Modeling Capabilities of Waveform-Based Bayesian Inference for Seismic Monitoring

Thomas Catanach, Robert Porritt, and Christopher Young | Sandia National Laboratories

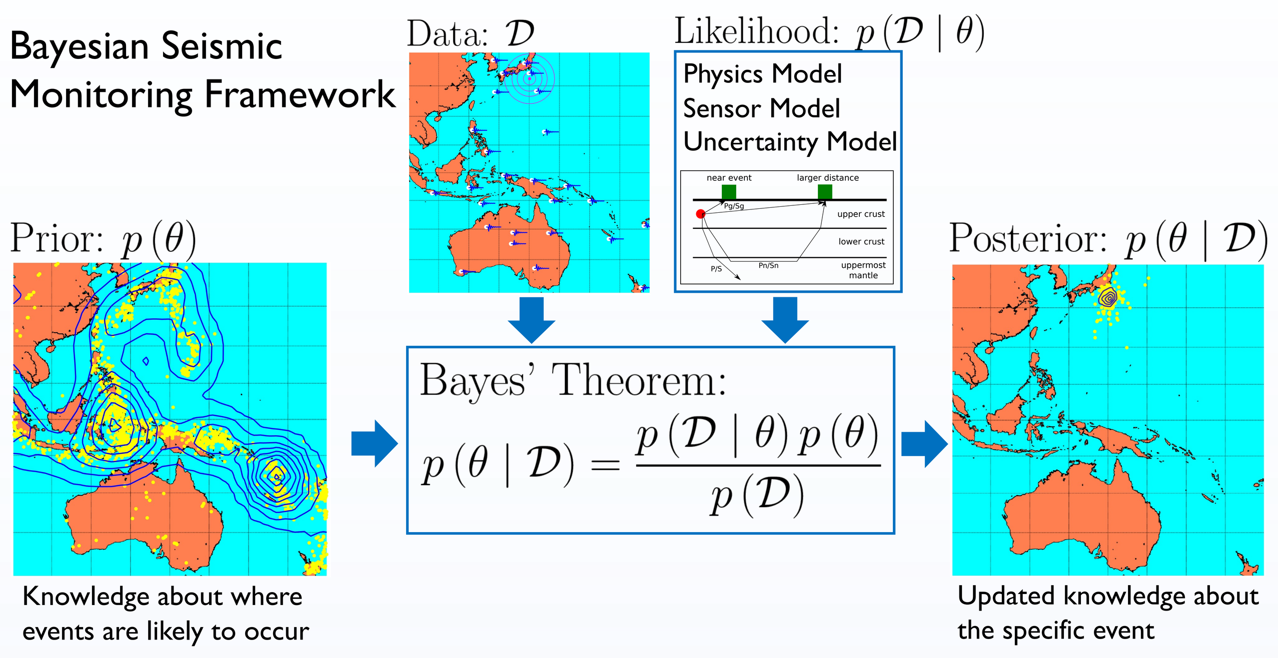
Evaluating Bayesian Seismic Monitoring

Goal:

- Computational tools allow high frequency (>1 Hz) seismic simulation; however, modeling uncertainties limits the accuracy of these results. How would refining these simulations to higher frequencies increase Bayesian seismic monitoring capabilities? Do we benefit from higher frequencies?

Bayesian Seismic Monitoring Problem:

- Infer event parameters with uncertainty: Longitude, Latitude, Depth, Origin Time, Source Time Function, and Moment Tensor
- Observations: Filtered seismic waveforms at various locations
- Uncertainty to integrate: Travel time uncertainty, earth structure heterogeneity, background noise process



Evaluation Approach:

