This paper describes objective technical results and analysis. Any subjective views or opinions that might be expressed in the paper do not necessarily represent the views of the U.S. Department of Energy or the United States Government.

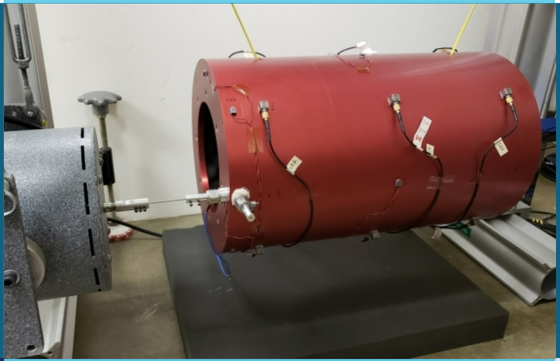
SAND2021-15662C

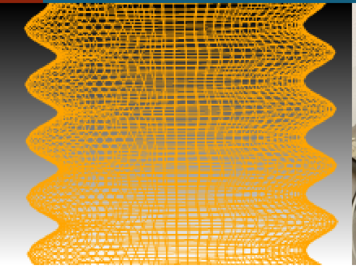
N-O-MAD Research Institute

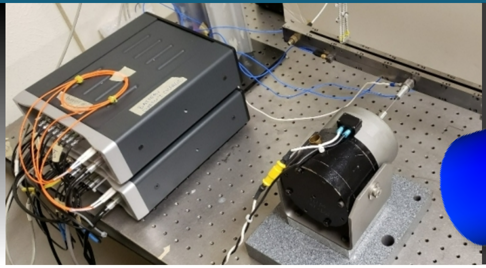
Sandia National Laboratories

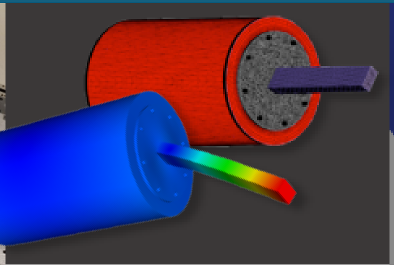
THE UNIVERSITY OF NEW MEXICO

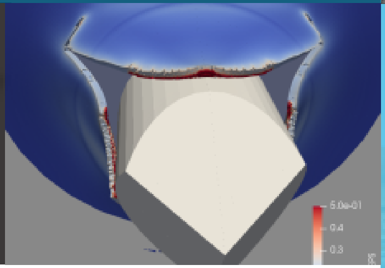
Experimental and computational investigation of nonlinear dynamics of a simplified bearing-and-shaft assembly











Presented by:

Nathaniel Goldberg, University of California-Berkeley

Co-authors:

Sarah Demsky (MIT), Abdelrahman Youssef (TAMU), Steven Carter (SNL), Deborah Fowler (SNL), Nathan Jackson (UNM), Robert Kuether (SNL), Andrew Steyer (SNL)

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

U.S. DEPARTMENT OF ENERGY NNSA National Nuclear Security Administration

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

es here

Background

Low-amplitude vibrations

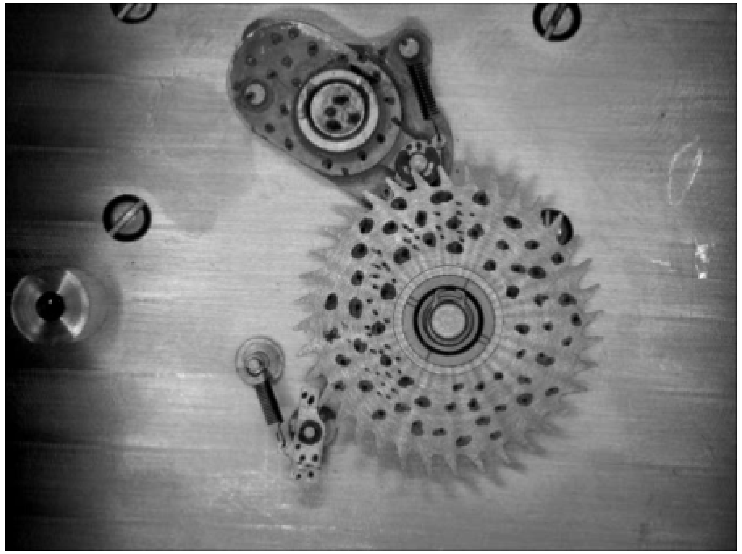
1. Long-duration random vibration
2. Linear responses produced
3. Classical modal analysis applicable

