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Effectiveness and nonlinear characterization of energy harvesting absorbers with mechanical and magnetic stoppers

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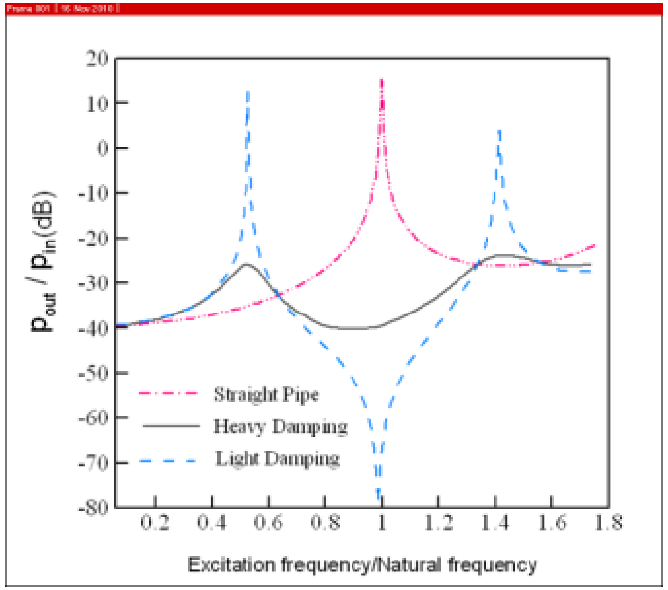
Overview

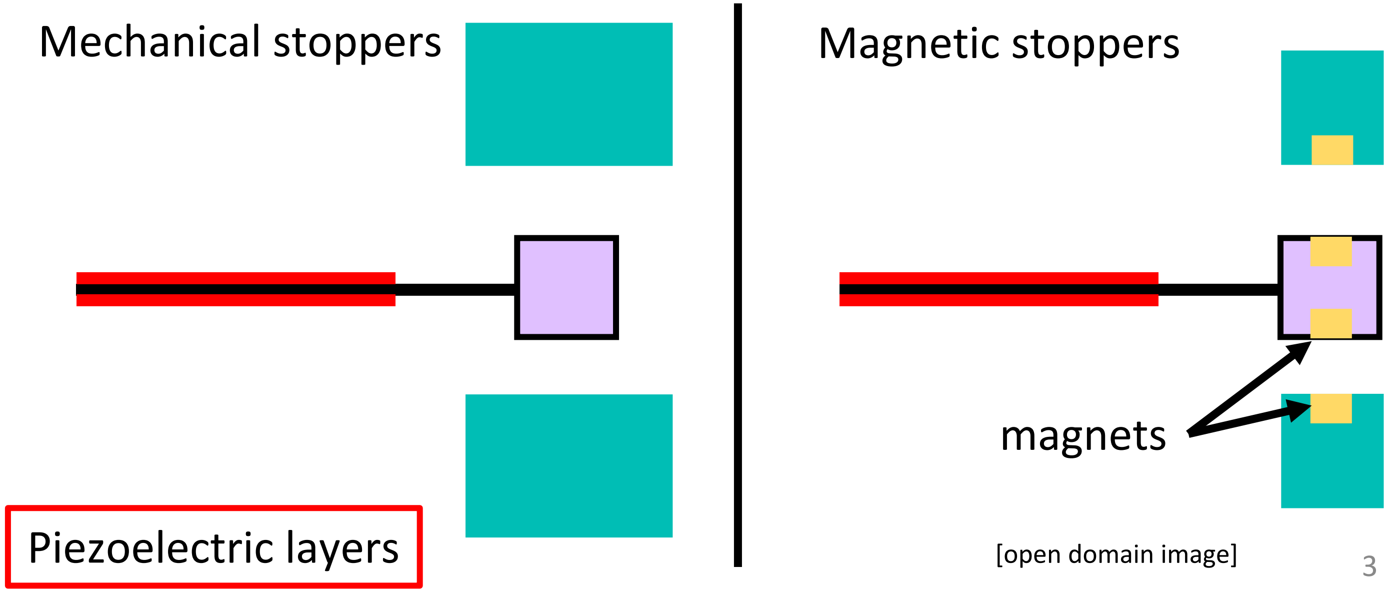
- Background & motivation
- Reduced-order modeling
- Results & discussion

Background & motivation

❑ Goals

- Design a cantilever vibrational absorber to control the motion of a structure under base excitation
- Adhere piezoelectric layers to the cantilever absorber to generate useful energy
- Optimize the amount of energy harvested by including amplitude stoppers
    - Determine whether mechanical or magnetic stoppers are more optimal





