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NEW HYBRID MODEL FOR EVALUATING THE FREQUENCY-DEPENDENT LEAKAGE INDUCTANCE OF A VARIABLE INDUCTANCE TRANSFORMER (VIT)

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- ✓ Variable Inductance Transformer (VIT)
- ✓ Frequency-independent (LF) Double-2-D Model
- ✓ Skin and Proximity Effects
- ✓ Frequency-dependent (HF) Dowell's 1-D Model
- ✓ Challenge Statement
- ✓ Proposed Hybrid Model
- ✓ Results
- ✓ Conclusion
- ✓ References

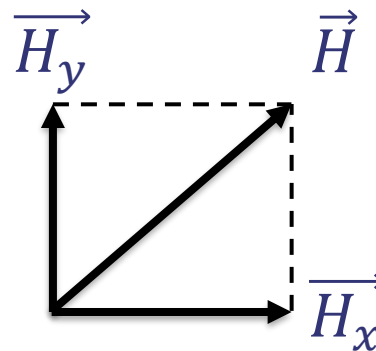
VARIABLE INDUCTANCE TRANSFORMER (VIT)

- *Partially-filled* transformer ($h_w < h$)
- One of the bobbins is *movable*
- First introduced in **ECCE 2021**
 - \vec{H} increases as \vec{H}_x increases
 - Magnetic energy *increases*
 - Leakage inductance *increases*
 - Needs a **2-D model**

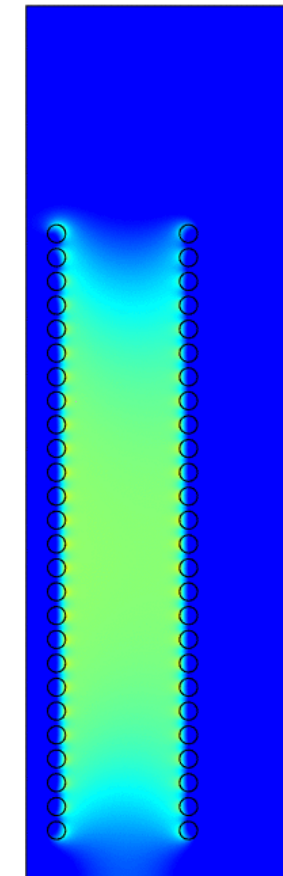
$$H^2 = H_x^2 + H_y^2$$

$$E_{mag} \propto H^2$$

$$L_{lk} \propto E_{mag}$$



g(1)=0 mm Time=2.5E-4 s Surface: Magnetic energy density (J/m³)



movable
bobbin

fixed
bobbin

0.6

0.5

0.4

0.3

0.2

0.1

LF DOUBLE-2-D MODEL

- ✓ Analyzes both \vec{H}_x and \vec{H}_y
- ✓ Analyzes both IW and OW planes
- ✓ Uses image method to calculate \vec{H}_x and \vec{H}_y

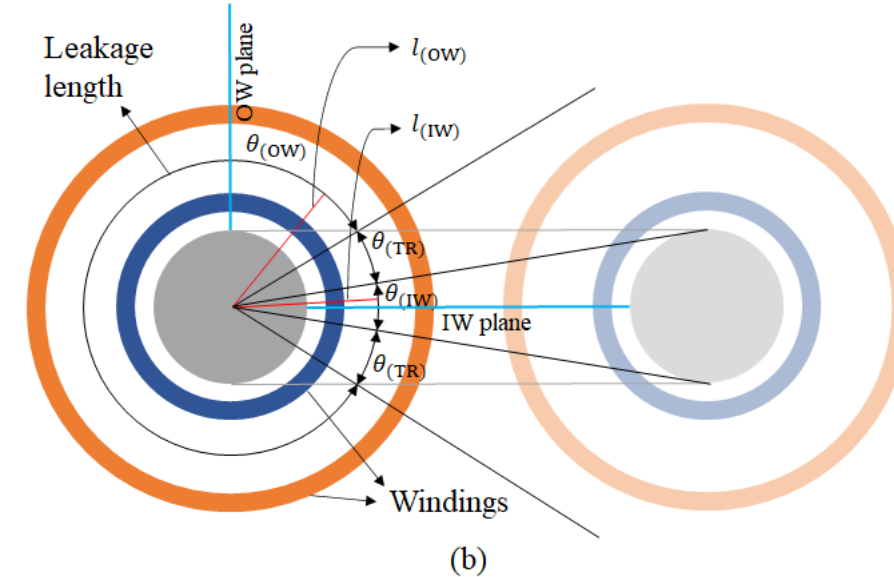
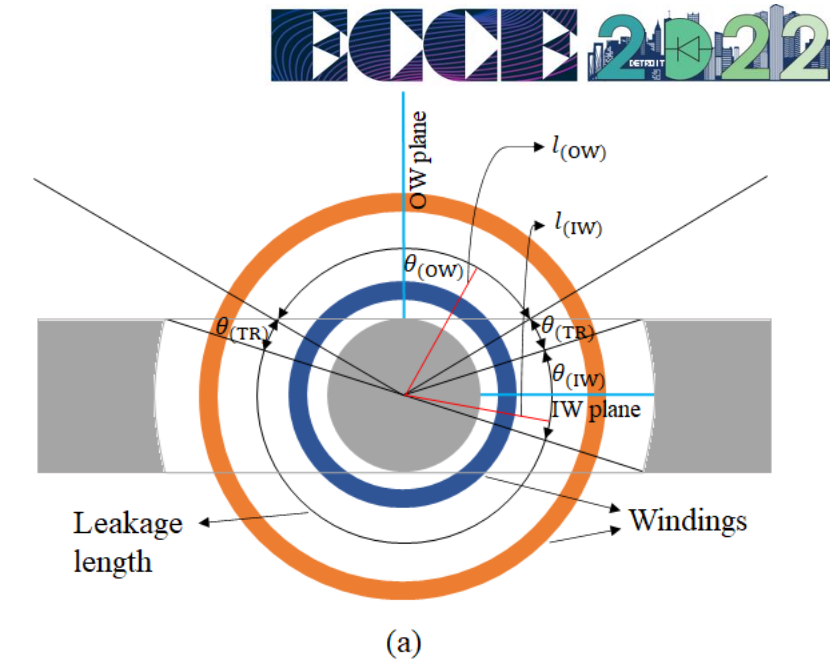
$$L_{lk, \text{double-2-D}} = s_c \left(L'_{2-D(IW)} d_{l(IW)} + L'_{2-D(OW)} d_{l(OW)} \right)$$

