

Characterization of the mean flow field in the far wake region behind ocean current turbines

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1 **Characterization of the mean flow field in the far wake region behind**
2 **ocean current turbines**

3 This paper forms, optimizes, and evaluates three numerical approaches for
4 characterizing mean velocities in far wake region behind ocean current turbines.
5 These approaches are derived from wake models originally developed for wind
6 turbines and are referred here as the Larsen/Larsen, Larsen/Ainslie, and
7 Jensen/Ainslie approaches based on the researchers originally credited with
8 developing the expressions for dependence of the mean wake velocity on centerline
9 and/or radial locations. The numerical coefficients utilized by these approaches are
10 optimized to best match Computational Fluid Dynamics (CFD) generated wake
11 velocity data. After optimizing the coefficients, this study finds that the
12 Larsen/Ainslie and Jensen/Ainslie approaches best match the CFD generated flow
13 data, with Larsen/Ainslie being the best match for an ambient turbulence intensity
14 (TI) of 3% and Jensen/Ainslie being the best match for TIs of 6% and 9%.

15 Keywords: marine renewable energy; ocean current turbine; in-stream
16 hydrokinetic; wake models; velocity deficit; turbulence intensity

17 **1. Introduction**

18 Ocean currents with time averaged kinetic energy fluxes above 0.5 kW/m^2 can be found
19 along the western boundaries of most of the world oceans (VanZwieten et al. 2013), with
20 values exceeding 2.0 kW/m^2 in several areas (VanZwieten et al. 2015). Multiple research
21 teams are actively pursuing the extraction of this renewable energy resource. It has been
22 estimated that ocean current based electricity production can feasibly approach 163
23 TWh/year in the US (GeorgiaTech Research Corp. 2013), which is equivalent to 4% of
24 2014 US electricity production (US EIA, 2015). Furthermore, average kinetic energy
25 fluxes higher than 3.0 kW/m^2 can be found in the ocean currents off SE Florida (Machado,
26 VanZwieten, and Pinos 2016). Ocean current turbine (OCT) prototypes being developed
27 to harness this resource are now approaching the offshore testing stage (VanZweiten et
28 al. 2014). Off Florida, these systems will likely be moored to the sea floor using compliant

29 mooring systems in 300+ meters of water, and will operate in the top 100 meters of the
30 water column (VanZwieten et al. 2013). To enable rapid transition from single device
31 testing to farm scale deployments, research directly related to turbine arrays is needed.

32 In a turbine array, the performance of downstream turbines are affected by the
33 wake of upstream turbines because of the associated decreased flow speed and increased
34 turbulence intensities. Several experimental studies (Mycek et al. 2014; Myers and Bahaj
35 2011) have been carried out to study the propagation of wake created by a single turbine,
36 as well as in an array setting. Experimental studies have shown that velocity deficit due
37 to wake are felt beyond 10 rotor diameters (D) downstream and even up to 20 D
38 downstream (Mycek et al. 2014; Myers and Bahaj 2011; Maganga et al. 2010; Bahaj et
39 al. 2007). Therefore, it is important to quantify velocity deficit in a turbine's wake to
40 evaluate the dependence of downstream turbine performance on device spacing.

41 Experimental studies discussed above were conducted in flume tanks that
42 represent conditions where tidal turbines are subjected to prominent boundary effects.
43 However, OCTs will likely be attached to the sea bed using mooring cables and will
44 operate in water column away from the boundaries (VanZwieten et al. 2013; IHI Corp.
45 2014). Therefore, boundary effects will not significantly impact these systems.
46 Computational Fluid Dynamics (CFD) can be used as a surrogate for estimating flow
47 downstream from OCTs that operate in deep water where boundary effects can be
48 neglected.

49 CFD analysis using Reynolds-Averaged Navier–Stokes (RANS) equations have
50 been accurately used to predict the time averaged velocities in the wake of a marine
51 turbine (Batten, Harrison, and Bahaj 2013). Furthermore, the commercial CFD software
52 package FLUENT has been used for simulating and analysing the fluid flow behind tidal

53 and wind turbines (Sun, Chick, and Bryden 2008; Mandas, Cambuli, and Carcangiu
54 2006).

55 Apart from CFD, several analytic wake models for quantifying the wake behind
56 wind turbines are also available (Jensen 1983; Larsen 1988; Frandsen et al. 2006). These
57 wake models contain mathematical expressions with empirical coefficients that were
58 originally estimated based on either wind turbine performance or experiments in wind
59 tunnels. Using these expressions, the mean velocity deficit downstream can be calculated
60 along the rotor centerline (Jensen 1983; Frandsen et al. 2006), as well as a function of
61 radial location from centerline (Larsen 1988; Ainslie 1988).

