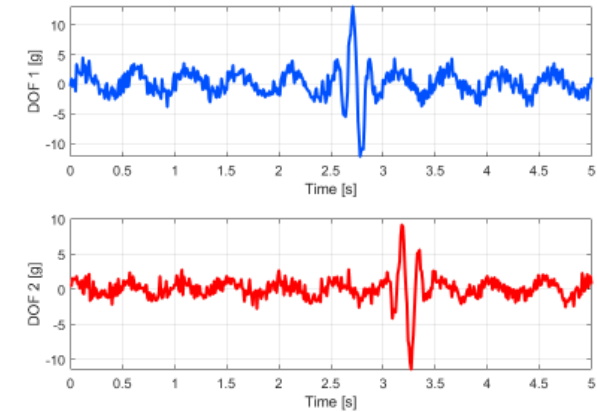
*MIMO Sine*



*MIMO Shock*



*MIMO TWR*



Multiple-Input/Multiple-Output (MIMO)  
Time Waveform Replication (TWR)

What is MDOF testing?

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Terminology

**MIMO random vibration  
theory**

MIMO control theory

Examining MDOF response  
data

# MIMO Random Vibration Theory

# What goes on in a MIMO test?

## *General MIMO Testing Process*

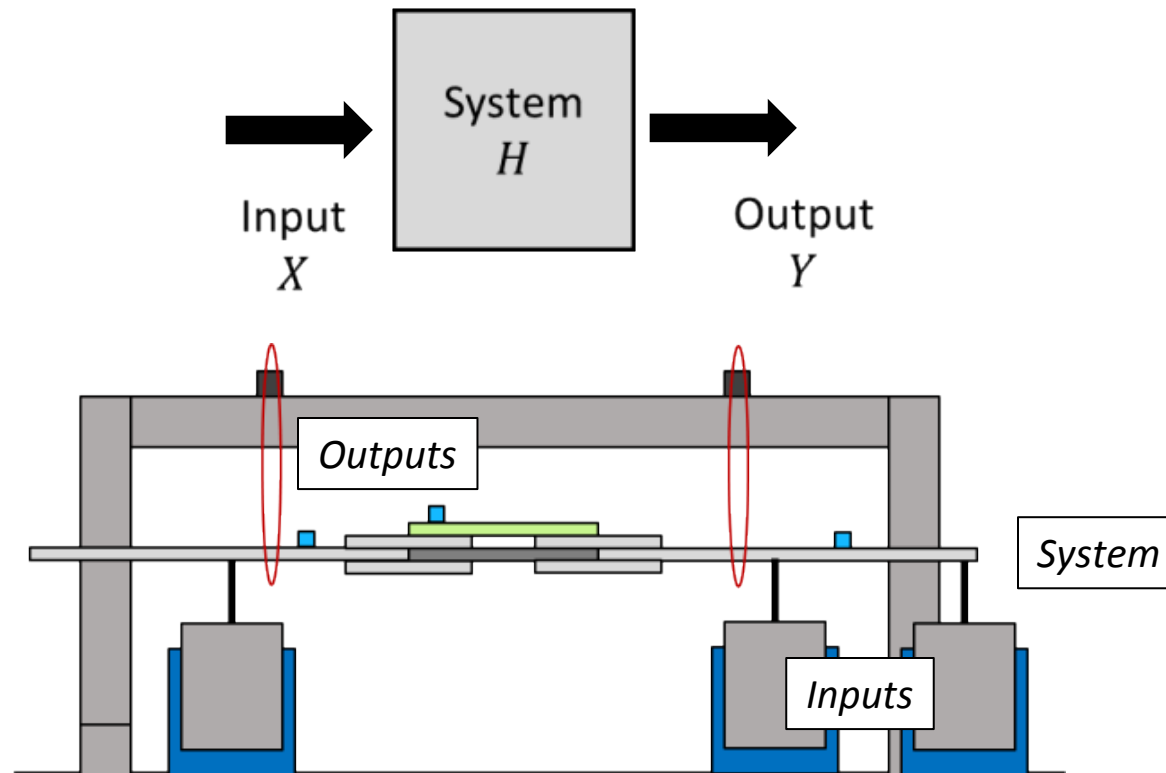
- Setup test & check instrumentation
- Measure system output/input relationship (FRF matrix) ←
- Import spec into control system
- Solve MIMO control problem, get shaker drive signals ←
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# MIMO Random Vibration Theory

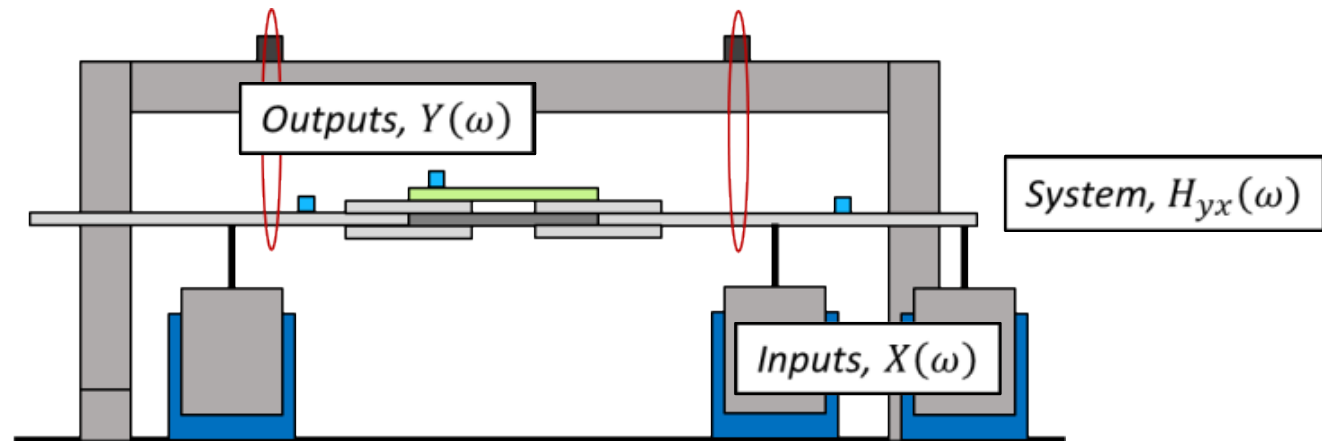
- The test system has inputs and outputs
  - Test system = DUT + fixtures + BCs + shakers + amplifiers
  - Outputs = DUT accelerometer responses
  - Inputs = Controller drive signals going to the amplifiers and shakers



# MIMO Random Vibration Theory

- The test system has inputs and outputs
  - Test system = DUT + fixtures + BCs + shakers + amplifiers
  - Outputs = DUT accelerometer responses
  - Inputs = Controller drive signals going to the amplifiers and shakers
- The system inputs and outputs can be modeled as a linear system:

$$\{Y(\omega)\} = [H_{yx}(\omega)]\{X(\omega)\}$$



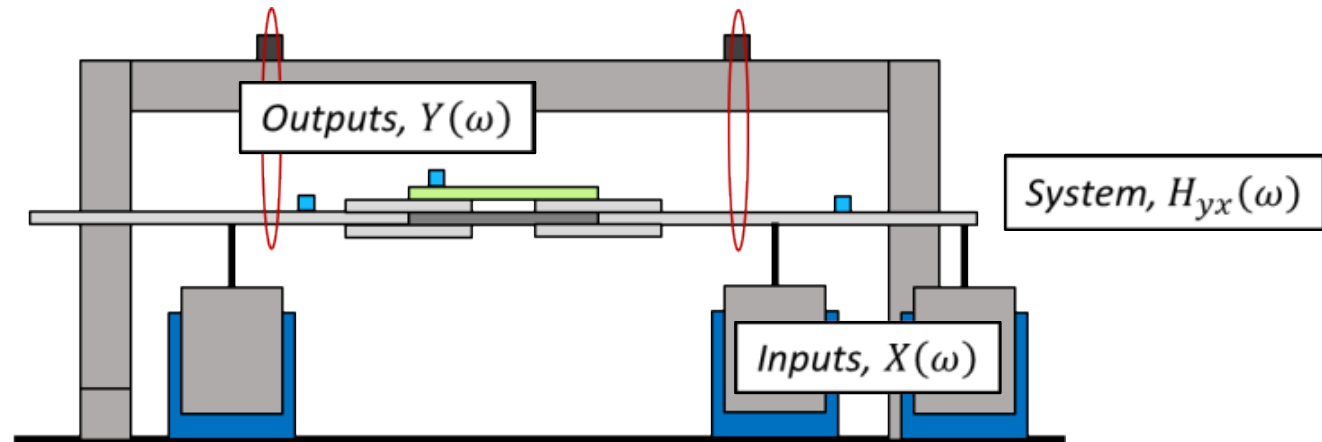
System FRF matrix,  $H_{yx}(\omega)$   
Output linear spectra,  $Y(\omega)$   
Input linear spectra,  $X(\omega)$   
Angular frequency,  $\omega$

# MIMO Random Vibration Theory

- The system inputs and outputs can be modeled as a linear system:

$$\{Y(\omega)\} = [H_{yx}(\omega)]\{X(\omega)\}$$

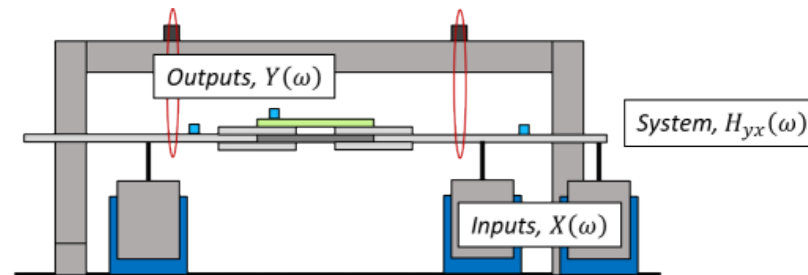
- Each of these is a 3-dimensional matrix of size:
  - $\{Y(\omega)\} = N_{outputs} \times 1 \times N_{freqs}$
  - $\{X(\omega)\} = N_{inputs} \times 1 \times N_{freqs}$
  - $[H_{yx}(\omega)] = N_{outputs} \times N_{inputs} \times N_{freqs}$



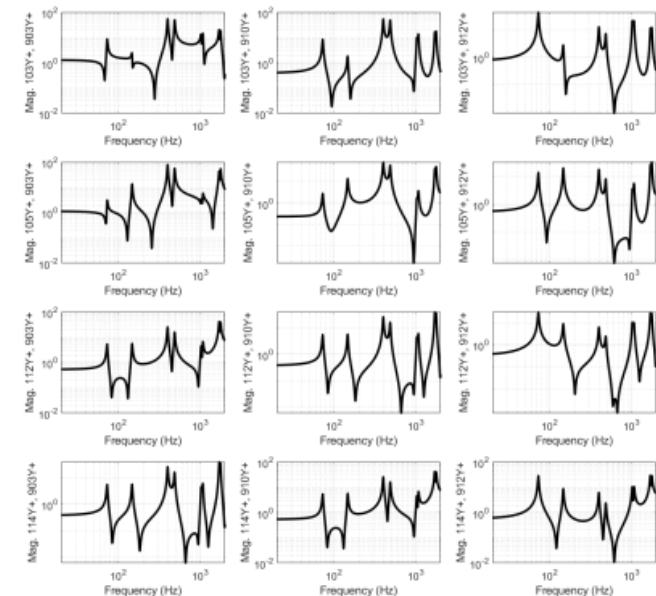
System FRF matrix,  $H_{yx}(\omega)$   
Output linear spectra,  $Y(\omega)$   
Input linear spectra,  $X(\omega)$   
Angular frequency,  $\omega$

# FRF Matrix

- System FRF matrix details:  $H_{yx}(\omega)$ 
  - $[H_{yx}(\omega)] = N_{outputs} \times N_{inputs} \times N_{freqs}$
  - Rows = output DOFs (e.g. accelerometer channels)
  - Columns = input DOFs (e.g. shaker drive voltages)
  - The i-th row and j-th column give the output-input relationship between the i-th output DOF and the j-th input DOF



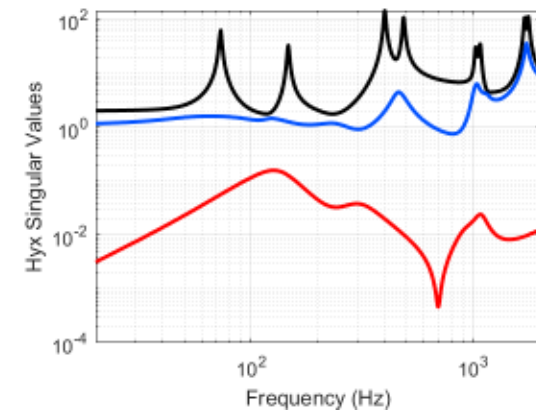
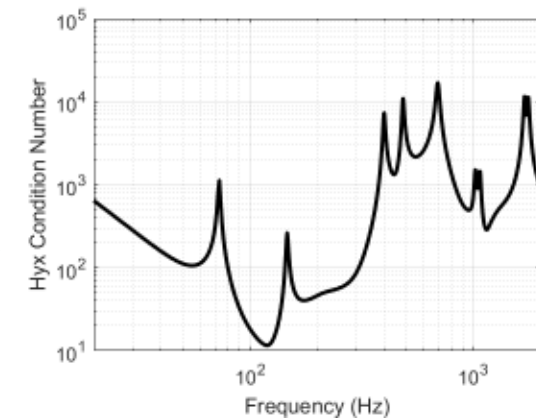
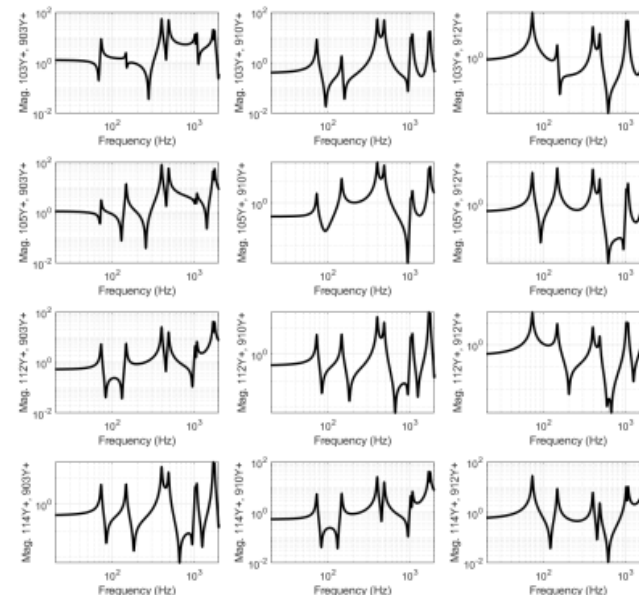
FRF magnitude,  $|H_{yx}(\omega)|$ , for a 4 output, 3 input system



# FRF Matrix

- System FRF matrix details:  $H_{yx}(\omega)$ 
  - Generally, good independence of outputs and inputs is needed for a high quality MIMO test
  - The matrix condition number is a useful measure of independence
  - Singular value decomposition helps to understand how many strong independent contributors there are in the FRF matrix

*FRF magnitude,  $|H_{yx}(\omega)|$ , for a 4 output, 3 input system*

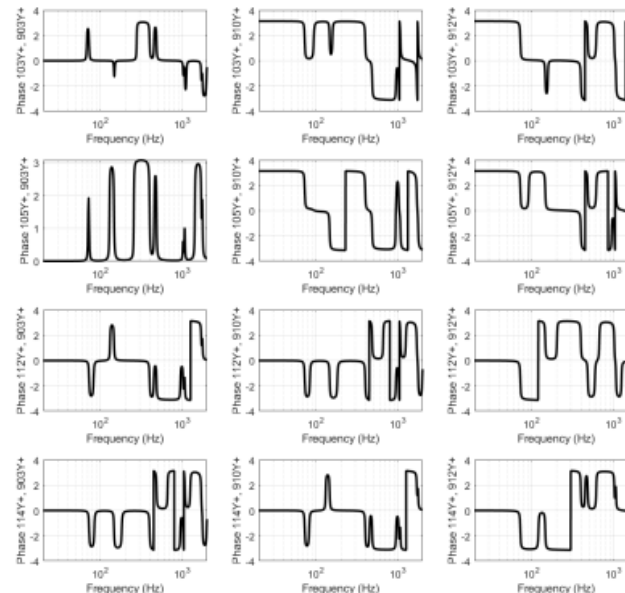


*3 Singular values ( $H_{yx}$  is rank 3)*

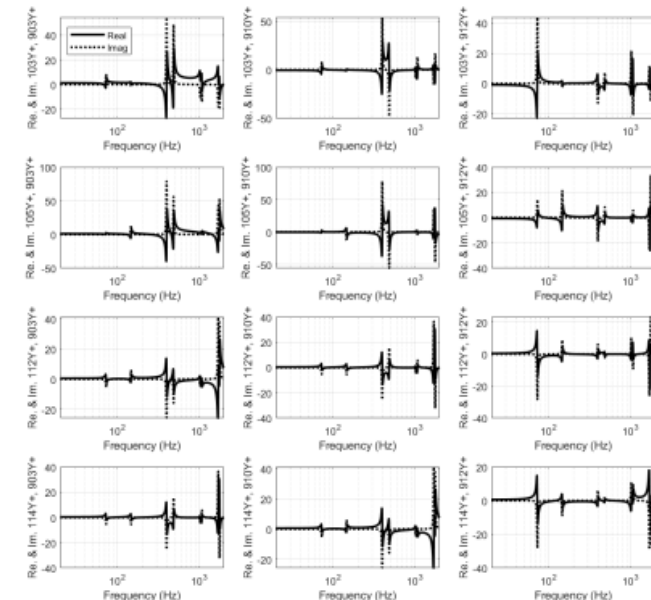
# FRF Matrix

- System FRF matrix details:  $H_{yx}(\omega)$ 
  - Typical units for accelerometer outputs and shaker drive inputs: [g/V]
  - These are not the FRFs you use for mode extraction, but you could measure the input forces and use those [g/lbf] FRFs for modal
  - FRF is complex valued and can be viewed in terms of magnitude, phase, real & imaginary, etc.

*FRF phase*



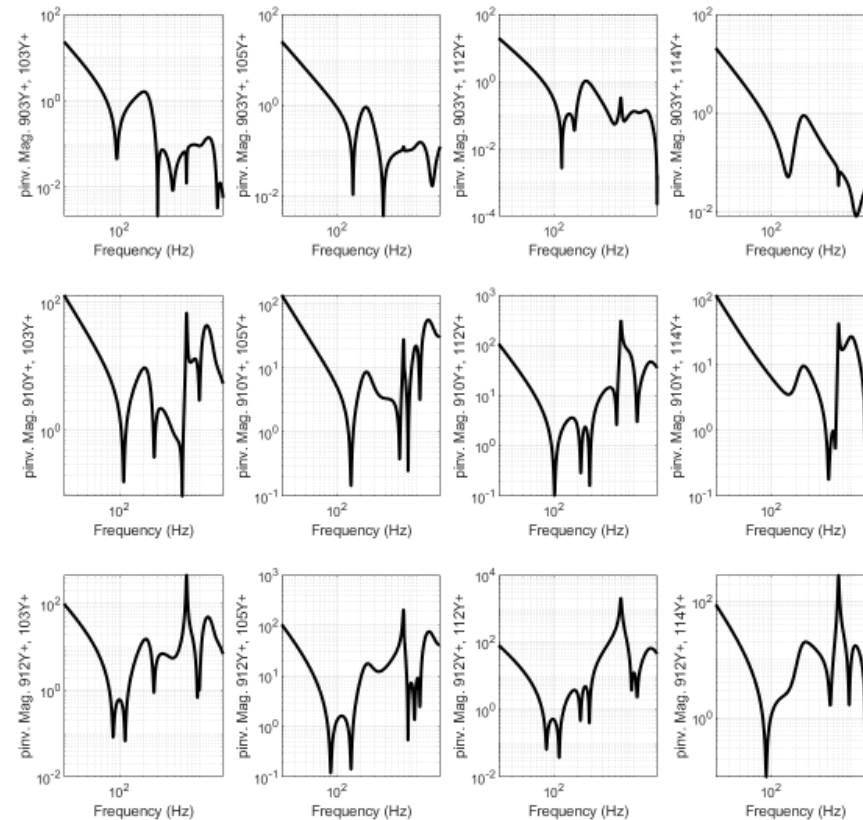
*FRF real & imaginary*



# Pseudo-Inverse FRF Matrix

- Pseudo-inverse of the FRF matrix,  $[H_{yx}(\omega)]^+$

*Pseudo-inverse FRF magnitude*



# Linear & Power Space

- Linear System: Linear & power space forms
  - Linear space = linear spectra with units of [g], [V], etc.
  - Power space = PSDs with units of [g<sup>2</sup>/Hz], [V<sup>2</sup>/Hz], etc.
  - Power space equation is an outer product of the linear space equation

*Linear Space  
MIMO Equation*

$$\{Y(\omega)\} = [H_{yx}(\omega)]\{X(\omega)\}$$

*Power Space  
MIMO Equation*

$$[S_{yy}(\omega)] = [H_{yx}(\omega)][S_{xx}(\omega)][H_{yx}(\omega)]^H$$



# Linear & Power Space

- Linear System: Linear- & power-space
  - Linear space = linear spectra with units of [g], [V], etc.
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*Linear Space  
MIMO Equation*

$$\{Y(\omega)\} = [H_{yx}(\omega)]\{X(\omega)\}$$

↑  
*Output  
linear  
spectra*

↑  
*System  
FRF  
Matrix*

↑  
*Input  
linear  
spectra*

$$Y = N_{outputs} \times 1 \times N_{freqs}$$

$$X = N_{inputs} \times 1 \times N_{freqs}$$

$$H_{yx} = N_{outputs} \times N_{inputs} \times N_{freqs}$$

# Linear & Power Space

- Linear System: Linear- & power-space
  - Linear space = linear spectra with units of [g], [V], etc.
  - Power space = PSDs with units of [g<sup>2</sup>/Hz], [V<sup>2</sup>/Hz], etc.

