

LA-UR-23-22299

Approved for public release; distribution is unlimited.

Title: Status of proton radius puzzle and QED radiative corrections for accelerator neutrinos

Author(s): Tomalak, Oleksandr Sergijovic

Intended for: Seminar at KIAS, 2023-04-05/2023-04-14 (Seoul, Korea, South)

Issued: 2023-07-18 (rev.1)



Los Alamos National Laboratory, an affirmative action/equal opportunity employer, is operated by Triad National Security, LLC for the National Nuclear Security Administration of U.S. Department of Energy under contract 89233218CNA000001. By approving this article, the publisher recognizes that the U.S. Government retains nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes. Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy. Los Alamos National Laboratory strongly supports academic freedom and a researcher's right to publish; as an institution, however, the Laboratory does not endorse the viewpoint of a publication or guarantee its technical correctness.

Korea Institute for Advanced Study
KIAS Seoul

13 April, 2023

1. Status of proton radius puzzle.
2. QED radiative corrections for
accelerator neutrinos

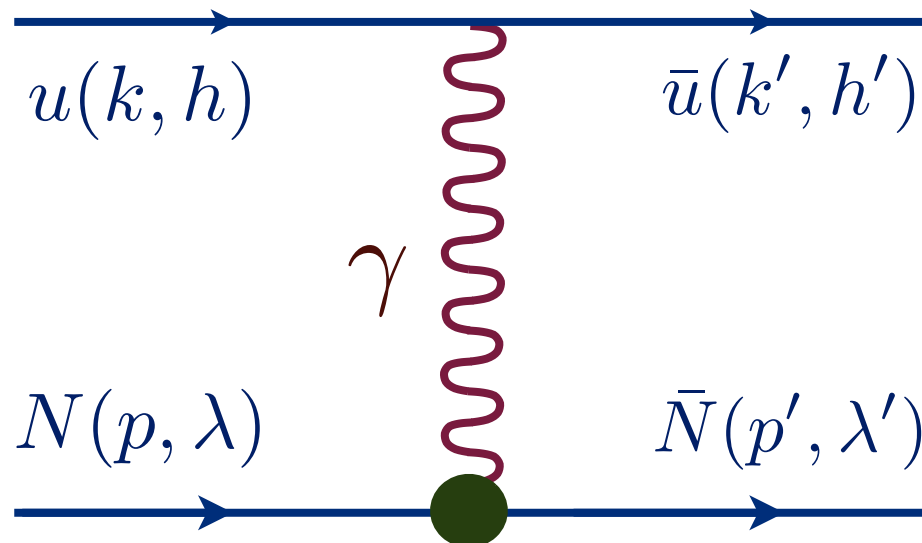


Oleksandr (Sasha) Tomalak
LA-UR-23-22299

Proton radius puzzle

2010-2023

Tool to explore the proton structure



photon-proton vertex

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_P(Q^2)$$

Dirac and Pauli form factors

lepton energy

ω

momentum transfer

$$Q^2 = -(k - k')^2$$

18 amplitude

$$T = \frac{e^2}{Q^2} (\bar{u}(k', h') \gamma_\mu u(k, h)) \cdot (\bar{N}(p', \lambda') \Gamma^\mu(Q^2) N(p, \lambda))$$

Form factors measurement

Sachs electric and magnetic form factors

$$G_E = F_D - \tau F_P \quad G_M = F_D + F_P$$

Rosenbluth separation

$$\frac{d\sigma^{\text{unpol}}}{d\Omega} \sim G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

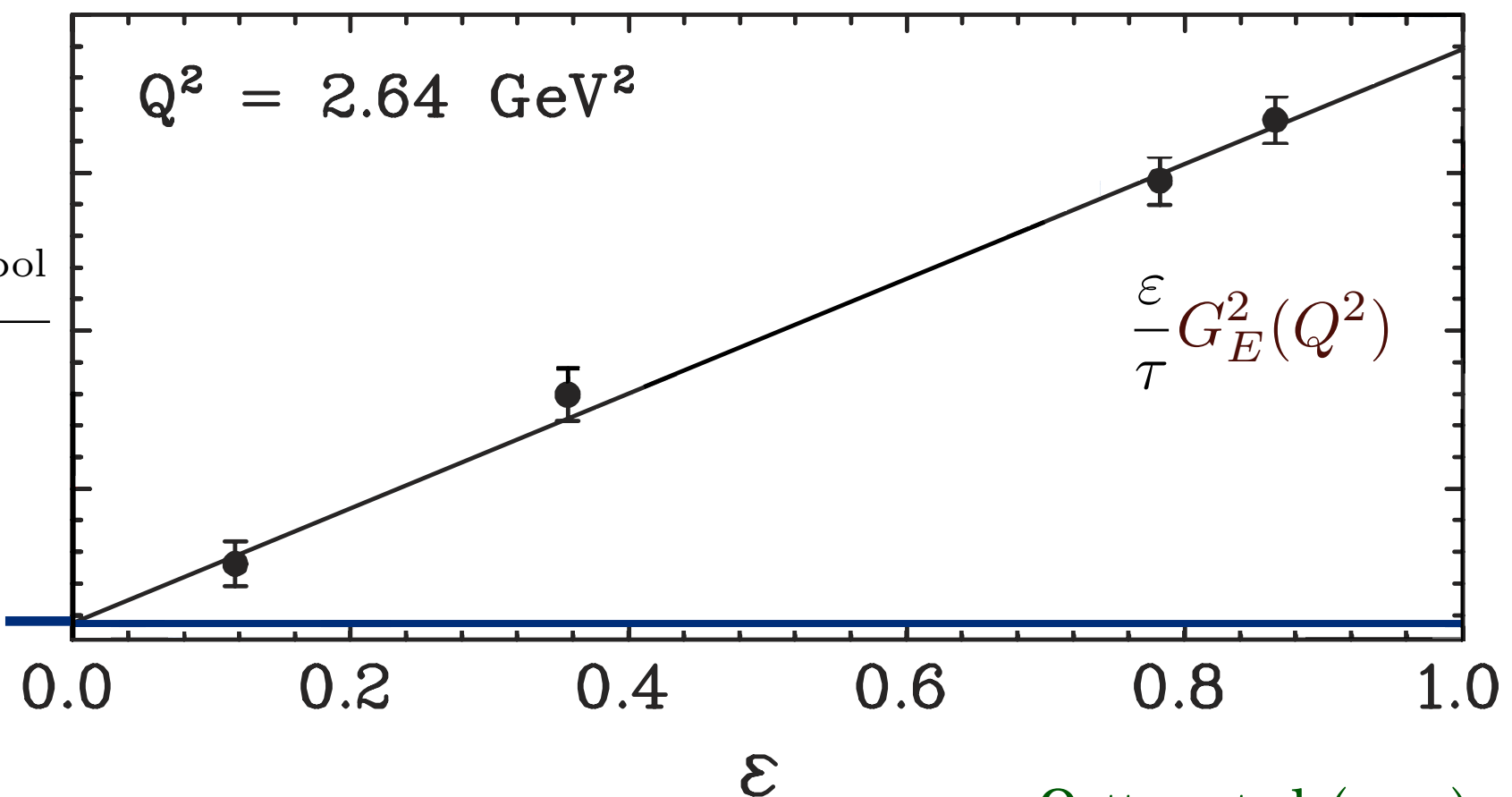
$$\tau = \frac{Q^2}{4M^2}$$

$$\frac{d\sigma^{\text{unpol}}}{d\Omega}$$

τ ε kinematical variables

$$\varepsilon \leftrightarrow \theta_{\text{lab}}$$

$$G_M^2(Q^2)$$



Qattan et al. (2005)

- Rosenbluth slope is sensitive to corrections beyond 18

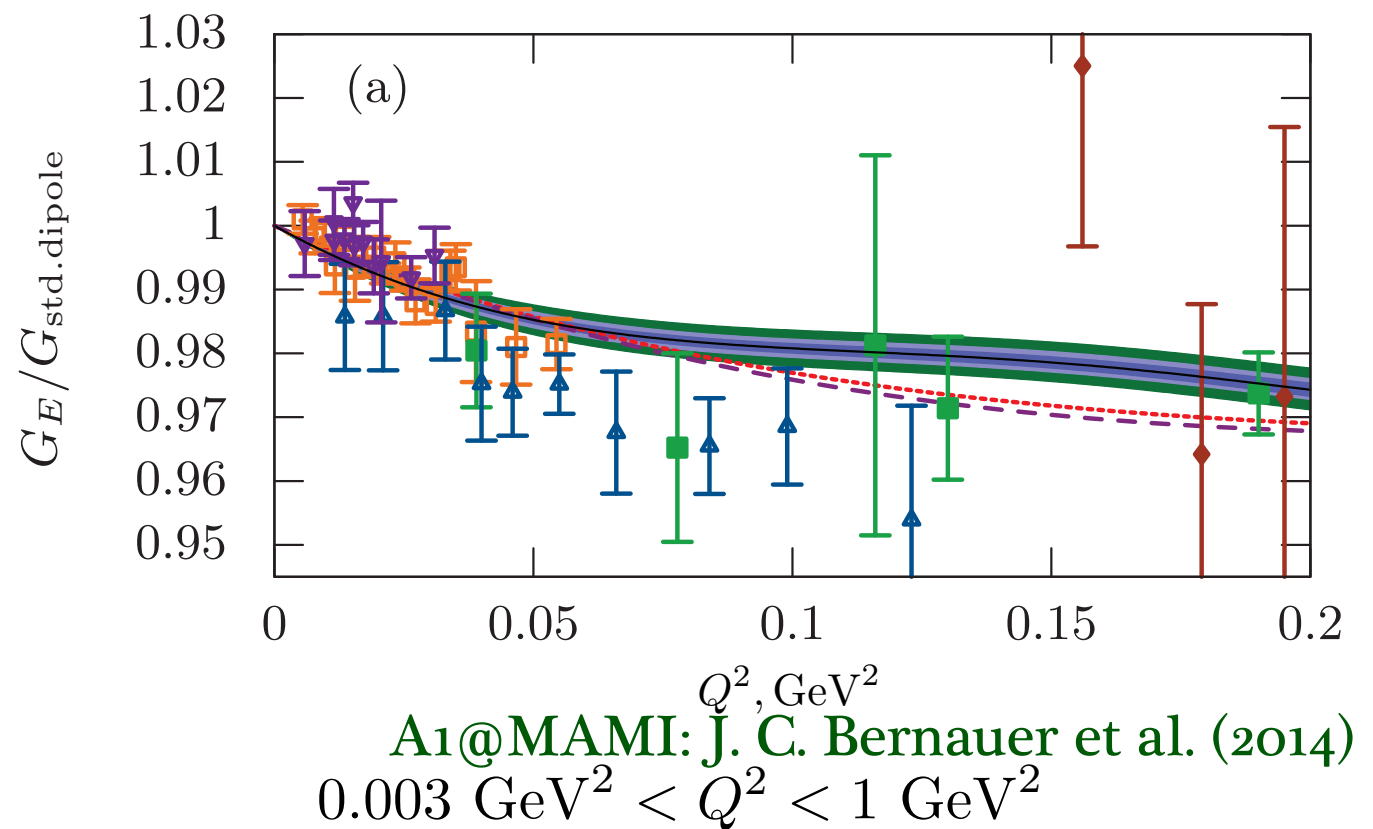
Proton radius

electric charge radius

$$\langle r_E^2 \rangle \equiv -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$

- ep elastic scattering

$$r_E = 0.879 \pm 0.008 \text{ fm}$$



$$G_{std.dipole} = \frac{1}{\left(1 + \frac{Q^2}{\Lambda^2}\right)^2}$$

Proton radius

electric charge radius

$$\langle r_E^2 \rangle \equiv -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$

- ep elastic scattering

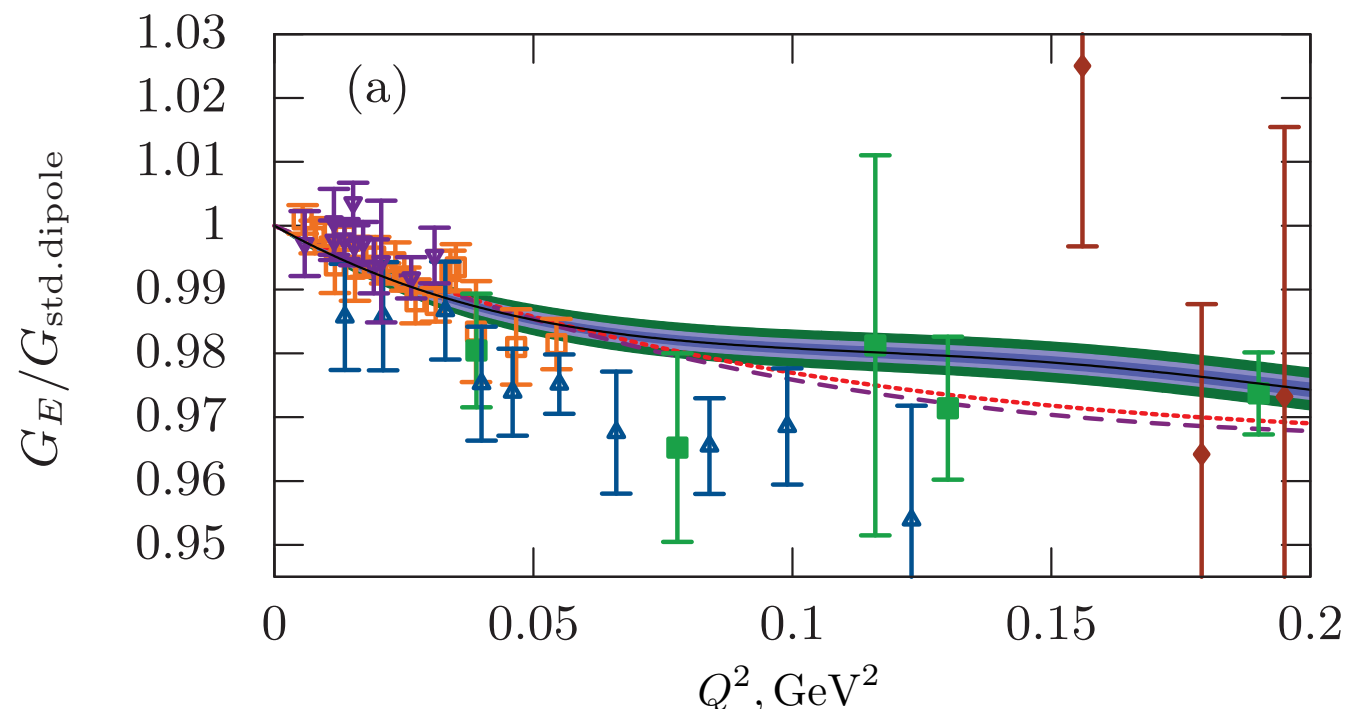
$$r_E = 0.879 \pm 0.008 \text{ fm}$$

- atomic spectroscopy

H, D spectroscopy

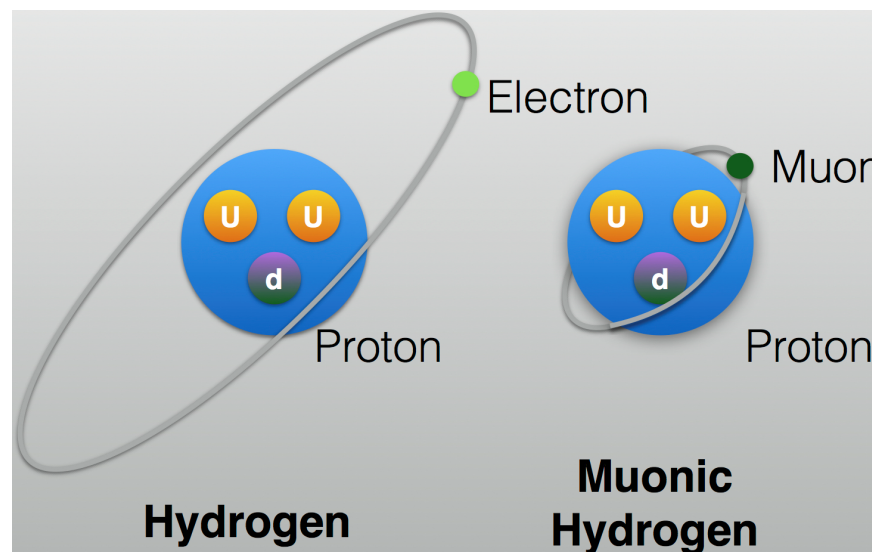
$$r_E = 0.8758 \pm 0.0077 \text{ fm}$$

CODATA 2010



A1@MAMI: J. C. Bernauer et al. (2014)

$$\nu_{nS} = -\frac{R_\infty}{n^2} + \frac{c_r m_r^3 \langle r_E^2 \rangle}{n^3} + \dots$$



μ H Lamb shift

$$r_E = 0.8409 \pm 0.0004 \text{ fm}$$

CREMA (2010, 2013)

Proton radius puzzle

electric charge radius

$$\langle r_E^2 \rangle \equiv -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$

- ep elastic scattering

$$r_E = 0.879 \pm 0.008 \text{ fm}$$

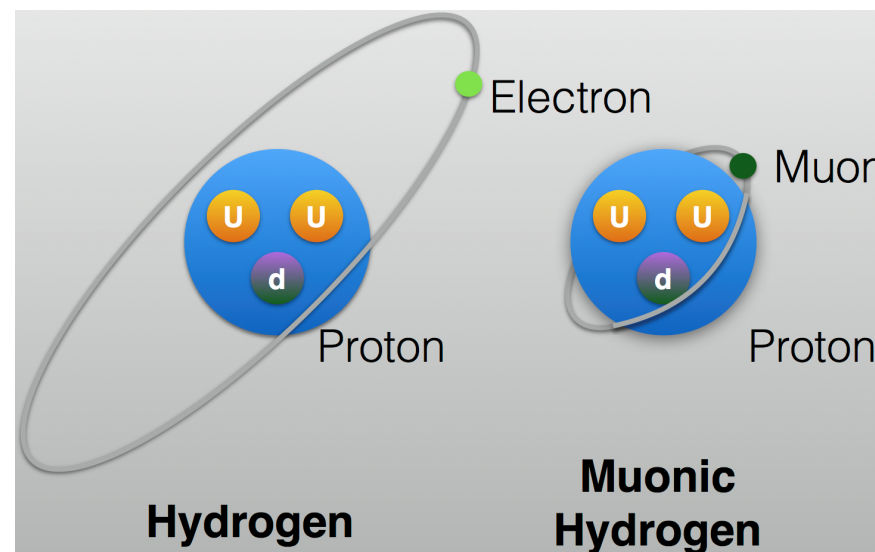
- atomic spectroscopy

$$\nu_{nS} = -\frac{R_\infty}{n^2} + \frac{c_r m_r^3 \langle r_E^2 \rangle}{n^3} + \dots$$

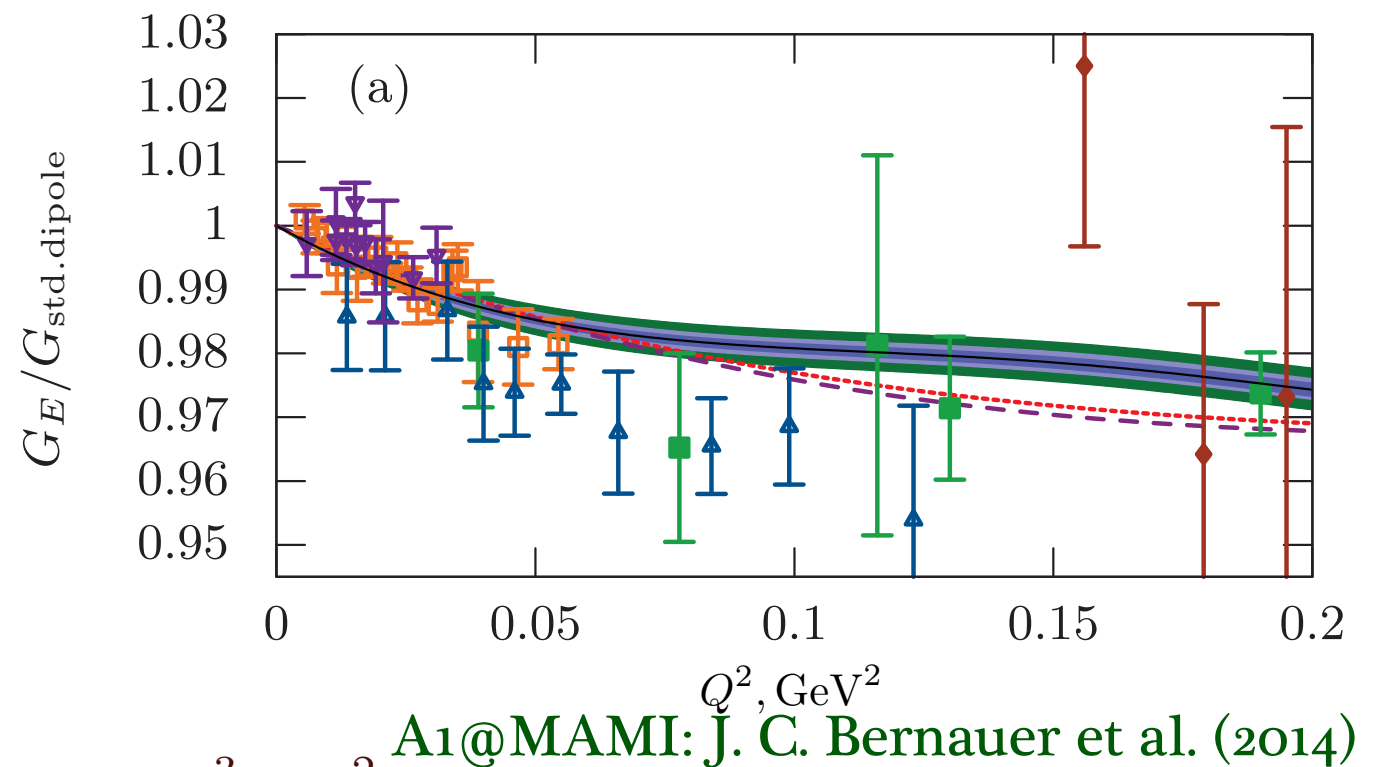
H, D spectroscopy

$$r_E = 0.8758 \pm 0.0077 \text{ fm}$$

CODATA 2010



5.6 σ difference !



μH Lamb shift

$$r_E = 0.8409 \pm 0.0004 \text{ fm}$$

CREMA (2010, 2013)

Proton radius puzzle

electric charge radius

$$\langle r_E^2 \rangle \equiv -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$

- ep elastic scattering

$$r_E = 0.879 \pm 0.008 \text{ fm} \quad ?$$

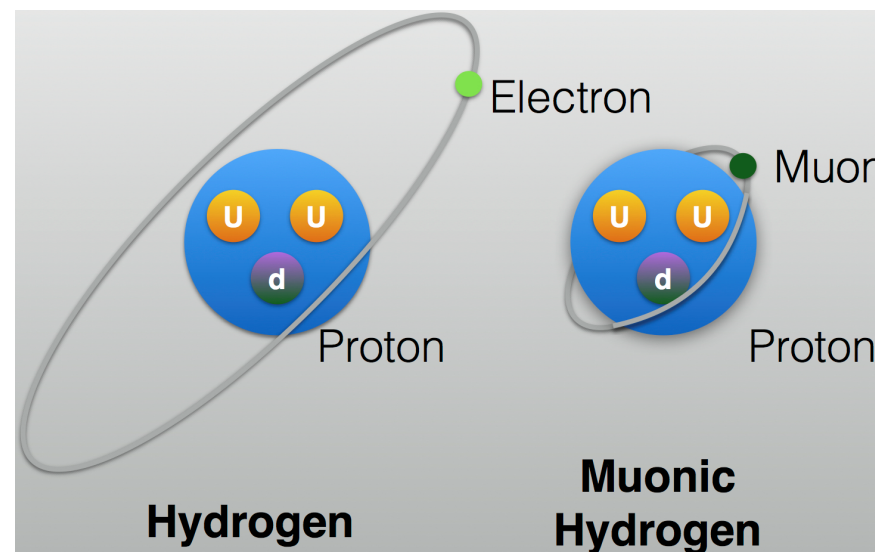
- atomic spectroscopy

$$\nu_{nS} = -\frac{R_\infty}{n^2} + \frac{c_r m_r^3 \langle r_E^2 \rangle}{n^3} + \dots$$

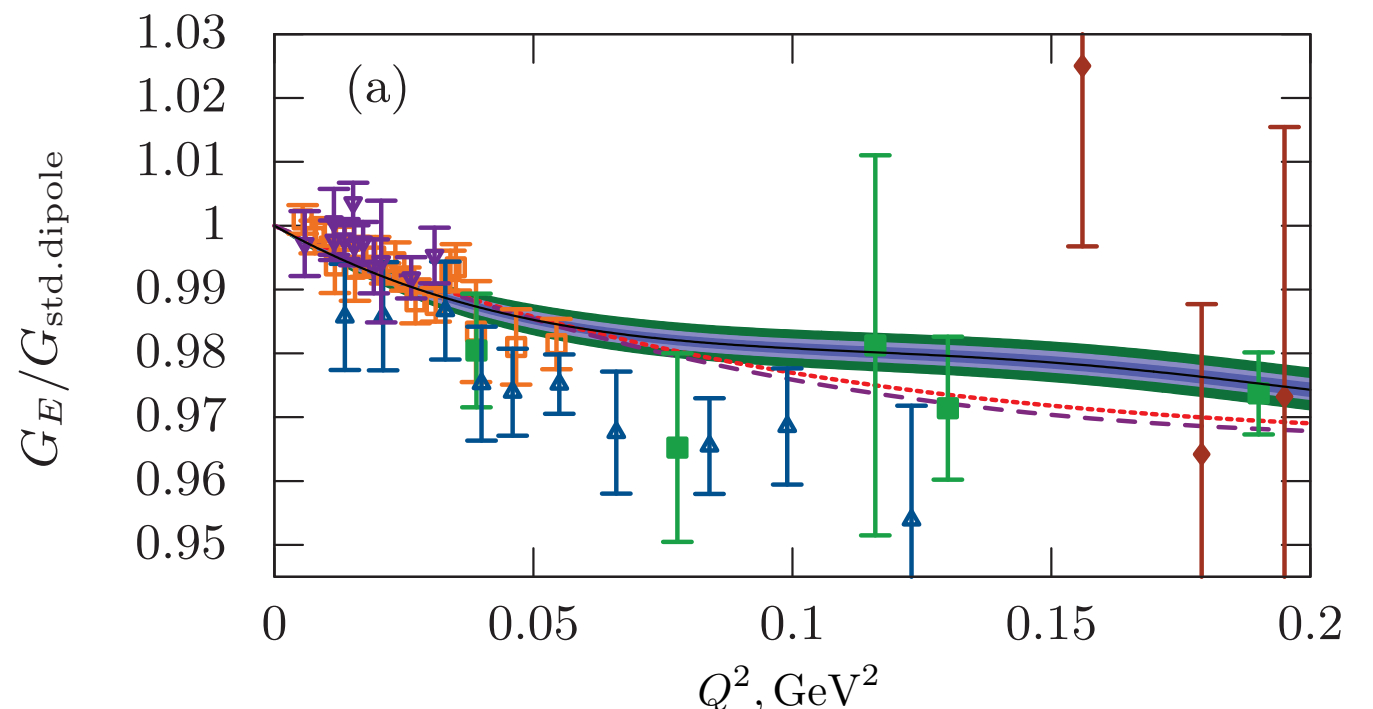
H, D spectroscopy

$$r_E = 0.8758 \pm 0.0077 \text{ fm}$$

CODATA 2010



5.6 σ difference !