62 This paper presents, optimizes and evaluates three analytic wake approaches,
63 which are based on expressions originally developed to characterize wake behind wind
64 turbines. These approaches are used to define the mean axial flow speed in the far wake
65 region ($\geq 5 D$) behind OCTs. First, CFD software ANSYS FLUENT is used to simulate
66 wake flow behind a representative OCT model up to $10 D$ downstream. Then, empirical
67 coefficients of the considered wind wake models are optimized to best fit the centerline
68 velocity deficit from these CFD data. To predict wake velocity as a function of radial
69 distance from rotor centerline, existing wind wake models are modified and combined to
70 suit OCT wake profiles to form three analytic wake approaches. Finally, wake velocity
71 data calculated using these approaches are compared with CFD results as a function of
72 radial location from centerline.

73 **2. Wake analysis approaches**

74 Three approaches for quantifying mean axial wake velocity behind OCTs are formed in
75 this section. These approaches are referred to as Larsen/Larsen, Larsen/Ainslie and
76 Jensen/Ainslie; based on the names of the researchers who developed original wind

77 turbine wake models that are modified in this paper to form OCT wake models. The
 78 Larsen model originally presented in Larsen (1988) is referred as the Larsen/Larsen
 79 approach in this paper for naming consistency, whereas expressions from the existing
 80 Larsen (1988), Ainslie (1988) and Jensen (1983) models are utilized here to create
 81 analytical expressions for wake velocity as a function of downstream and radial locations.
 82 These new expressions are referred to as the Jensen/Ainslie and Larsen/Ainslie
 83 approaches. It is noteworthy that the actual Ainslie model is a numerical scheme which
 84 solves RANS whereas the Larsen/Ainslie and Jensen/Ainslie approaches we have
 85 proposed are analytical expressions that do not solve RANS. These approaches are
 86 created in the present paper to utilize advantages of Jensen, Larsen and Ainslie models in
 87 order to calculate wake velocity at radial location from the centreline. These modified
 88 algorithms are presented below.

89 ***2.1. Larsen/Larsen approach***

90 The numerical algorithm termed Larsen/Larsen in this paper bases both the wake
 91 dependence on downstream centreline distance and radial distance from the rotor's
 92 centreline on the work of Larsen (1988). The mean axial wake deficit behind wind
 93 turbines were estimated by Larsen (Larsen 1988) using the assumption that Prandtl's
 94 turbulent boundary layer equations apply and the mean wake flow is both incompressible
 95 and stationary. The equation for the mean axial velocity deficit is given by:

$$96 \quad 1 - \frac{U_w}{U_o} = \frac{1}{9} (C_T A_d x^{-2})^{\frac{1}{3}} \left(y^{\frac{3}{2}} (3c_1 C_T A_d x)^{-\frac{1}{2}} - \left(\frac{35}{2\pi} \right)^{\frac{3}{10}} (3c_1^2)^{-\frac{1}{5}} \right)^2, \quad (1)$$

97 where U_o is free-stream velocity, C_T is thrust coefficient, A_d is rotor area, c_1 is non-
 98 dimensional mixing length (empirical coefficient), and U_w is wake velocity
 99 corresponding to centerline distance x and radial distance y from centerline.

100 Equation 1 is a function of both centerline distance and radial location, but can be
 101 reduced to represent centerline velocity deficit if y is set to zero to obtain the expression:

102
$$U_c^* = 1 - \frac{V_x}{U_o} = \frac{1}{9} (C_T A_d x^{-2})^{\frac{1}{3}} \left(\left(\frac{35}{2\pi} \right)^{\frac{3}{10}} (3c_1^2)^{-\frac{1}{5}} \right)^2, \quad (2)$$

103 where V_x is the centerline velocity at a distance x downstream from the rotor and U_c^* is
 104 axial centerline velocity deficit.

105 If centerline velocity deficits at different downstream distances are available from
 106 CFD simulation for an OCT with a known/calculated thrust coefficient and rotor area,
 107 empirical coefficient c_1 can be optimized by using Equation 2 to best fit the CFD data.
 108 The optimized value of c_1 can then be substituted in Equation 1 to calculate velocity
 109 deficit at any radial location.

