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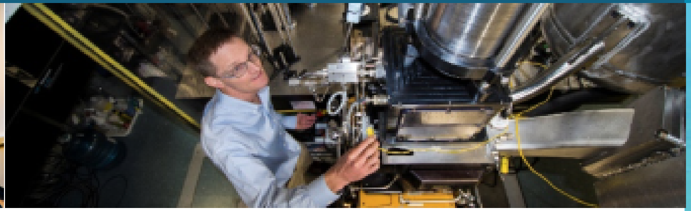
Sandia National Laboratories

Grid Tailored Reduced-Order Models for Steady Hypersonic Aerodynamics



PRESENTED BY

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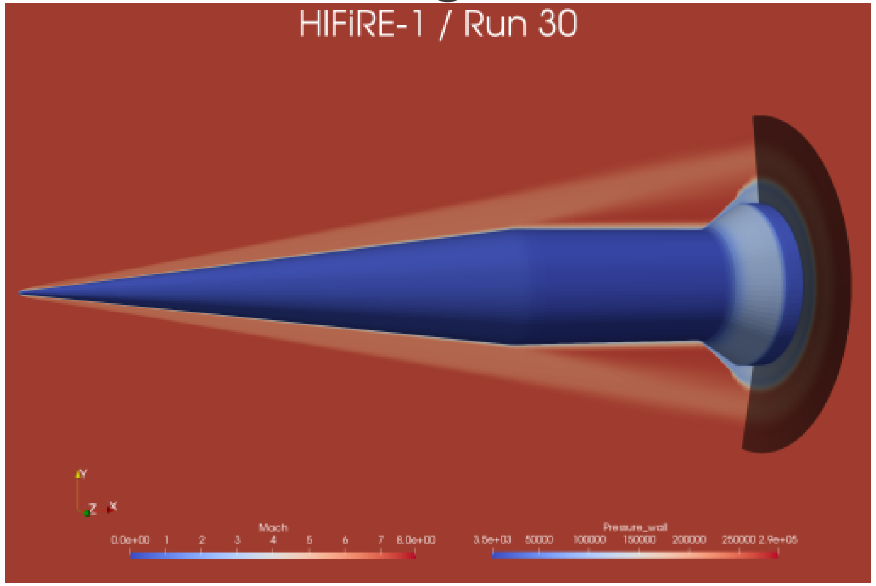
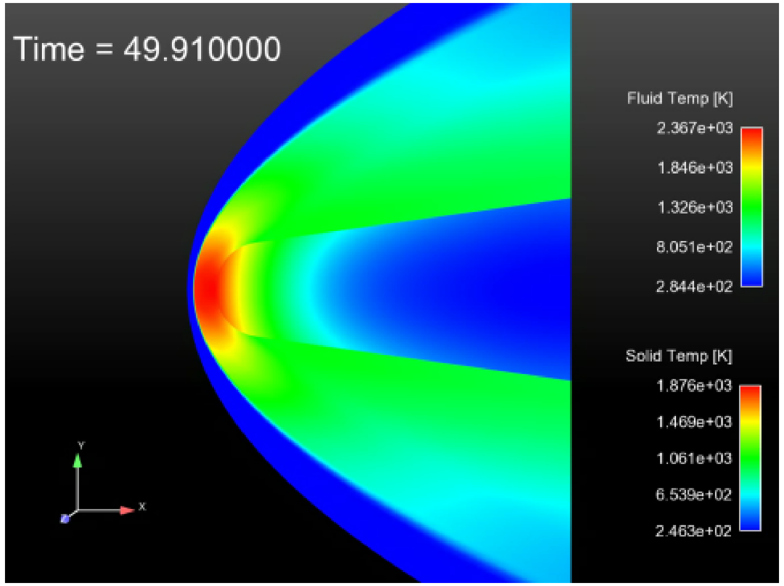
RANS simulations are crucial for hypersonic vehicle analysis and design



Mach number and pressure contours for the HIFiRE-1
(Courtesy M. Howard, SNL)



Temperature of a slender body in hypersonic flow
(Courtesy M. Howard, SNL)

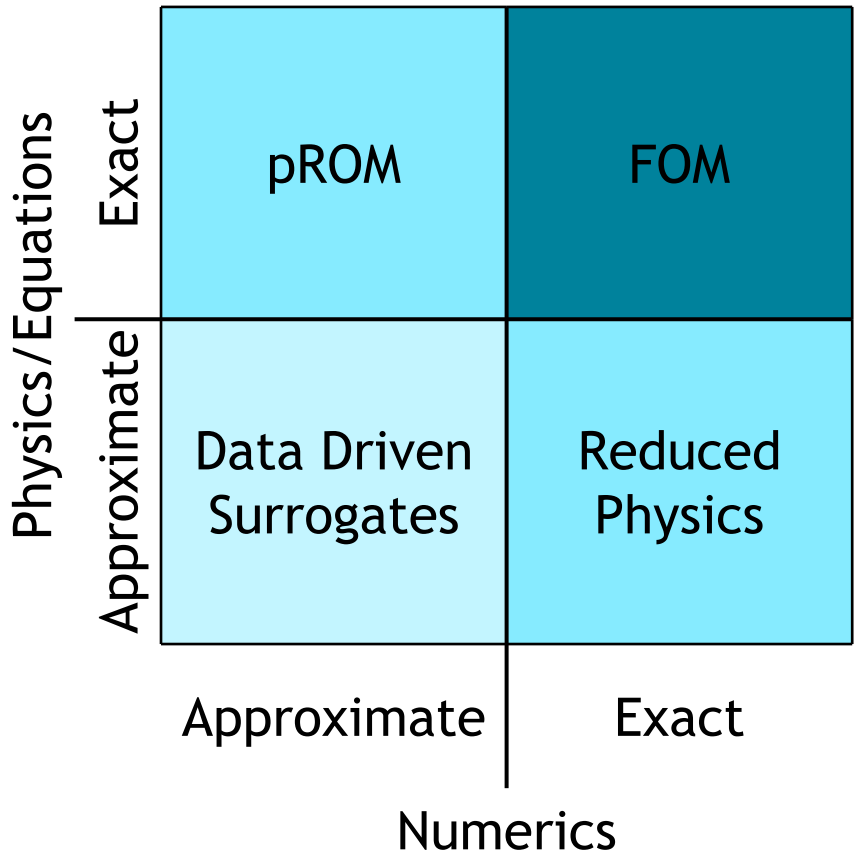
- RANS is our large-scale, nonlinear full-order model (FOM).
  - High cost: requires <u>hundreds</u> or <u>thousands</u> of CPU hours.

- High cost creates a “**computational barrier**” to the application of many-query and/or time -critical problems:
  - **Many-Query**: Design Optimization, Model Calibration, Uncertainty Propagation
  - **Time-Critical**: Path Planning, Model Predictive Control, Health Monitoring

A surrogate model is needed to break the computational barrier!