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Reduced-order modeling

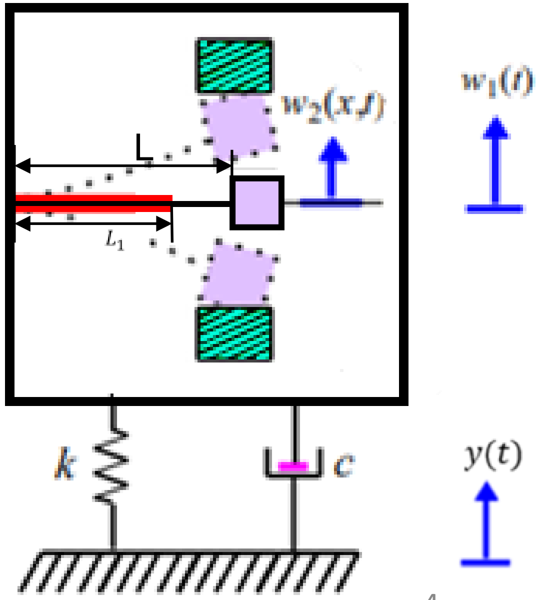
Reduced order model developed using the Euler Lagrange equations:

$$\mathcal{L} = T - \Pi\,, \qquad \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial v} - \frac{\partial \mathcal{L}}{\partial v} = \frac{\delta W_{nc}}{\delta v}$$

where $v = w_1\,, w_2\,, and\ \lambda$
$\dot{\lambda} = V$

$$T = \frac{1}{2}M\dot{w}_1^2 + \frac{1}{2}M_t\dot{r_2}^2 + \frac{1}{2}I_t w_2(L,t)'^2 + \frac{1}{2}M_{b1}\int_0^{L_1}\left(\dot{w}_1 + \dot{w}_2(x,t)\right)^2 dx + \frac{1}{2}M_{b2}\int_{L_1}^{L}\left(\dot{w}_1 + \dot{w}_2(x,t)\right)^2 dx$$

$$\Pi = EI_1\int_0^{L_1}\left(w_2''(x,t)\right)^2 dx + EI_2\int_{L_1}^{L}\left(w_2''(x,t)\right)^2 dx$$
$$+be_{31}\int_0^{L_1}(h_s + h_p)\left(w_2''(x,t)\right)V(t)dx - \frac{e_{31}bL_1}{h_p}V(t)^2 + \frac{1}{2}kw_1^2$$



where
$$\dot{r_2} = \dot{w}_1 + \dot{w}_2(L,t) + L_c\dot{w}_2'(L,t)$$
$$I_t = \frac{1}{3}M_t L_c^2$$
$$EI_1 = \frac{1}{12}bE_s h_s^3 + \frac{1}{2}bE_p h_s^2 h_p + bE_p h_s h_p^2 + \frac{2}{3}bE_p h_p^3$$
$$EI_2 = \frac{1}{12}bE_s h_s^3$$

Reduced-order modeling

Galerkin discretization: $w_2 = \sum_{i=1}^{\infty} \varphi_i(x) r_i(t)$

- Energy harvesting absorber mode shapes $\varphi_i(x)$
- General coordinate $r_i(t)$

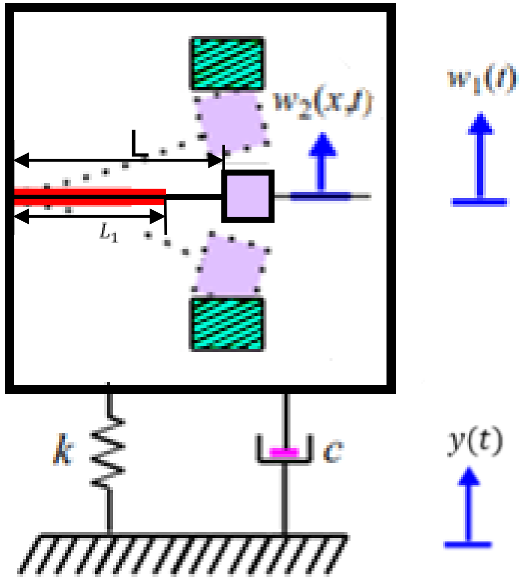
$$M_s \ddot{w}_1 + c\dot{w}_1 + kw_1 + \sum_{i=1}^{\infty}\left(M_t\varphi_i(L) + M_tL_c\varphi_i'(L) + M_b\int_0^L \varphi_i(x)dx\right)\ddot{r}_i = Fcos(\omega t + \phi)$$

$$\ddot{r}_i + 2\xi_i\omega_i\dot{r}_i + \omega_i r_i + \left(M_t\varphi_i(L) + M_tL_c\varphi_i'(L) + M_b\int_0^L \varphi_i(x)dx\right)\ddot{w}_1 - \theta_i V = \varphi_i F_{stopper}$$

