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NEW HYBRID MODEL FOR EVALUATING THE FREQUENCY-DEPENDENT LEAKAGE INDUCTANCE OF A VARIABLE INDUCTANCE TRANSFORMER (VIT)

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OUTLINE

- ✓ Variable Inductance Transformer (VIT)
- ✓ Frequency-independent (LF) Double-2-D Model
- ✓ Skin and Proximity Effects
- ✓ Frequency-dependent (HF) Dowell's 1-D Model
- ✓ Challenge Statement
- ✓ Proposed Hybrid Model
- ✓ Results
- ✓ Conclusion
- ✓ References

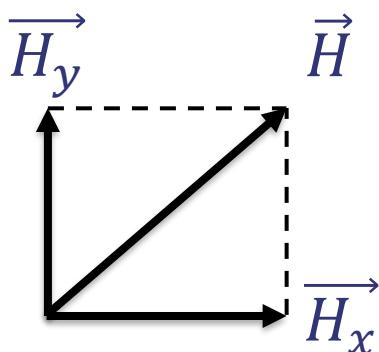
VARIABLE INDUCTANCE TRANSFORMER (VIT)

- *Partially-filled* transformer ($h_w < h$)
- One of the bobbins is *movable*
- First introduced in **ECCE 2021**
 - \vec{H} increases as \vec{H}_x increases
 - Magnetic energy *increases*
 - Leakage inductance *increases*
 - Needs a **2-D model**

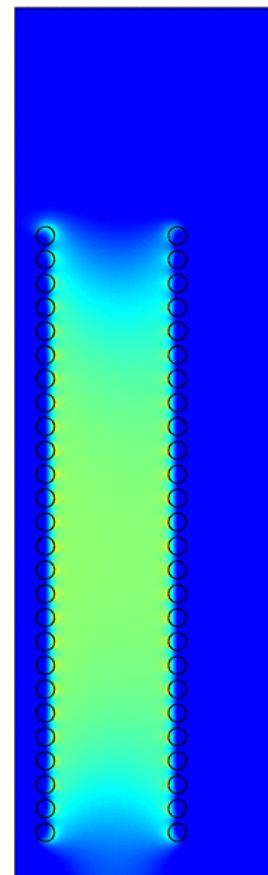
$$H^2 = H_x^2 + H_y^2$$

$$E_{mag} \propto H^2$$

$$L_{lk} \propto E_{mag}$$



g(1)=0 mm Time=2.5E-4 s Surface: Magnetic energy density (J/m³)



movable
bobbin

fixed
bobbin

fixed
bobbin

movable
bobbin

movable
bobbin

fixed
bobbin

fixed
bobbin

movable
bobbin

movable
bobbin

LF DOUBLE-2-D MODEL

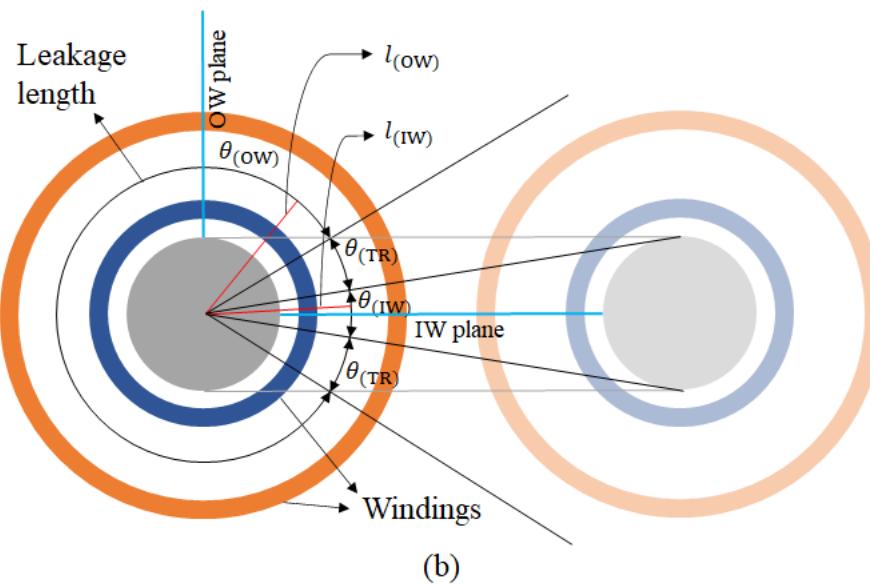
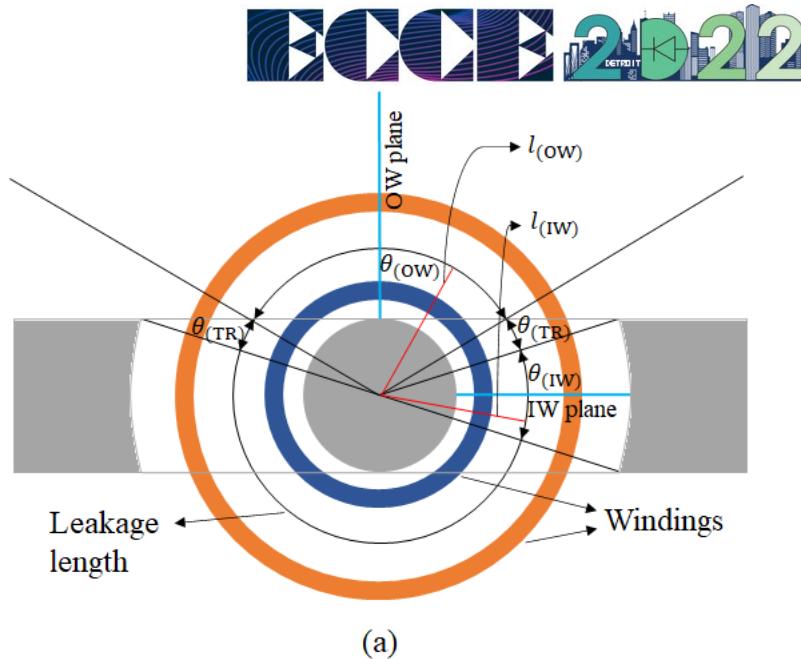
- ✓ Analyzes both \vec{H}_x and \vec{H}_y
- ✓ Analyzes both IW and OW planes
- ✓ Uses image method to calculate \vec{H}_x and \vec{H}_y