We use Projection-based Reduced Order Models (pROMs)

- pROMs are “physics-based” surrogates
  - Results are **explainable**
  - Evaluating **known dynamics** rather than *learning* unknown dynamics
- *A priori* and *a posteriori* error bounds
  - **Quantifying the uncertainty of the pROM is critical** for Sandia’s missions
- Hypersonic aero surrogates: mostly kriging or reduced physics
  - **[Blonigan et al. 2021]: LSPG for hypersonic RANS solver**

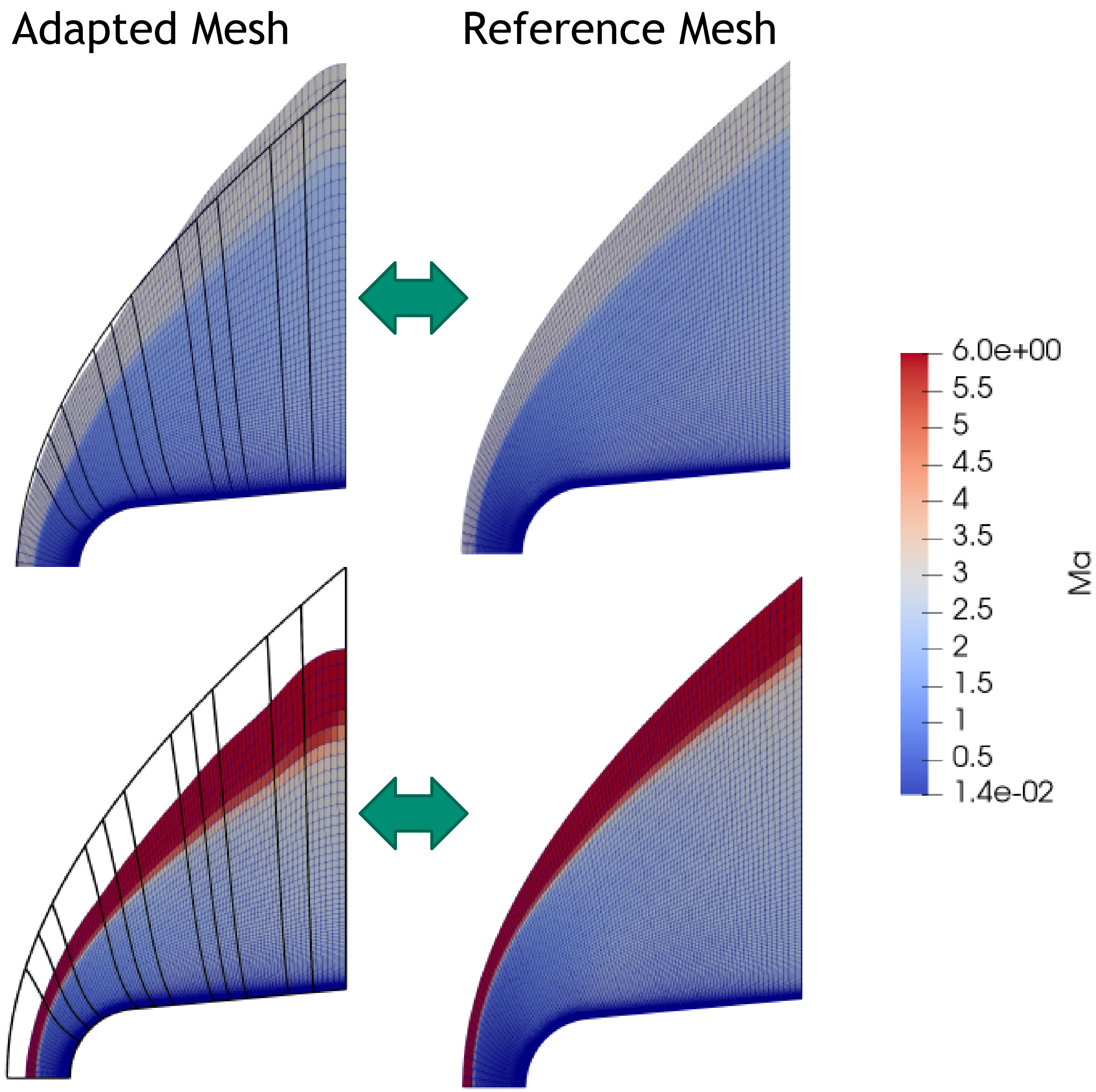


- Projection-based ROM implementations are very intrusive, difficult to set up
- <u>Steady simulations do not provide much training data</u>
- <u>Linear basis representations are not well suited for shock-dominated flows</u>

Our approach: leverage r-adaptation for improved basis quality

- We desire a basis that does not require much training data and can represent shock waves

- Idea: set up FOM to adapt mesh via grid-tailoring [Vinokur, 1983]
  - Bow shock wave is a fixed number of cells from the inlet
  - For FOM: Allows for greater robustness/accuracy when running at multiple inflow conditions and results in more accurate heat flux
  - For ROM: **R-adaptation provides mesh displacement data which can be used to create a mapping from a reference mesh to a tailored mesh**
    - Registration-based ROMs [Taddei 2020, Nair and Balajewicz 2019, Zahr et al. 2020, …].

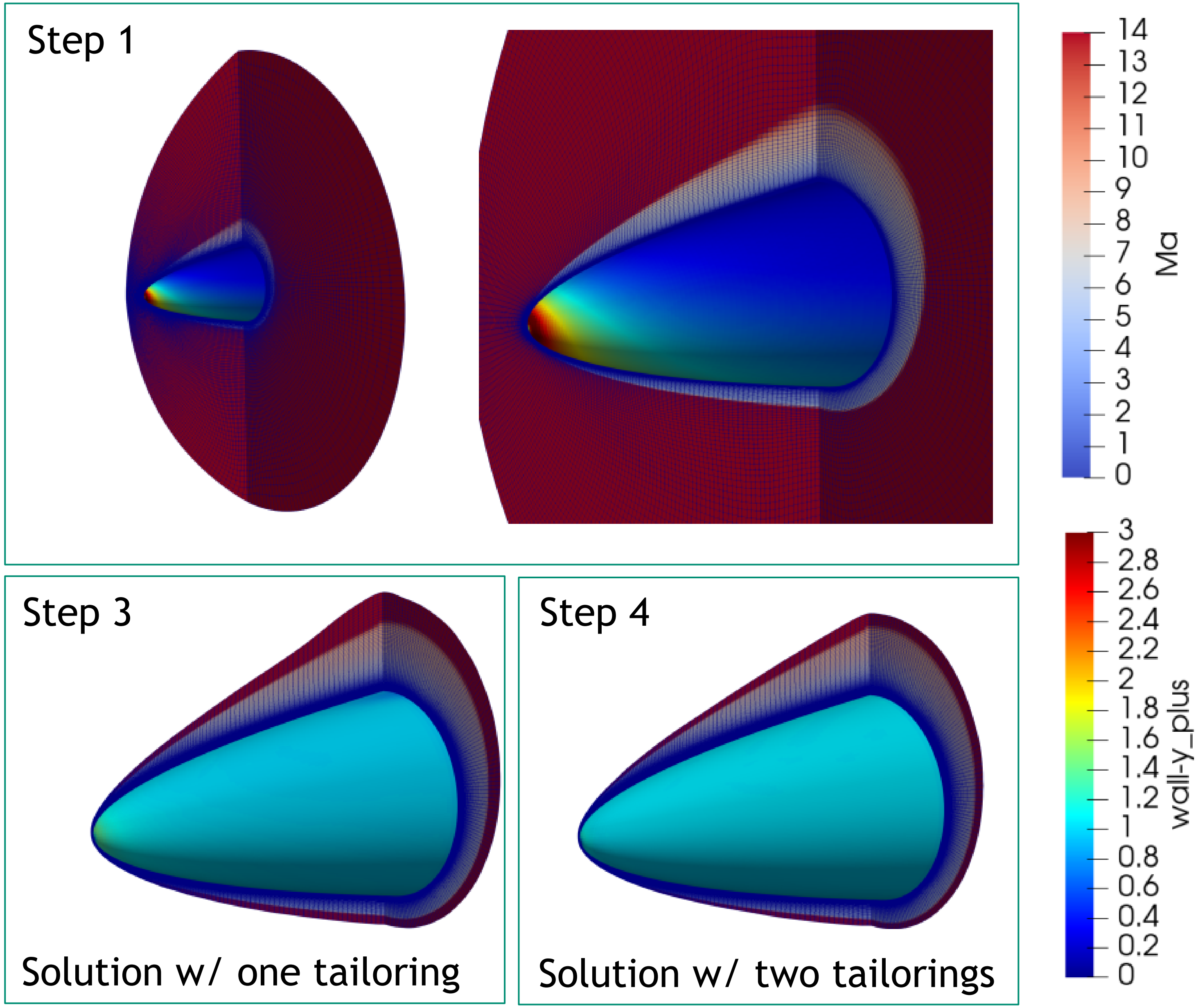


R-adaption is conducted with Grid-Tailoring [Vinokur, 1983]