110 **2.2. Larsen/Ainslie approach**

111 The numerical algorithm termed Larsen/Ainslie approach in this paper utilizes the work
 112 of Larsen (Larsen 1988) to quantify the wake's dependence on centerline distance and
 113 the work of Ainslie (Ainslie 1988) to define its dependence on the radial distance from
 114 the rotor's centreline. Numerical solutions of time averaged Navier-Stokes equations
 115 were used to describe wake behind wind turbines by J.F. Ainslie (Ainslie 1988). This
 116 model is based on solving differential momentum equation using eddy viscosity
 117 turbulence model to calculate wake flowfield. The initial wake velocity deficit at radial
 118 distance r and centreline distance $2D$ is defined as (Ainslie 1988):

119
$$1 - \frac{U_w}{U_o} = U_{deficit}^* e^{\left(-3.56 \left(\frac{y^*}{b} \right)^2 \right)}, \quad (3)$$

120 where $U_{deficit}^*$ is the centerline velocity deficit at 2 diameter (D) downstream, y^* is radial

121 distance coordinate (from wake centerline) that has been made non-dimensional by
 122 dividing radial distance by rotor diameter, b is wake width parameter, and U_w is wake
 123 velocity corresponding to non-dimensional radial coordinate y^* .

124 The wake width parameter, b , in Equation 3 was calculated by Ainslie (1988)
 125 using:

$$126 \quad b = \left[\frac{3.56 C_T}{8 U_{\text{deficit}}^* (1 - 0.5 U_{\text{deficit}}^*)} \right]^{1/2}. \quad (4)$$

127 Equations 3 and 4 were only used for $2 D$ downstream by Ainslie (1988).
 128 Equation 3 makes wake velocity deficit follow a Gaussian profile as a function of
 129 radial location from centreline. Since wake velocity deficit at radial locations from
 130 centreline is known to follow Gaussian profile (Jensen 1983; Sanderse et al. 2011), this
 131 paper seeks to examine applicability of Equations 3 and 4 for all centreline distances, and
 132 not just $2 D$ downstream. Therefore, these equations are modified as follows to test their
 133 prediction capabilities at any centerline distance downstream:

$$134 \quad 1 - \frac{U_w}{U_o} = U_c^* e^{\left(-3.56 \left(\frac{y}{2r_o b} \right)^2 \right)}, \quad (5)$$

$$135 \quad b = \left[\frac{3.56 C_T}{8 U_c^* (1 - 0.5 U_c^*)} \right]^{1/2}, \quad (6)$$

136 where y is the radial location from turbine centreline, r_o is radius of turbine and U_c^* is the
 137 centerline wake velocity deficits for any centreline distance x (not just $2 D$). The approach
 138 to quantify velocity deficit as a function of radial location (Equation 5) utilized with the
 139 value of U_c^* obtained from Equation 2 is termed the Larsen/Ainslie approach.

140 **2.3 Jensen/Ainslie approach**

141 An expression for the centerline wake behind a wind turbine was also developed by
142 Jensen (Jensen 1983). The near field was not modelled and the far field wake was treated
143 as a negative jet. The mean centerline wake velocity was calculated by the Jensen model
144 according to:

145
$$V_x = U_o \left(1 - 2a \left(\frac{r_o}{r_o + \alpha x} \right)^2 \right), \quad (7)$$

146 where α is an empirical coefficient (generally taken as 0.01 for wind turbines), and a is
147 the axial induction factor. For the velocity distribution in radial location, a Gaussian or
148 bell shaped profile was suggested (Jensen 1983) without presenting any characterizing
149 analytical expressions.

150 Equations 7 can be re-arranged to calculate centerline wake velocity deficit, U_c^* ,
151 by utilizing the relationship between axial induction factor and thrust coefficient (Hansen
152 2008) as:

153
$$U_c^* = 1 - \frac{V_x}{U_o} = \frac{(1 - \sqrt{1 - C_T})}{\left(1 + \frac{\alpha x}{r_o} \right)^2}. \quad (8)$$

154 The Jensen model also defines the wake radius, r_x , as:

155
$$r_x = r_o + \alpha x. \quad (9)$$

156 If velocity deficits at different distances downstream behind OCT are generated
157 through CFD simulation for a known thrust coefficient and rotor diameter, empirical
158 coefficient α can be optimized to best fit the CFD simulated data. The determination of
159 α enables calculation of U_c^* using Equation 8 to find centerline velocity. The calculated

160 U_c^* can be plugged into Equation 5 to find velocity deficits at radial location as a function
161 of centerline distance. This algorithm is termed as Jensen/Ainslie approach.