$$C_p\dot{V} + \frac{1}{R} + \sum_{i=1}^{\infty}\theta_i\dot{r}_i = 0$$

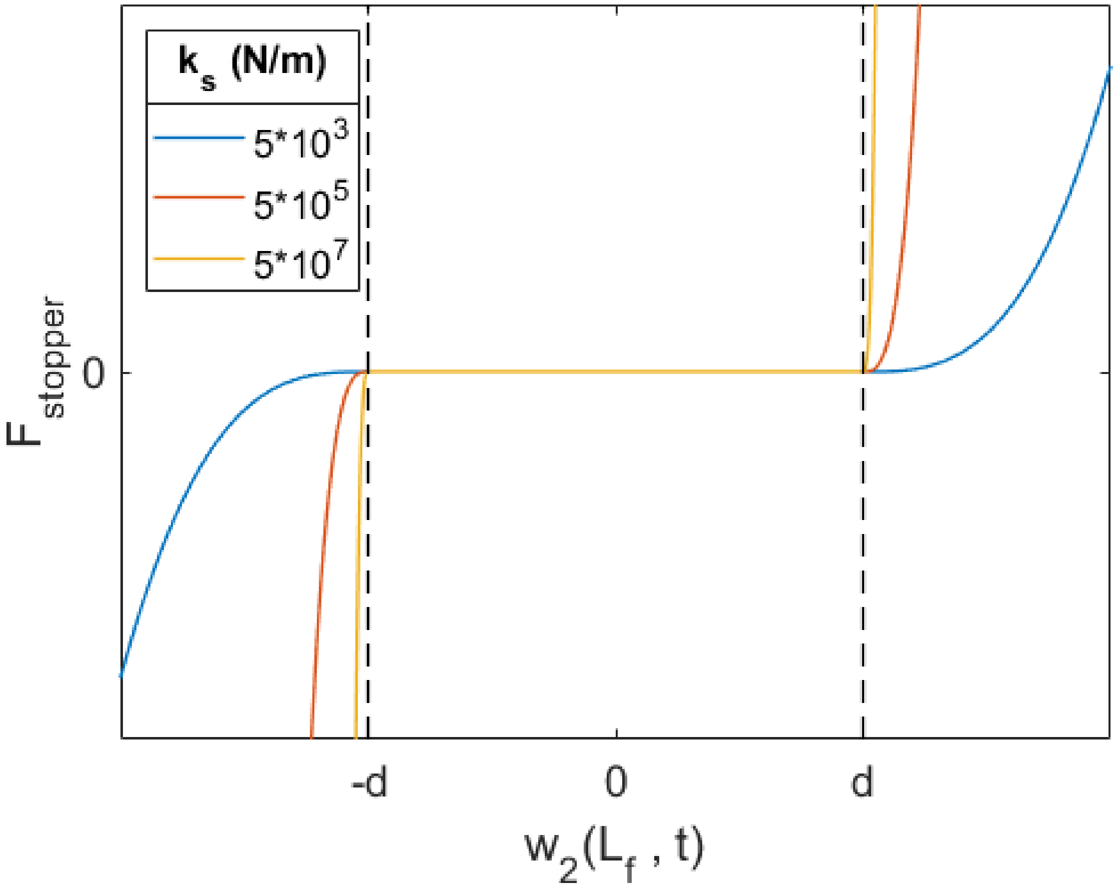
where: $M_S = M + M_t + M_{b_1}L_1 + M_{b2}(L - L_1)$, $C_p = 2\frac{e_{33}bL}{h_p}$,

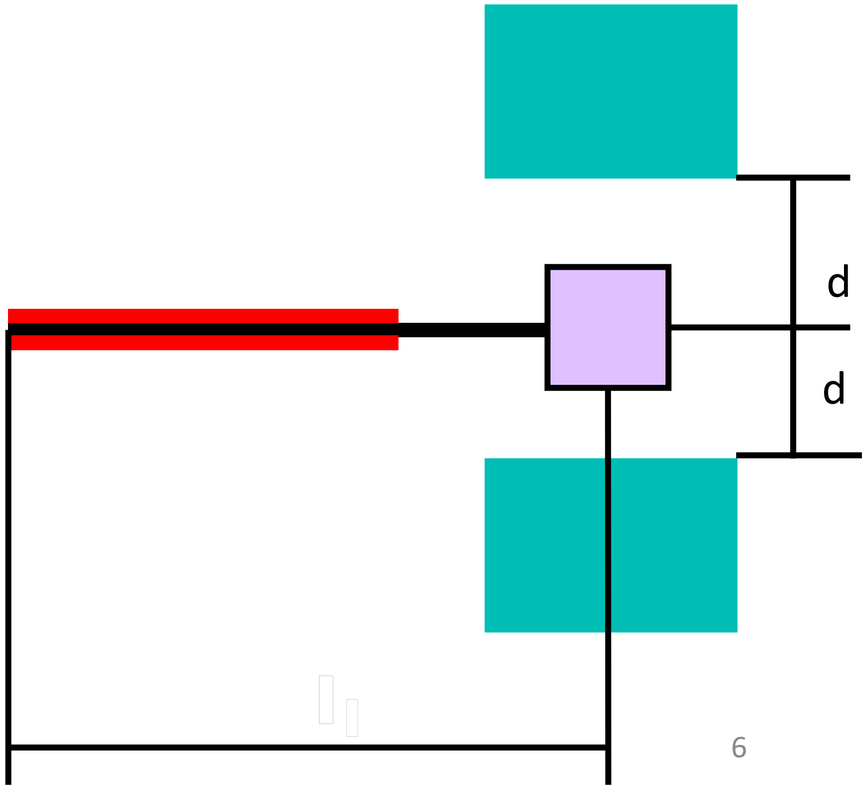
$$\theta = E_p d_{31} b(h_p + h_s)\varphi_i'(L)$$



