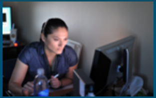
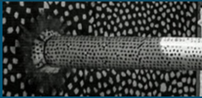
-0402C

Sandia National Laboratories

A Quantum Advantage for a Natural Streaming Problem





Presented by:

John Kallaugher[1]

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Primary quantity of interest is *space complexity*.

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In this talk: a quantum advantage for a natural one-pass streaming problem.

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- A quantum algorithm that beats it.

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- [**K.**, Price '17] Tight classical bounds (in some parameter regimes) from the *Boolean Hidden Matching* problem.
- [Gavinsky, Kempe, Kerenidis, Raz, Wolf '07] BHM *exponentially* separates quantum and classical one-way communication complexity.

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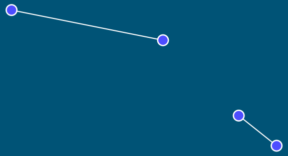

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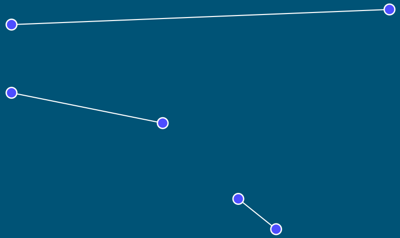

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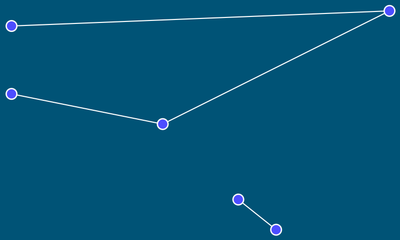

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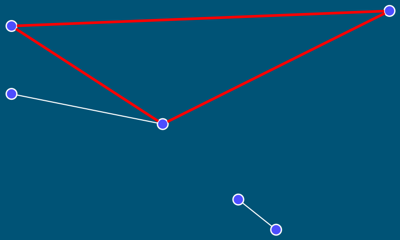

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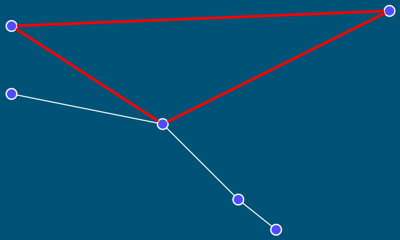

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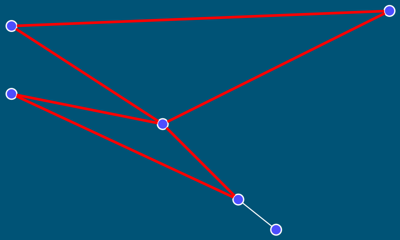

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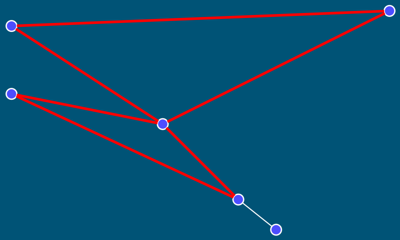

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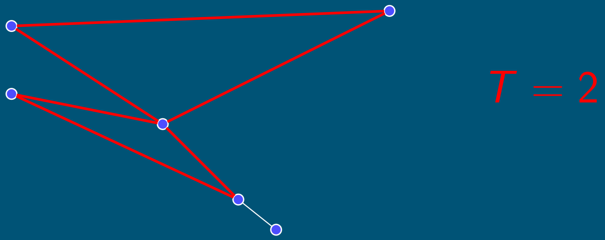

- [Braverman, Ostrovsky, Vilenchik '13] Non-trivial results require parametrization even if $T = \Omega(m)$.
- For this talk assume m edges, $T = \Theta(m)$ edge-disjoint triangles.