162 Thus, both Larsen/Ainslie and Jensen/Ainslie approaches use Equation 5. The
163 Larsen/Ainslie approach uses the U_c^* value obtained from Equation 2 whereas
164 Jensen/Ainslie uses the U_c^* value obtained from Equation 8.

165 **3. CFD data generation and model validation**

166 To optimize the coefficients α (Jensen model) and c_1 (Larsen model), which are
167 associated with wake propagation as a function of centerline distance, CFD data are
168 utilized. CFD data are utilized because experimental wake data are not available for
169 turbines operating at a distance from boundaries where their effects are negligible. To
170 generate these data, the commercial CFD code FLUENT v15.0 is used to solve the
171 incompressible RANS equations using a second-order-accurate finite-volume
172 discretization scheme. The RANS method utilized is real 3-D. The shear stress transport
173 (SST) $k-\omega$ turbulence model is selected to model the turbulence terms of the RANS
174 equations. The SST $k-\omega$ turbulence model is able to model the transport of turbulent shear
175 stress. It provides accurate predictions of the onset and amount of flow separation under
176 adverse pressure gradients, and has been successfully used in the CFD simulation of
177 wind/water turbines (Lawson, Li, and Sale 2011; Lee et al. 2015). The details of the CFD
178 data generation methodology and validation based on experimental results available in
179 Bahaj et al. (2007) are presented in Tian et al. (2016), and briefly described here for the
180 completeness of this paper.

181 **3.1. Data generation methodology**

182 In this CFD simulation, the computational domain is sized to allow for full development

183 of the upstream flow and to decrease the blockage ratio so that boundary effects are
184 negligible. This is done to simulate flow past an OCT operating away from boundaries,
185 as opposed to a tidal turbine where blockage ratios are high and boundary effects are
186 important. The computation domain is cylindrical with a diameter of $5 D$ and a length of
187 $14 D$, where D is diameter of turbine rotor being simulated. The turbine is placed in the
188 centerline of the cylinder, at a distance of $4 D$ from the inlet (Figure 1). This figure shows
189 the dimensions of the domains, with the overall domain split into two subdomains. The
190 first subdomain contains the grid elements surrounding the rotor and the second
191 subdomain contains the cells in the outer region.

192 A uniform and steady velocity profile of 1.6 m/s, which is the measured mean
193 velocity of the Florida Current at a water depth of 25 m (Duerr and Dhanak 2013), is
194 applied at the inlet of the computation domain. This results in a rotor diameter based
195 Reynold's number of approximately 5×10^6 . A sliding mesh model is used to simulate
196 the rotation of the rotor. Each simulation required six revolutions to allow for
197 convergence. The mean performances of the rotor, including thrust and power, are
198 averaged from those calculated during the last revolution. The time step for each
199 simulation is set so that 1° of rotor rotation is achieved at each time step.

200 Using the data generation methodology described above, simulations were carried
201 out for a reference 20kW three bladed OCT model, with rotor diameter 3 m and hub
202 diameter 0.6 m. The airfoil shapes used by this rotor are created from the FF-77-W airfoil
203 type and other rotor details are available in Tian et al. (2016). In addition to the 3 m rotor
204 utilized to create the primary results presented in this paper, a 0.8 m rotor is simulated
205 using the same CFD approach to validate the utilized methodology. This simulated 0.8 m
206 rotor is designed to match the one used in experimental setup in Mycek et al. (2014).
207 Further validation of the CFD approach utilized here was carried by comparing the CFD

208 approach with experimental setup in Bahaj et al. (2007) and is provided in detail in Tian
209 et al. (2016).

210 **3.2. CFD validation**

211 The CFD methodology presented in this paper was validated based on experimental
212 studies of Bahaj et al. (2007) and Mycek et al. (2014). The performance of turbine in
213 terms of thrust and power coefficients were validated by simulating experimental setup
214 of Bahaj et al. (2007), and this validation is presented in detail in Tian et al. (2016).
215 Likewise, the validation of wake profile generated by the CFD is carried out by comparing
216 the wake profile of the CFD with the experimental studies presented in Mycek et al.
217 (2014). This validation study has not been previously published and is presented here.
218 For this study the CFD simulation of the 0.8 m diameter turbine and experimental
219 condition utilized in Mycek et al. (2014) is carried out to validate wake profile obtained
220 through CFD simulation. It is noteworthy that the CFD simulation of this turbine is carried
221 out only for the validation of CFD generated wake profile and is different than the 3 m
222 diameter OCT CFD simulations described in Section 3.1. It is noted that the diameter
223 based Reynold number used for the CFD simulation is 5×10^6 , whereas the diameter
224 based Reynolds number in the experiment range between 0.28×10^6 and 0.84×10^6 . An
225 experimental study has shown that for a crossflow hydrokinetic turbine device
226 performance became essentially diameter based Reynolds number independent above 0.8
227 $\times 10^6$, but that no significant wake velocity deficit Reynolds number dependence was
228 found for diameter based Reynolds numbers between 0.3×10^6 and 1.3×10^6 (Bachant
229 and Wosnik 2014). Therefore, it is likely that the difference between the experiment and
230 CFD Reynolds numbers will have minimal effect on device mean wake propagation.