*Power Space  
MIMO Equation*

$$[S_{yy}(\omega)] = [H_{yx}(\omega)][S_{xx}(\omega)][H_{yx}(\omega)]^H$$

↑                      ↑                      ↑                      ↑

*Output                      System                      Input                      FRF*  
*CPSD                      FRF                      CPSD                      Matrix*  
*Matrix                      Matrix                      Matrix                      Hermitian*

$[\cdot]^H = \text{conjugate transpose (Hermitian)}$

$S_{yy} = N_{\text{outputs}} \times N_{\text{outputs}} \times N_{\text{freqs}}$

$S_{xx} = N_{\text{inputs}} \times N_{\text{inputs}} \times N_{\text{freqs}}$

$H_{yx} = N_{\text{outputs}} \times N_{\text{inputs}} \times N_{\text{freqs}}$

# Linear & Power Space

- Linear System: Why use the power-space representation for MIMO random vibration?
  - Welch-method averaging can be used to get much better estimates of the input and output quantities, reducing the effects of noise and making the control much better behaved

*Linear Space  
MIMO Equation*

$$\{Y(\omega)\} = [H_{yx}(\omega)]\{X(\omega)\}$$

*Single frame only = poor estimate  
of true outputs or inputs*

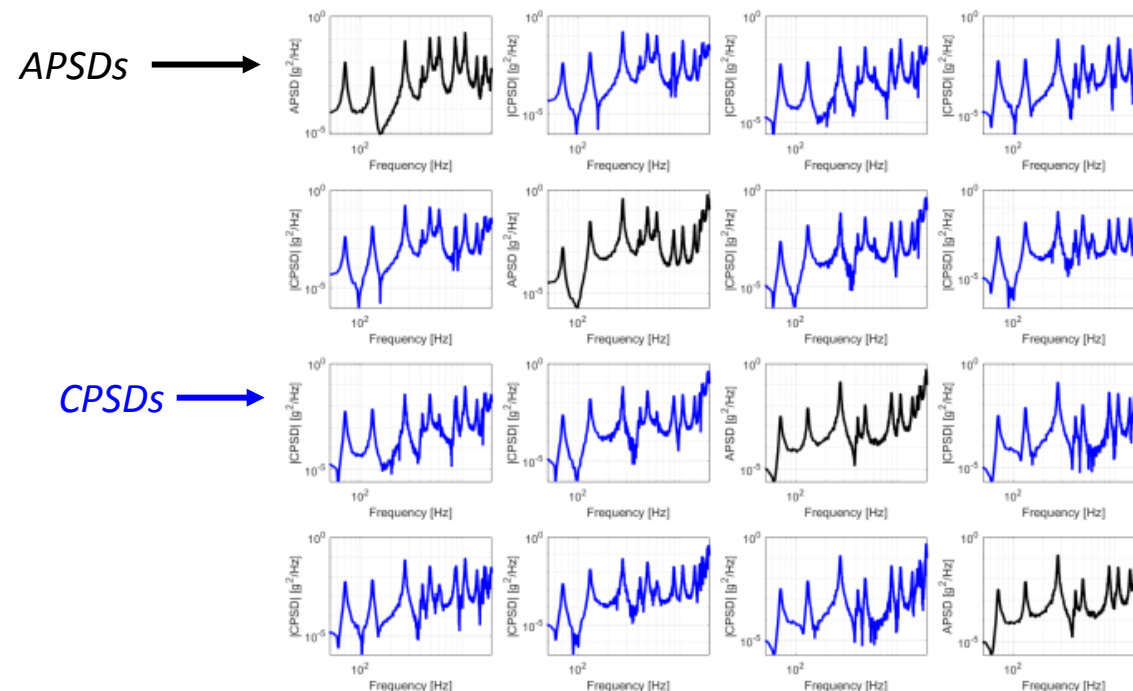
*Power Space  
MIMO Equation*

$$[S_{yy}(\omega)] = [H_{yx}(\omega)][S_{xx}(\omega)][H_{yx}(\omega)]^H$$

*Multiple frames averaged = good  
estimates of the true outputs or inputs*

# CPSD Matrices

- Cross-power spectra density (CPSD) matrices:
  - Square
  - Hermitian (conjugate symmetric)
  - Positive-definite (all positive eigenvalues)
  - Diagonals: auto-power spectral densities (APSDs)
  - Off-diagonals or cross-terms: blend of i-th and j-th DOF APSD and the coherence and phase between them



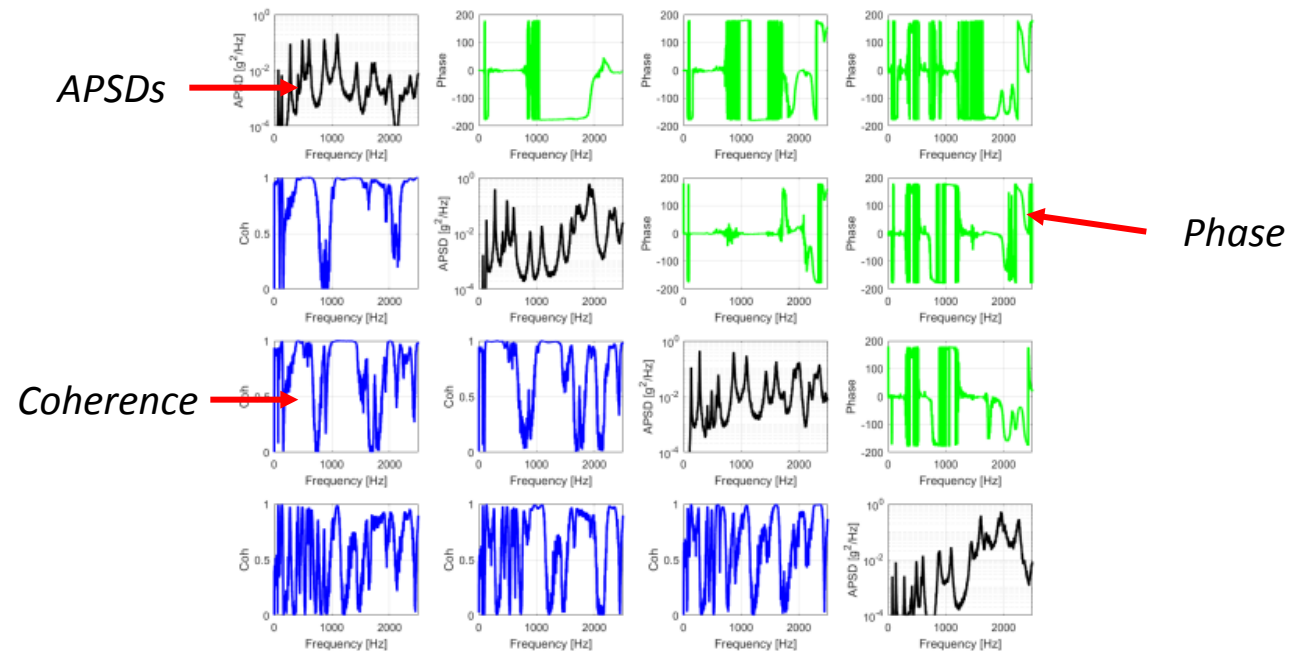
# CPSD Matrices

- Cross-power spectra density (CPSD) matrices:
  - CPSD is an averaged outer product of linear spectra:

$$[S_{yy}] = d_f\{Y\}\{Y\}^H$$

- Outer product means all CPSDs must be Hermitian positive definite
- Decompose CPSD in terms of APSD, coherence and phase:

$$S_{ij} = \sqrt{\gamma_{ij}^2 S_{ii} S_{jj}} e^{j\phi_{ij}}$$



# General Assumptions

- The system (DUT) and any amplifier or shaker equipment in the load path is linear
  - Output scales with input in a linear fashion
  - Amplifier is a linear system, shakers and shaker attachments are linear systems
- The system is time invariant
  - The output/input relationships do not change with time
  - Examples: bungee sag, temperature dependence, etc.
- Inputs to the system come from the controller and are measurable
  - There are no (strong) external sources
  - The drive voltages are measurable

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Terminology  
MIMO random vibration  
theory  
**MIMO control theory**  
Examining MDOF response  
data

# MIMO Control Theory

# MIMO Control Theory

**Objective: Determine MIMO test inputs which make the DUT vibrate like the specification**

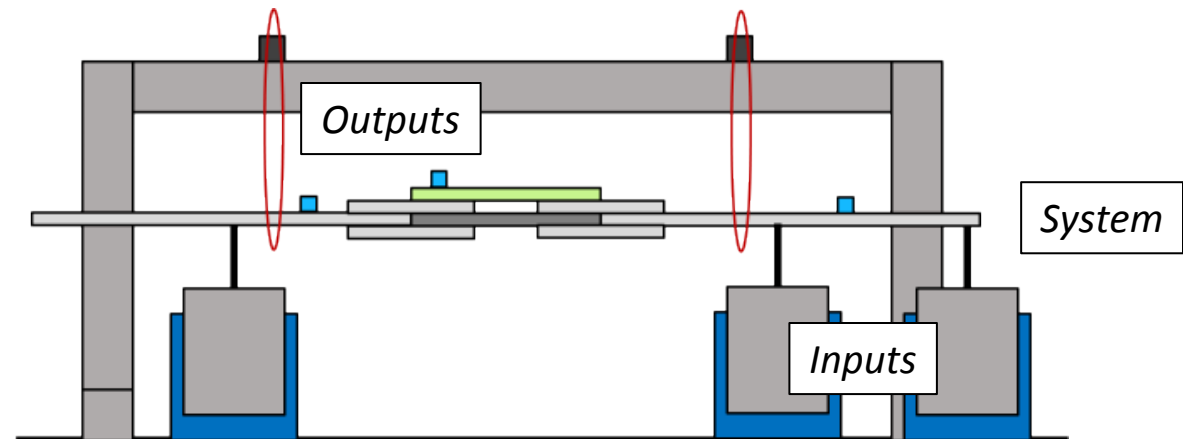
- Meaning: Solve an inverse problem using the MIMO test FRFs and the spec CPSD matrix
- Result: Shaker drive voltage CPSD matrix which should minimize the output response error

$$[S_{yy}(\omega)] = [H_{yx}(\omega)][S_{xx}(\omega)][H_{yx}(\omega)]^H$$

Known  
(spec)

Known  
(measured)

Unknown





# MIMO Control Theory

- Solve an inverse problem using the MIMO test FRFs and the spec CPSD matrix
  - Inverse problem: Input estimation, force estimation, control solution
  - At each frequency line, solve a direct inverse problem
  - Pseudo-inverse results in a least-squares solution, inputs which cause the smallest error between test output CPSD and spec CPSD in a least-squares sense
  - *Note: pseudo-inverse solution does not care what inputs result, only that they are inputs which minimize response error*

$$[S_{xx,T}(\omega)] = [H_{yx}(\omega)]^+ [S_{yy,S}(\omega)] [H_{yx}(\omega)]^{+H}$$

$[\cdot]^+ = \text{pseudo-inverse}$

$S_{yy,S} = \text{spec output CPSD}$


$S_{xx,T} = \text{test input CPSD}$

$H_{yx} = \text{test FRF matrix}$

# MIMO Control Theory

- Closed loop vs. Open loop control
  - Both begin with the same MIMO control solution. The difference is how the inputs are adjusted during the test
  - Open loop: estimate inputs and FRFs once prior to the test
  - Closed loop: update the inputs and FRFs during the test to minimize output error

$$\begin{array}{l}
 \text{Open Loop} \left\{ \begin{array}{l} [S_{xx,T}(\omega)] = [H_{yx}(\omega)]^+ [S_{yy,S}(\omega)] [H_{yx}(\omega)]^{+H} \\ \\ [S_{yy,E}(\omega)] = [S_{yy,S}(\omega)] - [S_{yy,T}(\omega)] \\ \\ [S_{xx,E}(\omega)] = [H_{yx}(\omega)]^+ [S_{yy,E}(\omega)] [H_{yx}(\omega)]^{+H} \\ \\ [S_{xx,U}(\omega)] = [S_{xx,T}(\omega)] + \alpha [S_{xx,E}(\omega)] \end{array} \right. \\
 \text{Closed Loop} \left\{ \begin{array}{l} \\ \\ \\ \\ \end{array} \right.
 \end{array}$$


 Weighted drive correction

# MIMO Control Theory

- Assumptions and limitations with the direct, pseudo-inverse MIMO control solution:
  - Pseudo-inverse solution only cares about minimizing output response error, not about drive limiting, minimizing forces, etc.
  - It is minimizing the error on the entire CPSD matrix – not just the APSDs but the cross-terms as well (there are way more cross-terms)
  - Effects of test design all get buried into the FRF matrix: choice of input and output DOFs, setup and BCs, etc.
    - These change the controllable space and the output/input efficiency of the MIMO test
    - It isn't clear what to change given some test results or predictions (i.e. where to move a shaker)
  - Not many “knobs to turn” with this solution – no way to emphasize or ignore different control DOFs, no way to put more force on one shaker vs. another

$$[S_{xx,T}(\omega)] = [H_{yx}(\omega)]^+ [S_{yy,S}(\omega)] [H_{yx}(\omega)]^{+H}$$

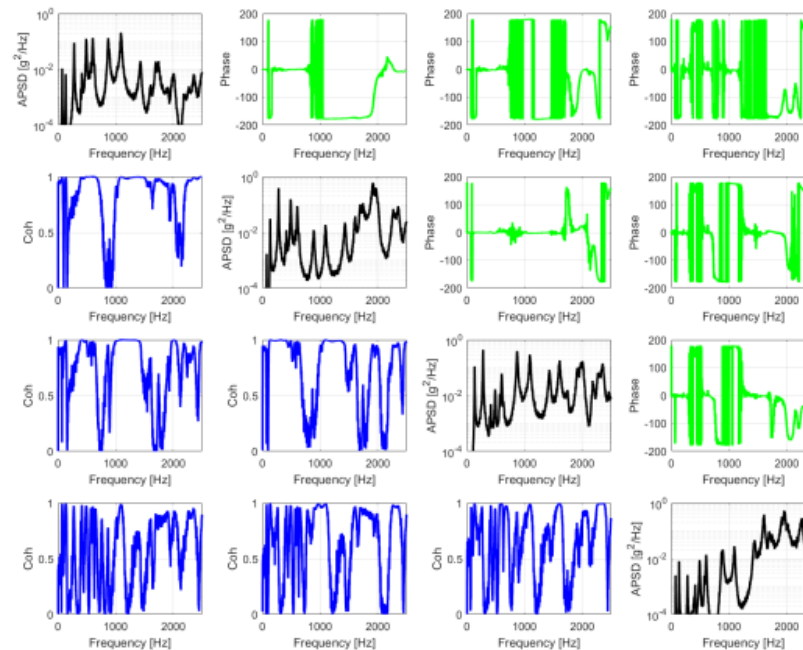
What is MDOF testing?  
Why is it good?  
Who is it for?  
Who is it not for?  
Terminology  
MIMO random vibration  
theory  
MIMO control theory  
**Examining MDOF response  
data**

# Examining MDOF Response Data

# Examining MDOF Response Data

- MDOF = multiple output DOFs
  - APSDs for each DOF ( $N_{outputs}$ )
  - CPSDs relating each DOF to every other DOF ( $N_{outputs}^2$ )
- This gets to be a lot of data for even modest numbers of output DOFs

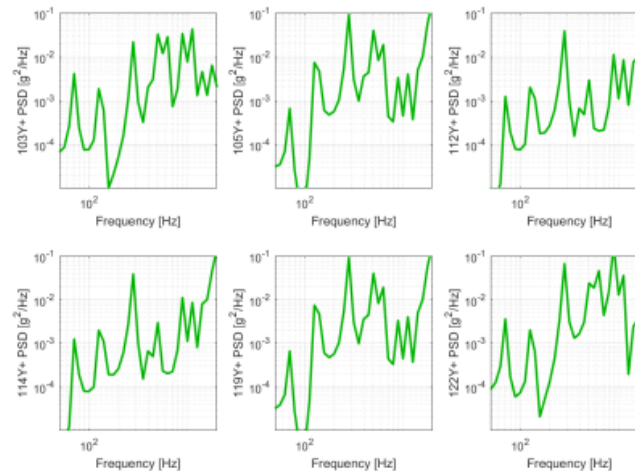
*CPSD matrix for 4 DOF test*



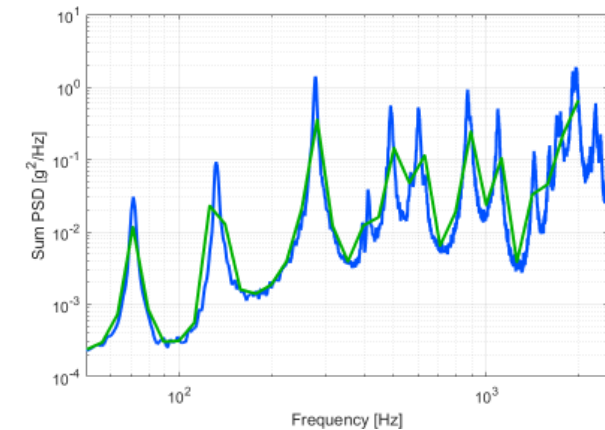
# Examining MDOF Response Data

- Generally convenient or necessary to condense down the output data to enable useful comparisons of test vs. spec
- Can be helpful to plot in terms of log-frequency or octave-averaged frequency

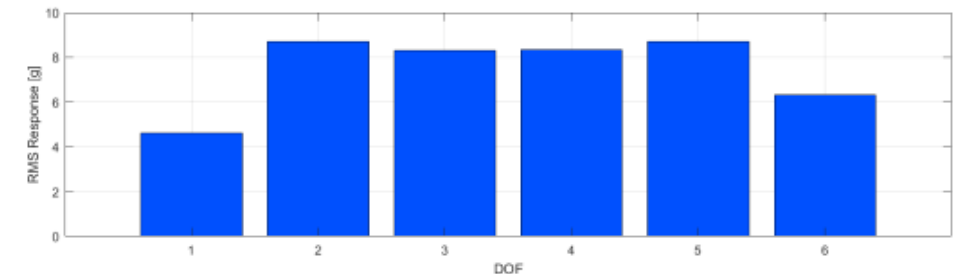
*APSDs for each DOF*



*Sum of APSDs*

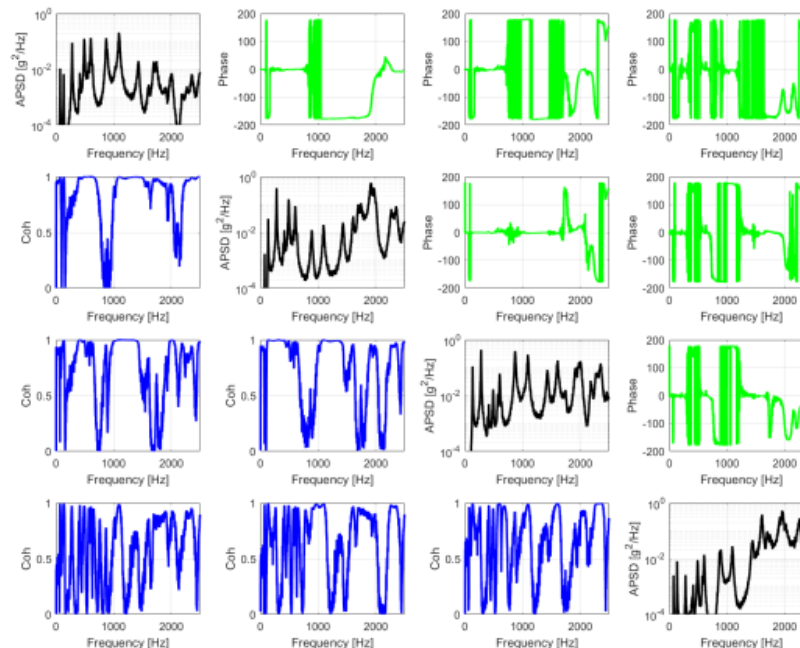


*RMS for each DOF*



# Examining MDOF Response Data

- What about phase?
- What about coherence?
- Are they important?
  - Yes, they are important
  - But it's  $N^2$  curves, and what does some error in coherence or phase mean?
  - We need to gain experience assessing phase and coherences so we can make useful assessments and not just ignore them



What is MDOF testing?  
Why is it good?  
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Who is it not for?  
Terminology  
MIMO random vibration  
theory  
MIMO control theory  
Examining MDOF response  
data

