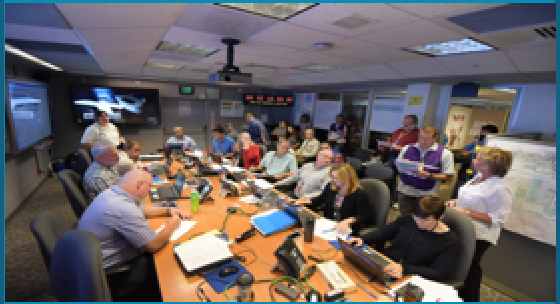
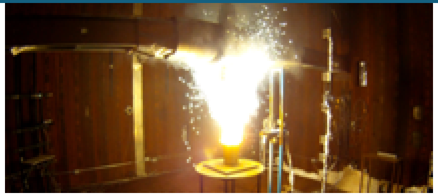
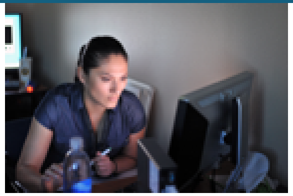
SAND2021-15877C

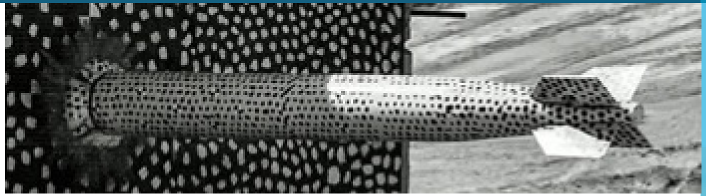
Sandia National Laboratories

Degree of Freedom Selection Approaches for MIMO Vibration Test Design









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IMAC XL

Presented By:

Christopher Beale

Authors:

Christopher Beale, Ryan Schultz, Chandler Smith, Timothy Walsh

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Outline

- Introduction
- Theory
- Model
- Results
- Conclusions

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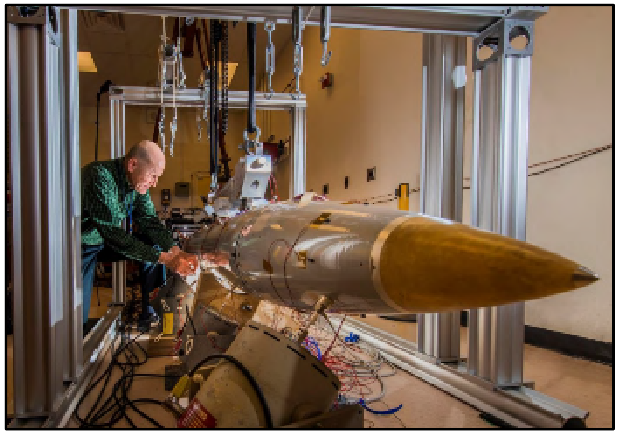
Introduction: Multiple Input Multiple Output (MIMO)

MIMO Definition:

- Test approach used to replicate a desired system response by controlling the inputs supplied to the system based on the outputs measured on the system.

Applications:

- Replicate a desired field response in a laboratory setting
  - Avoid performing costly and timely field tests
  - Perform system and component qualification



Challenges:

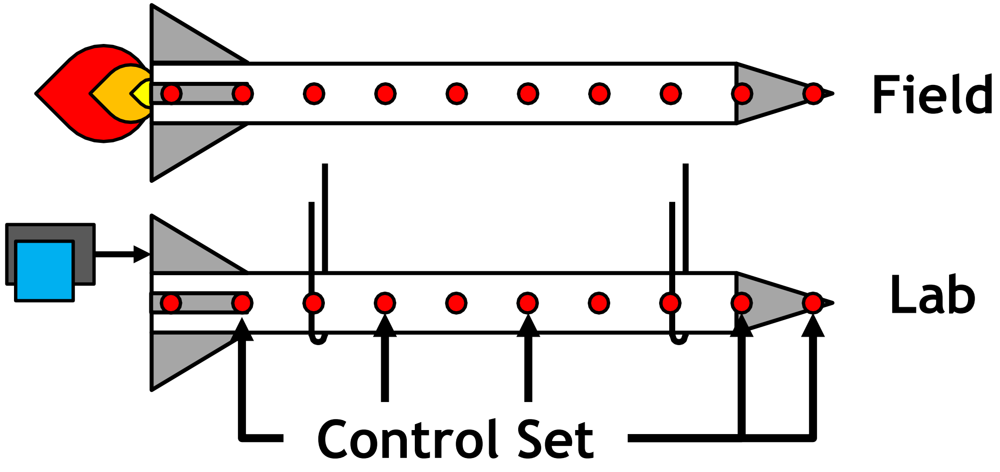
- Differences in Field vs. Laboratory system
  - Boundary conditions, design, variability, etc
  - Affects the ability to control and achieve the desired response
- Test Design
  - Instrumentation and which degrees of freedom to control
  - Inputs

[1-4]

Introduction: Objectives

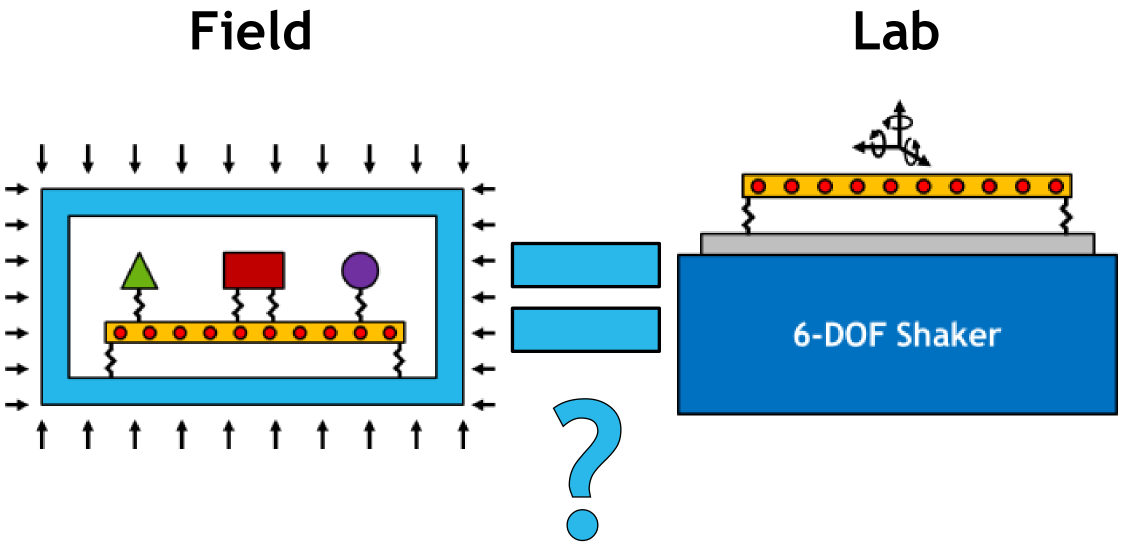
Objectives:

- Further demonstrate the capability of a Mean Square Error (MSE) based Degree of Freedom (DOF) selection approach to assist test engineers with MIMO test design
- Present an additional capability for DOF selection based on Optimal Experimental Design (OED)
- Demonstrate the capability of each approach considering:
  - Complex and realistic models
  - Complex and realistic environments
  - Differences in field vs. lab system



Approach:

- Define desired field responses from field models
- Apply the DOF selection techniques to laboratory models, with different boundary conditions than the "field" models.
- Compare the laboratory MIMO responses to the field responses using DOF selected from each approach.



[5]

Outline: Theory

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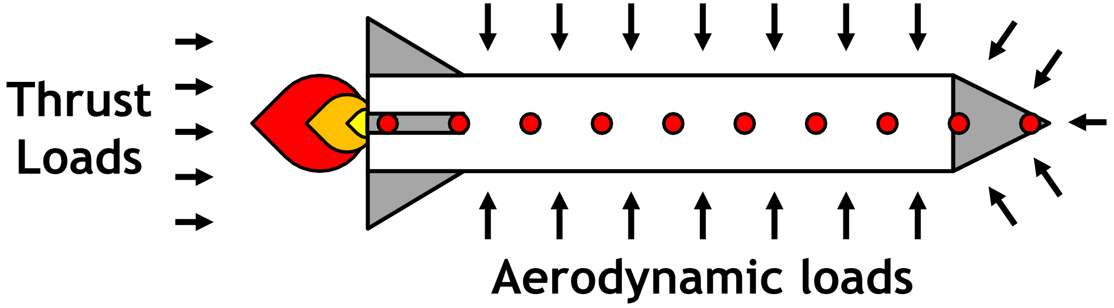
Theory: Multiple Input Multiple Output

Field Environment



$$S_{yy_0} = H_{yx_0} S_{xx_0} H^H_{yx_0}$$

Field Output CPSD Matrix

$$\begin{bmatrix} S_{11} & \cdots & S_{1M} \\ \vdots & \ddots & \vdots \\ S_{M1} & \cdots & S_{MM} \end{bmatrix}$$

