

LA-UR-23-25861

Approved for public release; distribution is unlimited.

Title: Rapid, Approximate Multi-Axis Vibration Testing

Author(s): Harvey, Dustin Yewell
Cramer, Ethan Savoy
Zhang, Richard

Intended for: Working Group

Issued: 2023-06-01



Los Alamos National Laboratory, an affirmative action/equal opportunity employer, is operated by Triad National Security, LLC for the National Nuclear Security Administration of U.S. Department of Energy under contract 89233218CNA00001. By approving this article, the publisher recognizes that the U.S. Government retains nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes. Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy. Los Alamos National Laboratory strongly supports academic freedom and a researcher's right to publish; as an institution, however, the Laboratory does not endorse the viewpoint of a publication or guarantee its technical correctness.

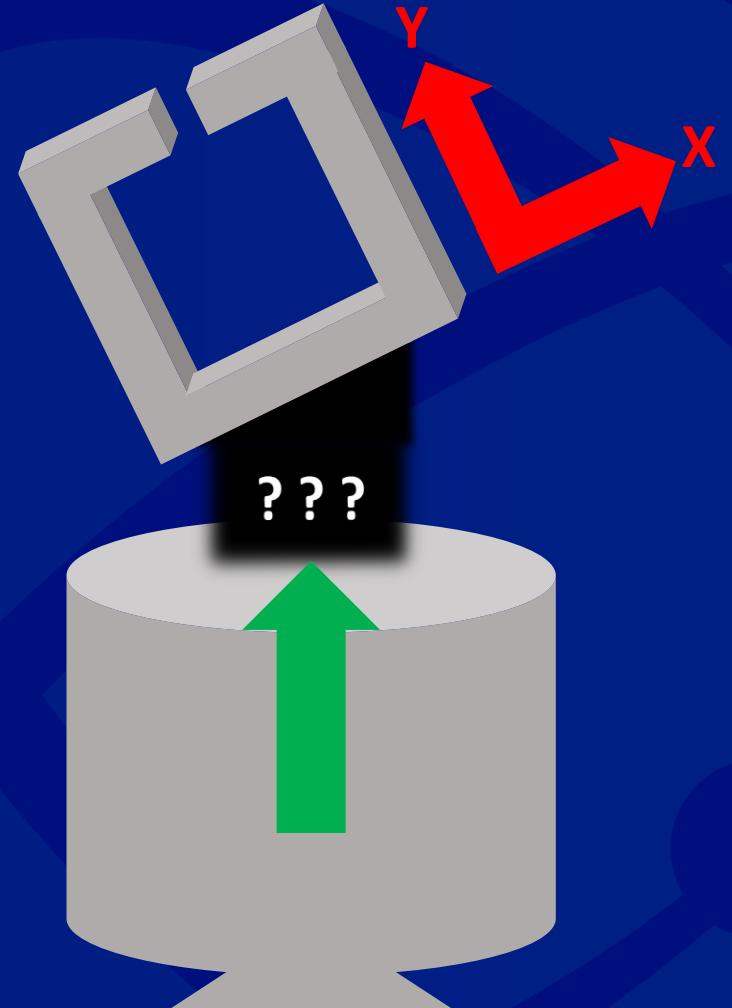
Rapid, Approximate Multi-Axis Vibration Testing