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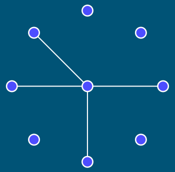

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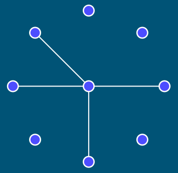

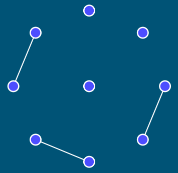

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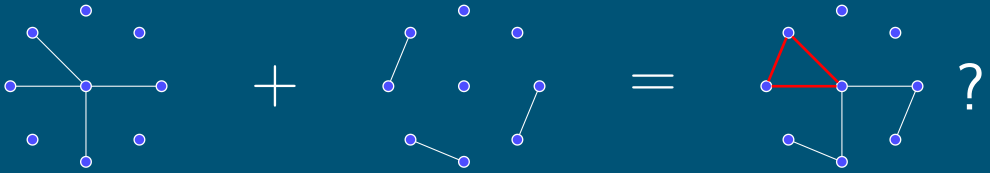

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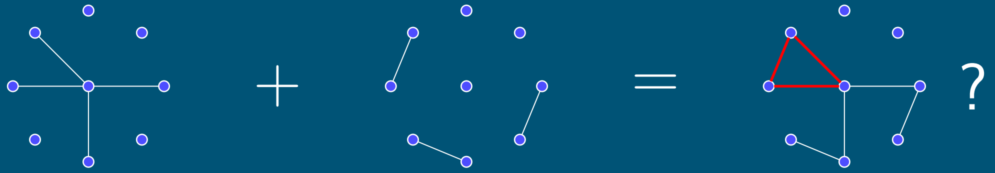

- Intuitively: if we keep k star edges, we have a $\sim T \times \left(\frac{k}{m}\right)^2 = \frac{k^2}{m}$ chance of getting both edges of at least one of $T = \Theta(m)$ triangles, so we need $\sqrt{m}$ samples.

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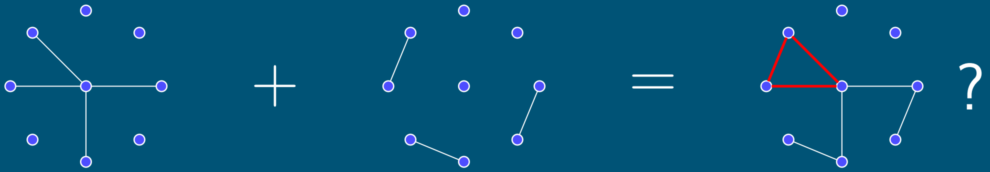

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- Reduction extends this to general (classical) algorithms.

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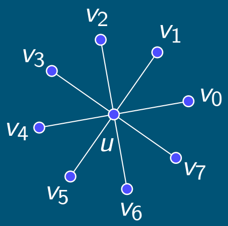

$$\Rightarrow \frac{1}{\sqrt{8}} \sum_{i=0}^{7} |v_i\rangle$$

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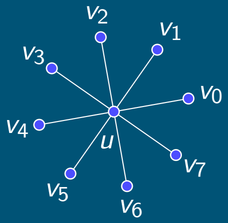

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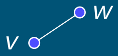

$$\Longrightarrow \frac{|v\rangle + |w\rangle}{\sqrt{2}}, \frac{|v\rangle - |w\rangle}{\sqrt{2}}$$

Bob measures with $(|v\rangle + |w\rangle)/\sqrt{2}$ and $(|v\rangle - |w\rangle)/\sqrt{2}$ for each vw in matching.

Quantum Idea

Measuring $\frac{1}{\sqrt{d_v}} \sum_{v \in N(u)} |v\rangle$ in the basis $\left\{ \frac{|v\rangle + |w\rangle}{\sqrt{2}}, \frac{|v\rangle - |w\rangle}{\sqrt{2}} : vw \in M \right\}$.

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- Otherwise, both basis vectors from vw equally likely.,
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- Gives a $\mathrm{O}(\log m)$ qubit protocol.