A1@MAMI: J. C. Bernauer et al. (2014)

μ H Lamb shift

$$r_E = 0.8409 \pm 0.0004 \text{ fm}$$

CREMA (2010, 2013)

Proton radius puzzle

electric charge radius

$$\langle r_E^2 \rangle \equiv -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$

- ep elastic scattering

$$r_E = 0.879 \pm 0.008 \text{ fm} \quad ?$$

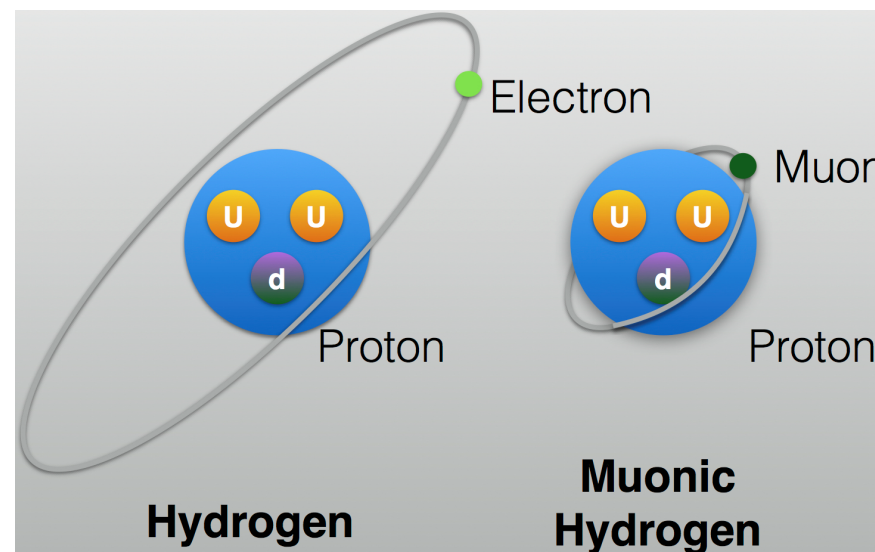
- atomic spectroscopy

$$\nu_{nS} = -\frac{R_\infty}{n^2} + \frac{c_r m_r^3 \langle r_E^2 \rangle}{n^3} + \dots$$

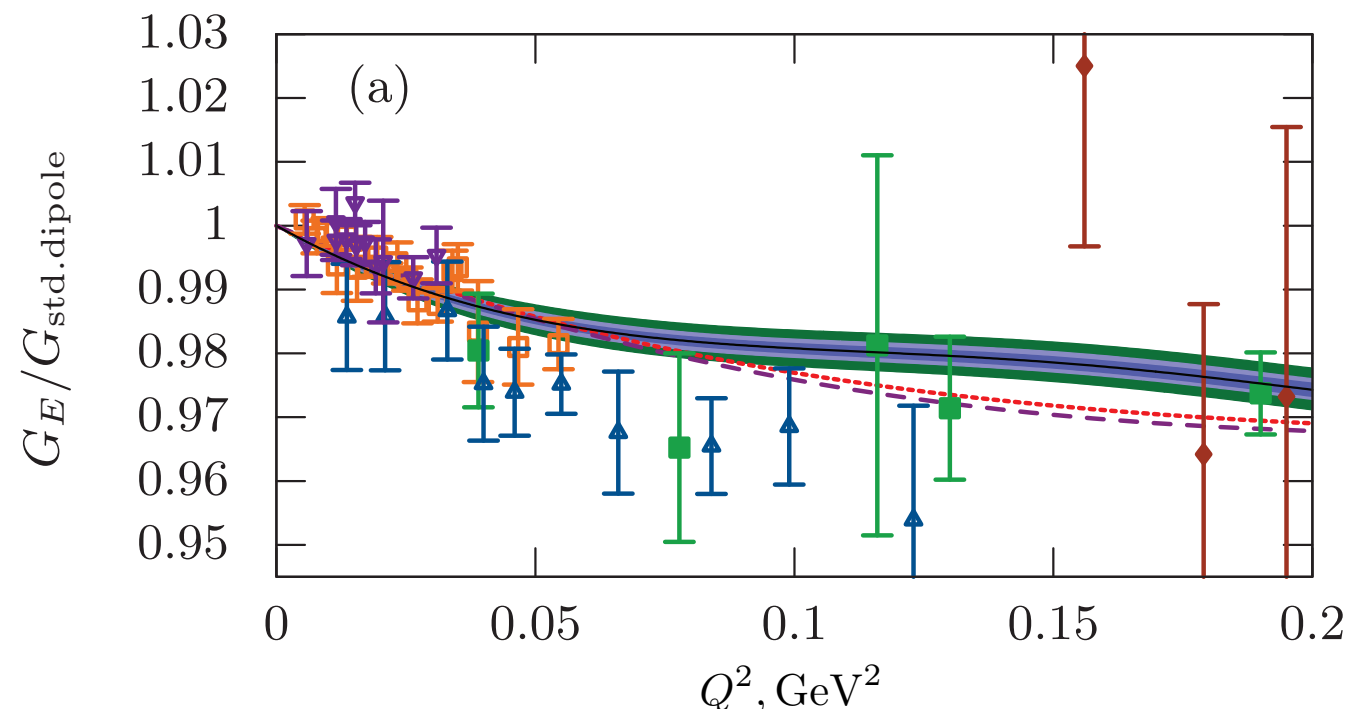
H, D spectroscopy

$$r_E = 0.8758 \pm 0.0077 \text{ fm}$$

CODATA 2010



5.6 σ difference !



A1@MAMI: J. C. Bernauer et al. (2014)

μ H Lamb shift

$$r_E = 0.8409 \pm 0.0004 \text{ fm}$$

CREMA (2010, 2013)

eH 2S-4P (Garching, 2017)

Proton radius puzzle

electric charge radius

$$\langle r_E^2 \rangle \equiv -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$

- ep elastic scattering

$$r_E = 0.879 \pm 0.008 \text{ fm} \quad ?$$

- atomic spectroscopy

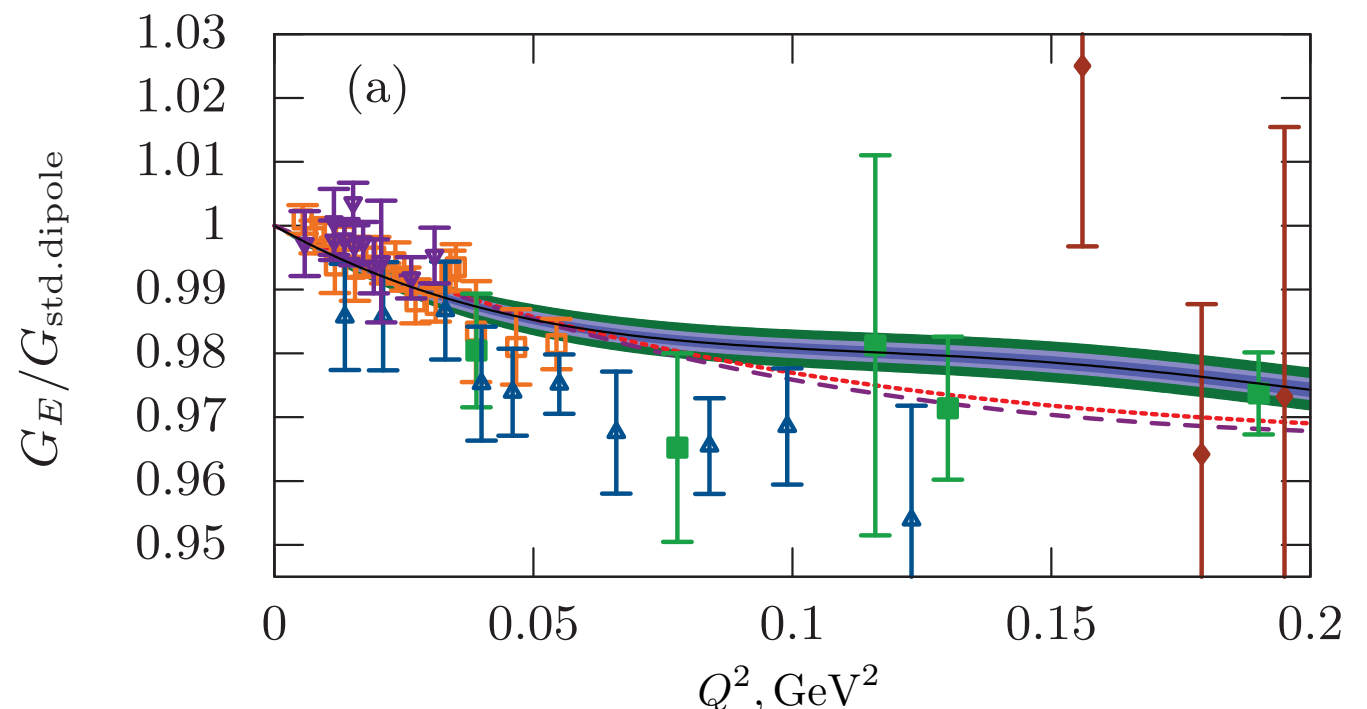
$$\nu_{nS} = -\frac{R_\infty}{n^2} + \frac{c_r m_r^3 \langle r_E^2 \rangle}{n^3} + \dots$$

H, D spectroscopy

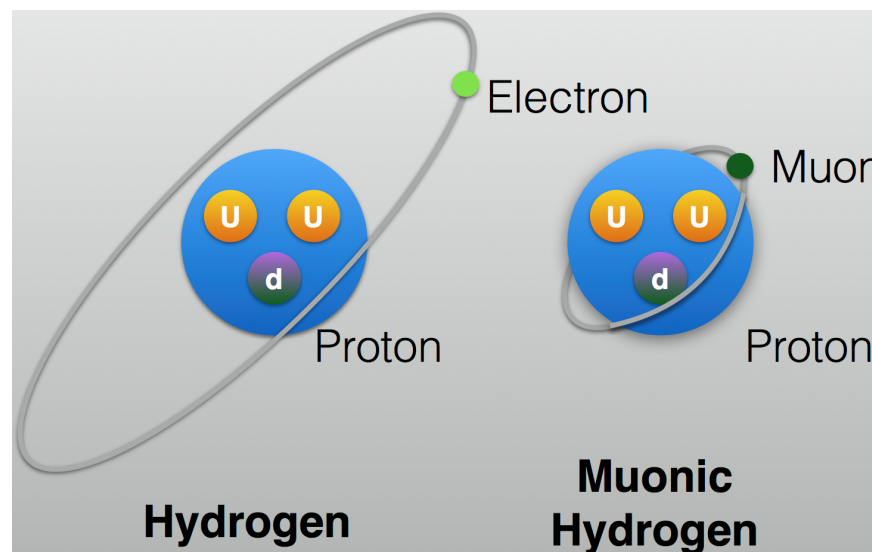
$$r_E = 0.8758 \pm 0.0077 \text{ fm}$$

CODATA 2010

eH 1S-3S (LKB, Paris, 2018)



A1@MAMI: J. C. Bernauer et al. (2014)



μ H Lamb shift

$$r_E = 0.8409 \pm 0.0004 \text{ fm}$$

CREMA (2010, 2013)

eH 2S-4P (Garching, 2017)

5.6 σ difference !

Proton radius puzzle

electric charge radius

$$\langle r_E^2 \rangle \equiv -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$

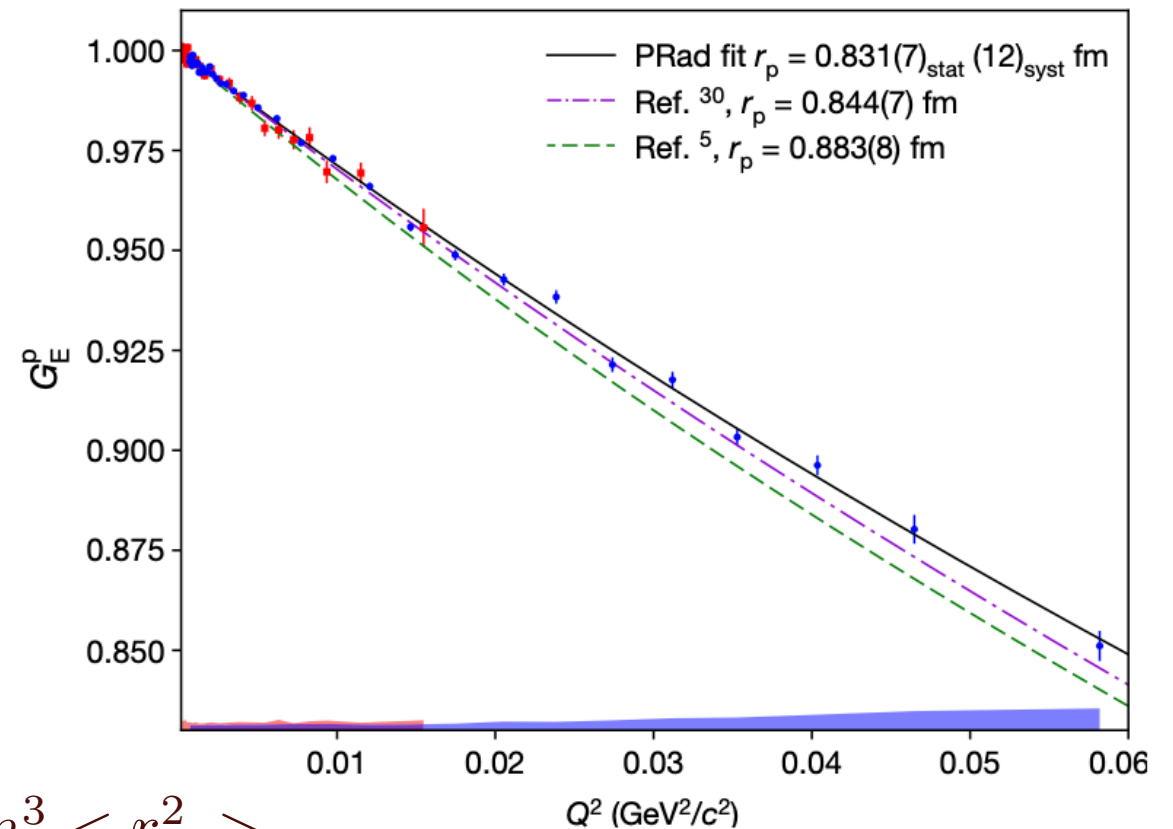
- ep elastic scattering

$$r_E = 0.879 \pm 0.008 \text{ fm} \quad ?$$

- atomic spectroscopy

$$\nu_{nS} = -\frac{R_\infty}{n^2} + \frac{c_r m_r^3 \langle r_E^2 \rangle}{n^3} + \dots$$

PRad (JLab, 2019)

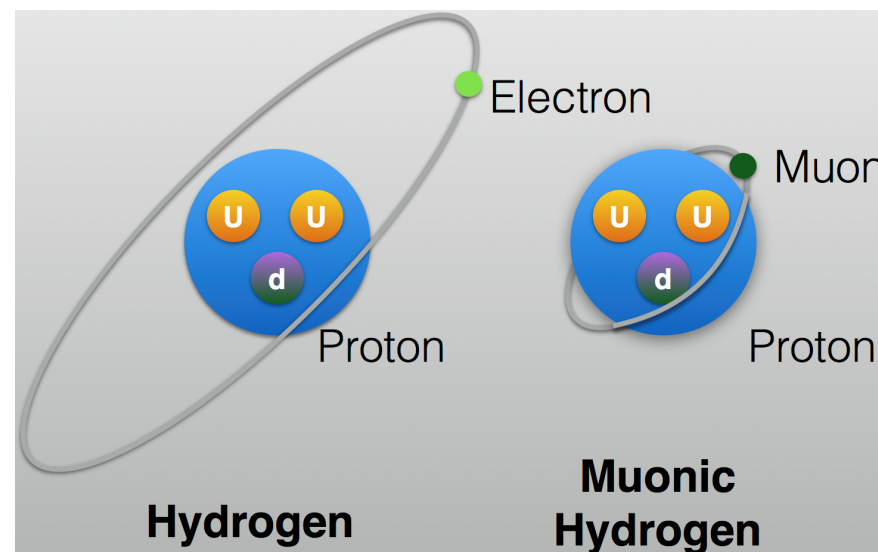


H, D spectroscopy

$$r_E = 0.8758 \pm 0.0077 \text{ fm}$$

CODATA 2010

eH 1S-3S (LKB, Paris, 2018)



μH , μD Lamb shift

$$r_E = 0.8409 \pm 0.0004 \text{ fm}$$

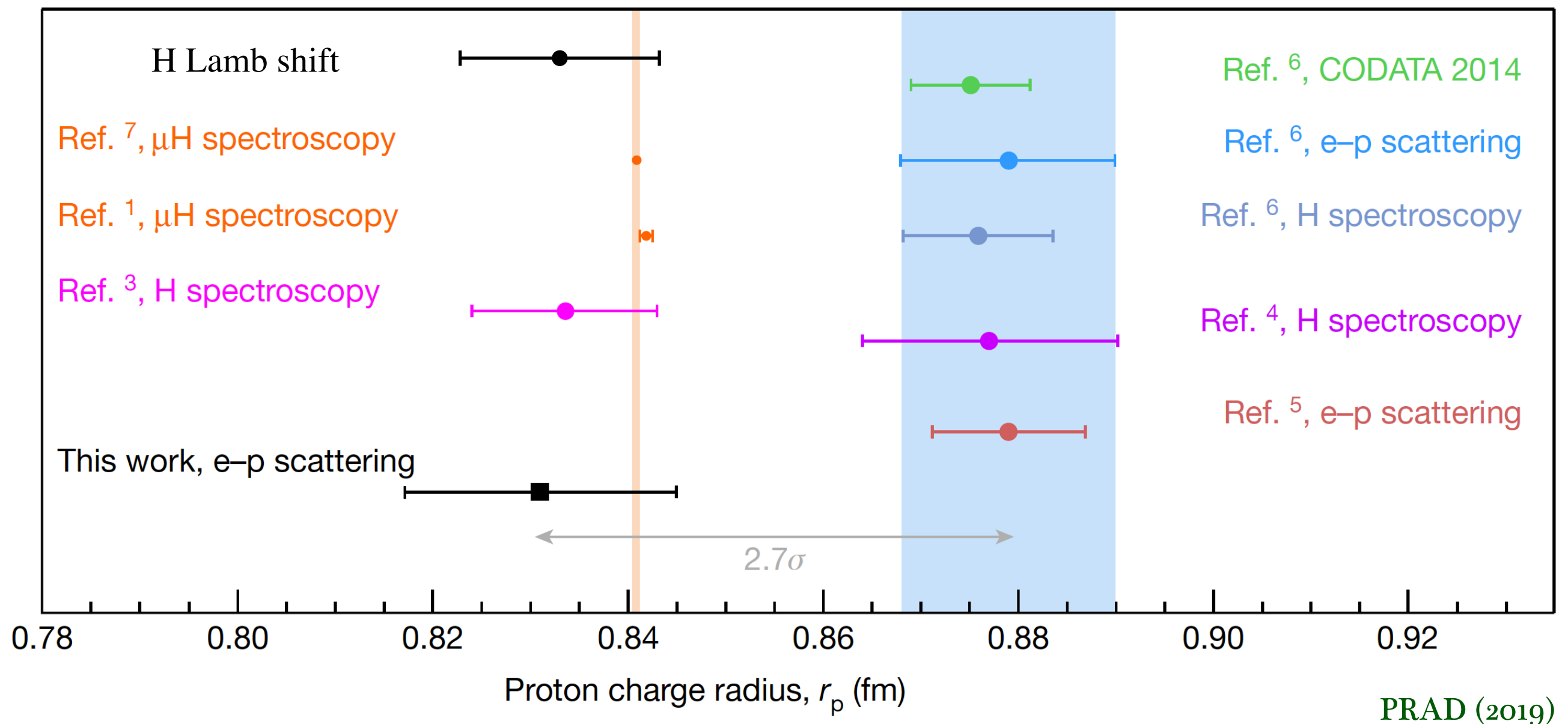
CREMA (2010, 2013)

eH 2S-4P (Garching, 2017)

eH 2S-2P (York U., 2019)

data prefers smaller radius

Proton radius puzzle



- no puzzle in atomic spectroscopy !!!
- scattering data is not completely understood

Possible solutions ?

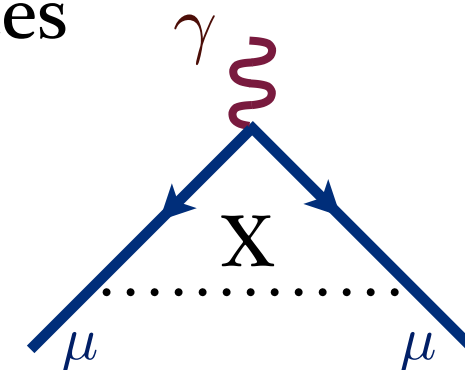
- drawback in experiment: not a single one !
- drawback in theory: many groups reevaluated rad. corrections, 2γ

Possible solutions ?

- drawback in experiment: not a single one !
- drawback in theory: many groups reevaluated rad. corrections, 2γ

BSM explanation ?

- lepton universality violating $\sim \text{MeV}$ muonic forces



- respect $(g-2)_\mu$, nuclear and particle physics constraints: fine tuning

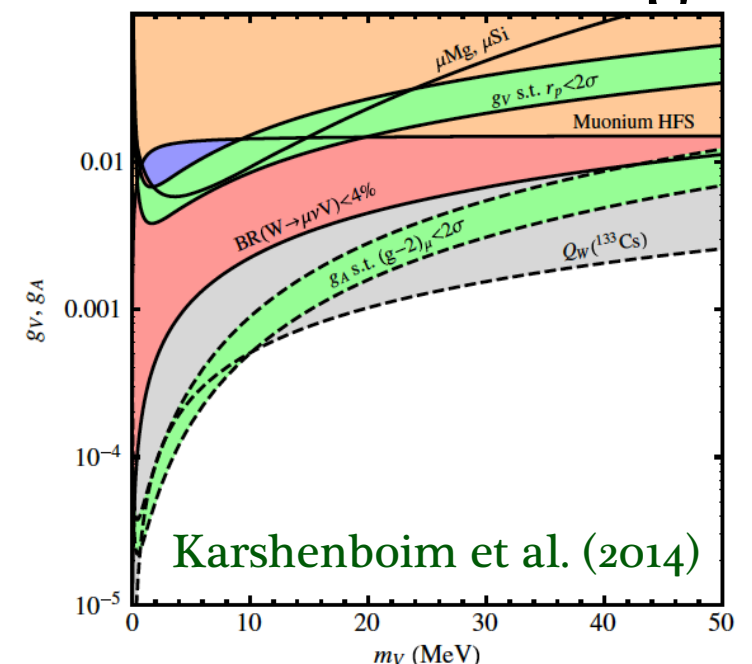
- vector particle: constraints from decay of W

- embedding in renormalizable theory

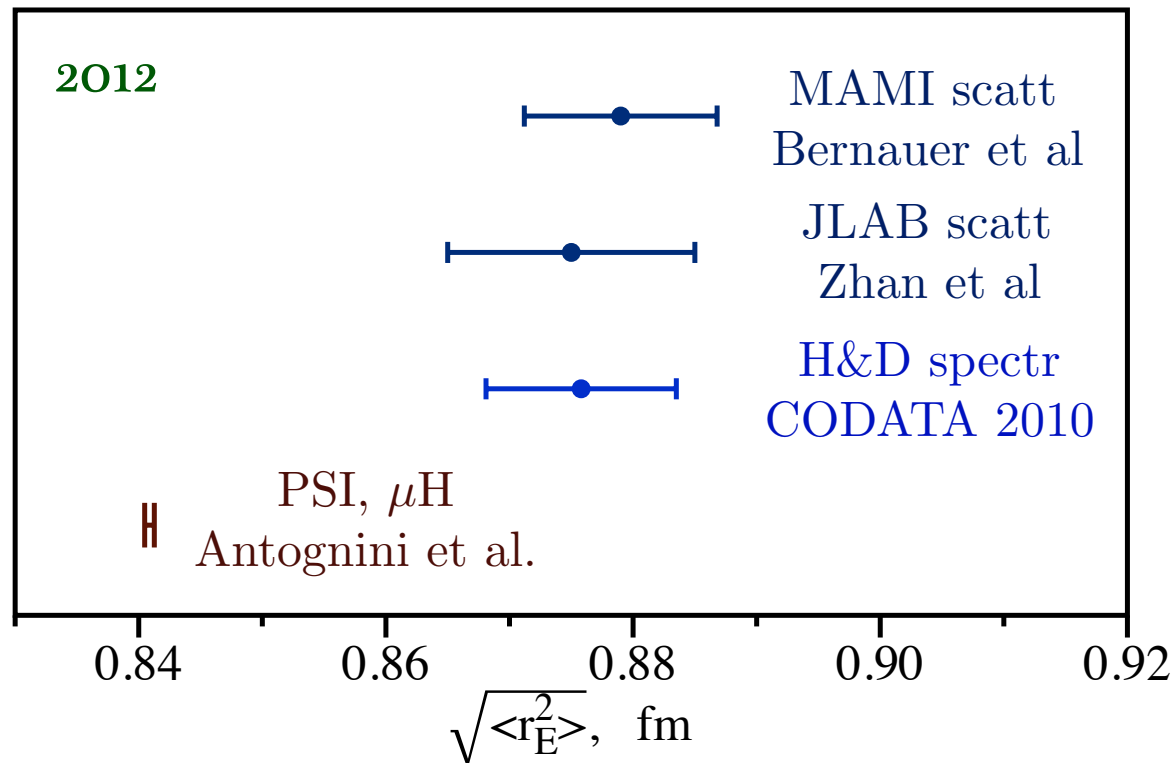
Carlson and Fried (2015)

- scalar particle: 200 keV - 3 MeV

Liu, Cloet and Miller (2018)