Mechanical stopper force representation

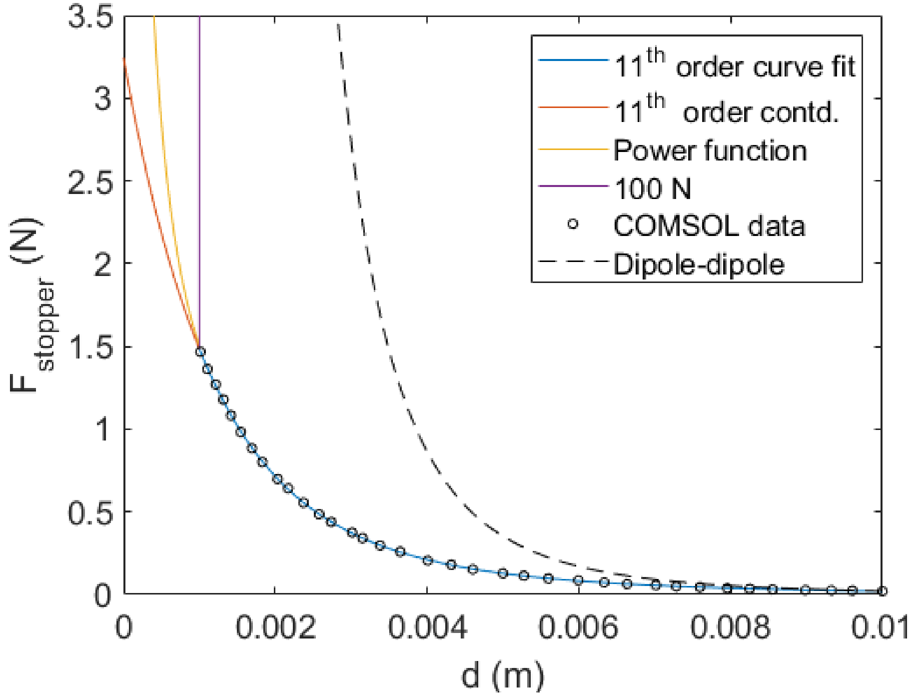
$$F_{stopper} = k\left[\sum_{i=1}^{\infty}\varphi_i(L_s)r_i(t) - \frac{1}{2}\left(\left|\sum_{i=1}^{\infty}\varphi_i(L_s)r_i(t) + d\right|\right) - \left(\left|\sum_{i=1}^{\infty}\varphi_i(L_s)r_i(t) - d\right|\right)\right]^3$$



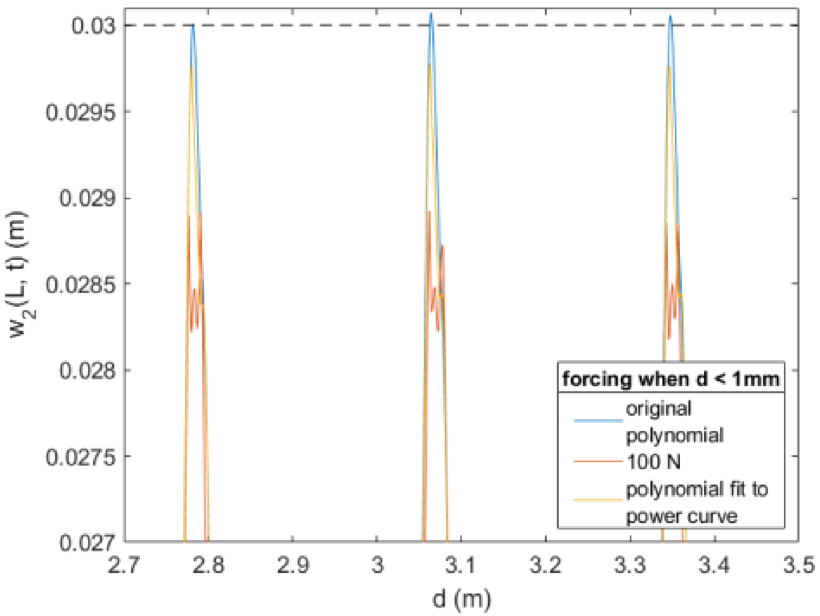


Magnetic stopper force representation

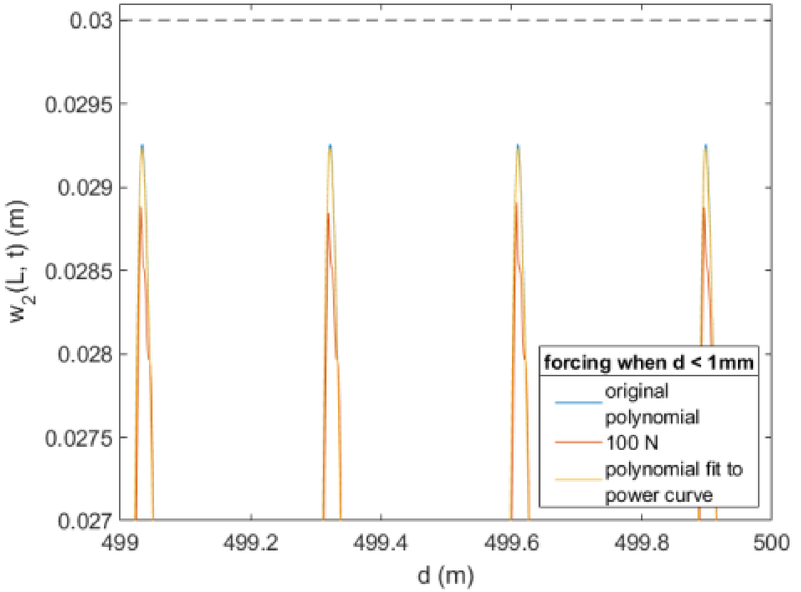
- Dipole-dipole representation is only accurate to a gap of 7 mm
- An 11th order curve fit to accurate COMSOL data is utilized for gaps smaller than 10 mm
- The COMSOL data is only available to 1mm, approximations are made for smaller gaps