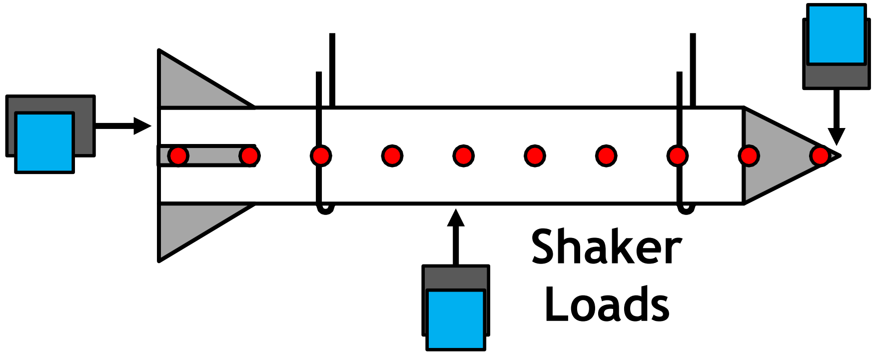
Field FRF Matrix

$$\begin{bmatrix} H_{11} & \cdots & H_{1N} \\ \vdots & \ddots & \vdots \\ H_{M1} & \cdots & H_{MN} \end{bmatrix}$$

Field Input CPSD Matrix

$$\begin{bmatrix} S_{11} & \cdots & S_{1N} \\ \vdots & \ddots & \vdots \\ S_{N1} & \cdots & S_{NN} \end{bmatrix}$$

Lab Environment



$$S_{yy_1} = H_{yx_1} S_{xx_1} H^H_{yx_1}$$

Lab Output CPSD Matrix

$$\begin{bmatrix} S_{11} & \cdots & S_{1M} \\ \vdots & \ddots & \vdots \\ S_{M1} & \cdots & S_{MM} \end{bmatrix}$$

Lab FRF Matrix

$$\begin{bmatrix} H_{11} & \cdots & H_{1N} \\ \vdots & \ddots & \vdots \\ H_{M1} & \cdots & H_{MN} \end{bmatrix}$$

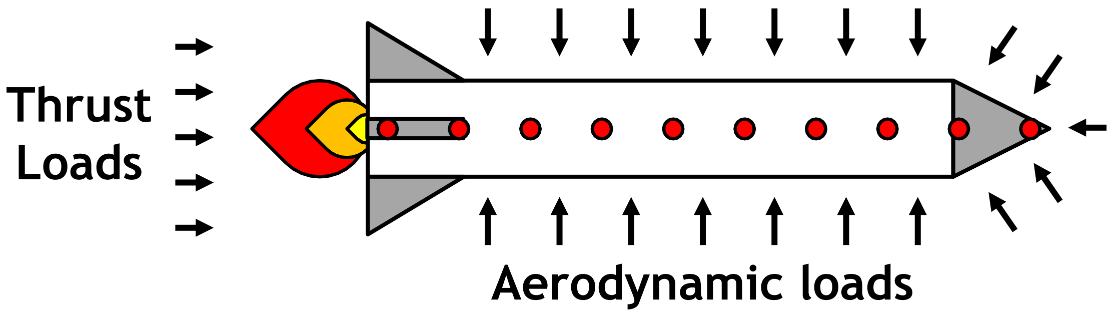
Lab Input CPSD Matrix

$$\begin{bmatrix} S_{11} & \cdots & S_{1N} \\ \vdots & \ddots & \vdots \\ S_{N1} & \cdots & S_{NN} \end{bmatrix}$$

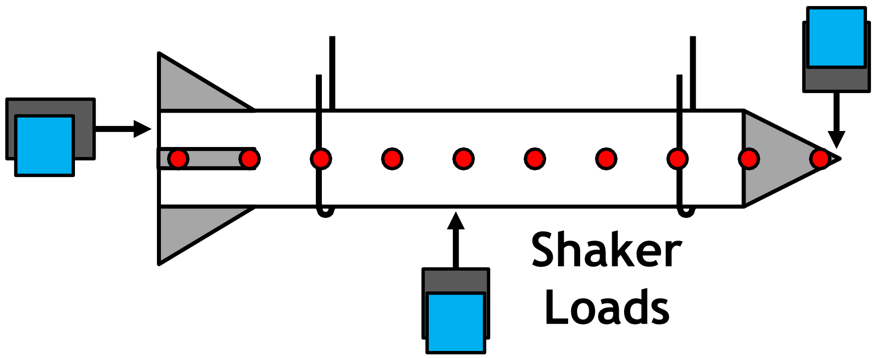
[5]

Theory: Multiple Input Multiple Output

Field Environment



Lab Environment



✓

$$S_{yy_0} = H_{yx_0} S_{xx_0} H_{yx_0}^H$$

Goal

$$S_{yy_0} = S_{yy_1}$$

Desired ✓ ? ✓

$$S_{yy_1} = H_{yx_1} S_{xx_1} H_{yx_1}^H$$

Issues:

- Field environment is not always replicated
- Difficult to control large DOF sets
- Some DOF work better than others for control

How do we select DOF best for MIMO?

Substitute in Field CPSD $S_{yy_0} \approx H_{yx_1} S_{xx_1} H_{yx_1}^H$

Solve for Lab Input CPSD $S_{xx_1} = H_{yx_1}^{+} S_{yy_0} H_{yx_1}^{+H}$

Obtain Lab CPSD $S_{yy_1} = H_{yx_1} S_{xx_1} H_{yx_1}^H$

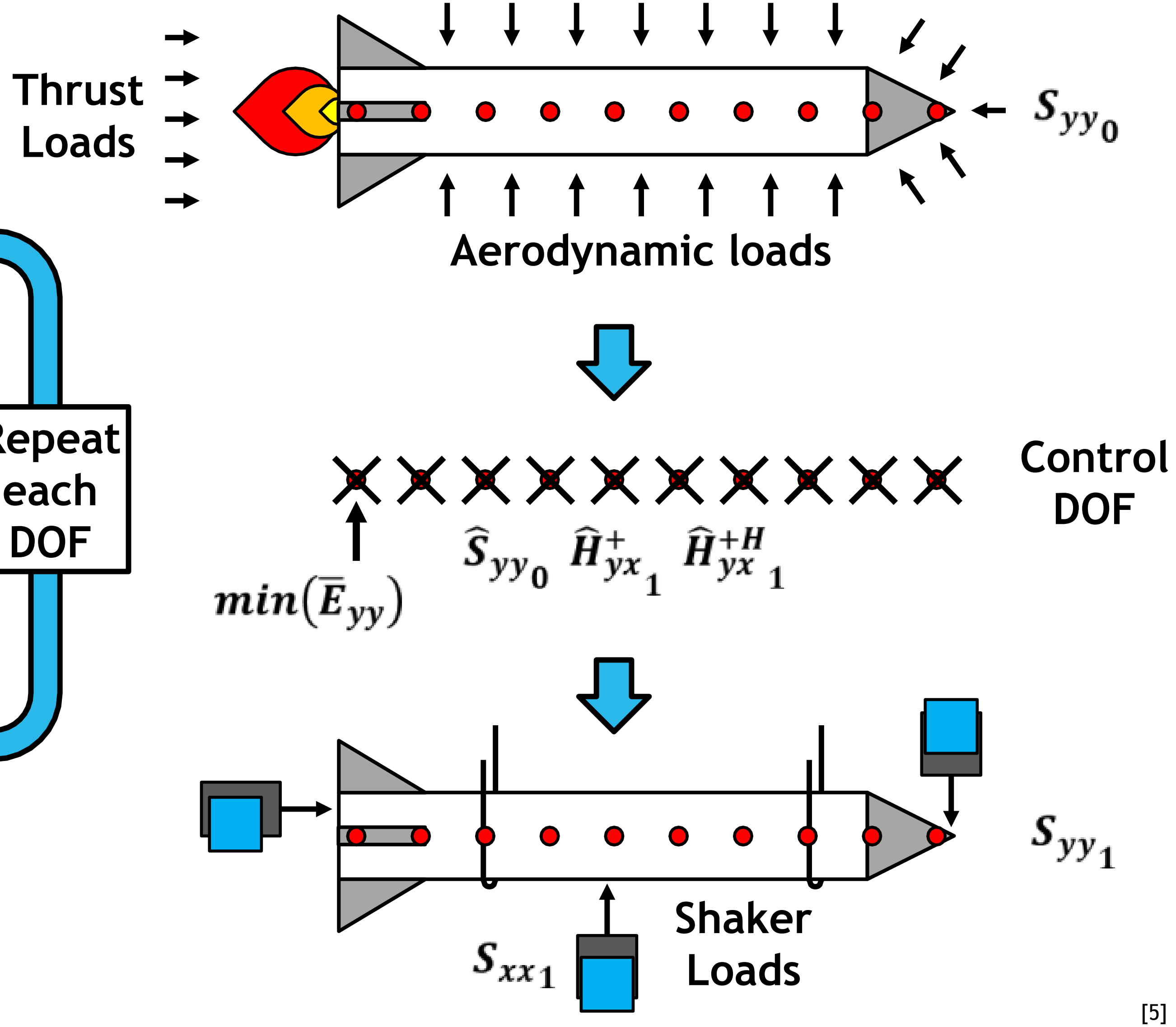
[5]