$$L_{lk,\text{double-2-D}} = s_c (L'_{2-D(\text{IW})} d_{l(\text{IW})} + L'_{2-D(\text{OW})} d_{l(\text{OW})})$$

$$s_c = \begin{cases} 1, & \text{core-type transformer} \\ 2, & \text{shell-type transformer} \end{cases}$$

where L'_{2-D} = leakage inductance per unit length
 d_l = partial leakage length

$$L'_{2-D} = \frac{\mu_0}{I_1^2} \iint H_{2-D}^2(x, y) dx dy$$



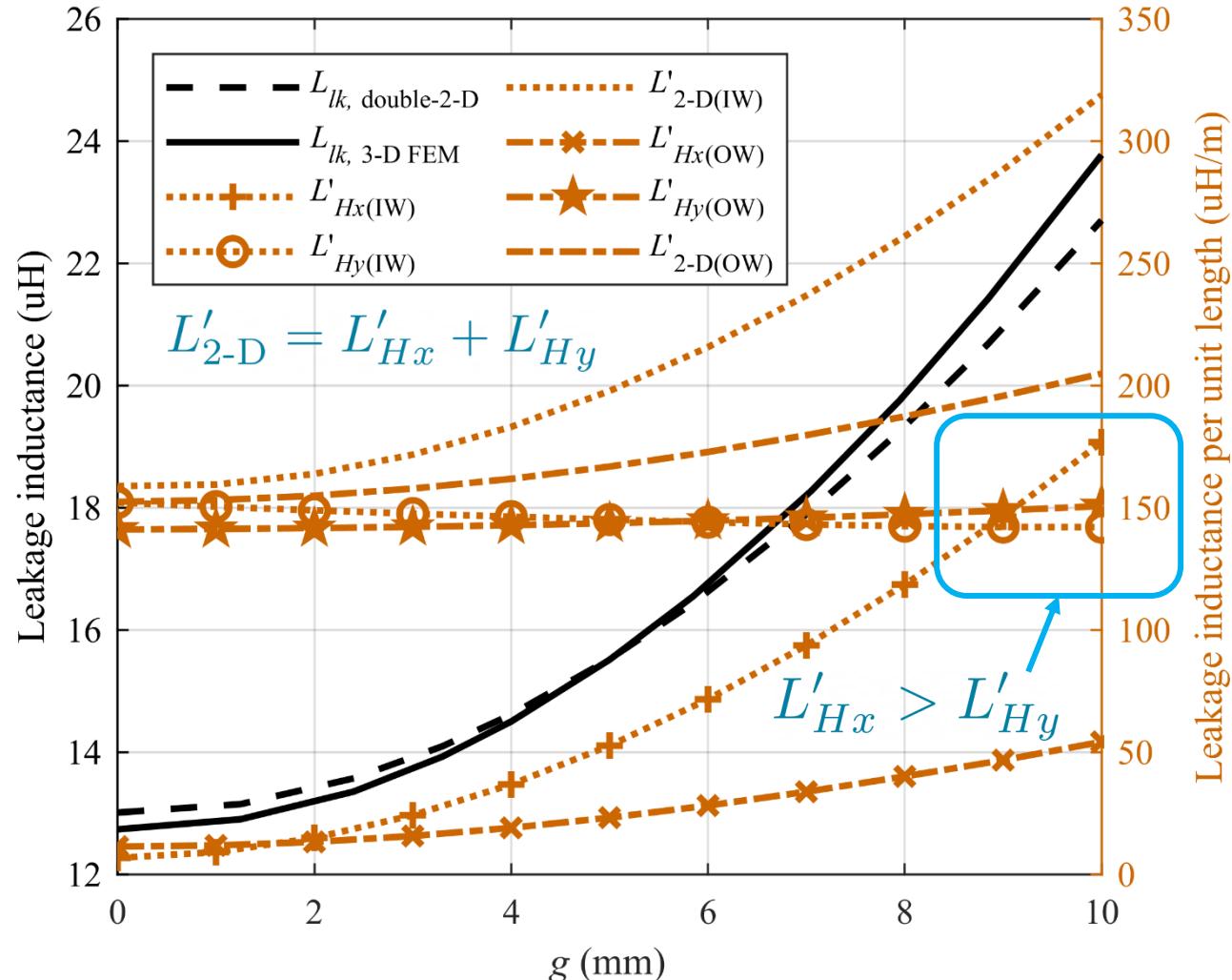
LF DOUBLE-2-D MODEL IN A VIT

Double-2-D model proved very effective for VITs

➤ Maximum error < 4.5%