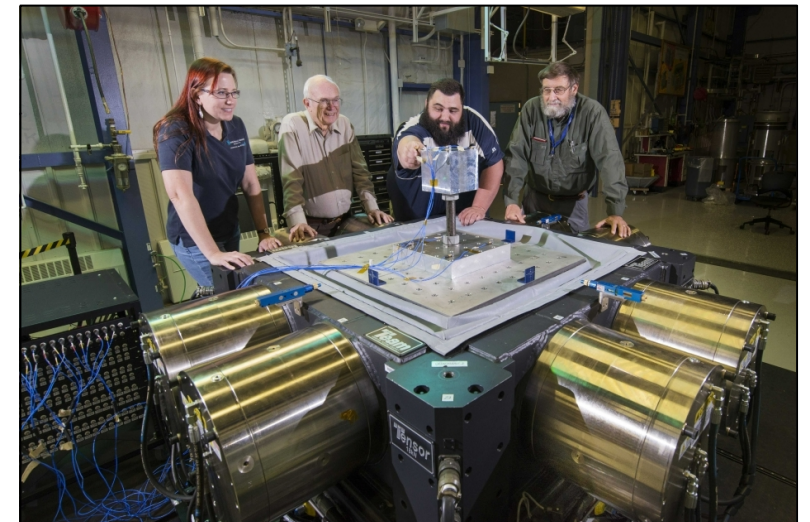
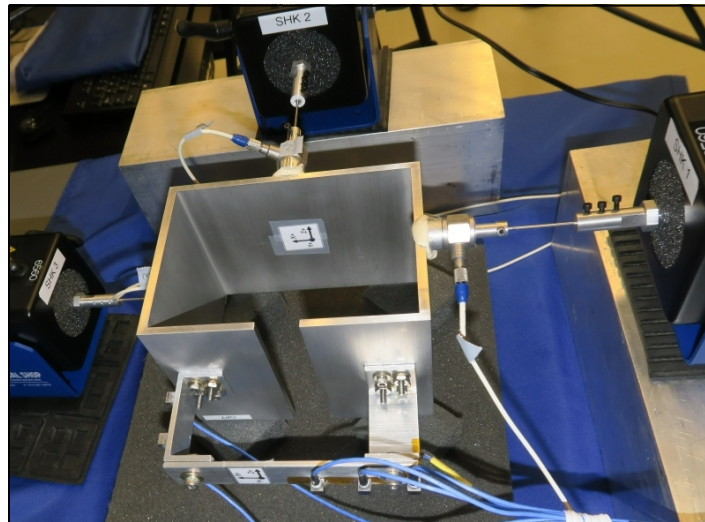
## Overview Wrap-Up



# Overview Wrap-Up

- Takeaways:
  - MDOF, MIMO, IMMAT, 6DOF are all slightly different versions of the same type of test
  - MIMO random vibration utilizes CPSD and FRF matrices
  - MIMO control relies on a pseudo-inverse direct solution
  - Assessing test results is challenging due to the CPSD form

$$[S_{xx,T}(\omega)] = [H_{yx}(\omega)]^+ [S_{yy,S}(\omega)] [H_{yx}(\omega)]^{+H}$$



# Schedule for Today

## Objective:

***Introduce MDOF Vibration Testing Concepts, Show You How it Works, and Discuss Why it is Useful***

| Section                    | Time          |
|----------------------------|---------------|
| Introduction               | 8:00 – 8:30   |
| General MDOF Overview      | 8:30 – 9:15   |
| Field vs. Lab Environments | 9:15 – 10:15  |
| Break                      | 10:15 – 10:30 |
| Example Problem            | 10:30 – 11:15 |
| Demo 1: Single-Axis Test   | 11:15 – 12:00 |
| Lunch                      | 12:00 – 1:00  |

| Section                   | Time        |
|---------------------------|-------------|
| Demo 2: Multi-Shaker Test | 1:00 – 2:00 |
| Test Design Methods       | 2:00 – 2:30 |
| Break                     | 2:30 – 2:45 |
| Rattlesnake Controller    | 2:45 – 3:30 |
| Common Issues             | 3:30 – 4:00 |
| 6DOF & 3DOF Testing       | 4:00 – 5:00 |
| Wrap-Up                   | 5:00 – 5:30 |

# Example Problem

*IMAC XL Short Course*

*Feb. 5<sup>th</sup> 2022*

*Ryan Schultz*

# Example Problem

## Objective:

***Create a simple but useful MIMO system and use it to demonstrate concepts and techniques in MIMO testing***

- Example “missile on a wing” system
- Model & test versions
- Multiple configurations (BCs)
- Field environment
- Simulating a MIMO test
- Choosing output and input DOFs
- Understanding effects of test BCs

## **Example system**

Model & test versions

Multiple configurations

Field environment

Simulating a MIMO test

Selecting output DOFs

Selecting input DOFs

Effects of MIMO test BCs

# **Example System: Missile on a Wing (or a beam on a plate)**

# Example System

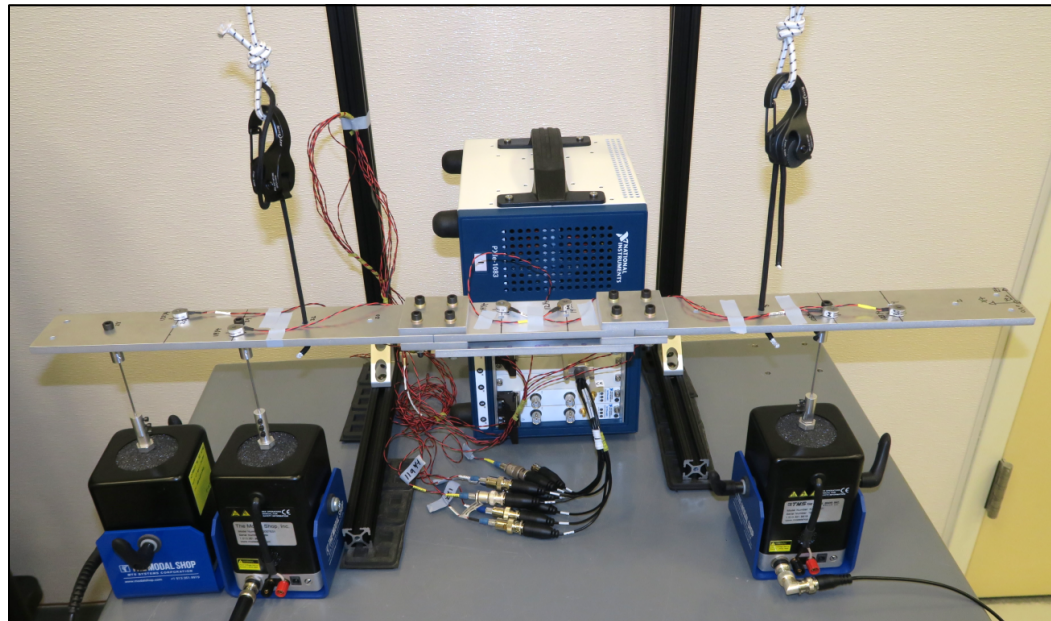
Desired features of this example system:

- Linear
- Simple & cheap
- Modular (easy BC changes)
- Simple setup
- Sensitive to changes

# Example System

Desired features of this example system:

- Linear
- Simple & cheap
- Modular (easy BC changes)
- Simple setup
- Sensitive to changes



Example system

**Model & test versions**

Multiple configurations

Field environment

Simulating a MIMO test

Selecting output DOFs

Selecting input DOFs

Effects of MIMO test BCs

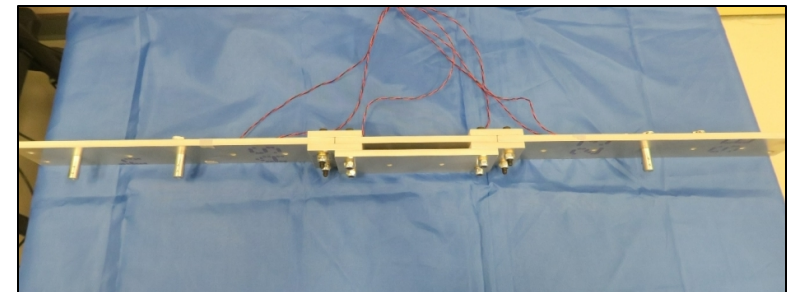
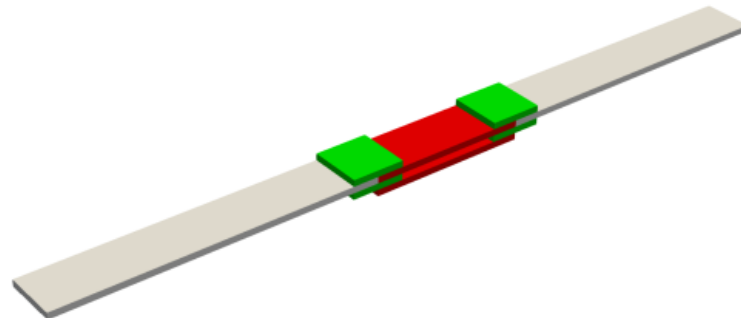
# Model & Test Versions



# Model & Test Versions of the Example System

Why have both model & test versions?

- In General:
  - Utilize the model to learn how to test
  - Understand how the test should work and how it should be run
  - Figure out metrics, etc. before you get to the lab
- Here:
  - Make it easier to iterate on configurations
  - Needed to figure out how this would look



Example system

Model & test versions

**Multiple configurations**

Field environment

Simulating a MIMO test

Selecting output DOFs

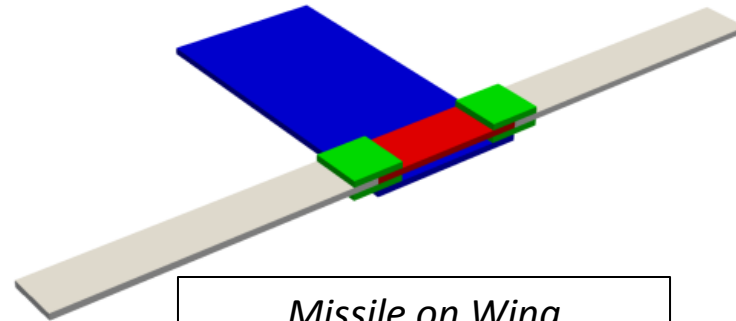
Selecting input DOFs

Effects of MIMO test BCs

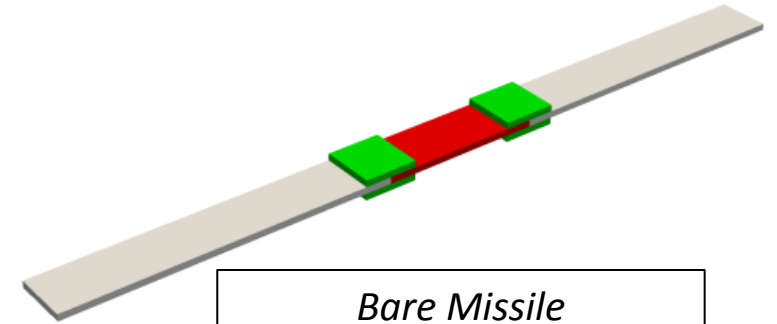
# Multiple Configurations

# Multiple Configurations

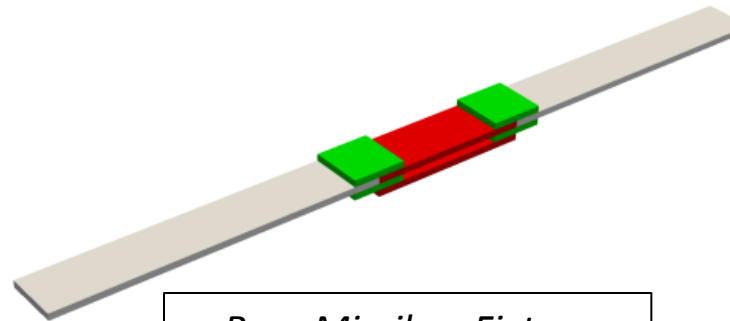
Simple assembly with variable system dynamics and boundary conditions



*Missile on Wing  
(Field Test Configuration)*



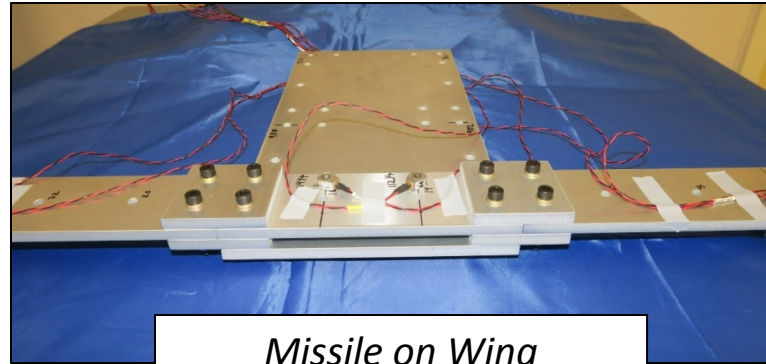
*Bare Missile  
(Lab Test Configuration 1)*



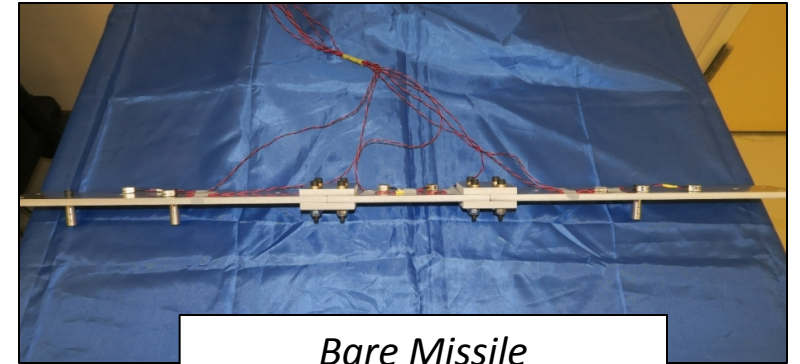
*Bare Missile + Fixture  
(Lab Test Configuration 2)*

# Multiple Configurations

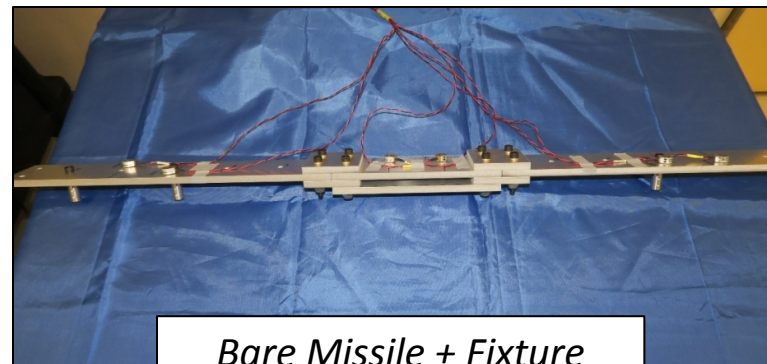
Simple bolted assembly enables quick change of configuration from field to lab to lab+fixture



*Missile on Wing  
(Field Test Configuration)*



*Bare Missile  
(Lab Test Configuration 1)*

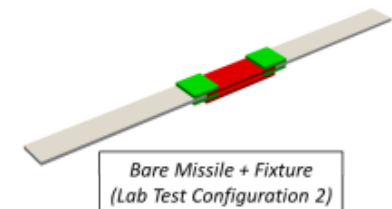
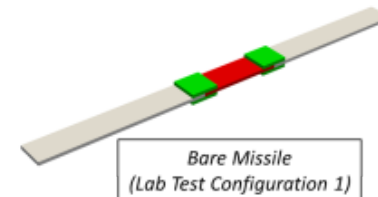
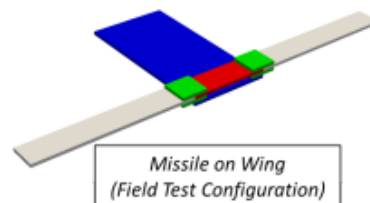
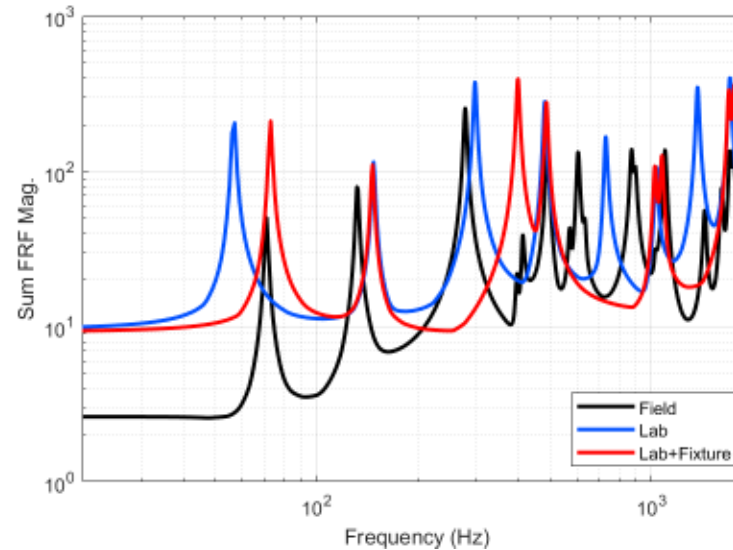


*Bare Missile + Fixture  
(Lab Test Configuration 2)*

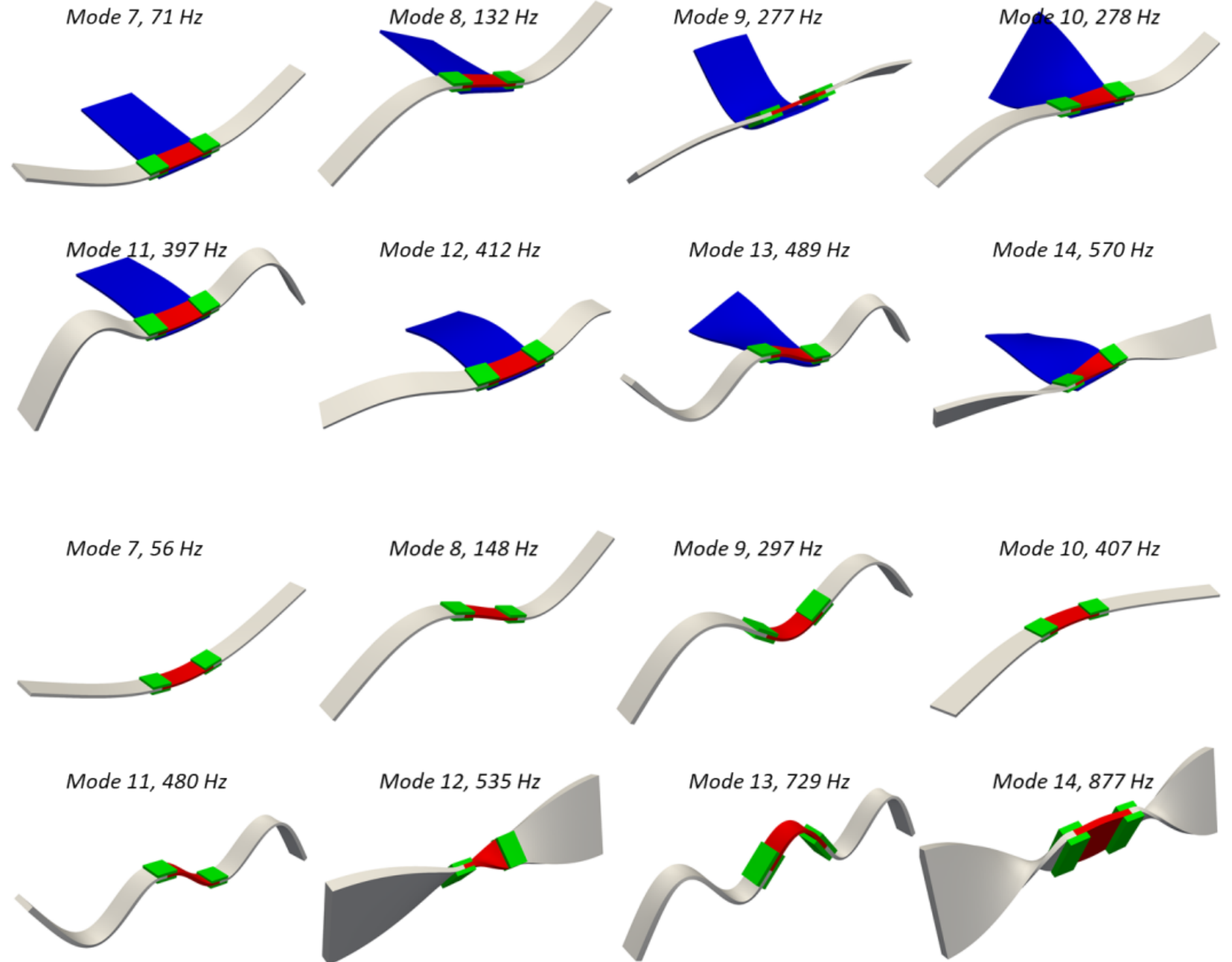
# Multiple Configurations

How different are the configurations?

