Stochastic Scheduling of Generating Units with Weekly Energy Storage: a Hybrid Decomposition Approach

Gonzalo E. Constante-Flores[a,*], Antonio J. Conejo[a,b], Ricardo M. Lima[c]

[a] Department of Electrical and Computer Engineering, The Ohio State University, 2015 Neil Avenue, Columbus, OH 43210, United States
[b] Department of Integrated Systems Engineering, The Ohio State University, 1971 Neil Avenue, Columbus, OH 43210, United States
[c] Computer, Electrical and Mathematical Sciences & Engineering Division at King Abdullah University, Thuwal 23955-6900, Saudi Arabia

Abstract

We propose a solution method for the large-scale stochastic unit commitment (SUC) problem with weekly-dispatched energy storage and significant weather-dependent stochastic generating capacity. Weekly storage facilities that mostly charge during weekends and discharge during weekdays require a weekly scheduling of generating units, which result in a large-scale optimization problem. This SUC problem is formulated as a two-stage stochastic model and we use the conditional value-at-risk as a risk measure. Using a Benders framework, the proposed solution method decomposes the problem into a mixed-integer linear master problem and linear and continuous subproblems. The master problem corresponds to the first-stage decisions throughout the week and includes all the commitment (binary) variables and their corresponding constraints. The subproblems correspond to the actual dispatch of the generating units on a weekly basis. Based on the success of column-and-constraint generation algorithms to solve robust optimization problems, we improve the low communication between the master problem and the subproblems in the standard Benders decomposition by adding primal variables and constraints from the subproblems to the master problem, which provides a better approximation of the recourse function. Our computational experiments demonstrate the effectiveness of the proposed decomposition method using an instance of the South Carolina synthetic system with 90 generating units under 40 scenarios.

Keywords: Benders decomposition, column-and-constraint generation algorithm, energy storage, risk control, two-stage stochastic programming, unit commitment.

1. Introduction

One of the key problems in the operation of power systems is the Unit Commitment (UC), also known as generation scheduling, which consists of determining the hourly start-up and shut-down schedule of all power plants for a given planning horizon (one week in this paper). Such schedule aims at supplying the electric demand while being economically efficient (i.e., minimizes the cost of scheduling and operating power plants) and satisfying operational constraints. Since starting up thermal units is costly and complex, the UC problem remains challenging in large-scale power systems. The growing share of weather-dependent generation into the generation mix provides important benefits to reduce the carbon footprint of electricity generation. Such generation has, however, raised important challenges to the unit commitment problem mainly due to its uncertainty. Deterministic models of weather-dependent generation may result in economically inefficient and unreliable commitment of generating units [1, 2, 3]. Hence, more accurate models of uncertain generation accounting for its stochastic nature need to be incorporated in the unit commitment formulation.

Numerous formulations of the unit commitment problem have been proposed in the literature to consider generation and demand uncertainties. The most common approach to deal with uncertainty in the model is based on two-stage stochastic programming, which explicitly considers the underlying probability distribution of the uncertainties using a discrete set of scenarios with associated probabilities [3, 4, 5]. In the first stage of the stochastic unit commitment, the commitment of generating units is modeled using binary decision variables. The second stage, which only contains continuous variables, models the actual generation dispatch, the curtailment of weather-dependent generation, and unserved energy for each scenario. For realistic power systems, the stochastic unit commitment problem is a large-scale two-stage mixed-integer linear optimization problem with continuous recourse, whose extensive formulation is in general unsolvable by state-of-the-art solvers within expected solution times even for a small number of scenarios. The main challenges of the two-stage stochastic unit commitment problem pertain to the binary nature of the commitment decision variables and

*Corresponding author
Email address: constanteflores.1@osu.edu (Gonzalo E. Constante-Flores)

Nomenclature

Sets

T	Time periods, $T = \{1, \ldots, \lvert T \rvert\}$.
T^{fill}	Time period(s) when the energy reservoirs of storage units have to be filled.
Λ^{g}	Combined cycle gas turbines.
Λ^{c}	Coal-fired power plants.
Λ^{ng}	Gas turbines.
Λ^{T}	Thermal power plants, $\Lambda^{\text{T}} = \Lambda_n^{\text{c}} \cup \Lambda_n^{\text{ng}} \cup \Lambda_n^{\text{g}}$.
Λ^{n}	Nuclear power plants.
Λ^{w}	Weather-dependent power plants.
Λ^{d}	Demands.
Λ^{s}	Storage units.
Ω	Scenarios indexed by ω.
$\Omega^{\text{M},(\nu)}$	Scenarios included in the master problem at iteration ν.
$\mathcal{I}_{\omega}^{(\nu)}(t)$	Active ramping constraints at iteration ν under scenario ω and at time period t.

Constants

Scenario independent

c_j	Linear cost coefficient of unit j (\$/MW).
$c_j^{\text{D}}/c_j^{\text{U}}$	Shutdown/Startup cost coefficient of unit j (\$/h).
$c_i^{\text{LOL}}(t)$	Value of unserved demand i in period t (\$/MW).
c_j^{NL}	No-load cost coefficient of unit j (\$/h).
$c_j^{\text{RU}}(t)/c_j^{\text{RD}}(t)$	Marginal cost of deploying up/down-reserves by unit j in period t (\$/MW).
$c_j^{\text{U}}(t)/c_j^{\text{D}}(t)$	Startup/shutdown cost of unit j in period t (h).
$D_i(t)$	Active power load of demand i in period t (MW).
$\overline{E}_{\ell}(t)/\underline{E}_{\ell}(t)$	Maximum/Minimum energy content of storage unit ℓ at the end of hour t (MWh).
E_{ℓ}^{ini}	Energy stored in storage unit ℓ at the beginning of the time period (MWh).
F_j	Equal to $[T_j^{\text{D}} - H_j^{\text{D}}][1 - v^{\text{ini}}]$ unless it exceeds the time horizon, in which case F_j equals $\lvert T \rvert$, $F_j = \min(\lvert T \rvert, [T_j^{\text{D}} - H_j^{\text{D}}][1 - v_j^{\text{ini}}])$
$H_j^{\text{D}}/H_j^{\text{U}}$	Number of hours that unit j is required to be off/on at the beginning of the time horizon (h).
L_j	Equal to $[T_j^{\text{U}} - H_j^{\text{U}}]v^{\text{ini}}$ unless it exceeds the time horizon, in which case L_j equals $\lvert T \rvert$, $L_j = \min(\lvert T \rvert, [T_j^{\text{U}} - H_j^{\text{U}}]v_j^{\text{ini}})$.
p_j^{ini}	Active power output of unit j at the beginning of the time horizon (MW).
$p_j^{\text{N}}(t)$	Active power output of nuclear unit j in period t (MW).
$\overline{P}_j/\underline{P}_j$	Maximum/minimum active power output of unit j (MW).
$R_j^{\text{D}}/R_j^{\text{U}}$	Maximum ramp-down/up rate of unit j (MW/h).
$S_j^{\text{U}}/S_j^{\text{D}}$	Maximum startup/shutdown ramp rate of unit j (MW/h).
$T_j^{\text{D}}/T_j^{\text{U}}$	Minimum number of time periods required for unit j to be off/on before it can be turned on/off (h).
v_j^{ini}	Initial commitment status of unit j (1 if online, and 0 otherwise).
$\vartheta_{\ell}^{\text{c}}/\vartheta_{\ell}^{\text{d}}$	Efficiency of storage unit ℓ in charging/discharging mode.
$\overline{\Delta}_{\ell}^{\text{ch}}(t)/\underline{\Delta}_{\ell}^{\text{ch}}(t)$	Maximum/Minimum power demand change of storage unit ℓ at the end of hour t (MW).
$\overline{\Delta}_{\ell}^{\text{dis}}(t)/\underline{\Delta}_{\ell}^{\text{dis}}(t)$	Maximum/Minimum power output change of storage unit ℓ at the end of hour t (MW).
β	CVaR weighting factor.
α	CVaR confidence level.

Scenario dependent

$D_{i\omega}(t)$	Power load of demand i at period t under scenario ω (MW).
$p_{j\omega}^{\text{W}}(t)$	Power production realization of weather-dependent unit j at time period t under scenario ω (MW).
$u_{j\omega}(t)$	Binary constant indicating whether or not unit j is out of service in period t and scenario ω.
π_{ω}	Probability of occurrence of scenario ω.

Variables

First-stage

$E_{\ell}(t)$	Energy content of storage unit ℓ at the end of time period t (MWh).
$p_{\ell}^{\text{ch}}(t)/p_{\ell}^{\text{dis}}(t)$	Demand/Output of the storage unit ℓ (charging/discharging mode) in hour t (MW).
$v_j(t)$	Commitment status at time t of unit j: $v_j(t)$ equals 1 if unit j is on at time period t, and 0 if it is off.
$y_j(t)$	Startup status at time t of unit j: $y_j(t)$ equals 1 if unit j starts up at the beginning of time t, and 0 otherwise.
$z_j(t)$	Shutdown status at time t of unit j: $z_j(t)$ equals 1 if unit j shuts down at the beginning of time t, and 0 otherwise.
ζ	Value at risk.