231 Figures 2 and 3 are presented to compare wake flow calculated using CFD with
232 the experimental results published in Mycek et al. (2014) for an ambient turbulence
233 intensity (TI) of 15%. Figure 2 shows variation of centerline velocity deficit as a function
234 of downstream distance, x , which is normalized as x/D . An important observation in
235 Figure 2 is that the CFD analysis predicted a faster centerline velocity decay than the
236 experimental results. Figure 3 shows variation of wake velocity as a function of radial
237 distance, y , for normalized centerline distances, x/D , ranging from 1-10. Here, wake
238 velocity is normalized by dividing wake velocity by freestream velocity. The horizontal
239 straight lines in Figure 3 are error bars as published in Mycek et al. (2014). It is seen that
240 CFD results are capable of calculating the mean axial wake flow field with enough
241 accuracy to appropriately optimize wake analysis approaches presented in Section 2,
242 especially after the transition region from the near wake to far wake field (around 5 D).

243 **4. Coefficient optimization approach**

244 This section presents the optimization of the analytic wake model empirical coefficients
245 using CFD generated data. The wake profile for ambient turbulence intensities of 3%, 6%
246 and 9% are simulated using the CFD methodology presented in Section 3.1 and in detail
247 in Tian et al. (2016). It is noteworthy that these CFD generated data are based on different
248 turbine and boundary conditions than the ones used for validation in Section 3.2. The
249 validation was conducted simulating the experimental setup of Mycek et al. (2014), where
250 the boundary conditions were more prominent and turbulence intensity was 15%.

251 The CFD data used for coefficient optimization are based on an OCT model
252 (Section 3.1) with the boundaries selected such that boundary effects are negligible.
253 These centerline velocity deficit data are utilized to optimize the coefficients α and c_1 in
254 the Jensen and Larsen models (Equations 2 and 8) respectively. For a given ambient

255 turbulence intensity, the root mean square error (*rmse*) for downstream distances from
256 5 - 10 D , at increments of 1 D , is minimized by iteratively tuning α and c_1 , resulting in
257 values of α and c_1 that are considered to be optimal for each turbulence intensity. This
258 *rmse* is defined as:

259
$$rmse = \sqrt{(U_s^* - U_c^*)^2}, \quad (10)$$

260 where U_s^* represents centerline velocity deficits simulated using CFD and U_c^* are the
261 corresponding centerline velocity deficits calculated from Equations 2 and 8. An iterative
262 search algorithm is utilized to vary the coefficients α and c_1 to minimize the *rmse*. Thus,
263 optimal values of α and c_1 are found for ambient turbulence intensities of 3%, 6% and
264 9%.

265 **5. Results**

266 In order to generate wake flow field data, CFD simulations with ambient Turbulence
267 Intensities (TIs) equal to 3%, 6% and 9%, are performed. The 3 m diameter OCT model
268 briefly discussed in Section 3.1 and presented in detail in Tian et al. (2016) and Borghi et
269 al. (2012) is used for the CFD simulation. Figure 4 presents two-dimensional contours of
270 axial velocity at the longitudinal cross-section planes. It can be seen that the expansion
271 rate of the width of wake increases with the increase in turbulence intensity. This means
272 that a turbine will have a wider wake at higher turbulence intensities, which is consistent
273 with previous observations (Bahaj et al. 2007).

274 The aim of this paper is to develop computationally inexpensive analytic
275 expressions for calculating the wake velocity behind OCTs without having to use CFD
276 simulation. The model coefficients (α and c_1) are first obtained using CFD, but these
277 coefficients will be used independently without having to generate the values of these

278 coefficients from CFD. CFD generated profiles with low blockage ratios were selected
279 for this study since OCTs will operate in conditions that are minimally affected by
280 boundary conditions and experimental data are not available for these low blockage ratio
281 conditions. Hence, we present a generic setup that is mainly governed by ambient
282 turbulence since there are not sufficient studies examining the effects of ambient
283 turbulence on mean wake profile.