μH Lamb shift and 2γ

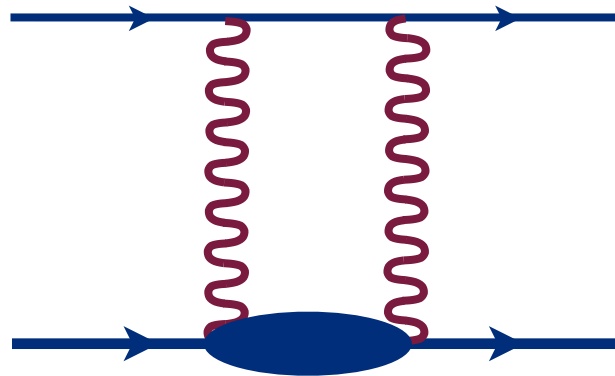


$2\text{P}-2\text{S}$ transition in μH

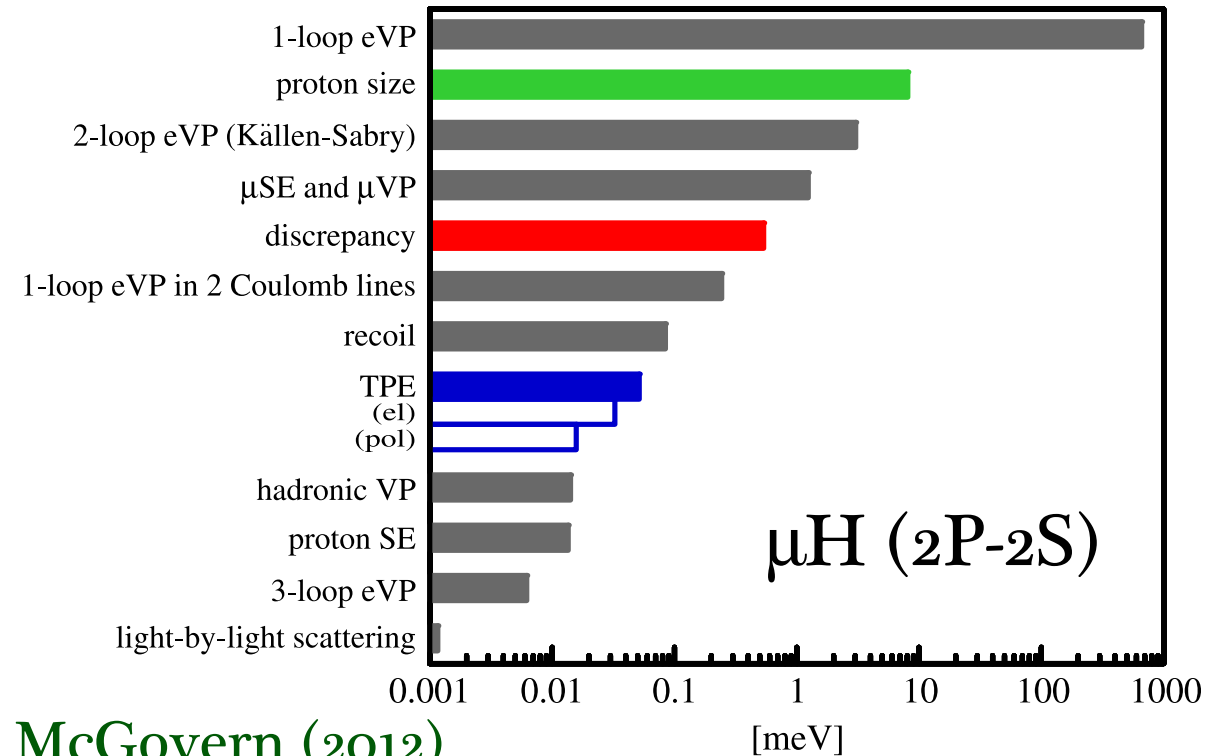
- discrepancy: $310 \mu\text{eV}$

- μH uncertainty: $2.5 \mu\text{eV}$

2γ hadronic correction



$$\Delta E_{2\text{P}-2\text{S}}^{2\gamma} = 33 \pm 2 \mu\text{eV}$$



C. Carlson, M. Vanderhaeghen (2011) + M. Birse, J. McGovern (2012)

A. Antognini et al. (2013)

- important to reduce ambiguities of 2γ

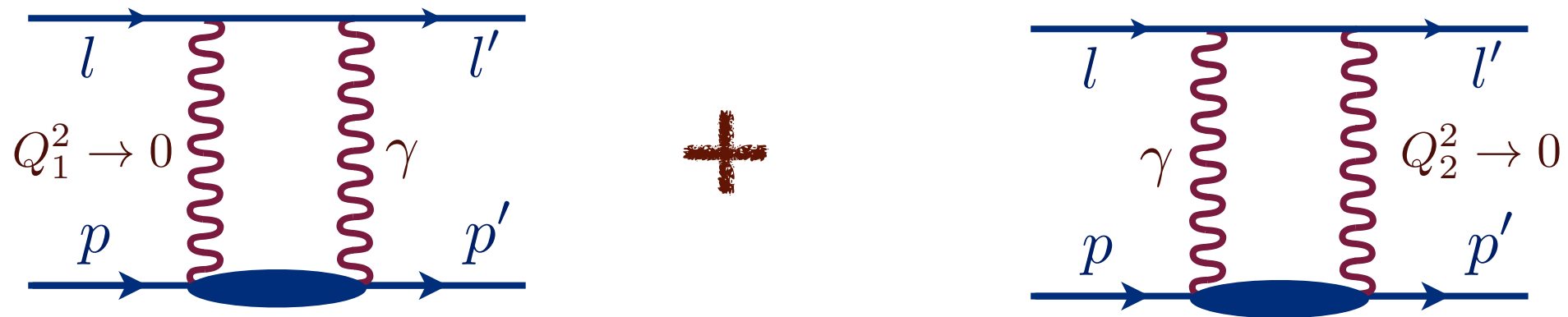
Elastic electron-proton scattering and two-photon exchange

Scattering experiments and 2γ

- 2γ is not among standard radiative corrections

$$\sigma^{\text{exp}} \equiv \sigma_{1\gamma}(1 + \delta_{\text{rad}} + \delta_{\text{soft}} + \delta_{2\gamma})$$

- soft-photon contribution is included



L.C. Maximon and J. A. Tjon (2000)

- hard-photon contribution modelled by Feshbach correction
- charge radius insensitive to 2γ model

- magnetic radius depends on 2γ model

A1@MAMI: J. C. Bernauer et al. (2014)

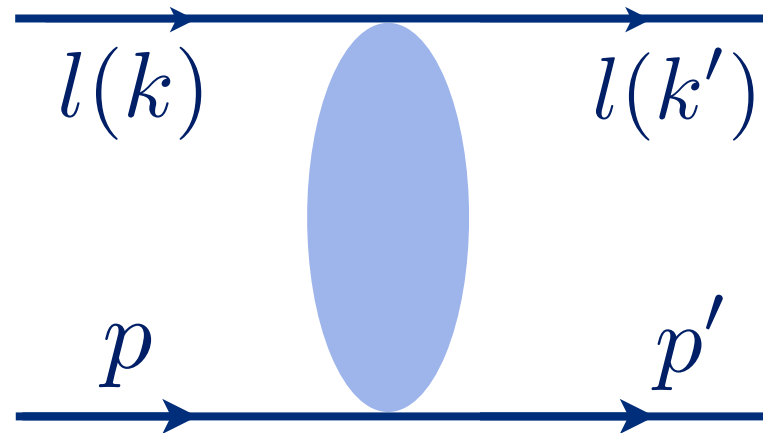
Elastic lepton-proton scattering and 2γ

momentum transfer

$$Q^2 = -(k - k')^2$$

crossing-symmetric
variable

$$\nu = \frac{(k, p + p')}{2}$$



photon polarization
parameter

$$\varepsilon$$

forward scattering

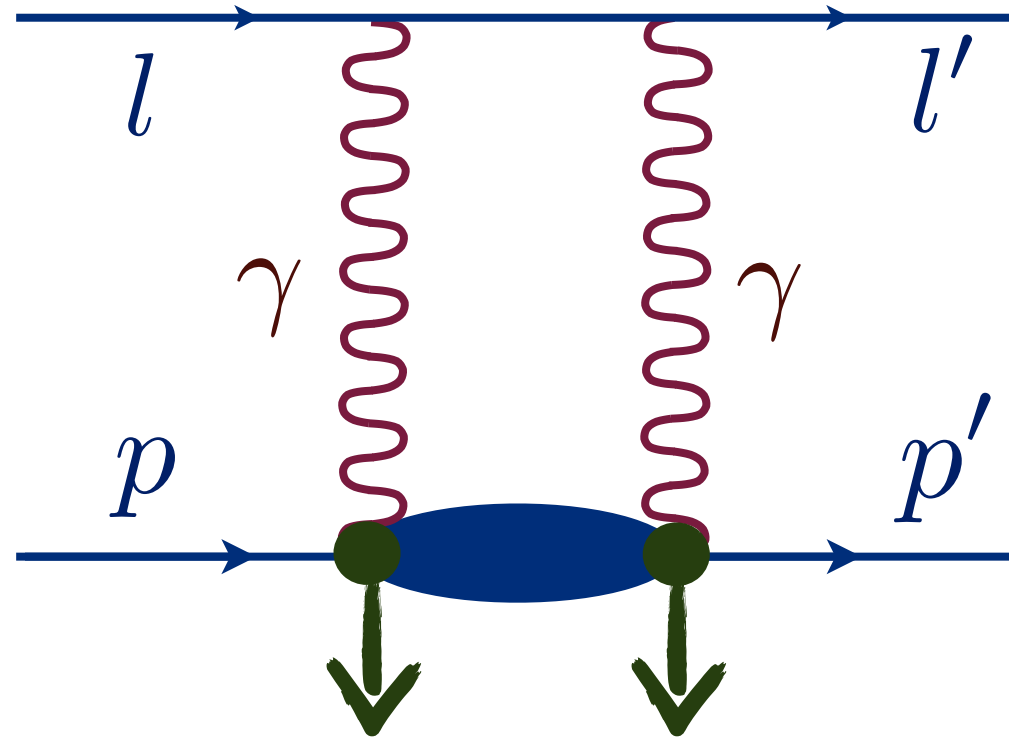
$$\varepsilon \rightarrow 1$$

- leading 2γ contribution: interference term

$$\left| \begin{array}{c} \text{Diagram 1: } l \rightarrow l' \text{ via } \gamma \text{ and } p \rightarrow p' \text{ via } \gamma \\ \text{Diagram 2: } l \rightarrow l' \text{ via } \gamma \text{ and } p \rightarrow p' \text{ via } \text{blob} \end{array} \right|^2 \rightarrow \delta_{2\gamma} = \frac{2 \sum_{\text{spin}} T^{1\gamma} \Re T^{2\gamma}}{\sum_{\text{spin}} |T^{1\gamma}|^2}$$

- 2γ correction to cross section is given by amplitudes real parts

non-forward scattering
at low momentum transfer

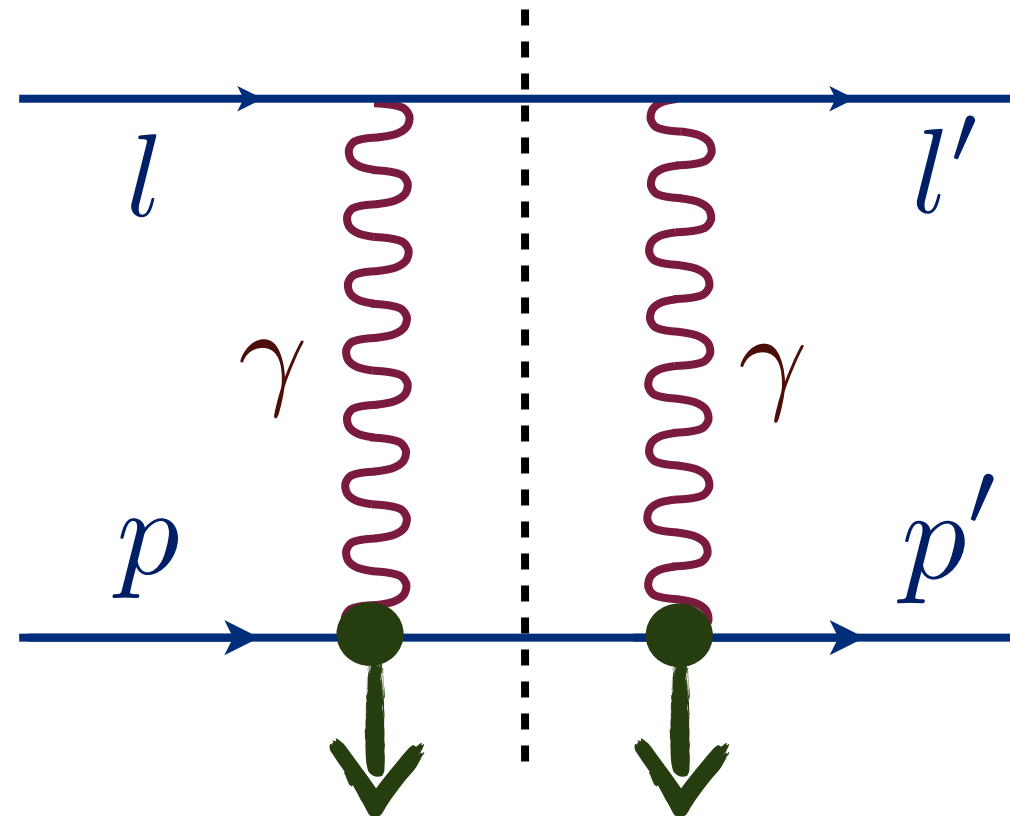


photoproduction vertex or Compton tensor

box diagram

assumption about the vertex

non-forward scattering
proton state



Dirac and Pauli form factors

box diagram

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_P(Q^2)$$

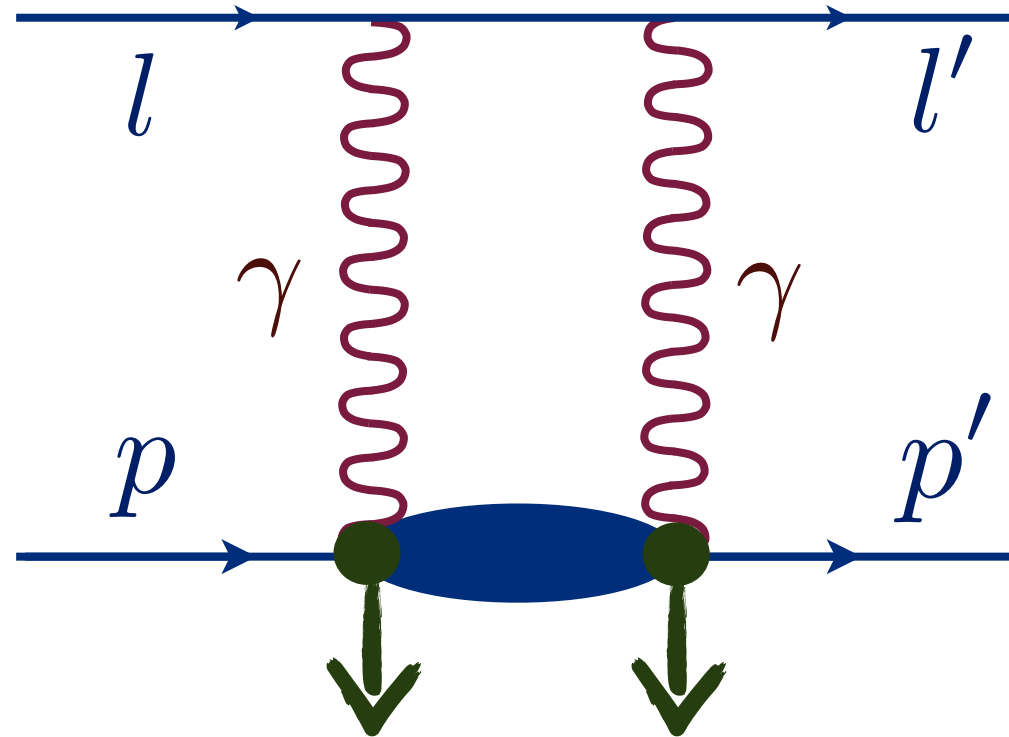
ep scattering: P. G. Blunden, W. Melnitchouk and J. A. Tjon (2003)

assumption about the vertex

violation of unitarity for resonances !

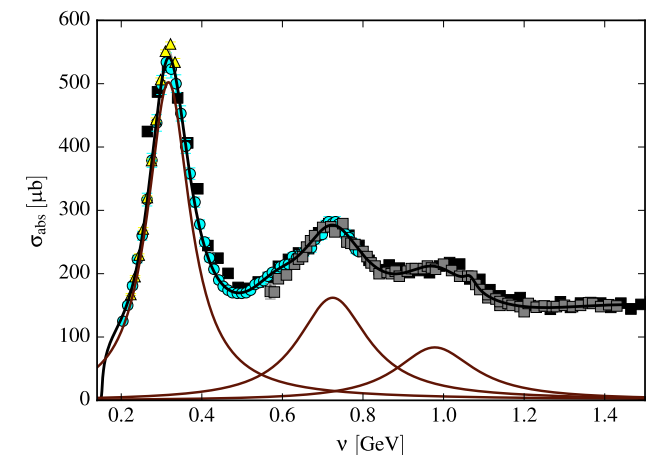
non-forward scattering
inelastic states

works at small
scattering angles



forward doubly-virtual Compton tensor

box diagram

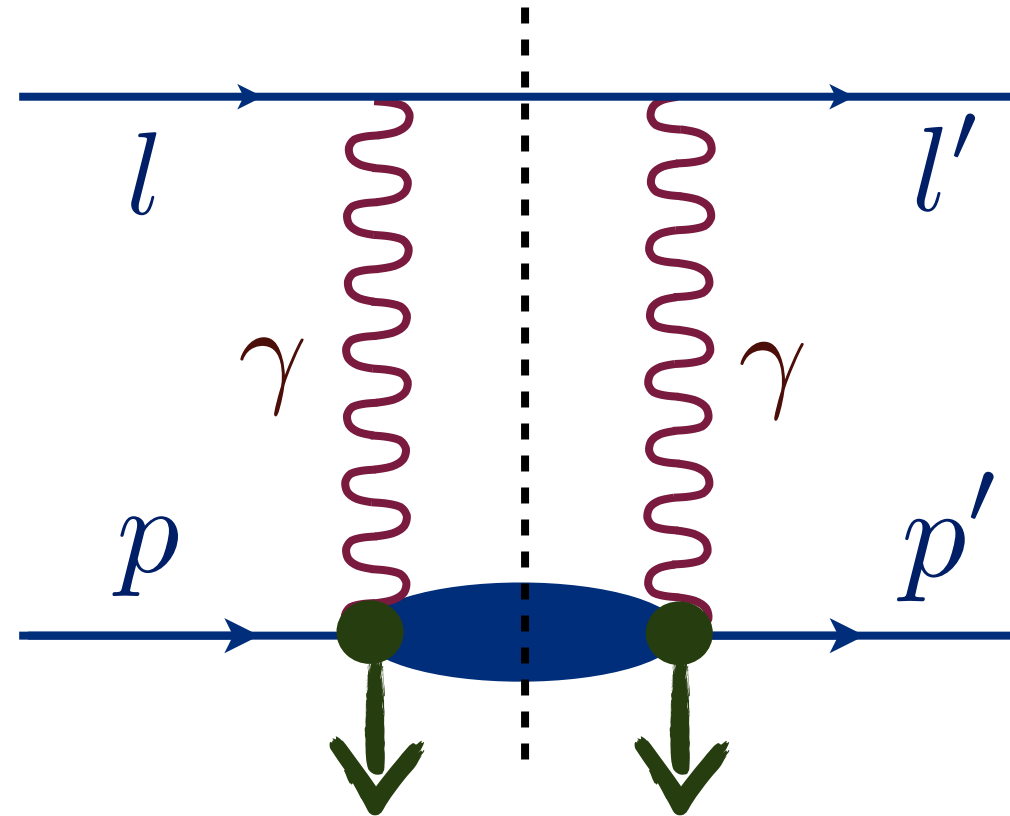


unpolarized proton structure

M. E. Christy, P. E. Bosted (2010)

proton + inelastic = total

non-forward scattering
at low momentum transfer



photoproduction vertex or Compton tensor

box diagram

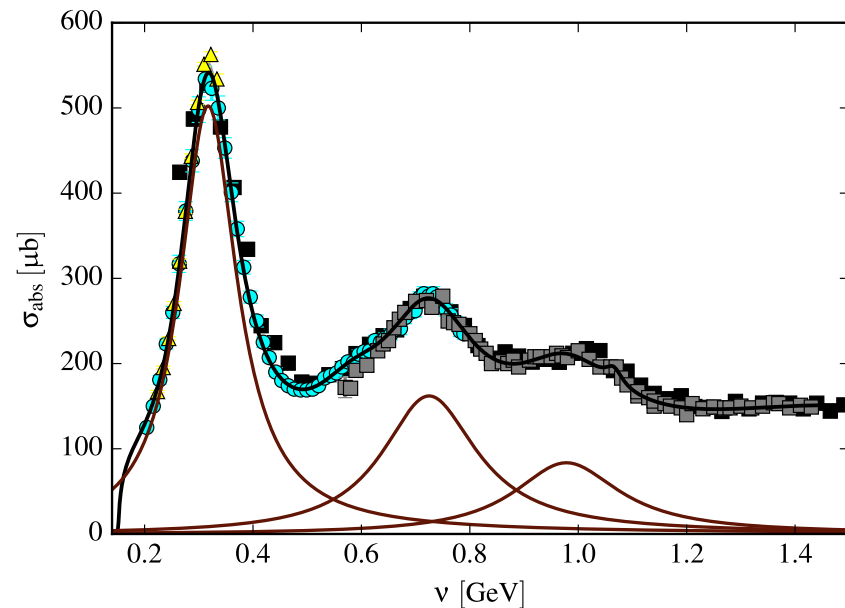
dispersion relations

assumption about the vertex

based on **on-shell** information

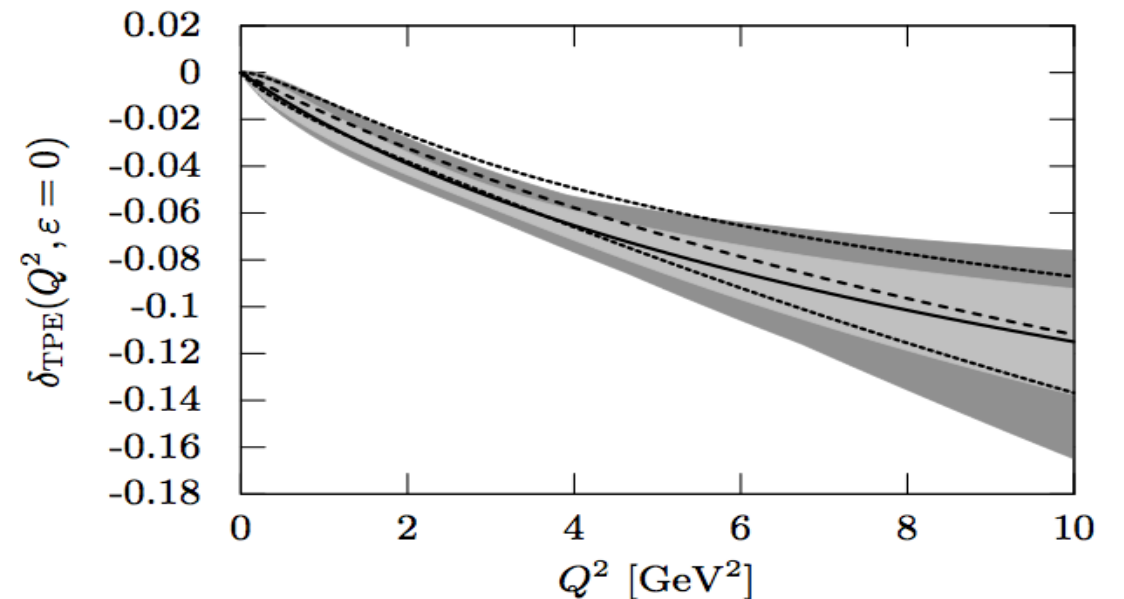
Fixed- Q^2 dispersion relation framework

on-shell 1γ amplitudes



experimental data

2γ prediction



elastic and πN

cross section correction

unitarity



2γ imaginary parts

$$\Re \mathcal{F}(\nu) = \frac{2\nu}{\pi} \mathcal{P} \int_{\nu_{\min}}^{\infty} \frac{\Im \mathcal{F}(\nu' + i0)}{\nu'^2 - \nu^2} d\nu'$$

disp. rel.

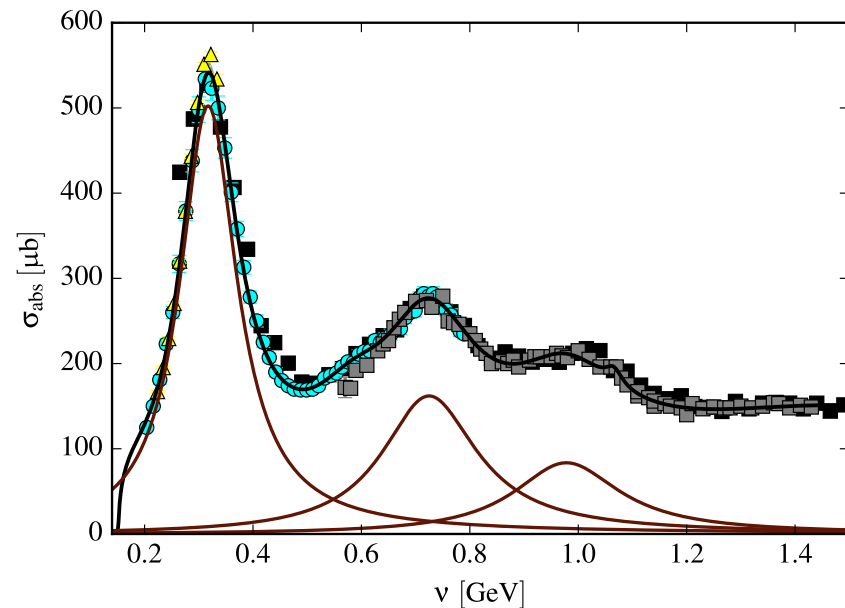


2γ real parts



Fixed- Q^2 dispersion relation framework

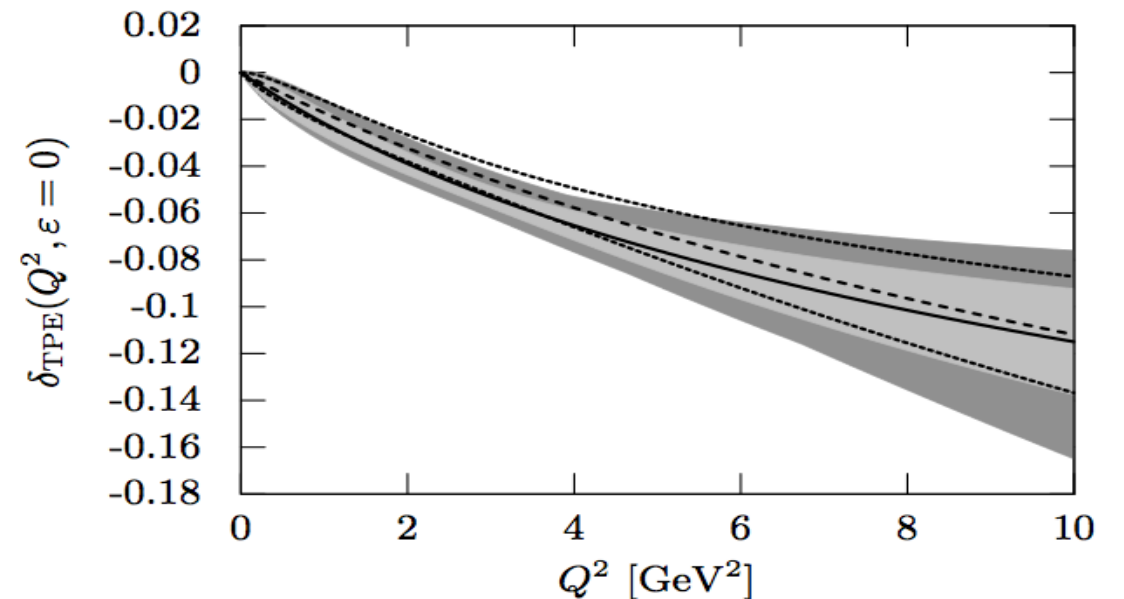
on-shell 1γ amplitudes



experimental data

elastic and πN

2γ prediction



cross section correction

unitarity



2γ imaginary parts

$$\Re \mathcal{F}(\nu) = \frac{2\nu}{\pi} \mathcal{P} \int_{\nu_{\min}}^{\infty} \frac{\Im \mathcal{F}(\nu' + i0)}{\nu'^2 - \nu^2} d\nu'$$

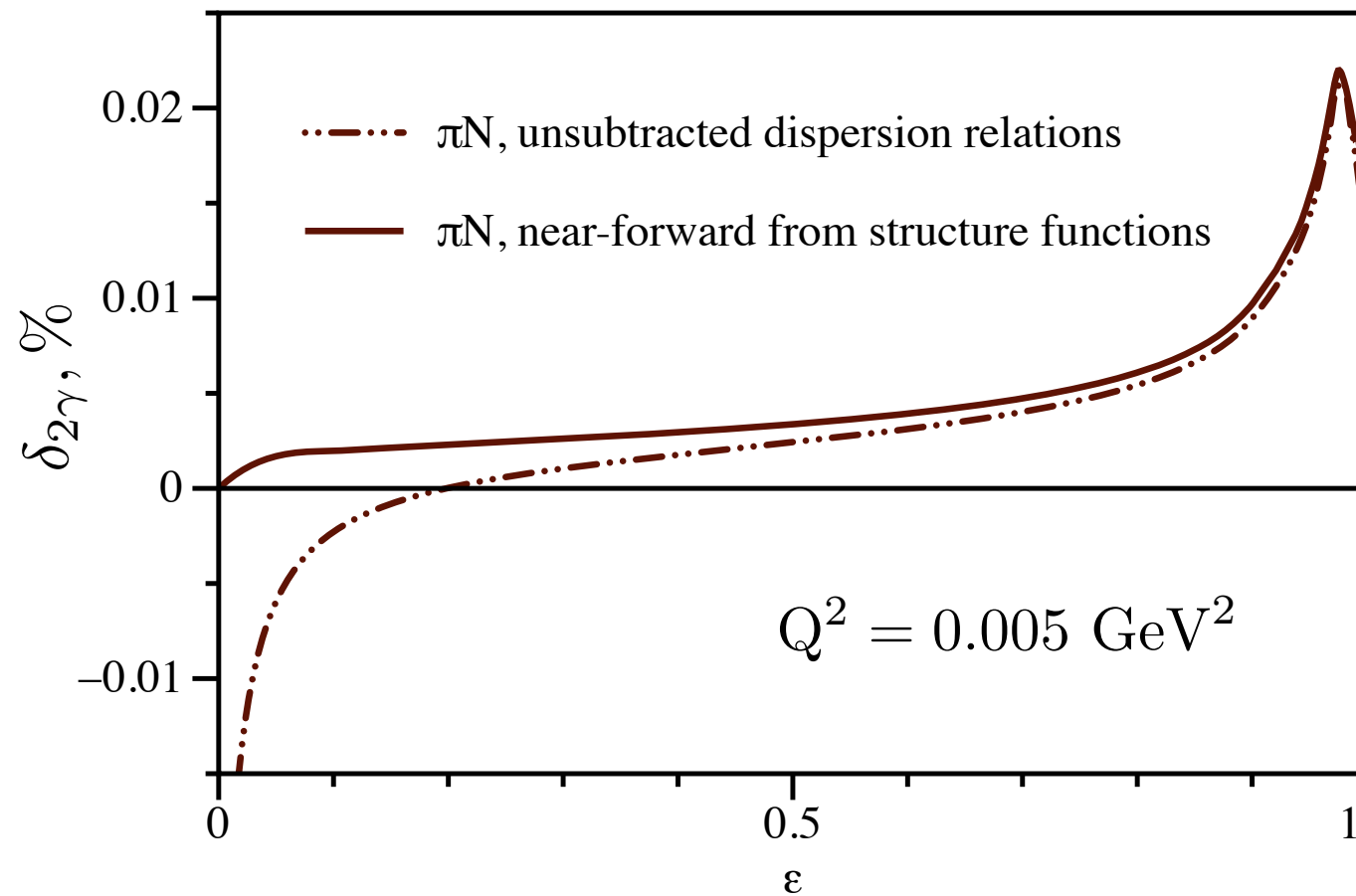
disp. rel.