Transient



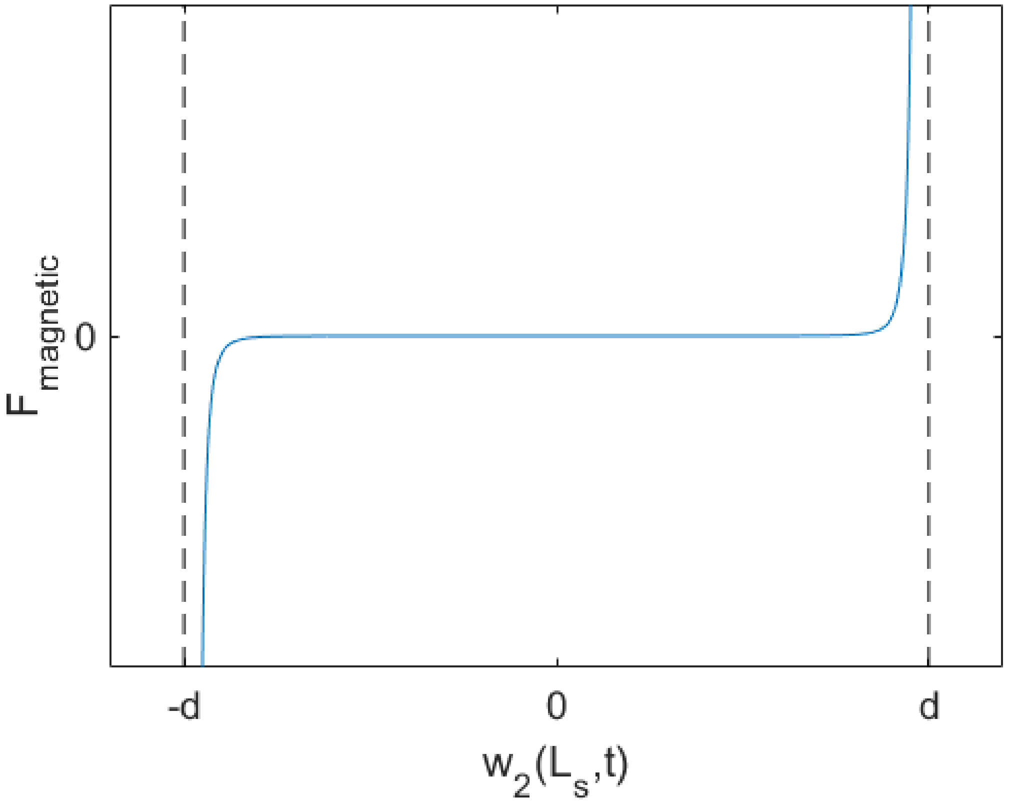
Steady-state



b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}
3.244	$-2.59 * 10^3$	$1.072 * 10^6$	$-2.87 * 10^8$	$5.33 * 10^{10}$	$-7.07 * 10^{12}$	$6.72 * 10^{14}$	$-4.52 * 10^{16}$	$2.1 * 10^{18}$	$-6.39 * 10^{19}$	$1.14 * 10^{21}$	$-9.09 * 10^{21}$

Upadrashta, D., Yang, Y., & Tang, L. (2015). Material strength consideration in the design optimization of nonlinear energy harvester. *Journal of Intelligent Material Systems and Structures*, *26*(15), 1980-1994.

Magnetic stopper force representation



$$F_{magnetic} = \begin{cases} -\dfrac{3\tau a_1 a_2}{2\pi y^4} & if\ w_2(L_f,t) > 10\ mm \\ b_0 + b_1 y + b_2 y^2 + \cdots + b_{11} y^{11} & if\ 1 \leq w_2(L_f,t) \leq 10\ mm \\ 0.0023 y^{-0.9386} & if\ w_2(L_f,t) < 1mm \end{cases}$$

where: $y = d - w_{2s} - w_2(L_f,t)$

b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}
3.244	$-2.59 * 10^3$	$1.072 * 10^6$	$-2.87 * 10^8$	$5.33 * 10^{10}$	$-7.07 * 10^{12}$	$6.72 * 10^{14}$	$-4.52 * 10^{16}$	$2.1 * 10^{18}$	$-6.39 * 10^{19}$	$1.14 * 10^{21}$	$-9.09 * 10^{21}$