$$s_c = \begin{cases} 1, & \text{core-type transformer} \\ 2, & \text{shell-type transformer} \end{cases}$$

where L'_{2-D} = leakage inductance per unit length

d_l = partial leakage length

$$L'_{2-D} = \frac{\mu_0}{I_1^2} \iint H_{2-D}^2(x, y) dx dy$$



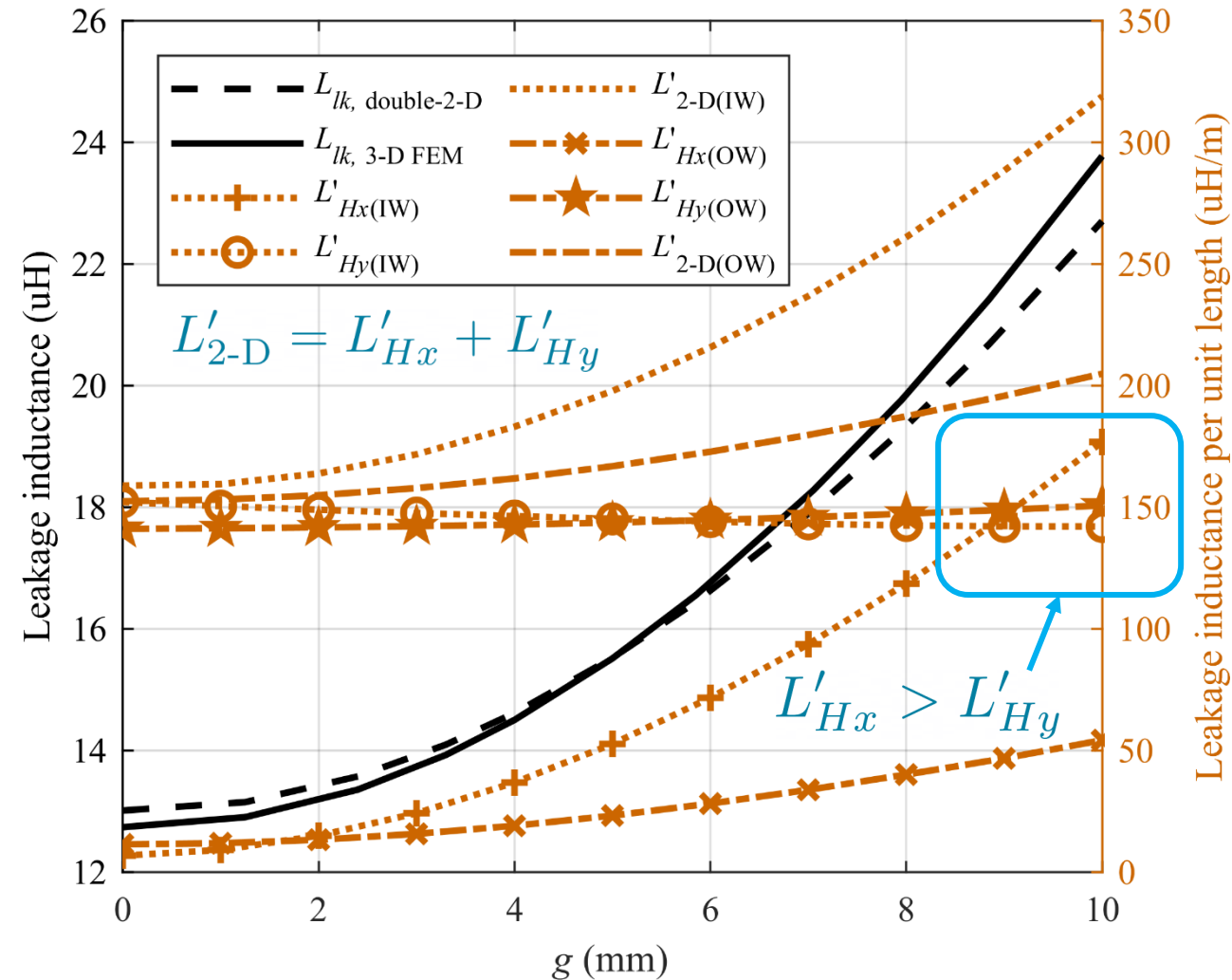
LF DOUBLE-2-D MODEL IN A VIT

Double-2-D model proved very effective for VITs

➤ Maximum error < 4.5%

Plot interpretation

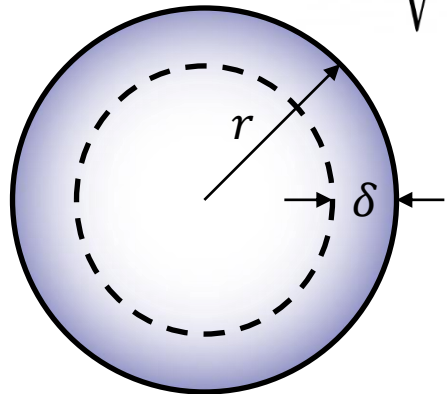
- L'_{Hx} increases with g , L'_{Hy} unchanged
- $L'_{Hx} > L'_{Hy}$ near $g = g_{max}$
- $L'_{Hx(IW)} > L'_{Hx(OW)}$
- L_{lk} increases due to increase in $\overrightarrow{H_x}$



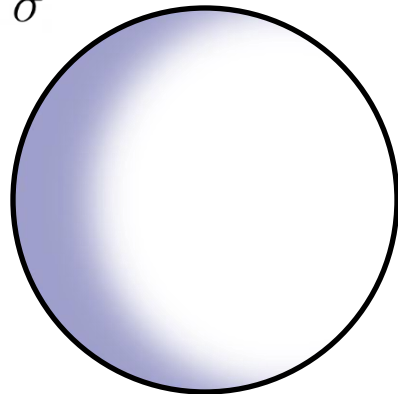
SKIN AND PROXIMITY EFFECTS

- *Non-linear* current density and magnetic field intensity across conductors
- *Reduction* in effective leakage inductance with frequency
- *Critical* to determine leakage inductance at the specific operating frequency range