Second-stage

$D_{i\omega}^{\text{sh}}(t)$	Power shed of demand i in time period t under scenario ω (MW).
$E_{\ell\omega}(t)$	Energy content of storage unit ℓ at the end of time period t under scenario ω (MWh).
$p_{j\omega}(t)$	Power produced by unit j in time period t under scenario ω (MW).
$p_{j\omega}^{\text{W,spill}}(t)$	Power spillage of weather-dependent unit j in time period t under scenario ω (MW).
$\Delta_{\ell\omega}^{\text{ch}}(t)/\Delta_{\ell\omega}^{\text{dis}}(t)$	Demand/Output change of storage unit ℓ at the end of hour t under scenario ω (MW).
η_{ω}	CVaR auxiliary variable.
ψ_{ω}	Under-approximation of the second-stage cost of scenario ω.

the problem size. The number of continuous and binary decision variables and constraints of the unit commitment problem grows with the number of generating units, time periods, and uncertainty scenarios [3, 6].

One alternative to solve the stochastic UC is using decomposition techniques. In particular, Benders decomposition [7], which exploits the L-shaped structure of the problem, breaks the problem into a master problem and many subproblems [8]. In the master problem, which is mixed-integer linear, the commitment (binary) decision variables and their constraints (minimum up/down time and commitment logic) are represented in detail whereas the second-stage costs are piece-wise linearly approximated. Once the first-stage variables are fixed, the subproblems, which correspond to the unrelated second-stage problems, can be solved independently and provide dual information (cuts) to the master problem to refine the approximations of the second-stage costs. In the basic form of Benders decomposition, a single cut is added to the master problem whereas in more advanced versions one cut for each subproblem is added. Adding multiple cuts per scenario generally provides a more accurate representation of the recourse function, at the cost of solving a larger master problem. In recent years, there has been significant attention to developing bundle methods that offer a trade-off between the multi-cut and single cut approaches [9, 10, 11, 12, 13].

In the context of the UC problem, some works have proposed strategies to accelerate the convergence of Benders decomposition. Wu and Shahidehpour [14] propose a method to generate multiple strong Benders cuts via multiple optimal dual solutions of the subproblems. Wang et. al. [15] propose adding second-stage constraints into the master problem in the first iteration and keeping them during the course of evolution of the algorithm. However, the authors do not provide a systematic way to include them. Zhang et. al. [16] propose adding an artificial scenario corresponding to the average scenario in the master problem. The authors use Jensen's inequality to show that adding such scenario does not cut any feasible solution, hence, the master problem still provides a valid lower bound.

Although there is a rich body of literature on solving the day-ahead stochastic UC using Benders decomposition (see [7] and references therein), there is a limited number of works addressing its week-ahead counterpart, which is required if weekly storage is present. The week-ahead unit commitment is relevant in power systems with important hydro resources whose water reservoirs are dispatched considering medium- to long-term time horizons [17]. In fact, pumped-hydro energy storage units are particularly useful, due to their flexibility, to address the pernicious effects induced by the uncertain weather-dependent generation [18, 19]. Such storage units are able to provide services in a wide range of time frames going from spinning reserves to weekly smoothing of load curves [20]. In the context of unit commitment, pumped-hydro energy storage units allow to reduce the number of startup and shutdowns operations of thermal power plants, reducing the scheduling and dispatch costs [21].

Most works focused on the weekly UC consider a deterministic approach, which could make its generation scheduling vulnerable to certain scenarios. In [22], the authors propose a hybrid decomposition approach based on outer approximation and Benders decomposition to address the stochastic UC with endogenous reserve determination considering forced outages of equipment. In [23], the authors propose a Benders decomposition to address the weekly two-stage robust UC problem considering the uncertain volume of water inflow in hydrothermal power systems. In [24], the authors propose a Benders decomposition algorithm to solve a risk-constrained day-ahead stochastic UC with energy storage and weather-dependent generation.

Despite of the popularity of Benders decomposition to address the deterministic and stochastic variants of the generation scheduling problem, such decomposition approach has several drawbacks, such as: (i) the master problem provides a loose relaxation of the original problem, (ii) poor initial iterations, and (iii) slow convergence of the upper and lower bounds [25]. The main reason for these drawbacks is that there is not a good communication between the master problem and the subproblems since the master problem does not consider scenario-wise information to make an informed scheduling decision. Such lack of communication may result in expensive or infeasible scheduling of generating units, particularly, in the initial iterations. To bridge this gap, we propose a Benders-like decomposition approach, which overcomes the main drawbacks of standard Benders decomposition, to address the risk-constrained stochastic unit commitment.

In this work, we aim at addressing the two-stage stochastic unit commitment problem with weekly storage and risk-control. In particular, our work is focused on the stochastic UC with weekly-dispatched energy storage (e.g., pumped-hydro storage), which requires a weekly scheduling horizon. The proposed model considers uncertain net power injections induced by demand and weather-dependent generation. Hence, we consider an hourly scheduling for a one-week time horizon. To hedge against high scheduling and dispatch costs, we incorporate the conditional value-at-risk (CVaR) to manage risk [3, 26, 27]. To solve the resulting risk-constrained stochastic unit commitment, we propose a hybrid decomposition technique based on Benders decomposition and the column-and-constraint generation algorithm.

Based on a multi-cut Benders framework, we decompose the problem into a master problem, which includes the first-stage decisions, and many subproblems corresponding to the second-stage decisions. The master problem, which is mixed-integer linear, includes all the binary commitment variables with their associated constraints (i.e., minimum up/down time and commitment logic). The subproblems, which are linear and continuous, correspond to the actual dispatch and include generation and ramping limits, power balance, unserved energy bounds, and weather-dependent generation spillage. At every iteration, dual information to refine the master problem is transferred from the subproblems via Benders' cuts. The communication between the master problem and the subproblems is weak even in the case of the multi-cut Benders framework. This drawback arises because the master problem is disconnected from the constraints on generating units, weather-dependent generation, and demand for each scenario.

To bridge this lack of communication and inspired by the column-and-constraint generation algorithm [28, 29], we propose to add primal information to the master problem corresponding to the primal constraints of a small subset of scenarios when the gap is not reducing effectively. The dispatch costs of such subset of scenarios are not approximated using Benders' cuts since they are actually embedded into the master problem. This strategy increases the size of the master problem and might increase its computation time. To alleviate the computational burden of the proposed master problem, we use an active set strategy on the generating ramping limits, which significantly reduces the size of the problem. At every iteration and for every scenario, we identify the set of violated ramping constraints and enforce them in the scenarios considered in the master problem. We illustrate the effectiveness of the proposed solution method using an instance of the South Carolina synthetic system under 40 uncertainty scenarios.

The main contributions of this paper are twofold:

1. To propose a Benders-like decomposition method, which overcomes the main drawbacks of the standard Benders decomposition approach, to address the risk-constrained stochastic unit commitment with weekly-dispatched energy storage. The effectiveness of the proposed method with respect to other Benders-like approaches lies in improving the communication between the master problem and the subproblems by effectively conveying primal information from a small subset of subproblems to the master problem.

2. To provide two strategies to select the scenarios to be included in the master problem based on: input data (data from weather-dependent scenarios) and solution data (solution of master problem and subproblems at every iteration of the decomposition method). The first strategy, based on the input data, selects a pre-specified number of scenarios, determined by the solution of a simple mixed-integer linear problem, to be added to the master problem at the first iteration and retained throughout the evolution of the solution method. The second strategy, based on the solution data, adds scenario information when the algorithm shows slow progress. Such strategy selects the scenarios based on the second-stage costs of the subproblems and the second-stage cost mismatch with the ones computed as a solution of the master problem.

The outline of the rest of this paper is as follows. In Section II, we present the formulation of the risk-averse two-stage stochastic unit commitment problem. In Section III, we describe and detail the proposed hybrid decomposition technique. Next, in Section IV, we demonstrate the effectiveness of the proposed algorithm using an instance of the South Carolina system under 40 scenarios. Lastly, we conclude the paper with some remarks in Section V.

2. Problem formulation

2.1. Assumptions

1. We model the commitment of coal-fired and combined cycle gas turbines using 3 binary variables.

2. The production cost of any nuclear unit is constant throughout the time horizon, therefore, we neglect it in the objective function.

3. We assume that there are not transmission bottlenecks that can impact the scheduling of generating units. Therefore, we do not consider the transmission network in the problem formulation. Additionally, this assumption allows us to focus on the intricacies of weekly-dispatched storage systems and uncertain weather-dependent generating units.

4. We assume that the no-load, start-up, and shutdown costs are per hour constant values incurred if the unit is operating, starts up of shuts down, respectively.

5. The risk measure of the cost corresponds to the conditional value-at-risk, which is a coherent measure that allows us to characterize the risk of unfavorable scenarios with high operating costs. The parameter $\beta \in [0, 1]$ controls the risk-aversion level. If $\beta = 0$, the risk is ignored and the model corresponds to a risk-neutral problem.