Theory: MSE-Based DOF Selection Approach

Field Output	S_{yy_0}
Reduced Matrices	$\hat{S}_{yy_0}$ $\hat{H}^{+}_{yx_1}$ $\hat{H}^{+H}_{yx_1}$
Input Estimation	$S_{xx_1} = \hat{H}^{+}_{yx_1}\hat{S}_{yy_0}\hat{H}^{+H}_{yx_1}$
Output Estimation	$S_{yy_1} = H_{yx_1}S_{xx_1}H^{H}_{yx_1}$
Error Metric	$E_{yy_m} = MSE\left(diag\left(S_{yy_0} - S_{yy_1}\right)\right)$
DOF Down Select	Remove: $min(\bar{E}_{yy})$

Repeat to desired DOF count

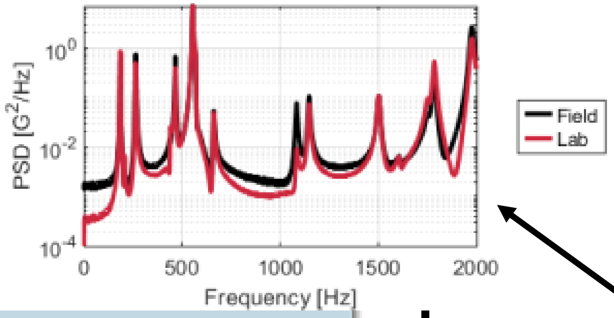
Repeat each DOF



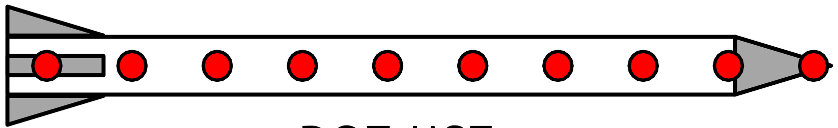
[5]

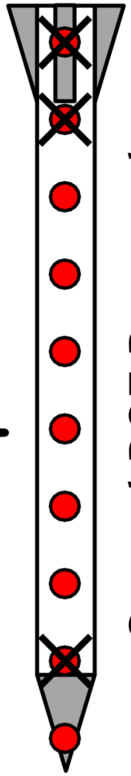
Theory: MSE-Based DOF Selection Approach

Field Output	S_{yy_0}
Reduced Matrices	$\hat{S}_{yy_0}$ $\hat{H}^{+}_{yx_1}$ $\hat{H}^{+H}_{yx_1}$
Input Estimation	$S_{xx_1} = \hat{H}^{+}_{yx_1} \hat{S}_{yy_0} \hat{H}^{+H}_{yx_1}$
Output Estimation	$S_{yy_1} = H_{yx_1} S_{xx_1} H^{H}_{yx_1}$
Error Metric	$E_{yy_m} = MSE\left(diag\left(S_{yy_0} - S_{yy_1}\right)\right)$
DOF Down Select	Remove: $min(\bar{E}_{yy})$



Error Matrix





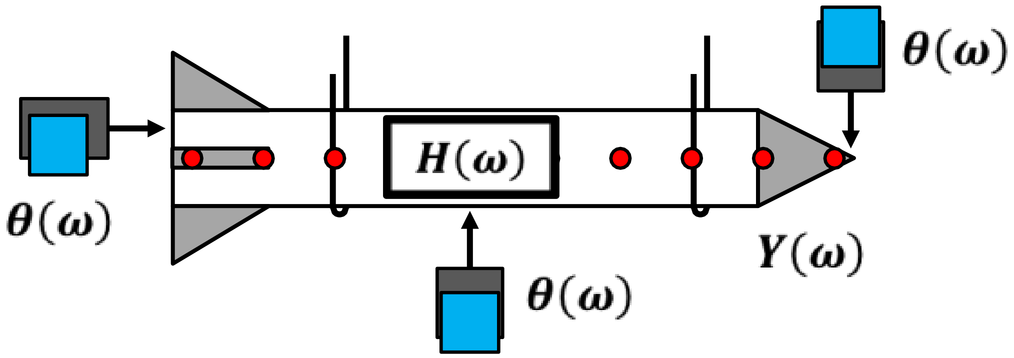
DOF MSE, Mean MSE

Control DOF Removed	1	2	3	4	5	6	7	8	9	10	Mean MSE
1	0.24	0.33	0.09	0.13	0.11	0.27	0.18	0.20	0.12	0.10	0.18
2	0.24	0.46	0.10	0.16	0.12	0.28	0.17	0.20	0.12	0.10	0.19
3	0.09	0.17	0.08	0.19	0.15	0.44	0.16	0.22	0.10	0.08	0.17
4	0.09	0.19	0.07	0.12	0.13	0.25	0.23	0.24	0.13	0.11	0.16
5	0.12	0.16	0.07	0.12	0.13	0.29	0.22	0.22	0.14	0.12	0.16
6	0.12	0.18	0.07	0.11	0.13	0.28	0.21	0.22	0.13	0.11	0.16
7	0.16	0.22	0.07	0.10	0.12	0.28	0.19	0.21	0.13	0.11	0.16
8	0.16	0.22	0.07	0.10	0.12	0.28	0.19	0.21	0.13	0.11	0.16
9	0.11	0.19	0.06	0.16	0.10	0.31	0.16	0.22	0.09	0.07	0.15
10	0.18	0.23	0.07	0.13	0.11	0.31	0.17	0.20	0.11	0.09	0.16

[5]

Theory: Optimal Experimental Design Approach

Structural Dynamics Model



$$Y(\omega) = H(\omega)\ \theta(\omega)$$

Output

$$\begin{bmatrix} y_1 \\ \vdots \\ y_M \end{bmatrix}$$

FRF

$$\begin{bmatrix} H_{11} & \cdots & H_{1N} \\ \vdots & \ddots & \vdots \\ H_{M1} & \cdots & H_{MN} \end{bmatrix}$$

Input

$$\begin{bmatrix} \theta_1 \\ \vdots \\ \theta_N \end{bmatrix}$$

1 1

Linear Regression

Reduce: $H_a - active\ dof$

Model: $y_a = \quad \theta +$

Estimate: $\hat{\theta} = \left(H_a^T H_a\right)^{-1} H_a^T (y_a)$

Prediction: $\hat{y} = H\,\hat{\theta}$

- Measurement noise Gaussian IID
- Each frequency is an observation with transfer matrix $H(\omega)$

Theory: Optimal Experimental Design Approach

DOF Weight Map:

$$H_a = PH$$

$$0 \leq p_i \leq 1,\ \sum_{i=1}^{M} p_i = 1$$

$$P = \begin{bmatrix} 1/M & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/M \end{bmatrix}$$

Applies weighting to each row (DOF) of the FRF matrix

Estimate:

$$\hat{\theta} = \left(H_a^T H_a\right)^{-1} H_a^T (y_a)$$

Unbiased Estimator Variance:

$$C = \mathbb{E}\left[\left(\hat{\theta} - \theta^*\right)\left(\hat{\theta} - \theta^*\right)^T\right]$$

$$C = \sigma^2 \left(H_a^T H_a\right)^{-1}$$

$$C = \begin{bmatrix} \sigma_{\theta_1}^2 & \cdots & \sigma_{\theta_1}\sigma_{\theta_N} \\ \vdots & \ddots & \vdots \\ \sigma_{\theta_N}\sigma_{\theta_1} & \cdots & \sigma_{\theta_N}^2 \end{bmatrix}$$

Covariance of the estimated inputs

Convex Program:

$$\text{Minimize } \Psi(C(P))$$

$$\Psi = \mathbb{E}[HCH^T] = \mathbb{E}\left[H\sigma^2\left(H_a^T H_a\right)^{-1} H^T\right] \rightarrow \begin{bmatrix} \sigma_{y_1}^2 & \cdots & \sigma_{y_1}\sigma_{y_M} \\ \vdots & \ddots & \vdots \\ \sigma_{y_M}\sigma_{y_1} & \cdots & \sigma_{y_M}^2 \end{bmatrix}$$

$$\Psi = \frac{1}{M}\sum_{i=1}^{M} \sigma_{y_i}^2$$

Average prediction variance (response)

- The OED algorithm iteratively updates the DOF weight map, P, to minimize the average prediction variance, Ψ.
- The diagonals of P that yield the minimum average prediction variance, identify the DOF that are most important.
- The DOF corresponding to the largest values of P are selected.