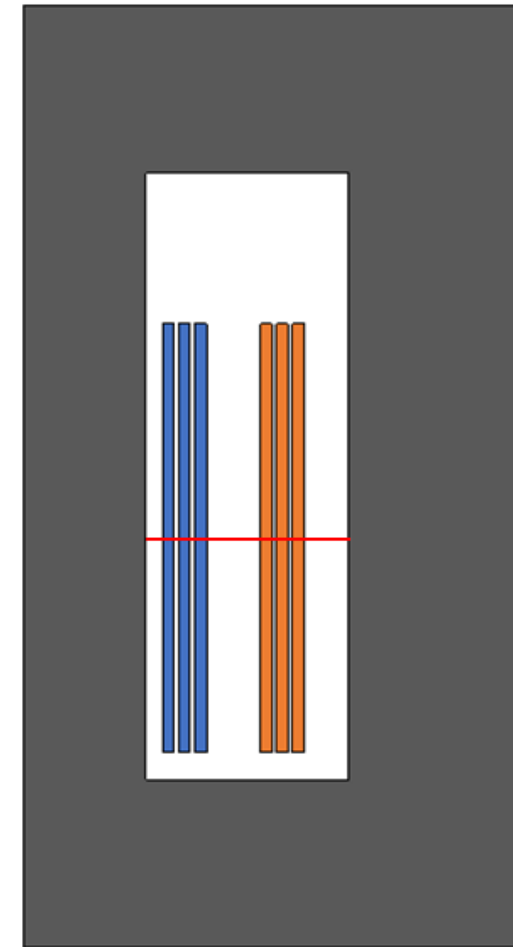
$$\delta = \sqrt{\frac{1}{\pi \mu f \sigma}}$$



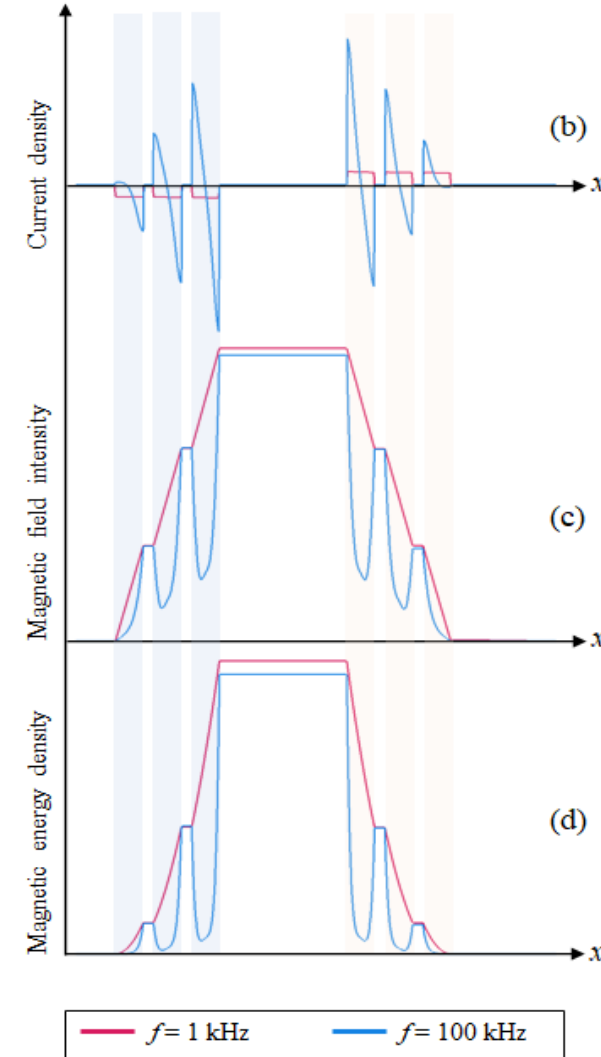
$r > \delta \rightarrow$ skin effect



proximity effect



(a) Primary winding Secondary winding



HF DOWELL'S 1-D MODEL

- Models both skin and proximity effects
- Models x-position of winding only
- Models \vec{H}_y only
- Models IW plane only
- *Winding height = window height*, otherwise requires porosity factors

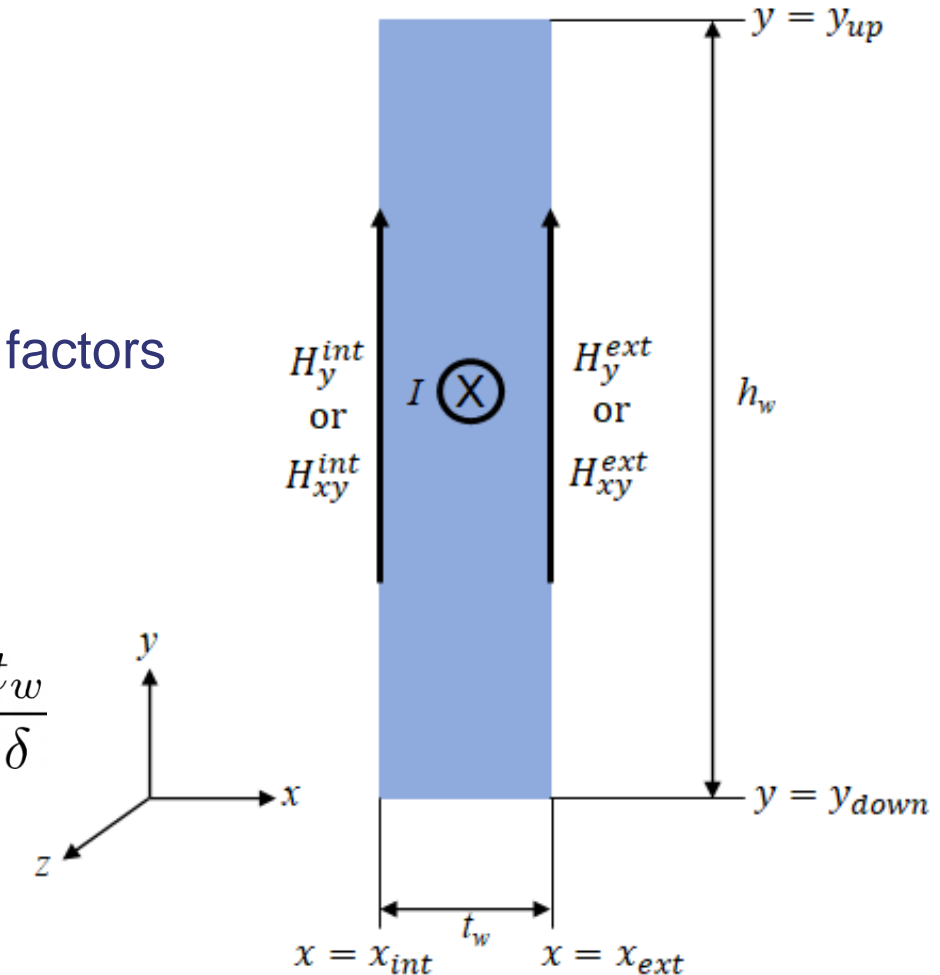
$$E''_{\text{foil}} = \frac{\mu_0 \delta}{4} \left(\left(H_y^{\text{ext}} + H_y^{\text{int}} \right)^2 \varphi_1 - 2 H_y^{\text{ext}} H_y^{\text{int}} \varphi_2 \right)$$