6. We assume that the proposed model is solved on a daily basis. Hence, the aim of the proposed model is to determine the day-ahead optimal scheduling of the power generating units and the week-ahead dispatch of the energy storage units.

2.2. Model formulation

The Risk-Constrained Stochastic Unit Commitment (RC-SUC) problem is formulated using three binary variables to represent the operating status of the generating units. The objective of the RC-SUC problem can be stated as follows:

$$\min_{\Xi} \sum_{t\in T} \sum_{j\in\Lambda^{\mathrm{c}}\cup\Lambda^{\mathrm{g}}} \Big(c_j^{\mathrm{NL}} v_j(t) + c_j^{\mathrm{U}} y_j(t) + c_j^{\mathrm{D}} z_j(t)\Big) + (1-\beta)\left[\sum_{\omega\in\Omega}\pi_\omega \sum_{t\in T}\left(\sum_{j\in\Lambda^{\mathrm{T}}} c_j p_{j\omega}(t) + \sum_{i\in\Lambda^{\mathrm{d}}} c_i^{\mathrm{LOL}} D_{i\omega}^{\mathrm{sh}}(t)\right)\right] + \beta\left(\zeta + \frac{1}{1-\alpha}\sum_{\omega\in\Omega}\pi_\omega\eta_\omega\right). \quad (1)$$

The objective is to minimize a combination of the expectation and CVaR of the system cost, where the system cost includes first-stage no-load, start-up, and shut-down costs, and second-stage costs, including operating costs of all the units and the cost of unserved energy. The operating cost of unit j is a linear approximation of the actual production cost. The no-load c_j^{NL}, start-up c_j^{U}, and production c_j cost coefficients are function of fuel prices.

2.2.1. First-stage constraints

$$v_j(t-1) - v_j(t) + y_j(t) - z_j(t) = 0; \quad \forall j \in \Lambda^{\mathrm{c}} \cup \Lambda^{\mathrm{g}}, \quad t = 2, \ldots, |T|, \tag{2}$$

$$v_j^{\mathrm{ini}} - v_j(1) + y_j(1) - z_j(1) = 0, \quad \forall j \in \Lambda^{\mathrm{c}} \cup \Lambda^{\mathrm{g}}, \tag{3}$$

$$\sum_{t=1}^{L_j} [1 - v_j(i)] = 0, \quad \forall j \in \Lambda^{\mathrm{c}} \cup \Lambda^{\mathrm{g}}, \tag{4}$$

$$\sum_{i=t}^{t+T_j^{\mathrm{U}}-1} v_g(i) \geq T_j^{\mathrm{U}} y_j(t), \forall j \in \Lambda^{\mathrm{c}} \cup \Lambda^{\mathrm{g}}; \quad \forall t \in [L_j + 1, \ \ldots, |T| - T_j^{\mathrm{U}} + 1], \tag{5}$$

$$\sum_{i=t}^{|T|} [v_j(i) - y_j(t)] \geq 0, \quad \forall j \in \Lambda^{\mathrm{c}} \cup \Lambda^{\mathrm{g}}; \quad \forall t \in [|T| - T_j^{\mathrm{U}} + 2, \ \ldots, |T|], \tag{6}$$

$$\sum_{t=1}^{F_j} v_j(t) = 0, \quad \forall j \in \Lambda^{\mathrm{c}} \cup \Lambda^{\mathrm{g}}, \tag{7}$$

$$\sum_{i=t}^{t+T_j^{\mathrm{D}}-1} [1 - v_j(i)] \geq T_j^{\mathrm{D}} z_g(t), \quad \forall j \in \Lambda^{\mathrm{c}} \cup \Lambda^{\mathrm{g}}, \quad \forall t \in [F_j + 1, \ \ldots, |T| - T_j^{\mathrm{D}} + 1], \tag{8}$$

$$\sum_{i=t}^{|T|} [1 - v_j(i) - z_j(t)] \geq 0; \quad \forall j \in \Lambda^{\mathrm{c}} \cup \Lambda^{\mathrm{g}}, \quad \forall t \in [|T| - T_j^{\mathrm{D}} + 2, \ \ldots, |T|], \tag{9}$$

$$0 \leq p_\ell^{\mathrm{ch}}(t) \leq \overline{P}_\ell^{\mathrm{ch}}, \quad \forall \ell \in \Lambda^{\mathrm{s}}, \ \forall t = 1, \ldots, |T| \tag{10}$$

$$0 \leq p_\ell^{\mathrm{dis}}(t) \leq \overline{P}_\ell^{\mathrm{dis}}, \quad \forall \ell \in \Lambda^{\mathrm{s}}, \ \forall t = 1, \ldots, |T| \tag{11}$$

$$E_\ell(t) - E_\ell(t-1) = \varphi_\ell^{\mathrm{c}} p_\ell^{\mathrm{ch}}(t) - \frac{1}{\varphi_\ell^{\mathrm{d}}} p_\ell^{\mathrm{dis}}(t), \quad \forall \ell \in \Lambda^{\mathrm{s}}, \ \forall t = 2, \ldots, |T| \tag{12}$$

$$E_\ell(1) - E_\ell^{\mathrm{ini}} = \varphi_\ell^{\mathrm{c}} p_\ell^{\mathrm{ch}}(1) - \frac{1}{\varphi_\ell^{\mathrm{d}}} p_\ell^{\mathrm{dis}}(1), \quad \forall \ell \in \Lambda^{\mathrm{s}} \tag{13}$$

$$\underline{E}_\ell(t) \leq E_\ell(t) \leq \overline{E}_\ell(t), \quad \forall \ell \in \Lambda^{\mathrm{s}}, \quad \forall t \in T \setminus T^{\mathrm{fill}} \tag{14}$$

$$E_\ell(t) = \overline{E}_\ell(t), \quad \forall t \in T^{\mathrm{fill}}, \forall \ell \in \Lambda^{\mathrm{s}}. \tag{15}$$

Constraints (2) and (3) ensure the start-up, shut-down and running logic of the power plant that require commitment variables. Constraints (4)–(6) enforce the minimum up time of coal-fired units and combined cycle gas turbines whereas their minimum down time are ensured by constraints (7)–(9).

Constraints (10) and (11) bound the charging/discharging power of storage units. The scheduled energy balance of storage units per time period is stated in constraints (12) and (13). The scheduled energy of storage units is upper and lower bounded by constraint (14). Constraint (15) ensures that the storage units are filled at a given time period. Constraints (10)–(15) are enforced since operators generally desire to schedule values for storage units and deviations upon them that satisfy operational constraints.

2.2.2. Second-stage constraints

The following second-stage constraints are defined for all $t = 1, \ldots, |T|$ and for all $\omega \in \Omega$ unless specified otherwise.

$$p_{j\omega}(t) - p_{j\omega}(t-1) \leq R_j^{\mathrm{U}} \, v_j(t-1) + S_j^{\mathrm{U}} \, y_j(t), \quad \forall j \in \Lambda^{\mathrm{c}} \cup \Lambda^{\mathrm{g}}, \ \forall t = 2, \ldots, |T|, \tag{16}$$

$$p_{j\omega}(1) - p_j^{\mathrm{ini}} \leq R_j^{\mathrm{U}} \, v_j^{\mathrm{ini}} + S_j^{\mathrm{U}} \, y_j(1), \ \forall j \in \Lambda^{\mathrm{c}} \cup \Lambda^{\mathrm{g}}, \tag{17}$$

$$p_{j\omega}(t-1) - p_{j\omega}(t) \leq R_j^{\mathrm{D}} \, v_j(t) + S_j^{\mathrm{D}} \, z_j(t), \quad \forall j \in \Lambda^{\mathrm{c}} \cup \Lambda^{\mathrm{g}}, \ \forall t = 2, \ldots, |T|, \tag{18}$$

$$p_j^{\mathrm{ini}} - p_{j\omega}(1) \leq R_j^{\mathrm{D}} \, v_j(1) + S_j^{\mathrm{D}} \, z_j(1), \quad \forall j \in \Lambda^{\mathrm{c}} \cup \Lambda^{\mathrm{g}}, \tag{19}$$

$$\underline{P}_j v_j(t) u_{j\omega}(t) \leq p_{j\omega}(t) \leq \overline{P}_j v_j(t) u_{j\omega}(t), \quad \forall j \in \Lambda^{\mathrm{c}} \cup \Lambda^{\mathrm{g}}, \tag{20}$$

$$0 \leq p_{j\omega}(t) \leq \overline{P}_j u_{j\omega}(t), \quad \forall j \in \Lambda^{\mathrm{ng}}, \tag{21}$$

$$\sum_{j \in \Lambda^{\mathrm{T}}} p_{j\omega}(t) + \sum_{j \in \Lambda^{\mathrm{w}}} \left(p_{j\omega}^{\mathrm{W}}(t) - p_{j\omega}^{\mathrm{W,spill}}(t) \right) + \sum_{j \in \Lambda^{\mathrm{n}}} p_j^{\mathrm{N}}(t) + \sum_{\ell \in \Lambda^{\mathrm{s}}} \left(p_\ell^{\mathrm{dis}}(t) + \Delta_{\ell\omega}^{\mathrm{dis}}(t) - (p_\ell^{\mathrm{ch}}(t) + \Delta_{\ell\omega}^{\mathrm{ch}}(t)) \right) = \sum_{i \in \Lambda^{\mathrm{d}}} D_{i\omega}(t) - \sum_{i \in \Lambda^{\mathrm{d}}} D_{i\omega}^{\mathrm{sh}}(t), \tag{22}$$