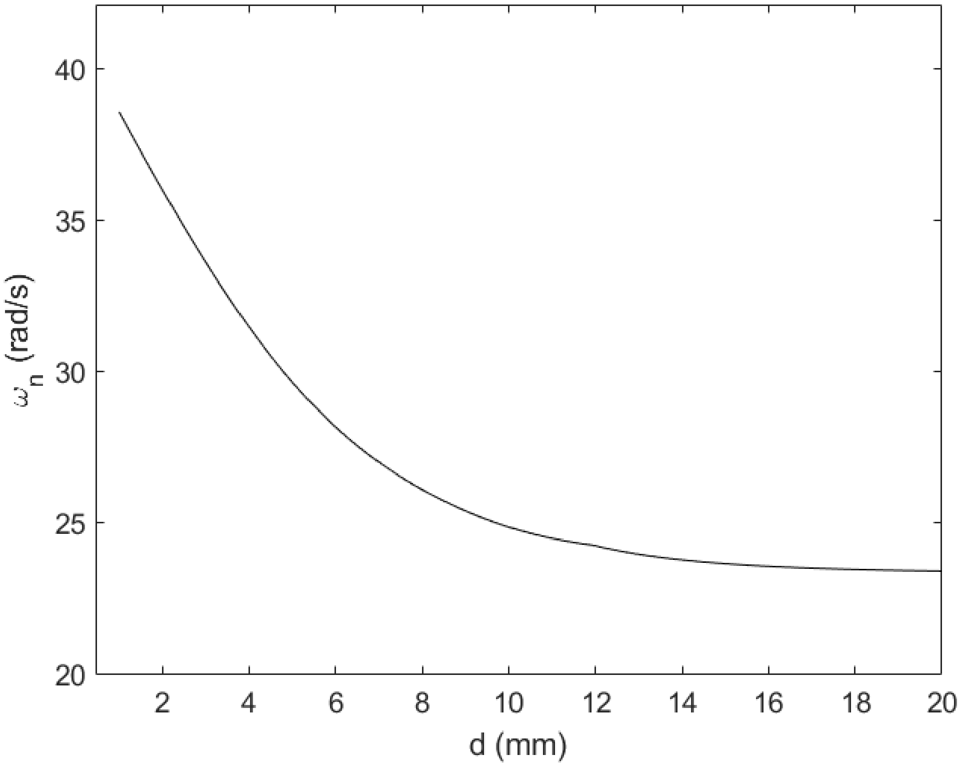
Magnetic stopper force representation

- Small static gap sizes cause a hardening behavior in the natural frequency of the absorber
- Extremely small gaps will cause the absorber to fail in controlling the primary structure's amplitude

<u>Orthogonality conditions:</u>

$$\int_0^{L_1} \varphi_{1s}(x) m_1 \varphi_{1r}(x) dx + \int_{L_1}^{L} \varphi_{2s}(x) m_2 \varphi_{2r}(x) dx + \varphi_{2s}(x) M_t \varphi_{2r}(x) + \varphi'_{2s}(x)(I_t + L_c^2)\varphi'_{2r}(x) + \varphi'_{2s}(x) M_t L_c \varphi_{2r}(x) + \varphi_{2s}(x) M_t L_c \varphi'_{2r}(x) = \delta_{rs}$$

$$\int_0^{L_1} \frac{d^2\varphi_{1s}(x)}{dx^2} EI_1 \frac{d^2\varphi_{1r}(x)}{dx^2} dx + \int_{L_1}^{L} \frac{d^2\varphi_{2s}(x)}{dx^2} EI_2 \frac{d^2\varphi_{2r}(x)}{dx^2} dx - \underline{F_{magnetic-linear}} = \delta_{rs}\omega_i^2$$