High-amplitude vibrations

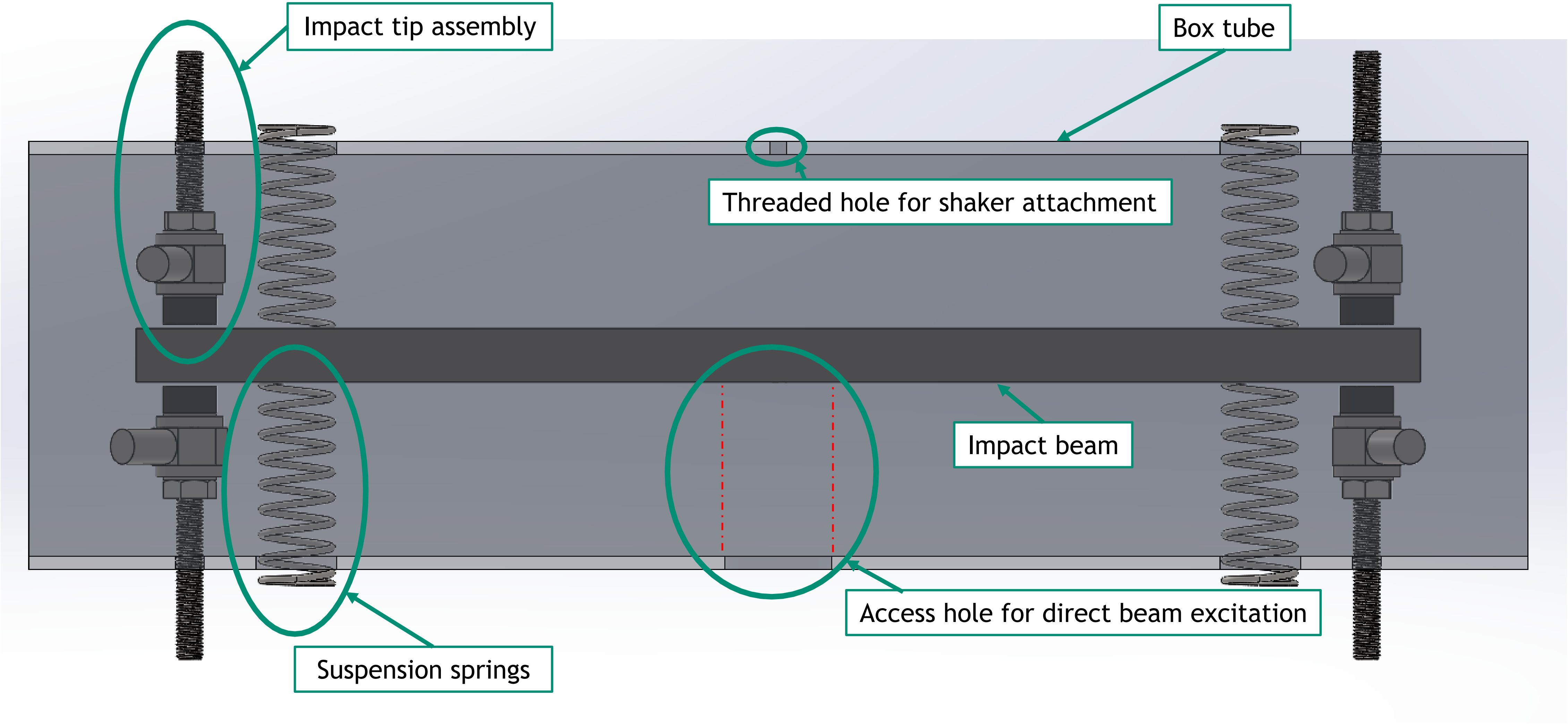
1. Short-duration mechanical shock
2. Nonlinear responses produced
3. Classical modal analysis not applicable*

- Many electromechanical assemblies have sources of nonlinearity stemming from contact impacts
  - Gears, roller bearings, clearance within small mechanical envelopes, etc..
- Nonlinearities limit or invalidate applicability of linear modal analysis techniques

Ratcheting Mechanism



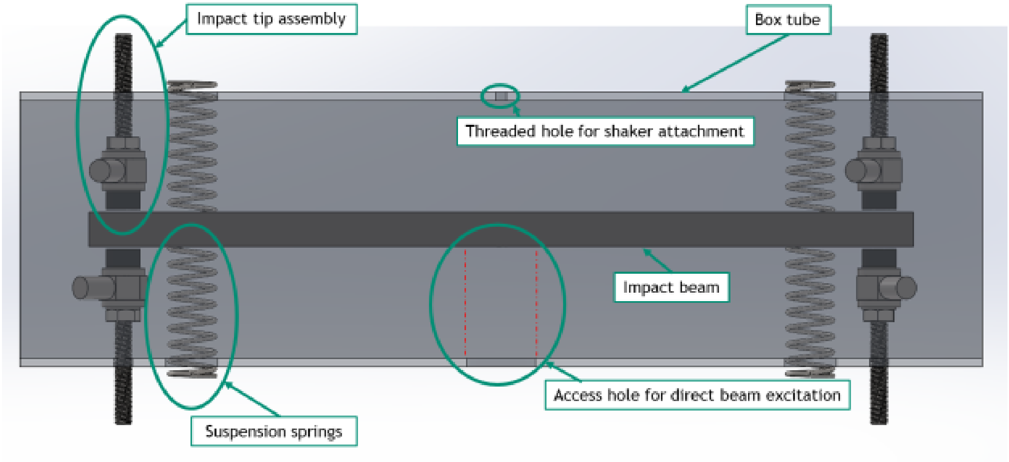
Experimental Apparatus – Simplified Bearing-and-Shaft Assembly



Hierarchy of Models

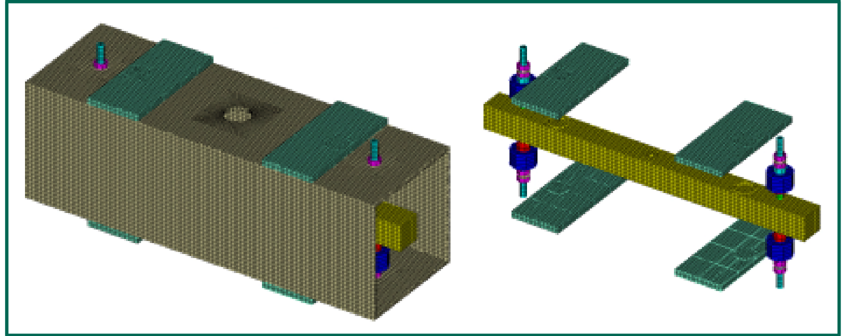
Shared characteristics:

- Symmetric pseudo-rigid body mode
- Antisymmetric pseudo-rigid body mode
- Symmetric bending mode