$$0 \leq p_{j\omega}^{\mathrm{W,spill}}(t) \leq p_{j\omega}^{\mathrm{W}}(t), \quad \forall j \in \Lambda^{\mathrm{W}}, \tag{23}$$

$$0 \leq D_{i\omega}^{\mathrm{sh}}(t) \leq D_{i\omega}(t), \quad \forall i \in \Lambda^{\mathrm{d}}, \tag{24}$$

$$0 \leq p_\ell^{\mathrm{ch}}(t) + \Delta_{\ell\omega}^{\mathrm{ch}}(t) \leq \overline{P}_\ell^{\mathrm{ch}}, \quad \forall \ell \in \Lambda^{\mathrm{s}}, \tag{25}$$

$$0 \leq p_\ell^{\mathrm{dis}}(t) + \Delta_{\ell\omega}^{\mathrm{dis}}(t) \leq \overline{P}_\ell^{\mathrm{dis}}, \quad \forall \ell \in \Lambda^{\mathrm{s}}, \tag{26}$$

$$\underline{\Delta}_\ell^{\mathrm{ch}} \leq \Delta_{\ell\omega}^{\mathrm{ch}}(t) \leq \overline{\Delta}_\ell^{\mathrm{ch}}, \quad \forall \ell \in \Lambda^{\mathrm{s}}, \tag{27}$$

$$\underline{\Delta}_\ell^{\mathrm{dis}} \leq \Delta_{\ell\omega}^{\mathrm{dis}}(t) \leq \overline{\Delta}_\ell^{\mathrm{dis}}, \quad \forall \ell \in \Lambda^{\mathrm{s}}, \tag{28}$$

$$E_{\ell\omega}(t) - E_{\ell\omega}(t-1) = \varphi_\ell^{\mathrm{c}} \left(p_\ell^{\mathrm{ch}}(t) + \Delta_{\ell\omega}^{\mathrm{ch}}(t) \right) - \frac{1}{\varphi_\ell^{\mathrm{d}}} \left(p_\ell^{\mathrm{dis}}(t) + \Delta_{\ell\omega}^{\mathrm{dis}}(t) \right), \quad \forall \ell \in \Lambda^{\mathrm{s}}, \quad \forall t = 2, \ldots, |T|, \tag{29}$$

$$E_{\ell\omega}(1) - E_\ell^{\mathrm{ini}} = \varphi_\ell^{\mathrm{c}} \left(p_\ell^{\mathrm{ch}}(1) + \Delta_{\ell\omega}^{\mathrm{ch}}(1) \right) -$$

$$\frac{1}{\varphi_\ell^{\mathrm{d}}}\Big(p_\ell^{\mathrm{dis}}(1) + \Delta_{\ell\omega}^{\mathrm{dis}}(1)\Big), \quad \forall \ell \in \Lambda^{\mathrm{s}}, \tag{30}$$

$$\underline{E}_\ell(t) \le E_{\ell\omega}(t) \le \overline{E}_\ell(t), \quad \forall \ell \in \Lambda^{\mathrm{s}},\, \forall t \in T \setminus T^{\mathrm{fill}}, \tag{31}$$

$$E_{\ell\omega}(t) = \overline{E}_\ell(t), \quad \forall t \in T^{\mathrm{fill}},\, \forall \ell \in \Lambda^{\mathrm{s}},. \tag{32}$$

Constraints (16) and (17) enforce the maximum ramping-up of generating units whereas their maximum ramping-down is ensured by constraints (18) and (19). Constraints (20) and (21) bound the actual generation of each generating unit. The power balance is enforced by constraint (22). The weather-dependent generation spillage is bounded by constraint (23). Constraint (24) bounds the unserved energy per scenario. The bounds and operation of storage units per scenario are stated in constraints (25)–(32).

2.2.3. CVaR constraints

$$\eta_\omega \ge \sum_{t\in T}\left(\sum_{j\in\Lambda^{\mathrm{T}}} c_j p_{j\omega}(t) + \sum_{i\in\Lambda^{\mathrm{d}}} c_i^{\mathrm{LOL}} D_{i\omega}^{\mathrm{sh}}(t)\right) - \zeta,$$
$$\forall \omega \in \Omega \tag{33}$$

$$\eta_\omega \ge 0, \quad \forall \omega \in \Omega. \tag{34}$$

Auxiliary constraints (33) and (34) are used to model the CVaR.

3. Solution Method

We propose a hybrid decomposition technique to address the risk-constrained stochastic unit-commitment problem, which blends ideas from Benders decomposition and the column-and-constraint generation algorithm. First, we decompose the RC-SUC problem using a Benders framework into a master problem, which includes the commitment variables and their constraints, and many subproblems, which correspond to the dispatch of the generating units for the corresponding uncertainty realizations. Note that the Benders' cuts, which provide an approximation of the second-stage costs, are based only on local dual information. Such characteristic results in weak communication between the master problem and the subproblems. Hence, inspired by the success of the column-and-constraint generation algorithm to solve robust optimization problems, we propose to enforce primal information to the master problem corresponding to the constraints of a small subset of scenarios. The proposed solution method is as follows:

1. In the first iteration, we identify an initial small subset of representative scenarios and incorporate them into the master problem, which helps to attain good quality first-stage variable values. These scenarios are included in the master problem for the remaining iterations.

2. Then, we solve the master problem, which includes all the binary commitment variables with their associated constraints (i.e., minimum up/down time and commitment logic). The second stage costs of the scenarios are approximated using Benders' cuts and further refined at every iteration. A lower bound is determined by the optimal value of the objective function of the master problem.

3. Next, we solve the subproblems, which are linear and continuous, and model the generation dispatch and include generation capacity and ramping limits, power balance constraints, and weather-dependent generation spillage limits.

4. To reduce the computational burden of the proposed master problem, we use an active set strategy on the generating ramping limits, which significantly reduces the size of the problem. At every iteration and for every scenario, we identify the set of binding ramping constraints of the scenarios considered in the master problem and enforce them in the next iteration. The set of active constraints at every iteration includes all the constraints that have been binding in previous iterations for the corresponding scenario.

5. Then, the CVaR is computed based on the solution of the first- and second-stage variables obtained from the solutions of the master problem and subproblems, respectively.

6. An upper bound is determined from the solution of the subproblems and the CVaR.

7. If the gap between the upper and lower bound is not effectively closing as measured through an improvement threshold, we append new scenarios to the master problem to provide a better approximation of the recourse function. Such new scenarios are identified using two criteria: (i) the scenario with the highest optimal objective value of the subproblems, and (ii) the scenario with the highest mismatch between the second-stage costs computed at the solution of the master problem and the subproblems.

8. If a convergence criterion has been satisfied, the algorithm stops. Otherwise, it returns to Step 2.

3.1. Master problem formulation

The master problem is formulated as follows:

$$\min_{\Xi^{\mathrm{M}}} \quad \sum_{t\in T}\sum_{j\in\Lambda^{\mathrm{c}}\cup\Lambda^{\mathrm{g}}} \Big(c_j^{\mathrm{NL}} v_j(t) + c_j^{\mathrm{U}} y_j(t) + c_j^{\mathrm{D}} z_j(t)\Big) +$$
$$(1-\beta)\Bigg[\sum_{\omega\in\Omega^{\mathrm{M},(\nu)}} \pi_\omega \sum_{t\in T}\Bigg(\sum_{j\in\Lambda^{\mathrm{T}}} c_j p_{j\omega}(t) +$$
$$\sum_{i\in\Lambda^{\mathrm{d}}} c_i^{\mathrm{LOL}} D_{i\omega}^{\mathrm{sh}}(t)\Bigg) +$$

$$\left.\sum_{\omega\in\Omega\setminus\Omega^{\mathrm{M},(\nu)}} \pi_\omega \psi_\omega \right] + \beta\left(\zeta + \frac{1}{1-\alpha}\sum_{\omega\in\Omega}\pi_\omega\eta_\omega\right) \tag{35}$$

$$\text{s.t.}\quad (2)-(15)$$

$$(16)-(19);\quad \forall j\in\mathcal{I}_\omega^{(\nu)}(t),\ \forall\omega\in\Omega^{\mathrm{M},(\nu)},\ \forall t\in T$$

$$(20)-(34);\quad \forall\omega\in\Omega^{\mathrm{M},(\nu)}$$

$$\psi_\omega \geq \psi_{\mathrm{down}};\quad \forall\omega\in\Omega\setminus\Omega^{\mathrm{M},(\nu)}, \tag{36}$$