- Implemented in US3D, SPARC

1. Run pseudo-time solver until shock location is steady
2. Determine shock location, boundary layer resolution
3. Redistribute cells so that
   - Shock is N cells away from the inlet
   - Boundary layer resolution requirements are satisfied
4. Repeat as needed

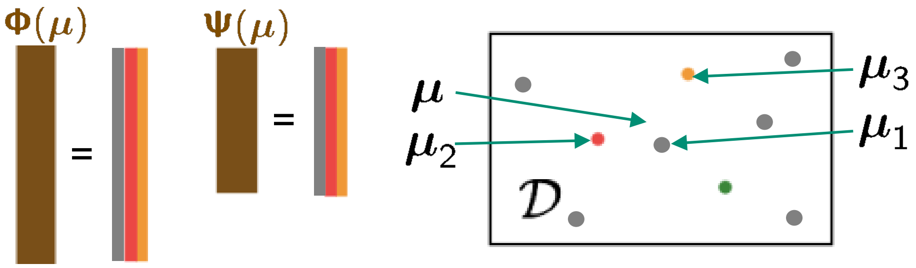
- Requires lines from wall to inlet to adapt along
- Aligns grid to outer-most shock
  - Will not address secondary shocks!



Our approach: perturb the mesh, then compute the pROM

- FOM Residual: $\mathbf{r}(\mathbf{x}; \mathbf{z}, \boldsymbol{\mu}) = \mathbf{0}$
  - **x**: state vector
  - **z:** nodal displacements for mesh
  - **μ:** input parameters
- Offline: solve FOM at various input parameters
- Online:
  1. Set up local basis for state, mesh displacements
  2. RBF interpolation over local basis to obtain mesh displacement
  3. RBF interpolation over local basis to obtain state initial guess
  4. Apply Least-squares Petrov—Galerkin (LSPG) projection with reduced basis on the **morphed mesh**

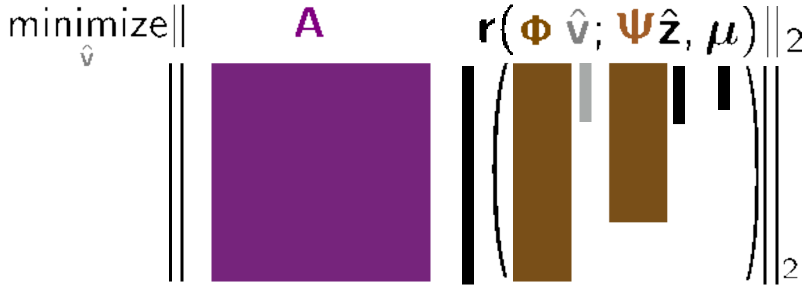
1. **Local Dictionary basis**



2. Interpolate for mesh deformation: $\hat{\mathbf{z}}(\boldsymbol{\mu}) = \sum_{i=0}^{N} \beta_i(\boldsymbol{\mu})\hat{\mathbf{z}}(\boldsymbol{\mu}_i) \Rightarrow \mathbf{z} = \boldsymbol{\Psi}(\boldsymbol{\mu})\hat{\mathbf{z}}(\boldsymbol{\mu})$

3. Interpolate for initial guess: $\hat{\mathbf{x}}^{IG}(\boldsymbol{\mu}) = \sum_{i=0}^{N} \alpha_i(\boldsymbol{\mu})\hat{\mathbf{x}}^{IG}(\boldsymbol{\mu}_i) \Rightarrow \mathbf{x}^{IG} = \boldsymbol{\Phi}(\boldsymbol{\mu})\hat{\mathbf{x}}^{IG}(\boldsymbol{\mu})$

4. Solve Residual minimization for $\hat{\mathbf{v}}$



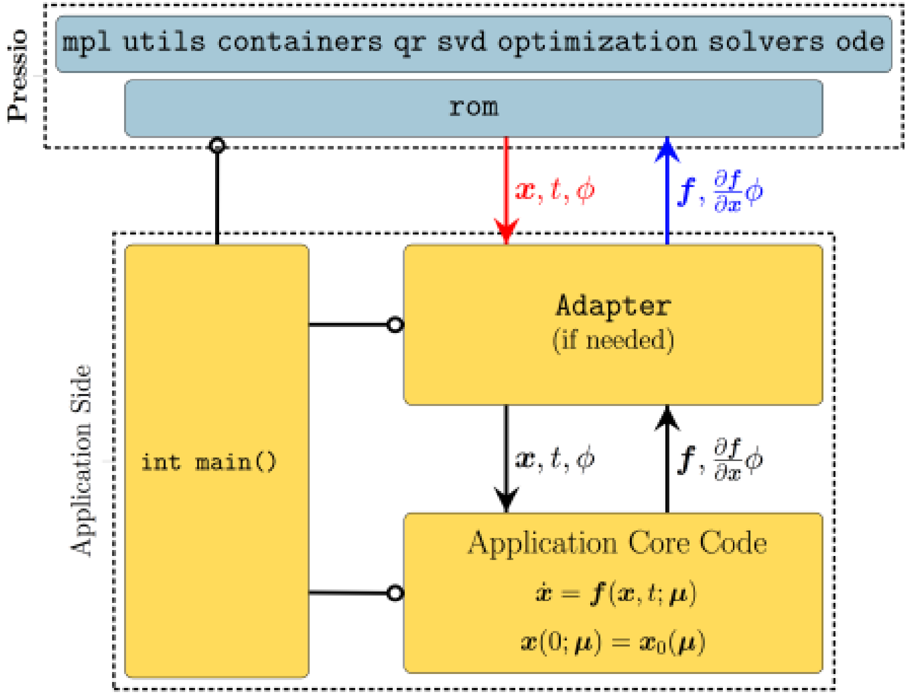
Codes

Sandia Parallel Aerodynamics and Reentry Code (SPARC)

- Compressible CFD code focused on aerodynamics and aerothermodynamics in the Transonic and Hypersonic regimes
- Emphasis on performance portability
- See AIAA 2017-4407 for more details.