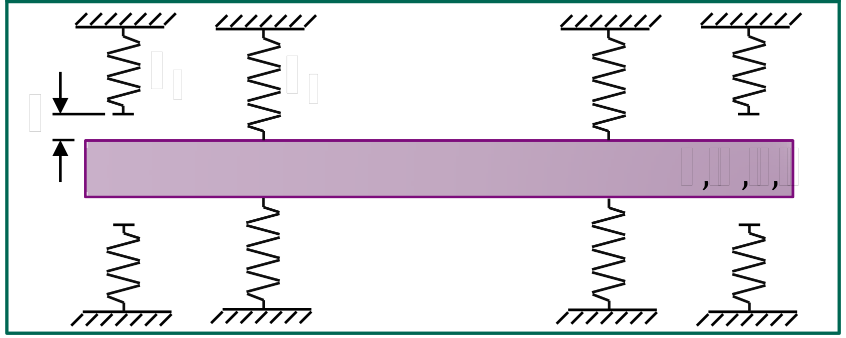


CAD of experimental apparatus

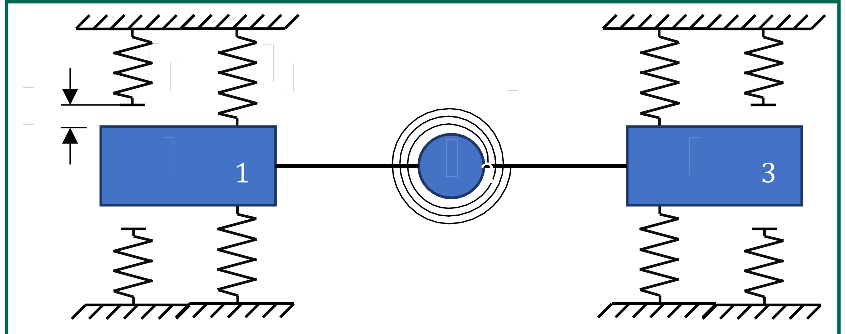
Finite Element Model



Euler-Bernoulli Beam Model

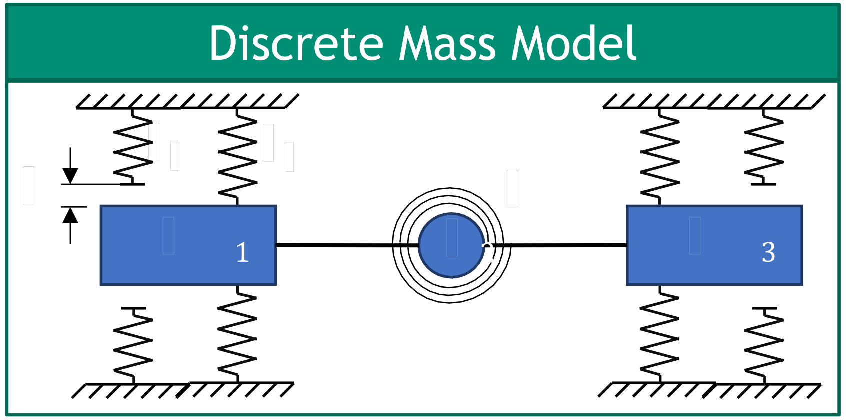


Discrete Mass Model



Objective

- Investigate the nonlinear dynamics in response to high-amplitude broadband excitation
- Identify the spectral content of the ring-down response and correlate to the underlying nonlinear normal modes of the system
  - Key Question – how to treat boundary conditions between impacts? Linearize about open/closed state? Or model as nonlinear displacement dependent boundary condition?
- Observe how the low frequency pseudo-rigid body modes evolve into higher-frequency elastic modes of the structure



Low-energy vibrations:

- No impacts occur
- Linear response

High-energy vibrations:

- Impacts make response nonlinear
- Frequency and energy are related

Nonlinear Normal Mode Theory

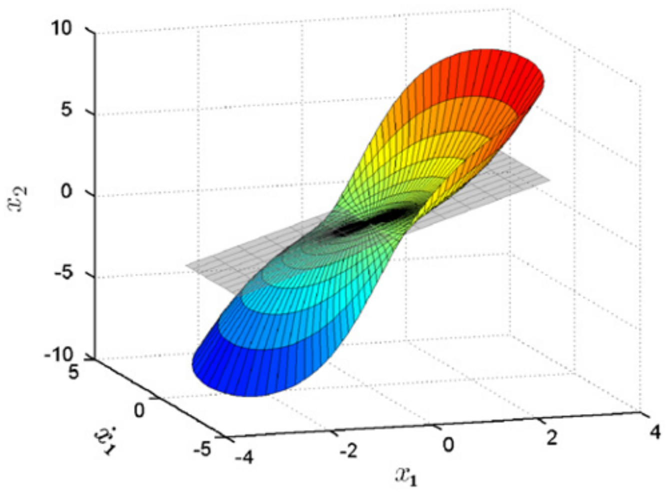
Many definitions exist for either damped or undamped systems [1-4]

For a conservative (undamped) system, a nonlinear normal mode (NNM) is defined as a ***not necessarily synchronous periodic response of the undamped nonlinear system***

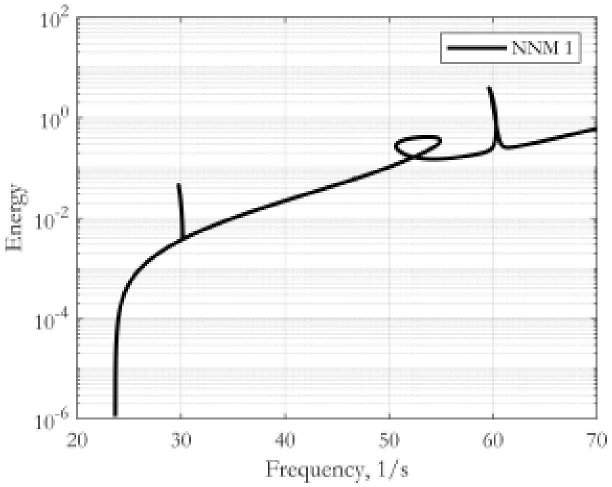
$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) + \mathbf{f}_{nl}\big(\mathbf{x}(t)\big) = \mathbf{0}$$