$$\begin{aligned}
\psi_\omega \geq \sum_{t\in T}\Bigg[& \sum_{j\in\Lambda^{\mathrm{T}}} c_j p_{j\omega}^{(\mu)}(t) + \sum_{i\in\Lambda^{\mathrm{d}}} c_i^{\mathrm{LOL}} D_{i\omega}^{\mathrm{sh},(\mu)}(t) + \\
& \sum_{j\in\Lambda^{\mathrm{c}}\cup\Lambda^{\mathrm{g}}}\Bigg(\gamma_{j\omega}^{v,(\mu)}(t)\left(v_j(t)-v_j^{(\mu)}(t)\right) + \\
& \gamma_{j\omega}^{y,(\mu)}(t)\left(y_j(t)-y_j^{(\mu)}(t)\right) + \\
& \gamma_{j\omega}^{z,(\mu)}(t)\left(z_j(t)-z_j^{(\mu)}(t)\right)\Bigg) + \\
& \sum_{\ell\in\Lambda^{\mathrm{s}}}\Bigg(\gamma_{\ell\omega}^{\mathrm{ch},(\mu)}(t)\left(p_\ell^{\mathrm{ch}}(t)-p_\ell^{\mathrm{ch},(\mu)}(t)\right) + \\
& \gamma_{\ell\omega}^{\mathrm{dis},(\mu)}(t)\left(p_\ell^{\mathrm{dis}}(t)-p_\ell^{\mathrm{dis},(\mu)}(t)\right) + \\
& \gamma_{\ell\omega}^{E,(\mu)}(t)\left(E_\ell(t)-E_\ell^{(\mu)}(t)\right)\Bigg)\Bigg],
\end{aligned}$$

$$\forall\mu=1,\ldots,\nu-1,\ \forall\omega\in\Omega\setminus\Omega^{\mathrm{M},(\nu)}, \tag{37}$$

$$\begin{aligned}
\sum_{t\in T}\Bigg(& \sum_{j\in\Lambda^{\mathrm{T}}} c_j p_{j\omega}(t) + \sum_{i\in\Lambda^{\mathrm{d}}} c_i^{\mathrm{LOL}} D_{i\omega}^{\mathrm{sh}}(t)\Bigg) \geq \\
\sum_{t\in T}\Bigg[& \sum_{j\in\Lambda^{\mathrm{T}}} c_j p_{j\omega}^{(\mu)}(t) + \sum_{i\in\Lambda^{\mathrm{d}}} c_i^{\mathrm{LOL}} D_{i\omega}^{\mathrm{sh},(\mu)}(t) + \\
& \sum_{j\in\Lambda^{\mathrm{c}}\cup\Lambda^{\mathrm{g}}}\Bigg(\gamma_{j\omega}^{v,(\mu)}(t)\left(v_j(t)-v_j^{(\mu)}(t)\right) + \\
& \gamma_{j\omega}^{y,(\mu)}(t)\left(y_j(t)-y_j^{(\mu)}(t)\right) + \\
& \gamma_{j\omega}^{z,(\mu)}(t)\left(z_j(t)-z_j^{(\mu)}(t)\right)\Bigg) + \\
& \sum_{\ell\in\Lambda^{\mathrm{s}}}\Bigg(\gamma_{\ell\omega}^{\mathrm{ch},(\mu)}(t)\left(p_\ell^{\mathrm{ch}}(t)-p_\ell^{\mathrm{ch},(\mu)}(t)\right) + \\
& \gamma_{\ell\omega}^{\mathrm{dis},(\mu)}(t)\left(p_\ell^{\mathrm{dis}}(t)-p_\ell^{\mathrm{dis},(\mu)}(t)\right) + \\
& \gamma_{\ell\omega}^{E,(\mu)}(t)\left(E_\ell(t)-E_\ell^{(\mu)}(t)\right)\Bigg)\Bigg],
\end{aligned}$$

$$\forall\mu=1,\ldots,\nu-1,\ \forall\omega\in\Omega^{\mathrm{M},(\nu)}, \tag{38}$$

$$\eta_\omega \geq \psi_\omega - \zeta,\ \forall\omega\in\Omega\setminus\Omega^{\mathrm{M},(\nu)}, \tag{39}$$

$$\eta_\omega \geq 0;\quad \forall\omega\in\Omega\setminus\Omega^{\mathrm{M},(\nu)}. \tag{40}$$

where the optimization variables are elements of the set

$$\begin{aligned}
\Xi^{\mathrm{M}} = & \left\{v_j(t), y_j(t), z_j(t), E_\ell(t), p_\ell^{\mathrm{ch}}(t), p_\ell^{\mathrm{dis}}(t), \zeta\right\}\cup \\
& \left\{P_{j\omega}(t), p_{j\omega}^{\mathrm{W,spill}}(t), D_{i\omega}^{\mathrm{sh}}(t), E_{\ell\omega}(t), \Delta_{\ell\omega}^{\mathrm{ch}}(t), \Delta_{\ell\omega}^{\mathrm{dis}}(t),\right. \\
& \left.\eta_\omega\ \forall\omega\in\Omega^{\mathrm{M},(\nu)}\right\}\cup\left\{\eta_\omega, \psi_w\ \forall\omega\in\Omega\setminus\Omega^{\mathrm{M},(\nu)}\right\}.
\end{aligned}$$

The objective function (35) includes scheduling costs, actual dispatch costs of the scenarios included in the master problem (i.e., $\omega\in\Omega^{\mathrm{M}}$), and approximated costs of the remaining scenarios (i.e., $\omega\in\Omega\setminus\Omega^{\mathrm{M}}$). Constraint (36) enforces a lower bound to the approximated costs ψ_ω to avoid unboundedness. Constraints (37) and (38) correspond to the Benders' cuts of the scenarios considered and approximated in the master problem, respectively. Note that the master problem is a relaxed version of the original RC-SUC problem since a reduced number of scenarios in the second stage are considered, i.e., $\Omega^{\mathrm{M}}\subset\Omega$. Note as well that even though Benders' cuts (38) are not strictly necessary, they provide additional information to the master problem about the solutions of previous iterations. The contribution for the CVaR calculation of the scenarios considered in the master problem is stated in constraints (33) and (34) whereas the contribution of the remaining scenarios, which are not considered in the master problem, is stated in constraints (39) and (40).

3.2. Subproblem formulation

Each scenario of the second stage can be solved independently since the only linking variables correspond to the first-stage ones, which are computed in the master problem. Once such variables are fixed, the problem is decomposed in easy-to-solve subproblems since they are linear, continuous, and small-scale. The formulation of the subproblem corresponding to scenario ω is:

$$\min_{\Xi^{\mathrm{S}}}\quad \sum_{t\in T}\left(\sum_{j\in\Lambda^{\mathrm{T}}} c_j p_{j\omega}(t) + \sum_{i\in\Lambda^{\mathrm{d}}} c_i^{\mathrm{LOL}} D_{i\omega}^{\mathrm{sh}}(t)\right) \tag{41}$$

$$\text{s.t.}\quad (16)-(32)$$

$$v_j(t) = v_j^{(\nu)}(t) : (\gamma_{j\omega}^{v,(\nu)}),\ \forall j\in\Lambda^{\mathrm{c}}\cup\Lambda^{\mathrm{g}},\ \forall t\in T, \tag{42}$$

$$y_j(t) = y_j^{(\nu)}(t) : (\gamma_{j\omega}^{y,(\nu)}),\ \forall j\in\Lambda^{\mathrm{c}}\cup\Lambda^{\mathrm{g}},\ \forall t\in T, \tag{43}$$

$$z_j(t) = z_j^{(\nu)}(t) : (\gamma_{j\omega}^{z,(\nu)}),\ \forall j\in\Lambda^{\mathrm{c}}\cup\Lambda^{\mathrm{g}},\ \forall t\in T, \tag{44}$$

$$p_\ell^{\mathrm{ch}}(t) = p_\ell^{\mathrm{ch},(\nu)}(t) : (\gamma_{\ell\omega}^{\mathrm{ch},(\nu)}),\ \forall\ell\in\Lambda^{\mathrm{s}},\ \forall t\in T, \tag{45}$$

$$p_\ell^{\mathrm{dis}}(t) = p_\ell^{\mathrm{dis},(\nu)}(t) : (\gamma_{\ell\omega}^{\mathrm{dis},(\nu)}),\ \forall\ell\in\Lambda^{\mathrm{s}},\ \forall t\in T, \tag{46}$$

$$E_\ell(t) = E_\ell^{(\nu)}(t) : (\gamma_{\ell\omega}^{E,(\nu)}),\ \forall\ell\in\Lambda^{\mathrm{s}},\ \forall t\in T, \tag{47}$$

where

$$\Xi^{\mathrm{S}} = \left\{p_{j\omega}(t), p_{j\omega}^{\mathrm{W,spill}}(t), D_{i\omega}^{\mathrm{sh}}(t), E_{\ell\omega}(t), \Delta_{\ell\omega}^{\mathrm{ch}}(t), \Delta_{\ell\omega}^{\mathrm{dis}}(t)\right\}.$$

Fixing the first-stage variables in the above subproblem may result in infeasible cases. Therefore, we introduce non-negative slack variables to the ramping constraints (16)–(19) and penalize them in the objective function with a cost coefficient equal to the cost of unserved energy c_i^{LOL}.