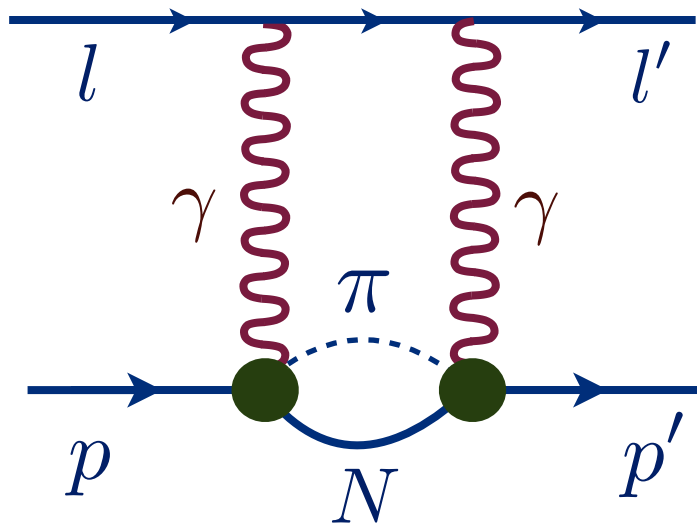
2γ real parts



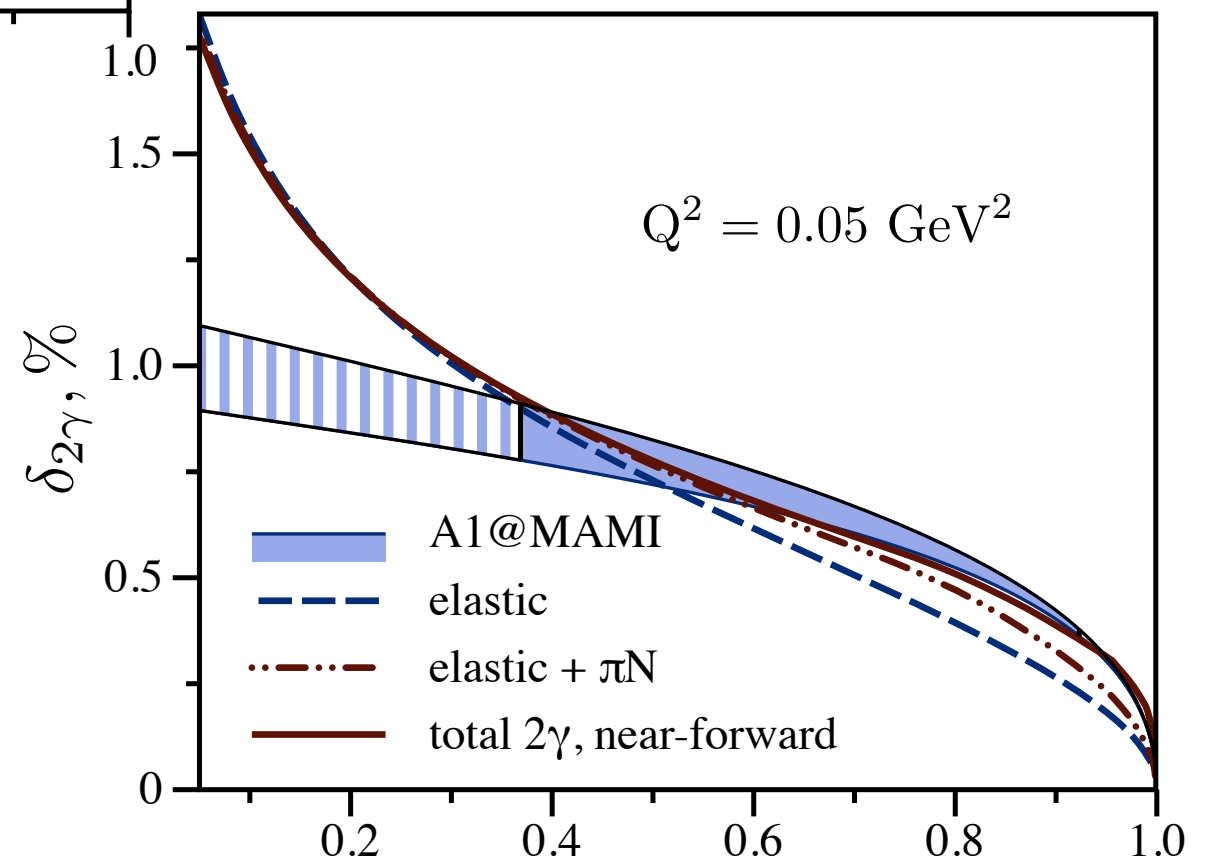
πN in dispersive framework (e-p)



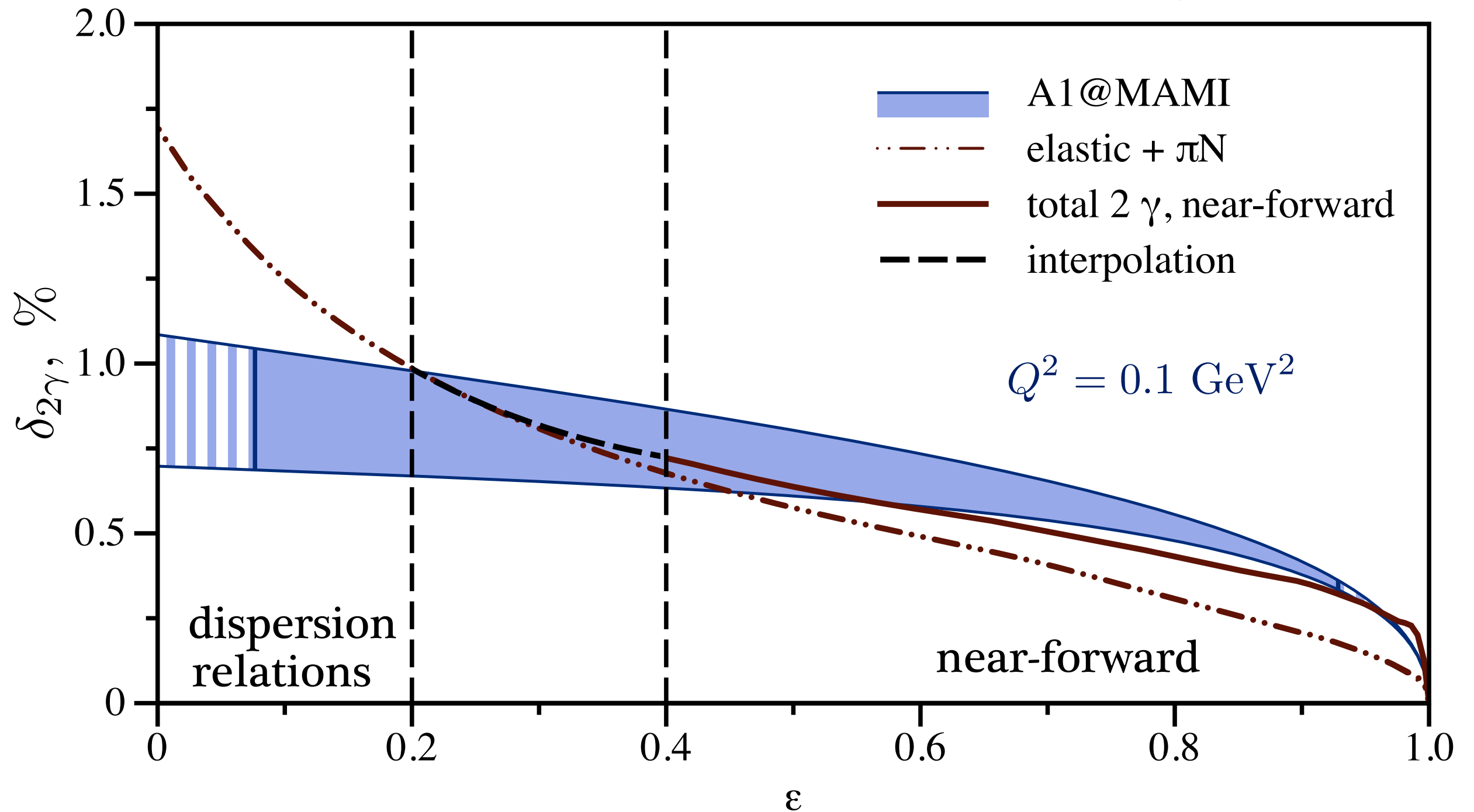
- dispersion relations agree with near-forward at large ε



- πN is dominant inelastic 2γ

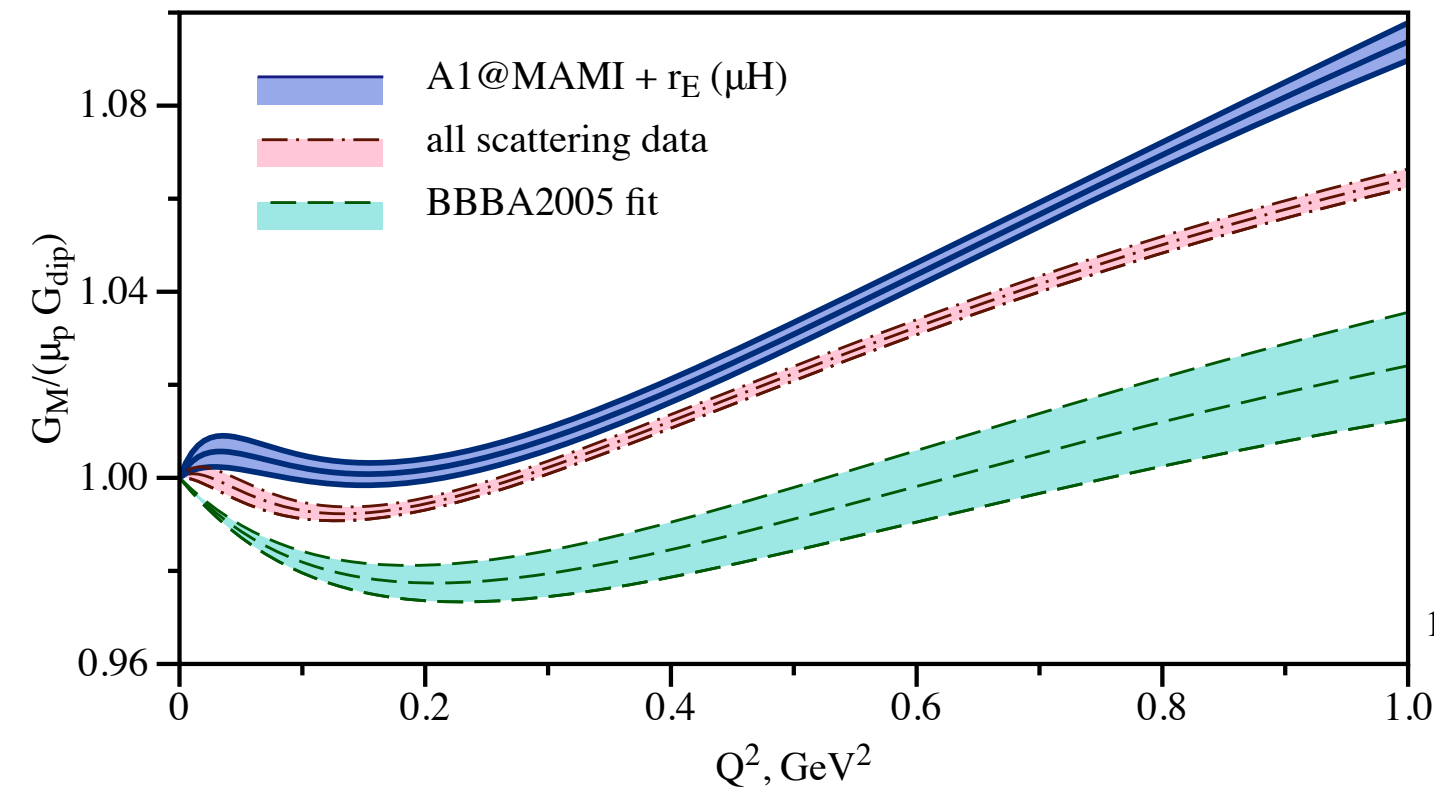


Our best 2γ knowledge



- small Q^2 : near-forward at large ϵ , all inelastic states
- $Q^2 \lesssim 1 \text{ GeV}^2$: elastic+ πN within dispersion relations
- intermediate range: interpolation

Applications to nucleon form factors



z expansion fit

$$z(Q^2) = \frac{\sqrt{t_{cut} + Q^2} - \sqrt{t_{cut} - t_0}}{\sqrt{t_{cut} + Q^2} + \sqrt{t_{cut} - t_0}}$$

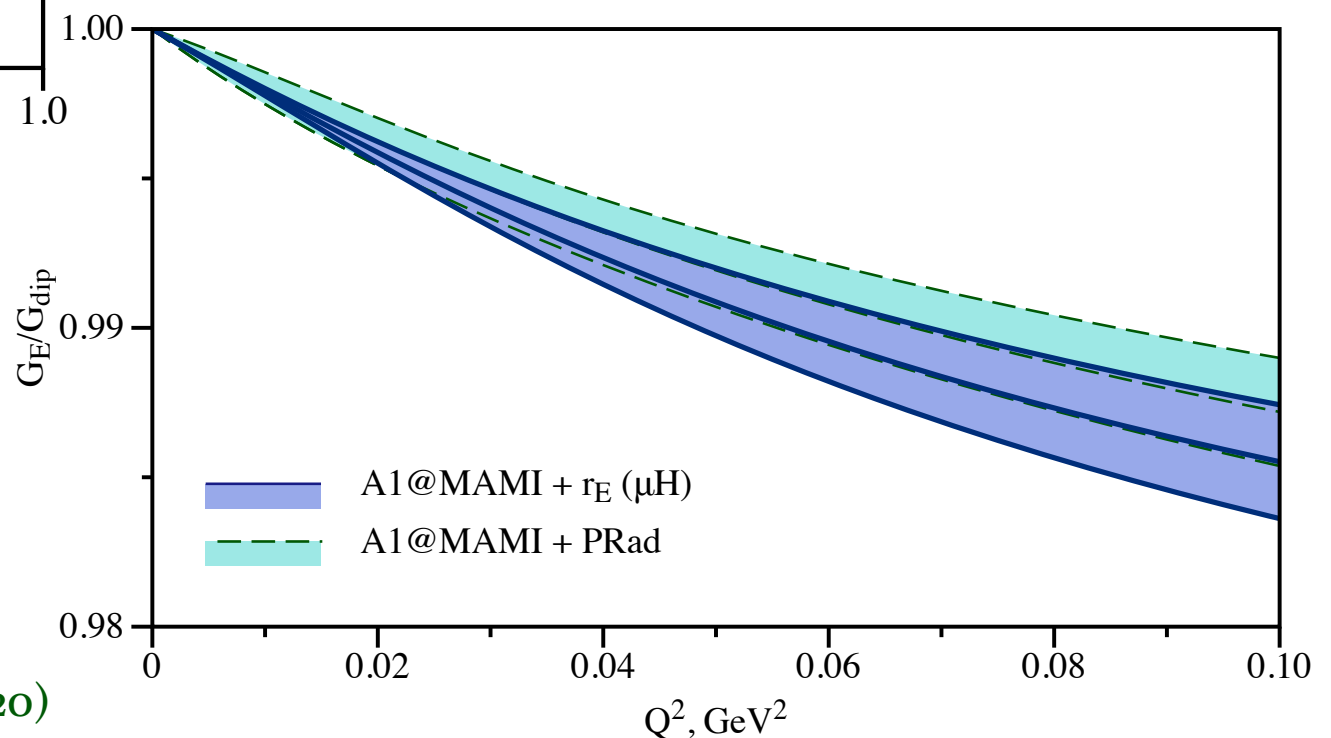
with 4-5 independent coefficients

$$G(Q^2) = \sum_{k=0}^{k_{max}} a_k z(Q^2)^k$$

BBBA: fit used in neutrino physics

PRad data or μH charge radius ?

K. Borah, G. Lee, R. J. Hill and O. T. (2020)



- first model-independent fits presenting covariance matrix
- 2γ provides nontrivial hadronic radiative correction
- proton charge radius as a constraint

Elastic muon-proton scattering and two-photon exchange

Elastic muon-proton scattering

- charge radius extractions:

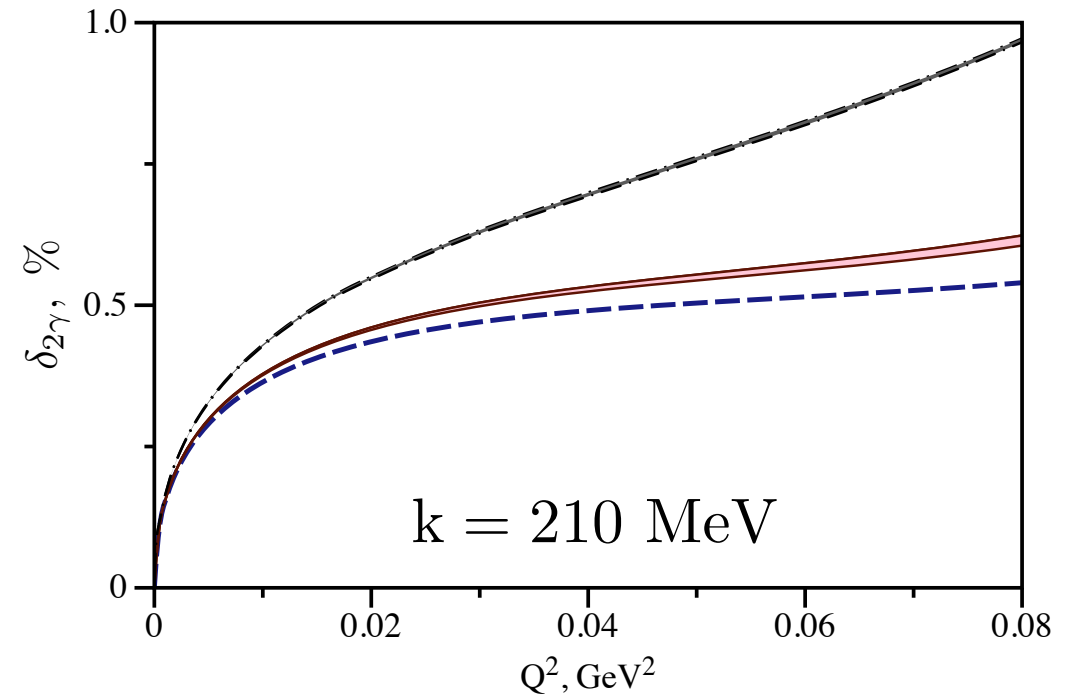
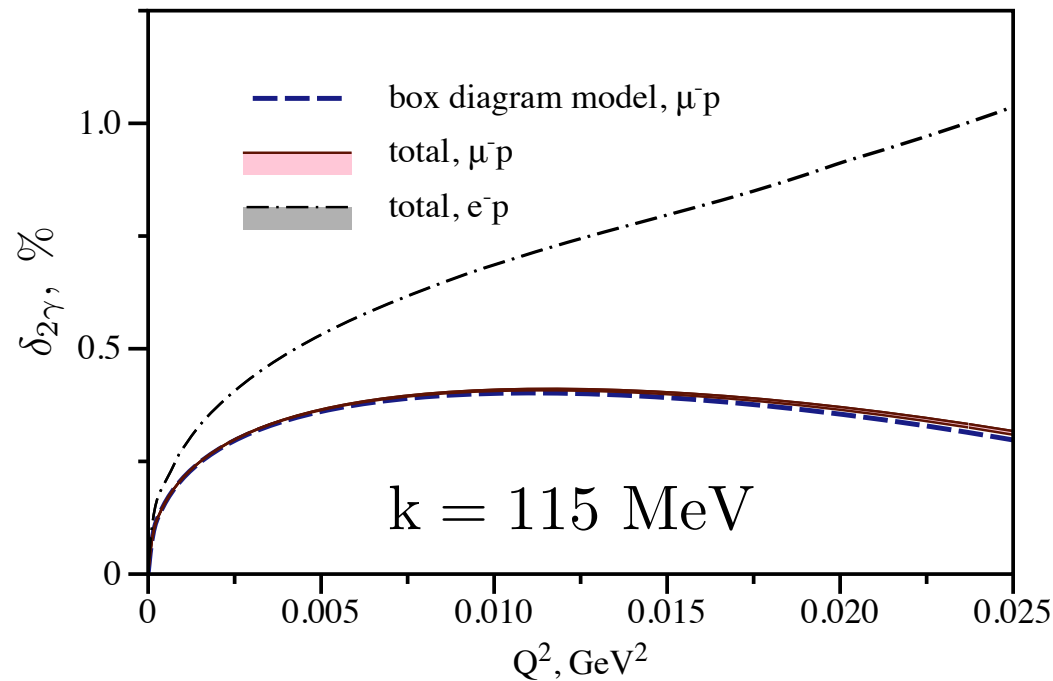
eH, eD spectroscopy	ep scattering
μ H, μ D spectroscopy	μ p scattering ????

- μ p elastic scattering is planned by **MUSE@PSI(2018-19)**
measure with both electron/muon charges
- three nominal beam energies: 115, 153, 210 MeV, $Q^2 < 0.1 \text{ GeV}^2$

- 2γ correction in MUSE ?

MUSE@PSI (2018-19) estimates (μ -p)

- proton box diagram model + inelastic 2γ



O. T. and M. Vanderhaeghen (2014, 2016)

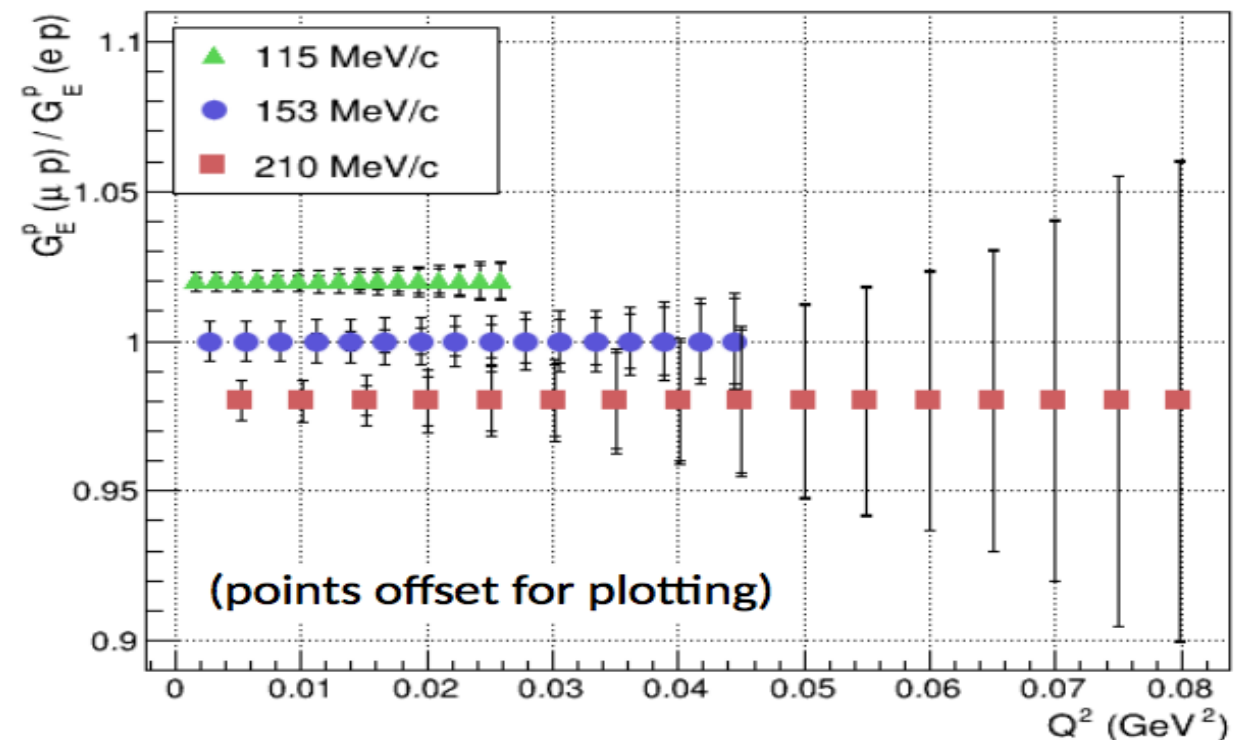
- expected muon over electron ratio

small inelastic 2γ



small 2γ uncertainty

- MUSE can test r_E in one charge channel



COMPASS proton radius experiment

- elastic μp scattering at SPS with 100 GeV beam
- measure $G_E^2 + \tau G_M^2$ at forward angles

2γ corrections?