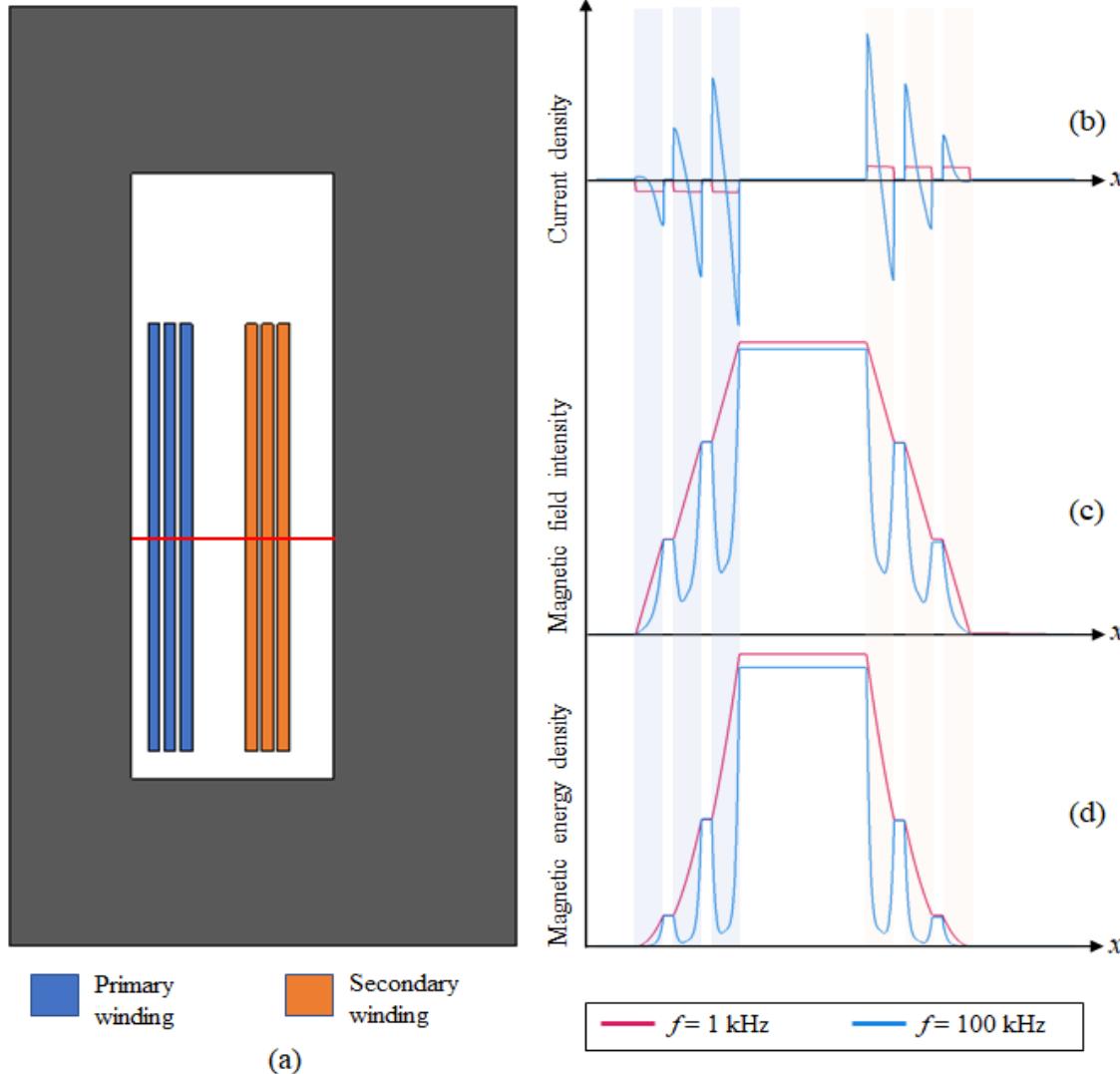
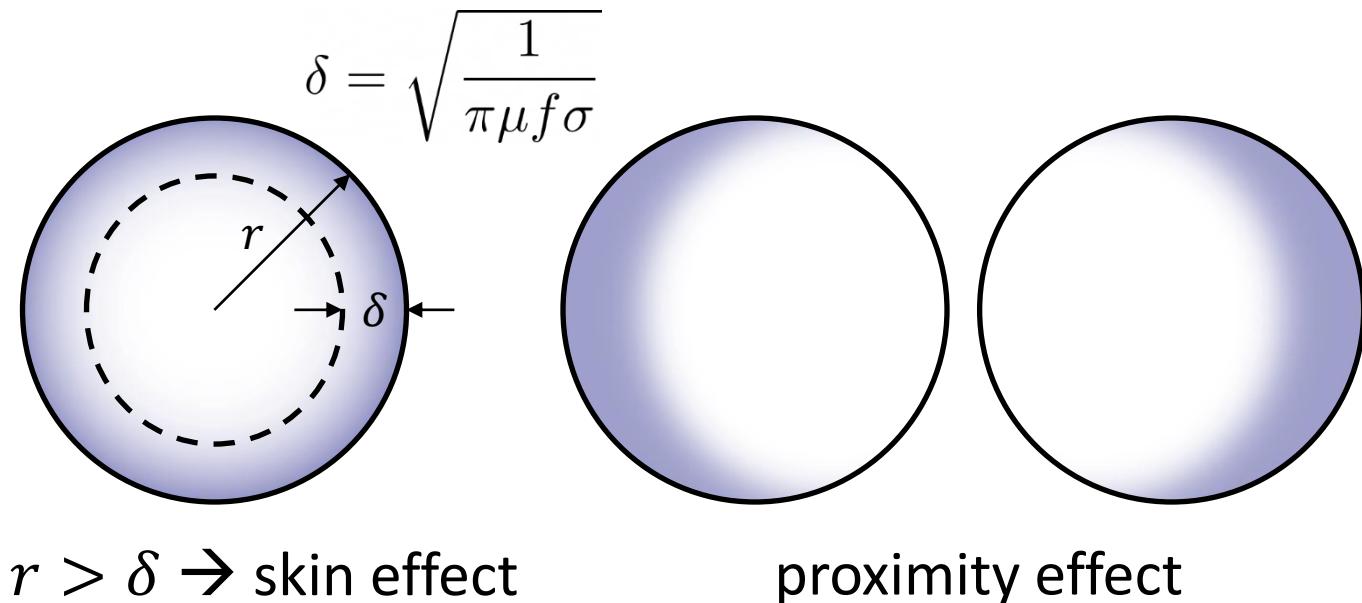
Plot interpretation

- L'_{Hx} increases with g , L'_{Hy} unchanged
- $L'_{Hx} > L'_{Hy}$ near $g = g_{max}$
- $L'_{Hx(IW)} > L'_{Hx(OW)}$
- L_{lk} increases due to increase in $\overrightarrow{H_x}$



SKIN AND PROXIMITY EFFECTS

- *Non-linear* current density and magnetic field intensity across conductors
- *Reduction* in effective leakage inductance with frequency
- *Critical* to determine leakage inductance at the specific operating frequency range