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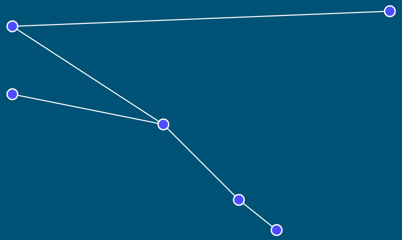
- Generalize to Alice and Bob holding arbitrary graphs.
- Go from two player protocol to full streaming.

Generalizing the Graph: Alice

Now let Alice have a general graph that Bob might somehow complete to triangles.



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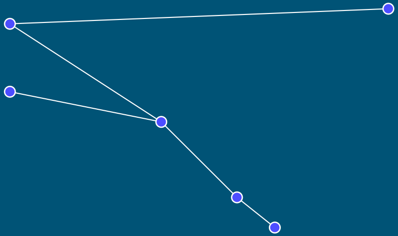

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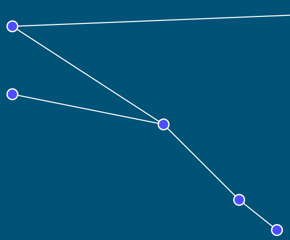

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- For each of Bob's edges vw we now measure with the basis vectors $\frac{|uv\rangle + |uw\rangle}{\sqrt{2}}, \frac{|uv\rangle - |uw\rangle}{\sqrt{2}}$ for every $u \in V$.

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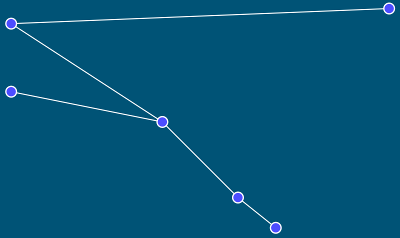
$$\Longrightarrow \frac{|u\rangle + |v\rangle}{\sqrt{2}} \not\perp \frac{|v\rangle + |w\rangle}{\sqrt{2}}$$

Now the basis vectors generated are no longer orthogonal.

Subsampling

If Bob's graph has max degree d, we can subsample with probability $\Theta(1/d)$.



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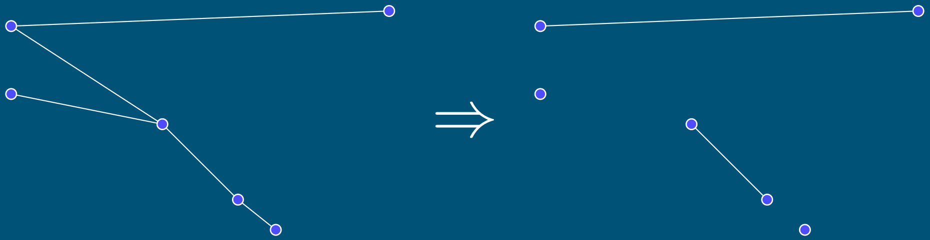

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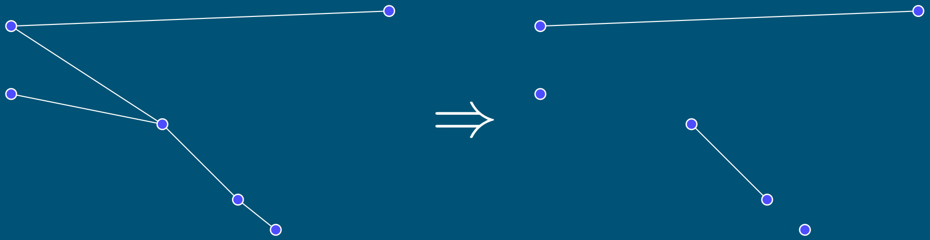