For an MDOF system, there exists *N* NNM solution branches that are extensions of linear normal modes at low energy [2]

Nonlinear Normal Modes



Mode Shapes
(Manifolds from [2])



Frequency-Energy Plots

[1] Rosenberg, R.M., *The normal modes of nonlinear n-degree-of-freedom systems*. Journal of Applied Mechanics, 1962.
[2] Kerschen, G., et al., *Nonlinear normal modes. Part I. A useful framework for the structural dynamicist.* Mechanical Systems and Signal Processing, 2009.
[3] Shaw, S.W., Pierre, C., *Non-linear normal modes and invariant manifolds*. Journal of Sound and Vibration, 1991.
[4] Haller, G., Ponsioen, S., *Nonlinear normal modes and spectral submanifolds: Existence, uniqueness, and use in model reduction*. Nonlinear Dynamics, 2016.

Equations of Motion

Assuming small rotations in torsional spring (undamped):

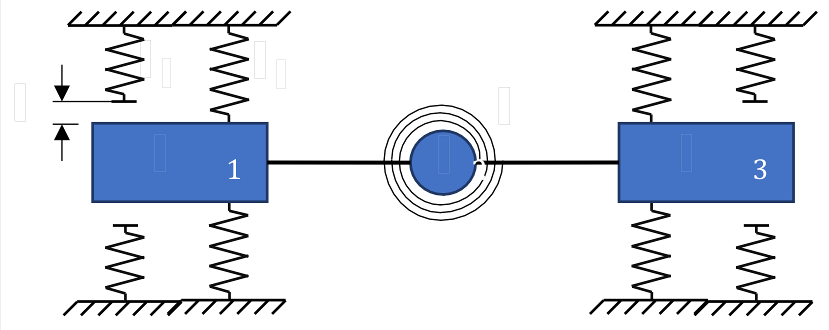
$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} 2k_s + \kappa/L^2 & -2\kappa/L^2 & \kappa/L^2 \\ -2\kappa/L^2 & 4\kappa/L^2 & -2\kappa/L^2 \\ \kappa/L^2 & -2\kappa/L^2 & 2k_s + \kappa/L^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} f(x_1) \\ 0 \\ f(x_3) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

where

$$f(x) = \begin{cases} k_g(x-g), x > g \\ 0, -g \le x \le g \\ k_g(x+g), x < -g \end{cases}.$$

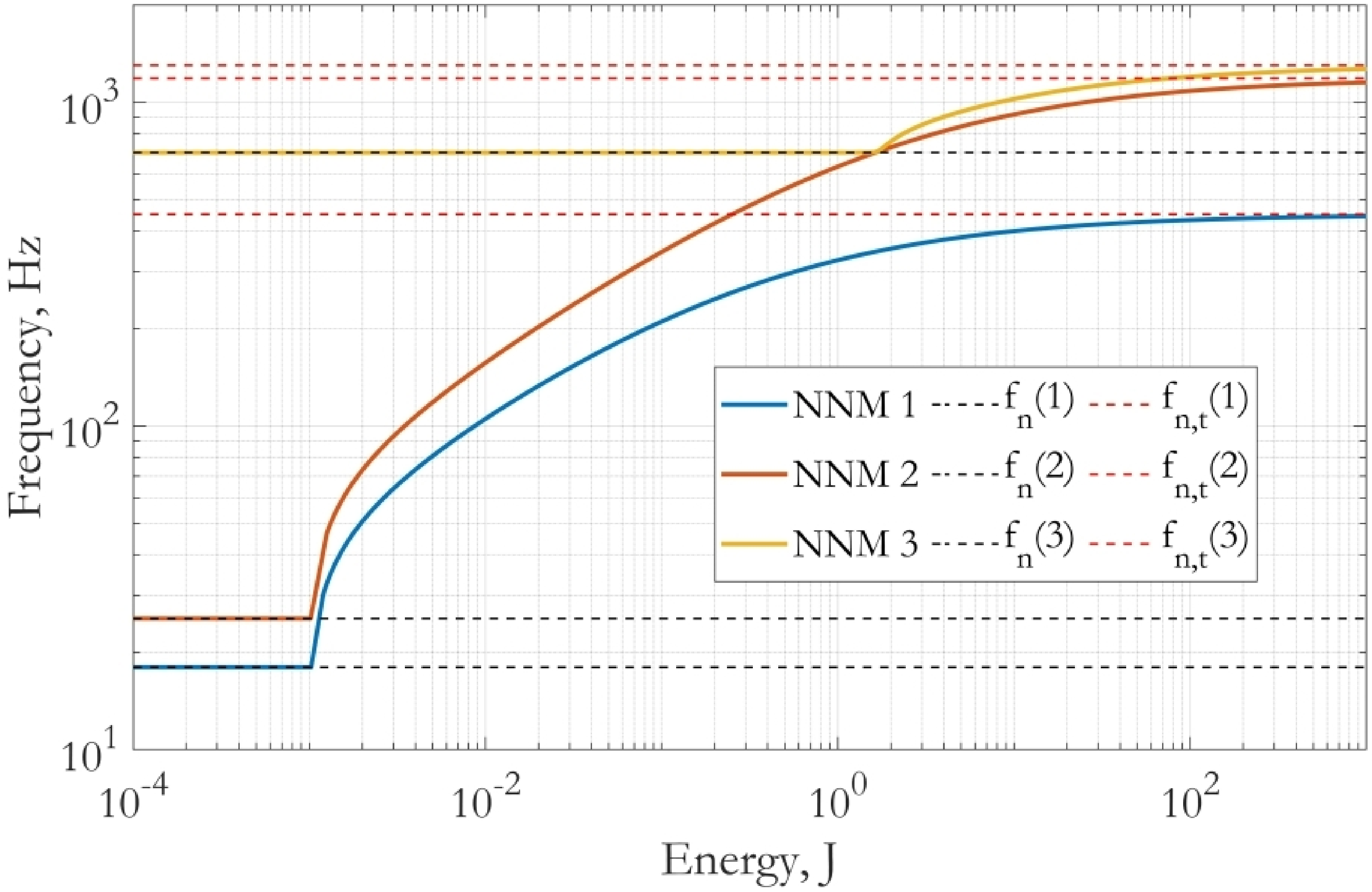
For transient simulations, we include proportional damping matrix $c\boldsymbol{I}$.