HF DOWELL'S 1-D MODEL

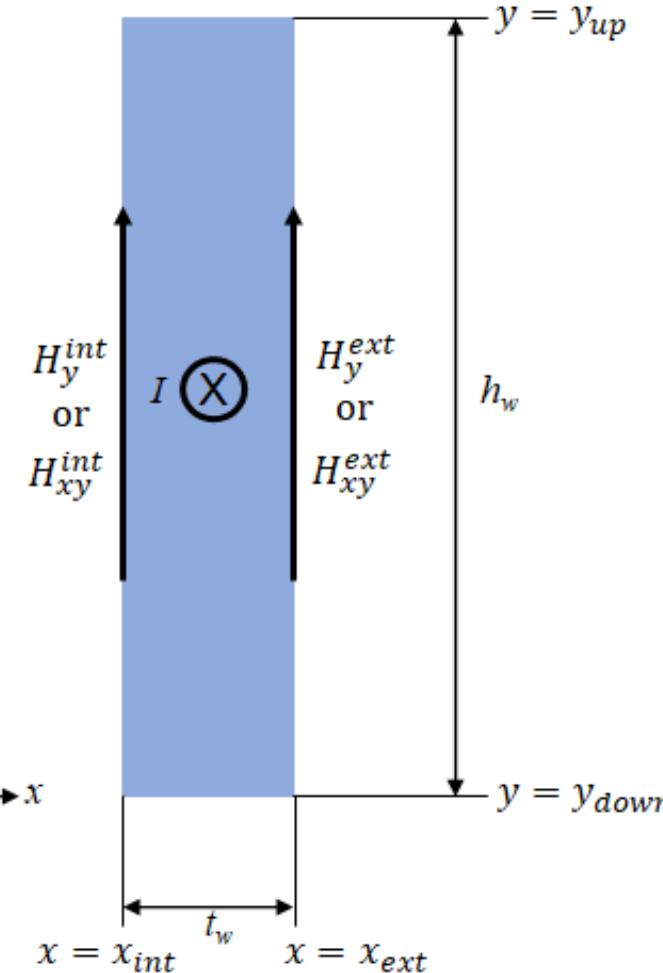
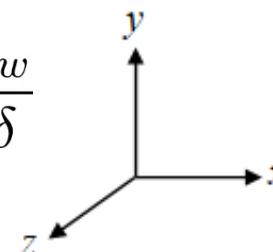
- Models both skin and proximity effects
- Models x -position of winding only
- Models $\overrightarrow{H_y}$ only
- Models IW plane only
- *Winding height = window height*, otherwise requires porosity factors

$$E''_{\text{foil}} = \frac{\mu_0 \delta}{4} \left(\left(H_y^{\text{ext}} + H_y^{\text{int}} \right)^2 \varphi_1 - 2 H_y^{\text{ext}} H_y^{\text{int}} \varphi_2 \right)$$

$$\varphi_1 = \frac{\sinh(2\Delta) - \sin(2\Delta)}{\cosh(2\Delta) - \cos(2\Delta)}, \quad \varphi_2 = \frac{\sinh(\Delta) - \sin(\Delta)}{\cosh(\Delta) - \cos(\Delta)}$$

$$E_{\text{foil}} = h_w \times \text{MLT} \times E''_{\text{foil}}$$

$$\Delta = \frac{t_w}{\delta}$$



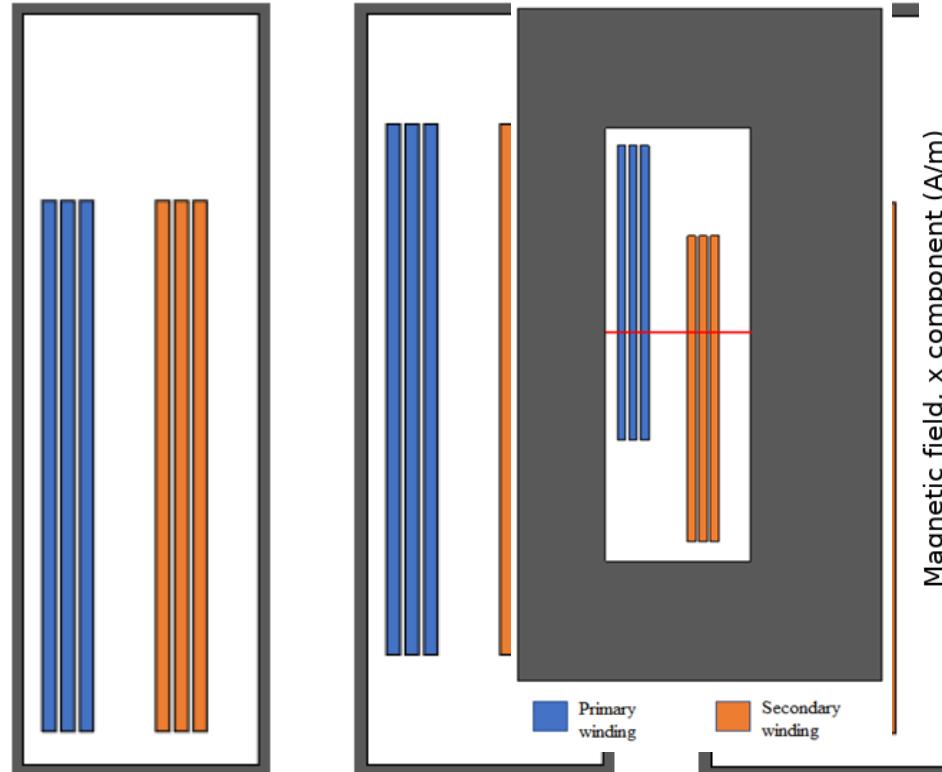
φ_1 = skin effect factor, φ_2 = proximity effect factor, Δ = penetration ratio, δ = skin depth

CHALLENGE STATEMENT

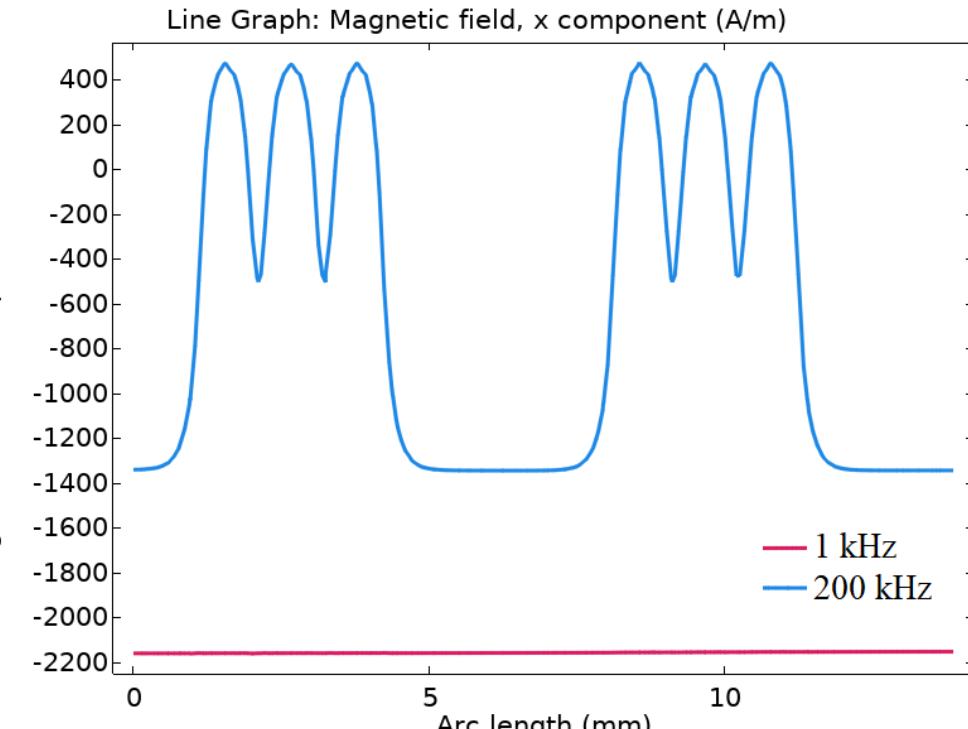
No 2-D HF model has been proposed so far