[6-8]

Outline: Model

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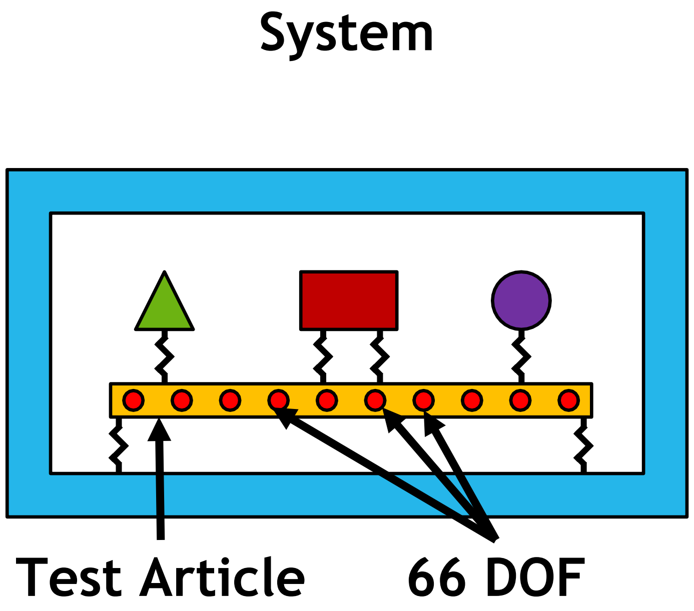
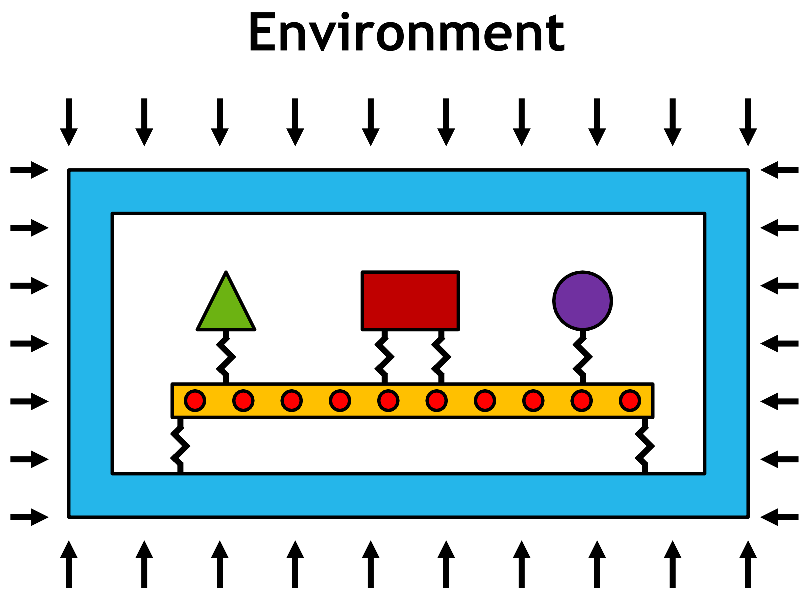
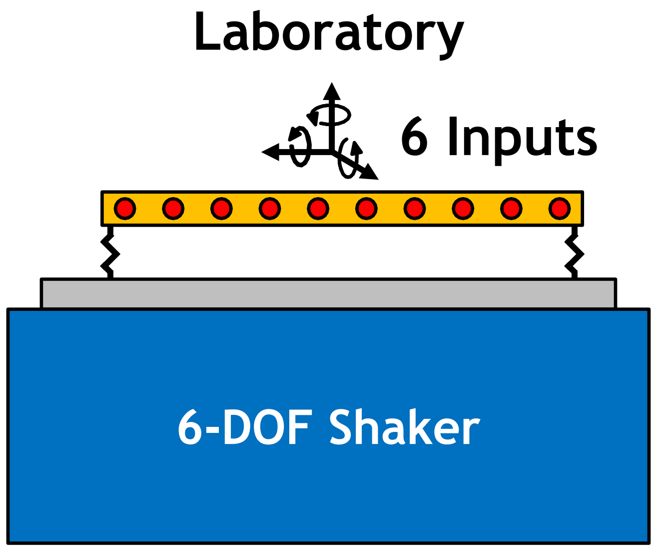
Model: Overview





S_{yy_0}



S_{yy_1}

- Generic Aerospace System
- Several Subassemblies and Components
- Single component selected as test article
- 100,000+ Elements
- 1,000,000+ Nodes

Goal

$$S_{yy_0} = S_{yy_1}$$

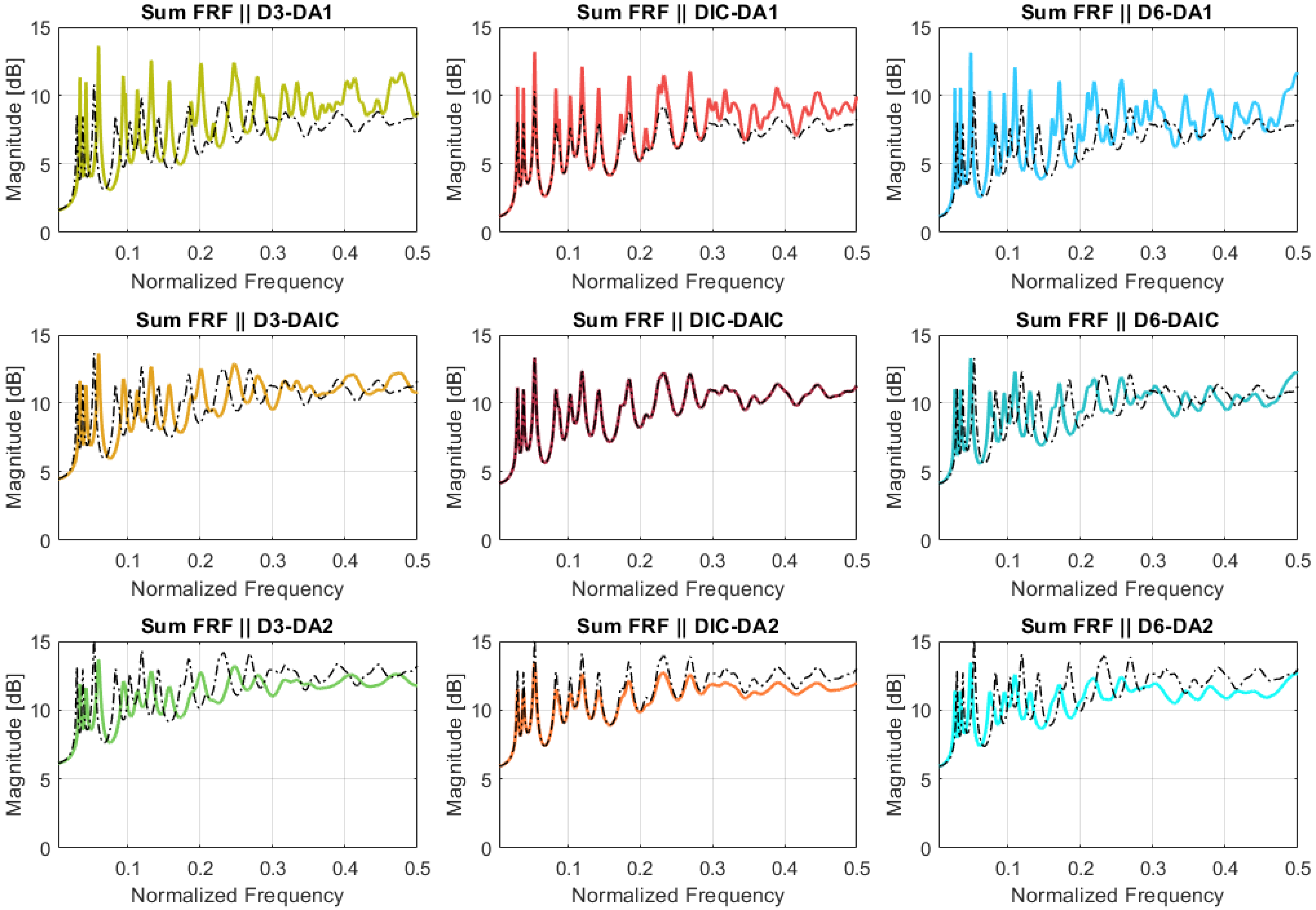
- Test article mounted to 6-DOF shaker
- Model used to simulate laboratory test

Model: Variants

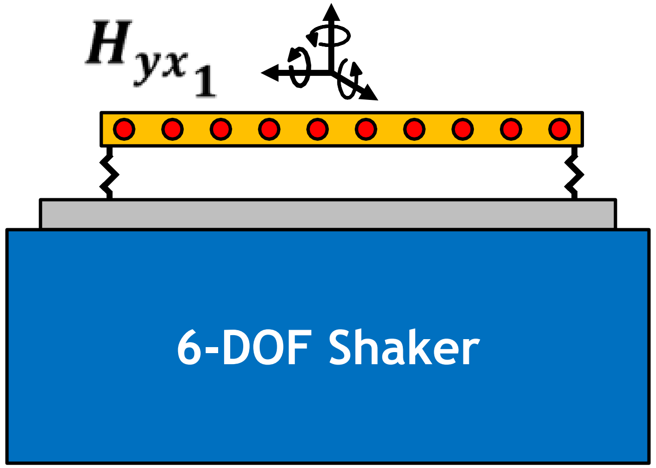
Component Density

D3 (0.75x) | DIC (1.00x) | D6 (1.25x)