Pressio

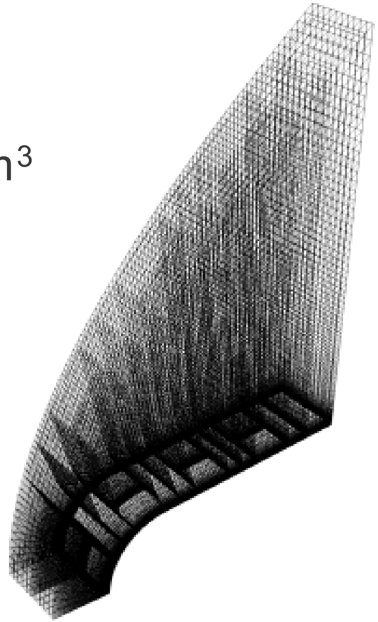
- Minimally intrusive API for model reduction
- Open source! https://github.com/Pressio

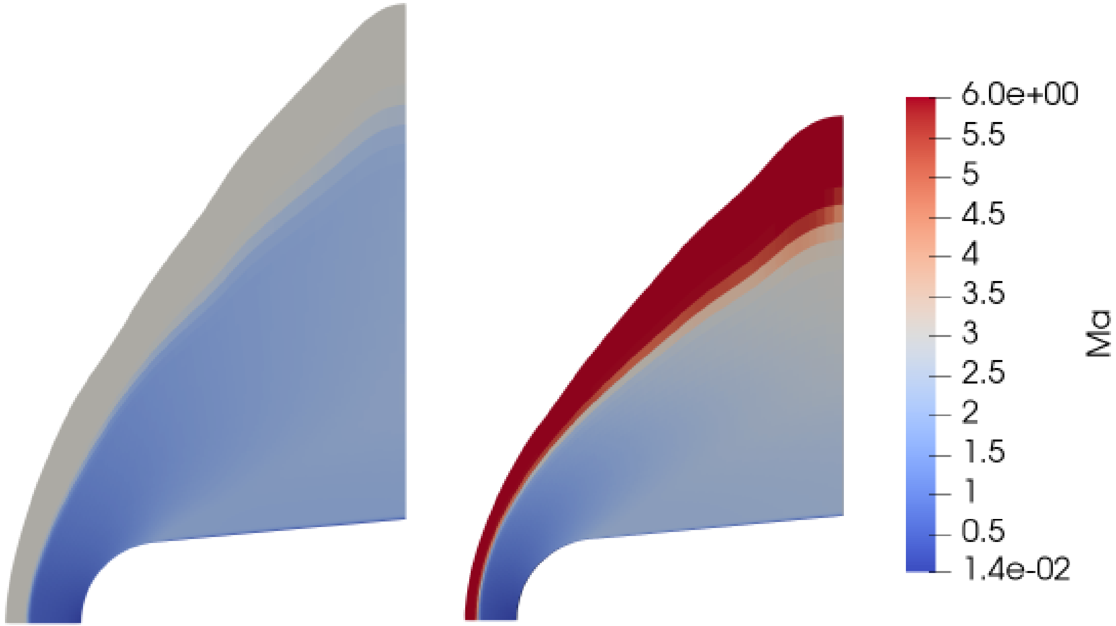


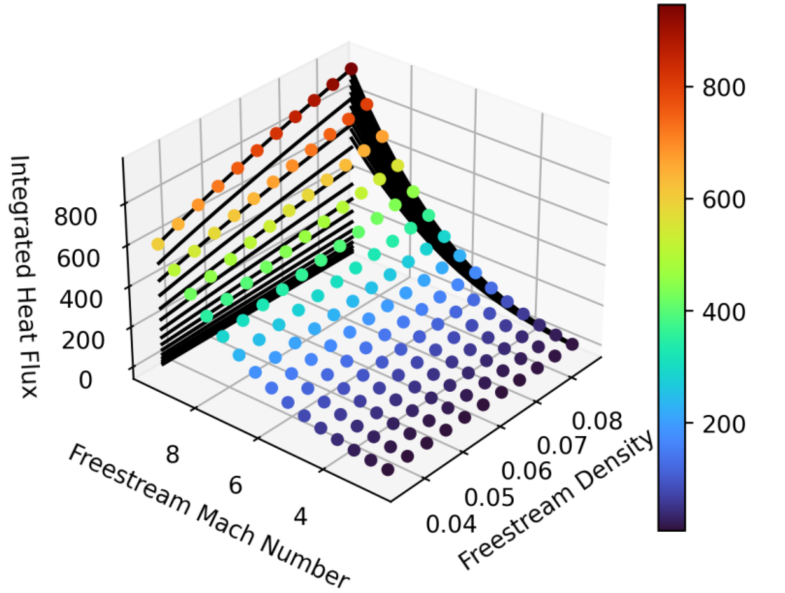
Schematic of Pressio software workflow

Test case: two-dimensional wedge geometry

- Parameters:
  - Freestream Mach number: 3.0 to 9.5
  - Freestream Density: 0.03 to 0.09 kg/m^3
- FOM:
  - 100,000 cell mesh
  - Perfect Gas
  - Laminar flow
  - Steady-state
  - Run grid tailoring twice
- ROMs:
  - Basis constructed from 12 solutions
  - Mesh perturbed using local dictionary basis
  - LSPG with local dictionary basis
    - RBF for initial guess
    - Gauss-Newton with QR-decomposition
  - pROM is 27 to 50 times faster than FOM without hyper-reduction
- Quantities of Interest (QoIs):
  - Axial force
  - Integrated Wall Heat Flux



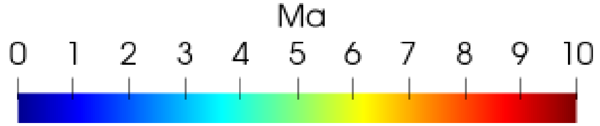




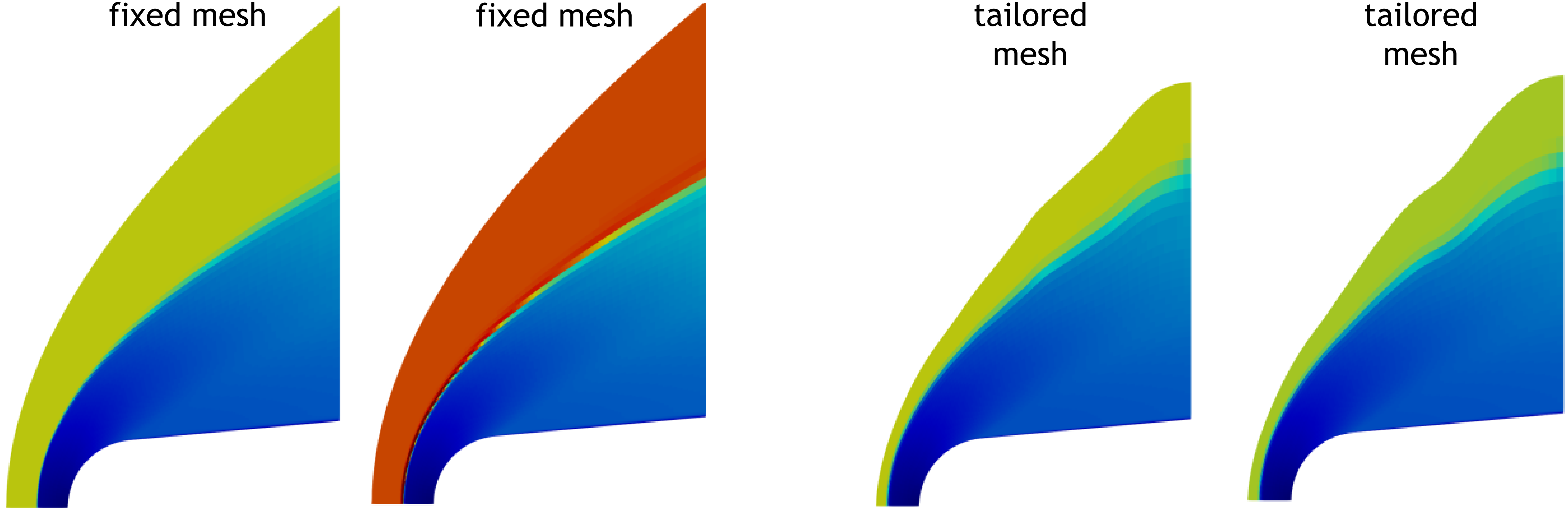
Mesh adaptation can prevent oscillations near shocks