3.3. CVaR computation

To compute an upper bound of the objective function, we need to compute the actual CVaR based on the solution of the master problem and subproblems. First, we fix the first-stage and second stage decision variables to the solution of the master problem and subproblems, respectively. Then, we compute the CVaR by solving the following problem:

$$\min_{\eta_\omega,\zeta} \quad \zeta + \frac{1}{1-\alpha}\sum_{\omega\in\Omega}\pi_\omega\eta_\omega \tag{48}$$

$$\text{s.t.} \quad \eta_\omega \geq \sum_{t\in T}\left(\sum_{j\in\Lambda^{\mathrm{T}}} c_j p_{j\omega}^{(\nu)}(t) + \sum_{i\in\Lambda^{\mathrm{d}}} c_i^{\mathrm{LOL}} D_{i\omega}^{\mathrm{sh},(\nu)}(t)\right) - \zeta, \quad \forall\omega\in\Omega, \tag{49}$$

$$\eta_\omega \geq 0, \quad \forall\omega\in\Omega. \tag{50}$$

3.4. Scenario selection

We use a strategy to determine which scenarios to include in the master problem in the first iteration based on the concept of row covering presented in [30]. Such concept allows us to determine a fixed number of scenarios with the highest net demand (i.e., scenarios that cover other scenarios the most) in a systematic way, which ensures that enough committed capacity is available in the initial iterations. First, we let the net power injection per time period be defined as follows:

$$p_\omega^{\mathrm{net}}(t) = \sum_{i\in\Lambda^{\mathrm{d}}} D_{i\omega}(t) - \sum_{j\in\Lambda^{\mathrm{w}}} p_{j\omega}^{\mathrm{W}}(t), \quad \forall t\in T,\ \forall\omega\in\Omega.$$

Next, we define the parameter $\delta_{\tilde{\omega}\omega}(t)$, $\omega\in\Omega$, $\tilde{\omega}\in\Omega$, $t\in T$ such that $\tilde{\omega}\neq\omega$, as follows:

$$\delta_{\tilde{\omega}\omega}(t) = \begin{cases} 1 & \text{if } p_{\tilde{\omega}}^{\mathrm{net}}(t) \geq p_\omega^{\mathrm{net}}(t) \\ 0 & \text{otherwise.} \end{cases}$$

Then, we use a binary variable r_ω, $\omega\in\Omega$, which is equal to 1 if scenario ω is to be included in the master problem and 0 otherwise. The number of scenarios to be included in the master problem are controlled by the integer parameter R.

Finally, the following simple mixed-integer problem is solved to obtain $\Omega^{\mathrm{M},(0)}$:

$$\max_{r_\omega} \quad \sum_{\omega\in\Omega}\sum_{t\in T}\sum_{\tilde{\omega}\in\Omega}\delta_{\tilde{\omega}\omega}(t) r_{\tilde{\omega}} \tag{51}$$

$$\text{s.t.} \quad \sum_{\omega\in\Omega} r_\omega = R, \tag{52}$$

$$r_\omega \in \{0,1\}, \quad \forall\omega\in\Omega. \tag{53}$$

The objective of problem (51)–(53), which is easily solved, is to maximize the number of scenarios that are covered by the scenarios to be included in the master problem. The proposed optimization problem determines the scenarios to be included in the master problem solely based on known information of the demand and weather-dependent available generation capacity.

The number of scenarios to be included in the master problem, which is determined by R, depends on the size of the problem. Smaller problems allow to add more scenarios in the master problem without facing computational limitations. Conversely, solving the SUC problem for larger systems limits the number of scenarios to be included due to the high computational burden to solve the master problem, which could make it too time-consuming or intractable to solve.

Additionally, we propose to dynamically enrich the scenario information incorporated into the master problem during the course of evolution of the proposed decomposition method. To determine which scenarios to add to the master problem as the algorithm progresses, we use the solutions of the master problem and the subproblems. In particular, we use the following criteria:

(i) Add the scenario with the highest second-stage cost as computed in the subproblem, which allows us to control scenarios with potential infeasibilities that increase the upper bound of the algorithm

(ii) Add the scenario with the highest mismatch between the second-stage costs computed in the master problem and the subproblems, which allows us to better represent such scenarios in the master problem and to improve the lower bound provided by the master problem.

The proposed hybrid decomposition approach is depicted in Algorithm 1.

4. Simulation Results

In this section, we report computational experiments showcasing the effectiveness of the proposed solution method using the South Carolina synthetic system with 90 generating units reported in [32]. The proposed approach has been implemented on a Windows-based laptop with an Intel Core i7 processor clocking at 2.60 GHz and 16 GB of RAM under JuMP 0.21.3. We use CPLEX 12.10.0 for the master MILP problem and the linear subproblems.

We consider a weekly horizon to capture the weekly nature of the storage system; however, the studied generation scheduling model is solved and implemented on a daily basis. Therefore, the exact system's operation for the days beyond day one is not necessary. We relax the binary variables for $t = 25, \ldots, |T|$, which provides a good approximation of the operation of the system for the days beyond day one and increases the computational tractability of the problem.

4.1. Data

The South Carolina synthetic system has 90 generating units (coal, gas, run-of-the-river hydro, solar, and nuclear) [32, 33]. In our computational experiments, we assume that all thermal (coal and natural gas) units require commitment (binary) variables, resulting in a total of 45 units.

Algorithm 1 Proposed Hybrid Decomposition Algorithm

Input: δ_{tol}, δ, $\Omega^{\text{M},(0)}$.

$\nu \leftarrow 1$, $Z_{\text{down}}^{(1)} = -\infty$, $Z_{\text{up}}^{(1)} = \infty$, $\epsilon^{(0)} \leftarrow \infty$.

Initialize the set of active ramping constraints. $\mathcal{I}_{\omega}^{(\nu)}(t) \leftarrow \emptyset$, $t = 1, \ldots, |T|$, $\omega \in \Omega$

repeat

Solve and update lower bound: Set $(\boldsymbol{v}^{(\nu)}, \boldsymbol{y}^{(\nu)}, \boldsymbol{z}^{(\nu)}, \boldsymbol{p}^{\text{ch},(\nu)}, \boldsymbol{p}^{\text{dis},(\nu)}, \boldsymbol{E}^{(\nu)})$ to the solution of the master problem under the set of active ramping constraints $\mathcal{I}^{(\nu)}(t), \forall t \in T$ and the subset of scenarios $\Omega^{\text{M},(\nu)}$. Update $Z_{\text{down}}^{(\nu)}$ to be the optimal objective-function value of the master problem.

Solve: Set $(\boldsymbol{p}_{\omega}^{(\nu)}, \boldsymbol{D}_{\omega}^{\text{sh},(\nu)})$ to the solution of the subproblem for each scenario ω, and set $(\boldsymbol{\gamma}^{v,(\nu)}, \boldsymbol{\gamma}^{y,(\nu)}, \boldsymbol{\gamma}^{z,(\nu)}, \boldsymbol{\gamma}^{\text{ch},(\nu)}, \boldsymbol{\gamma}^{\text{dis},(\nu)}, \boldsymbol{\gamma}^{E,(\nu)})$ to the dual variables of constraints (42)–(47), respectively.

for $t = 1, \ldots, |T|$, $\omega \in \Omega$ **do**

Update the set of active ramping constraints. Set $\mathcal{I}_{\omega}^{(\nu+1)}(t)$ to the set of binding ramping constraints (16)–(19) of scenario ω in period t. $\mathcal{I}_{\omega}^{(\nu+1)}(t) \leftarrow \mathcal{I}_{\omega}^{(\nu)}(t) \cup \mathcal{I}_{\omega}^{(\nu+1)}(t)$.

end for

Solve: Set $(\boldsymbol{\eta}_{\omega}^{(\nu)}, \zeta^{(\nu)})$ to the solution of the CVaR problem (48)–(50).