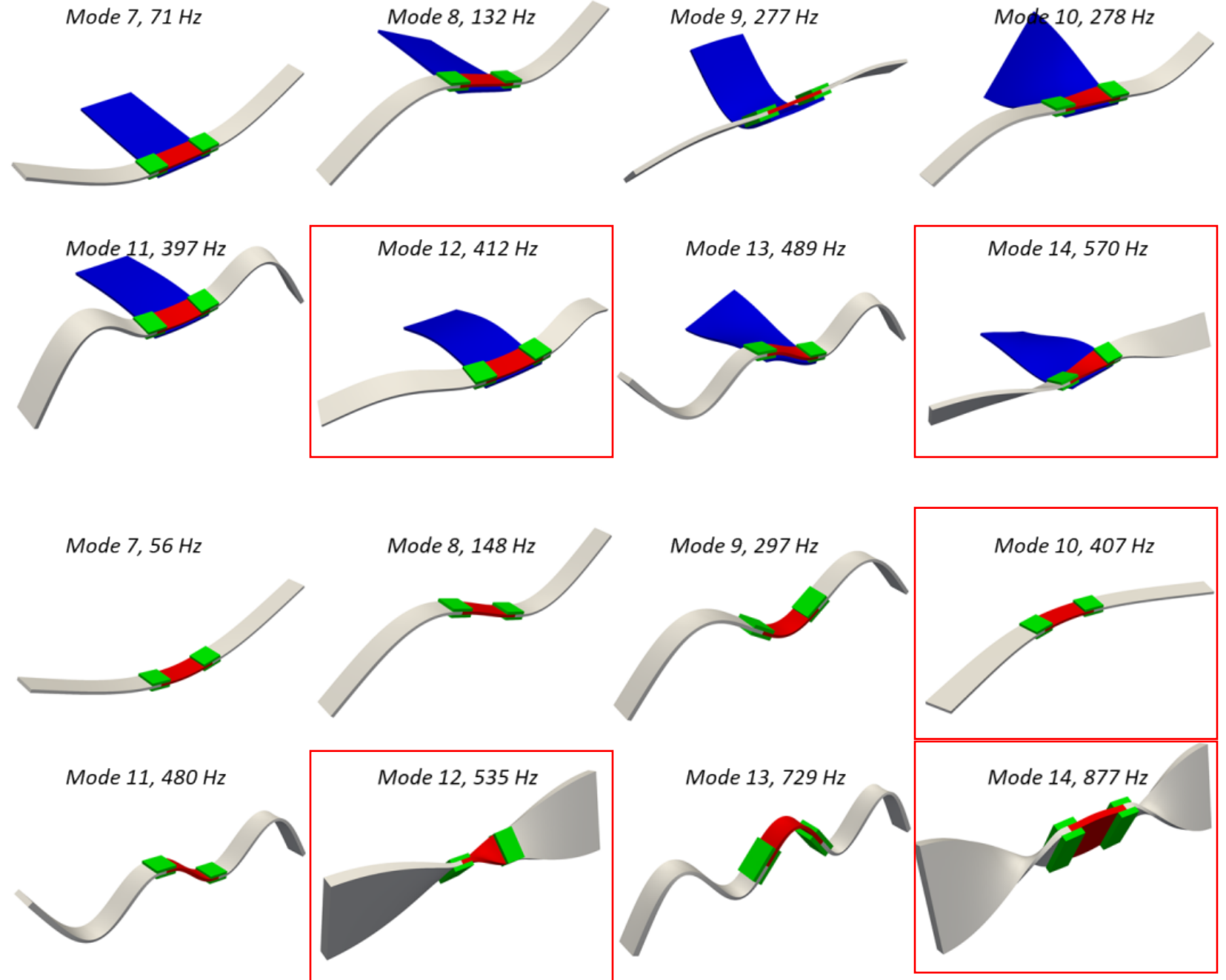
- Simplified view: sum FRF for 6 output, 2 inputs on each of the 3 configurations
- See the addition of the fixture makes the first mode match better, but doesn't make it perfect



# Modes of Field and Lab Configurations



# Modes of Field and Lab Configurations



Example system

Model & test versions

Multiple configurations

**Field environment**

Simulating a MIMO test

Selecting output DOFs

Selecting input DOFs

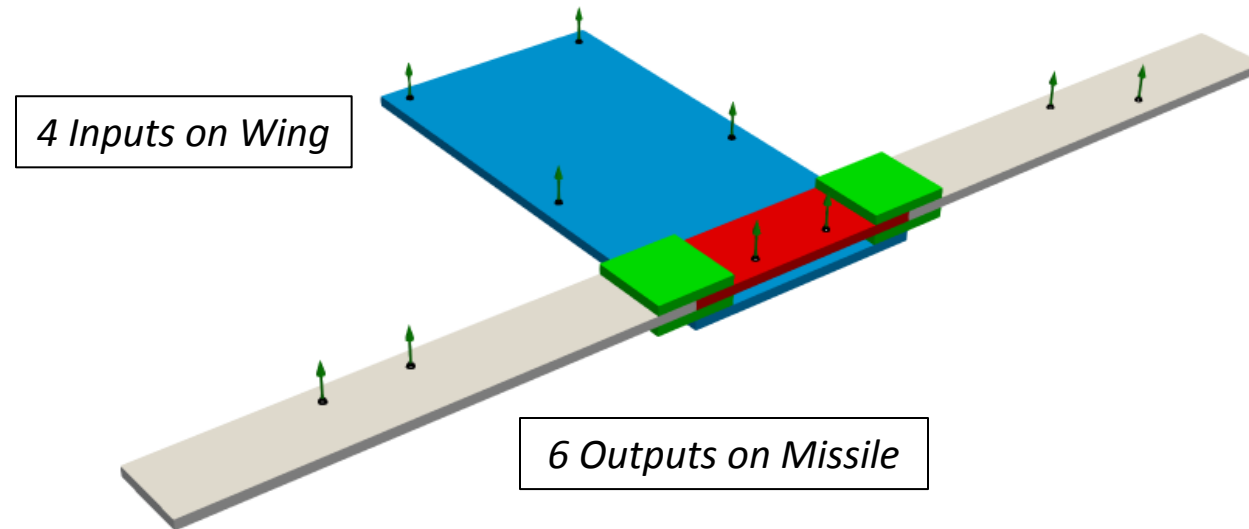
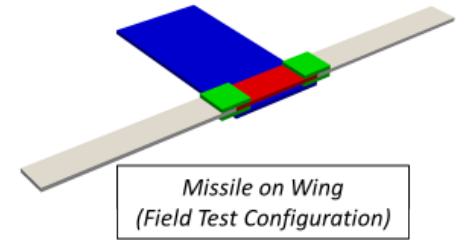
Effects of MIMO test BCs

# Field Environment



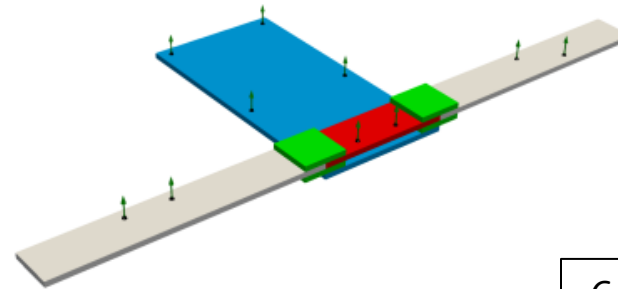
# Field Environment

- Configuration: Missile on the wing
- Multiple inputs on the wing
  - Load path: Shaker-Wing-Missile

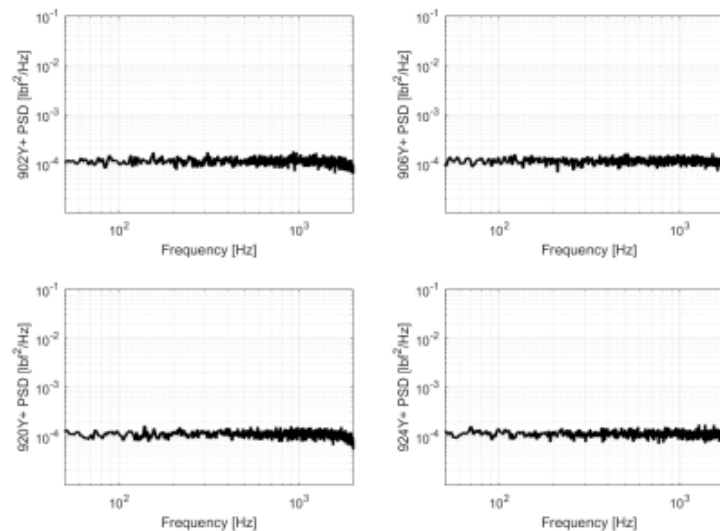


# Simulated Field Environment

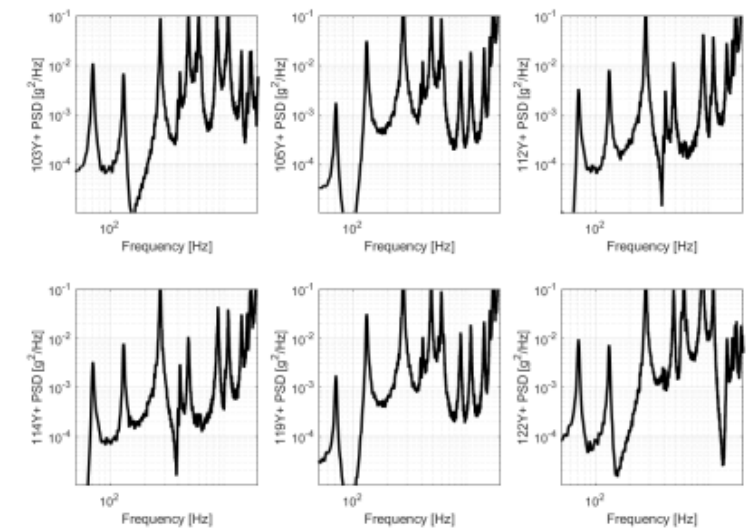
- Inputs simulated as 4 uncorrelated, constant-amplitude, broadband forces
- Modal transient simulation with added noise
- Response is rich: broadband with some differences DOF to DOF



*4 Inputs on Wing*

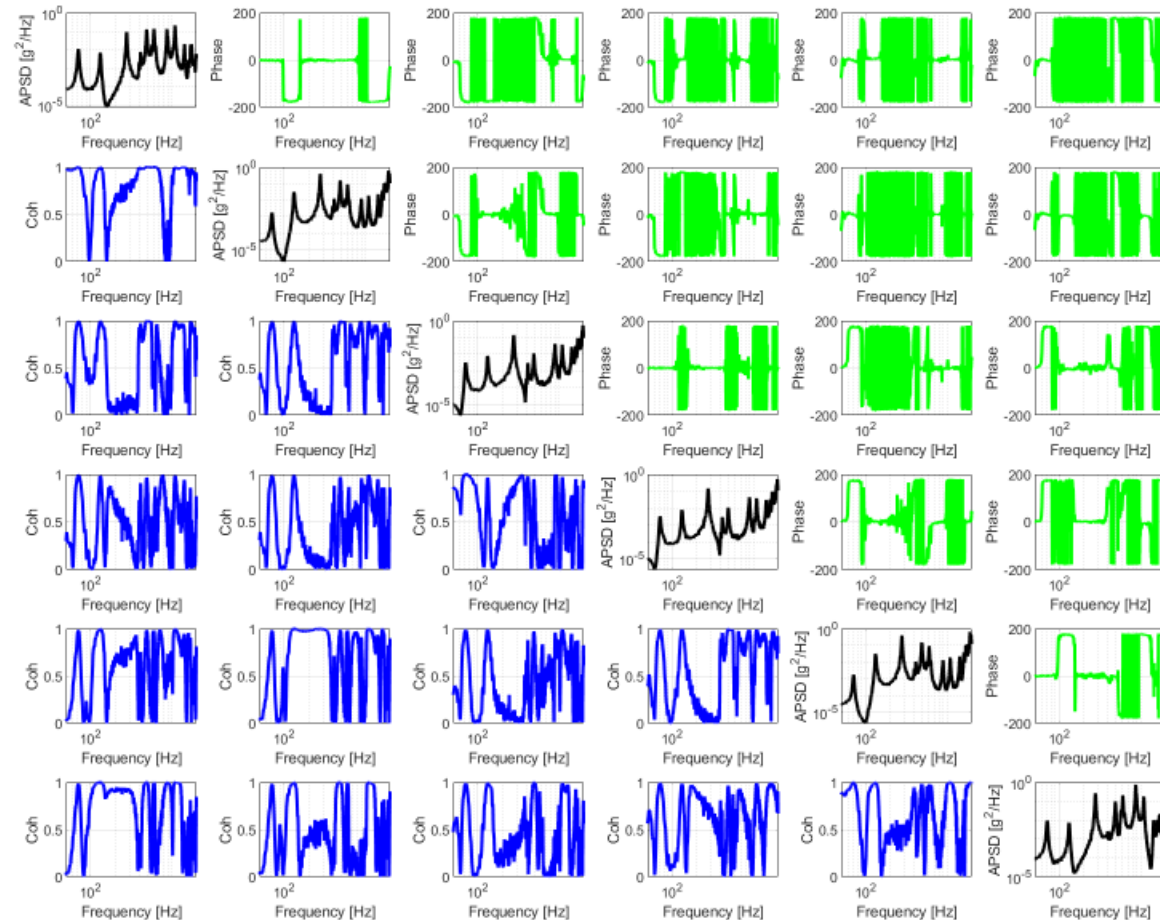


*6 Outputs on Missile*



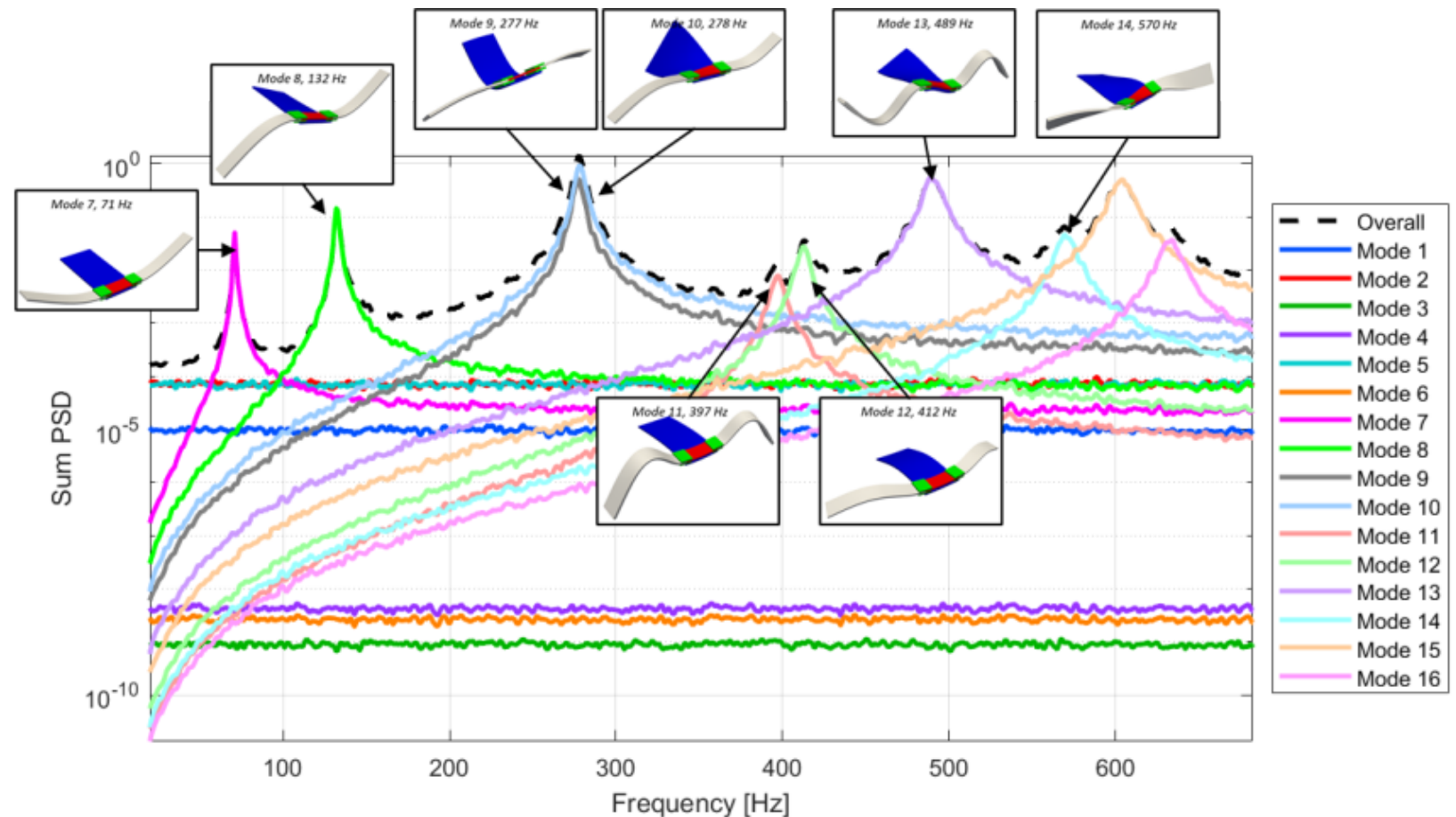
# Simulated Field Environment

- 6 responses = 6x6 CPSD matrix
- This is the MIMO test specification



# Field Response Mode-by-Mode

- Overall response is a combination of responses from the various modes
  - Some modes are not well excited by the input forces and load paths



Example system

Model & test versions

Multiple configurations

Field environment

**Simulating a MIMO test**

Selecting output DOFs

Selecting input DOFs

Effects of MIMO test BCs

# Simulating a MIMO Test

# Simulating a MIMO Test

## How to Simulate a MIMO Test

- Assuming a nice linear system
- Just need FRF matrix and spec CPSD matrix

- Run the Test

- Forward problem (given inputs, get outputs):

$$[S_{yy}(\omega)] = [H_{yx}(\omega)][S_{xx}(\omega)][H_{yx}(\omega)]^H$$

- MIMO Control

- Inverse problem (given outputs, get inputs):

$$[S_{xx,T}(\omega)] = [H_{yx}(\omega)]^+ [S_{yy,S}(\omega)][H_{yx}(\omega)]^{+H}$$

# Simulating a MIMO Test

## How to Simulate a MIMO Test

- Control with different DOFs (control set vs. validation set)
- Different FRF matrices with different output DOFs:

- Control:  $[H_{yx,C}]$

- Validation:  $[H_{yx,V}]$

- Total set:  $[H_{yx,T}] = \begin{bmatrix} [H_{yx,C}] \\ [H_{yx,V}] \end{bmatrix}$

- Inverse problem (given outputs, get inputs):

$$[S_{xx}] = [H_{yx,C}]^+ [S_{yy,C}] [H_{yx,C}]^{+H}$$

- Forward problem (given inputs, get outputs):

$$[S_{yy,T}] = [H_{yx,T}] [S_{xx}] [H_{yx,T}]^H$$

# Simulating a MIMO Test

## Example Code

```
% SOLVE MIMO CONTROL:
for index=1:Nfreqs
    % GRAB SPEC CPSD FOR THIS FREQ LINE:
    SyySpec_i = SyySpec(:, :, index) ;

    % GRAB FRF FOR THIS FREQ LINE:
    Hyx_i = Hyx(:, :, index) ;

    % TAKE PINV OF FRF MATRIX:
    pinvHyx_i = pinv(Hyx_i) ;

    % ESTIMATE INPUTS TO BEST MATCH SPEC OUTPUTS:
    SxxTest_i = pinvHyx_i*SyySpec_i*pinvHyx_i' ;

    % PREDICT OUTPUTS GIVEN THESE INPUTS:
    SyyTest_i = Hyx_i*SxxTest_i*Hyx_i' ;

    % STORE THESE RESULTS:
    SxxTest(:, :, index) = SxxTest_i ;
    SyyTest(:, :, index) = SyyTest_i ;
end ; clear index
```



# Simulating a MIMO Test

Simple, right? Where to add complexity:

```
% SOLVE MIMO CONTROL:
```

```
for index=1:Nfreqs
```

```
    % GRAB SPEC CPSD FOR THIS FREQ LINE:
```

```
    SyySpec_i = SyySpec(:, :, index) ;
```

*Modify the Cross-Terms  
(Buzz Method)*

```
    % GRAB FRF FOR THIS FREQ LINE:
```

```
    Hyx_i = Hyx(:, :, index) ;
```

```
    % TAKE PINV OF FRF MATRIX:
```

```
    pinvHyx_i = pinv(Hyx_i) ;
```

*Change the pinv Method  
(Regularization)*

```
    % ESTIMATE INPUTS TO BEST MATCH SPEC OUTPUTS:
```

```
    SxxTest_i = pinvHyx_i*SyySpec_i*pinvHyx_i' ;
```

```
    % PREDICT OUTPUTS GIVEN THESE INPUTS:
```

```
    SyyTest_i = Hyx_i*SxxTest_i*Hyx_i' ;
```

*Error Corrections  
(Closed Loop)*

```
    % STORE THESE RESULTS:
```

```
    SxxTest(:, :, index) = SxxTest_i ;
```

```
    SyyTest(:, :, index) = SyyTest_i ;
```

```
end ; clear index
```

# Simulating a MIMO Test

What do we get from a simulated MIMO test?

- Input predictions
- Output predictions
- Control accuracy predictions

Why is this data helpful?

- Understanding required input levels
- Understanding control accuracy
- Determining which DOF are or are not important
- Determining which inputs are more important
- Set expectations for the test
- Likely won't be a perfectly accurate prediction, but general trends, effects will map to the test

# Simulating a MIMO Test

What don't we get from a simulated MIMO test?