284 **5.1. Calculation of empirical coefficients**

285 Empirical coefficients (α and c_1) are optimized based on minimizing $rmse$ of centerline
286 wake velocity deficits obtained using Equation 10 for each Turbulence Intensity (TI). The
287 optimized coefficients are then plugged in Equations 2 and 8 to compare how well results
288 from these equations match the CFD data. Since 3 sets of TIs (TI= 3%, 6% and 9%) are
289 considered, 3 corresponding values of α and c_1 are obtained (Table 1).

290 It is seen that Larsen model (Equation 2) is the closest match to CFD results for
291 TI of 3%, with the $rmse$ values for Jensen and Larsen models being 0.021 and 0.019
292 respectively. Likewise, Jensen model (Equation 8) is closest match to CFD results for TIs
293 of 6% and 9%. The $rmse$ values for Jensen model for TI of 6% and 9% are 0.027 and
294 0.034 whereas the corresponding values for Larsen model are 0.032 and 0.043.

295 Our study shows that the empirical coefficients (α and c_1) are strongly affected
296 by TI and non-linear relationships exist between the empirical coefficients and TI (Figure
297 5). Since the orders of polynomials that relate TI and the values of the coefficients are not
298 clear yet, further experimental and/or numerical studies are required to characterize the
299 dependence of wake profile on TI. This will provide additional data points that will enable
300 the development of mathematical expressions that relate TI with the empirical
301 coefficients.

302 Figure 6 presents centerline velocity deficit results found using CFD and the
303 Jensen and Larsen models with the optimized coefficients (Table 1) for TIs of 3%, 6%
304 and 9%. It is noted that both wake models predict a slower convergence towards free
305 stream water velocities, and thus more persistent wake profiles, than the CFD generated
306 data. Similar observation is made in Section 3.2 where CFD predicted faster convergence
307 towards free stream than experimental results (Mycek et al. 2014). Therefore, the
308 discrepancy between decay rate from CFD results and wake models do not necessarily
309 indicate a limitation of the analytic models but may also indicate limitation of RANS
310 based simulation of wake profiles. However, as no experimental data of OCT that operate
311 in deep water where boundary conditions can be neglected exist as of now, the present
312 study has utilized RANS simulation despite its potential limitation.

313 Overall, it is to be noted that Jensen and Larsen models were originally developed
314 for wind turbines but the results in Figure 6 show that these models can also be utilized
315 for hydrokinetic turbines after optimizing empirical coefficients of these models to suit
316 for hydrokinetic turbines.

317 **5.2. Evaluation of wake approaches**

318 Comparison of wake velocity profiles at radial locations for six downstream centerline
319 distances calculated using the wake approaches (see Section 2) and CFD are presented
320 here. These wake profiles are quantified in terms of normalized velocity, U_w/U_o , for
321 centerline distances from $5 D - 10 D$. Figures 7-9 present the results for TI of 3%, 6%
322 and 9% respectively. Radial location is presented as a normalized parameter y/D , where
323 y is radial distance from the centerline. Similarly, axial downstream distance (centerline)
324 is presented as x/D .

325 It can be noted from Figures 7-9 that Larsen/Ainslie and Jensen/Ainslie approach
326 predict very similar velocity profiles as a function of radial locations because they both
327 base their radial dependence on Equation 5. Both these approaches better match the CFD
328 data than Larsen/Larsen approach suggesting that modified expressions from Ainslie
329 model predict the dependence on radial location for ocean current turbines more
330 accurately than the Larsen model. The horizontal straight lines shown in Figures 7-9 are
331 wake radii defined by Jensen model using Equation 9. This wake radius is seen to provide
332 a reasonable estimate of the radial cut-off point beyond which wake approaches cannot
333 be used.

344 It is observed that the wake velocity is lowest at the turbine centerline and velocity
345 gradually increases with radial distance (Figures 7-9), eventually reaching back to free
346 stream value. Nearly axisymmetric Gaussian curves of wake velocity profiles are
347 observed in Figures 7-9. As the downstream distance x/D increases, wake velocity
348 recovers closer to free-stream and the Gaussian shape gets less prominent. For TI of 9%
349 and $x/D = 10$ (Figure 9), it is seen that the wake velocities at radial locations are almost
350 a constant as Gaussian shape has turned into a near straight line. This is due to the fact
351 that wake velocity has nearly recovered to free-stream.