- Feshbach correction (+ recoil)

$$\delta_{2\gamma} = \frac{\alpha\pi Q}{2\omega} \left(1 + \frac{m}{M}\right) \quad \rightarrow \quad \text{2-3 orders below MUSE}$$

- inelastic states: kinematically enhanced

- sub per mille level of 2γ in COMPASS kinematics

1S-2S transition in hydrogen and 2γ

- measurements of 1S-2S transition in eH with 4×10^{-15} accuracy:

$$\nu_{1S-2S}(H) = 2466061413187018(11) \text{ Hz} \quad \text{2010th}$$

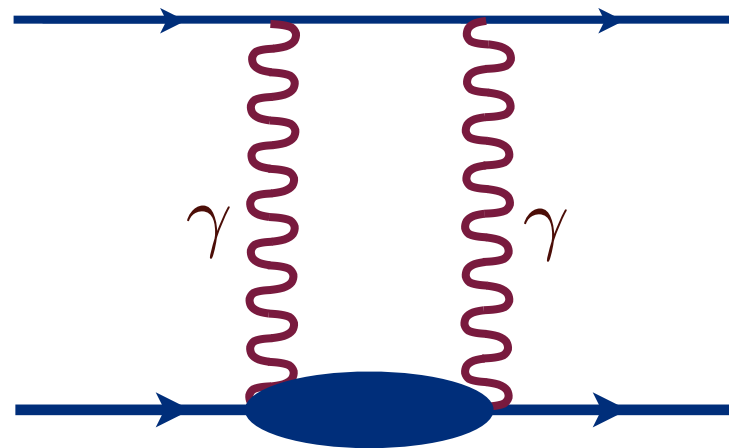
A. Matveev et al. (2013)

- more precise than recent Lamb shift measurement (**error: 3.2 kHz**)

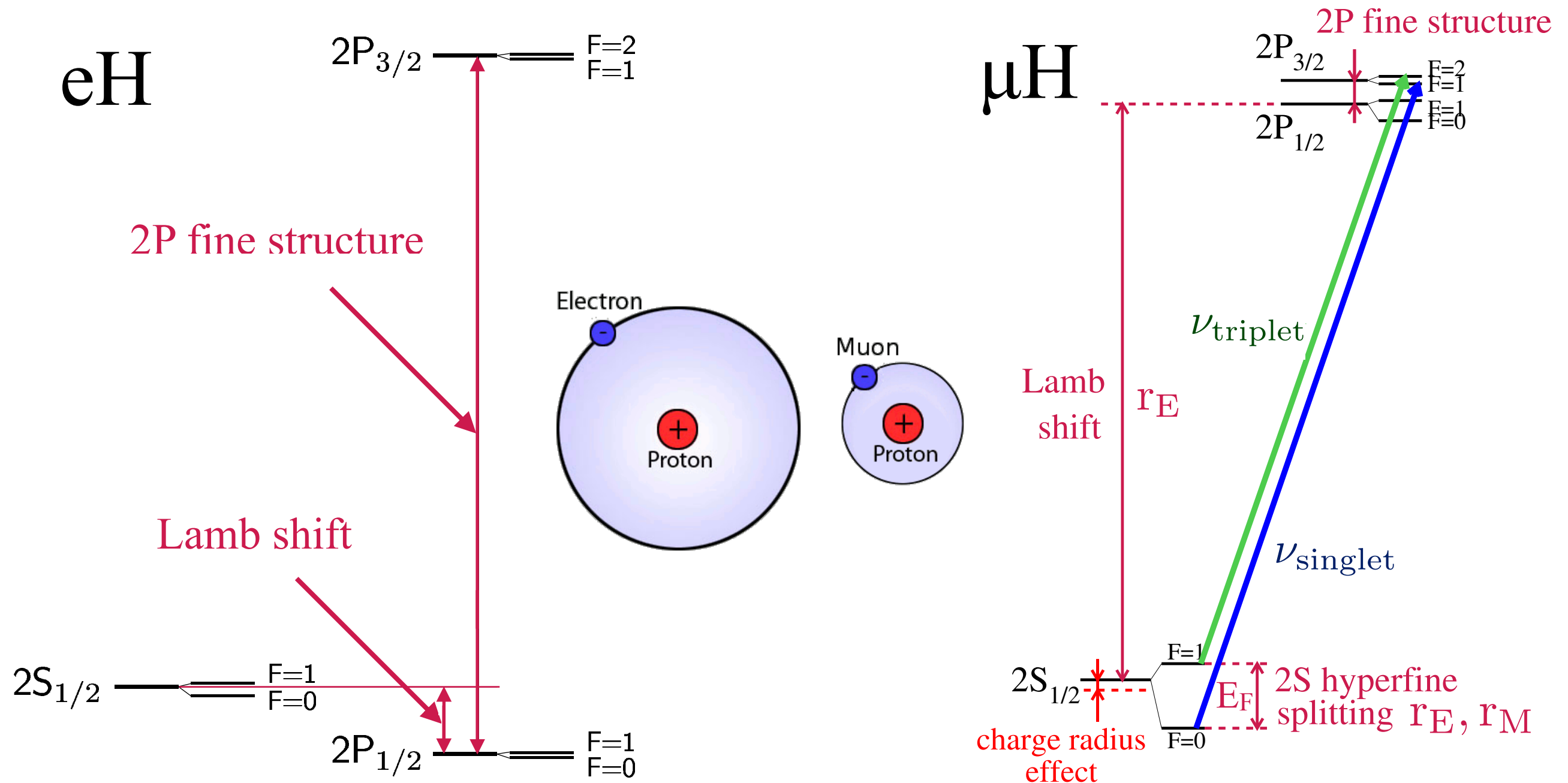
N. Bezginov et al. (2019)

$$\nu_{nS} = -\frac{R_\infty}{n^2} + \frac{L_{1S}(r_E)}{n^3}$$

- main input to determine Rydberg constant



Lamb shift and hyperfine splitting in H

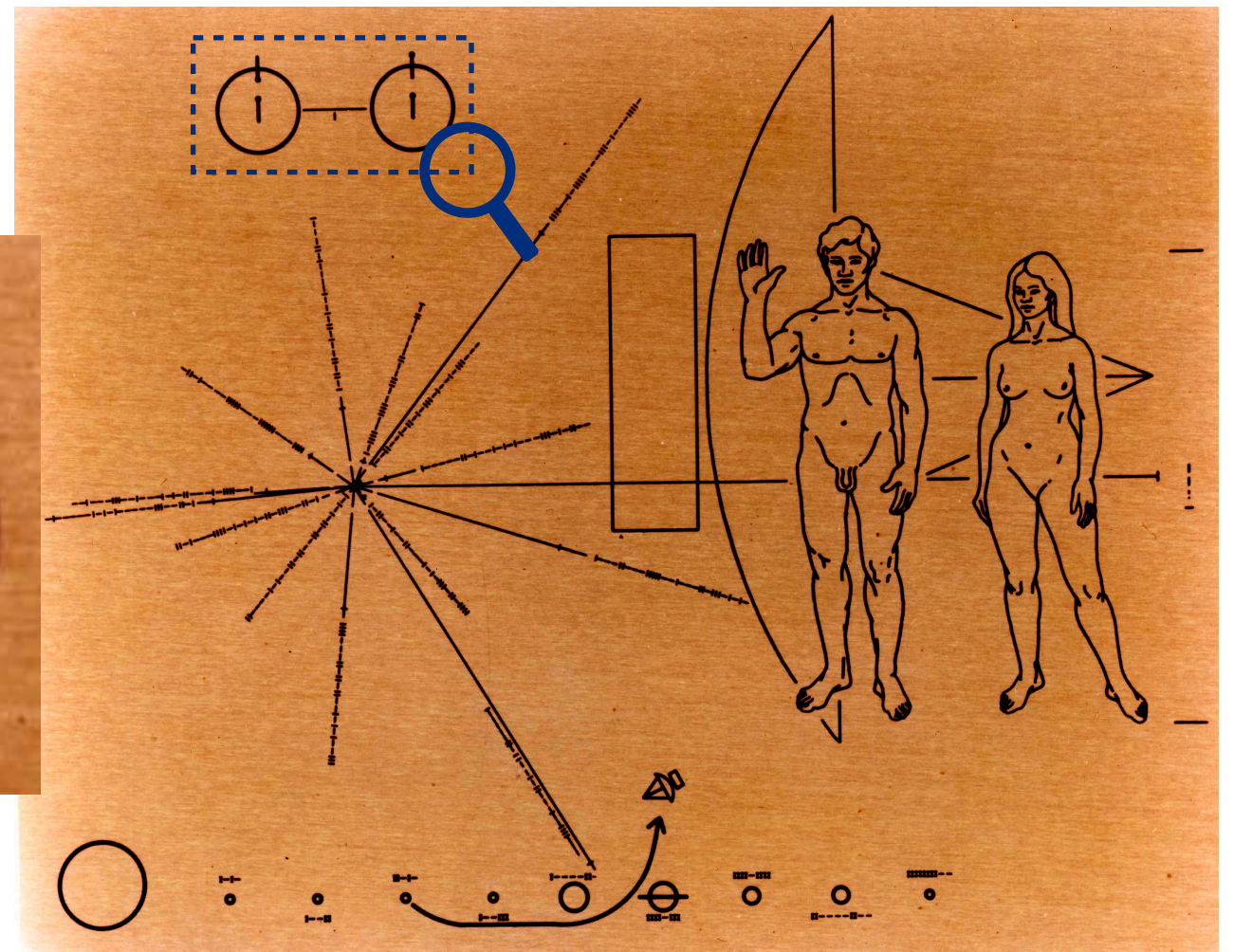
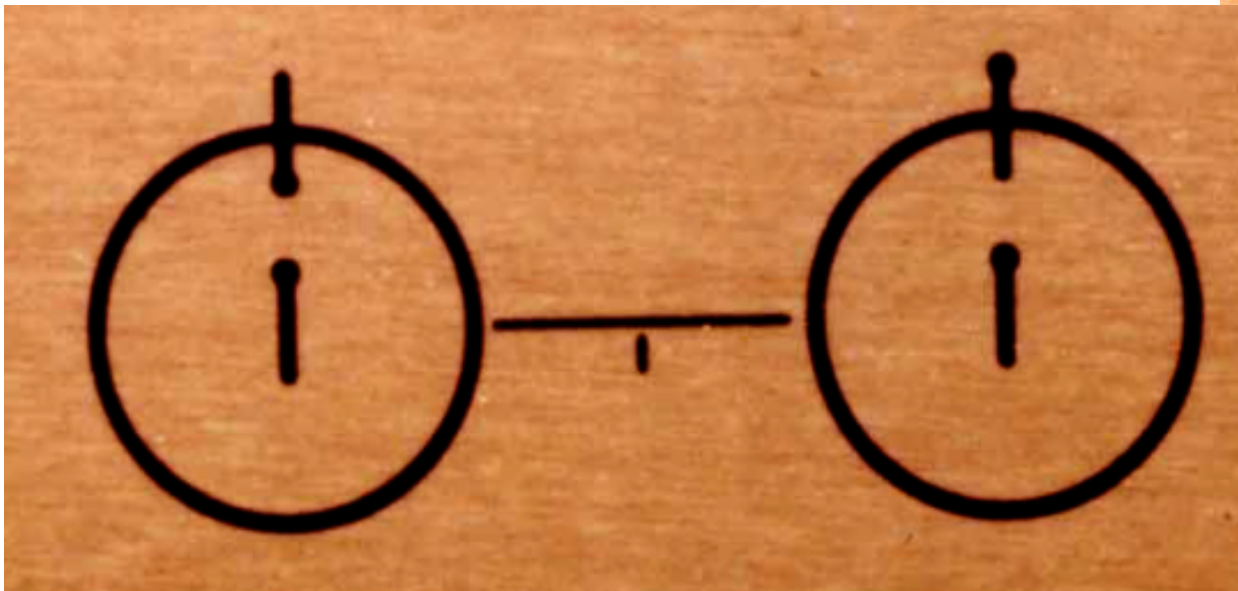


- 1S HFS in μH with 1 ppm accuracy at PSI, J-PARC, RIKEN-RAL

μH 1S HFS from eH 1S HFS

- measurements of 1S HFS in eH (21 cm line):

$$\nu_{\text{HFS}}(\text{H}) = 1420.4057517667(9) \text{ MHz} \quad 1970\text{th}$$



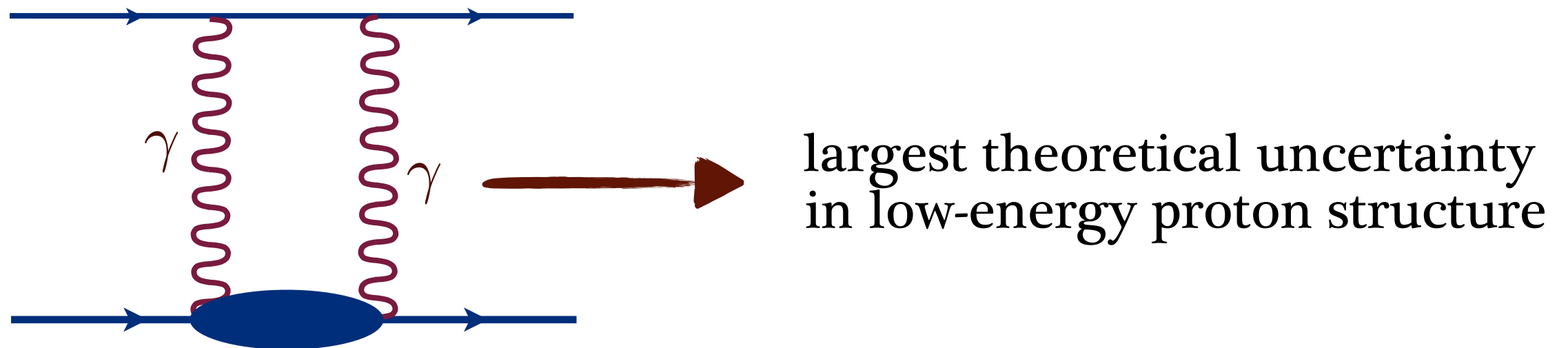
Pioneer plaque, NASA Ames

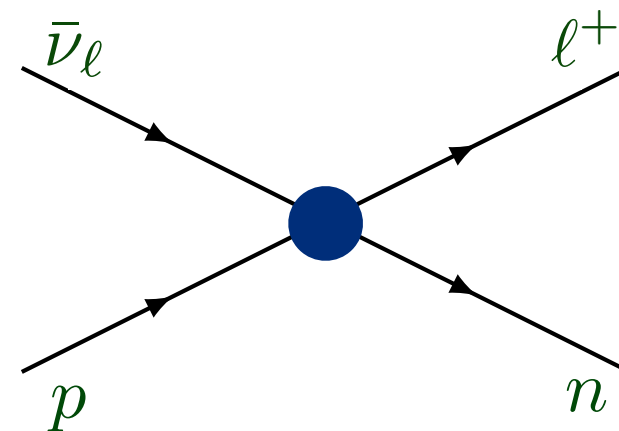
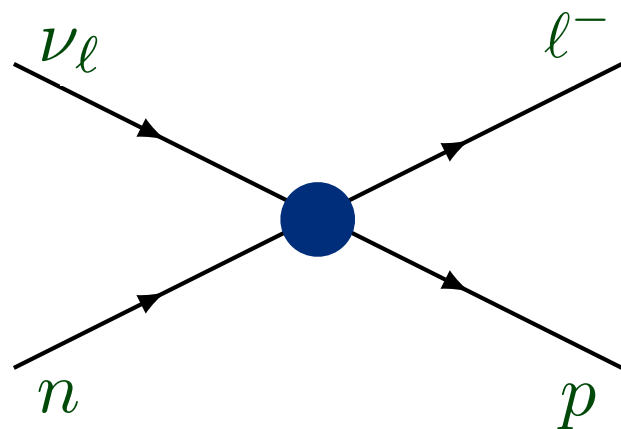
- relation between eH and μH through g_1 and g_2 in 2γ

O. T., Eur.Phys.J.A 55 (2019) 5, 64

Conclusions

- proton charge radius puzzle dissolves with new measurements
- tensions in scattering data are not resolved
- forthcoming muon scattering data will shed new light

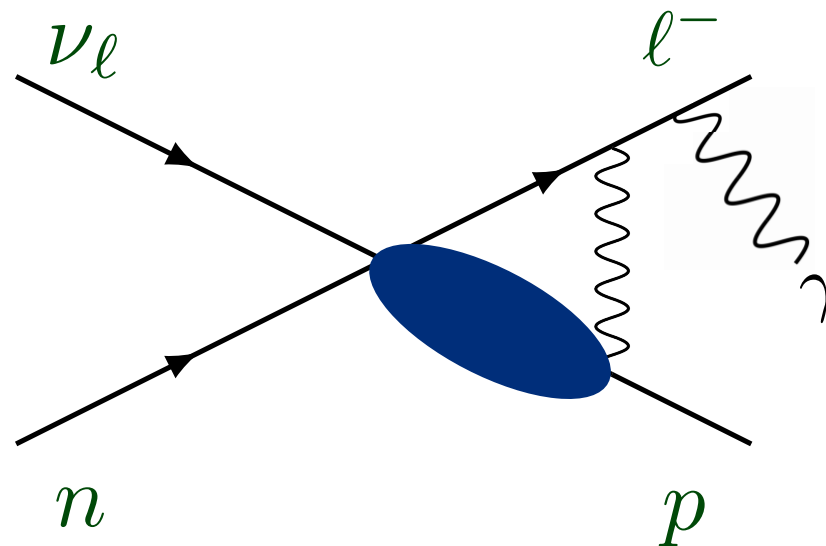




O. T., Qing Chen, Richard J. Hill and Kevin S. McFarland, Nature Commun. 13 (2022), 1, 5286

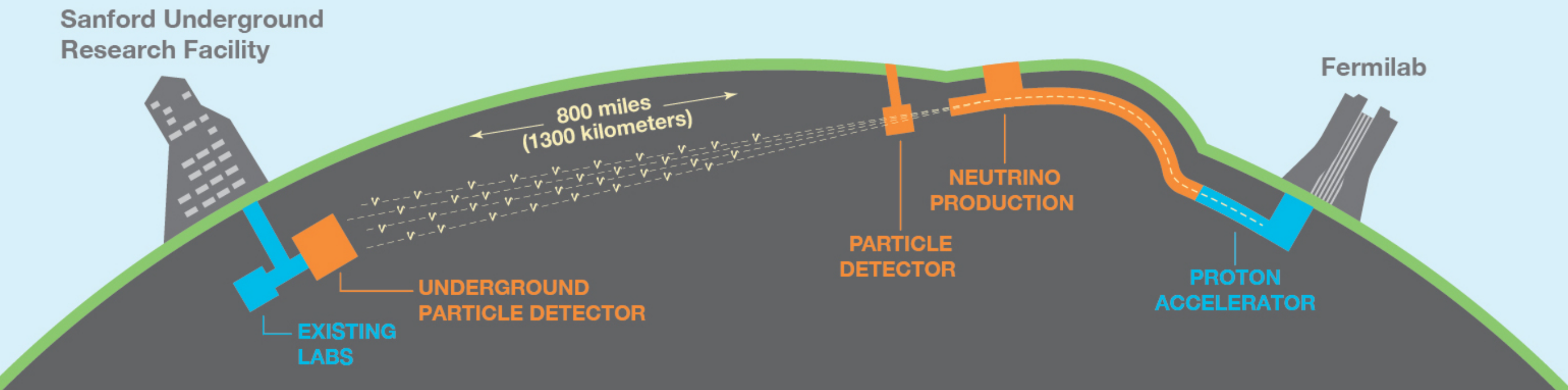
Radiative corrections in charged-current elastic scattering on free nucleons

O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret
editors suggestion in Phys. Rev. D (2022)



Neutrino experiments

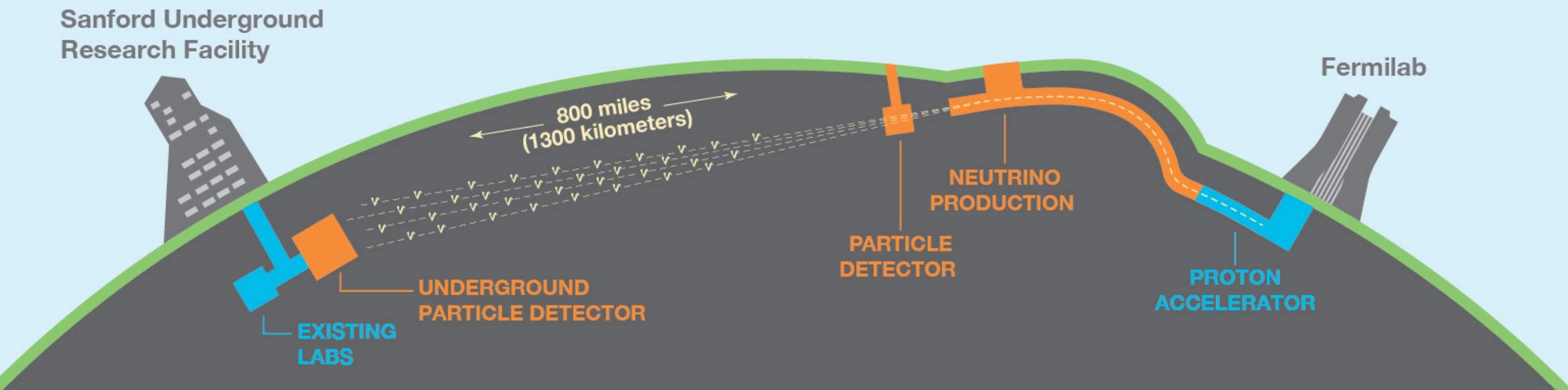
- **DUNE** and Hyper-K: leading-edge ν science experiments



- origin of matter-antimatter asymmetry δ_{CP}
- mass hierarchy and oscillation parameters PMNS matrix, Δm_{31}^2
- Grand Unified Theories proton decay
- dynamics of supernova explosion wait for one;)

Neutrino experiments

- **DUNE** and Hyper-K: leading-edge ν science experiments

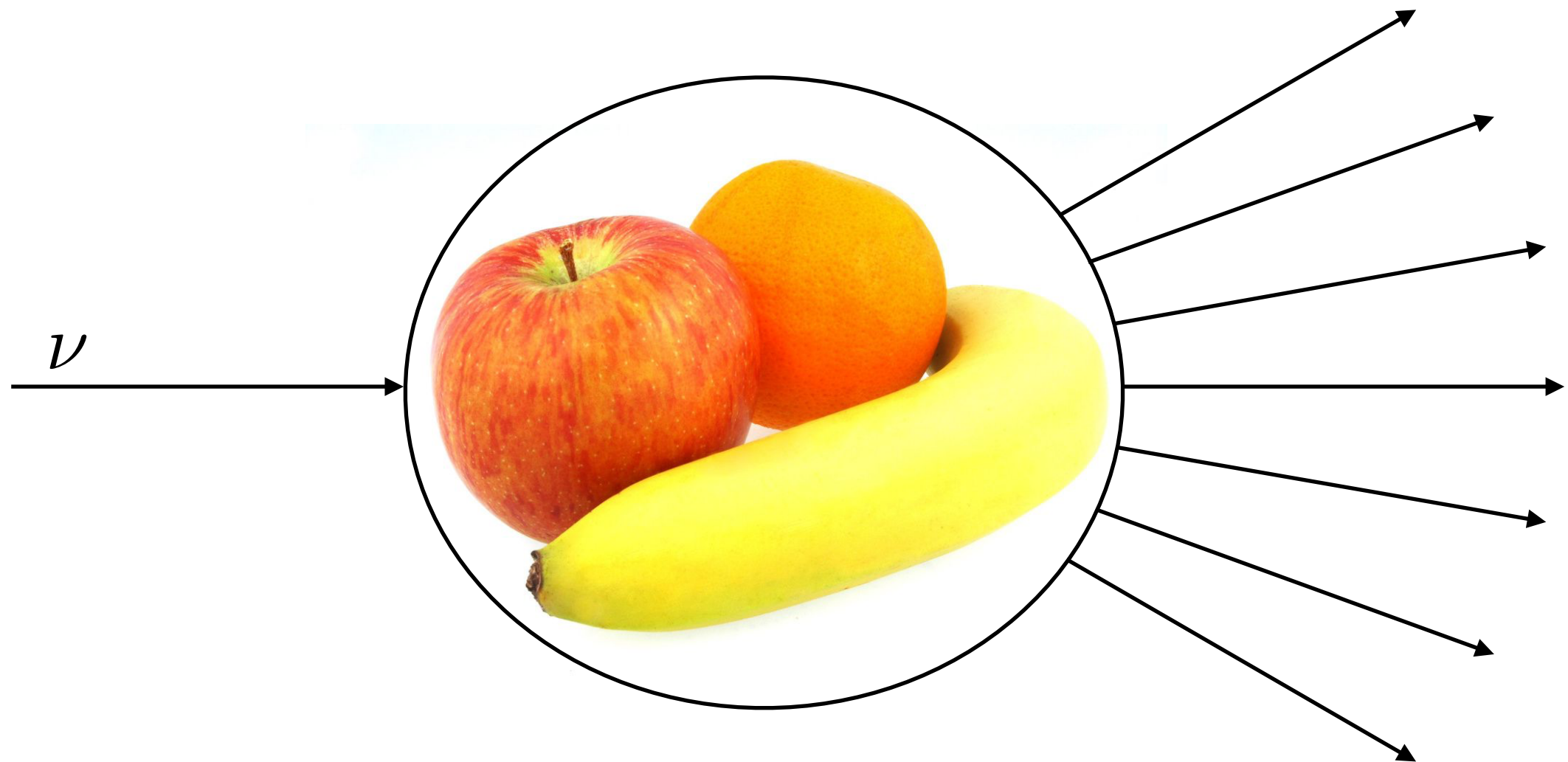


- measurement of $\nu_\mu(\bar{\nu}_\mu)$ disappearance and $\nu_e(\bar{\nu}_e)$ appearance