Mach 6.0, Density 0.076 kg/m^3

- ROM basis is 4 nearby FOM solutions



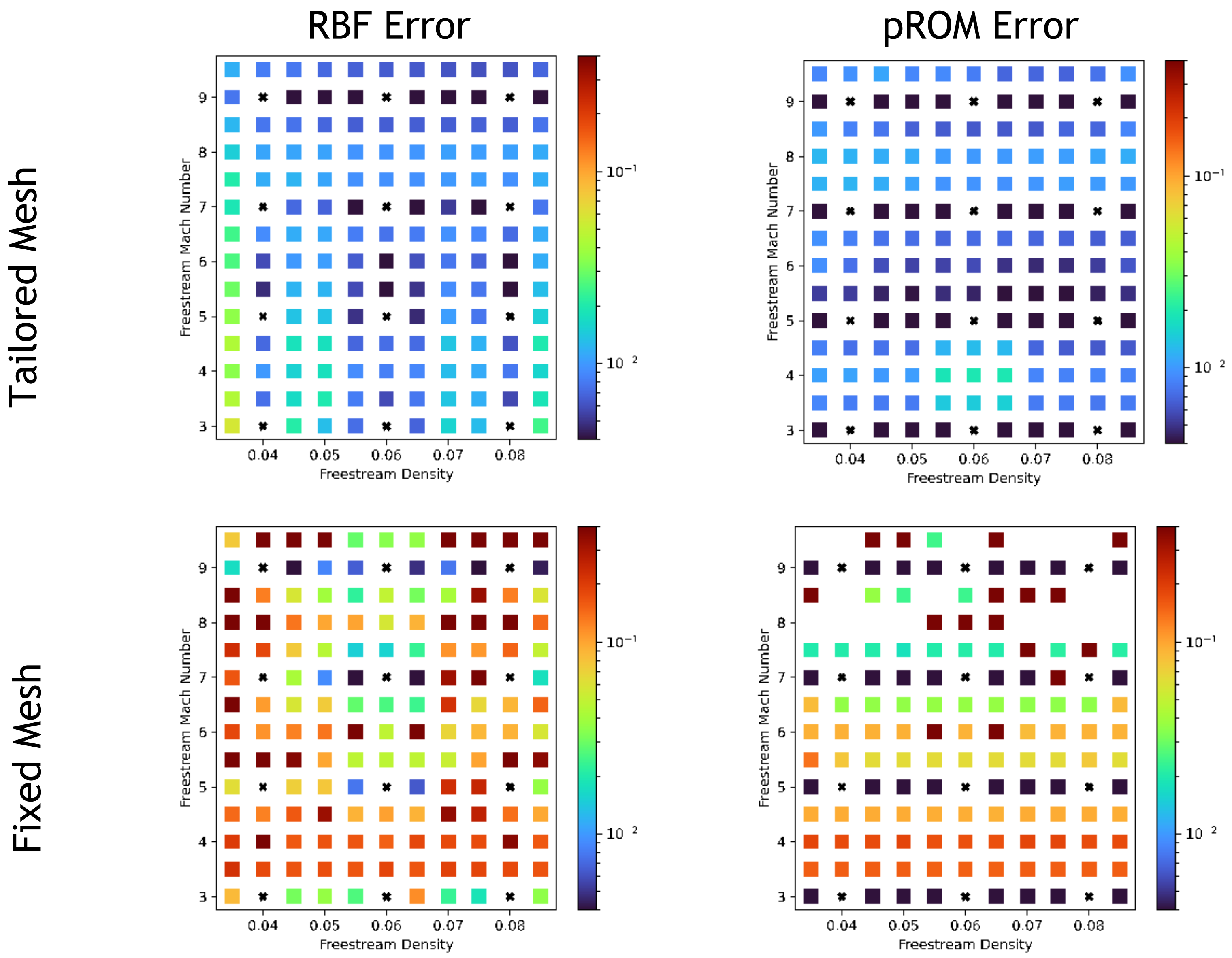
FOM with fixed mesh | pROM with fixed mesh | FOM with tailored mesh | pROM with tailored mesh



Large oscillation near shock without adaptation

No oscillation near shock with adaptation

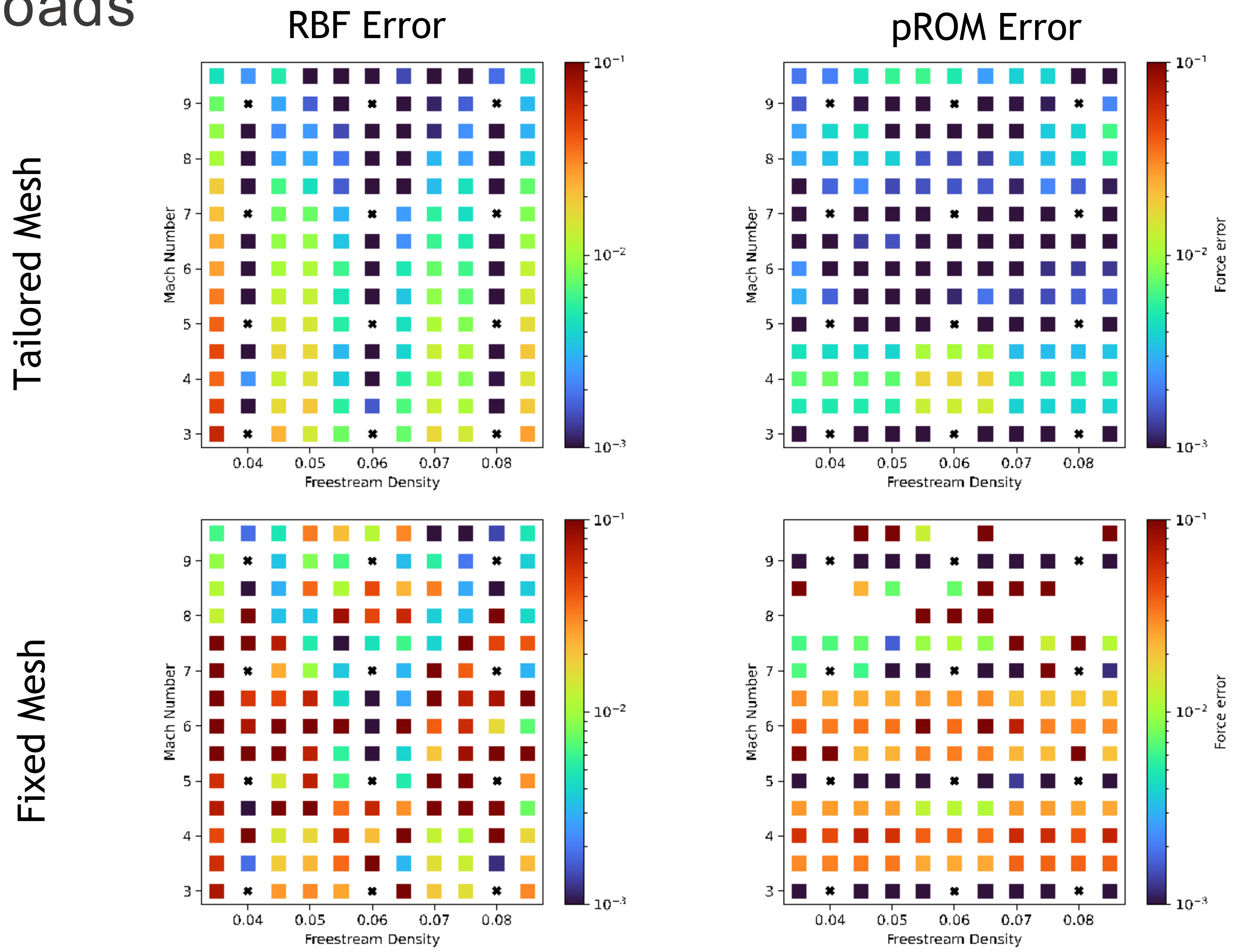
State L2 Error shows benefits of tailored mesh pROM