Damping: DA1 (0.5x) | DAIC (1.0x) | DA2 (1.5x)

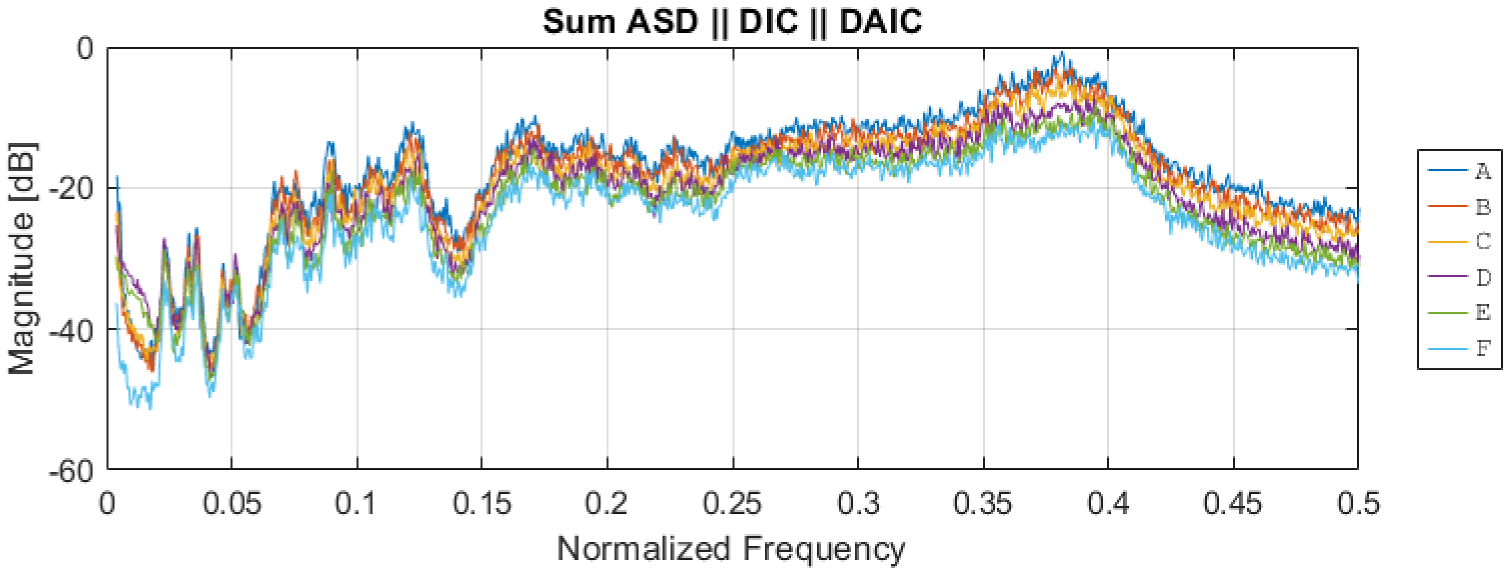


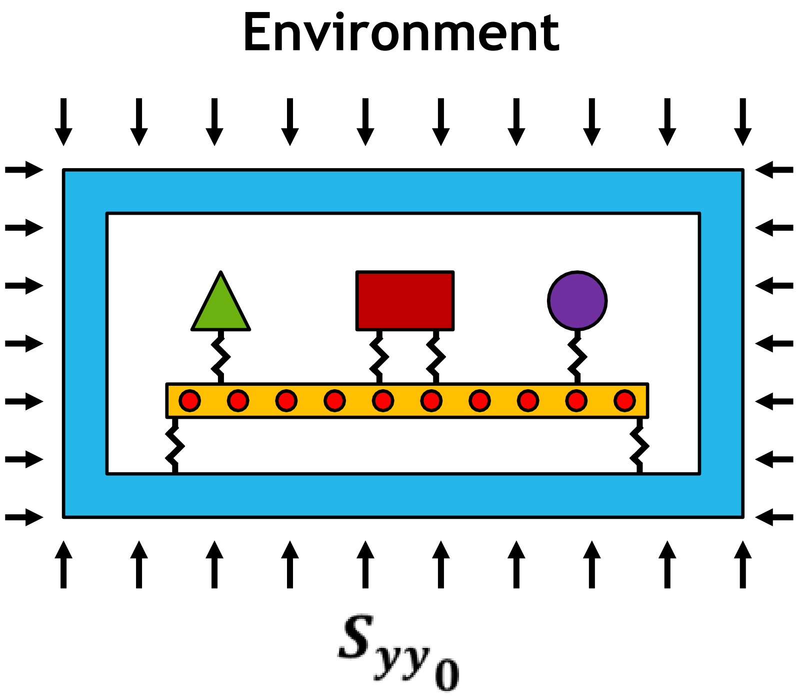
Laboratory



- 9 Model Variants:
  - 3 Damping Cases
  - 3 Component Density Cases
- Demonstrate versatility of each approach to differences in the field component vs. the lab component

Model: Field Response





- Considered 6 complex and realistic aerospace environments labeled A - F.
- Full system response was simulated for all model variants and environment variants to use as target environments in the MIMO test:
  - 54 total field responses (S_{yy_0})
- Demonstrate the versatility of approach to different field environments

Outline: Results

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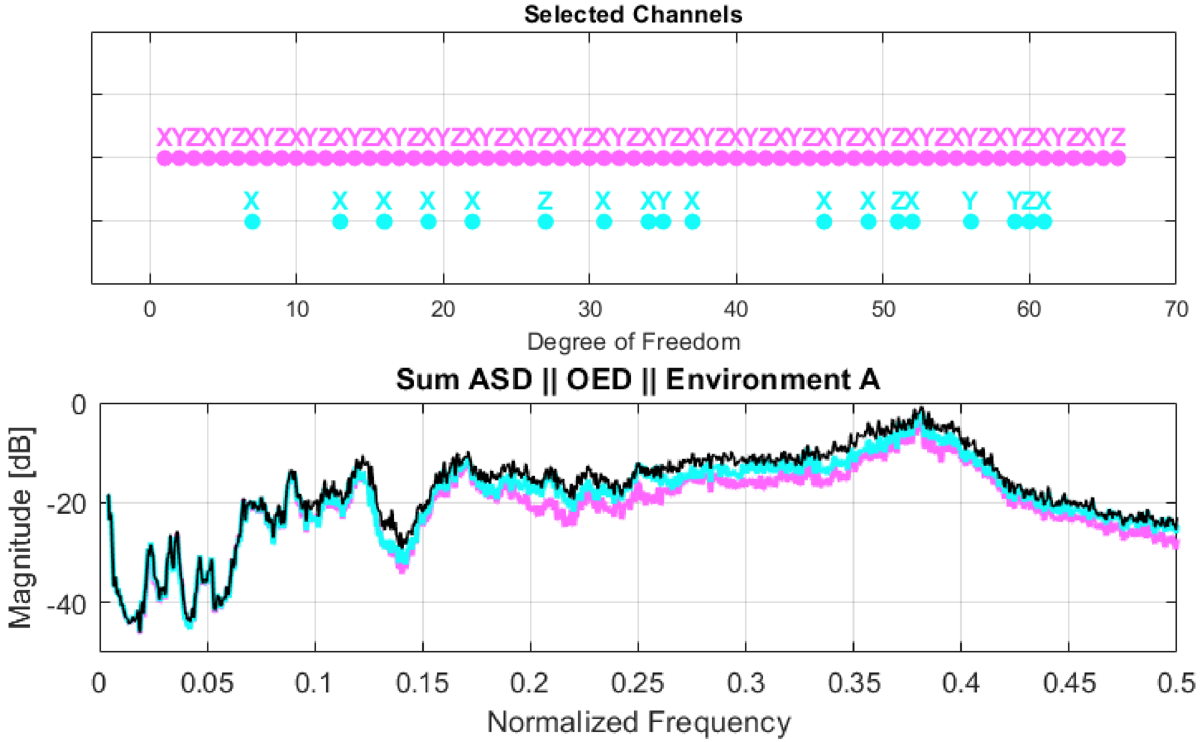
Results: Nominal Model – Nominal Field – Environment A