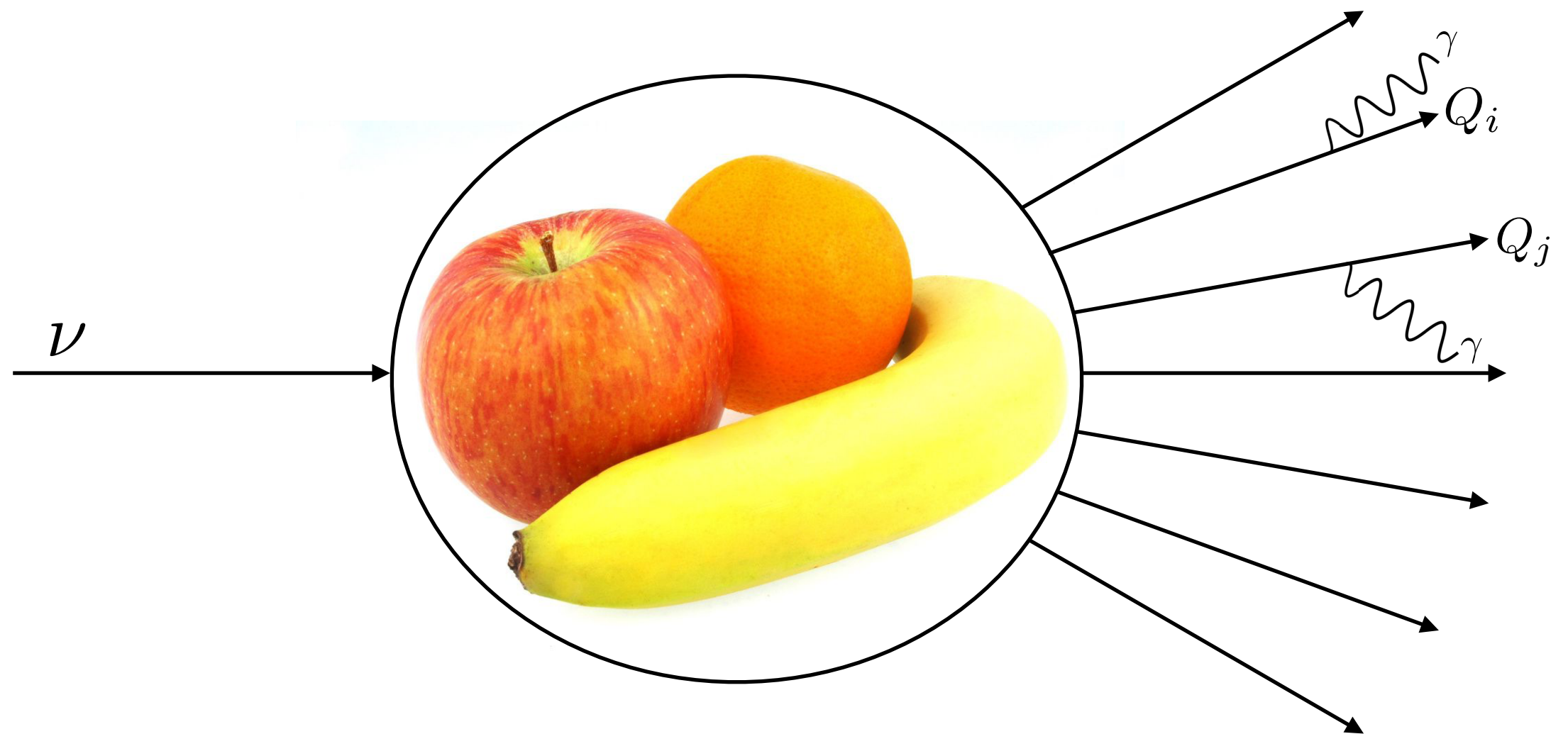
$$N_\nu \sim \int dE_\nu \Phi_\nu(E_\nu) \times \sigma(E_\nu) \times R(E_\nu, E_\nu^{\text{rec}})$$

- near detector: determine flux and cross sections

Neutrino interactions

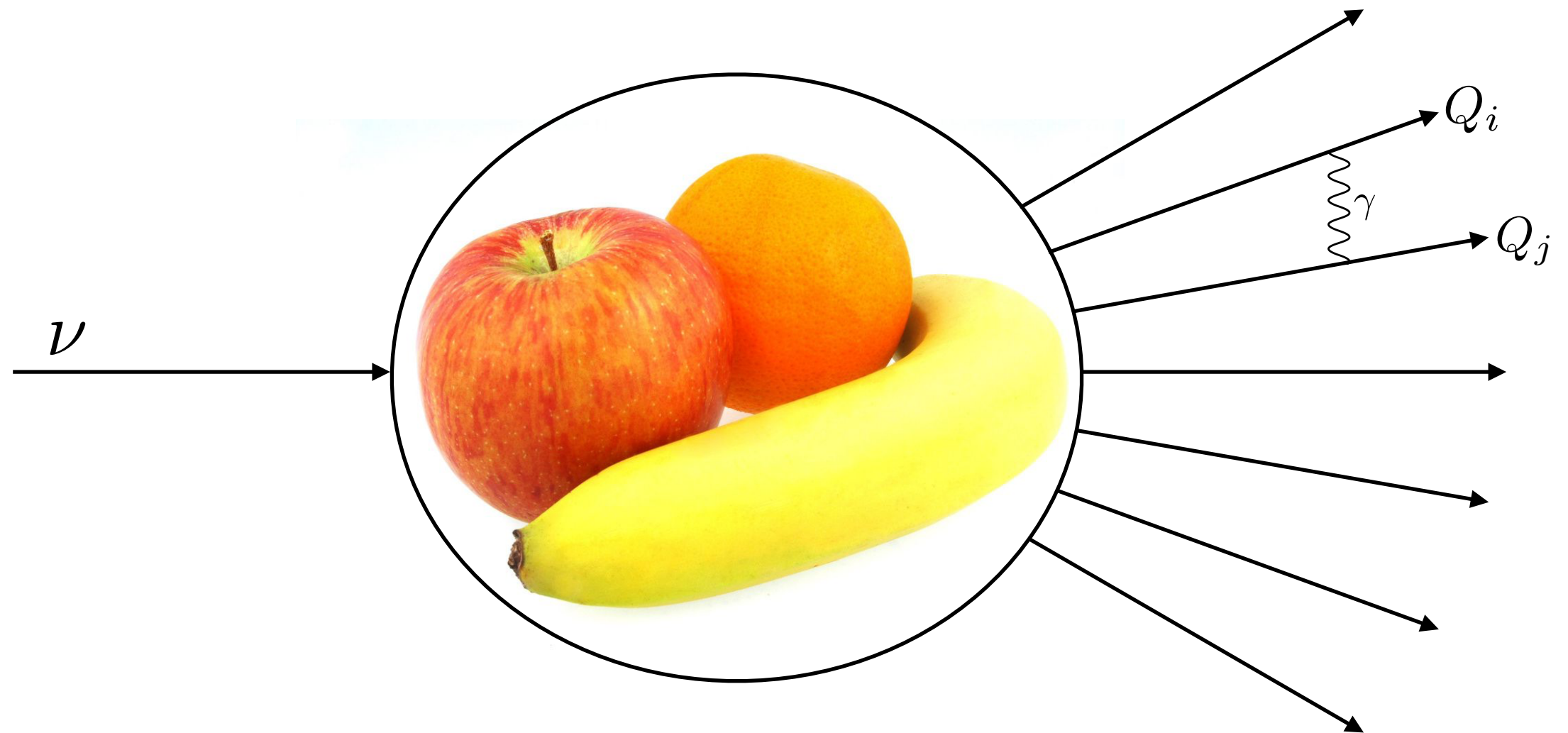


QED corrections



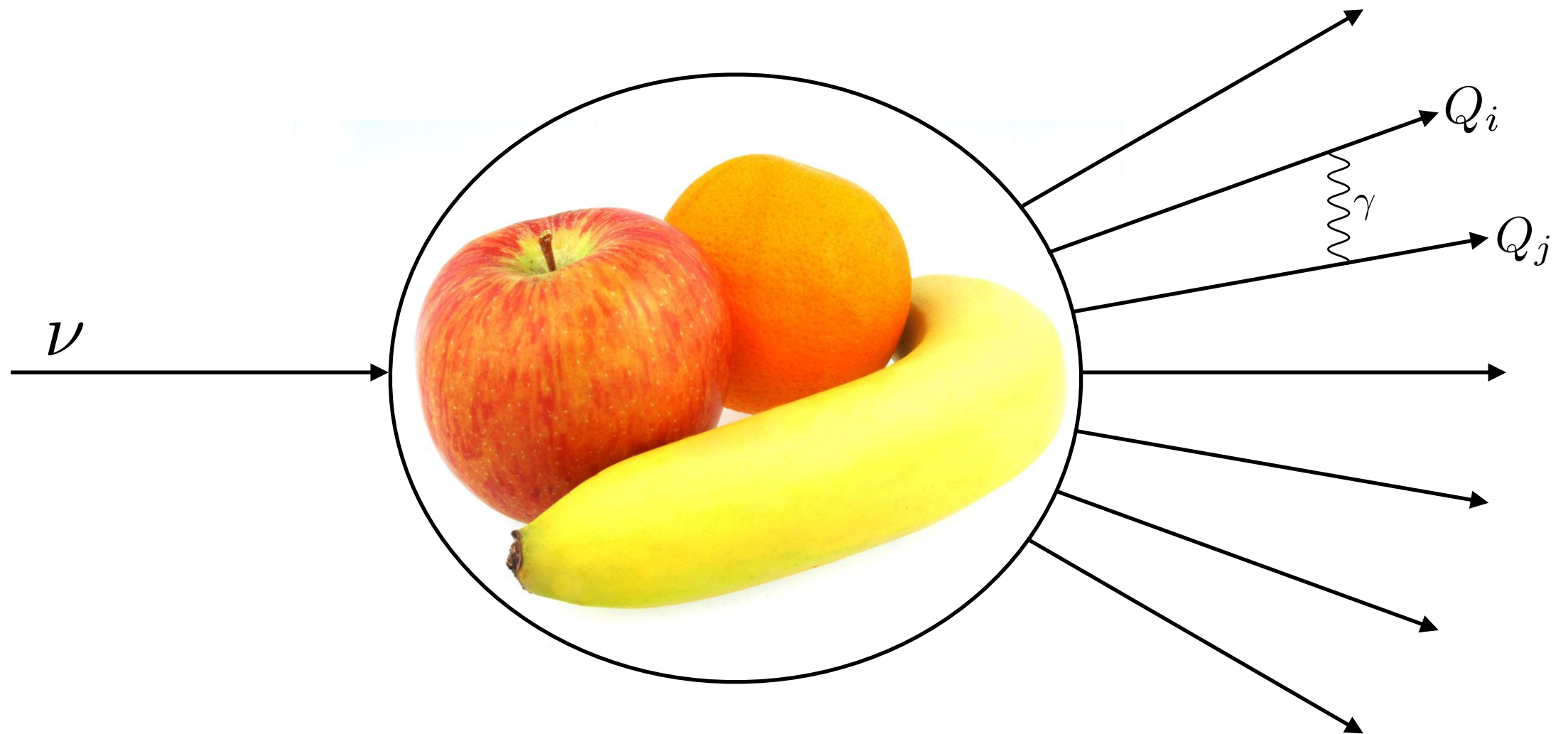
- all charged particles couple to real and virtual photons

QED corrections



- all charged particles couple to real and virtual photons

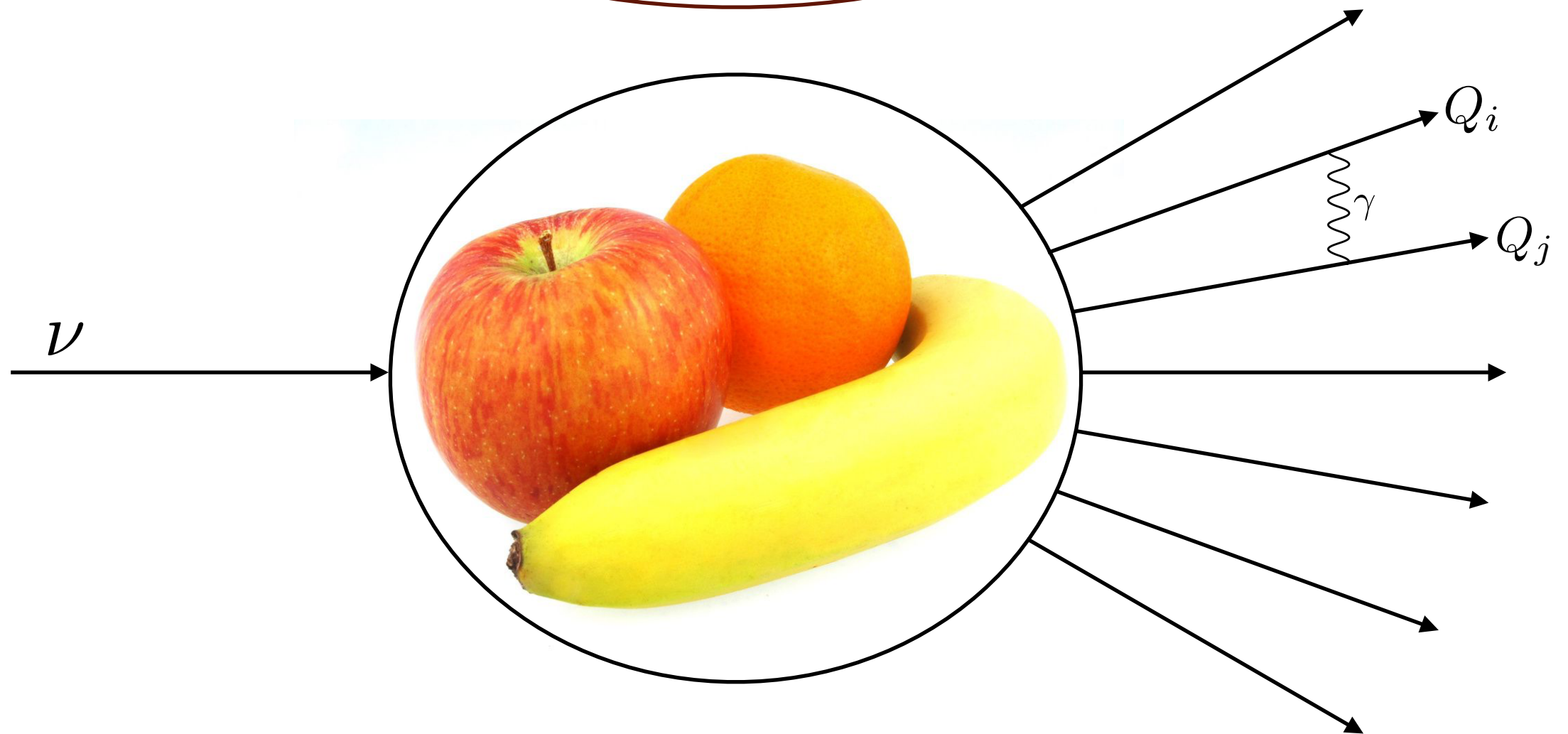
QED corrections



- $\frac{\alpha}{\pi} \sim 0.2 \%$ suppression by electromagnetic coupling constant

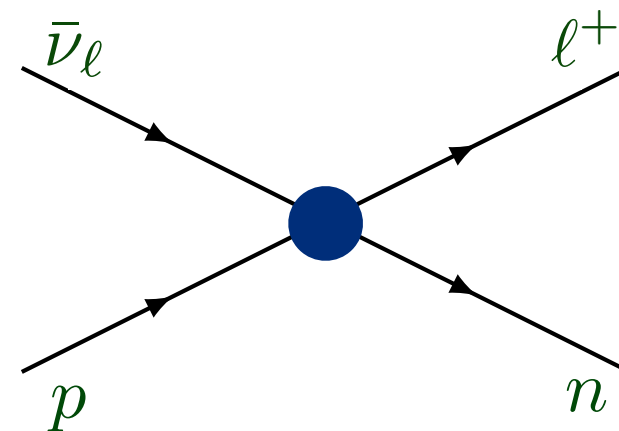
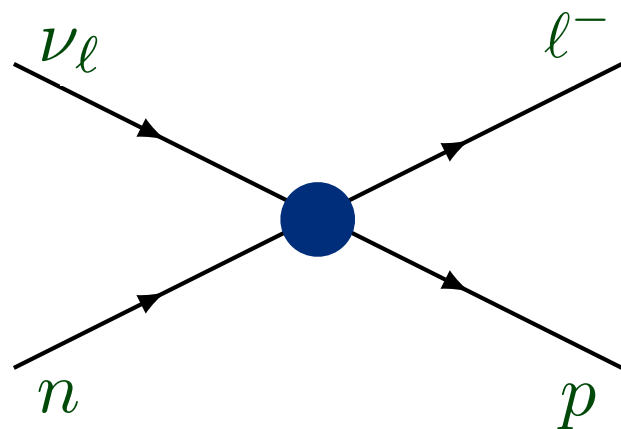
QED corrections

$$m_e \ll m_\mu \ll E_\nu$$



$$\frac{\alpha}{\pi} \sim 0.2 \% \text{ multiplied by } \ln \frac{E_\nu}{m_e} \sim 6 - 10 \text{ or } \ln^2 \frac{E_\nu}{m_e} \sim 36 - 100$$

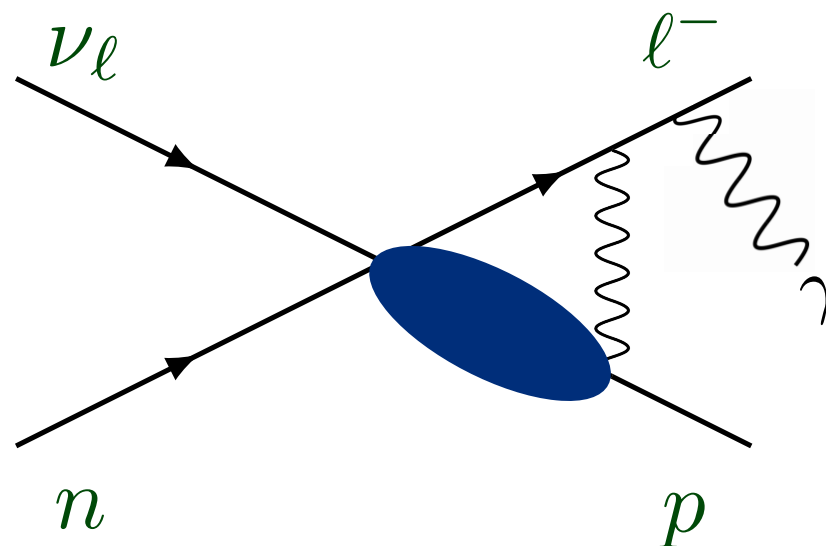
- scale separation introduces large flavor-dependent QED logarithms



O. T., Qing Chen, Richard J. Hill and Kevin S. McFarland, Nature Commun. 13 (2022), 1, 5286

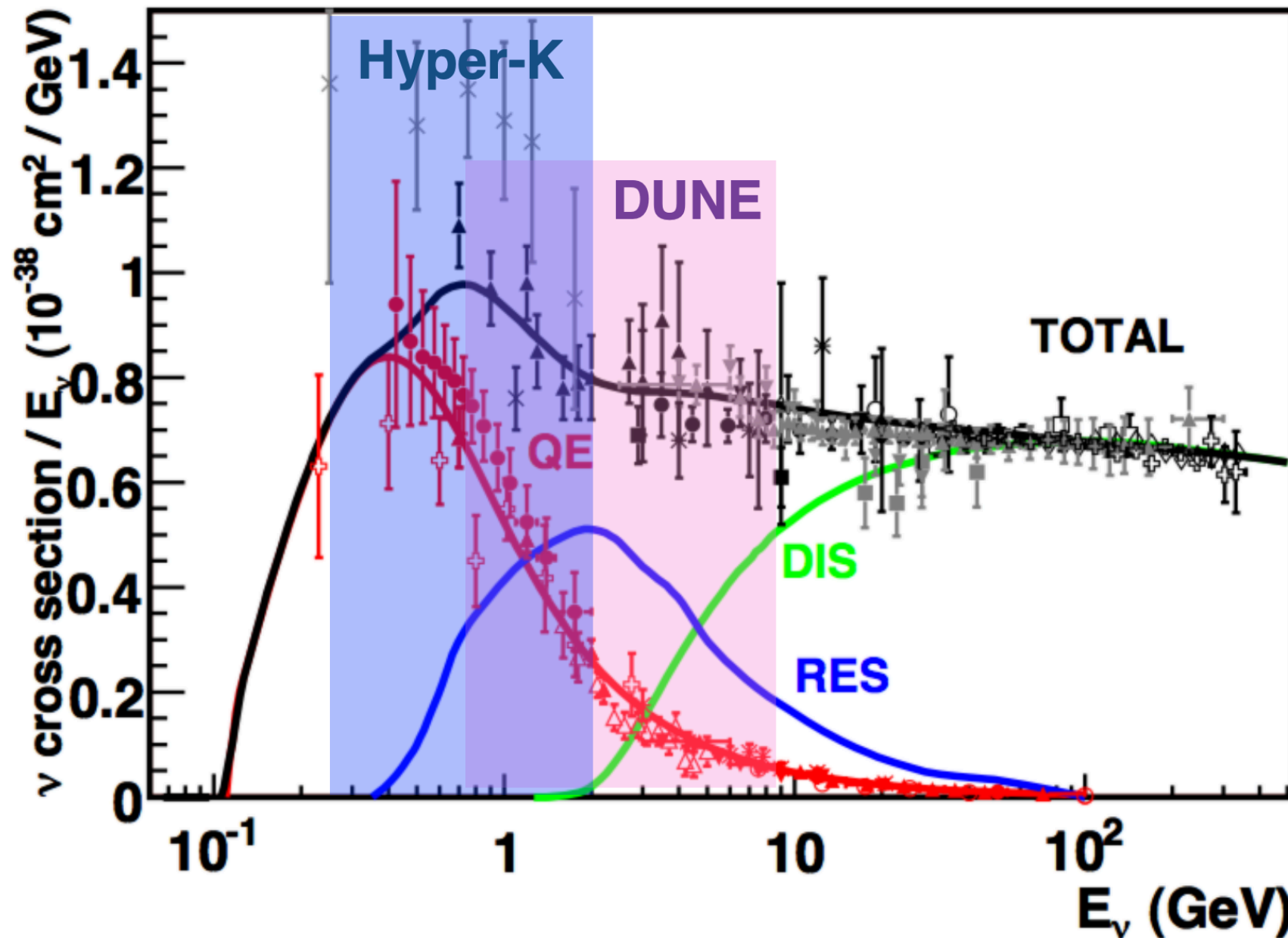
Radiative corrections in charged-current elastic scattering on free nucleons

O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret
editors suggestion in Phys. Rev. D (2022)



CCQE. Why should we care?

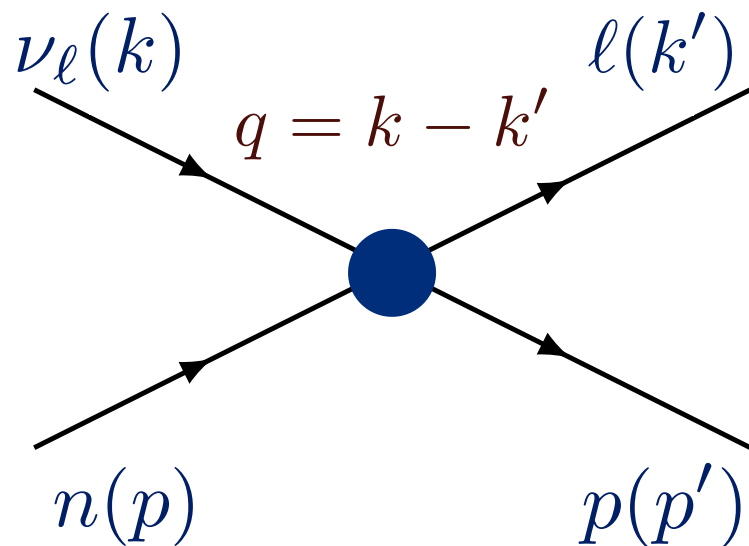
- neutrino-nucleus cross sections and future accelerator-based fluxes



Formaggio
and Zeller
(2013)

- basic process: bulk of events at Hyper-K and DUNE
- channel for reconstruction of neutrino energy

CCQE scattering on free nucleon



neutrino energy

$$E_\nu$$

momentum transfer

$$Q^2 = -q^2$$

contact interaction at GeV energies

- assuming isospin symmetry, nucleon current:

$$\Gamma^\mu(Q^2) = \langle p | \bar{u} (\gamma^\mu - \gamma^\mu \gamma_5) d | n \rangle$$

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D^V(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_P^V(Q^2) + \gamma^\mu \gamma_5 F_A(Q^2) + \frac{q^\mu}{M} \gamma_5 F_P(Q^2)$$

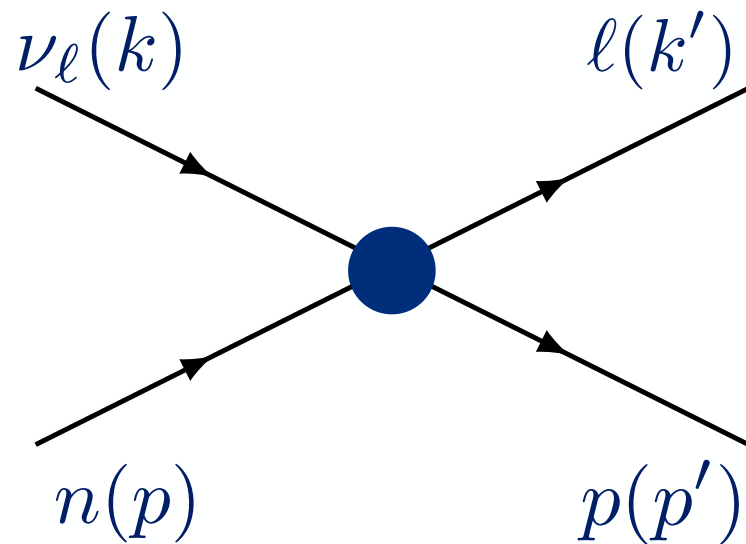
form factors: isovector Dirac and Pauli axial and pseudoscalar

$$F_{D,P}^V = F_{D,P}^p - F_{D,P}^n$$

tree-level amplitude

$$T = \frac{G_F V_{ud}}{\sqrt{2}} (\bar{\ell}(k') \gamma_\mu (1 - \gamma_5) \nu_\ell(k)) (\bar{p}(p') \Gamma^\mu(Q^2) n(p))$$

CCQE scattering on free nucleon



$$\nu = E_\nu / M - \tau - r^2$$

$$\underline{r = \frac{m_\ell}{2M}} \quad \tau = \frac{Q^2}{4M^2}$$

unpolarized cross section

$$\frac{d\sigma}{dQ^2} \sim \frac{M^2}{E_\nu^2} \left((\tau + r^2) A(Q^2) - \nu B(Q^2) + \frac{\nu^2}{1 + \tau} C(Q^2) \right)$$

Llewellyn Smith (1972)

- structure-dependent functions

$$A = \tau (G_M^V)^2 - (G_E^V)^2 + (1 + \tau) F_A^2 - \underbrace{r^2 \left((G_M^V)^2 + F_A^2 - 4\tau F_P^2 + 4F_A F_P \right)}$$

$$B = \pm 4\tau F_A G_M^V$$

$$C = \tau (G_M^V)^2 + (G_E^V)^2 + (1 + \tau) F_A^2$$

- **pseudoscalar** form factor contribution is suppressed by lepton mass
- cross section is sensitive to both **vector** and **axial** contributions

Elastic scattering on free nucleon

- only 3 experiments performed with deuterium bubble chamber
- direct access to form-factor shape

ANL 1982: 1737 events

BNL 1981: 1138 events

FNAL 1983: 362 events

world data: ~3200 events



Fermilab bubble chamber, Richard Drew

- axial form factor extracted based on electromagnetic structure

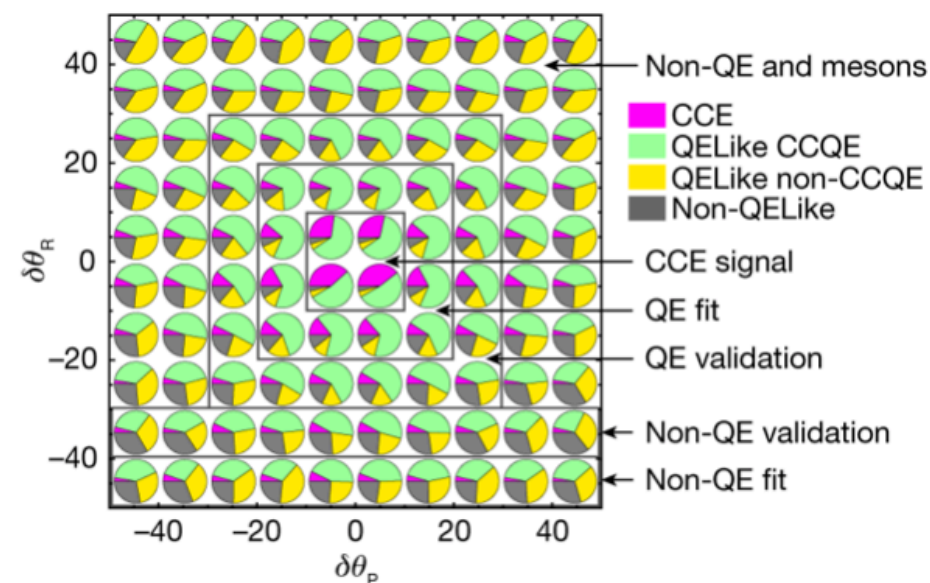
A.S. Meyer, M. Betancourt, R. Gran and R.J. Hill (2016)

MINERvA result with free protons

- idea of scattering on molecular hydrogen realized !!!