Discrete Mass Model



Description	Symbol	Value
Suspension spring stiffness	k_s	8.03e3 N/m
Torsional spring stiffness	κ	79161 N m/rad
Gap spring stiffness	k_g	3.502e7 N/m
Half-length	L	0.1614 m
Gap	g	2.54e-4 m
Left mass	m_1	0.629 kg
Middle mass	m_2	1.258 kg
Right mass	m_3	0.629 kg
Damping coefficient	c	4 N s/m

Approximate Frequency-Energy Relations from NNM Theory



Exact Frequency-Energy Relation for Antisymmetric Motions

When $m_1 = m_3$, antisymmetric initial conditions yield antisymmetric responses (i.e., $x_1(t) = x_3(t)$ and $x_2(t) = 0$)

EOM simplify to a single EOM:

$$m_1\ddot{x}_1 + 2k_s x_1 + f(x_1) = 0\,.$$

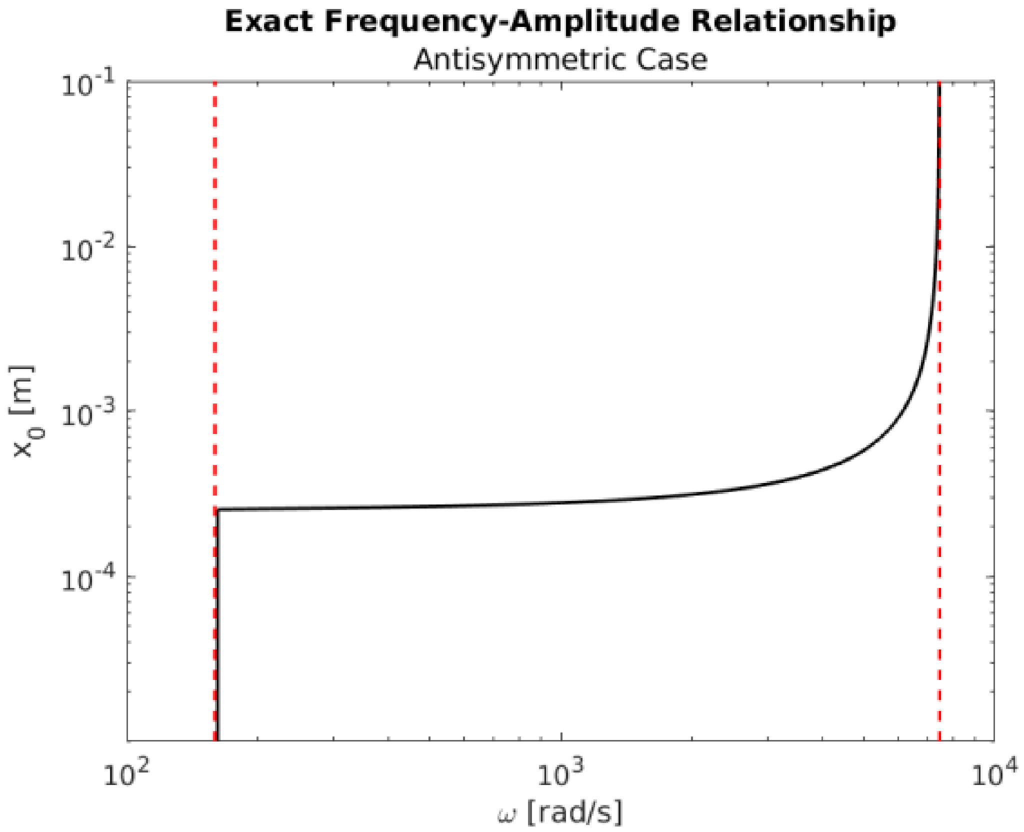
Piecewise linear, can be studied analytically.

Open-gap natural frequency: $\omega_0 = \sqrt{2k_s/m_1}$.