352 The $rmse$ averaged over $5 - 10 D$ for radial locations within radius defined by
353 Jensen model (Equation 9) at TI of 3% are 0.34, 0.34 and 0.46 for Jensen/Ainslie,
354 Larsen/Ainslie and Larsen/Larsen approaches respectively. It is noteworthy that $rmse$ in
355 this section is mean error/differences at radial locations and not centreline locations as in
356 Equation 10. These $rmse$ values are calculated using CFD results as baseline. For TI of
357 6%, average $rmse$ values for Jensen/Ainslie, Larsen/Ainslie and Larsen/Larsen
358 approaches are 0.16, 0.17 and 0.28 whereas the corresponding values for TI of 9% are
359 0.16, 0.19 and 0.27.

350 **6. Conclusions**

351 Two new approaches, Larsen/Ainslie and Jensen/Ainslie, are presented in this
352 paper which utilize analytical expression from Larsen, Jensen and Ainslie wind wake
353 models. Our study indicates that these wind turbine based analytic wake models can be
354 applied to ocean current turbines after tuning the empirical coefficients present in the
355 wind wake models. The present study calculates the empirical coefficient values for three
356 ambient turbulence intensities and shows that a non-linear relationships exists between
357 these empirical coefficients and ambient turbulence intensity.

358 CFD analyses are used to generate wake velocity flow data behind an OCT. These
359 data are used to optimize coefficients α and c_1 of Jensen and Larsen models. The
360 optimized coefficients are then utilized in three presented approaches to characterize
361 wake, both as a function of centerline and radial locations.

362 The Larsen/Ainslie and Jensen/Ainslie approaches are found to be the closest
363 matches to the CFD generated data for TI of 3% with equal *rmse* values, whereas
364 Jensen/Ainslie approach is found to be the best match of the CFD generated data for TI
365 of 6% and 9%. Overall, Jensen/Ainslie approach is found to be most suitable to predict
366 wake velocity behind OCT.

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372 **References**

373 Ainslie J (1988) Calculating the flowfield in the wake of wind turbines. *J. Wind*
374 *Eng.&Industrial Aerodynamics*, 27, 213-224.

375 Bachant P, Wosnik M (2014) Reynolds number dependence of cross flow turbine
376 performance and near-wake characteristics. Proc. 2nd Marine Energy Technology
377 Symposium, METS 2014, Seattle, WA.

378 Bahaj A, Molland A, Chaplin J, Batten W (2007) Power and thrust measurements of
379 marine current turbines under various hydrodynamic flow conditions in a
380 cavitation tunnel and a towing tank. Renewable Energy, 32, 407-26.

381 Bahaj AS, Myers LE, Thomson MD, Jorge N (2007) Chararcterizing the wake of
382 horizontal axis marine current turbines. Proc. 7th European Wave Tidal Energy
383 Conference, Porto, Portugal.

384 Batten WMJ, Harrison ME, Bahaj AS (2013) Accuracy of the actuator disc-RANS
385 approach for predicting the performance and wake of tidal turbines. Philosophical
386 Transcations A, The Royal Society Publishing.

387 Borghi, M, Kolawole F, Gangadharan S, Engblom A, VanZwieten JH, Alsenas G, Baxley
388 W, Ravenna S (2012) Design, fabrication and installation of a hydrodynamic rotor
389 for a small-scale experimental ocean current turbine. Proc. IEEE SoutheastCon,
390 Orlando, FL, 2012.

391 Duerr AES, Dhanak MR (2010) Hydrokinetic power resource assessment of the Florida
392 Current. Proc. MTS/IEEE Oceans Conference, Seattle, September 20-23, 2010.

393 Frandsen S, Barthelmie R, Pryor S, Rathmann O, Larsen S, Højstrup J, Thøgersen M,
394 (2006) Analytical Modelling of Wind Speed Deficit in Large Offshore Wind
395 Farms. Wind Energy, 40-48.

396 GeorgiaTech Research Corp (2013) Assessment of Energy Production Potential from
397 Ocean Currents along the United States Coastline. Atlanta, GA.

398 Hansen, MOL (2008) Aerodynamics of Wind Turbines. Earthscan, pp. 31-33.

399 IHI Corporation (2014) Power Generation Using the Kuroshio Current: Development of
400 floating type ocean current turbine system. IHI Engineering Review, Vol. 46, No.
401 2.