Existing 1-D HF models are inapplicable in a partially filled-transformer like VIT

- Movement of the *y-position* of the
 - both *x*- and *y*-positions must be *non* linear
- L'_{Hx} increases with *y*
 - $\vec{H_x}$ must be *non* linear
- $L'_{Hx(IW)} > L'_{Hx(OW)}$
 - both *IW* and *OW* must be *non* linear



4 different partially-filled winding geometries in IW plane

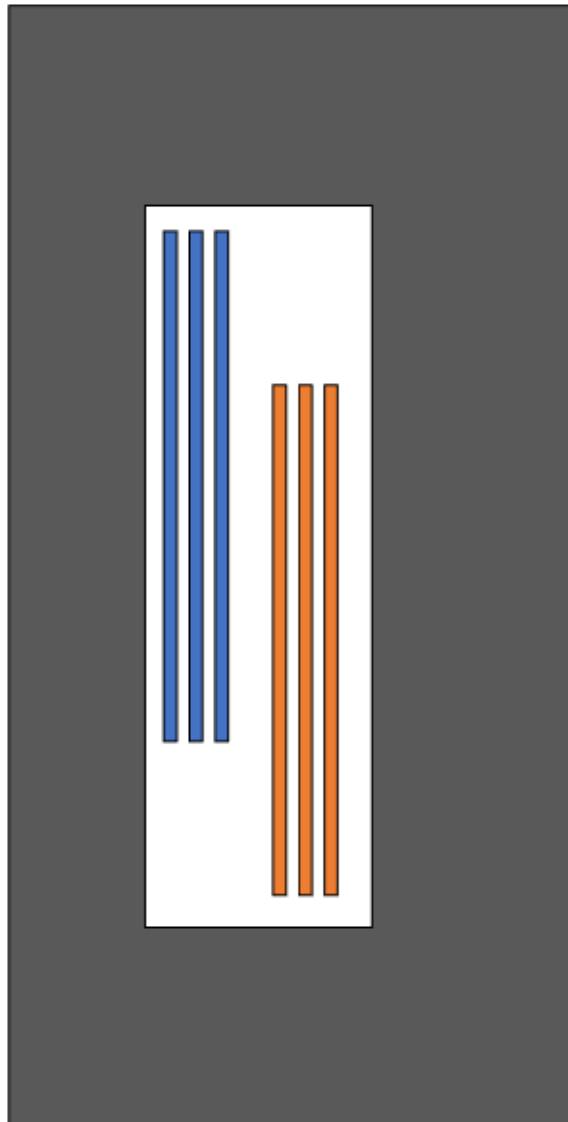


PROPOSED HYBRID MODEL

- *Pseudo 2-D model*, built on the double-2-D model platform
- *Assumption* is that skin and proximity effects influence the magnetic energy densities across the *winding cross-sections*
- Uses *superposition* to combine HF Dowell's model with LF double-2-D model

Hybrid model = double-2-D + modified Dowell's model

Region	Frequency dependency	Model
winding cross-sections	dependent	modified Dowell's
non-winding spaces	independent	double-2-D



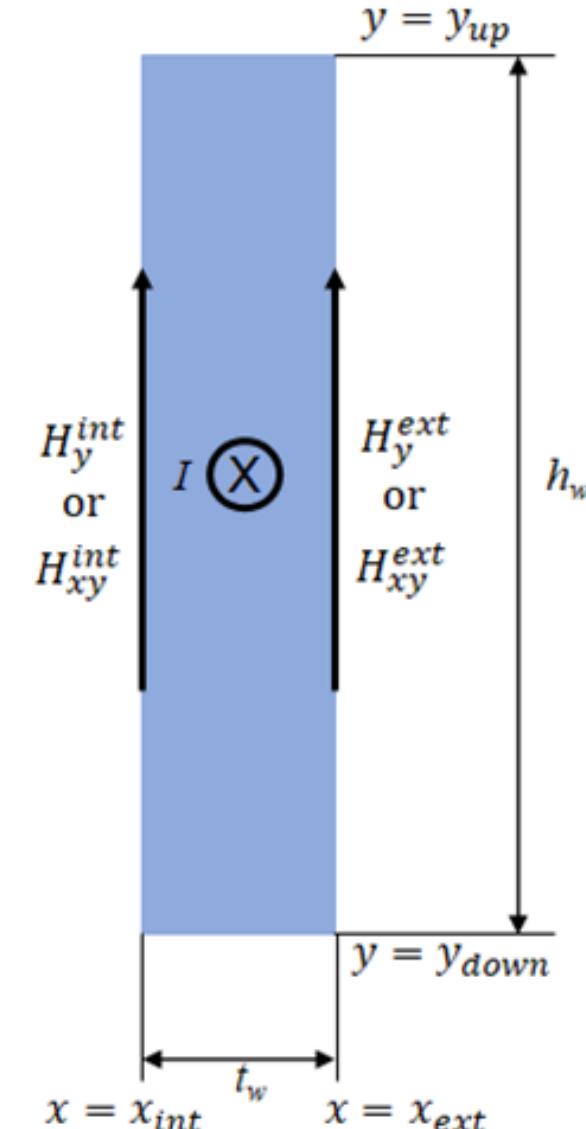
MODIFIED DOWELL'S MODEL

Closed-form solution of the modified Dowell's model for a foil,

$$E'_{\text{foil}} = \frac{\mu_0 \delta}{4} \int_{y_{\text{down}}}^{y_{\text{up}}} \left(\left(H_{xy}^{\text{ext}}(y) + H_{xy}^{\text{int}}(y) \right)^2 \varphi_1 - 2H_{xy}^{\text{ext}}(y)H_{xy}^{\text{int}}(y)\varphi_2 \right) dy$$