- Nonlinear behavior
- Effects of test-specific noise
- Shaker coupling
- Shaker table dynamics
- Perfectly accurate predictions

Example system

Model & test versions

Multiple configurations

Field environment

Simulating a MIMO test

**Selecting output DOFs**

Selecting input DOFs

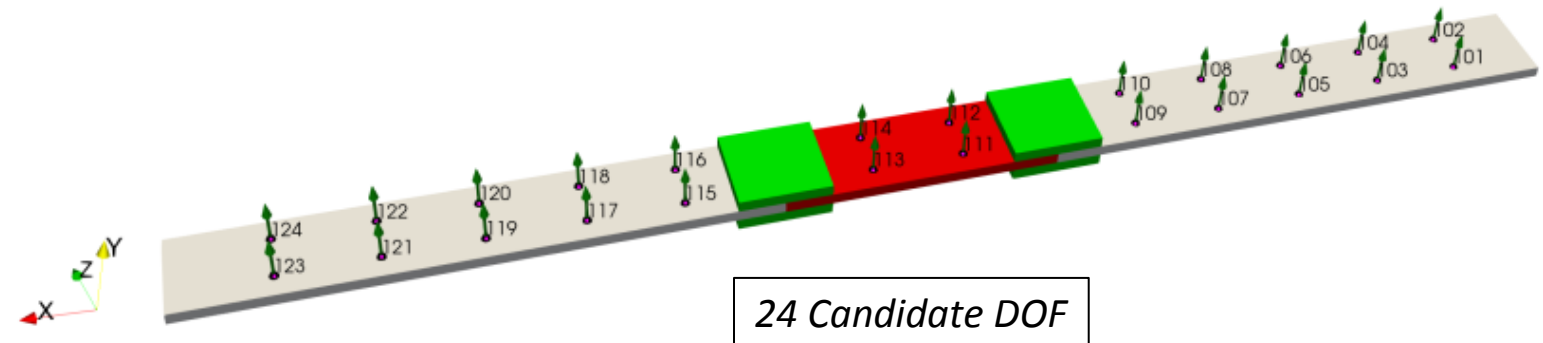
Effects of MIMO test BCs

# Selecting Output DOFs

# Selecting Output DOFs

Objective: Choose a small number of accelerometer locations which will enable good MIMO control of the DUT

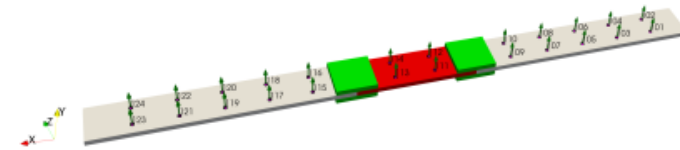
- Need to choose this instrumentation set prior to the field test since we need the spec at these test DOFs
- Many approaches exist to do this, here is just one example
- Approach:
  - Don't know the true field response, so just create some close-ish environment with the model (uncorrelated inputs to the middle 2 input DOFs)
  - Use the effective independence (EFI) method to remove one DOF at a time until the desired number of output DOF is reached
  - To choose the modes to use in the EFI method, the top M contributing modes were chosen ( $M \text{ modes} < N \text{ DOFs}$ ) based on the simulated response



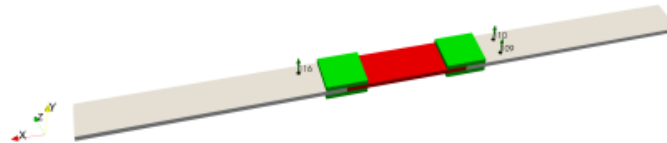
# Selecting Output DOFs

## Results: Selected output DOF using EFI

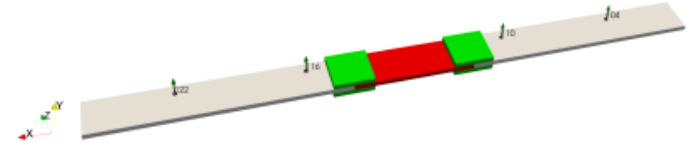
- The modal matrix used in the EFI approach changes based on the number of output DOF you want
- Chose optimized sets of 3, 4, 6, and 12 DOF



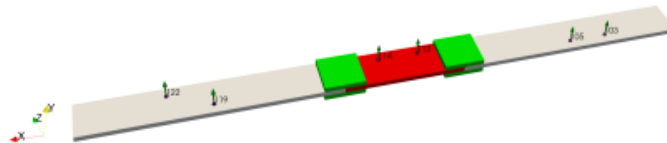
*24 Candidate DOF*



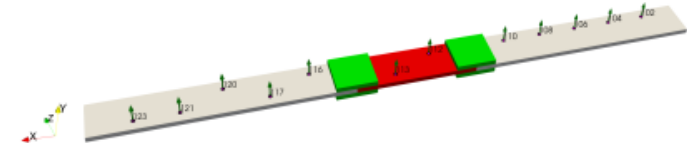
*3 Selected DOF*



*4 Selected DOF*



*6 Selected DOF*

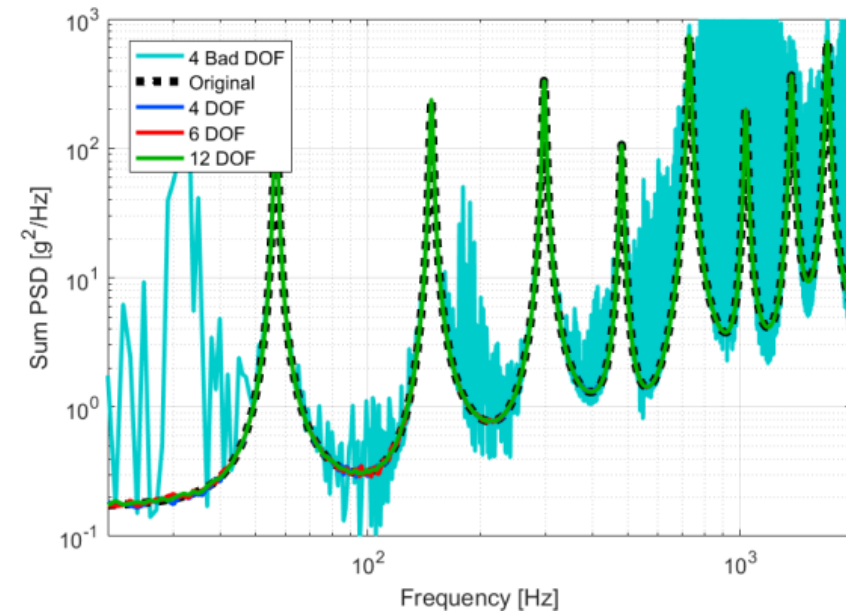
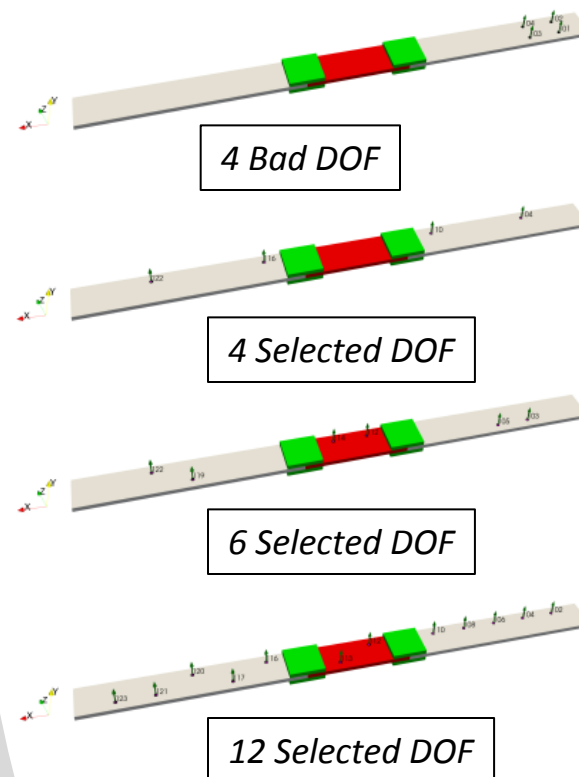


*12 Selected DOF*

# Comparing Results Using Different Output DOFs

Want to verify the chosen output (control) DOFs are sufficient to fully capture (observe) the response everywhere on the system

- Approach: Use transmissibility to predict the response at non-control output DOFs given the response at the selected control DOFs
- If the response prediction is good at non-control DOFs, then the control DOF set is likely sufficient for a MIMO test

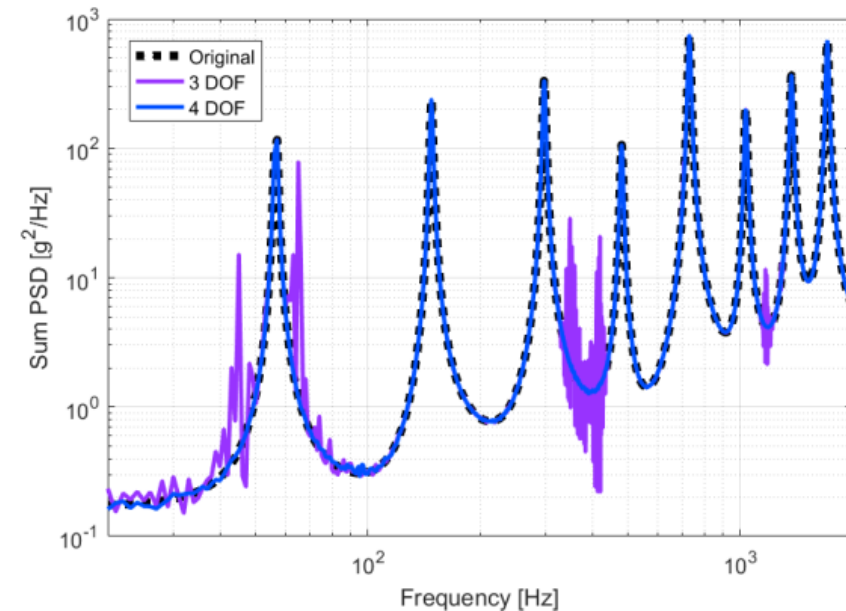
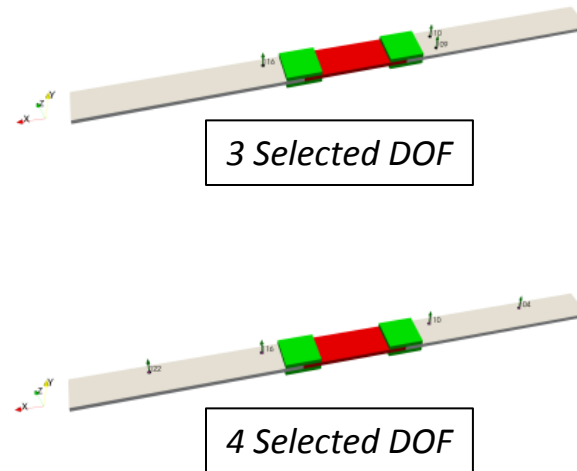


*Response predictions using transmissibilities and response at selected output DOFs*

# How Many Output DOFs Do You Need?

Some minimum number of good control DOF is required to be able to properly control the response

- This minimum number depends on various factors including the modal density, how the system is excited, noise, and independence of the output DOFs
- Here we see a sharp degradation in controllability when going from 4 to 3 control DOFs



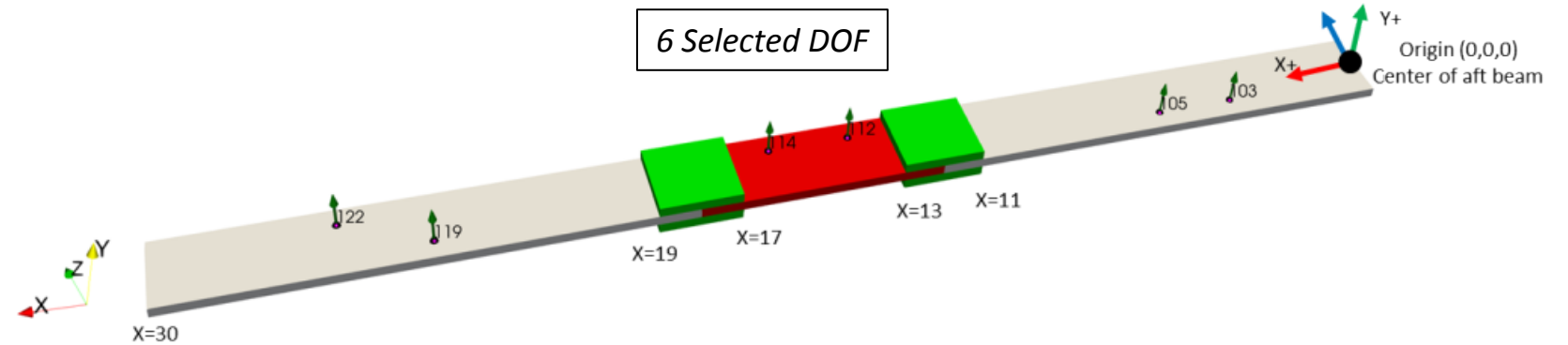
*Response predictions using transmissibilities and response at selected output DOFs*



# Selected Output DOFs

6 output DOF were selected for use on this system

- 4 looks pretty good, add 2 more just in case



Example system  
Model & test versions  
Multiple configurations  
Field environment  
Simulating a MIMO test  
Selecting output DOFs  
**Selecting input DOFs**  
Effects of MIMO test BCs

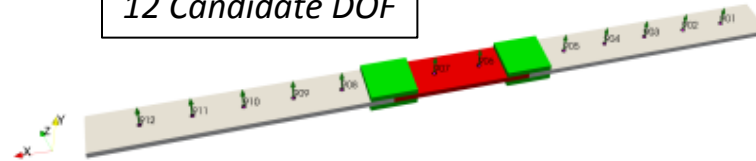
# Selecting Input DOFs

# Selecting Input DOFs

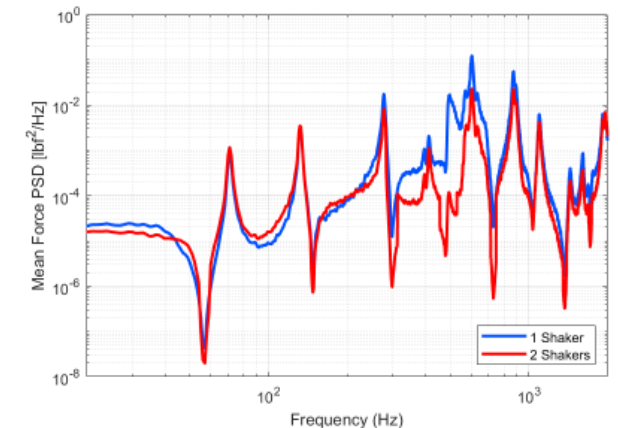
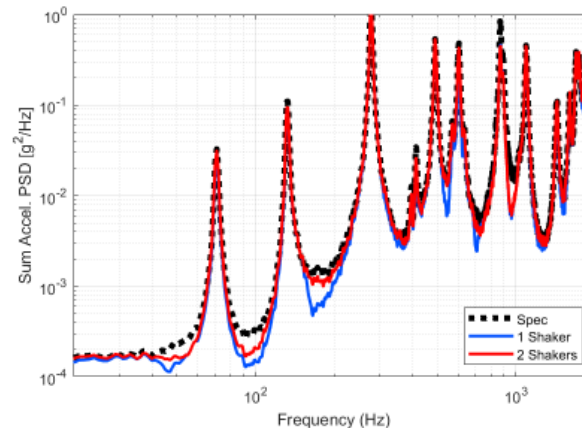
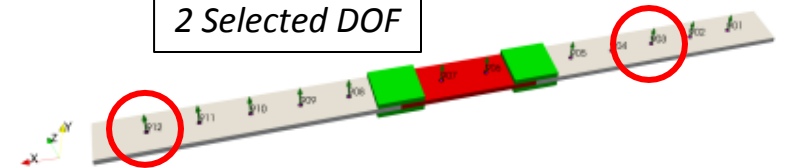
Objective: Choose shaker input locations which allow for an accurate replication of the field response as measured by the chosen output DOFs

- Pre-test design phase: Given the DUT (or a model of the DUT) and the field environment response specification, figure out how to run a good MIMO test
- Many possible methods to do this. Experience shows there are many good input sets, a few slightly better sets, and a few really bad sets
- Approach: Iteratively add one shaker at a time to minimize response error while keeping the shaker forces below a limit
- Simulate a MIMO test with each candidate set, compare results, choose the best shaker to add, then move to the next iteration

12 Candidate DOF



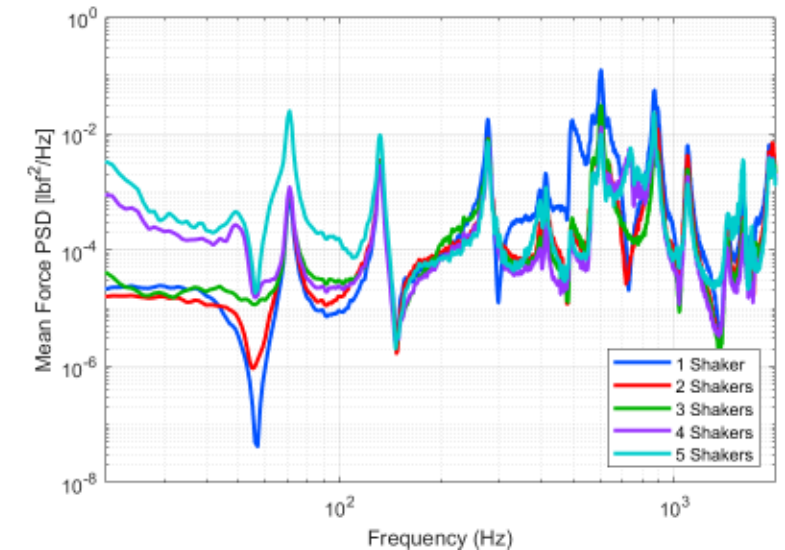
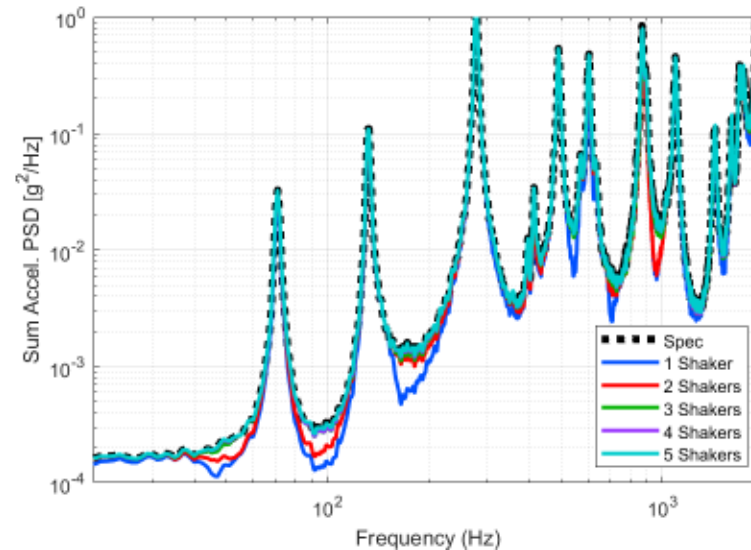
2 Selected DOF



# Selecting Input DOFs

Can continue to add more shakers and see how response accuracy and input forces trend with more inputs

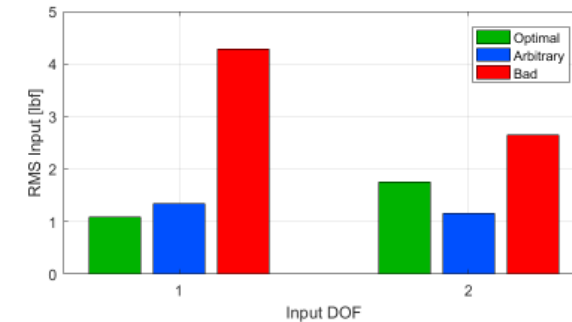
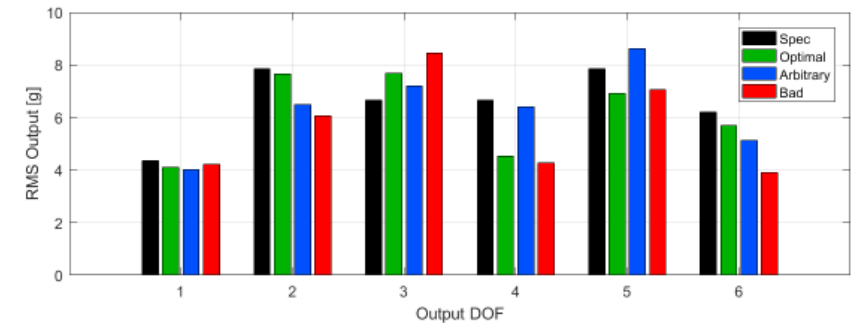
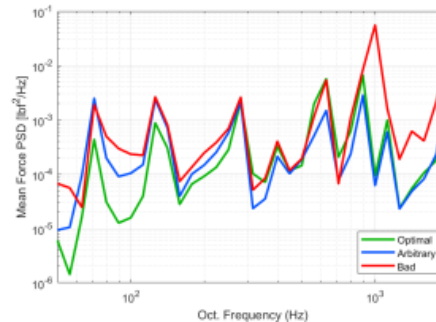
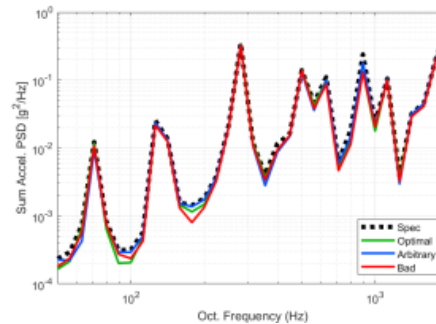
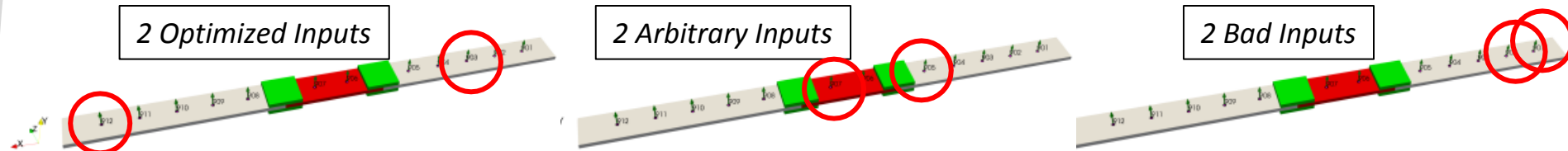
- Response accuracy (as viewed by the sum of PSDs) converges pretty quickly – adding additional shakers does not dramatically improve results
- Input force can actually increase when adding additional shakers
- More inputs = larger controllable space



# Good vs. Bad Input DOFs

Optimal input (shaker) locations should provide better results than randomly chosen locations, but how much better?