$$\varphi_1 = \frac{\sinh(2\Delta) - \sin(2\Delta)}{\cosh(2\Delta) - \cos(2\Delta)}, \quad \varphi_2 = \frac{\sinh(\Delta) - \sin(\Delta)}{\cosh(\Delta) - \cos(\Delta)}$$

$$\Delta = \frac{t_w}{\delta}$$

$$E_{\text{foil}} = h_w \times MLT \times E''_{\text{foil}}$$

φ_1 = skin effect factor, φ_2 = proximity effect factor, Δ = penetration ratio, δ = skin depth

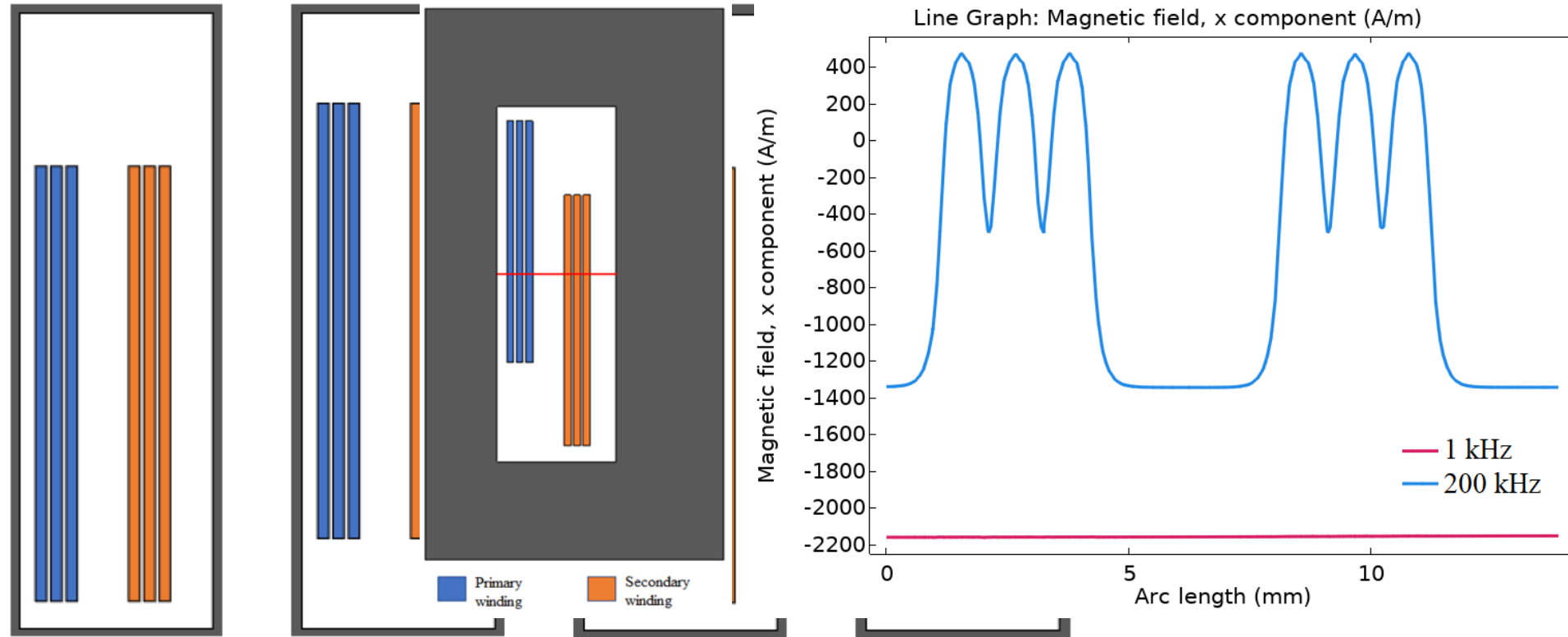


CHALLENGE STATEMENT

No 2-D HF model has been proposed so far

Existing 1-D HF models are inapplicable in a partially filled-transformer like VIT

- Movement of the y -position of the
 - both x -ar must be n
- L'_{Hx} increases wi
 - \vec{H}_x must b
- $L'_{Hx(IW)} > L'_{Hx(OW)}$
 - both IW a must be n