402 Jensen N (1983) A note on wind generator interaction. Risø National Laboratory,
403 Roskilde, Denmark.

404 Larsen G (1988) A Simple Wake Calculation Procedure. Risø National Laboratory,
405 Roskilde, Denmark.

406 Lawson M, Li Y, Sale DC (2011) Development and verification of a computational fluid
407 dynamics model of a horizontal-axis tidal current turbine. Proc 30th International

408 Conference Ocean, Offshore, Arctic Engineering, Rotterdam, Netherlands, June
409 19-24, 2011.

410 Lee NJ, Kim IC, Kim CG, Hyun BS, Lee YH (2015) Performance study on a counter-
411 rotating tidal current turbine by CFD and model experimentation. Renewable
412 Energy, 79,122-126.

413 Machado MCPM, VanZwieten JH, Pinos I (2016) A Measurement Based Analyses of the
414 Hydrokinetic Energy in the Gulf Stream. J Ocean Wind Energy, ISOPE, 3(1), 25-
415 30.

416 Maganga F, Germain G, King J, Pinon G, Rivoalen E (2010) Experimental
417 characterisation of flow effects on marine current turbine behaviour and on its
418 wake properties. IET Renewable Power Generation, vol. 4, no. 6.

419 Mandas N, Cambuli F, Carcangiu CE (2006) Numerical prediction of horizontal axis
420 wind turbine flow. Proc European Wind Energy Conference & Exhibition,
421 Athens.

422 Mycek P, Gaurier B, Germain G, Pinon G, Rivoalen E (2014) Experimental study of the
423 turbulence intensity effects on marine current turbines behaviour, Part I: One
424 single turbine. Renewable Energy, 66, 729-746.

425 Myers LE, Bahaj AS (2011) An experimental investigation simulating flow effects in first
426 generation marine current energy converter arrays. Renewable Energy, 37, 28-36.

427 Sanderse B, van der Pijl SP, Koren B (2011) Review of computational fluid dynamics for
428 wind turbine wake aerodynamics. Wind Energ. 2011; 14:799–819.

429 Sun X, Chick JP, Bryden IG (2008) “Laboratory-scale simulation of energy extraction
430 from tidal currents. Renewable Energy, Vol. 33, Issue 6, pp. 1267–1274.

431 Tian W, VanZwiten JH, Pyakurel P, Li Y (In Press) “Influences of yaw angle and
432 turbulence intensity on the performance of a 20kW in-stream hydrokinetic
433 turbine. Energy.

434 U.S. Energy Information Administration (2015) Annual Energy Review. Web.
435 <http://www.eia.gov/>. Accessed 1 June 2016.

436 VanZwieten JH, Baxley WE, Alsenas GM, Meyer I, Muglia M, Lowcher C, Bane J, Gabr
437 M, He R, Hudon T, Steven R, Duerr AES (2015) SS Marine Renewable Energy –
438 Ocean Current Turbine Mooring Considerations. Proc Offshore Technology
439 Conference, Texas, 2015.

440 VanZwieten JH, Duerr A, Alsenas G, Hanson H (2013) Global ocean current energy
441 assessment: an initial look. Proc Marine Energy Technology Symposium,
442 Washington D.C., April 10-11, 2013.

443 VanZwieten JH, McAnally W, Ahmad J, Davis T, Martin J, Bevelhimer M, Cribbs A,
444 Lippert R, Hudon T, Trudeau M (2014) In-Stream Hydrokinetic Power – A
445 Review and Appraisal. J Energy Engineering, no. 04014024.

446 Table 1. Empirical coefficients and *rmse* variation with TI

TI (%)	Equation 2		Equation 8	
	<i>a</i>	<i>rmse</i>	<i>c₁</i>	<i>rmse</i>
3	0.0325	0.0216	0.1178	0.0194
6	0.0477	0.0274	0.1656	0.0328
9	0.0679	0.0343	0.2450	0.0430

447

448 Figure 1. Domains, boundary conditions and mesh on the surface of the turbine

449 Figure 2. Centerline velocity deficit.

450 Figure 3. Normalized axial wake velocity at radial locations for different downstream
451 distances.

452 Figure 4. Contours of the axial velocity at the mid plane for TI=3% (top), 6% (middle)
453 and 9% (bottom).

454 Figure 5. Variation of model coefficients with TI

455 Figure 6. Axial velocity deficits for CFD and analytical models for turbulence
456 intensities of 3%, 6% and 9%.

457 Figure 7. Wake profiles at radial locations from CFD and wake approaches for TI=3%.

458 Figure 8. Wake profiles at radial locations from CFD and wake approaches for TI=6%.

459 Figure 9. Wake profiles at radial locations from CFD and wake approaches for TI=9%.