H_{xy}^{ext} and H_{xy}^{int} are readily available from the double-2-D model by evaluating $H_x(x, y)$ and $H_y(x, y)$ at $x = x_{\text{int}}$ and $x = x_{\text{ext}}$

$$H_{xy} = \sqrt{(H_x^2 + H_y^2)}$$

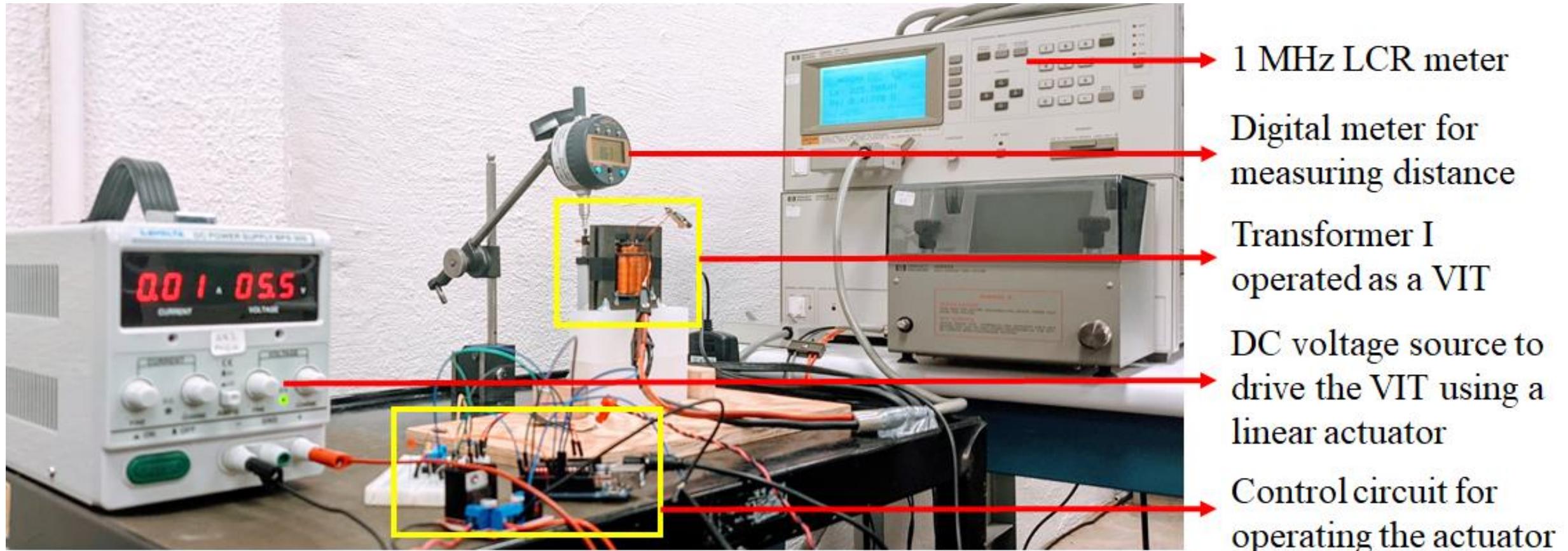


VIT SPECIFICATIONS

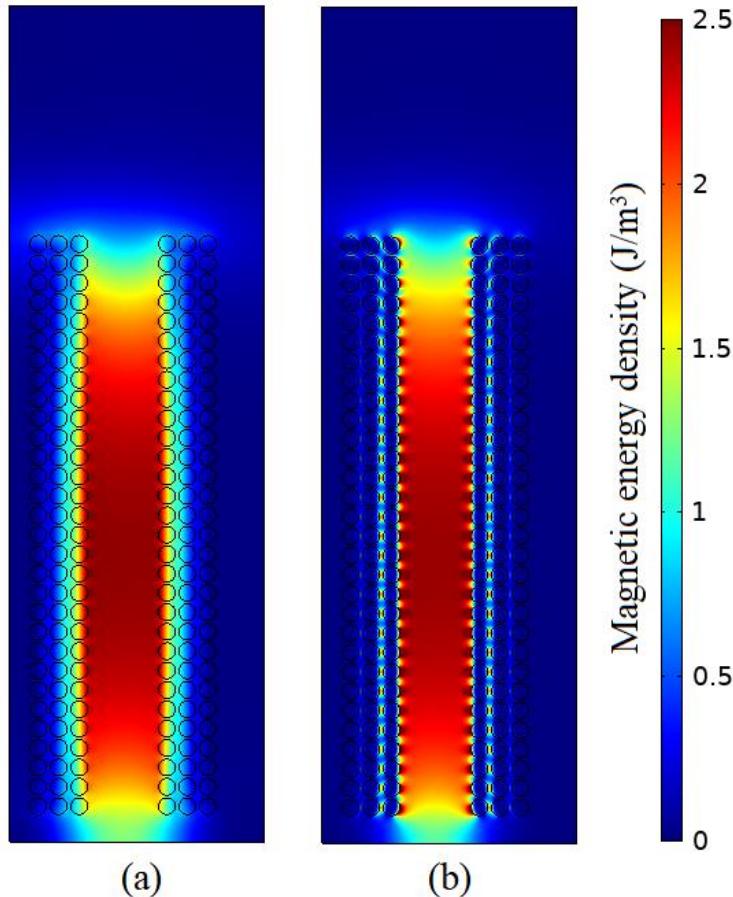
Turns ratio	1:1
Conductor type, size	Round, AWG 19
Number of turns per layer	30
Number of layers per winding	3
Core geometry	EC 70
Winding height	31.50 mm
External diameter of the movable bobbin	19 mm
External diameter of the fixed bobbin	33 mm
Insulation gap between layers	0.20 mm
Fill-factor	19.04 %
Maximum travel of the movable bobbin	11 mm
Test frequency range	1 – 200 kHz
Air cube (for OW plane boundary)	80 ³ mm ³