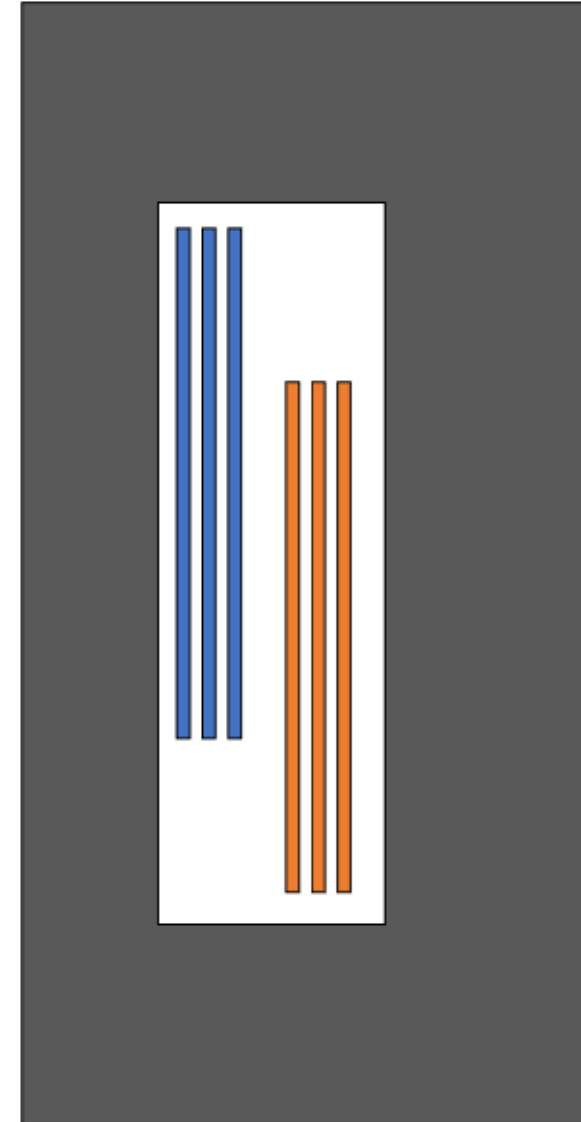
4 different partially-filled winding geometries in IW plane

PROPOSED HYBRID MODEL

- *Pseudo 2-D model*, built on the double-2-D model platform
- *Assumption* is that skin and proximity effects influence the magnetic energy densities across the *winding cross-sections*
- Uses *superposition* to combine HF Dowell's model with LF double-2-D model

Hybrid model = double-2-D + modified Dowell's model

| Region | Frequency dependency | Model |
|------------------------|----------------------|-------------------|
| winding cross-sections | dependent | modified Dowell's |
| non-winding spaces | independent | double-2-D |



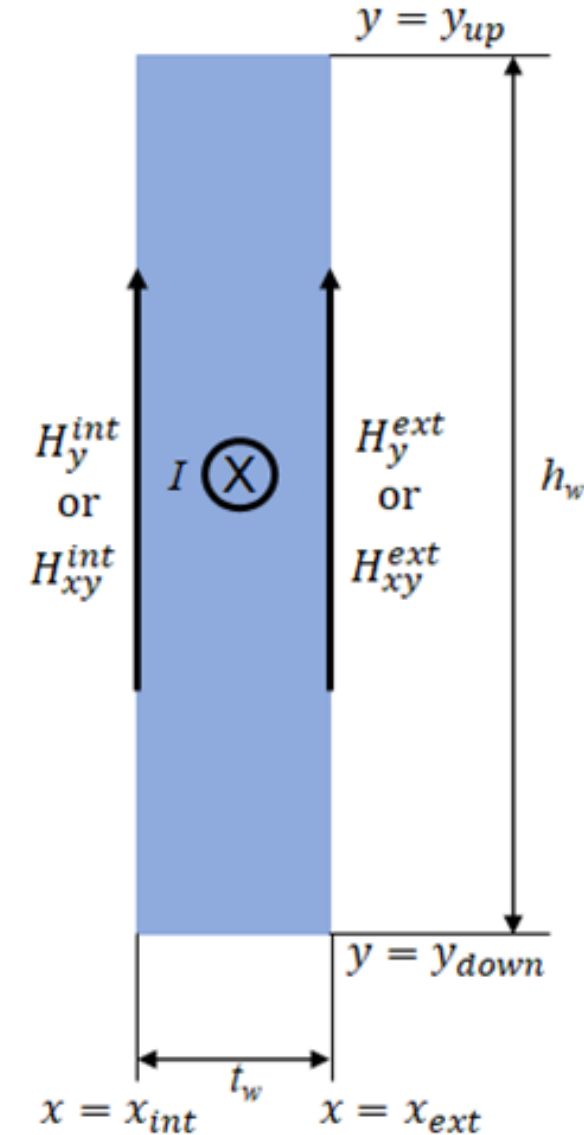
MODIFIED DOWELL'S MODEL

Closed-form solution of the modified Dowell's model for a foil,

$$E'_{\text{foil}} = \frac{\mu_0 \delta}{4} \int_{y_{\text{down}}}^{y_{\text{up}}} \left(\left(H_{xy}^{\text{ext}}(y) + H_{xy}^{\text{int}}(y) \right)^2 \varphi_1 - 2H_{xy}^{\text{ext}}(y)H_{xy}^{\text{int}}(y)\varphi_2 \right) dy$$

H_{xy}^{ext} and H_{xy}^{int} are readily available from the double-2-D model by evaluating $H_x(x, y)$ and $H_y(x, y)$ at $x = x_{\text{int}}$ and $x = x_{\text{ext}}$