18 out of 66 DOF selected
Same Variant Field + Lab Models

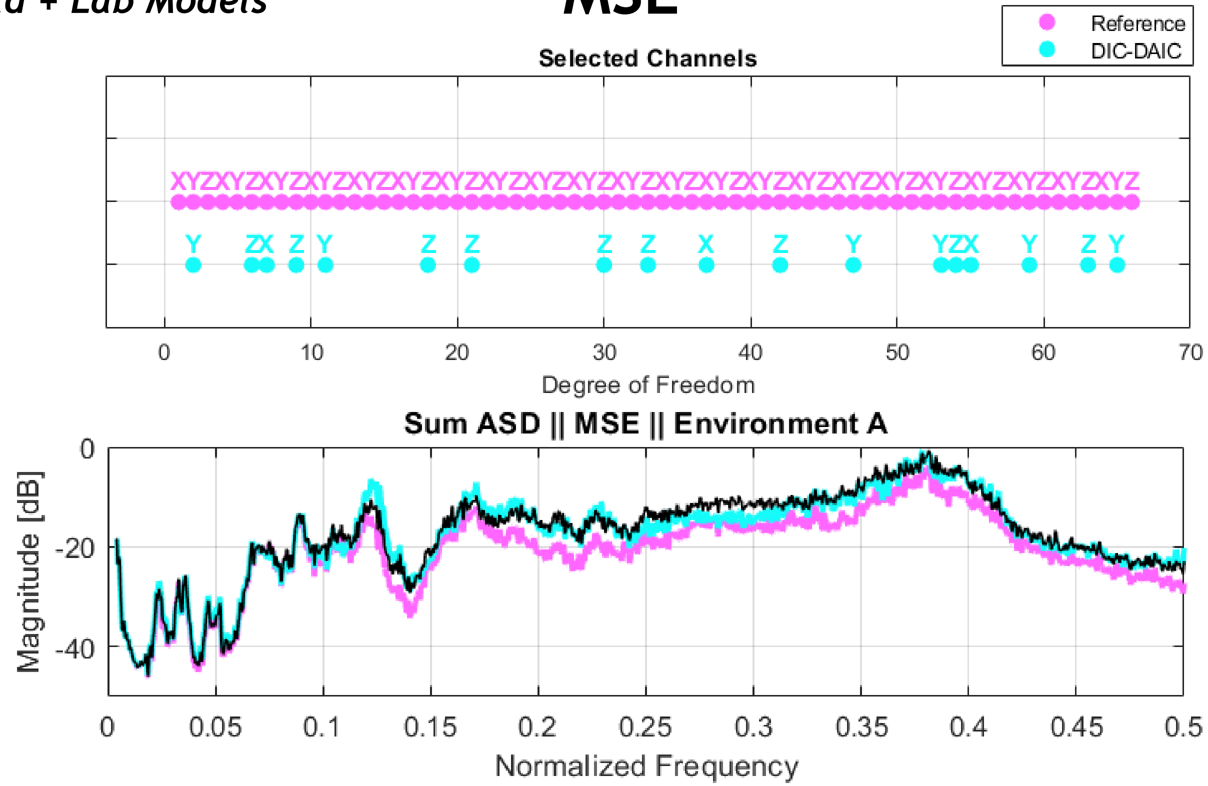
OED



Reference: All DOF in control set

	RMS	MSE
Reference	0.118	13.82
DIC-DAIC	0.144	04.17
Field	0.181	00.00

MSE



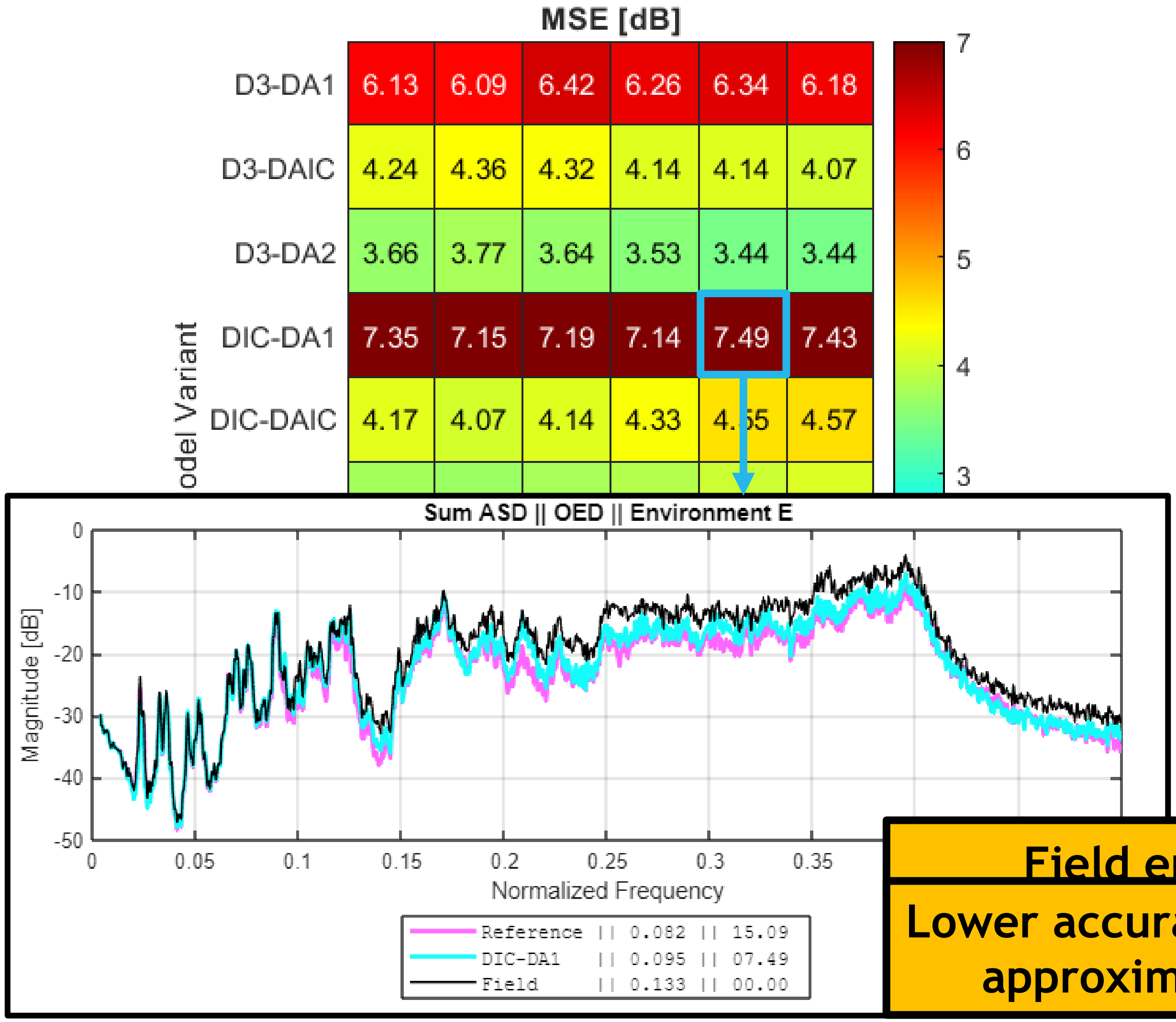
	RMS	MSE
Reference	0.118	13.82
DIC-DAIC	0.174	02.67
Field	0.181	00.00

Both approaches accurately reproduce the responses to the field environment loads