Dustin Harvey, LANL, E-14

Ethan Cramer, LANL, E-14

Richard Zhang, University of North Texas

LA-UR-TBD

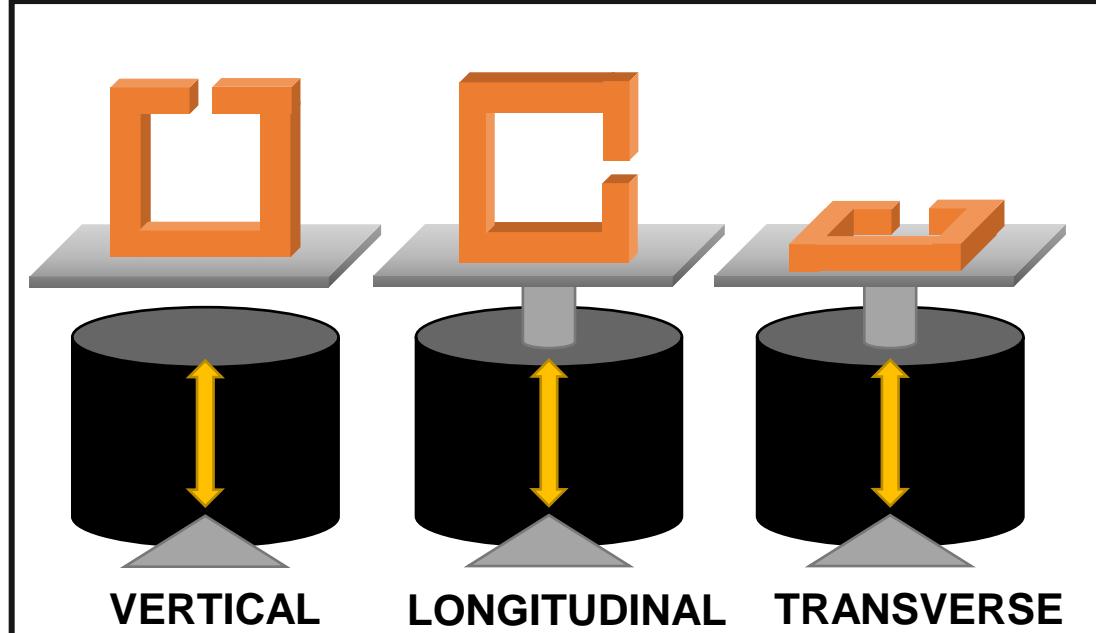


Problem:

Sequential single-axis testing provides a poor approximation of service environments.

Multi-axis vibration tests are not yet standard practice.

Sequential Single-Axis Testing

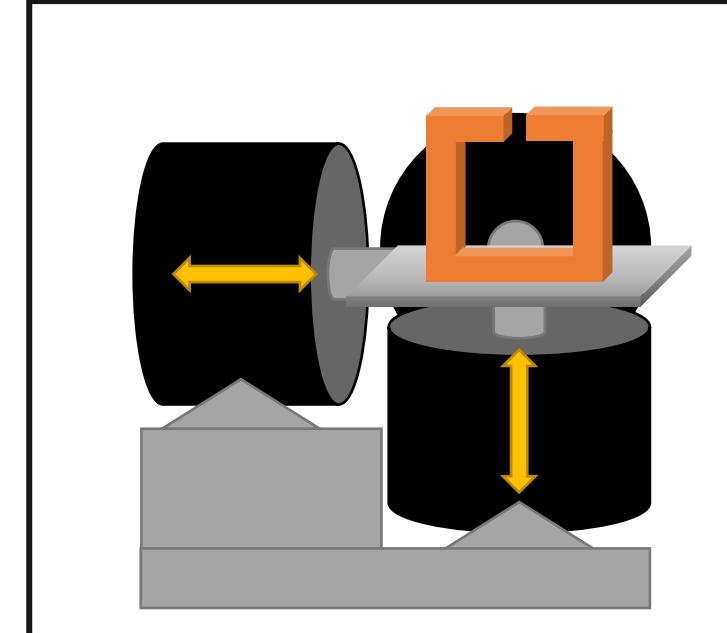


Question:

How good do multi-axis tests need to be to replace single-axis testing in practice?

Is the simplest possible multi-axis test close to single-axis testing in test quality?

Multi-Axis Testing



Problem:

Sequential single-axis testing provides a poor approximation of service environments.

Multi-axis vibration tests are not yet standard practice.

Question:

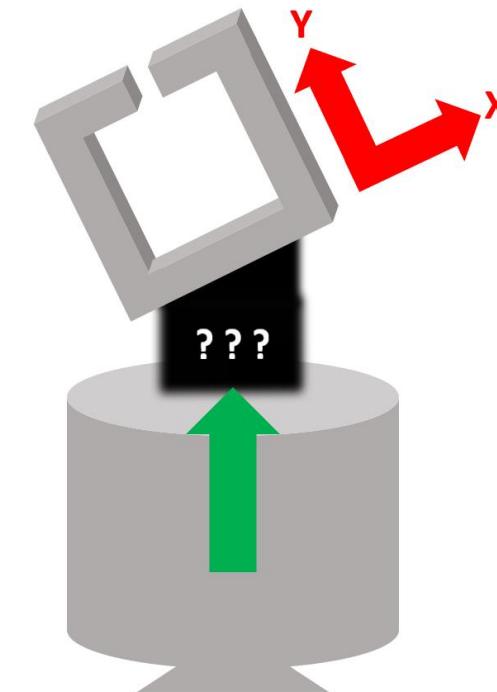
How good do multi-axis tests need to be to replace single-axis testing in practice?