Mean State Error

	RBF	pROM
Tailored	0.0108	0.0069
Fixed	2052	inf

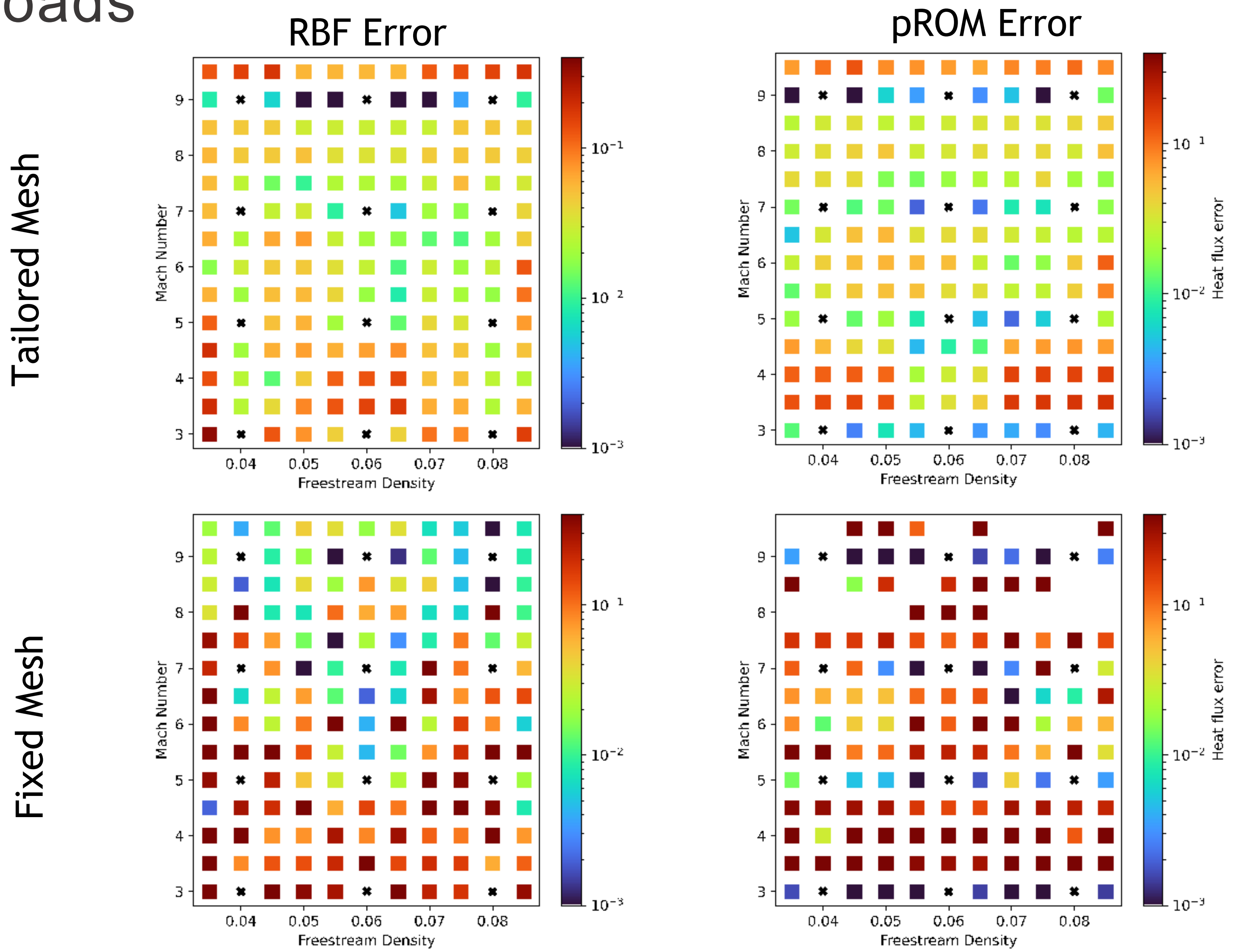
Tailored mesh pROM computes accurate aerodynamic loads