Closed-gap nat. freq.: $\bar{\omega}_0 = \sqrt{(2k_s + k_g)/m_1}$

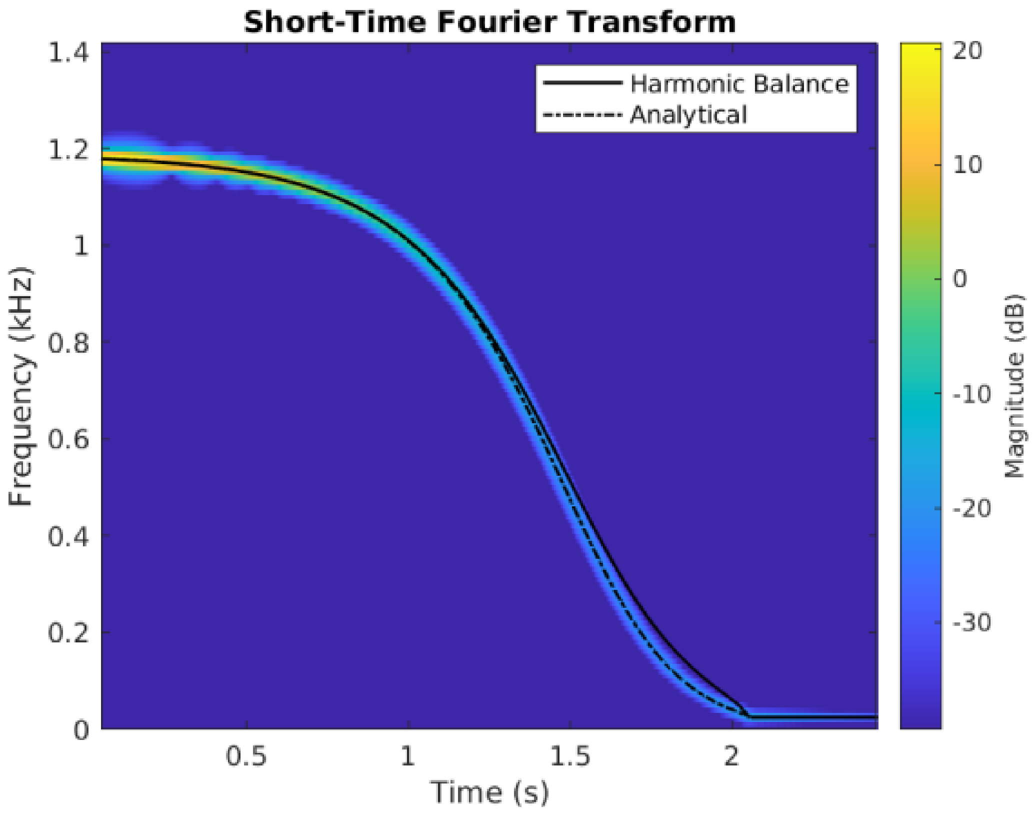
Frequency-amplitude relationship:

$$\widehat{\omega}(x_0) = \begin{cases} 2\pi\left[T_g(x_0) + T_s(x_0)\right]^{-1} & x_0 > g \\ \omega_0 & x_0 < g \end{cases}.$$



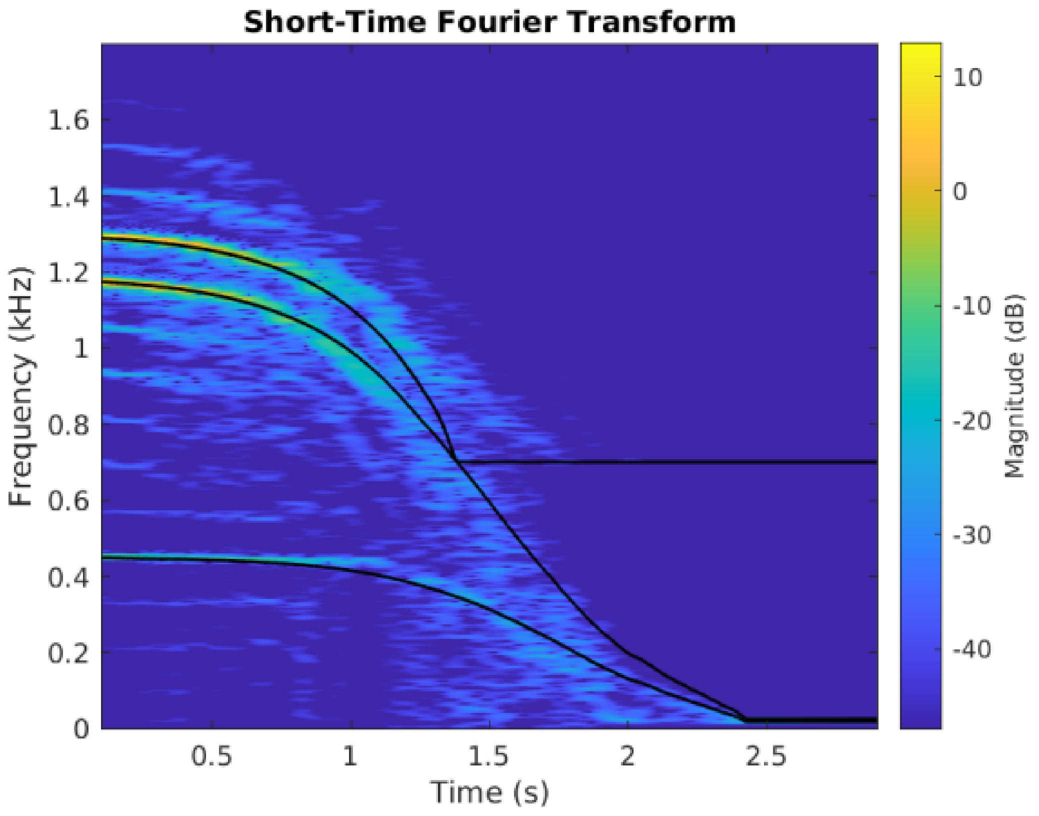
Lightly Damped Ringdown: Antisymmetric Initial Conditions

1. Solve (damped) antisymmetric EOM with $x_1(0) = 100g$ and $\dot{x}_1(0) = 0$.
2. Create spectrogram of $x_1(t)$.
3. Compute total mechanical energy $E(t)$.
4. Use exact and approximate frequency-energy relations to overlay $\widehat{\omega}(E(t))/(2\pi)$ and $\breve{\omega}_1(E(t))/(2\pi)$.