$$H_{xy} = \sqrt{(H_x^2 + H_y^2)}$$



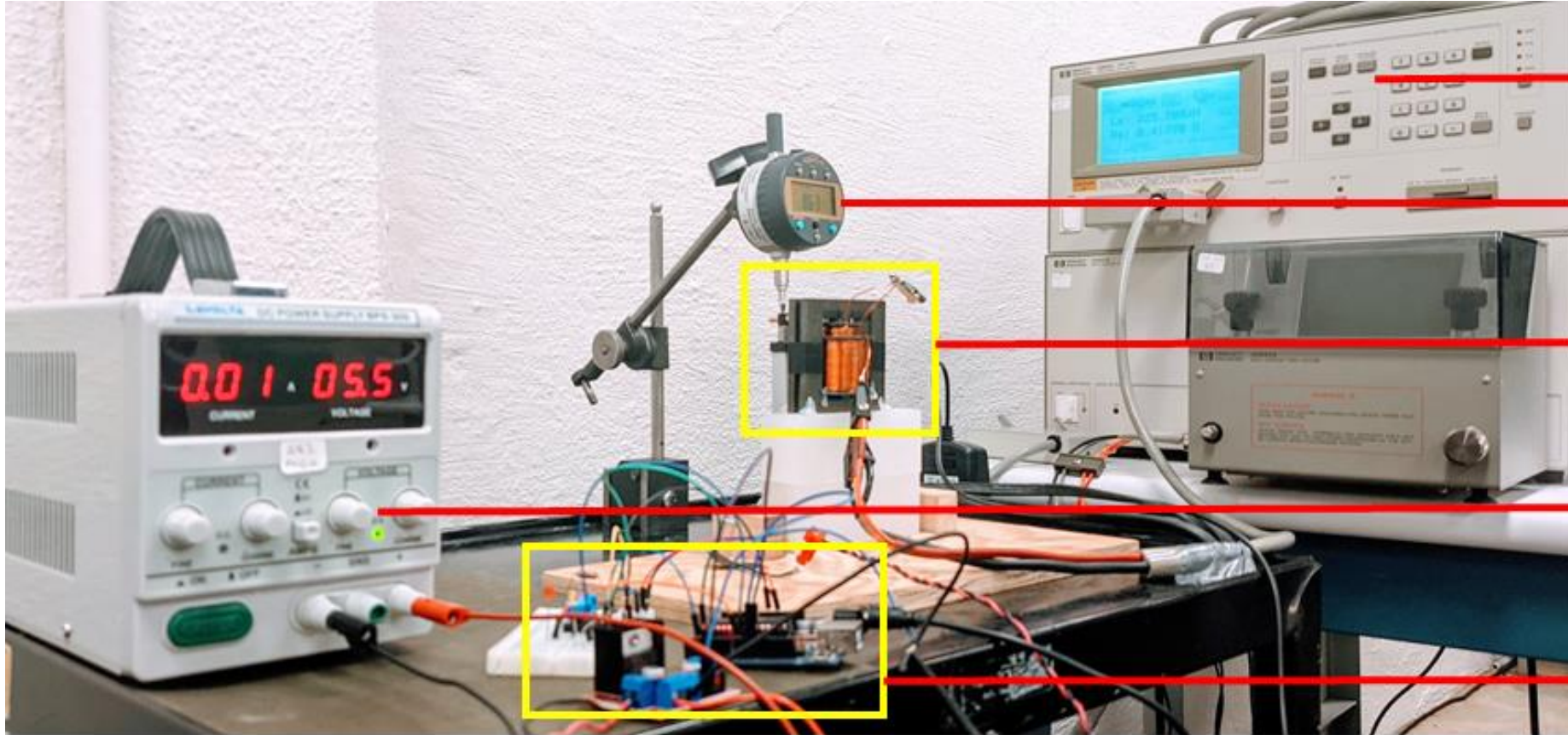
VIT SPECIFICATIONS

| | |
|---|---------------------|
| Turns ratio | 1:1 |
| Conductor type, size | Round, AWG 19 |
| Number of turns per layer | 30 |
| Number of layers per winding | 3 |
| Core geometry | EC 70 |
| Winding height | 31.50 mm |
| External diameter of the movable bobbin | 19 mm |
| External diameter of the fixed bobbin | 33 mm |
| Insulation gap between layers | 0.20 mm |
| Fill-factor | 19.04 % |
| Maximum travel of the movable bobbin | 11 mm |
| Test frequency range | 1 – 200 kHz |
| Air cube (for OW plane boundary) | 80^3 mm^3 |