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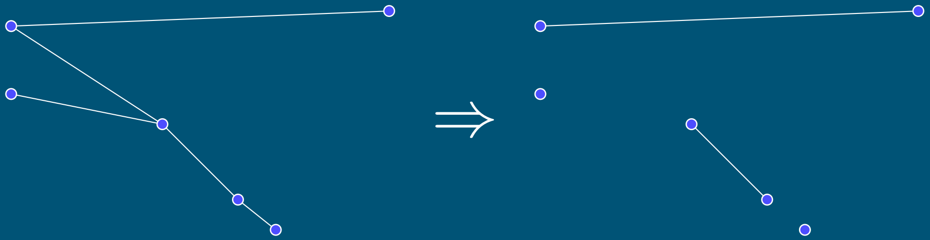
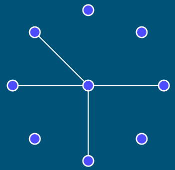
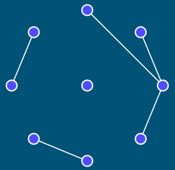


- The expectation of our estimator is now $2T/md$, so we need d^2 as many repeats.
- Not very helpful if d is unbounded.

How Much do High-Degree Vertices Matter?

Consider the original "hard case" but with Bob possibly having higher degree vertices.





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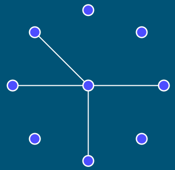
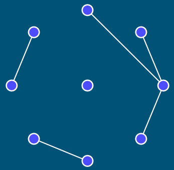
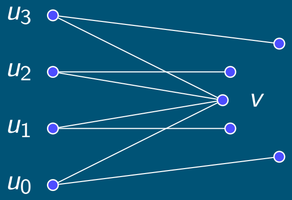


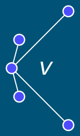


If all of the potential triangles are in one star Alice holds, at most a few of them are incident to Bob's high degree vertices.

Interpolating

For more triangles to involve Bob's high degree vertices (here v), they must be split among more of Alice's vertices (here u_i).





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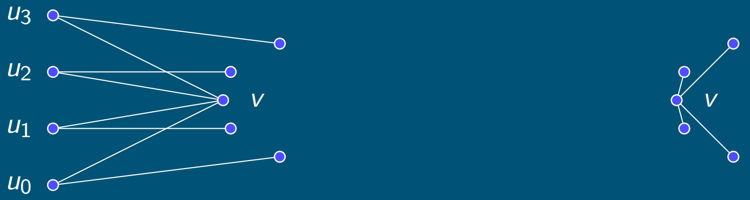

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- But this makes counting them *classically* easier!
- Interpolating gives us an $m^{2/5}$ (instead of $\sqrt{m}$) space algorithm.

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- Perform the “Bob” measurement one edge at a time by measuring with three projectors, onto $\text{span}(\{|uv\rangle + |uw\rangle\}_{u\in V})$, $\text{span}(\{|uv\rangle - |uw\rangle\}_{u\in V})$ and the remaining space, whenever an edge vw arrives.

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- Treat every edge as both an "Alice" edge and a "Bob" edge.
- Prepare the "Alice" superposition $\frac{1}{\sqrt{2m}}\sum_{u\in V}\sum_{v\in N(u)}|uv\rangle$ by starting with a superposition over $2m$ "dummy" variables and swapping when an edge arrives.
- Perform the "Bob" measurement one edge at a time by measuring with three projectors, onto $\text{span}(\{|uv\rangle + |uw\rangle\}_{u\in V})$, $\text{span}(\{|uv\rangle - |uw\rangle\}_{u\in V})$ and the remaining space, whenever an edge vw arrives.
- Now for each triangle the third edge to arrive will act as a "Bob edge" and the first two as "Alice edges".

Conclusion

Theorem (Informal)

There is a quantum streaming algorithm that uses

$$O\left(\frac{m^{8/5}}{T^{6/5}}\Delta_E^{4/5}\log n \cdot \frac{1}{\varepsilon^2}\log\frac{1}{\delta}\right)$$

space to (ε, δ)*-count triangles in the stream, where* m *is the number of edges,* T *the number of triangles, and* Δ_E *the greatest number of triangles sharing any given edge.*

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- Best lower bound for quantum is $\Omega\left(\frac{m\Delta_E}{T}\right)$. Is exponential separation possible?