Is the simplest possible multi-axis test close to single-axis testing in test quality?

Preliminary Study:

Can single-axis testing techniques provide a sufficient rapid approximation to a multi-axis test?

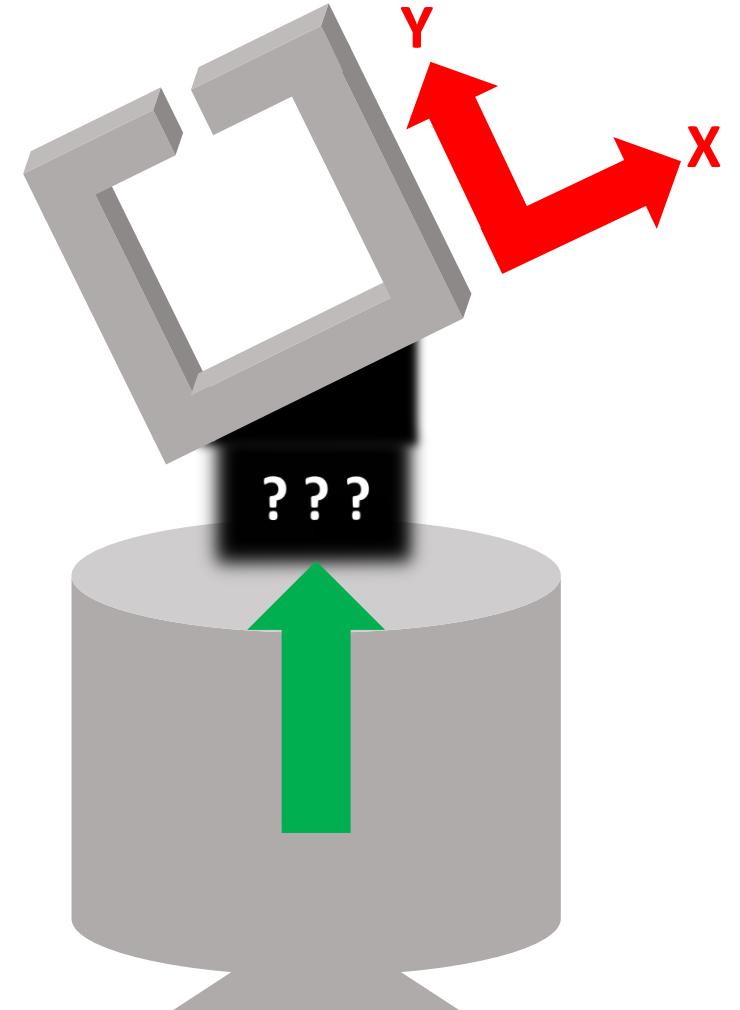
How much can we improve multi-axis test quality by optimizing passive test fixture hardware instead of adding active excitation sources?



Approach

Three steps to approximate a multi-axis test:

1. Employ **single-input, multiple output (SIMO)** test strategy with outputs in multiple axes
2. Optimize excitation angle
3. Optimize test fixture design for each test environment



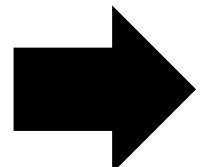
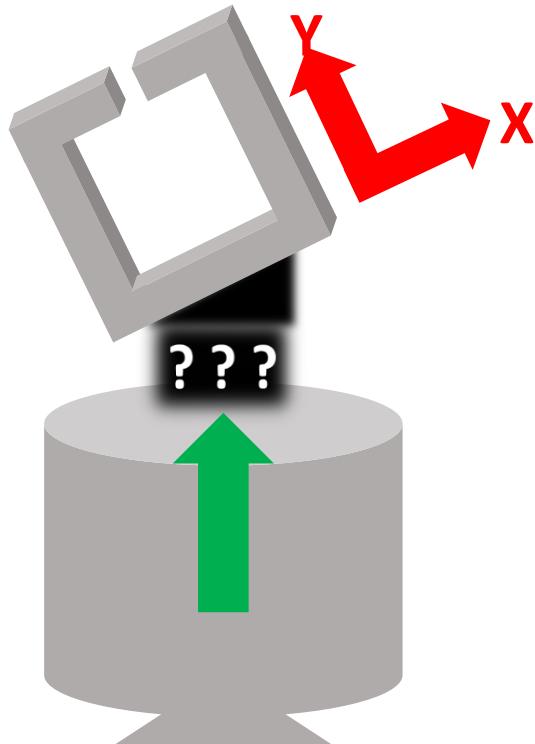
Objective

Assess viability of a rapid, approximate multi-axis vibration testing technique utilizing standard single-axis testing hardware.