□ MATLAB R2019a is used for all semi-analytical calculations

□ COMSOL Multiphysics 5.5 is used for obtaining the 2-D FEM results

EXPERIMENTAL SETUP WITH THE VIT



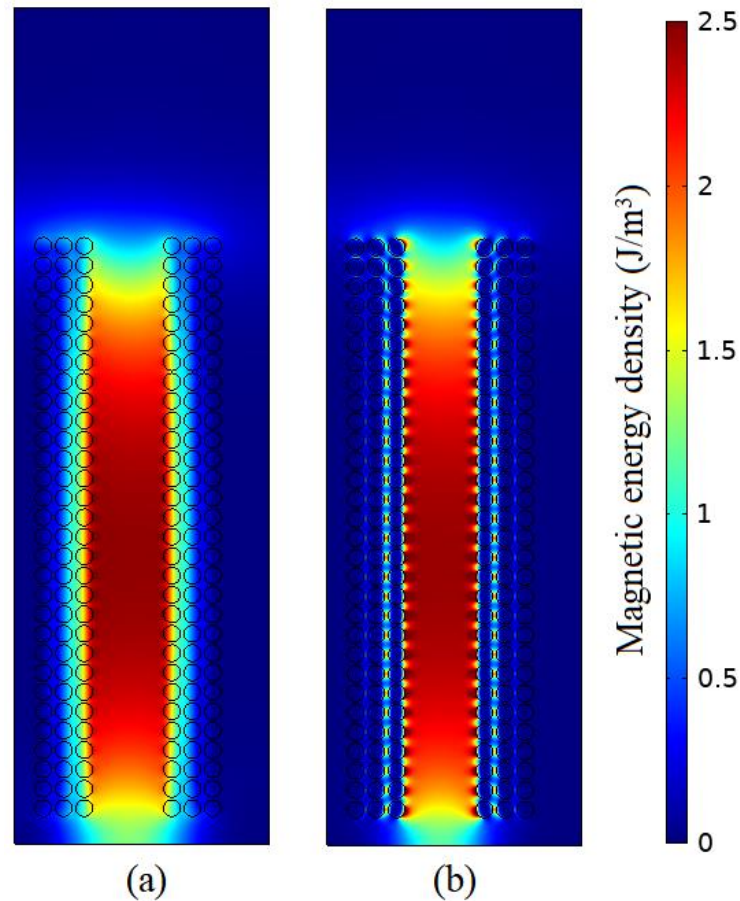
1 MHz LCR meter

Digital meter for measuring distance

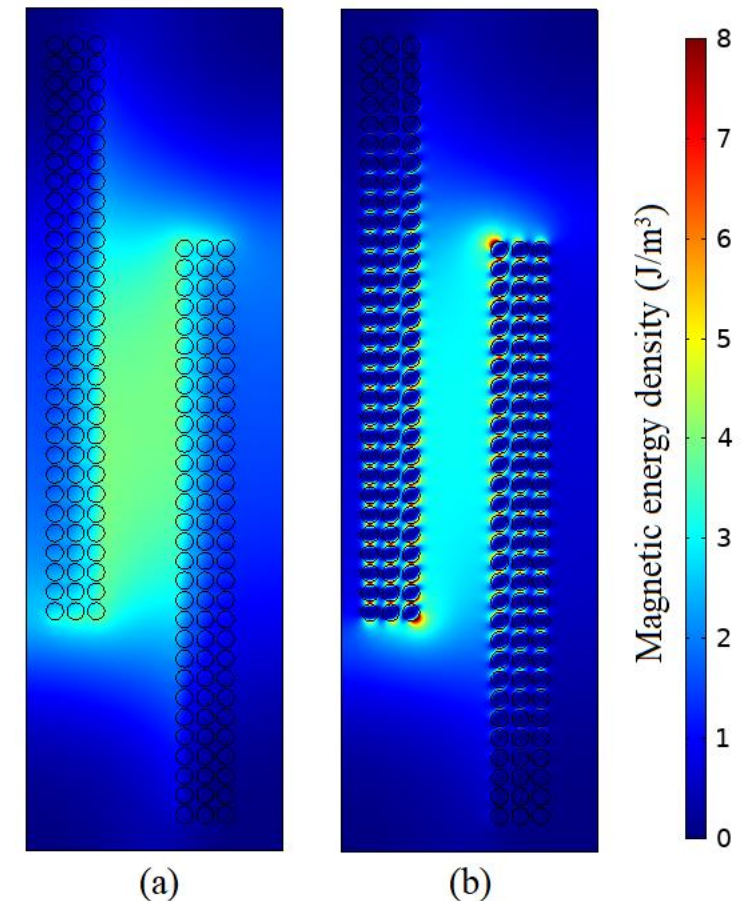
Transformer I operated as a VIT

DC voltage source to drive the VIT using a linear actuator

Control circuit for operating the actuator

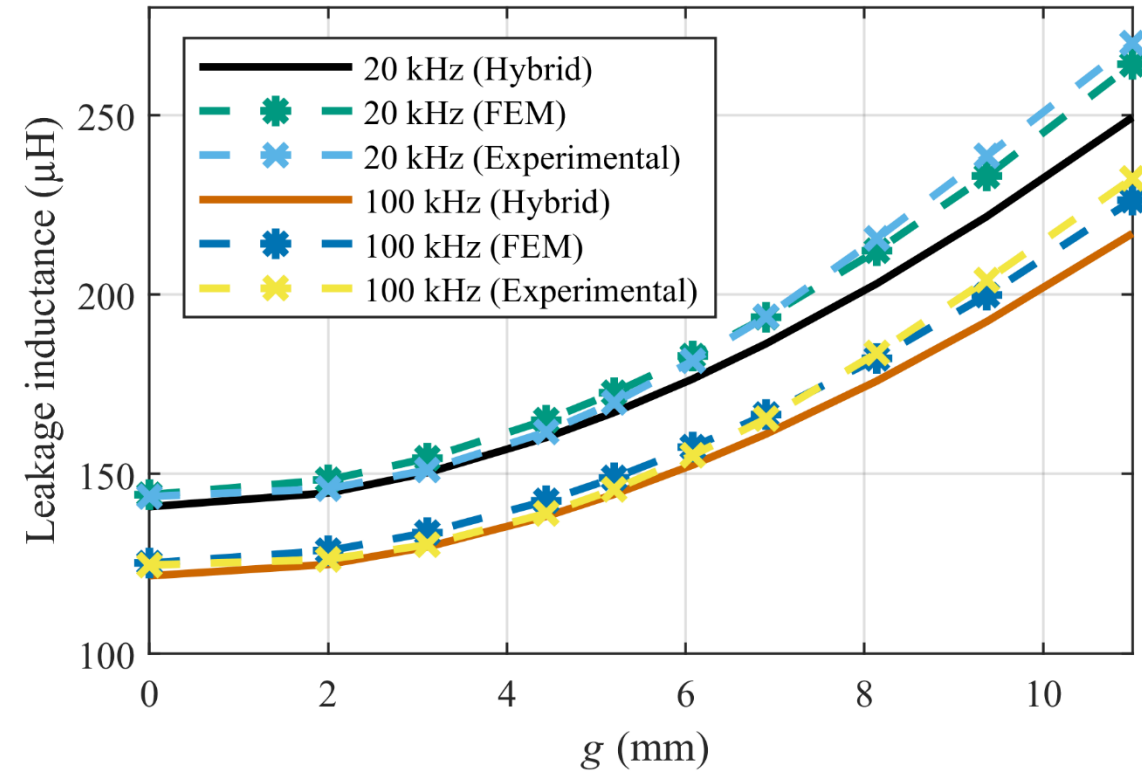
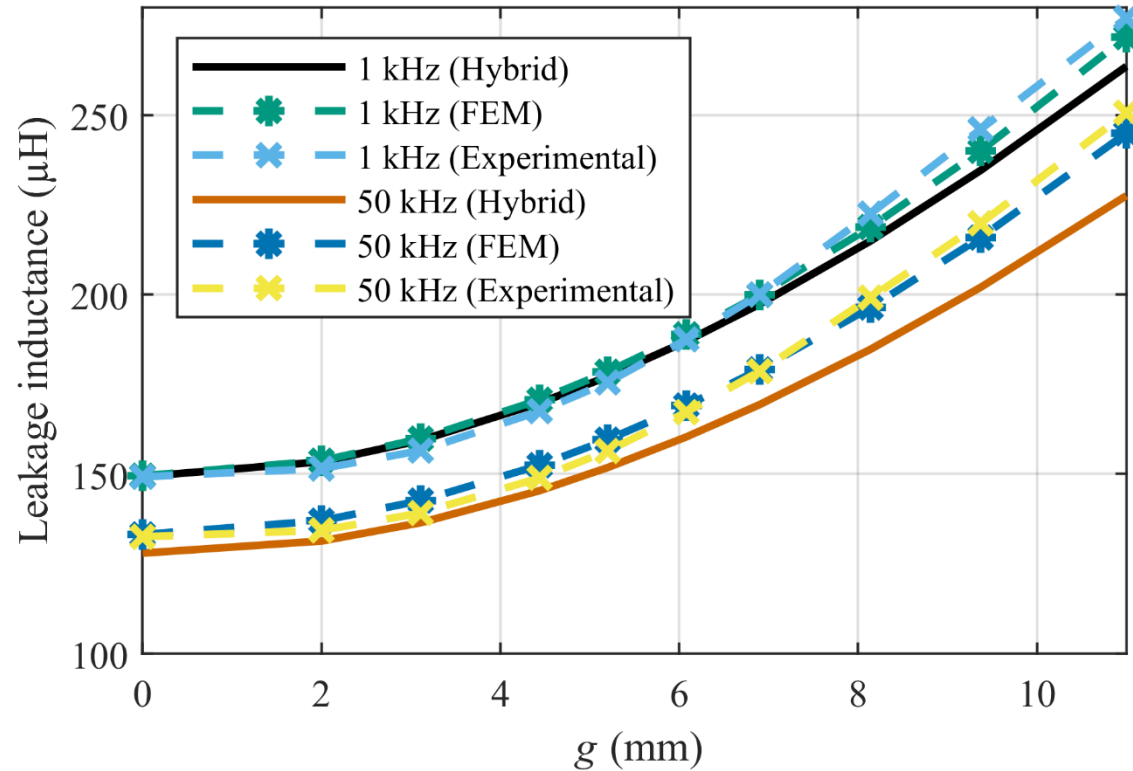