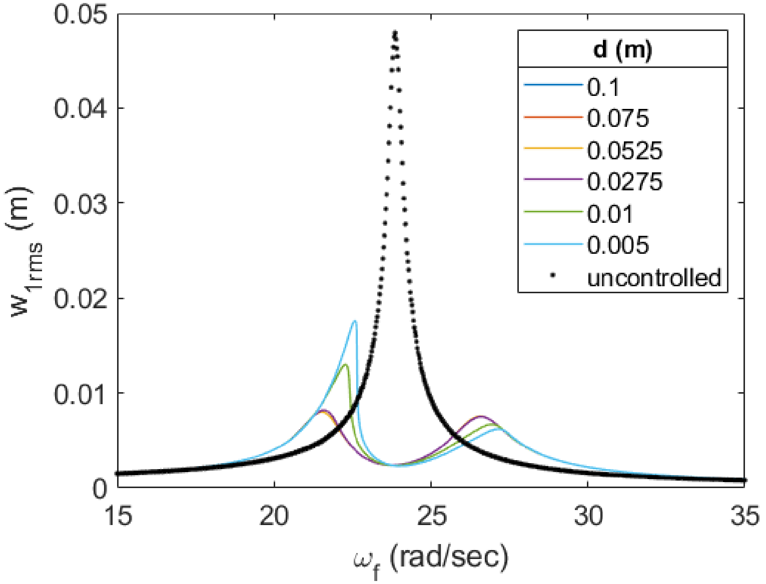


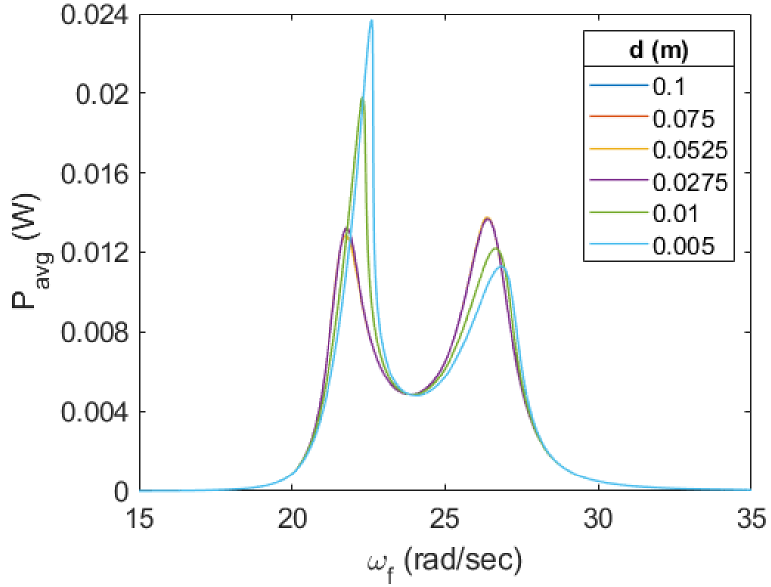
$\underline{F_{magnetic-linear}}$ is the linear portion of the Taylor series expansion of the magnetic force around $y = 0$

Results – Mechanical stoppers' gap effects

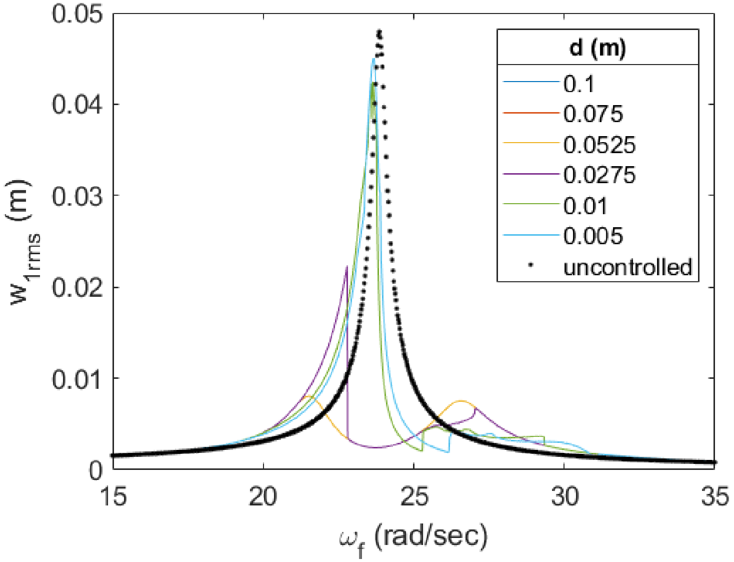
Parameter Values	
	2 cm
h	0.5 mm
h	0.267 mm
	$10^{4.73}$
	18 cm
	18 cm
	0.048 kg