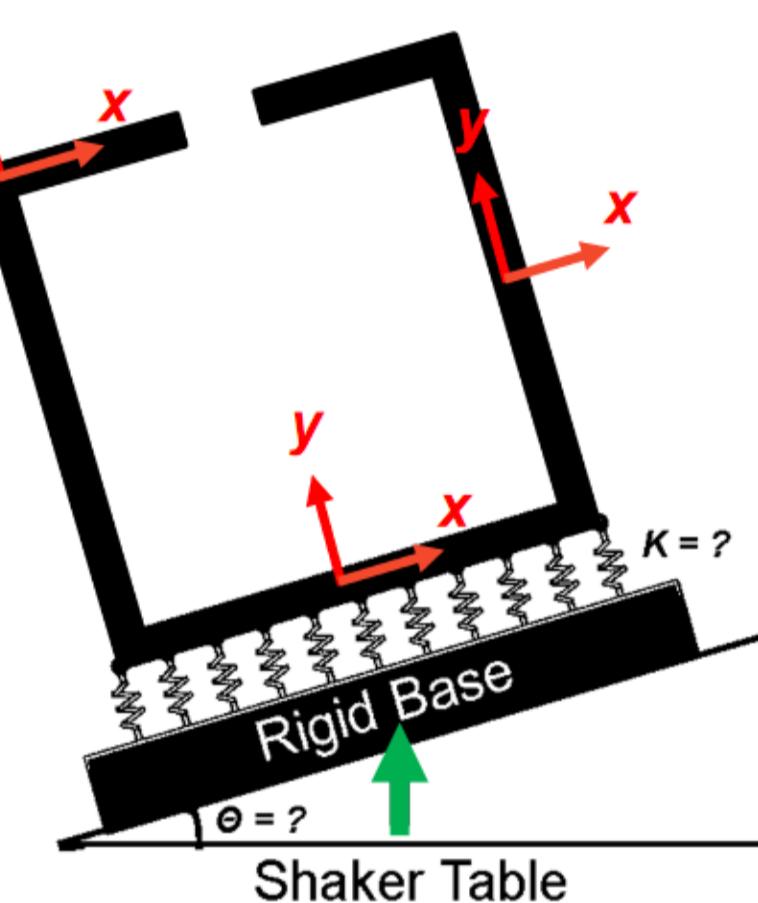
- Rapid Test Setup and Execution
 - Single test event instead of three sequential single-axis test setups and executions
 - Much simpler compared to multi-excitation schemes and MIMO control strategies.
- Approximate
 - Error is expected as in all multiple-output vibration tests.
 - How much can error be mitigated through test fixture design?
 - What level of error is acceptable compared to traditional single-axis testing practice?
- Multi-Axis
 - All response directions are intentionally excited by a single source by optimally orienting excitation axis for each test environment.

Simulation Study

The proposed method was modeled for a 2D BARC base test article as shown.