- What happens if a bad set of input DOFs are used?



# Selecting Input DOFs

## Things to note on shaker location selection

- Locations matter but they are not critical – there are many sets of locations with similar, good performance
- How you solve the MIMO control solution in the simulation will change the selected set (best is dependent on the problem setup and solution method)
- Required input force does not scale linearly with the number of shakers
- More shakers = larger controllable space = ability to more closely match cross-terms = high forces
- With more shakers a bad set can require tons of force to get a good solution

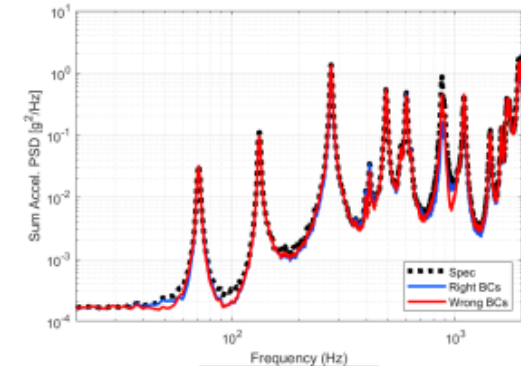
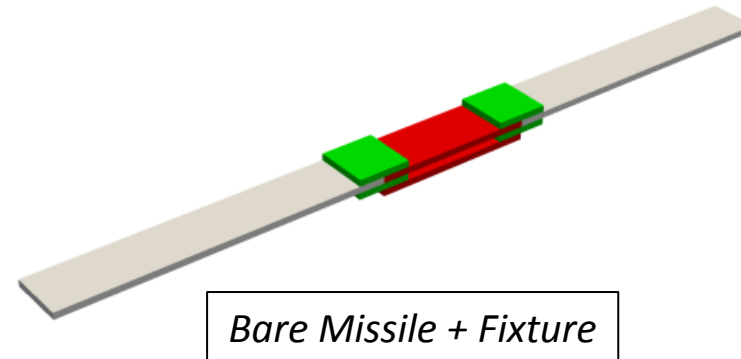
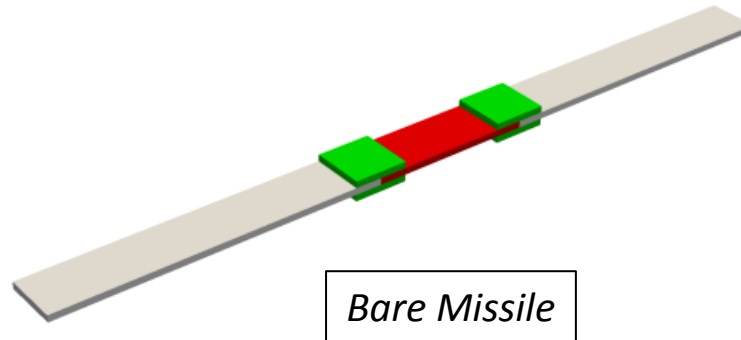
Example system  
Model & test versions  
Multiple configurations  
Field environment  
Simulating a MIMO test  
Selecting output DOFs  
Selecting input DOFs  
**Effects of MIMO test BCs**

# Effects of Boundary Conditions

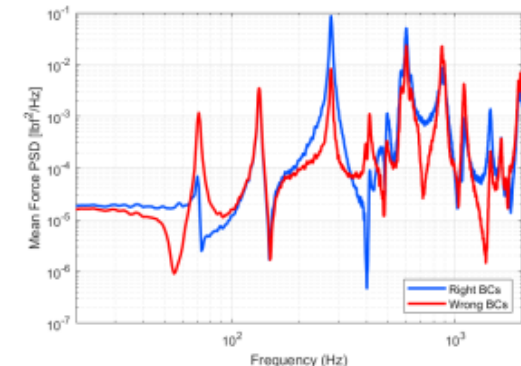
# Effects of MIMO Test Boundary Conditions

Boundary conditions matter. Getting them right should help the shakers accurately control the response

- Comparison of the 2 lab test configurations: bare missile and missile + fixture
- No impedance-matching design was attempted, just an approximation of the next level assembly



*Outputs*



*Inputs*



# Takeaways

## Objective:

***Create a simple but useful MIMO system and use it to demonstrate concepts and techniques in MIMO testing***

- While simple, this “missile on a wing” system is an effective demonstrator for many concepts in MIMO
- To run a test (or design a demo), input and output DOFs need to be chosen, and there are good DOF sets and bad DOF sets
- Using a model to do this design work is fast and effective

# Schedule for Today

## Objective:

***Introduce MDOF Vibration Testing Concepts, Show You How it Works, and Discuss Why it is Useful***

| Section                    | Time          |
|----------------------------|---------------|
| Introduction               | 8:00 – 8:30   |
| General MDOF Overview      | 8:30 – 9:15   |
| Field vs. Lab Environments | 9:15 – 10:15  |
| Break                      | 10:15 – 10:30 |
| Example Problem            | 10:30 – 11:15 |
| Demo 1: Single-Axis Test   | 11:15 – 12:00 |
| Lunch                      | 12:00 – 1:00  |

| Section                   | Time        |
|---------------------------|-------------|
| Demo 2: Multi-Shaker Test | 1:00 – 2:00 |
| Test Design Methods       | 2:00 – 2:30 |
| Break                     | 2:30 – 2:45 |
| Rattlesnake Controller    | 2:45 – 3:30 |
| Common Issues             | 3:30 – 4:00 |
| 6DOF & 3DOF Testing       | 4:00 – 5:00 |
| Wrap-Up                   | 5:00 – 5:30 |

# MIMO Test Design Methods

*IMAC XL Short Course*

*Feb. 5<sup>th</sup> 2022*

*Ryan Schultz*

# MIMO Test Design Methods

## Objective:

*Present some techniques for choosing output and input DOFs and other aspects of test design to help make for a better, easier MIMO test*

- MIMO test design parameters
- Output DOF selection methods
- Input DOF selection methods
- Control solution method selection
- Put it all together: Simulate the test

## **MIMO test parameters**

Why design a MIMO test?

Test design techniques

Things to consider

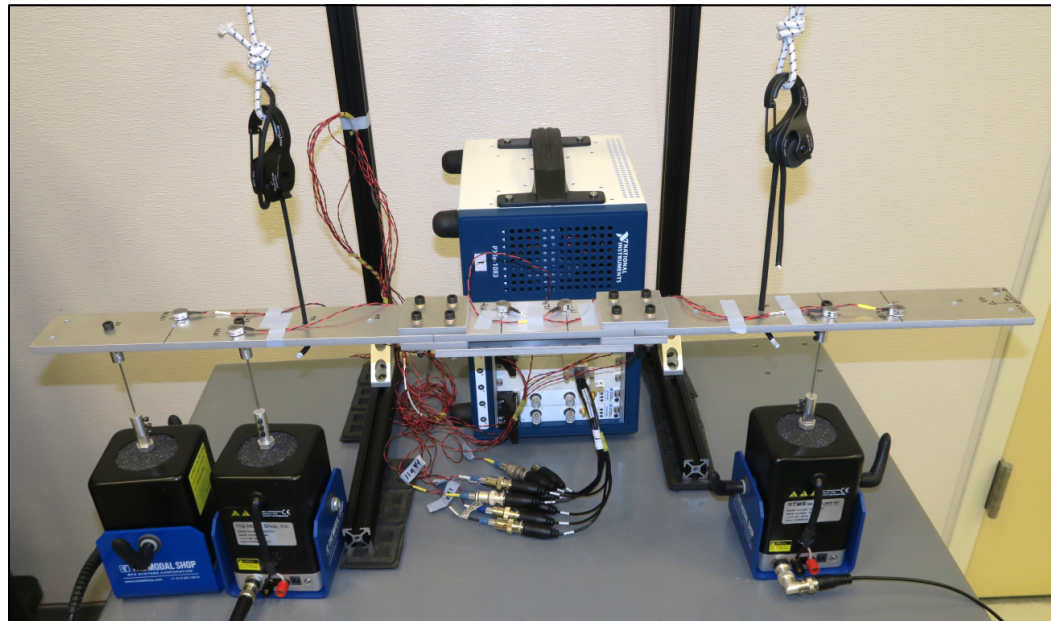
My approach

# **MIMO Test Parameters**

# MIMO Test Parameters

What things need to be designed?

- Boundary conditions
- Output & control DOFs
- Input DOFs
- Controller and solution methods



MIMO test parameters

**Why design a MIMO test?**

Test design techniques

Things to consider

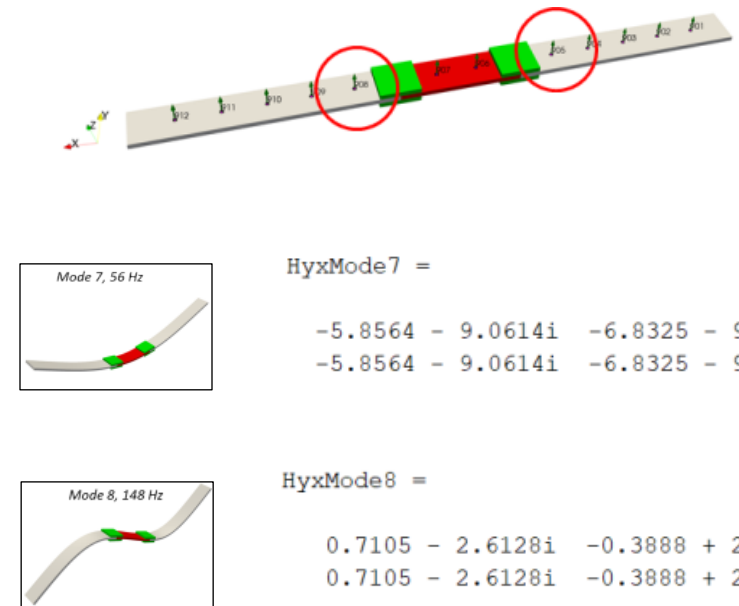
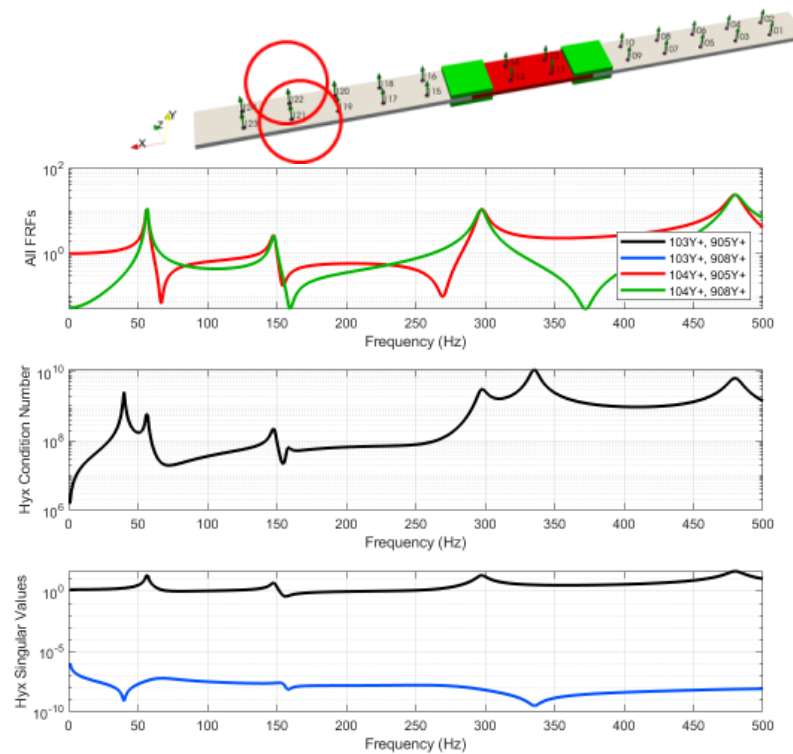
My approach

# Why Design a MIMO Test?

# How DOF Selection Affects the FRF Matrix

The chosen gauge and shaker locations affect the FRF matrix conditioning or independence

- Locations that are similar or symmetric may have very similar FRFs
- Rows or columns in the FRF matrix which are too similar make the FRF matrix non-independent, or near-singular
- Singular matrices cause problems when inverted (as in the MIMO control problem)
- Inverting poorly-conditioned (i.e. singular) matrices results in noise amplification in the inverse solution



HyxMode7 =

$$\begin{bmatrix} -5.8564 & -9.0614i & -6.8325 & -9.0612i \\ -5.8564 & -9.0614i & -6.8325 & -9.0612i \end{bmatrix}$$

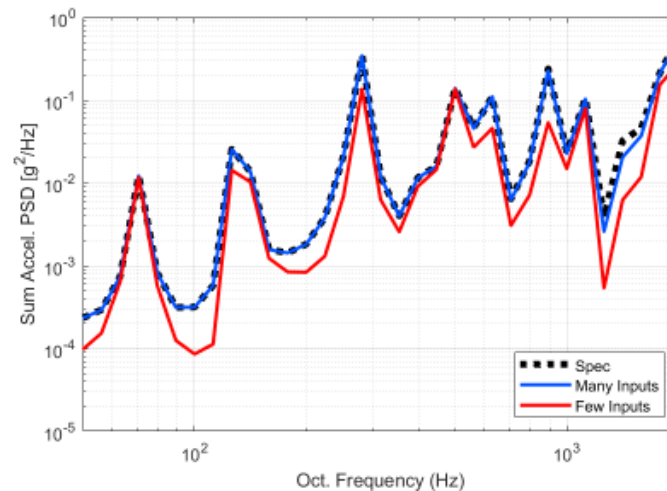
HyxMode8 =

$$\begin{bmatrix} 0.7105 & -2.6128i & -0.3888 & +2.6084i \\ 0.7105 & -2.6128i & -0.3888 & +2.6084i \end{bmatrix}$$

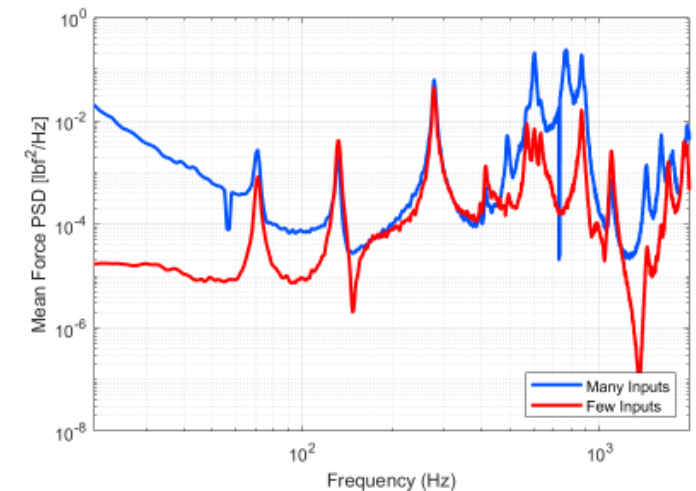


# Many vs. Few Inputs

- To have a good MIMO test you need enough of the right DOFs
  - Enough, good output DOFs to fully observe the system response
  - Enough, good input DOFs to allow the system to respond as needed
- If you don't have enough, good DOFs the test will not work
  - Insufficient Output DOFs:
    - High errors at non-control locations or inability to accurately control
  - Insufficient Input DOFs:
    - Inability to accurately control or hit test levels



*Output PSDs*

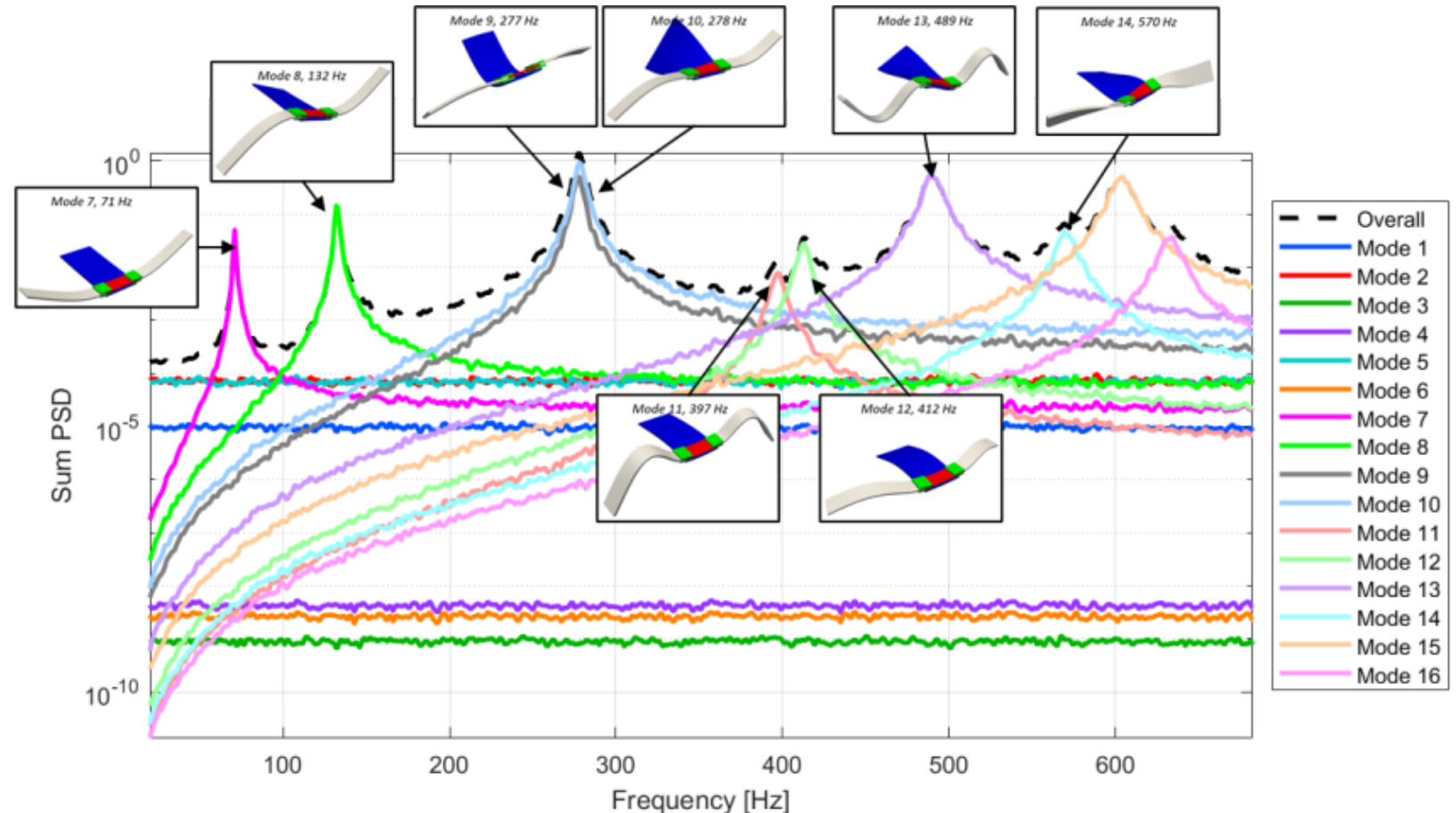


*Input PSDs*

# Test Design Phase is an Opportunity to Deep-Dive into the System Dynamics and the Spec Response

Utilize models or modal tests to examine the content in the spec

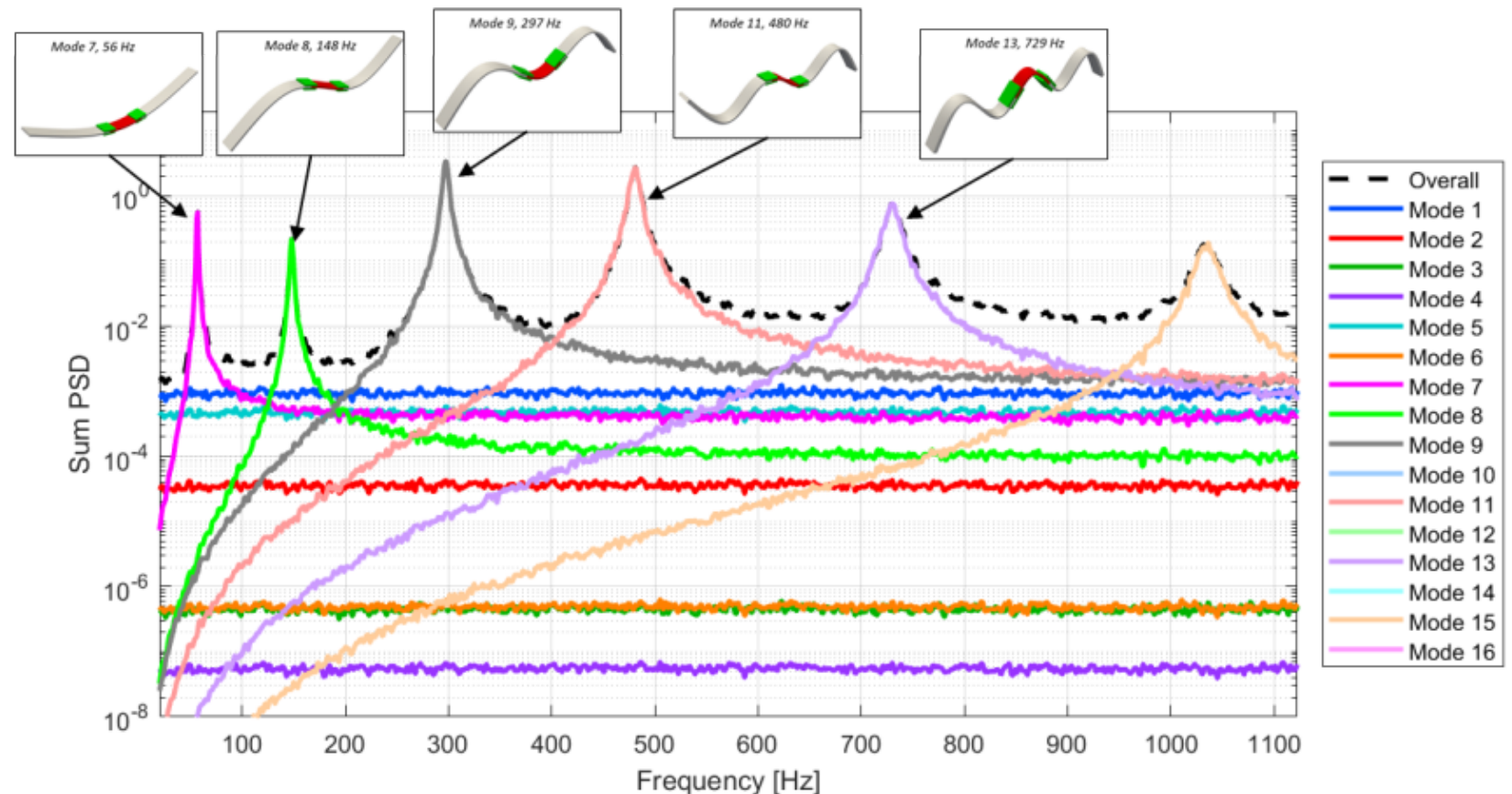
- Overall response is a combination of responses from the various modes
- Some modes are not well excited by the input forces and load paths
- Not necessary, but can provide useful insight