- ❑ MATLAB R2019a is used for all semi-analytical calculations
- ❑ COMSOL Multiphysics 5.5 is used for obtaining the 2-D FEM results

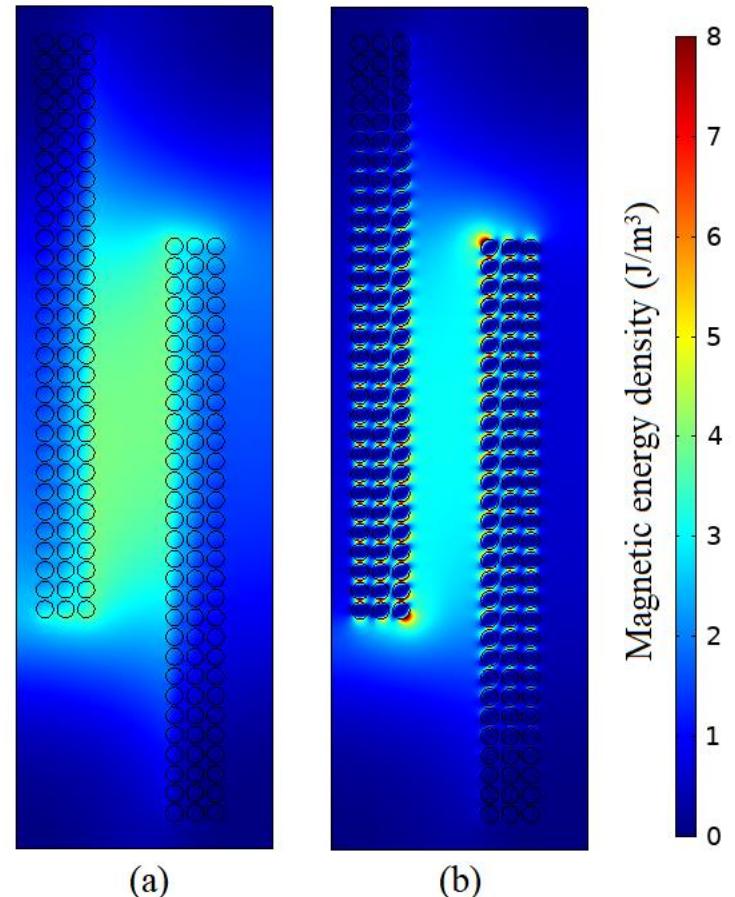
EXPERIMENTAL SETUP WITH THE VIT



FEM PLOTS

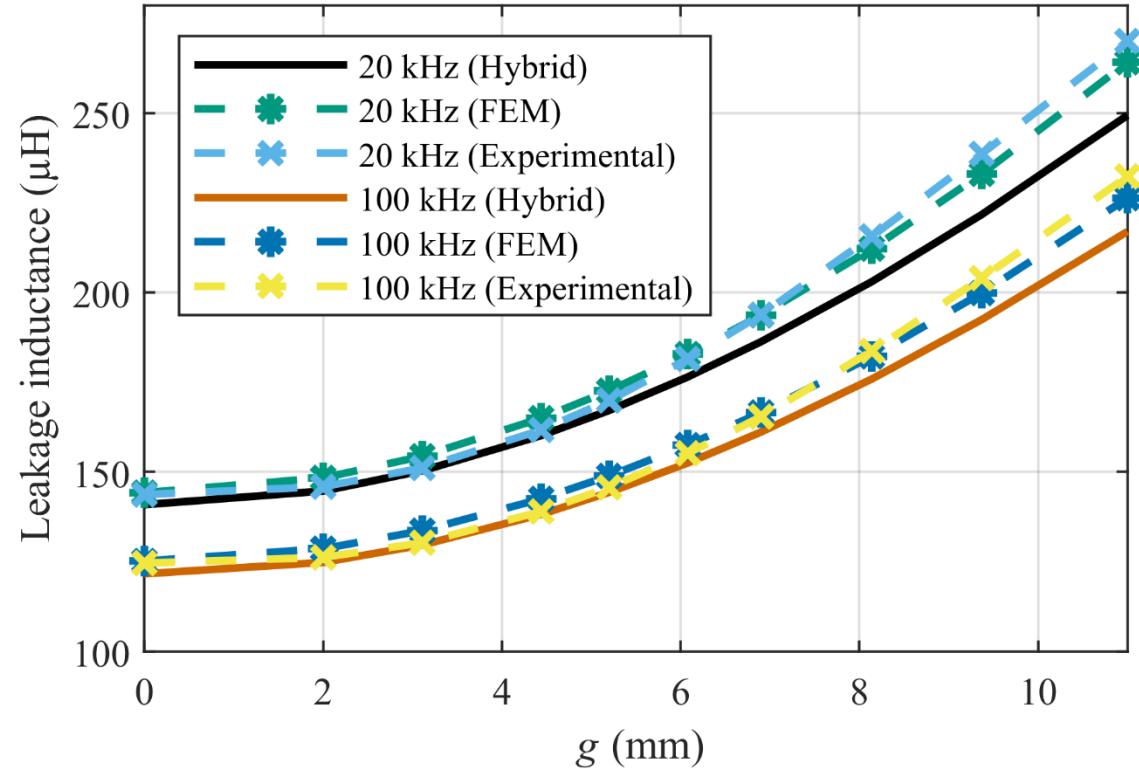
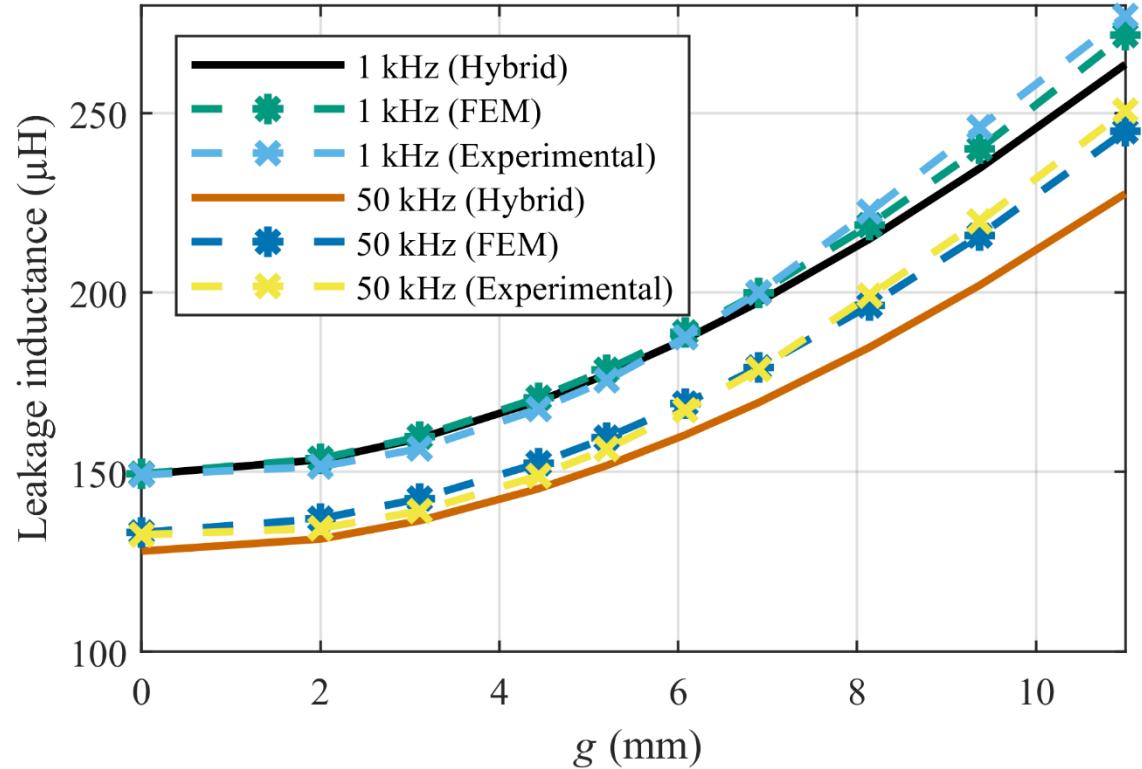