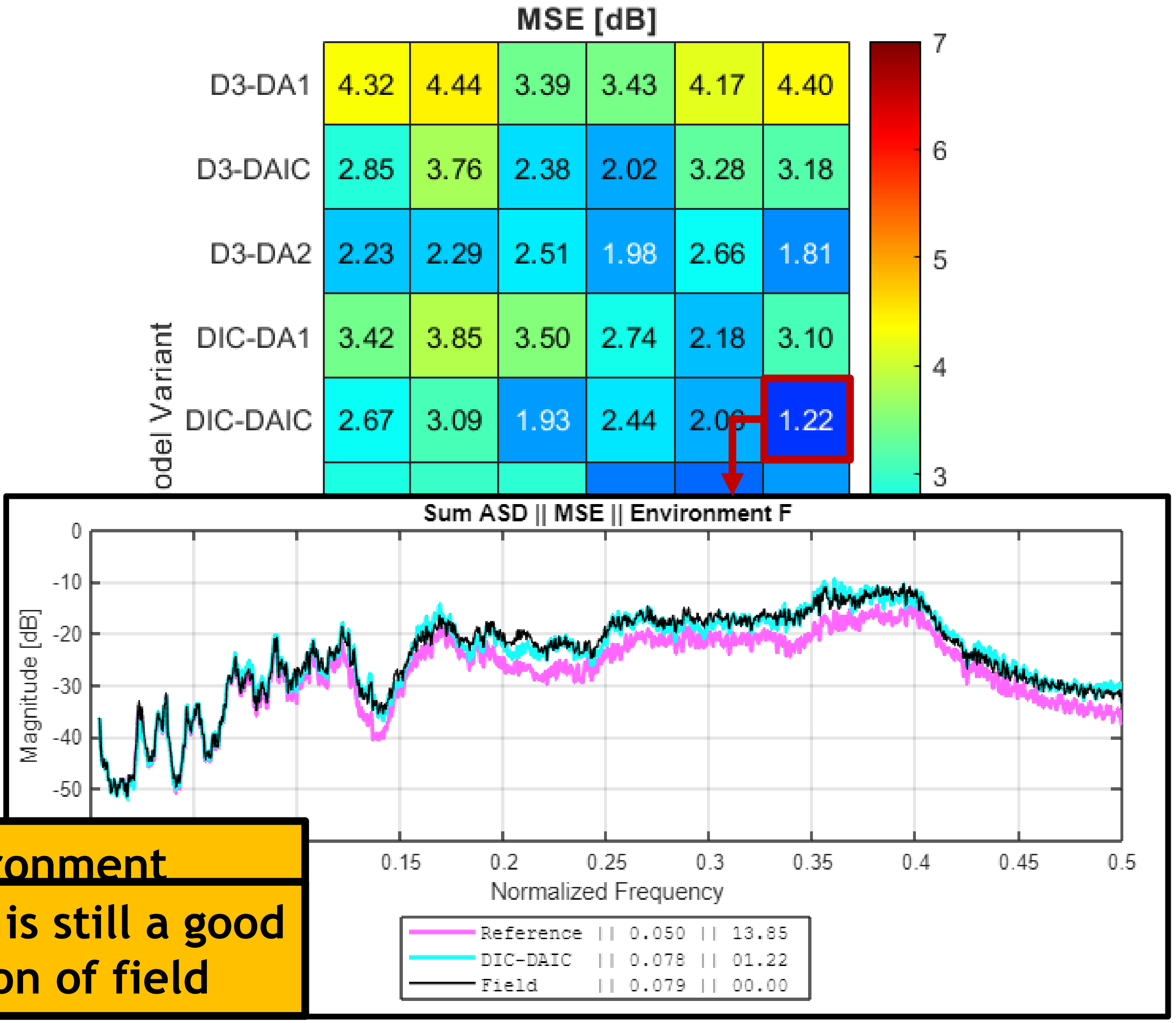
Results: Overall Performance

18 out of 66 DOF selected
Same Variant Field + Lab Models

OED



MSE



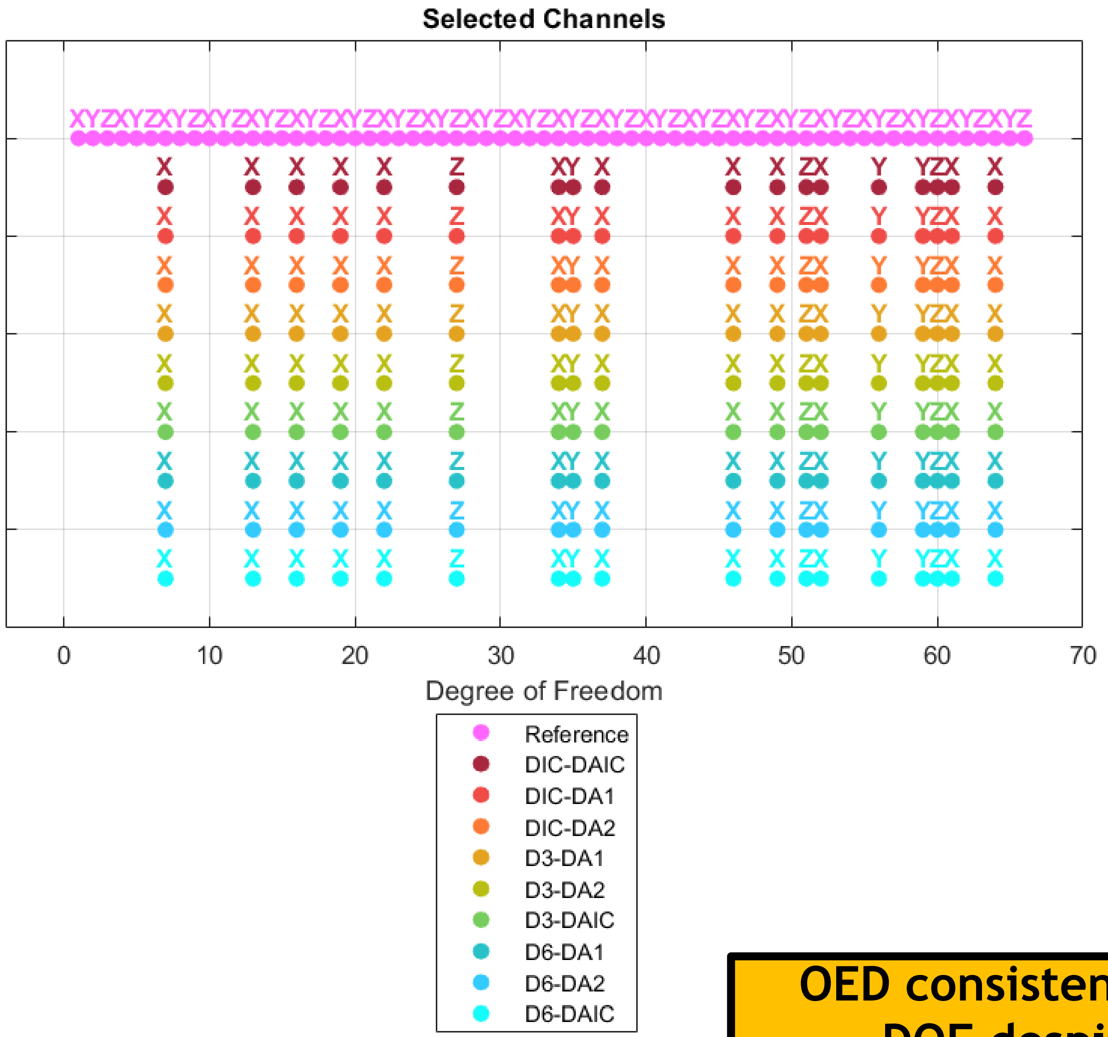
Field environment

Lower accuracy is still a good approximation of field

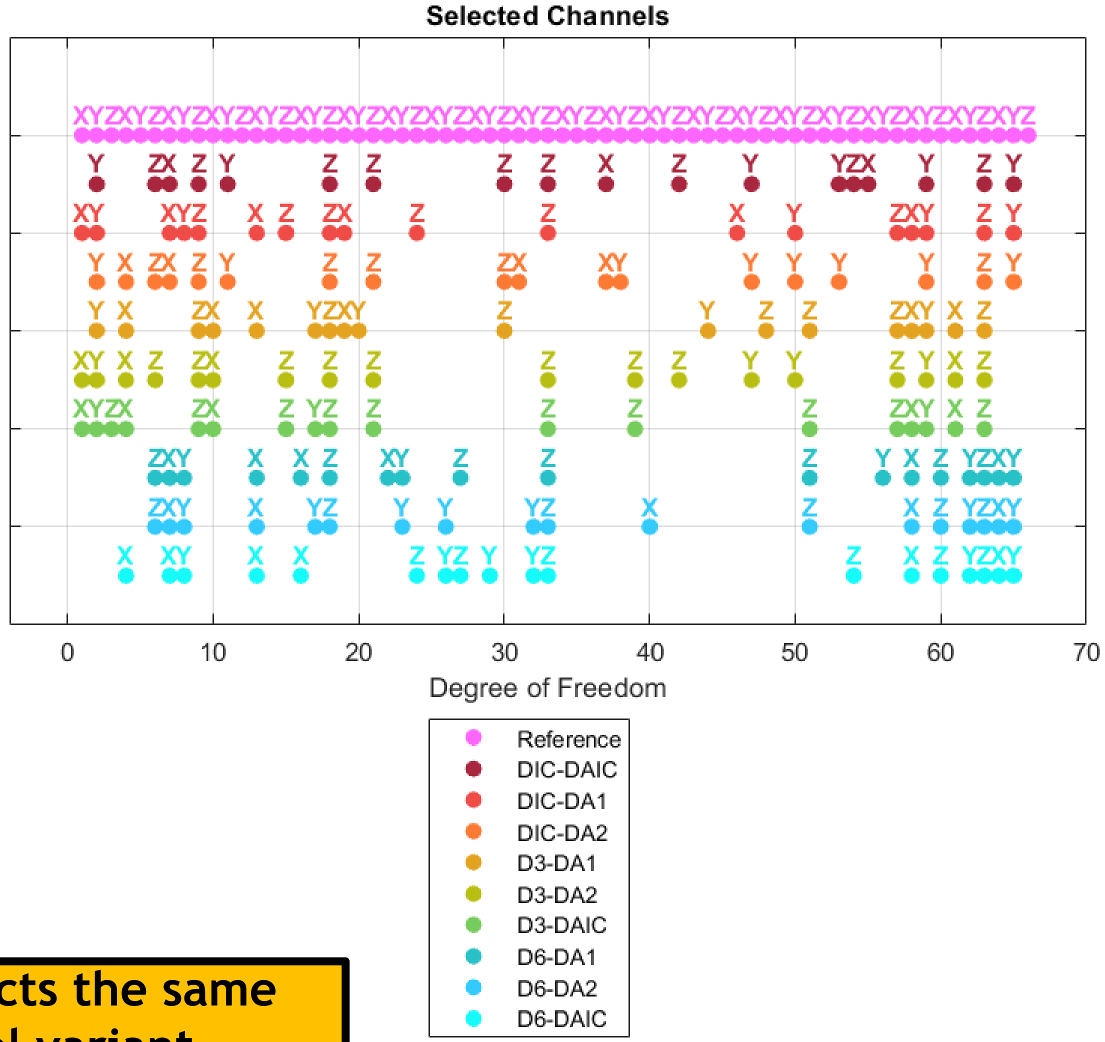
Results: Model Variant – Nominal Field – Environment A - DOF