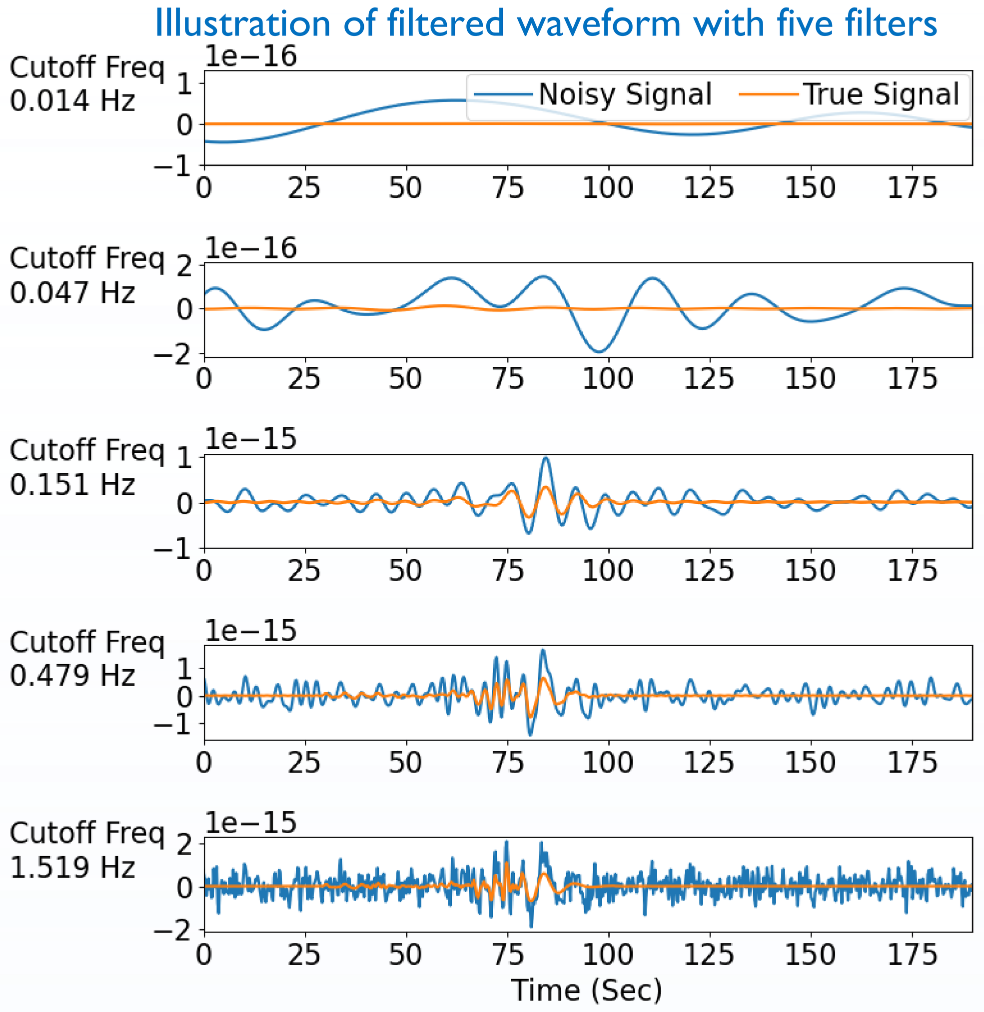
- We use the approach of Bayesian experimental design where we estimate the expected information gain of the Bayesian inference problem under different low-pass filter frequency assumptions. This replicates the effect of limiting our simulations to certain frequencies.
- Expected information gain (see Mathematical Methods for definition) is an information theoretic quantify that broadly measures the change from the prior parameter distribution to the posterior distribution.
- Algorithm Outline:
  1) Create a representative set of seismic sources with different locations and source properties
  2) Simulate high-frequency waveforms for each of these sources and add background noise from a known noise model
  3) For each waveform apply a set of low pass filters with different cutoff frequencies
  4) For each representative event and filtered waveform solve the Bayesian inference problem to find the posterior distribution on the event parameters and compute the information gain
  5) Average the information gain over all events for each of the filters to capture the effect of frequency content on seismic monitoring.

Experiment and Results

Setup:

- AxiSEM/Instaseis simulations with an AK135 earth model, sample rate 3.95 Hz
- Regional domain $\pm 2^\circ$ in latitude and longitude, 40km in depth
- Exponentially distributed $\log_{10}$ moment magnitude factor between -2 to 2 and isotropic moment tensor
- Source time function width 0 to 5 sec and origin time 0 to 152 sec
- Additive white background noise
- Low pass filtered using sinc filter
- Simulated 100 representative events