Update upper bound. Update $Z_{\text{up}}^{(\nu)}$ as follows:

$$Z_{\text{up}}^{(\nu)} \leftarrow \sum_{t\in T} \sum_{j\in\Lambda^{\text{c}}\cup\Lambda^{\text{g}}} \Big(c_j^{\text{NL}} v_j^{(\nu)}(t) + c_j^{\text{U}} y_j^{(\nu)}(t) + c_j^{\text{D}} z_j^{(\nu)}(t) \Big) + (1-\beta) \left[\sum_{\omega\in\Omega} \pi_\omega \sum_{t\in T} \left(\sum_{j\in\Lambda^{\text{T}}} c_j p_{j\omega}^{(\nu)}(t) + \sum_{i\in\Lambda^{\text{d}}} c_i^{\text{LOL}} D_{i\omega}^{\text{sh},(\nu)}(t) \right) \right] + \beta \left(\zeta^{(\nu)} + \frac{1}{1-\alpha} \sum_{\omega\in\Omega} \pi_\omega \eta_\omega^{(\nu)} \right).$$

Update ϵ. $\epsilon^{(\nu)} \leftarrow 100 \cdot (Z_{\text{up}}^{(\nu)}/Z_{\text{down}}^{(\nu)} - 1)$

if $1 - \epsilon^{(\nu)}/\epsilon^{(\nu-1)} \le \delta$ **then**

Identify the scenario with the highest recourse cost.

$$\vartheta^1 \leftarrow \underset{\omega\in\Omega\setminus\Omega^{\text{M},(\nu)}}{\operatorname{argmax}} \left\{ \sum_{t\in T} \left[\sum_{j\in\Lambda^{\text{T}}} c_j p_{j\omega}^{(\nu)}(t) + \sum_{i\in\Lambda^{\text{d}}} c_i^{\text{LOL}} D_{i\omega}^{\text{sh},(\nu)}(t) \right] \right\}.$$

Identify the scenario with the highest cost mismatch.

$$\vartheta^2 \leftarrow \underset{\omega\in\Omega\setminus\Omega^{\text{M},(\nu)}}{\operatorname{argmax}} \left\{ \sum_{t\in T} \left[\sum_{j\in\Lambda^{\text{T}}} c_j p_{j\omega}^{(\nu)}(t) + \sum_{i\in\Lambda^{\text{d}}} c_i^{\text{LOL}} D_{i\omega}^{\text{sh},(\nu)}(t) \right] - \psi_\omega \right\}.$$

Update Ω^{M}. $\Omega^{\text{M},(\nu+1)} \leftarrow \Omega^{\text{M},(\nu)} \cup \vartheta^1 \cup \vartheta^2$

end if

Update ν. $\nu \leftarrow \nu + 1$

until $\epsilon^{(\nu)} \le \delta_{\text{tol}}$

We include a pumped-hydro energy storage unit of 400 MW of installed capacity, which represents around 3% of the total generation capacity, and 4970 MWh of maximum energy. We also assume that at the beginning of the time horizon, the energy storage unit is charged at maximum. The charging and discharging efficiency rates are set to 96% and 85%, respectively. No transmission bottleneck is considered.

Solar generation uncertainty scenarios are modeled based on data obtained from the System Advisor Model [34]. We consider 40 equiprobable scenarios and solar spillage is allowed. Figure 1 depicts the system demand considered deterministic and the solar generation scenarios.

The parameters for the proposed decomposition technique are set as follows: algorithm optimality tolerance $\delta_{\text{tol}} = 0.3\%$ and gap evolution tolerance $\delta = 0.05$. The extensive formulation of this problem has 1.8178×10^6 continuous variables, 3285 binary variables, and 3.917×10^6 constraints.

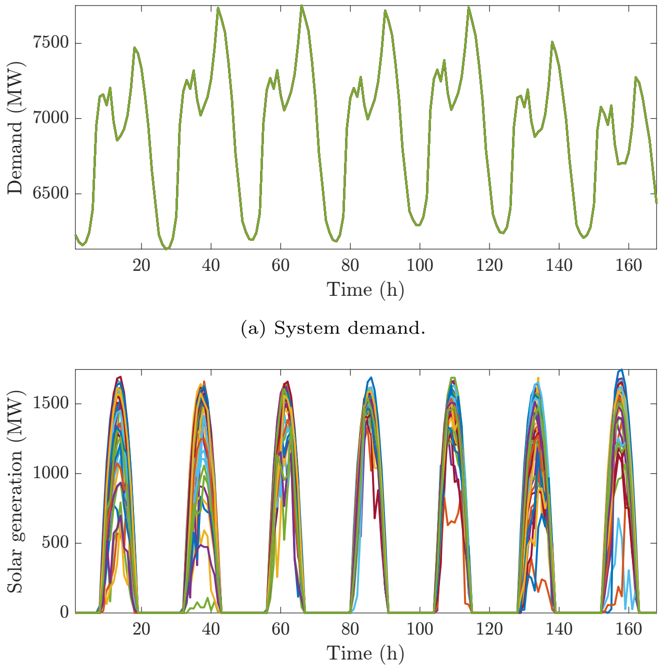
Figure 2 depicts the 5 scenarios determined by the proposed strategy. Such strategy is able to identify the subset of scenarios that cover the others most of the time. Note as well that a scenario is not necessarily going to show the highest net demand all the time. For example, the scenario shown in blue shows the highest net demand in days 1 and 2, but it has one of the smallest net demands in the remaining days. Also, the high penetration of solar generation induces a significant requirement of ramping up/down capability from the thermal units.

4.2. Computational performance

To show the effectiveness of the proposed approach, we compare its performance with the one of a tailored Benders decomposition, which enforces the same initial subset of scenarios in the master problem in the first iteration, for a stressed system operating condition. However, unlike our proposed decomposition technique, the tailored Benders decomposition does not enforce more scenarios into the master problem in the subsequent iterations. Such subset of scenarios is selected based on the formulation presented in Section III.D.

The evolution of the best upper and lower bounds for both algorithms is depicted in Fig. 3 for a risk-neutral case (risk-averse cases behave similarly). The proposed approach outperforms the tailored Benders decomposition since the former closes the gap below 0.3% in 4.58 hours whereas the latter is unable to reduce the gap below 0.6% in 6 hours. Note as well that the proposed algorithm, in less than 15 minutes, is able to find a better upper bound than the one provided by the tailored Benders decomposition after more than 4 hours.

The comparison of the gap evolution of both algorithms is shown in Fig. 4. The Benders' cuts used in the tailored Benders decomposition to approximate the second-stage costs are highly ineffective to close the gap beyond 0.7%, which is depicted by the slow progress of the gap reduction. This effect is even more significant when dealing



(a) System demand.

(b) Solar generation scenarios.

Figure 1: Uncertainty scenarios.

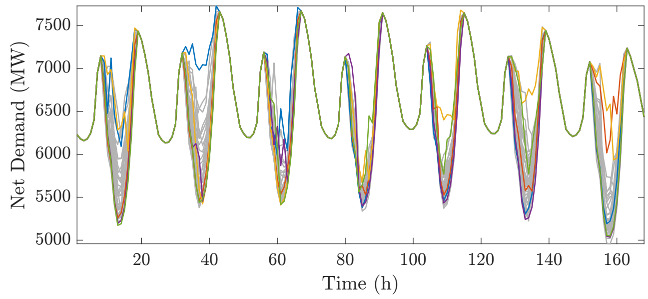


Figure 2: Scenario selection.

with larger systems, as the number of constraints needed to approximate the second-stage costs increases. That is, in larger systems the Benders' cuts start being ineffective even at earlier iterations with higher gaps. Conversely, the primal constraints enforced in the proposed approach provide an effective way of reducing the gap by providing a better approximation of the second-stage costs in the master problem.

Additionally, we compare the performance of the proposed decomposition against a commercial solver. Using CPLEX 12.10.0 to solve the extensive formulation of the problem takes 2.44 hours to compute the root relaxation and the gap of the first integer solution is 91.02%. Within the same time period, the proposed decomposition is able to compute an integer solution within 0.5% optimality gap.

4.3. Deterministic vs. Stochastic Model

We compare the solution of the proposed risk-constrained model with respect to the solution of a deterministic model. To this end, we consider three cases:

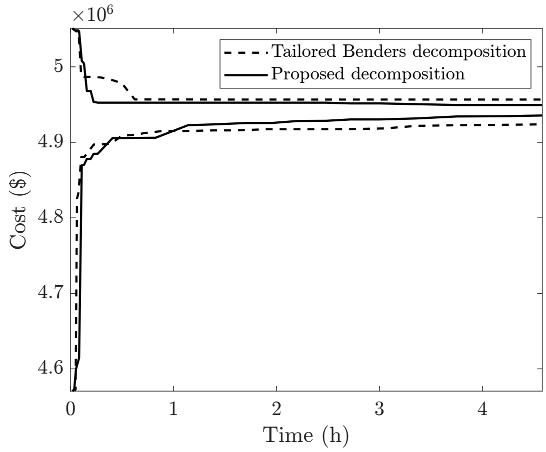


Figure 3: Comparison of algorithms' bounds evolution.

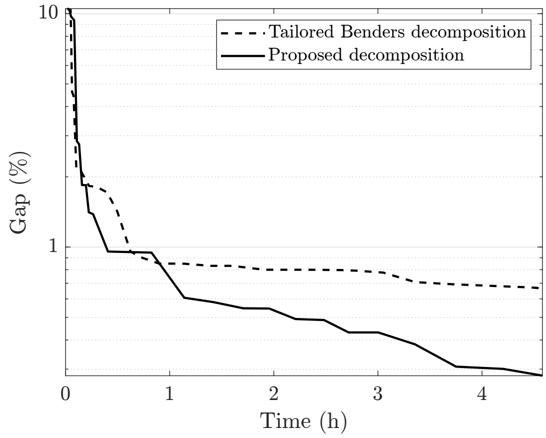


Figure 4: Comparison of algorithms' gap evolution.