18 out of 66 DOF selected
Nominal Variant Field Model

OED



MSE

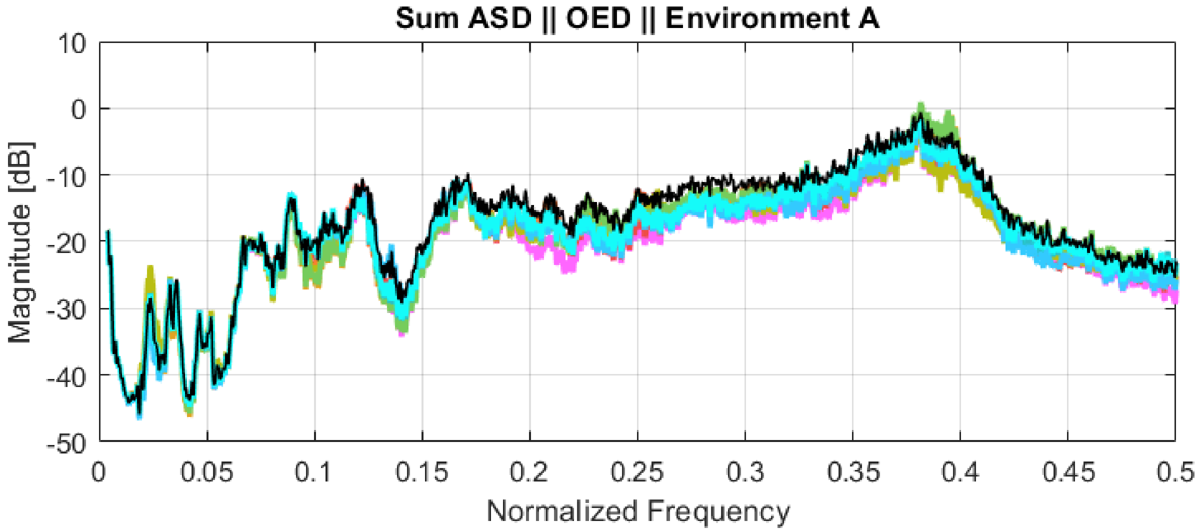


OED consistently selects the same DOF despite model variant

Results: Model Variant – Nominal Field – Environment A - S_{yy}

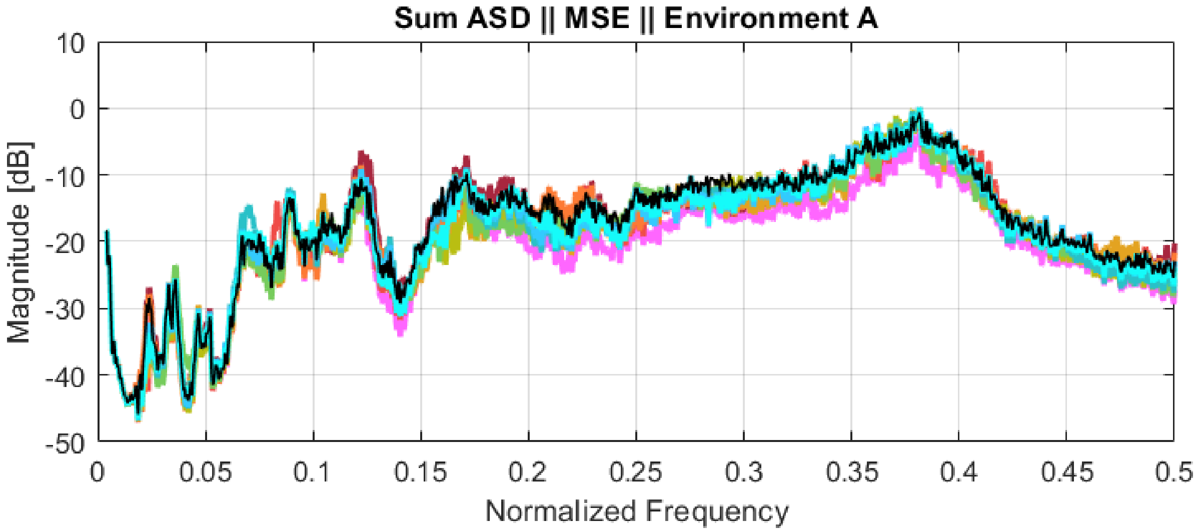
18 out of 66 DOF selected
Nominal Variant Field Model

OED



	RMS	MSE
Reference	0.118	13.82
DIC-DAIC	0.144	04.17
DIC-DA1	0.132	06.77
DIC-DA2	0.146	04.10
D3-DAIC	0.164	05.11
D3-DA1	0.127	07.69
D3-DA2	0.171	04.55
D6-DAIC	0.147	04.72
D6-DA1	0.136	06.24
D6-DA2	0.151	03.92
Field	0.181	00.00

MSE



	RMS	MSE
Reference	0.118	13.82
DIC-DAIC	0.174	02.67
DIC-DA1	0.167	03.88
DIC-DA2	0.171	02.73
D3-DA1	0.160	04.77
D3-DA2	0.169	04.93
D3-DAIC	0.163	04.31
D6-DA1	0.176	02.45
D6-DA2	0.175	02.24
D6-DAIC	0.174	02.48
Field	0.181	00.00

Field responses to environment loads are accurately reproduced despite differences in Field vs. Lab system

Outline: Conclusions

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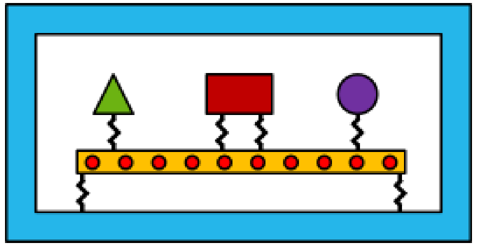
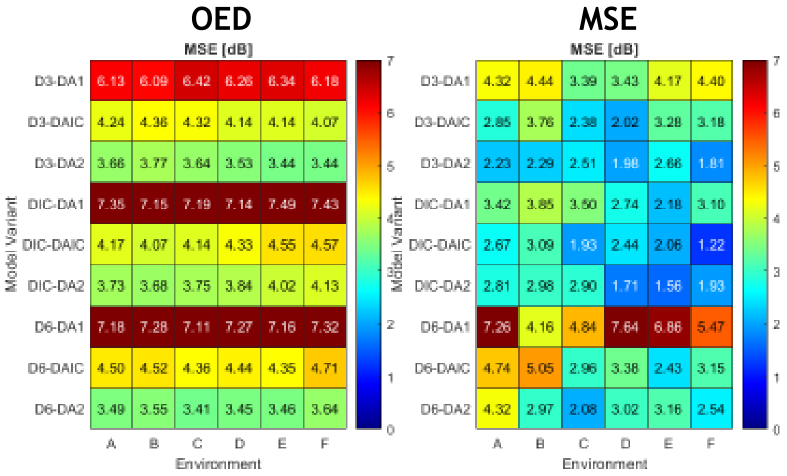
Conclusions

Summary:

- Two DOF selection approaches for MIMO test design were presented and demonstrated on complex models of a practical system.

- Laboratory responses obtained using DOF selected by each approach matched the field responses well for all test cases considering:
  - Complex field environments
  - Boundary condition differences
  - Dynamic property differences

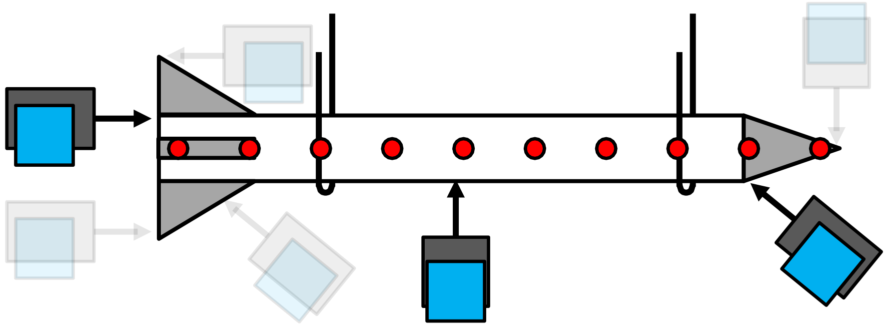




Future Work:

- Extend the approach to selection of input locations

- Apply the approach to degree of freedom selection for field



Conclusions

Acknowledgments

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The authors also acknowledge Drew Kouri at Sandia for providing the OED algorithms in ROL and Wilkins Aquino at Duke for many technical discussions on OED.

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