Magnetic energy densities across the IW plane at $g = 0$ mm: (a) 1 kHz, and (b) 200 kHz.



Magnetic energy densities across the IW plane at $g = 11$ mm: (a) 1 kHz, and (b) 200 kHz.

RESULTS

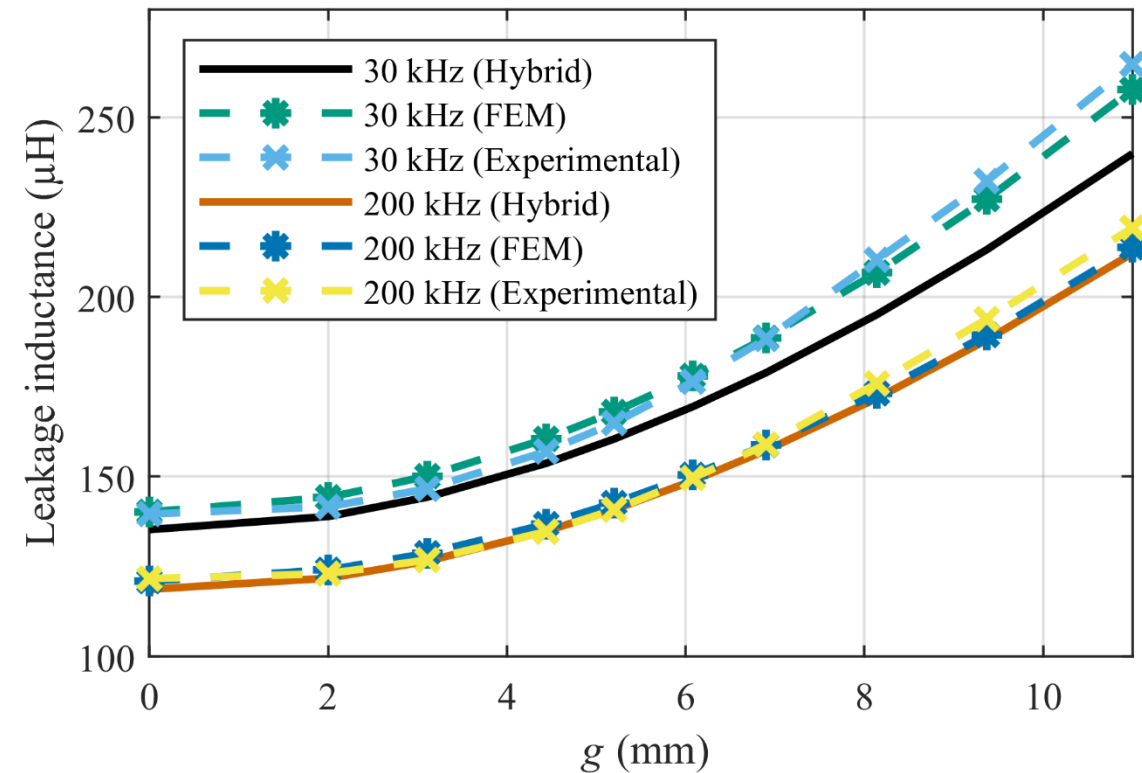


6 different frequencies are investigated within 1 – 200 kHz.

For the specific wire gauge, the frequency at which δ equals the radius of the conductor is 20.4 kHz.

RESULTS

- Leakage inductance increases with g while decreases with frequency
- Leakage inductance at $g = 11$ mm is 80 % higher than that at $g = 0$ mm across all frequencies
- Maximum error with the hybrid model is 7.1 % observed at 50 kHz and $g = 11$ mm
- Existing HF models give a flat horizontal curve for varying overlaps
- Between 1 and 200 kHz, the leakage inductance dropped by more than 18 % across all overlaps



CONCLUSION

- A *new hybrid model* is proposed – first attempt towards 2-D HF modeling
- It uses *superposition* to combine Dowell's model with the double-2-D model
 - A modified Dowell's model calculates the leakage inductance contributions from the winding cross-sections
 - The double-2-D model calculates the leakage inductance contributions from the non-winding spaces
- Semi-analytical results are validated through FEM modeling and experimentation
- With a *maximum error of only 7.1 %*, the hybrid model is promising for any partially filled transformer geometries
- It can aid *multi-objective optimization-based designs* of power electronic converters employing transformers with integrated magnetics

ACKNOWLEDGMENT

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- 1) **A. Sharma** and J. W. Kimball, “Novel transformer with variable leakage and magnetizing inductances,” in *2021 IEEE Energy Conversion Congress and Exposition (ECCE)*, 2021, pp. 2155–2161.
- 2) **A. Sharma** and J. W. Kimball, “Evaluation of transformer leakage inductance using magnetic image method,” *IEEE Transactions on Magnetics*, vol. 57, no. 11, pp. 1–12, 2021.
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- 5) Z. Ouyang, J. Zhang, and W. G. Hurley, “Calculation of leakage inductance for high-frequency transformers,” *IEEE Transactions on Power Electronics*, vol. 30, no. 10, pp. 5769–5775, 2015.

“Thank you”

Questions?

Please email me at asc4v@mst.edu