- Deterministic. This case models the capacity of uncertain weather-dependent generation as forecasted values. We consider that such forecasted values are determined as the expected value of the scenarios considered for the stochastic model.
- Risk neutral ($\beta = 0$). This case corresponds to the solution of the two-stage stochastic problem. The decision maker aims at minimizing the scheduling and dispatch costs without considering any risk constraint.
- Risk averse ($\beta = 0.7$). This case enforces a high level of risk aversion to hedge against high second-stage costs (i.e., dispatch costs) due to uncertain weather-dependent generation.

To compare the solution's quality of the deterministic model with respect to the risk-neutral and risk-averse models, we solve the deterministic problem and then fix the scheduling decisions in the risk-neutral stochastic model.

In Table 1, we present a detail of the costs for the three cases. The model approach results in a more expensive solution, in terms of scheduling cost, balancing cost, and CVaR of the cost, with respect to the risk-neutral problem.

The risk-neutral case shows the lowest total costs, however, its risk is moderate with respect to the risk-averse counterpart. The risk-averse case reduces the CVaR by increasing the scheduling and balancing costs. Despite of having the same scheduling costs, the CVaR of the deterministic and risk-averse cases differ by 1.028 million dollars.

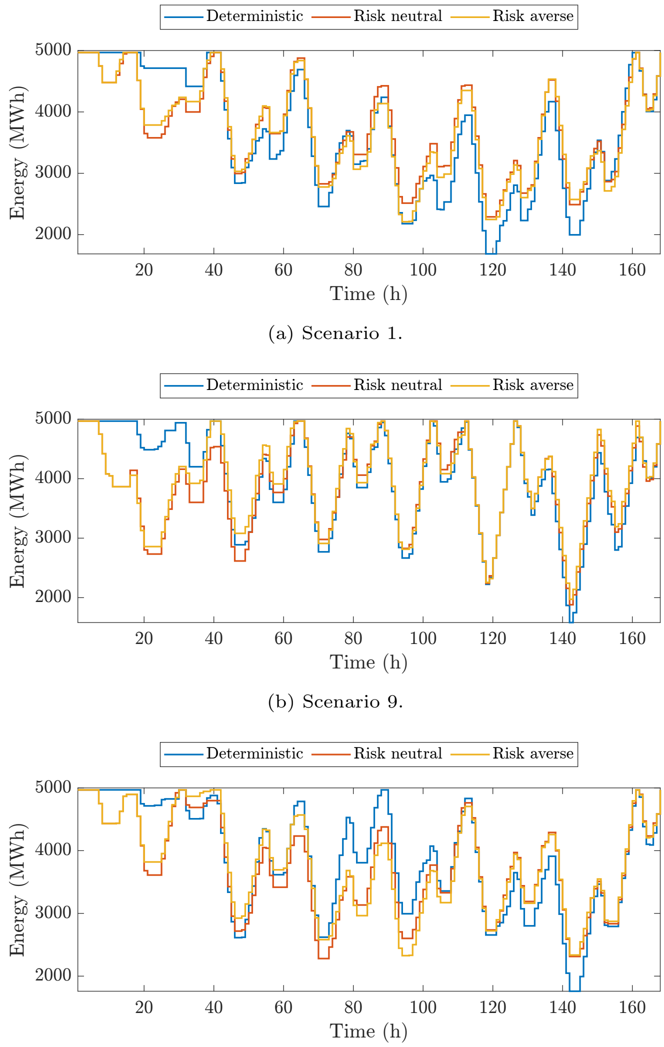
One of the advantages of the risk-averse model is its ability to reduce the unserved demand, which is materialized when insufficient generation capacity is committed. Such ability is highly desirable in the daily operation of power grids. In Table 2, we present the average and maximum load shedding for the three cases. The latter one corresponds to the highest unserved demand seen in the scenarios. The deterministic case shows the worst performance in terms of mean and maximum load shedding. In the two stochastic cases, the maximum load shedding is the same. The risk-averse case is able to reduce the expected unserved demand by 82% and 49% with respect to the deterministic and risk-neutral cases, respectively.

We show the energy content of the storage unit in Fig. 5 for three selected scenarios (scenarios 1,9, and 20). The scheduling dispatch of the energy storage units shows a clear charging/discharging pattern. During the peak demand hours and lower weather-dependent generation variability, the storage units are discharged to supply electricity whereas the unit is charged in the hours of less demand, and high solar generation availability and variability.

It is noteworthy that the deterministic model shows a deeper discharge of the storage unit with respect to the stochastic case in the three selected scenarios, which could be detrimental on the system's capacity to respond against contingencies or uncertainties.

In Fig. 6, we present the charging/discharging pattern of the three cases for a selected scenario (scenario 1). The different levels of risk aversion impact the scheduled dispatch of the storage units. For instance, the risk averse case shows the highest usage of the storage units in terms of the charging cycles. The risk-neutral case shows the shallowest charge/discharge pattern since the unit is never charged and discharged at its maximum power level.

Note that despite not using binary variables or penalty terms to avoid the simultaneous behavior of charging and discharging the energy storage units [35], our numerical results do not show simultaneous charging/discharging behavior for any time period or risk-aversion level. Such simultaneous behavior is uneconomical since the round-trip efficiency of pumped-hydro storage is less than 80%. Additionally, we do not consider network congestion since the network is not modeled.



(a) Scenario 1.

(b) Scenario 9.

(c) Scenario 20.

Figure 5: Energy content of storage unit.

5. Conclusions

This paper proposes a hybrid decomposition approach to address the weekly risk-constrained stochastic unit commitment with energy storage. The proposed solution technique relies on a multi-cut Benders framework to decompose the large-scale problem into a mixed-integer linear master problem and many linear continuous subproblems. Inspired by the effectiveness of the column-and-constraint generation algorithm, we improve the communication between the master problem and the subproblems by enforcing primal constraints corresponding to a selected subset of scenarios. Also, we provide a strategy to select such subset of scenarios based on solving an inexpensive mixed-integer optimization problem.

Using an instance of the South Carolina synthetic system with 40 scenarios, we numerically show the effectiveness of the proposed solution approach with respect to a tailored Benders decomposition, which enforces a subset of scenarios in the master problem and refines it only using Benders' cuts. The proposed approach outperforms the performance of the tailored Benders decomposition and attained a high-quality upper bound in less than 15 minutes in the studied instance.

In future work, we plan to extend the proposed solution strategy to address the network- and risk-constrained

Table 1: Total generation scheduling costs

Model	Total cost (thousands of $)	Scheduling cost (thousands of $)	Expected balancing cost (thousands of $)	CVaR (thousands of $)
Deterministic	23,387	8,116	15,270	16,567
Risk neutral	23,142	8,040	15,102	15,915
Risk averse	23,588	8,130	15,270	15,538

Table 2: Total unserved demand

Model	Mean (MW)	Maximum (MW)
Deterministic	1.72	259.42
Risk neutral	0.61	225.63
Risk averse	0.31	225.63

stochastic unit commitment problem. In such problem, the proposed active set strategy to speed up the solution time of the decomposition method can be used not only on the generation ramping constraints but also on the transmission capacity limits.

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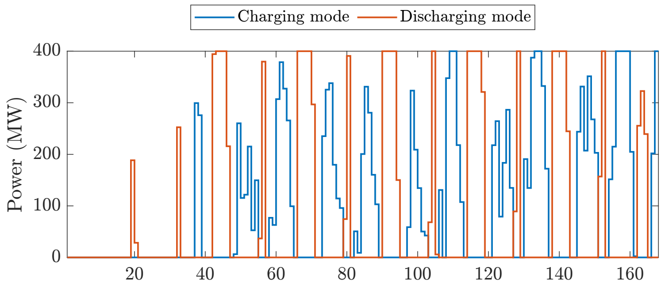
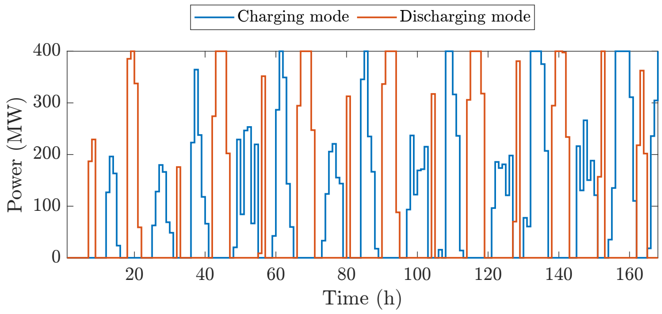
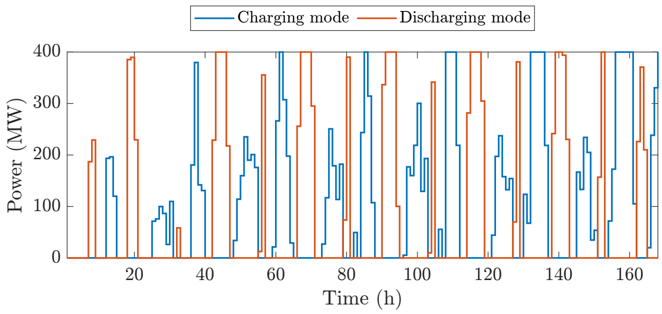


(a) Deterministic.



(b) Risk neutral.



(c) Risk averse.

Figure 6: Demand/output of storage unit.

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