# Boundary Conditions & Inputs Affect the Modal Response in the Lab Test

The field environment excites some torsional modes but our lab test setup (BCs and shaker locations) are not allowing those modes to be activated

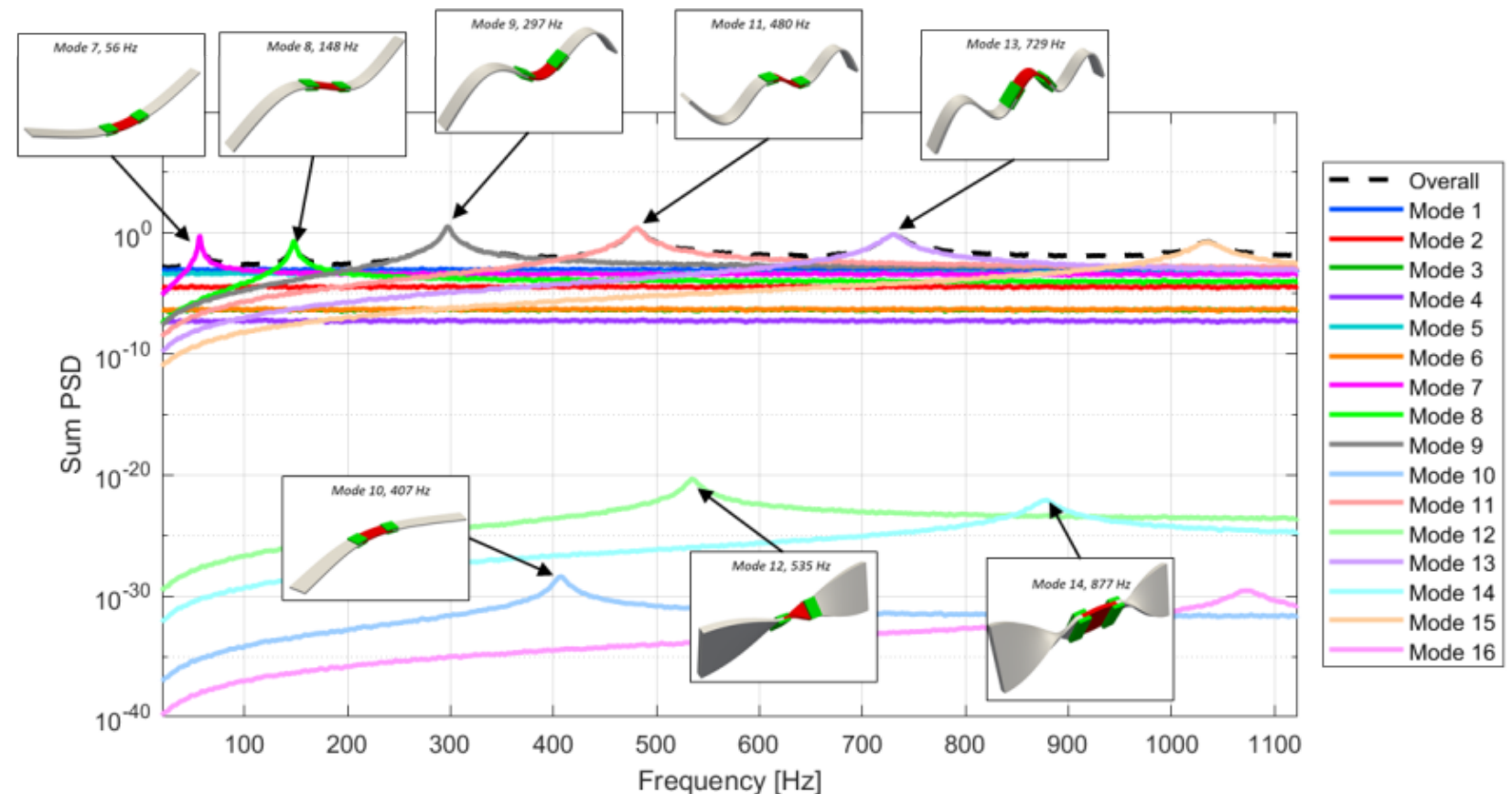
- Could modify BCs or inputs to change what modes are activated or how efficiently various modes are excited



# Boundary Conditions & Inputs Affect the Modal Response in the Lab Test

The field environment excites some torsional modes but our lab test setup (BCs and shaker locations) are not allowing those modes to be activated

- Could modify BCs or inputs to change what modes are activated or how efficiently various modes are excited



# Pre-Test Design Can Set Expectations for the Test

Doing a little bit of model-based test design early on can help predict results

- Response accuracy, achievable test levels
- De-risk a complex test series
- Get more equipment if needed (e.g. more shakers)
- Determine if a MIMO test is even a good idea given the problem setup (available field test data, DUT configuration, etc.)

MIMO test parameters  
Why design a MIMO test?  
**Test design techniques**  
Things to consider  
My approach

# Test Design Techniques

# Choosing Input and Output DOFs

## Output DOFs (gauge locations & directions):

- Don't need one DOF per mode – just enough to have an complete view of the full system response at each frequency line
- Try to leave out some validation gauges from the control

## Input DOFs (shaker locations & directions):

- Goal is enough input DOFs to allow the response to be accurately matched within the shaker capabilities
- Shakers need to work together – best set is not the best individual locations

## Optimization is helpful but this is a challenging problem:

- Good DOFs at one frequency may not be good for other frequencies
- There are many good sets of DOFs, a few bad sets, not really any perfect sets
- Mostly helpful for avoiding problematic locations and directions

# Output DOF Selection

Some techniques for choosing sensor locations:

- Modal test methods: Effective independence, condition number minimization, min off-diagonal MAC, etc.
- FRF methods: OED, iteratively solve MIMO simulations
- General approach: Start with many candidate DOFs and remove the bad ones to get to a set of good DOFs that meet your sensor budget

The size of the problem can be prohibitively large:

- Combinatorial problem
- Example: choose 8 gauges from 30 possible locations: nearly 6 million combinations
- Iterative approaches are nearly optimal (add one at a time)

Keep in mind response levels in addition to location independence

- A unique location that has very low response is not helpful. May need to add some penalty terms to the objective function



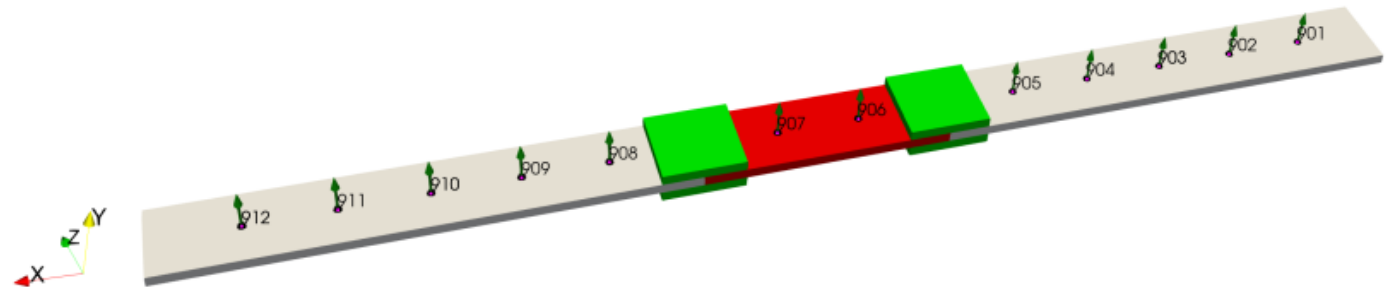
# Input DOF Selection

Choosing shaker locations is very similar in general approach to the output DOF selection problem

- Iterative solutions are nearly as good as global optimization solutions
- The search space can be huge – same combinatorial problem
- Shaker electro-mechanical models can be substructured into the DUT model to account for shaker coupling and predict shaker voltages and currents

Multi-variable objective functions can be used

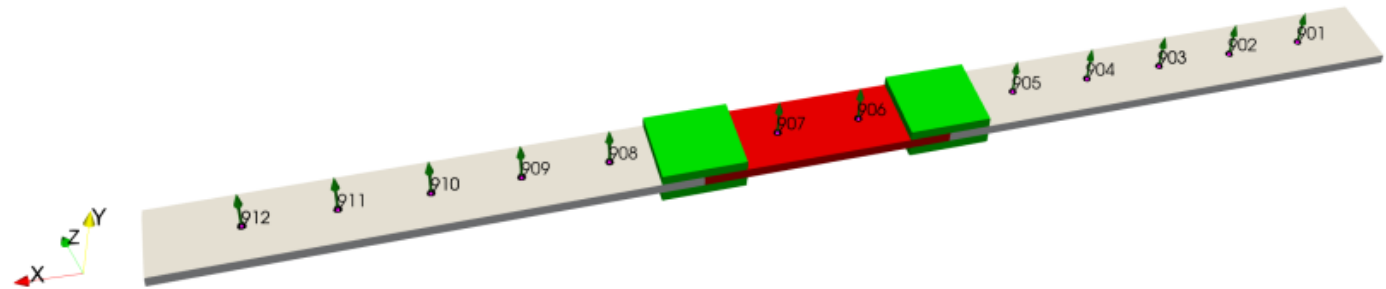
- Likely care about multiple factors: response accuracy, shaker forces, shaker stroke, voltage, current, etc.
- Different location sets will be chosen based on the objective function



# Selecting Input DOFs

## Things to note on shaker location selection

- Locations matter but they are not critical – there are many sets of locations with similar, good performance
- How you solve the MIMO control solution in the simulation will change the selected set (best is dependent on the problem setup and solution method)
- Required input force does not scale linearly with the number of shakers
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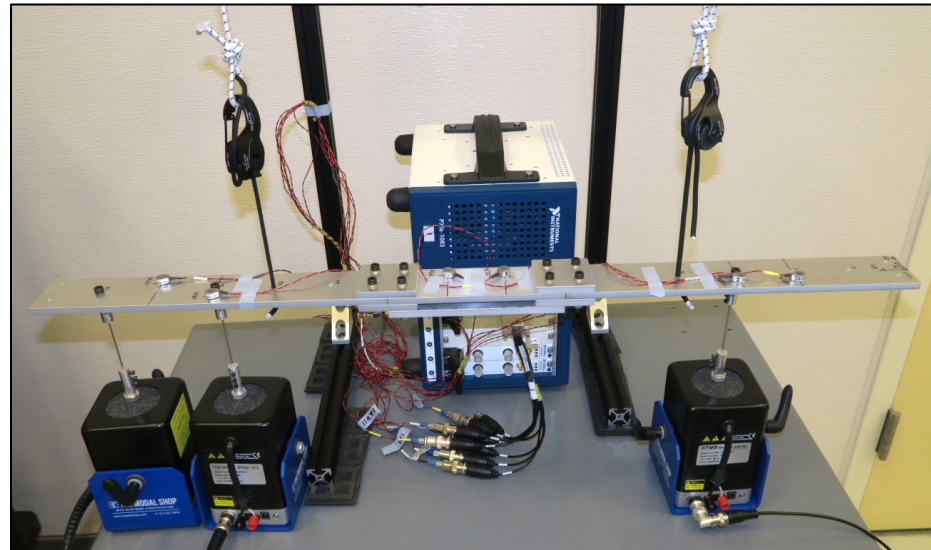
# Test Boundary Conditions

## Boundary conditions:

- DUT configuration
- Next-level assembly
- Fixtures and suspension (bungees)

## How to design boundary conditions:

- Challenging to do during testing (each change is time consuming)
- Challenging to optimize (TO for dynamics is still R&D)
- Use N+1 or similar approaches to approximate the impedance to the next level assembly



MIMO test parameters  
Why design a MIMO test?  
Test design techniques  
**Things to consider**  
My approach

# Things to Consider in MIMO Test Design

# Things to Consider

A model is useful but not necessary for test design

- Hammer tap data could be used to form a large FRF matrix with candidate DOFs

How you solve the MIMO control problem will affect design results

Optimized designs are helpful but may not be doable

- Often practical limitations prevail
- Typically some adjustment is needed based on how shakers fit around the item and how they can be supported

May not have a choice in the output DOFs

- Existing field test data is what it is
- At least determine if a fixed set of gauges is sufficient to do a MIMO test

# Things to Consider

Not every gauge will have great data

- Need to have more than the optimal, minimum set of gauges in case one is broken or is noisy
- Ideally, have some ranking of what gauges are most useful to the controller or most necessary to the test

All BC and DUT variations must be made up for with additional input forces

- Shakers need to work extra hard if the lab system is different than the field system
- Try and get these as close as possible to the field configuration (DUT assembly, BCs, load paths)

MIMO test parameters  
Why design a MIMO test?  
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**My approach**

# My Approach

# An Approach to MIMO Test Design

Determine a decent set of accel locations & directions

- Determine which can be removed or are least necessary
- Estimate how many are necessary, then add some more

Approximate the BCs

- Free-free with bungees or bolted to a rigid fixture
- Use N+1 fixture approach

Use iterative shaker selection method to choose input DOFs

- Change various control settings and run optimization several times
- Get a feel for which locations/combinations are better or worse

Dial-in the controller settings and make pre-test predictions

- Once the design is (mostly) known, figure out control method, spec changes, control DOFs, etc.
- Practice assessing the results

Setup the test & adjust the design

- Adjust shaker locations based on what is doable in setup
- Use design results as guidance



# Test Design Takeaways

- There are multiple considerations in MIMO test design
- A good MIMO test requires good design
  - Enough, good outputs (control gauges)
  - Enough, good inputs (shakers)
  - Good controller and settings
  - Close BCs
- You can simulate the test to make predictions of responses and input levels
  - Helpful to understand what settings to use, what to expect from a test

# Schedule for Today

## Objective:

***Introduce MDOF Vibration Testing Concepts, Show You How it Works, and Discuss Why it is Useful***

| Section                    | Time          |
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| Introduction               | 8:00 – 8:30   |
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| 6DOF & 3DOF Testing       | 4:00 – 5:00 |
| Wrap-Up                   | 5:00 – 5:30 |

# **Data Quality:**

## **Considerations and Typical Issues in MIMO Testing**

*IMAC XL Short Course*

*Feb. 5<sup>th</sup> 2022*

*Ryan Schultz*

# MIMO = More Complexity = Opportunities for Problems

## Objective:

*Present several of the “gotchas” and common issues encountered in MIMO testing so you are aware of them*

- Bookkeeping
- Modifying CPSD and FRF matrices
  - Changing signs and DOF
  - Interpolating to different frequencies
- FRF conditioning and regularization
- Data quality
  - Identifying good data and bad data
  - Noisy measurements
  - Nonlinearities

## **Bookkeeping**

Modifying matrices

FRF conditioning

Data quality

# **Bookkeeping**

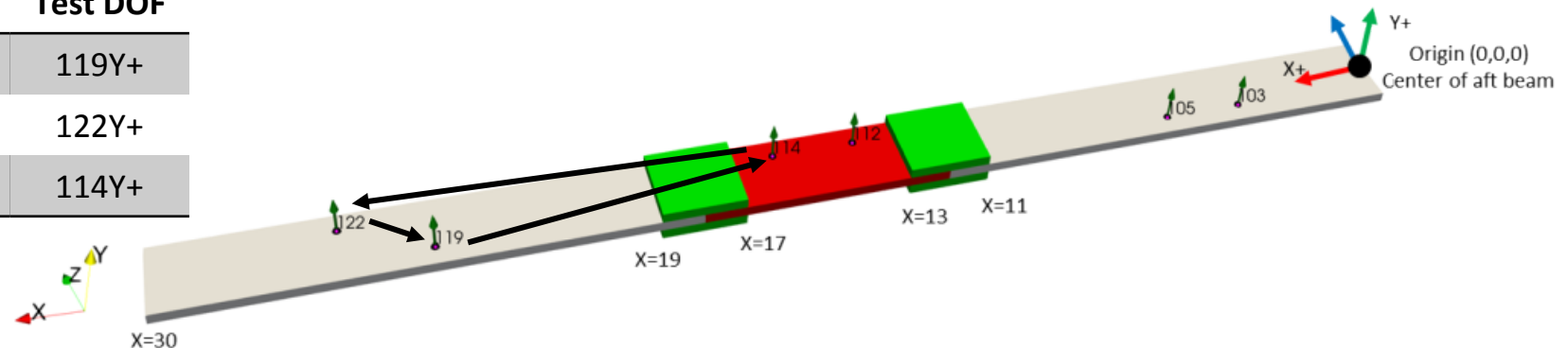
# MIMO = More DOFs

- Multiple-Inputs/Multiple-Outputs = many DOF to keep track of
- The DOF must be correct
  - Can't get a good solution if you think you're controlling to DOF A but really you're controlling to DOF B
- The DOF must match exactly
  - Meaning: the location AND direction are as required
  - Orientation of the gauge is just as important as the location
  - Flipping a gauge is NOT ok

# Common Causes of Bookkeeping Errors

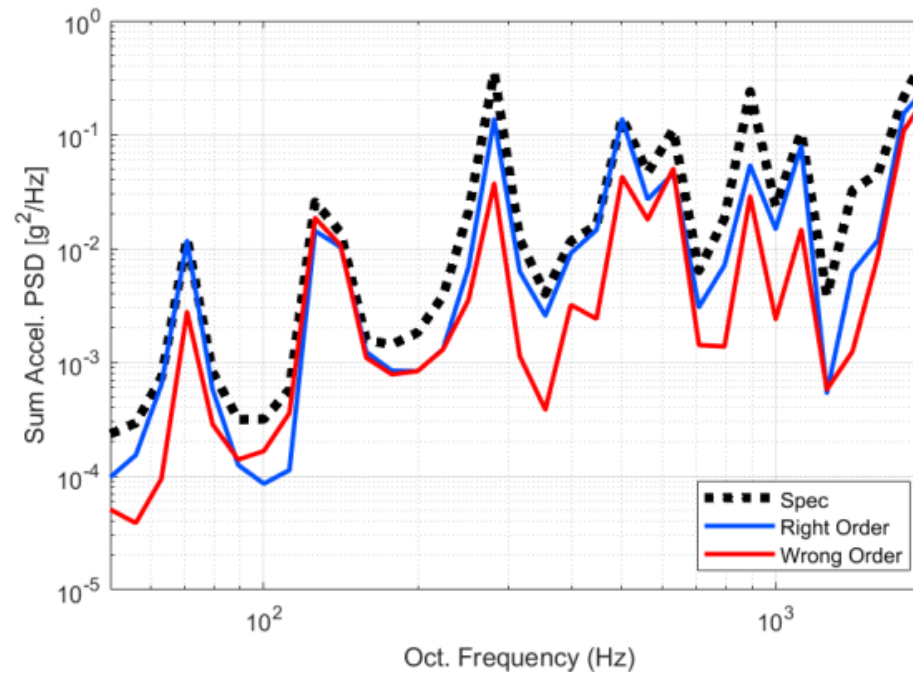
- Channels plugged into the DAS in the wrong order
- Not ensuring the specification CPSD matrix and FRF matrix have the same DOF in the same order
- Gauge is oriented in the opposite direction
  - Or doesn't exactly match the specification DOFs
- Data used to derive the specification is uncertain
  - Not 100% sure how the prior field test was instrumented
- DUT instrumentation is uncertain
  - Not 100% sure how each gauge was installed

| Spec DOF | Test DOF |
|----------|----------|
| 114Y+    | 119Y+    |
| 119Y+    | 122Y+    |
| 122Y+    | 114Y+    |