Magnetic energy densities across the IW plane at $g = 0$ mm: (a) 1 kHz, and (b) 200 kHz.



Magnetic energy densities across the IW plane at $g = 11$ mm: (a) 1 kHz, and (b) 200 kHz.

RESULTS

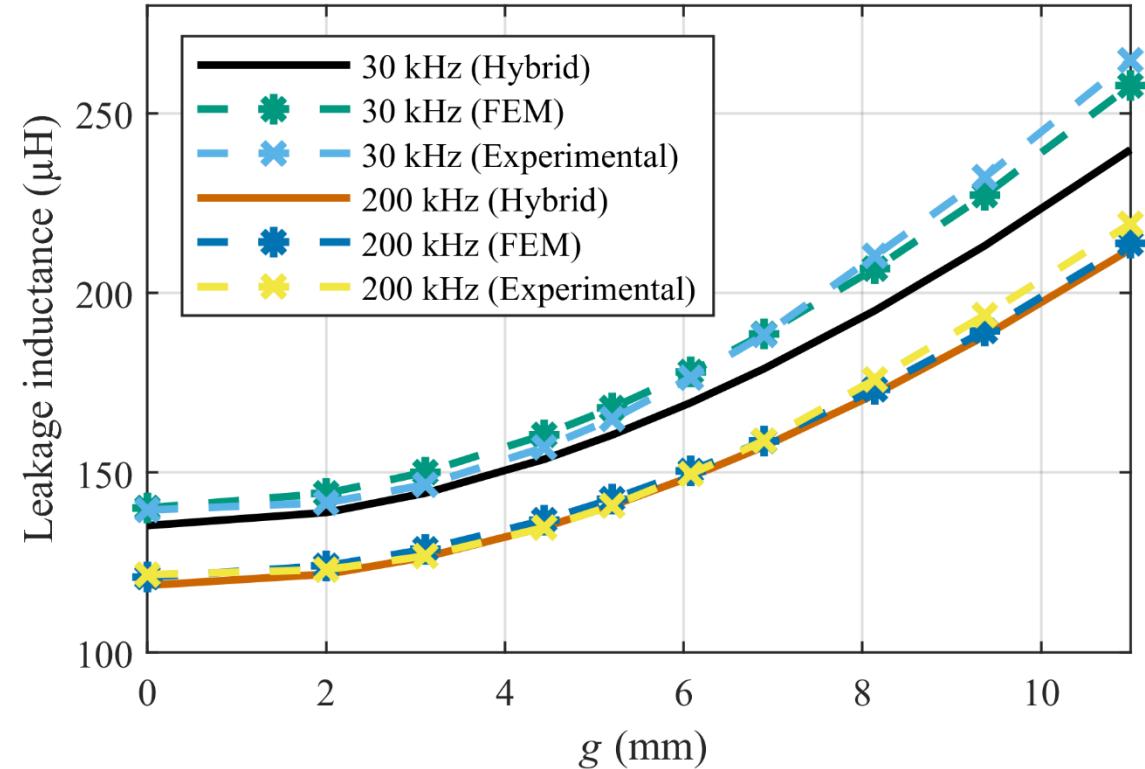


6 different frequencies are investigated within 1 – 200 kHz.

For the specific wire gauge, the frequency at which δ equals the radius of the conductor is 20.4 kHz.

RESULTS

- Leakage inductance increases with g while decreases with frequency
- Leakage inductance at $g = 11$ mm is 80 % higher than that at $g = 0$ mm across all frequencies
- Maximum error with the hybrid model is 7.1 % observed at 50 kHz and $g = 11$ mm
- Existing HF models give a flat horizontal curve for varying overlaps
- Between 1 and 200 kHz, the leakage inductance dropped by more than 18 % across all overlaps



CONCLUSION

- A new *hybrid model* is proposed – first attempt towards 2-D HF modeling
- It uses *superposition* to combine Dowell's model with the double-2-D model
 - A modified Dowell's model calculates the leakage inductance contributions from the winding cross-sections
 - The double-2-D model calculates the leakage inductance contributions from the non-winding spaces
- Semi-analytical results are validated through FEM modeling and experimentation
- With a *maximum error of only 7.1 %*, the hybrid model is promising for any partially filled transformer geometries
- It can aid *multi-objective optimization-based designs* of power electronic converters employing transformers with integrated magnetics

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Questions?

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