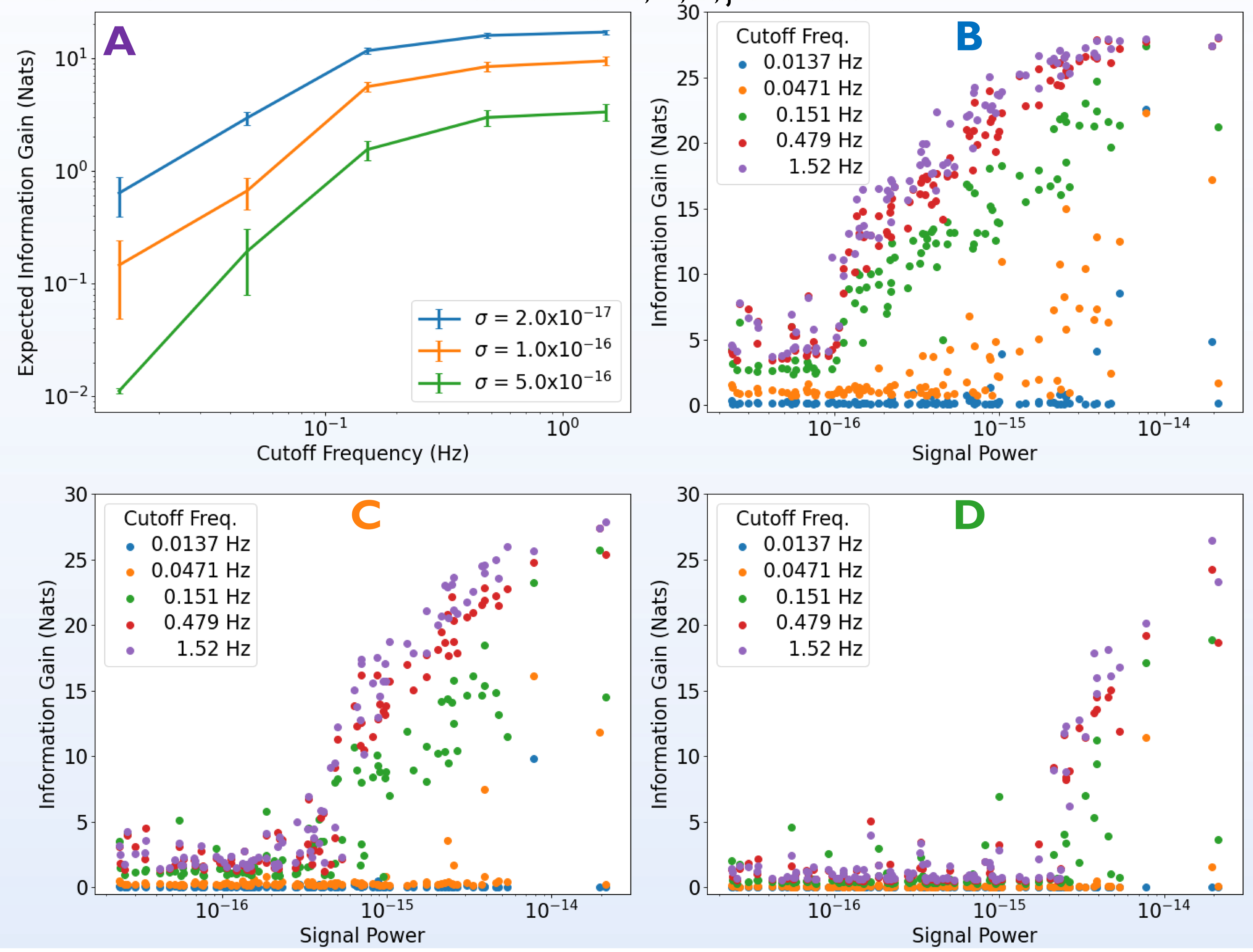
Illustration of filtered waveform with five filters



Results:

Panel A – Expected information gain for different cutoff frequencies at different background noise levels. We see in all cases there is little change in information gain between 0.479 -1.52 Hz

Panel B, Panel C, Panel D – Information gain for each representative event for the five filters vs the true signal power. Background level noise: Panel B $\sigma=2\times10^{-17}$, Panel C $\sigma=1\times10^{-16}$, and Panel D $\sigma=5\times10^{-16}$. We see similar trends between B, C, D, just shifted.



Conclusion

Discussion:

- Using the framework of Bayesian experimental design we can quantify the utility of different frequency content in high fidelity seismic simulation for seismic monitoring.
- Under the assumptions of an AK135 earth model and white background noise process, we observe that the contribution of higher frequency information is limited above 0.5 Hz for this regional setup.

Future Directions:

- We would like to extend this analysis to frequencies up to 10Hz. To do that we will need to leverage higher fidelity simulation codes and choose a representative earth model with more complexity than AK135.
- We made simple assumptions about the background and source mechanism to facilitate computation, relaxing the assumptions and using more realistic models may influence these results.

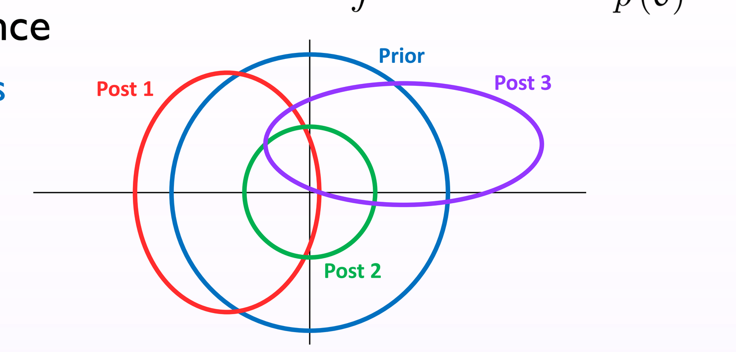
Mathematical Methods

Quantifying Information Gain :