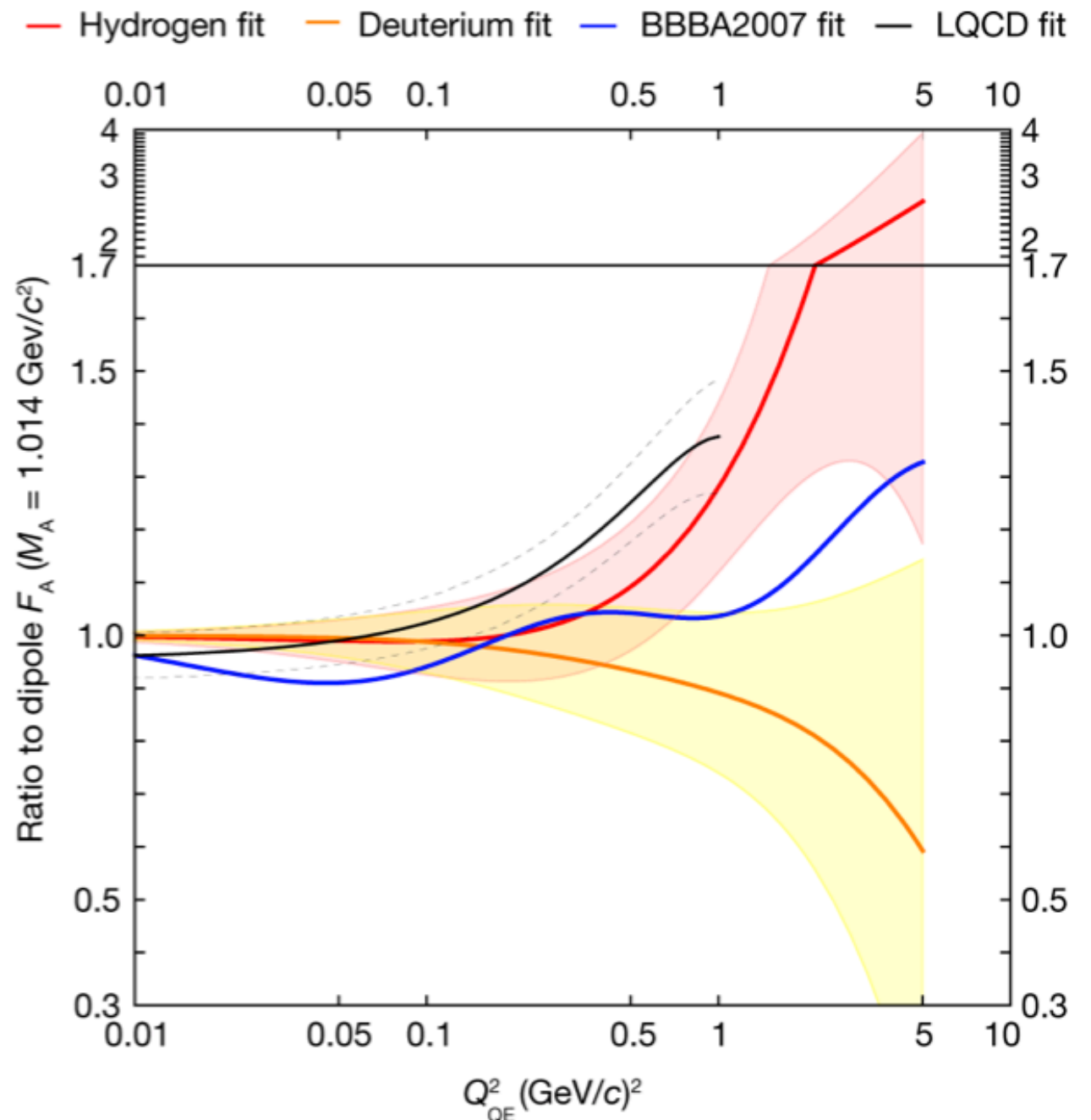
$$\bar{\nu}_\mu p \rightarrow \mu^+ n$$

muon kinematic selection



5580 events over
12500 background

background nuclear events
constrained by scattering of ν

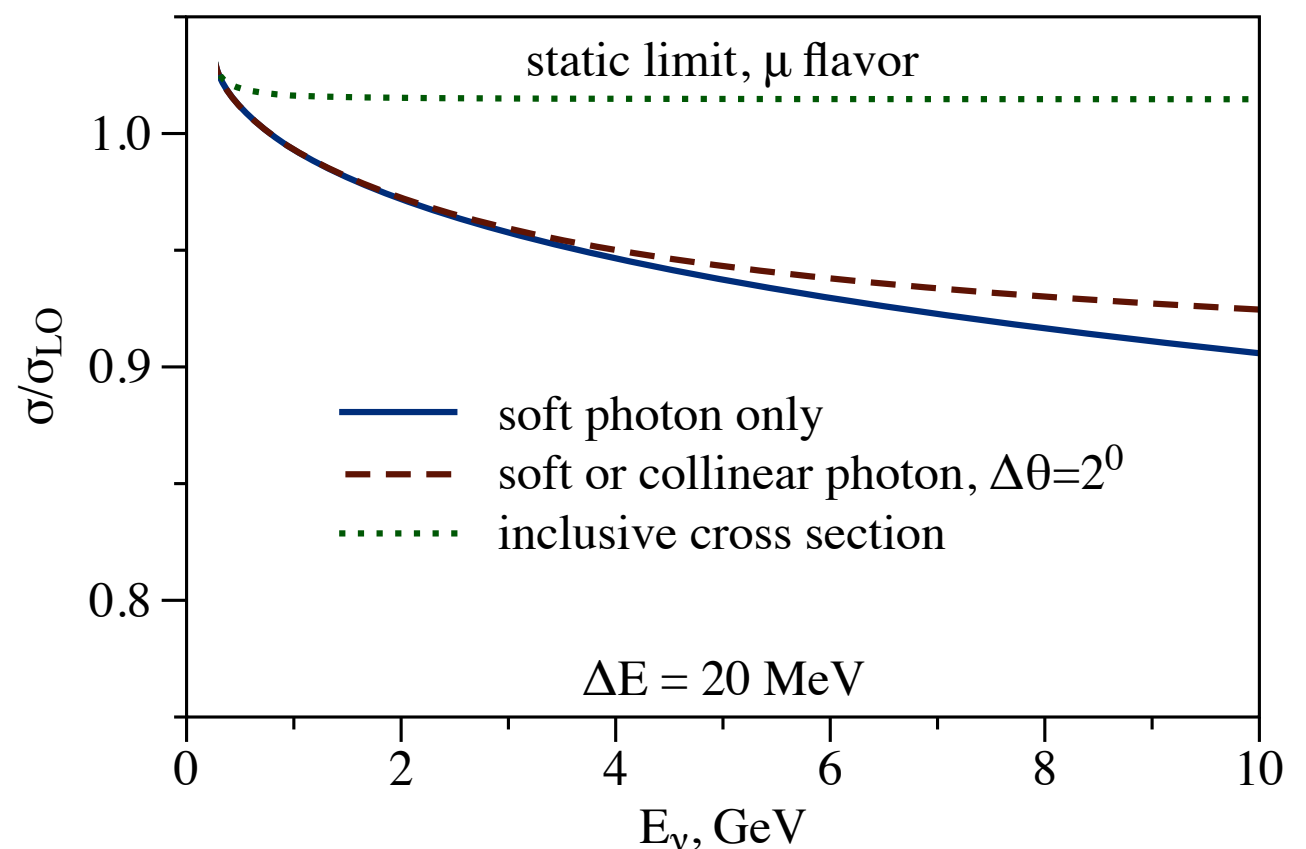
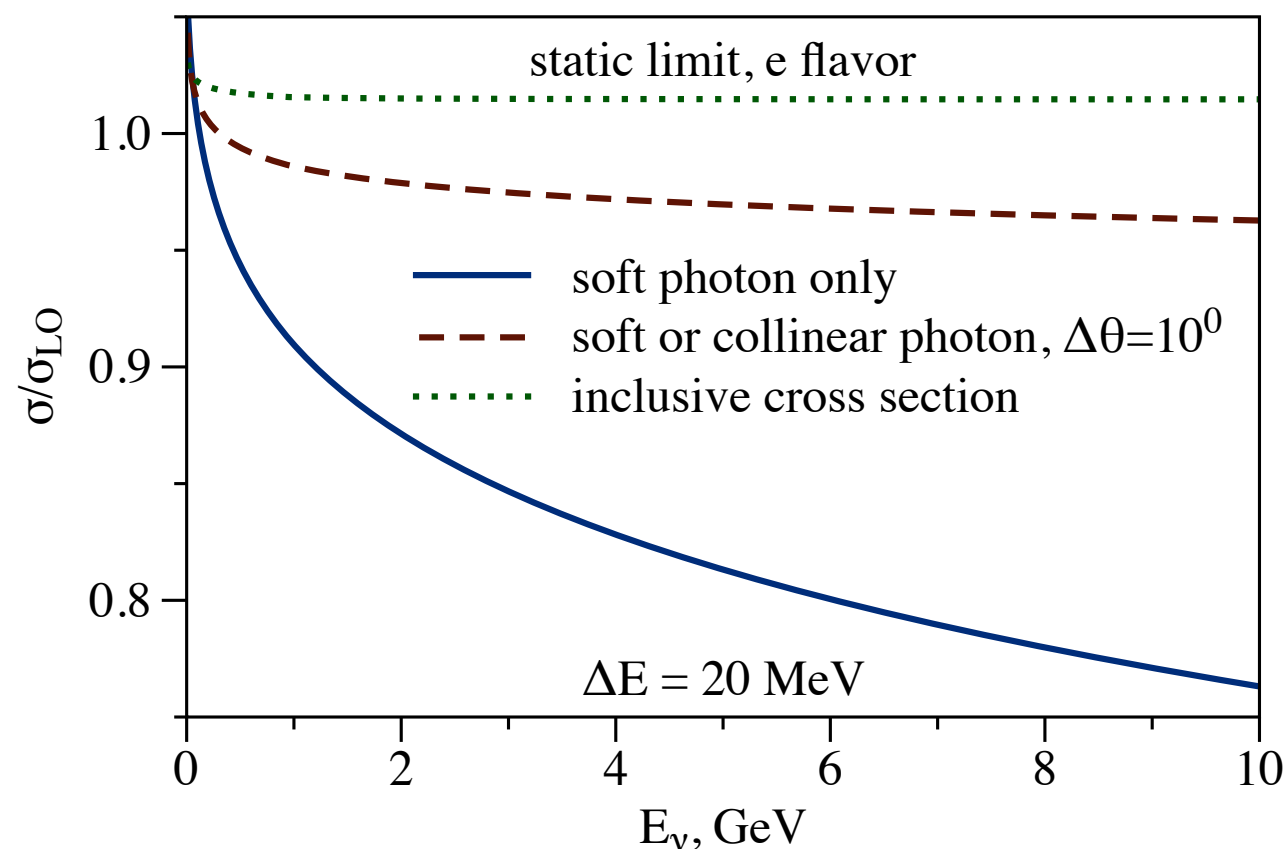


- 1st measurement of axial form factor on “free” protons $\bar{\nu}_\mu p \rightarrow \mu^+ n$

T. Cai et al., MINERvA Collaboration, Nature (2023), 614, 48-53

Static nucleon limit

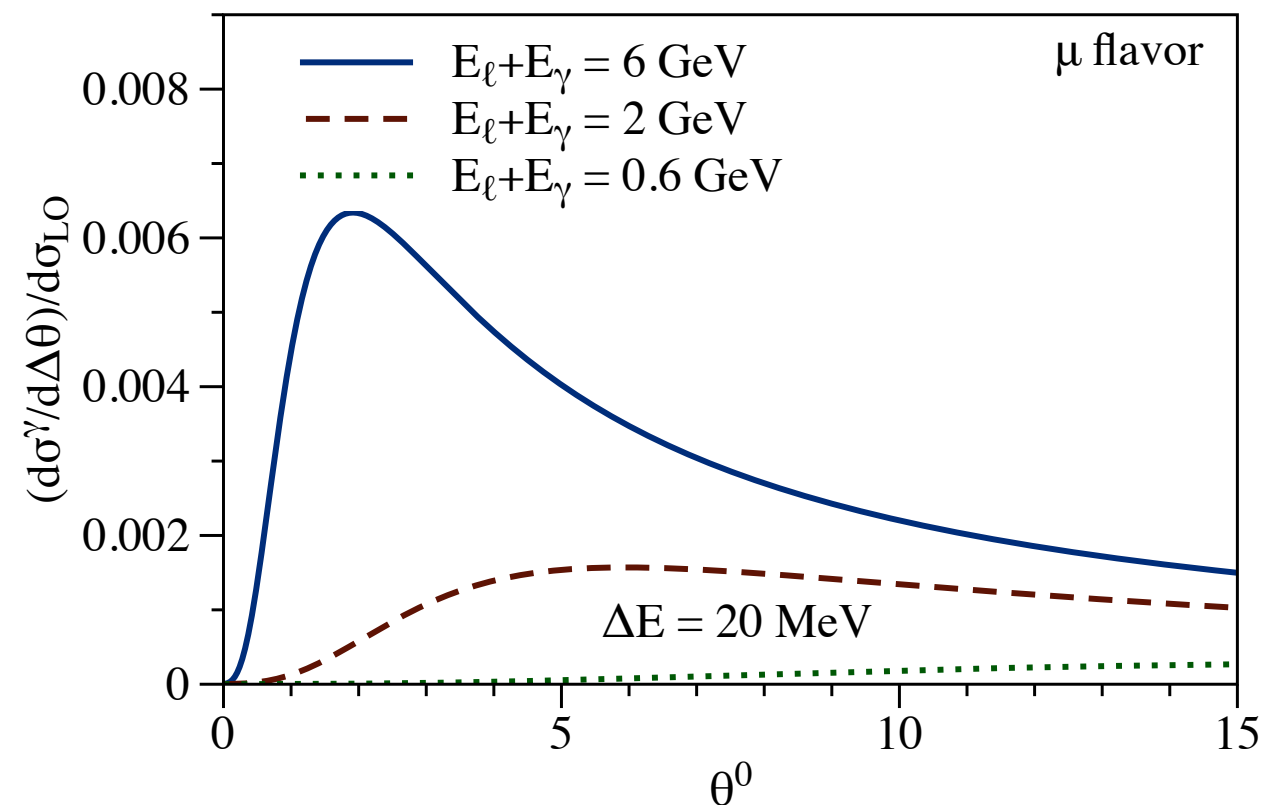
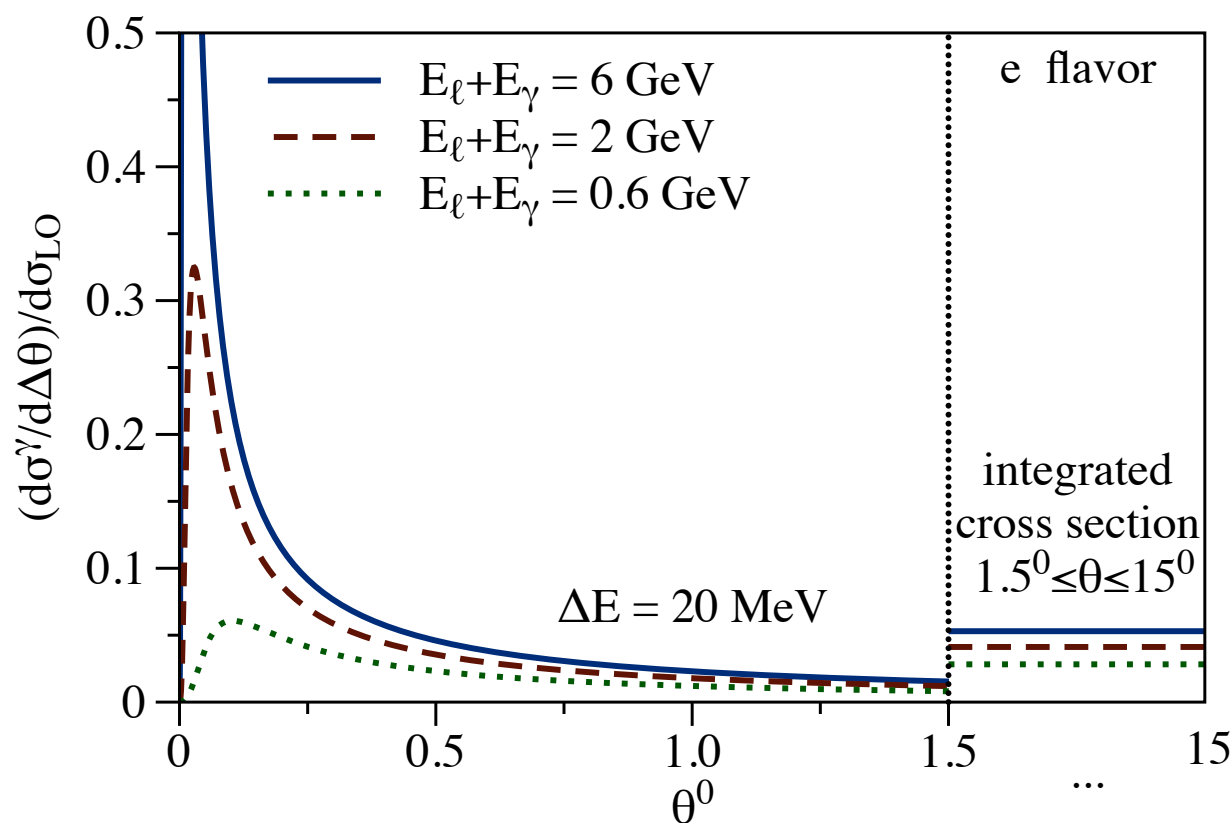
- formal limit of infinitely heavy nucleus $m_\ell \ll E_\ell \ll M$
- provides correct soft and collinear logarithms
- soft-photon energy < 20 MeV, jet size: 10° for electron and 2° for muon



- flavor-dependent effect, same for $\nu_\ell n \rightarrow \ell^- p$ vs $\bar{\nu}_\ell p \rightarrow \ell^+ n$
- collinear observable: cancellation of virtual vs real logs
- inclusive observables ($+\gamma$): few % level, flavor independent

Electron vs muon jets

- factorization for radiation of collinear photons
- cone angle is defined to lepton direction
- photons of energy > 20 MeV, fixed energy in the cone



- flavor-dependent effect, same for $\nu_\ell n \rightarrow \ell^- p$ vs $\bar{\nu}_\ell p \rightarrow \ell^+ n$
- forward-peaked radiation for electron flavor
- negligible radiation for muons with shifted peak position

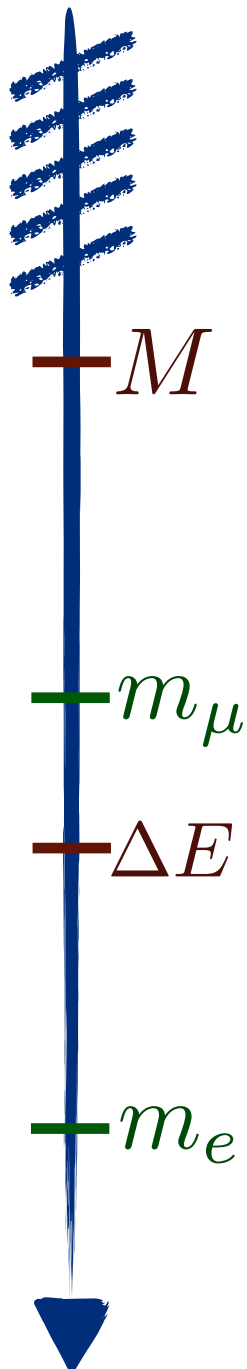
Factorization approach

- cross section is given by **factorization formula**

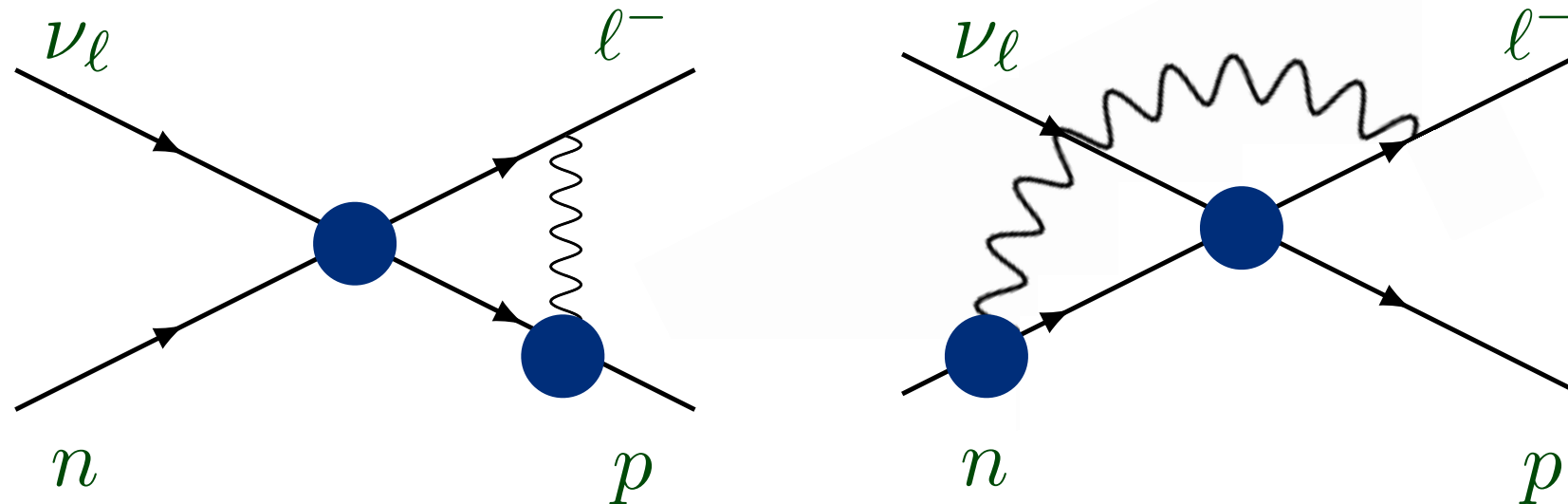
$$d\sigma \sim S \left(\frac{\Delta E}{\mu} \right) J \left(\frac{m_\ell}{\mu} \right) H \left(\frac{M}{\mu} \right)$$

- determine **hard function** at hard scale by matching experiment or hadronic model to the theory with heavy nucleon

- **soft and collinear functions** are evaluated **perturbatively**



Hadronic model at GeV scale



- exchange of photon between the charged lepton and nucleons
- assume **onshell form** for each interaction with dipole form factors
discussed for neutrino-nucleon scattering: Graczyk (2013)
- add **self energy** for charged particles
- reproduce soft and collinear regions of SCET