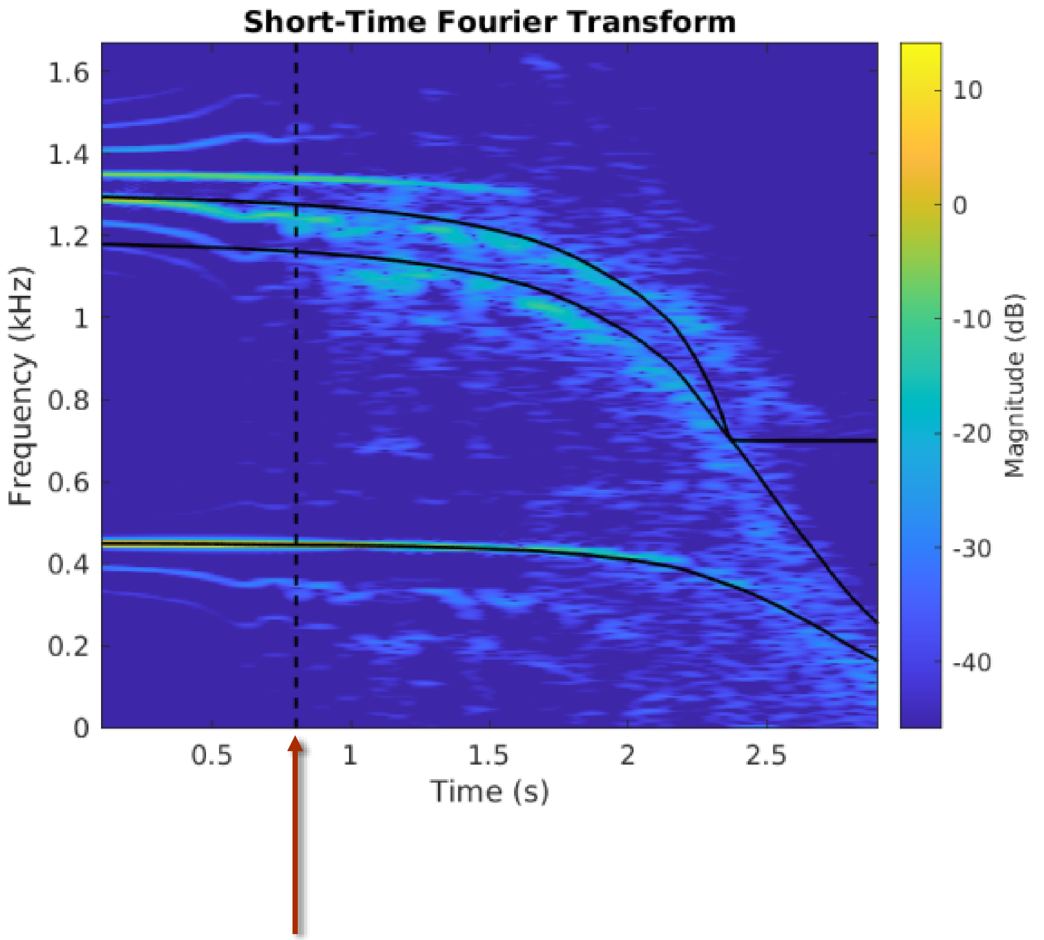
Lightly Damped Ringdown: Asymmetric Initial Conditions

$$\boldsymbol{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{m}, \qquad \dot{\boldsymbol{x}}(0) = \begin{bmatrix} 0 \\ 0 \\ 200 \end{bmatrix} \text{m/s}$$



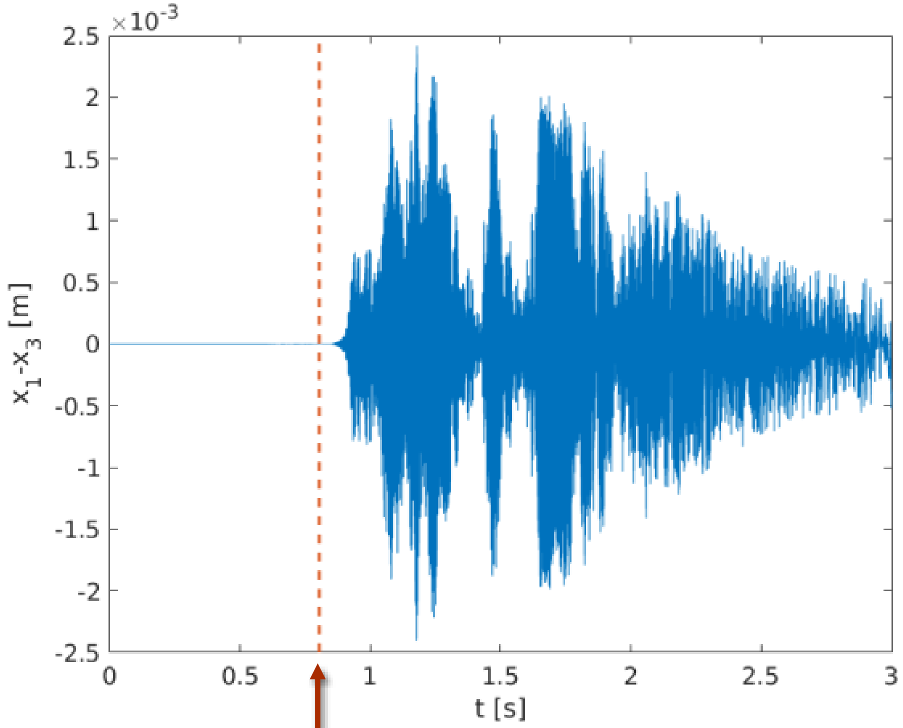
Lightly Damped Ringdown: Symmetric Initial Conditions

$$\boldsymbol{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ m}, \qquad \dot{\boldsymbol{x}}(0) = \begin{bmatrix} 0 \\ 200 \\ 0 \end{bmatrix} \text{m/s}$$