- Bayesian Inference: $$p(\theta \mid \mathcal{D}) = \frac{p(\mathcal{D} \mid \theta)\, p(\theta)}{p(\mathcal{D})}$$
- Kullback-Leibler (KL) Divergence measures information due to inference $$\mathrm{KL}\left[p(\theta \mid \mathcal{D}) \,||\, p(\theta)\right] = \int p(\theta \mid \mathcal{D}) \log \frac{p(\theta \mid \mathcal{D})}{p(\theta)} d\theta$$

Illustration of information gain in bits for three posteriors

Prior → Post 1	0.5 Bits
Prior → Post 2	1 Bit
Prior → Post 3	1 Bit



- Expected Information Gain (EIG) from a model with filter frequency (S):

$$\mathcal{I}(\mathcal{S}) = \mathrm{E}\left[\mathrm{KL}\left[p(\theta \mid \mathcal{D}) \,||\, p(\theta)\right] \mid \mathcal{D} \sim p(\mathcal{D} \mid \mathcal{S})\right]$$
$$= \int p(\mathcal{D} \mid \mathcal{S}) \int p(\theta \mid \mathcal{D}, \mathcal{S}) \log \frac{p(\theta \mid \mathcal{D}, \mathcal{S})}{p(\theta)} d\theta d\mathcal{D}$$

Distribution of hypothetical data $p(\mathcal{D} \mid \mathcal{S}) = \int p(\mathcal{D} \mid \theta', \mathcal{S})\, p(\theta')\, d\theta'$

KL Divergence to measure information gain

Likelihood model in the frequency domain:

- Let ω_j be the Discrete Fourier Transform (F) of the predicted waveform at frequency j for an event characterized by (Lat-Lon $\mathcal{L}$, Depth z, Magnitude m, Origin Time t_o, and STF width λ)

$$\omega_j(\mathcal{L}, z, m, t_o, \lambda) = F_j \bar{w}(\mathcal{L}, z, m, t_o, \lambda)\, \mathbb{1}\left[s_j < f\right]$$

- Then the likelihood of the observed Discrete Fourier Transform (ξ) up to frequency f given the predicted waveform is

$$p(\xi_1 \ldots \xi_{n_f} \mid \mathcal{L}, z, m, t_o, \lambda) = p(\gamma_{01} = \mathrm{Real}\left[\xi_1 - \omega_1(\mathcal{L}, z, m, t_o, \lambda)\right]) \times \prod_{j=2}^{n_f} p(\gamma_{0j} = \mathrm{Real}\left[\xi_j - \omega_j(\mathcal{L}, z, m, t_o, \lambda)\right])\, p(\gamma_{1j} = \mathrm{Imag}\left[\xi_j - \omega_j(\mathcal{L}, z, m, t_o, \lambda)\right])$$

- Where $\gamma_{01} \sim \mathcal{N}(0, n\sigma^2), j = 1$; $\gamma_{11} = 0, j = 1$; $\gamma_{0j} \sim \mathcal{N}\left(0, \frac{n}{2}\sigma^2\right), j = 2 \ldots n$; $\gamma_{1j} \sim \mathcal{N}\left(0, \frac{n}{2}\sigma^2\right), j = 2 \ldots n$

This research was funded by the National Nuclear Security Administration, Defense Nuclear Nonproliferation Research and Development (NNSA DNN R&D). The authors acknowledge important interdisciplinary collaboration with scientists and engineers from LANL, LLNL, MSTS, PNNL, and SNL. Sandia National Laboratories is a multimission laboratory managed and operated by National Technology &Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

References

Nissen-Meyer, Tarje, et al. "AxiSEM: broadband 3-D seismic wavefields in axisymmetric media." *Solid Earth* 5.1 (2014): 425-445.
van Driel, Martin, et al. "Instaseis: Instant global seismograms based on a broadband waveform database." *Solid Earth* 6.2 (2015): 701-717.
Kennett, B. L. N. "Seismological tables: ak135." *Research School of Earth Sciences, Australian National University Canberra, Australia* (2005): 1-289.
Trabant, Chad, et al. "Data products at the IRIS DMC: Stepping stones for research and other applications." *Seismological Research Letters* 83.5 (2012): 846-854.
IRIS DMC (2015), Data Services Products: IRIS Syngine supported 1-D Earth models, https://doi.org/10.17611/DP/EMCSYNG.1.
Huan, Xun, and Youssef M. Marzouk. "Simulation-based optimal Bayesian experimental design for nonlinear systems." *Journal of Computational Physics* 232.1 (2013): 288-317.