Percentage of cases with $\varepsilon_{F_x} < 1\%$

	RBF	pROM
Tailored	72%	94%
Fixed	35%	32%

Tailored mesh pROM computes most accurate thermal loads

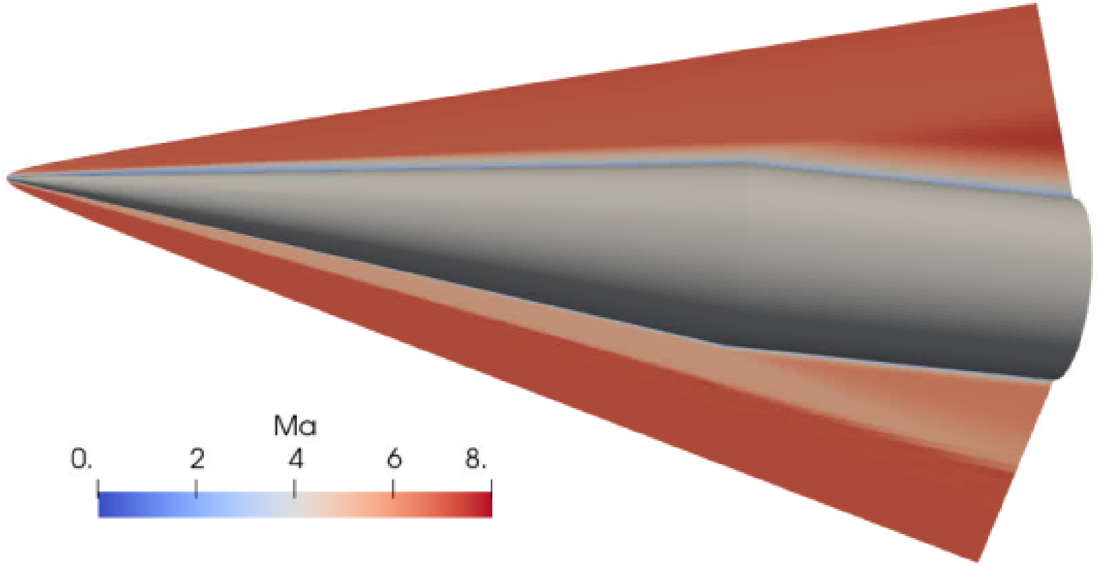


Percentage of cases with $\varepsilon_{Q_{wall}} < 5\%$

	RBF	pROM
Tailored	61%	69%
Fixed	44%	28%

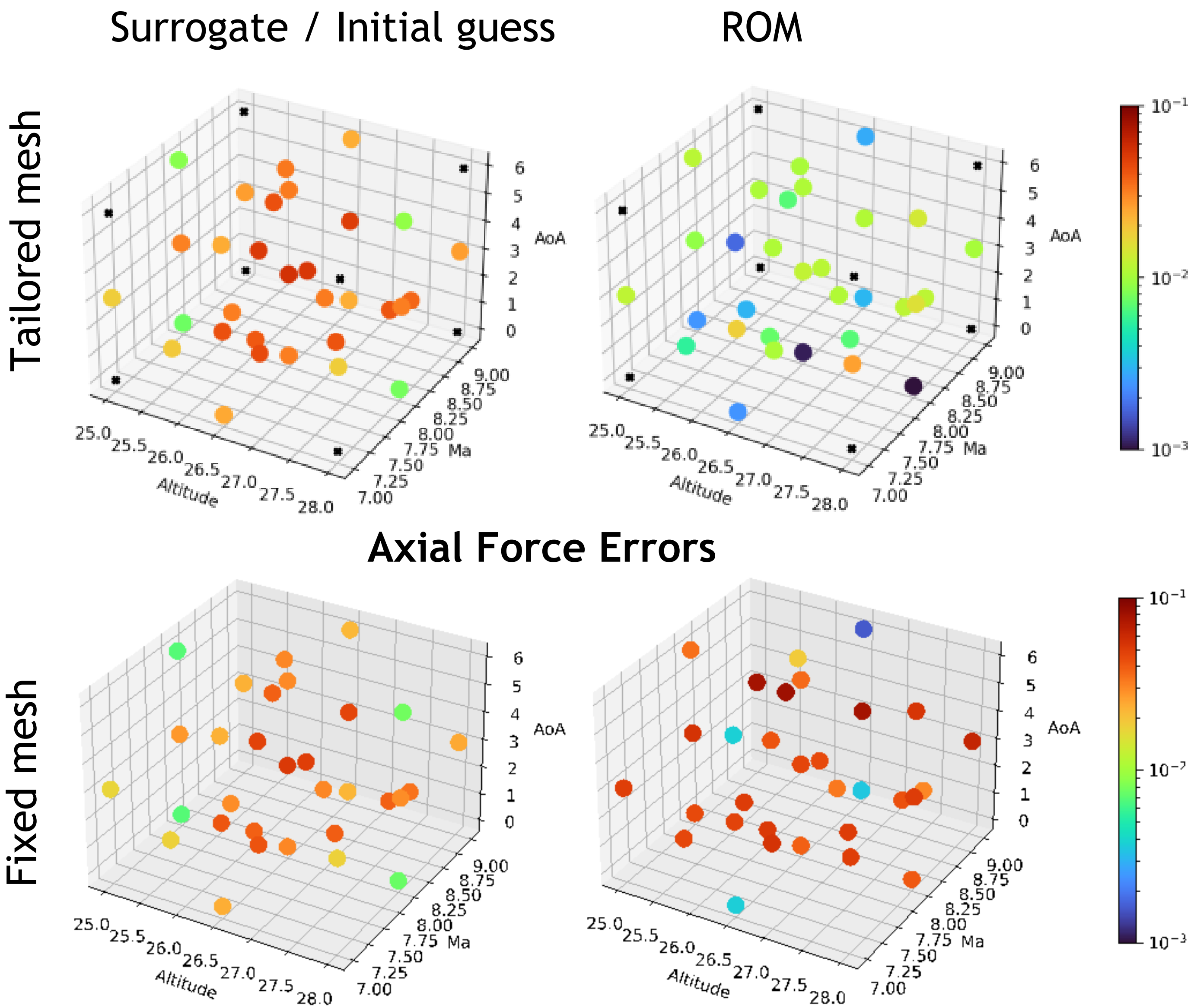
3D HIFiRE-1 case

Demonstrated grid-tailored ROM for 3D HIFiRE-1 with 3 input parameters



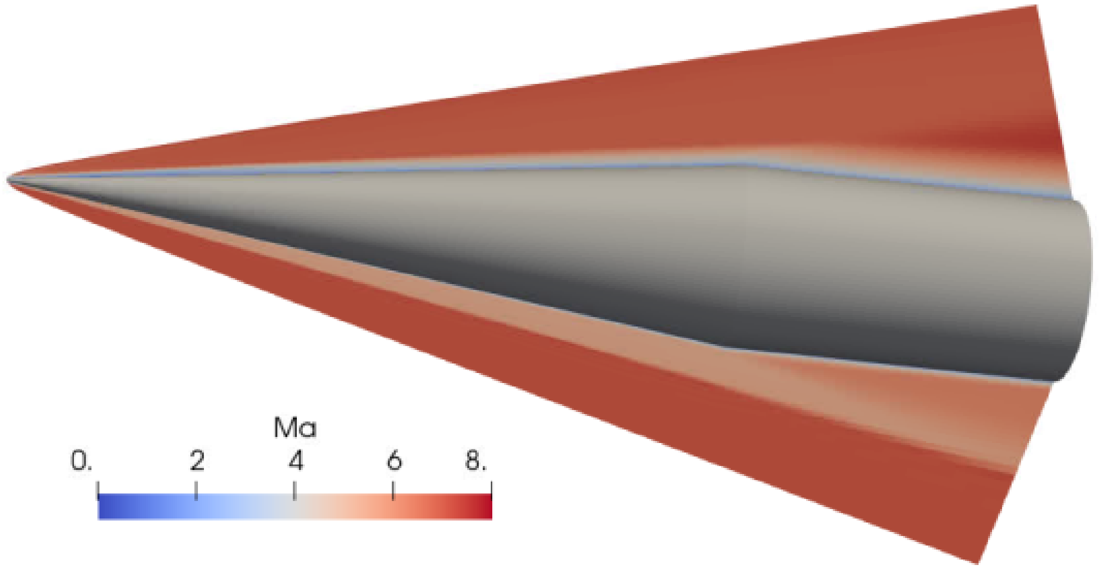
Percentage of cases with $\varepsilon_{F_x} < 1\%$

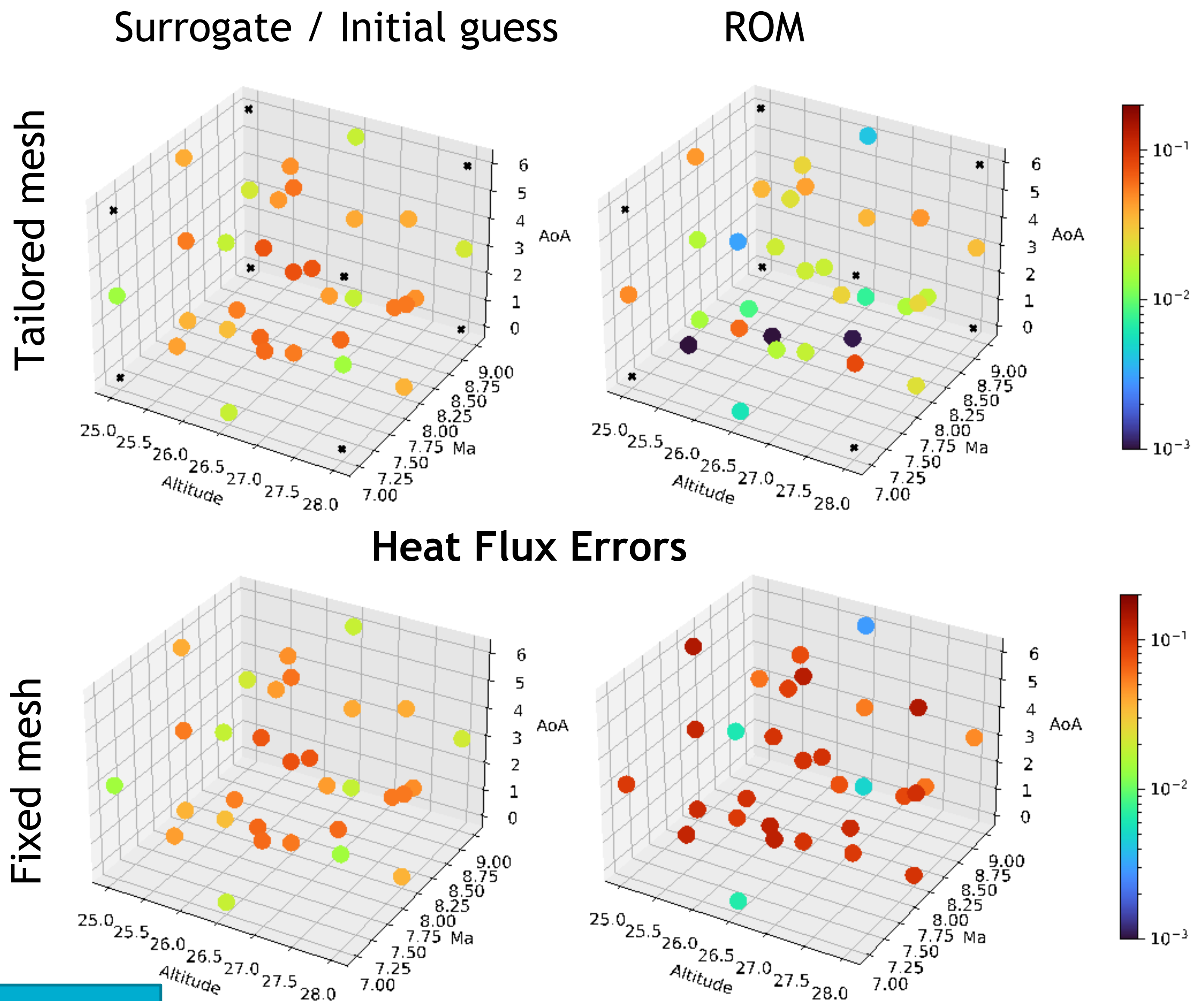
	RBF	pROM
Tailored	13%	41%
Fixed	13%	12%