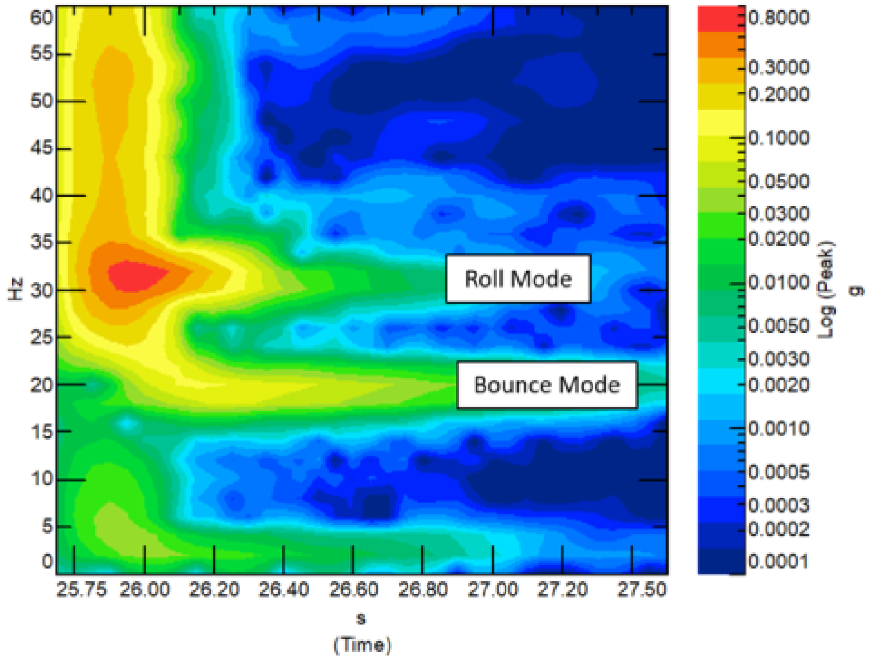
What is special about this time?

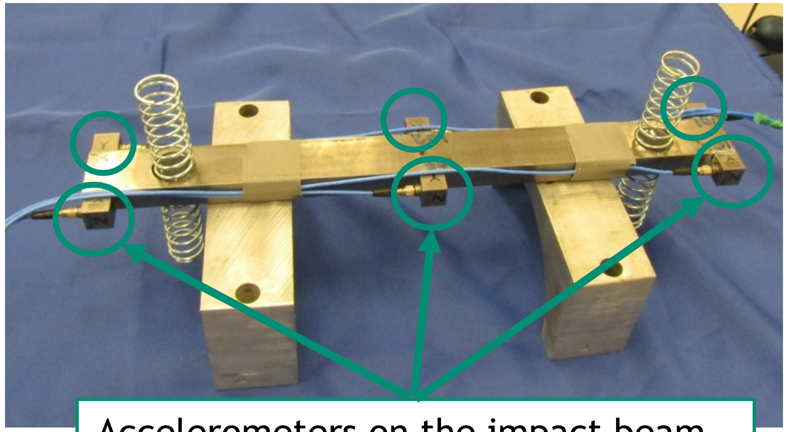
Lightly Damped Ringdown: Symmetry Breaking



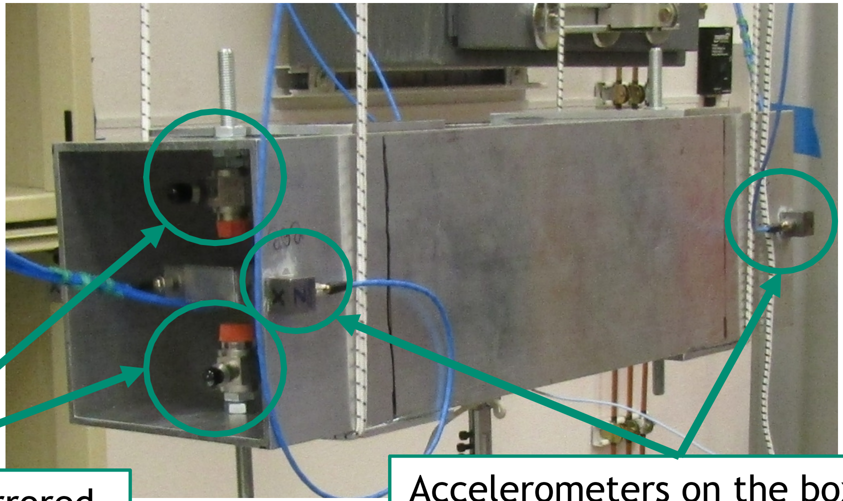
Left-right symmetry is spontaneously broken!

Evidence of Ringdown After Impact





Accelerometers on the impact beam



Impact load cells (mirrored on the other end)

Accelerometers on the box tube (mirrored on the opposite side)

Conclusions

- Developed a simple discrete mass system to study the nonlinear dynamics of an idealized bearing-and-shaft assembly
- Nonlinear normal modes reveal the evolution of the resonant behavior starting from pseudo-rigid body motion at low energy to elastic mode deformation at high energy
  - Help inform operating range for modeling assumptions (linear vs. nonlinear boundary conditions)
- Transient free-response simulations show how the NNM frequencies contribute to the response to high-amplitude broadband excitation
  - Observed resonant frequency can depend on energy of excitation
- Future work to further investigate the dynamics of the experimental apparatus to identify spectral content of transient ring-down

Acknowledgements

This research was conducted at the 2021 Nonlinear Mechanics and Dynamics Research Institute hosted by Sandia National Laboratories and the University of New Mexico.

Supported by the Laboratory Directed Research and Development program at Sandia National Laboratories, a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia LLC, a wholly owned subsidiary of Honeywell International Inc. for the U.S. Department of Energy’s National Nuclear Security Administration under contract DE-NA0003525.