# Bookkeeping Problem Example

- Example problem: DOF ordering error
  - Say the channel list (DOFs) is different between spec and test
  - Output DOF in the FRF matrix is different from the spec CPSD matrix





# Bookkeeping: Some Advice

- Know that bookkeeping is absolutely critical in MIMO testing
  - Impress on everyone working on your test that this must get attention
  - Minimize the number of people modifying your channel table, specification, test setup, etc.
- Have some tools to be able to modify your CPSD and FRF matrices to account for changes to DOFs
  - Re-order or pick a subset of DOFs
  - Change signs
  - A verified tool to do this is much better than doing this manually as-needed
- Document your test setup as much as possible
  - Photos and notes to know the location and direction of every channel
  - Label and document the local CS for each gauge, and understand what CS is being used in the test (gauge vs. local vs. global)

Bookkeeping

**Modifying matrices**

FRF conditioning

Data quality

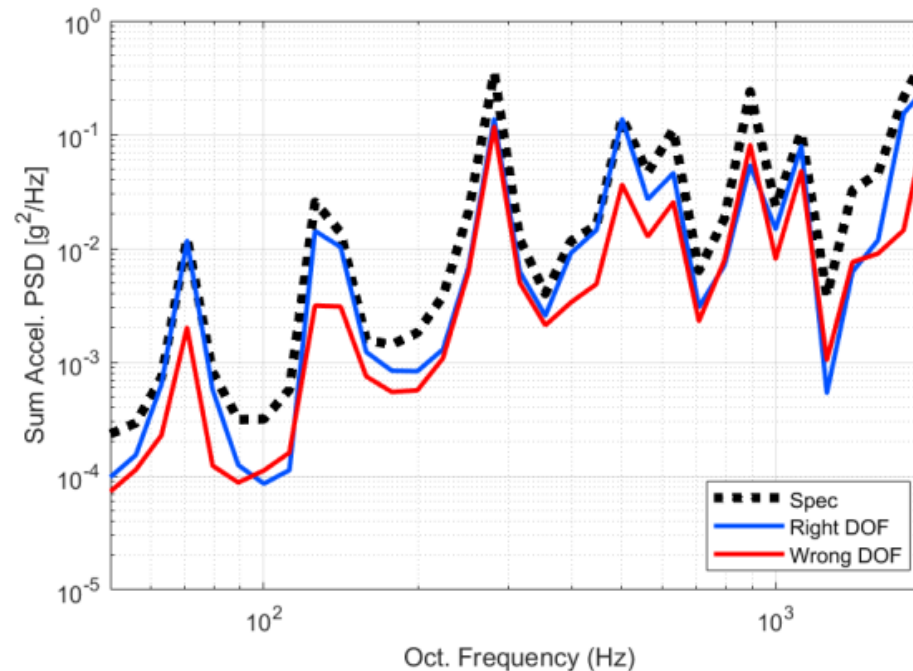
# Modifying CPSD and FRF Matrices

# Changing DOFs in CPSD and FRF Matrices

- Changing DOFs = changing rows or columns
  - Changing order of rows or columns,  $[1\ 2\ 3] \rightarrow [2\ 3\ 1]$
  - Picking out subsets of rows or columns,  $[1\ 2\ 3] \rightarrow [1\ 2]$
  - Changing signs = multiply rows or columns by -1
- For CPSD matrices:
  - Same DOFs on the rows and the columns
  - Do the same thing to the rows and the columns
  - Sign change is applied to both the row and column
- For FRF matrices:
  - Different DOFs on the rows and columns (outputs and inputs)
  - Do different things to the rows and columns
  - Sign change is applied only to the row or only to the column

# Changing DOFs in CPSD and FRF Matrices

- Example problem: DOF sign error
  - Say the signs on 3 DOFs got changed in the test but the spec wasn't updated to reflect the change
  - What does this mean? The spec response is not in the controllable space of the FRF vectors because of the sign error, you cannot control the desired response with this new, incorrect vector space



# CPSD Interpolation

- Common situation:
  - Obtained the MIMO spec CPSD matrix from someone at frequency spacing  $df_S$ . No time data is available.
  - Controller used in the MIMO test only has a few options for frequency spacing so we pick the closest one but  $df_T \neq df_S$
  - The spec imported into the controller must be at  $df_T$
- Solution:
  - Modify the spec CPSD matrix to be at the test frequency spacing
  - Interpolate – but do it carefully
  - Check that the new CPSD matrix to ensure it is Hermitian and positive-definite

# Modifying CPSD and FRF Matrices: Some Advice

- Changing DOFs and signs is possible and straightforward
  - Best if you have a tool to do this automatically
  - Helpful if you have some ways to check if the changes were done correctly (don't just put a negative sign on the data)
- While you can interpolate CPSD and FRF matrices to different frequency lines, if possible go back to the time data and re-derive
  - Avoids potential positive definiteness errors
  - Avoids potential errors due to the interpolation process

Bookkeeping  
Modifying matrices  
**FRF conditioning**  
Data quality

# FRF Conditioning and Regularization

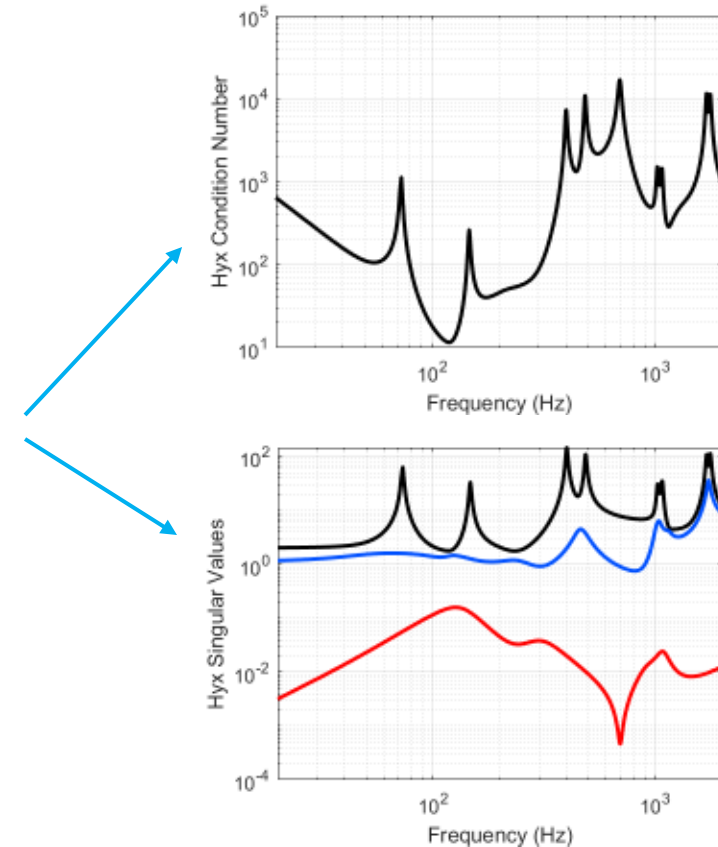
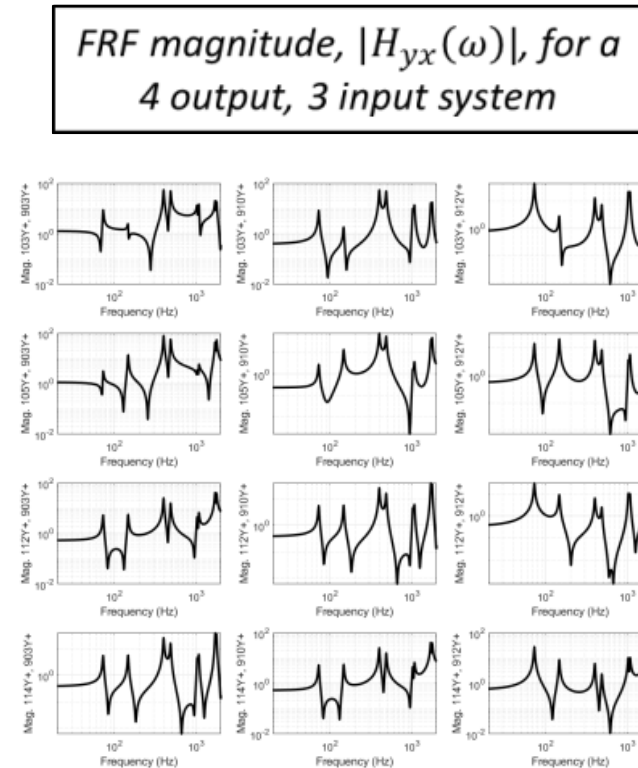
# FRF Conditioning

- The independence of the rows or columns of a matrix can be assessed by the condition number of a matrix
  - Ratio of the largest to the smallest singular value
  - Big condition number = potential for numerical errors in the inverse
- In MIMO control the FRF matrix is being inverted
  - Poorly conditioned FRF matrix = amplification of errors due to noise
  - Noise = anything not linear in the measured system (actual noise, system nonlinearities, response from unmeasured inputs, etc.)
- A bad condition number is subjective and system-dependent
  - For typical IMMAT-type MIMO tests, keep the condition number under 1000



# FRF Conditioning

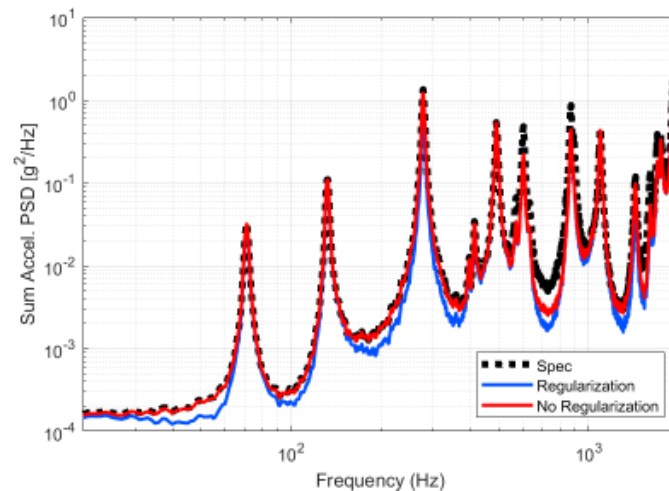
- Example FRF matrix condition number and singular values



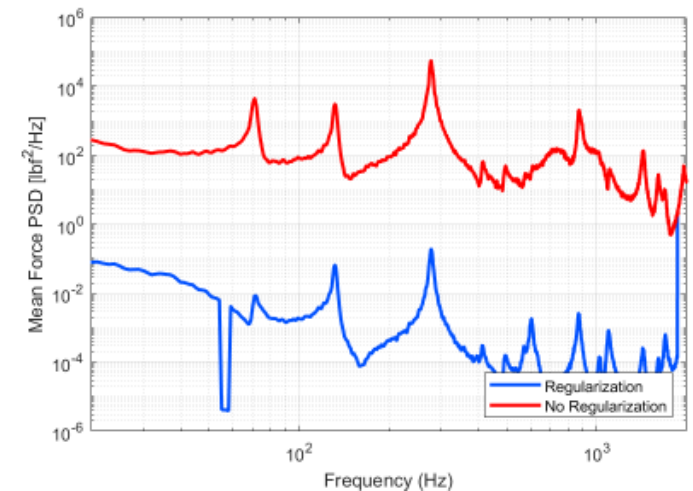
3 Singular  
values ( $H_{yx}$   
is rank 3)

# Regularized Solutions

- To minimize effects of poor conditioning, utilize regularized inverse solutions
  - Common methods: SVD rejection or truncation, Tikhonov regularization
- Too much regularization or applying regularization when it is not needed can be bad as well
  - You're essentially modifying the output-input relationships when applying regularization, so doing this too much changes the system



*Output PSDs*



*Input PSDs*

# FRF Conditioning and Regularized Solutions: Some Advice

- Always understand the conditioning of your test system (FRFs)
  - Collect response due to uncorrelated inputs (e.g. system ID)
  - Compute the condition number and singular value decomposition of the test FRF matrix
  - Examine to understand which frequency ranges may be poorly conditioned or if you have poor independence of outputs or inputs
- Always do some kind of regularized solution in the MIMO control process (if possible)
  - Figure out how much regularization is needed and don't do too much
  - Avoids the problem “blowing up”
- Understand that regularization can reduce the rank (controllable space) of your system
  - Expect that it may reduce the accuracy of your solution, but require lower input levels

Bookkeeping  
Modifying matrices  
FRF conditioning  
**Data quality**

# Data Quality

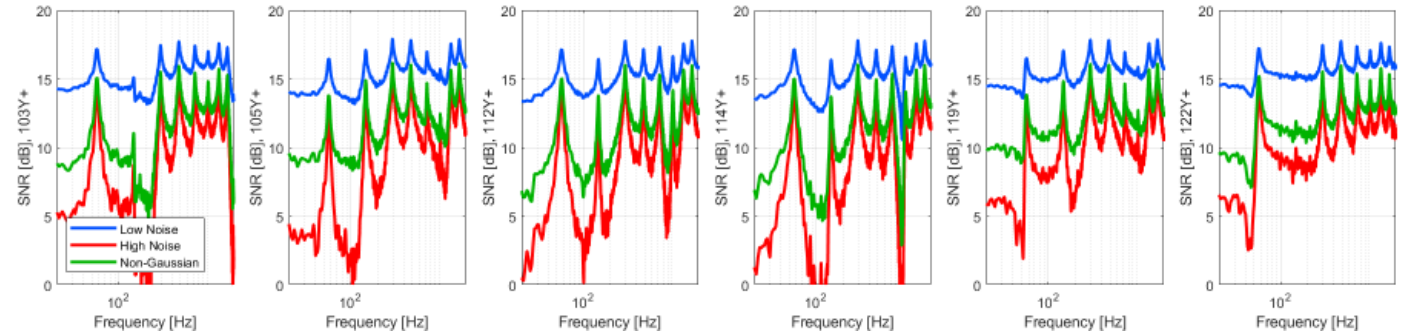
# Data Quality

- MIMO random vibration needs a few things to work well:
  - Linearity
  - Good measurements
  - Gaussian inputs and outputs
  - Sufficient independence of inputs
- If your data doesn't exhibit these traits, then there is some problem that you need to solve before trying to run a test (or trying to run a test with those bad channels)
  - First, need to identify the bad channels (be able to look at the data and compute metrics to assess those properties)
  - Next, need to remove bad channels from the control DOF set – don't want to try and control to bad data, or use bad FRFs in the control solution
- It is very important to spend the time figuring out if the data is good or bad – don't just assume the controller is going to do this for you

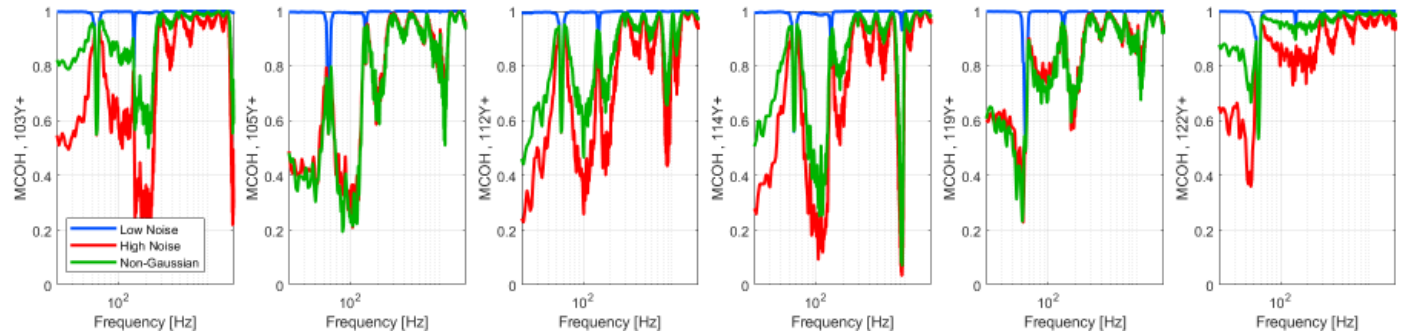
# Data Quality

- Example Data

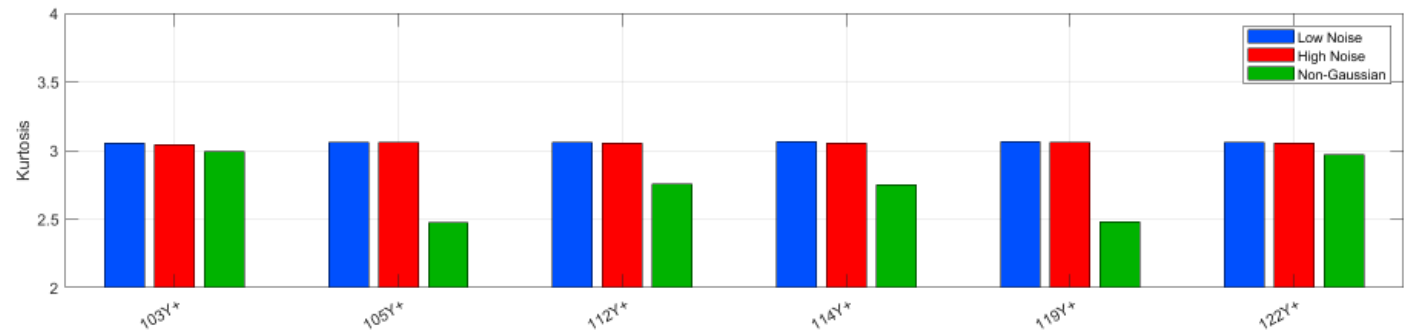
*Signal/Noise Ratio*



*Multiple Coherence*



*Kurtosis*



## Data Quality: Some Advice

- A good test requires good data
- Be able to assess your test data quickly
- Data quality can mean various things:
  - Signal/noise ratio
  - Linearity (coherence)
  - Gaussian
- Identify DOFs with poor data quality and remove them from the control set
  - You must not use bad data in the MIMO control solution
- There are ways to clean up poor FRFs or CPSDs but only do this as a last resort

Bookkeeping  
Modifying matrices  
FRF conditioning  
Data quality

# Key Takeaways



# Key Takeaways

- MIMO is more complicated than single-axis testing and some things become critical:
  - Bookkeeping
  - Gauge directions (sign)
  - FRF matrix conditioning
  - Data quality
- You can modify FRF and CPSD matrices, but take caution and make sure to verify the modified matrices are correct and valid
- Good data quality is critical to a good MIMO solution so be able to assess data quality during testing

# Schedule for Today

## Objective:

***Introduce MDOF Vibration Testing Concepts, Show You How it Works, and Discuss Why it is Useful***

| Section                    | Time          |
|----------------------------|---------------|
| Introduction               | 8:00 – 8:30   |
| General MDOF Overview      | 8:30 – 9:15   |
| Field vs. Lab Environments | 9:15 – 10:15  |
| Break                      | 10:15 – 10:30 |
| Example Problem            | 10:30 – 11:15 |
| Demo 1: Single-Axis Test   | 11:15 – 12:00 |
| Lunch                      | 12:00 – 1:00  |

| Section                   | Time        |
|---------------------------|-------------|
| Demo 2: Multi-Shaker Test | 1:00 – 2:00 |
| Test Design Methods       | 2:00 – 2:30 |
| Break                     | 2:30 – 2:45 |
| Rattlesnake Controller    | 2:45 – 3:30 |
| Data Quality              | 3:30 – 4:00 |
| 6DOF & 3DOF Testing       | 4:00 – 5:00 |
| Wrap-Up                   | 5:00 – 5:30 |

# Wrap Up

*IMAC XL Short Course*

*Feb. 5<sup>th</sup> 2022*

*Randal Mayes*

*Dan Rohe*

*Dr. Ryan Schultz*

# Wrap Up

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# Key Takeaways

- There are various MIMO random vibration test methods, they all use a similar theoretical basis and approach and they can provide some significant benefits vs. traditional single-axis shaker testing
- More inputs, more outputs means more complication and complexity
- MIMO is not for every DUT, every environment – the complexity may not be worth it, the objective of the test may not warrant a refined MIMO test, or data to derive a MIMO specification may not exist
- This remains an active area of research and we're excited to get more people involved in expanding the application space and the state-of-the-art

# Before You Go

- Fill out the course critique forms
  - What worked well
  - What did not
  - What was too short or too long
  - What wasn't clear
  - What should we add or subtract
- Clean up your space
- Let us know if you have any questions!