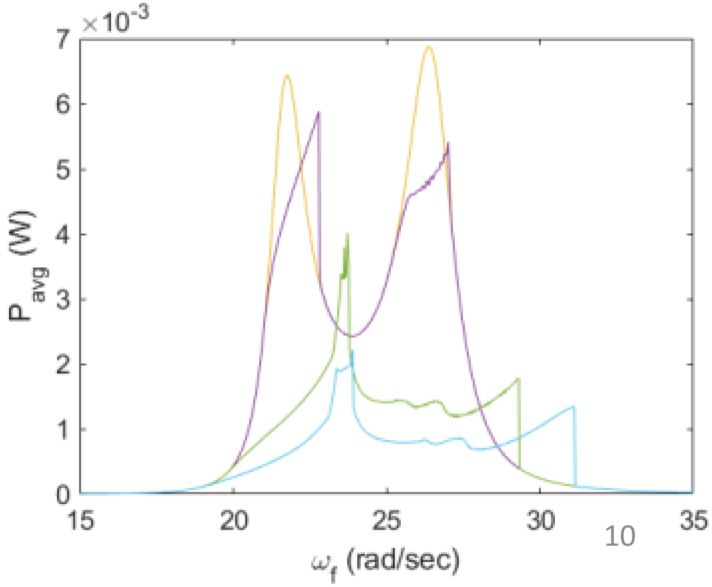
$= 5 * 10^3 \; /$





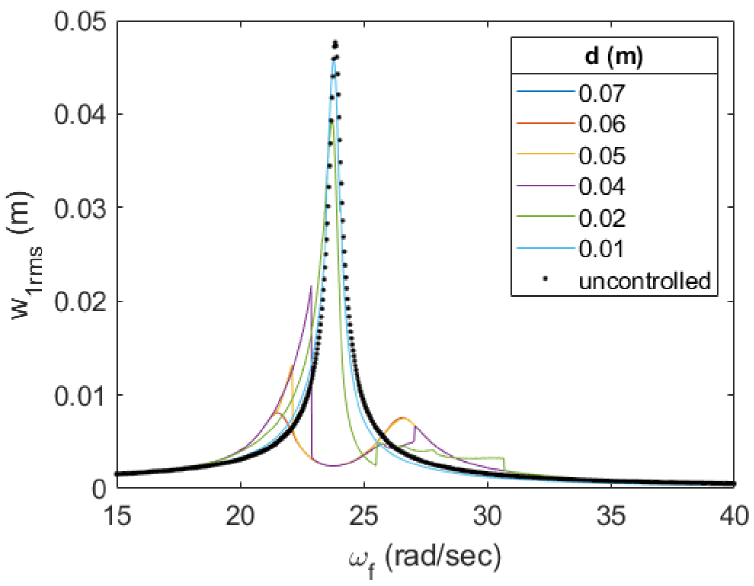
$= 5 * 10^5 \; /$

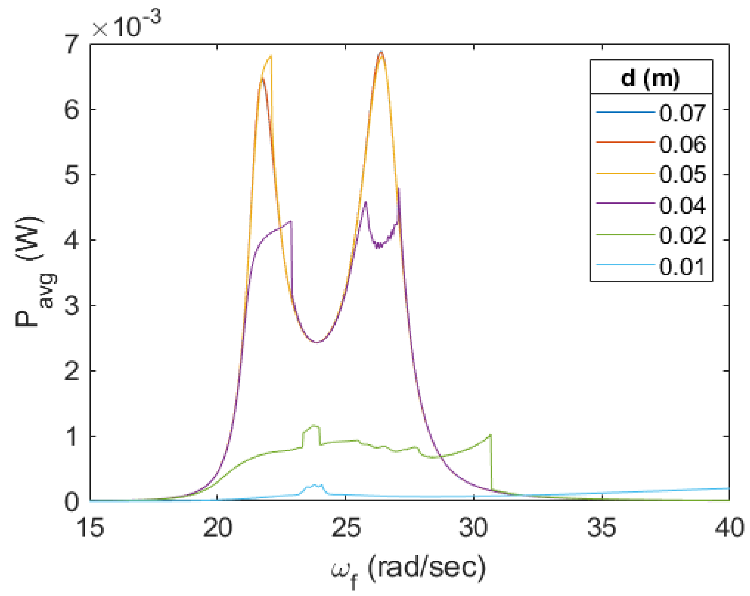


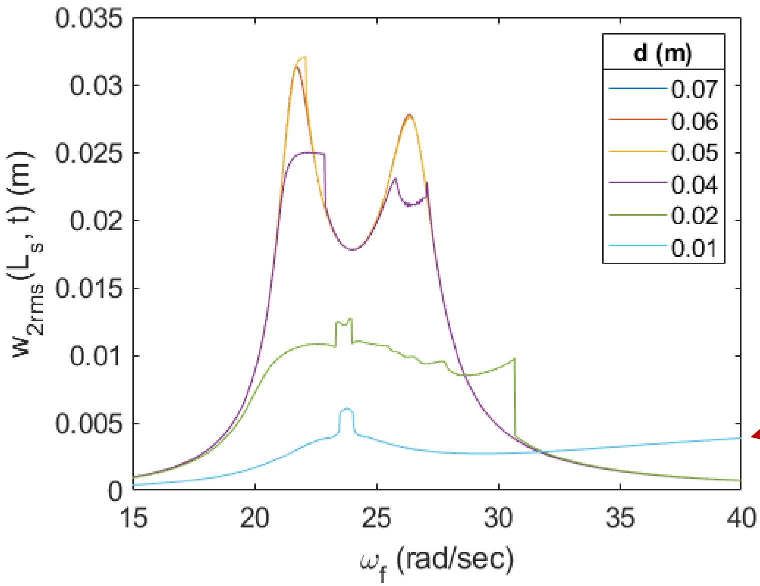


Results – Magnetic stoppers' gap effects

Parameter Values	
L_c	2 cm
h_s	0.5 mm
h_p	0.267 mm
R	$10^{4.73}\ \Omega$
L	18 cm
L_s	18 cm
M_t	0.048 kg
τ	$4\pi * 10^{-7}$ H/m
a_1	$0.0192\ Am^2$
a_2	$0.0192\ Am^2$







The gap of 1cm sees a shift of the absorber's natural frequency to 45 rad/s

Conclusions

- Mechanical stoppers with a stiffness of $5 * 10^3\ N/m$ increases the peak and average energy harvested while maintaining great control of the primary structure's amplitude.
- Mechanical stoppers with a stiffness of $5 * 10^5\ N/m$ has a decrease in energy harvested, as well as aperiodic regions developing.
- Magnetic stoppers with a gap of 0.05 m sees a small increase in peak power
- Magnetic stoppers with gaps smaller than 0.05 m have sharp decrease in peak power and aperiodic regions present
- Magnetic stoppers with a gap of 0.01 m has a shift in the absorber's natural frequency, detuning with the primary structure.

Acknowledgements

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Questions

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