3D HIFiRE-1 case

Demonstrated grid-tailored ROM for 3D HIFiRE-1 with 3 input parameters





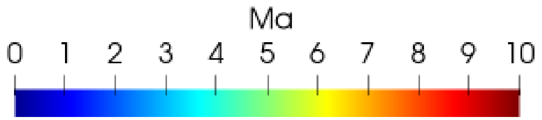
Percentage of cases with $\varepsilon_{Q_{wall}} < 5\%$

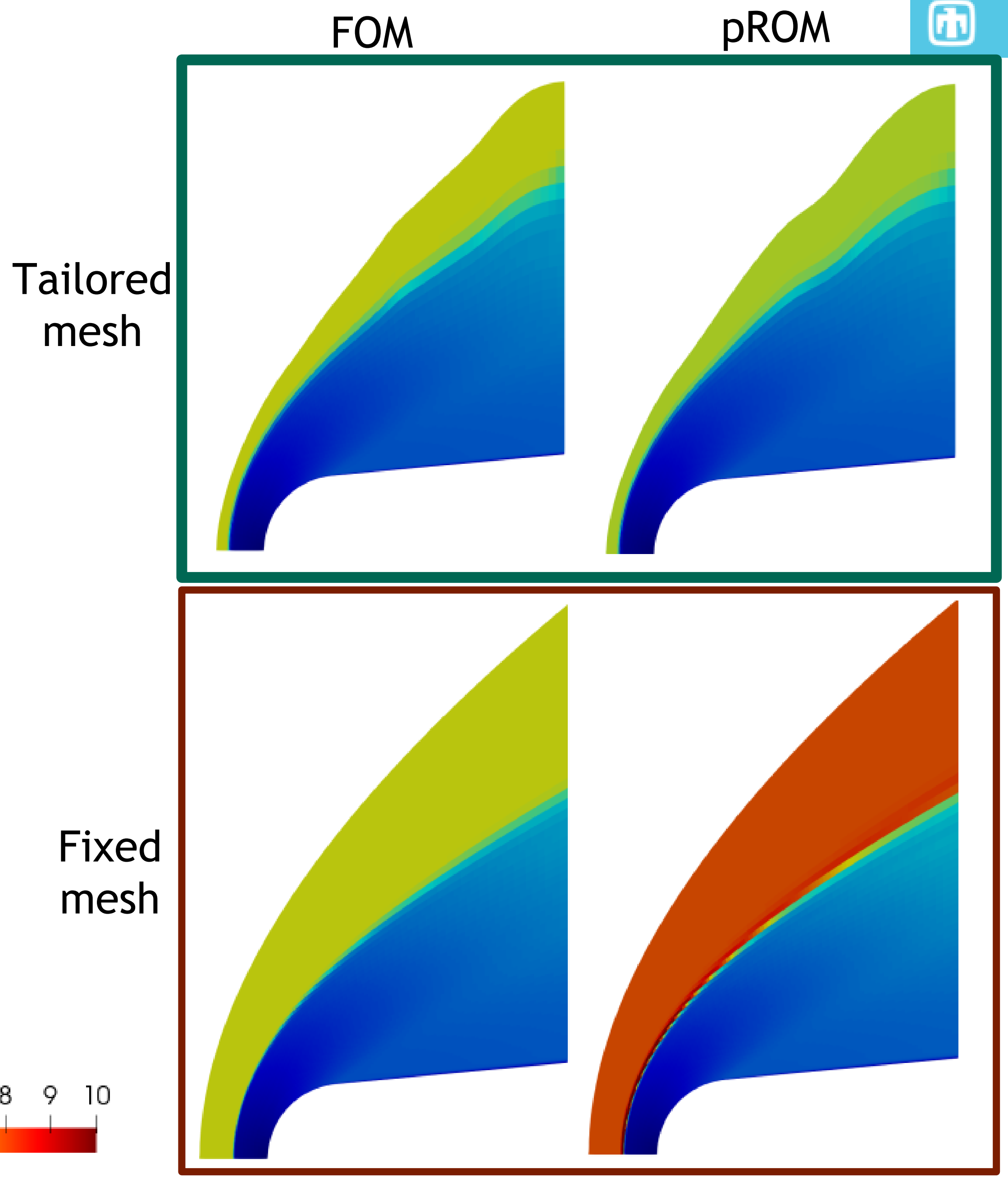
	RBF	pROM
Tailored	61%	93%
Fixed	61%	16%

Grid tailored ROM works for 3D flight vehicle geometries

Conclusions

- Presented results of pROM trained on r-adapted full model data for hypersonic aerodynamics
- For a given training data set, pROM with grid-tailoring is more accurate than RBF snapshot interpolation
- Steady pROMs achieve speed-up even without hyper-reduction





Current and Future Work

Current work:

- Reacting gas flows with non-equilibrium chemistry
- Hyper-reduction for further performance gains
- L1 residual minimization

Future work:

- Further increase ROM robustness
- Integration of mesh perturbations into ROM residual minimization
- Generalization of grid-tailoring for more complex geometries and flows

Relevant Papers

- Blonigan P., Rizzi F., Howard M., Fike J., and Carlberg K., “Model Reduction for Steady Hypersonic Aerodynamics via Conservative Manifold Least-Squares Petrov–Galerkin Projection”, AIAA Journal 2021 59:4, 1296-1312
- Rizzi F., Blonigan P., and Carlberg K., “Pressio: Enabling projection-based model reduction for large-scale nonlinear dynamical systems,” Preprint: arXiv:2003.07798, 2021.
- https://github.com/Pressio
- Howard M., Bradley A., Bova S., Overfelt J., Wagnild R., Dinzl D., Hoemmen M., and Klinvex A., “Towards Performance Portability in a Compressible CFD Code,” 23rd AIAA Computational Fluid Dynamics Conference, AIAA Paper 2017-4407, 2017.
- Vinokur M., “On one-dimensional stretching functions for finite-difference calculations,” Journal of Computational Physics, Volume 50, Issue 2, 1983, Pages 215-234, ISSN 0021-9991
- Nair, N., Balajewicz, M., “Transported snapshot model order reduction approach for parametric, steady-state fluid flows containing parameter-dependent shocks”. Int. J. Numer. Methods Eng. 2019; 117: 1234– 1262.