- best determination of hard function

Factorization approach

- cross section is given by **factorization formula**

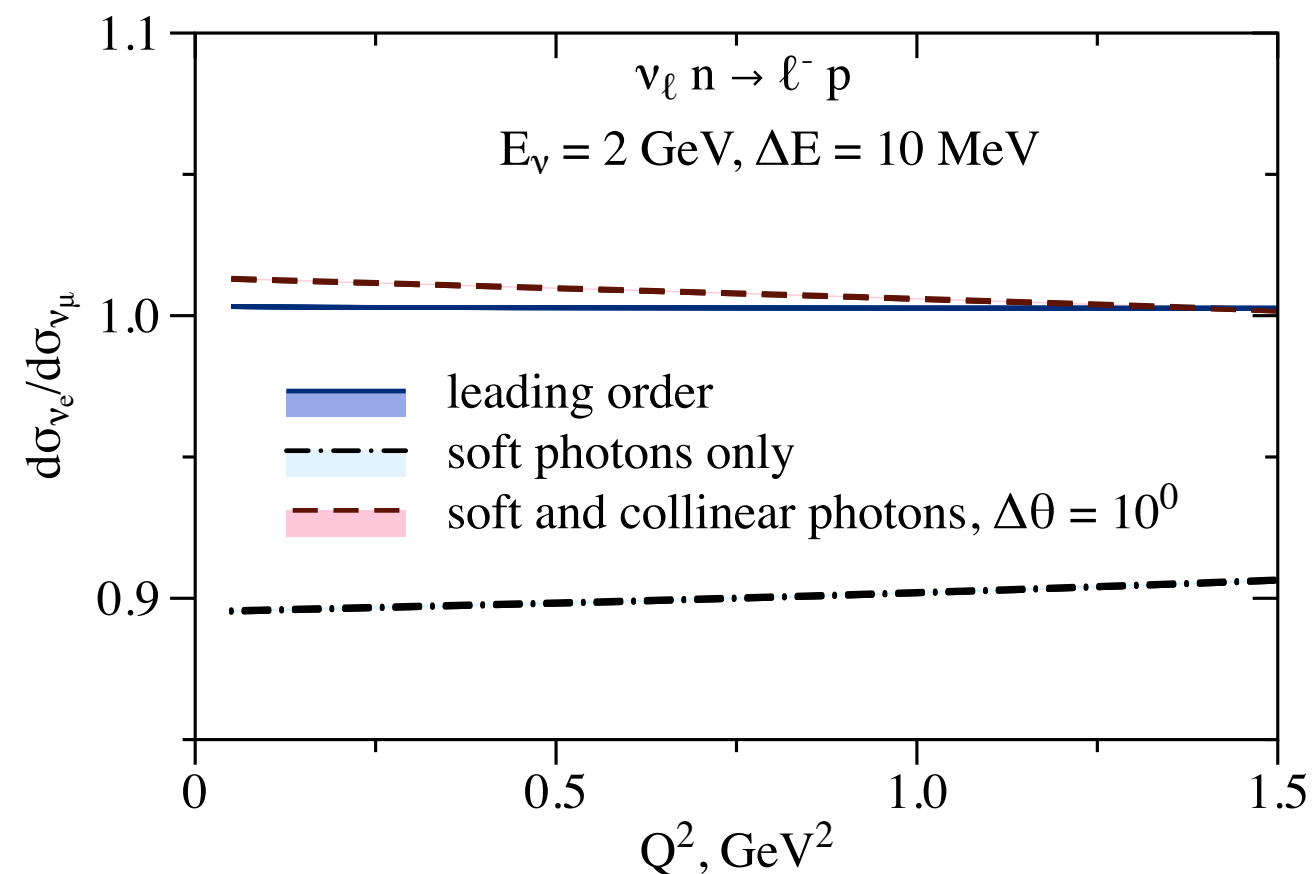
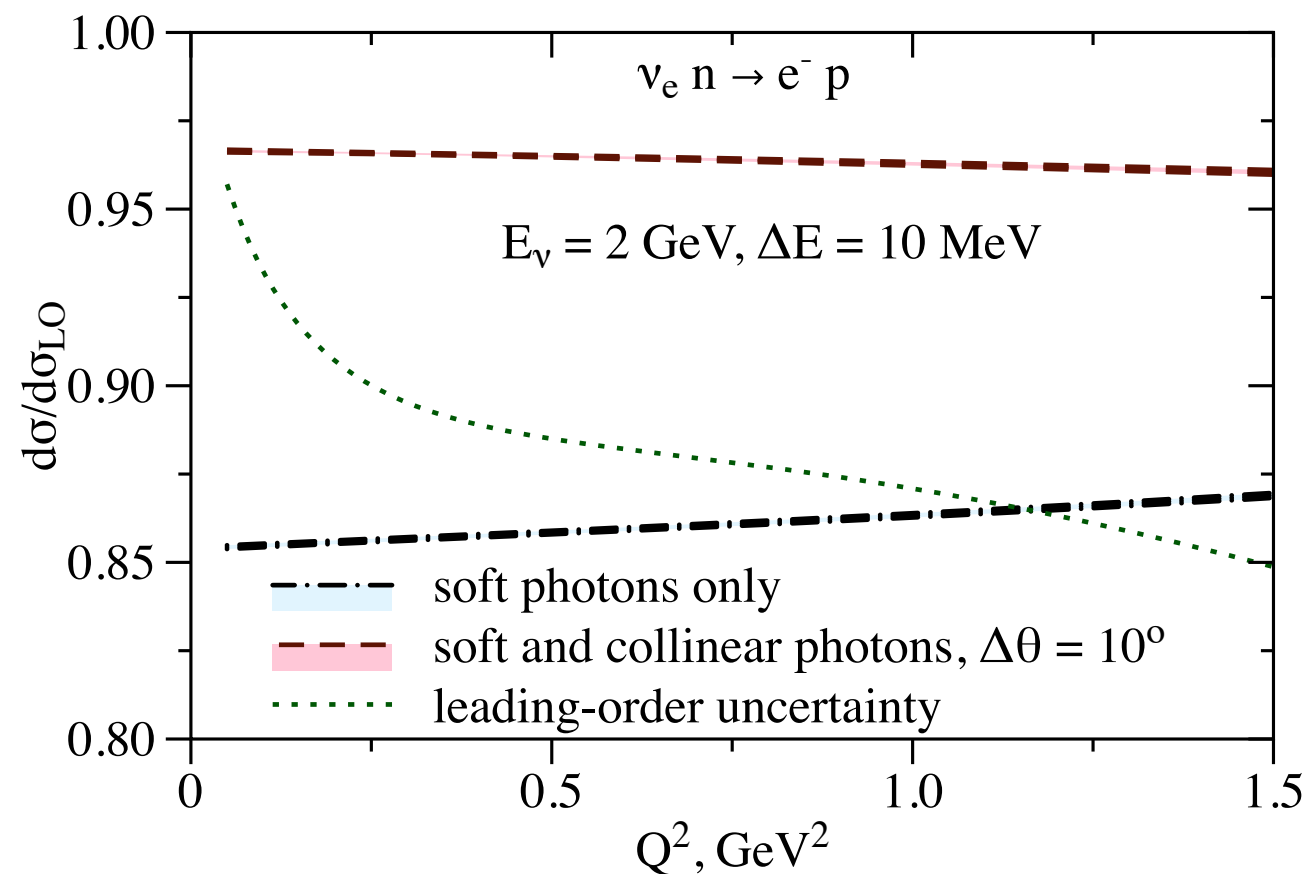
$$d\sigma \sim S \left(\frac{\Delta E}{\mu} \right) J \left(\frac{m_\ell}{\mu} \right) H \left(\frac{M}{\mu} \right)$$

- 
- determine **hard function** at hard scale by matching experiment or hadronic model to the theory with heavy nucleon
 - **RGE evolution** of the hard function to scales $\Delta E, m_\ell$
 - **soft and collinear functions** are evaluated **perturbatively**
 - calculate cross section at low energies accounting for **all large logs**
ep scattering with soft radiation only: Richard J. Hill (2016)

- **soft and collinear functions** determined **analytically**
- **hard function** describes physics at GeV energies

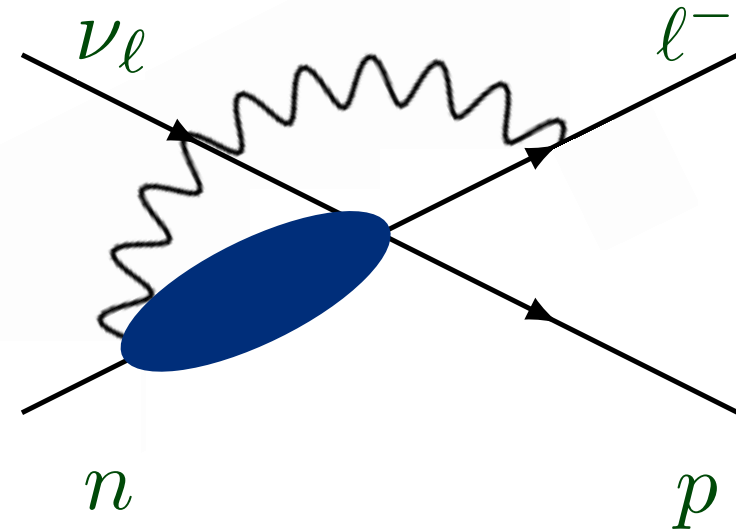
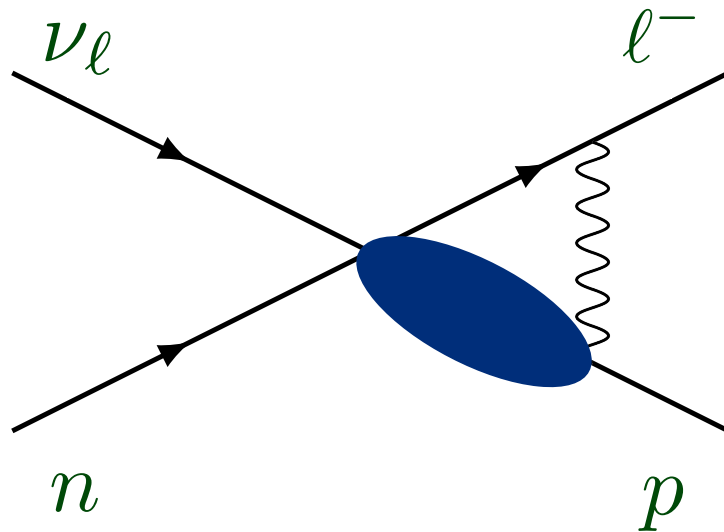
Exclusive observables

- cancellation of uncertainties from hard function for e/μ and ratio to LO

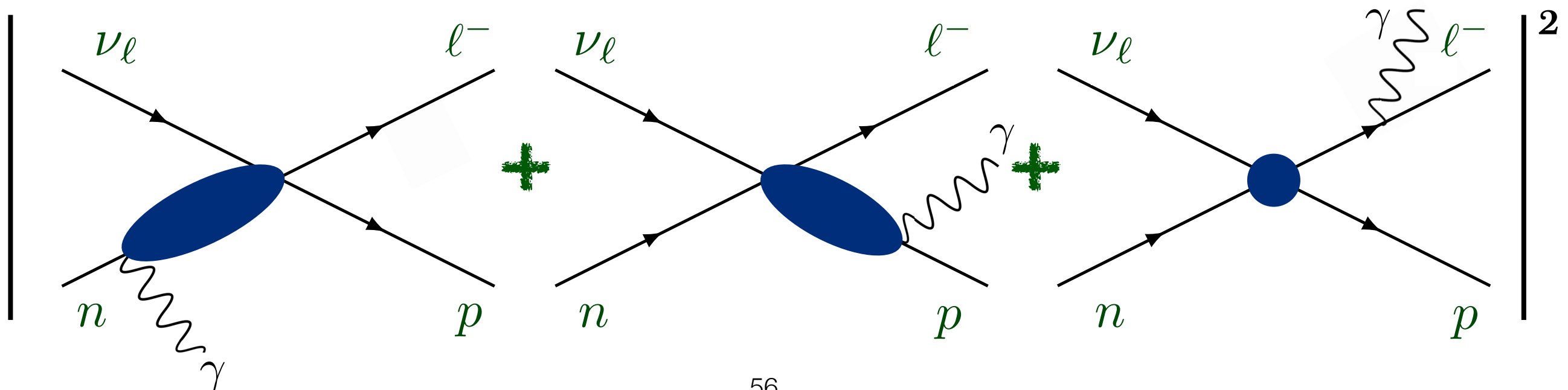


- ratios: cancellation of uncertainty from hard function

Inclusive observables

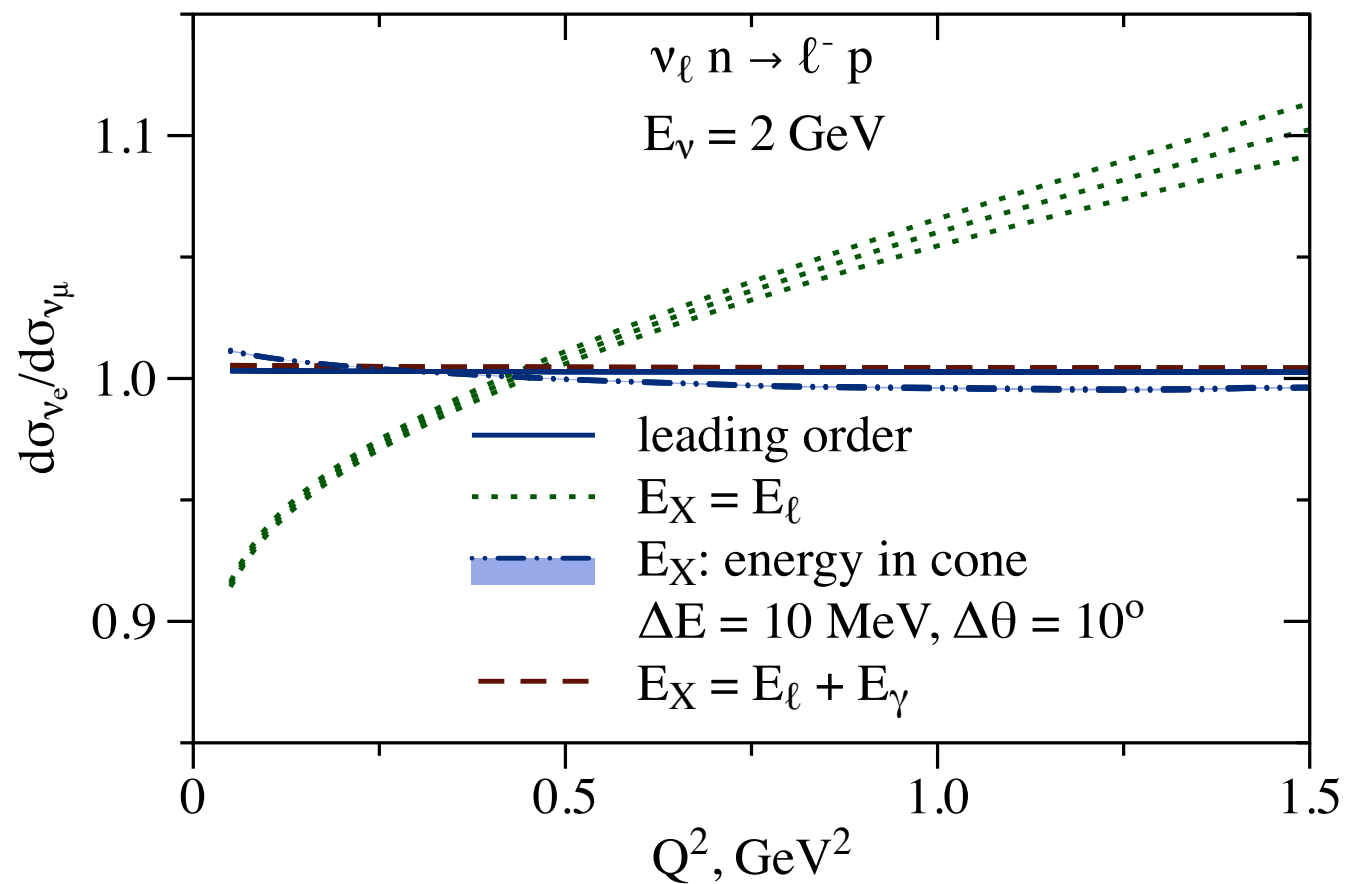
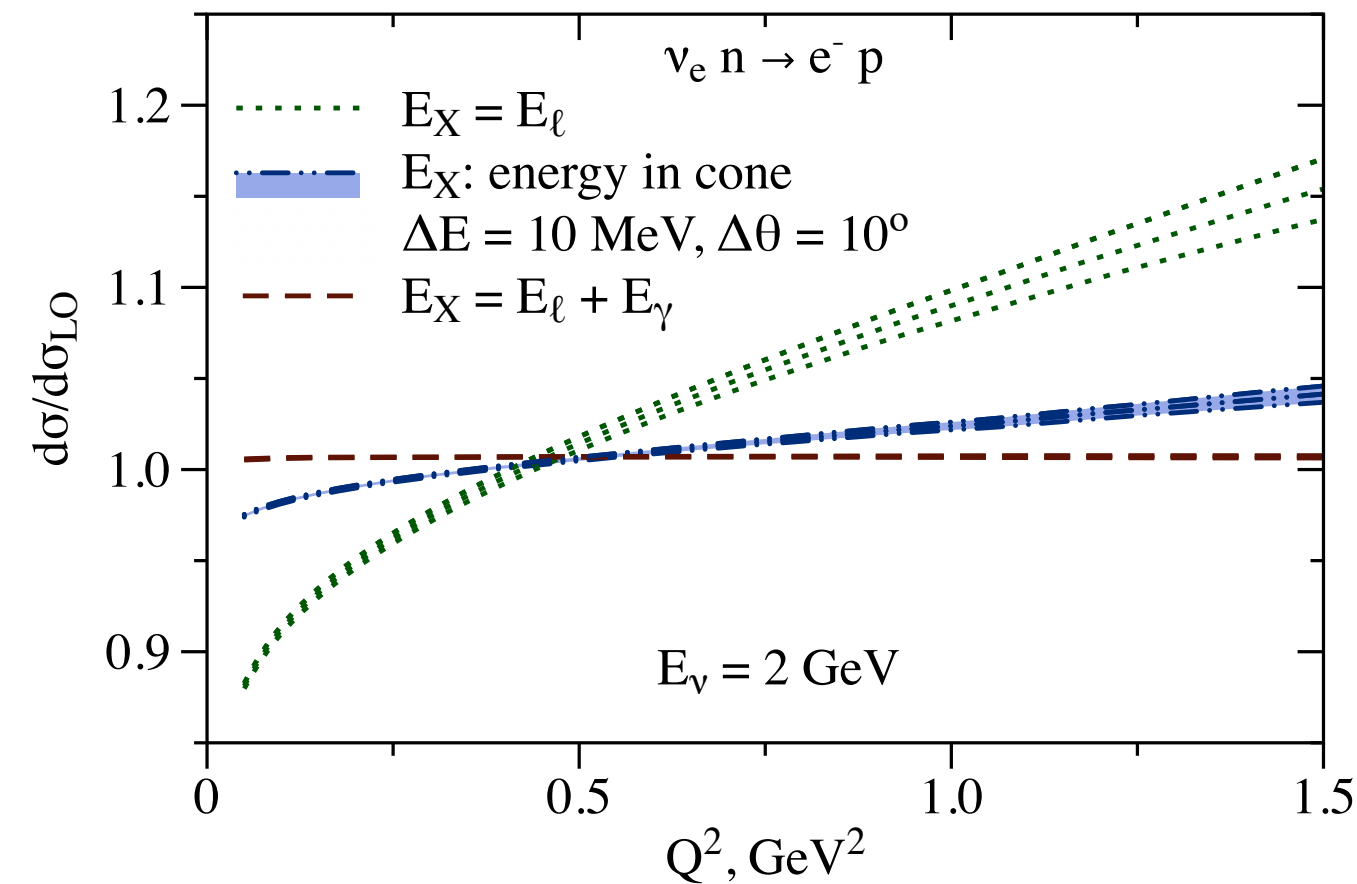


- the same gauge-invariant model for the real radiation
- arbitrary hard photons are part of the observable



Inclusive observables

- kinematics $Q^2 = 2M (E_\nu - E_X)$ is reconstructed with 3 different E_X



- dependence on reconstruction of kinematics and cuts
- predict σ_{ν_e} from σ_{ν_μ} measurements with neutrino beam



Electron/muon ratio

	E_ν , GeV		$\left(\frac{\sigma_e}{\sigma_\mu} - 1\right)_{\text{LO}}, \%$	$\frac{\sigma_e}{\sigma_\mu} - 1, \%$
T2K/HyperK	0.6	ν	2.47 ± 0.06	$2.84 \pm 0.06 \pm 0.37$
		$\bar{\nu}$	2.04 ± 0.08	$1.84 \pm 0.08 \pm 0.20$
NOvA/DUNE	2.0	ν	0.322 ± 0.006	$0.54 \pm 0.01 \pm 0.22$
		$\bar{\nu}$	0.394 ± 0.003	$0.20 \pm 0.01 \pm 0.19$

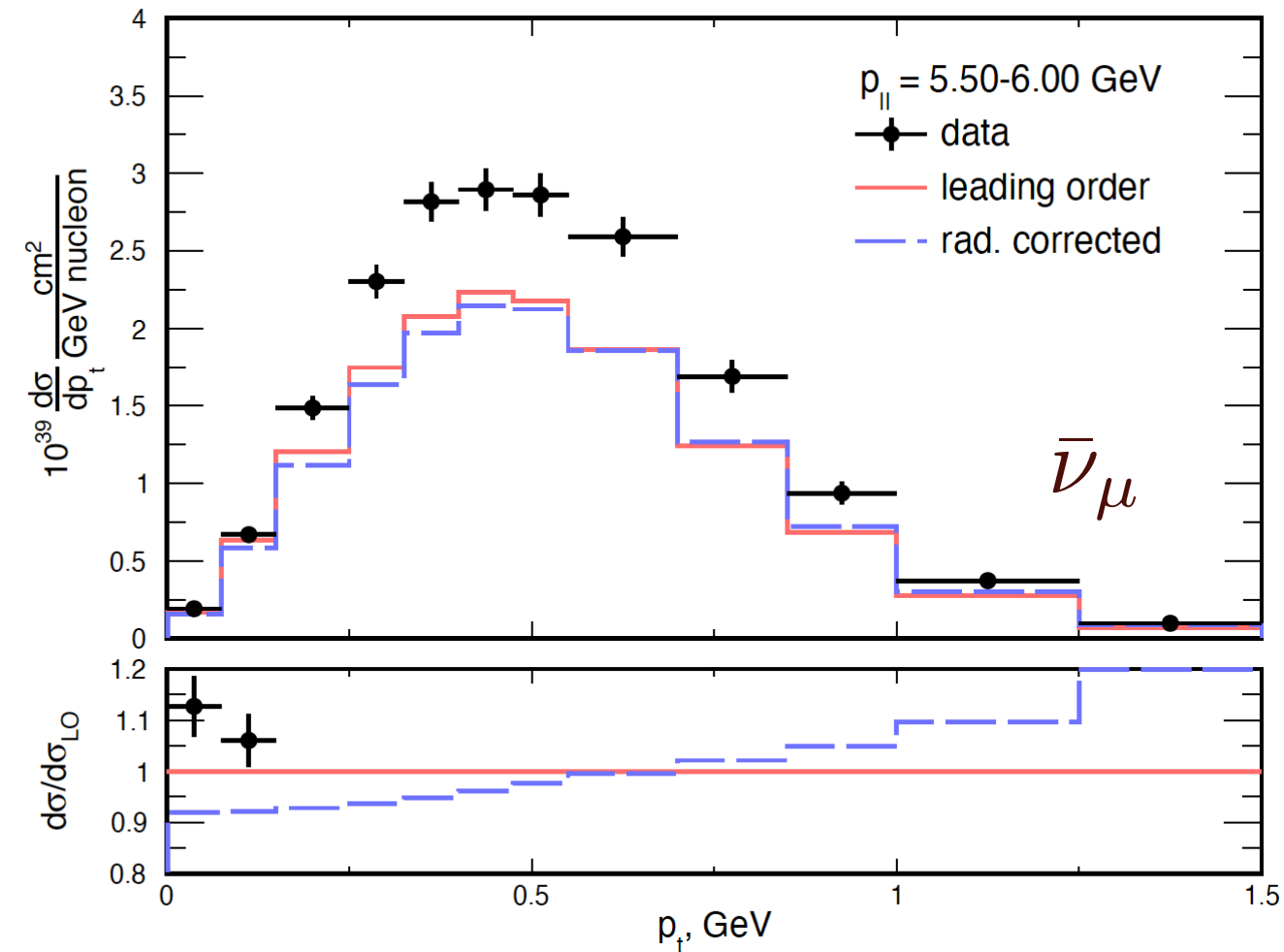
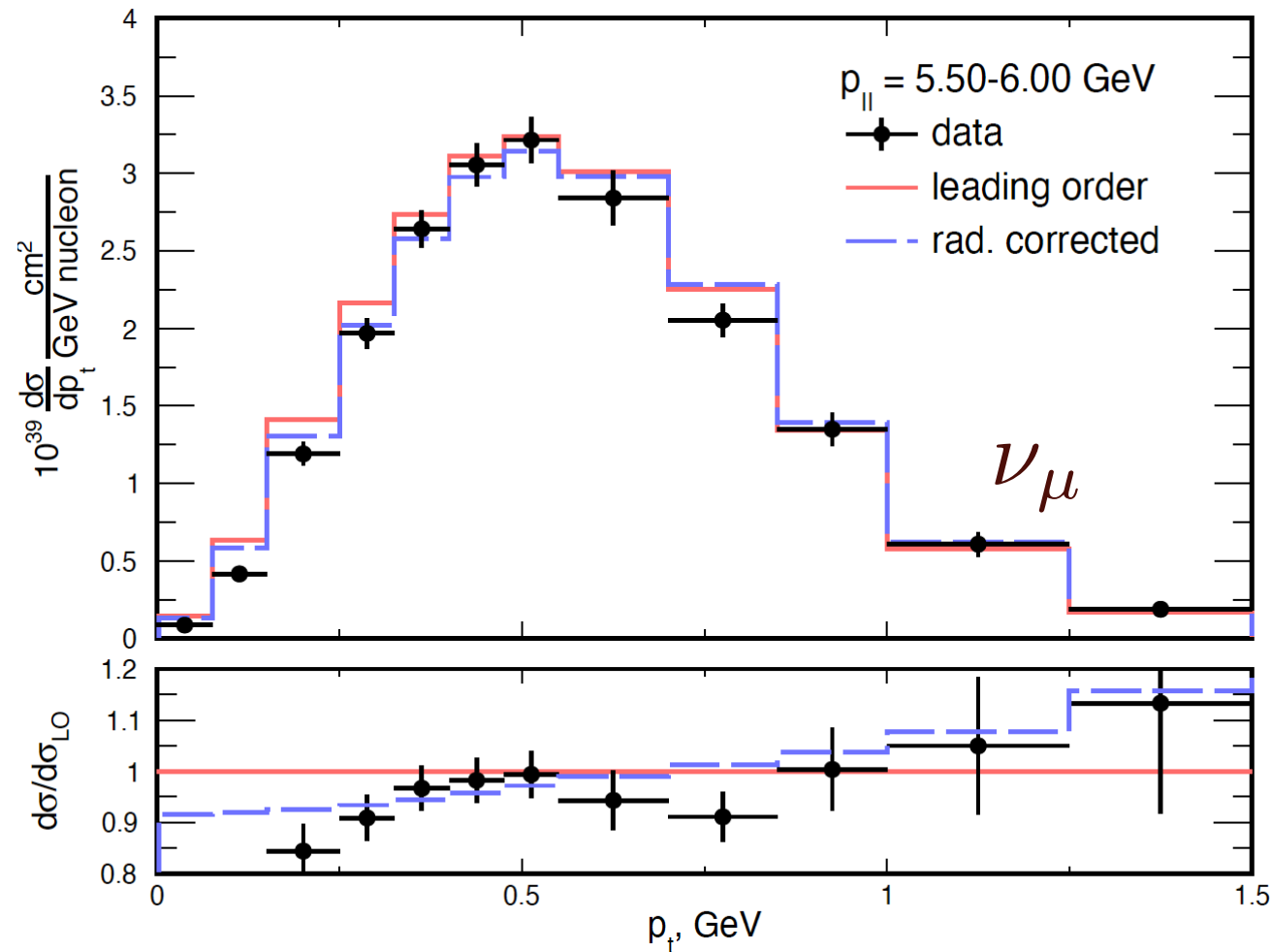
TABLE II: Inclusive electron-to-muon cross-section ratios for neutrinos and antineutrinos without kinematic cuts. Uncertainties at leading order are from vector and axial nucleon form factors. For the final result, we include an additional hadronic uncertainty from the one-loop correction to the first uncertainty, and provide a second uncertainty as the magnitude of the radiative correction.

$$\frac{\sigma(m_\ell \rightarrow 0)}{\sigma(m_\ell = 0)} \approx 1 + Am_\ell^2 + \alpha Bm_\ell^2 \ln m_\ell$$

- inclusive cross sections and flavor ratios determined by KLN
- nuclear effects: suppressed by expansion parameters squared

Comparison to data

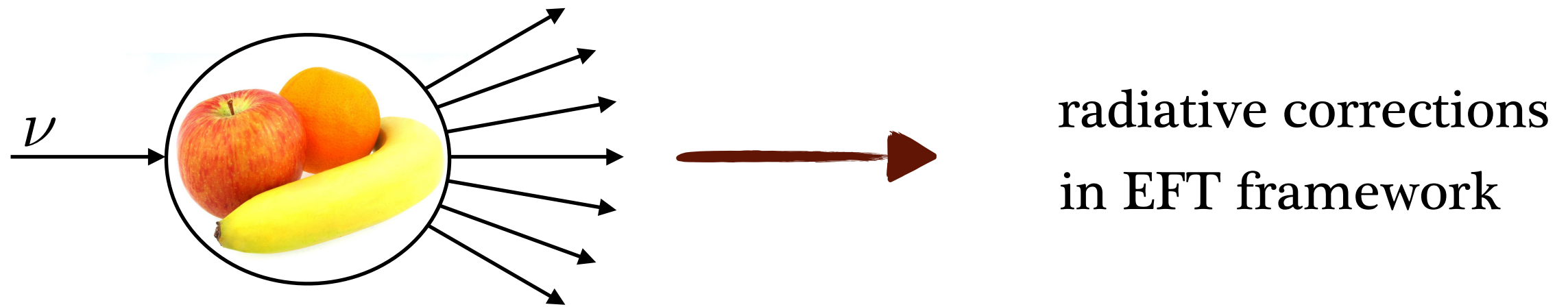
- medium-energy flux data from MINERvA@FERMILAB



O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret
editors suggestion, Phys. Rev. D 106, 093006 (2022)

- electron flavor: measurements are uncertain
- muon flavor: comparable to experimental precision

Conclusions



- radiative corrections to neutrino-nucleon cross sections formulated in factorization framework
- charged-current elastic electron vs muon cross-section ratios evaluated from theory with sub-percent uncertainty
- ongoing work on applications

Thanks for your attention !!!