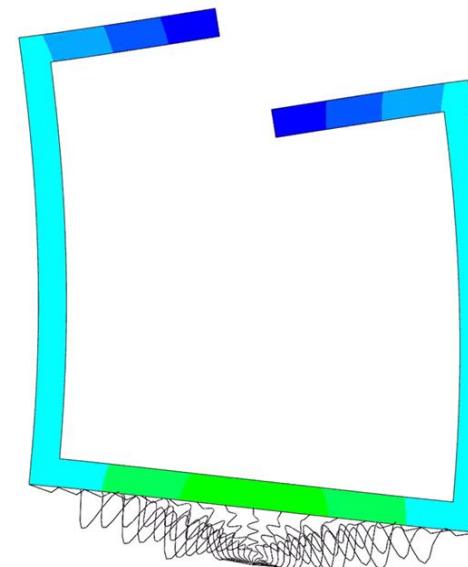
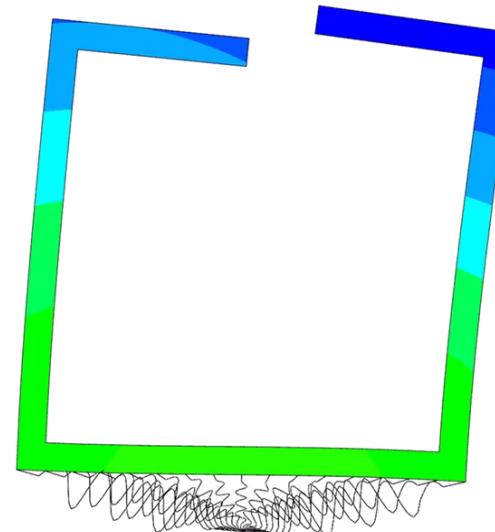
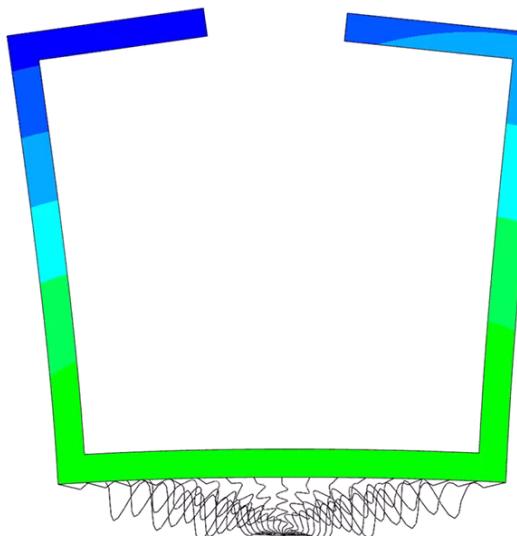
SIMO Multi-Axis Test



Simulation Study

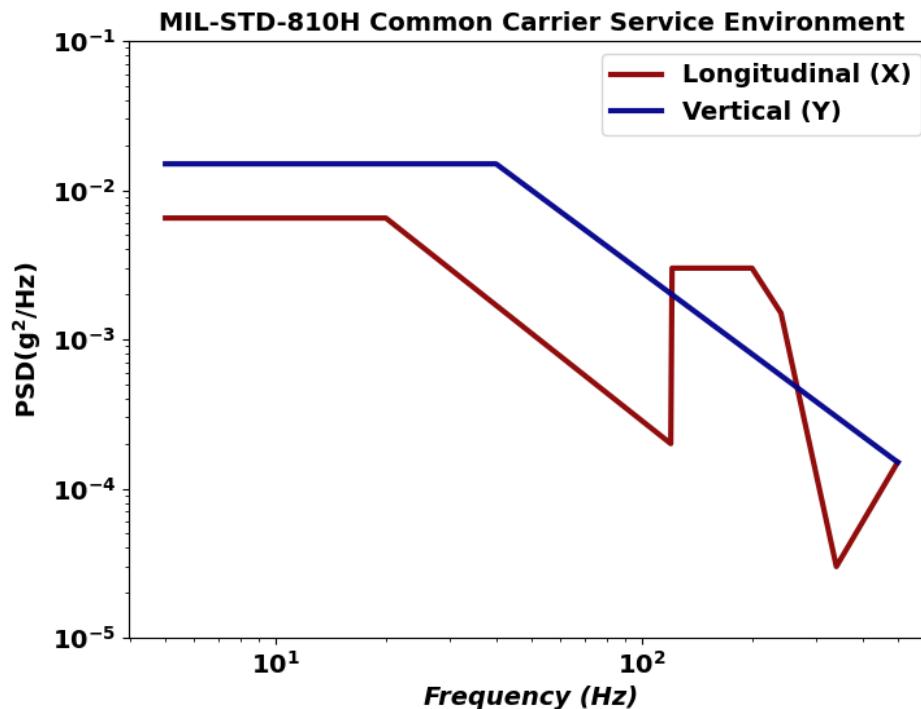
Simulations were carried out using an Abaqus FEA model consisting of:

1. 2-D BARC without the removable component
2. Test fixture, idealized by sixteen springs
3. Rigid base, representing a vertically-oriented, single-axis shaker



Service Environment

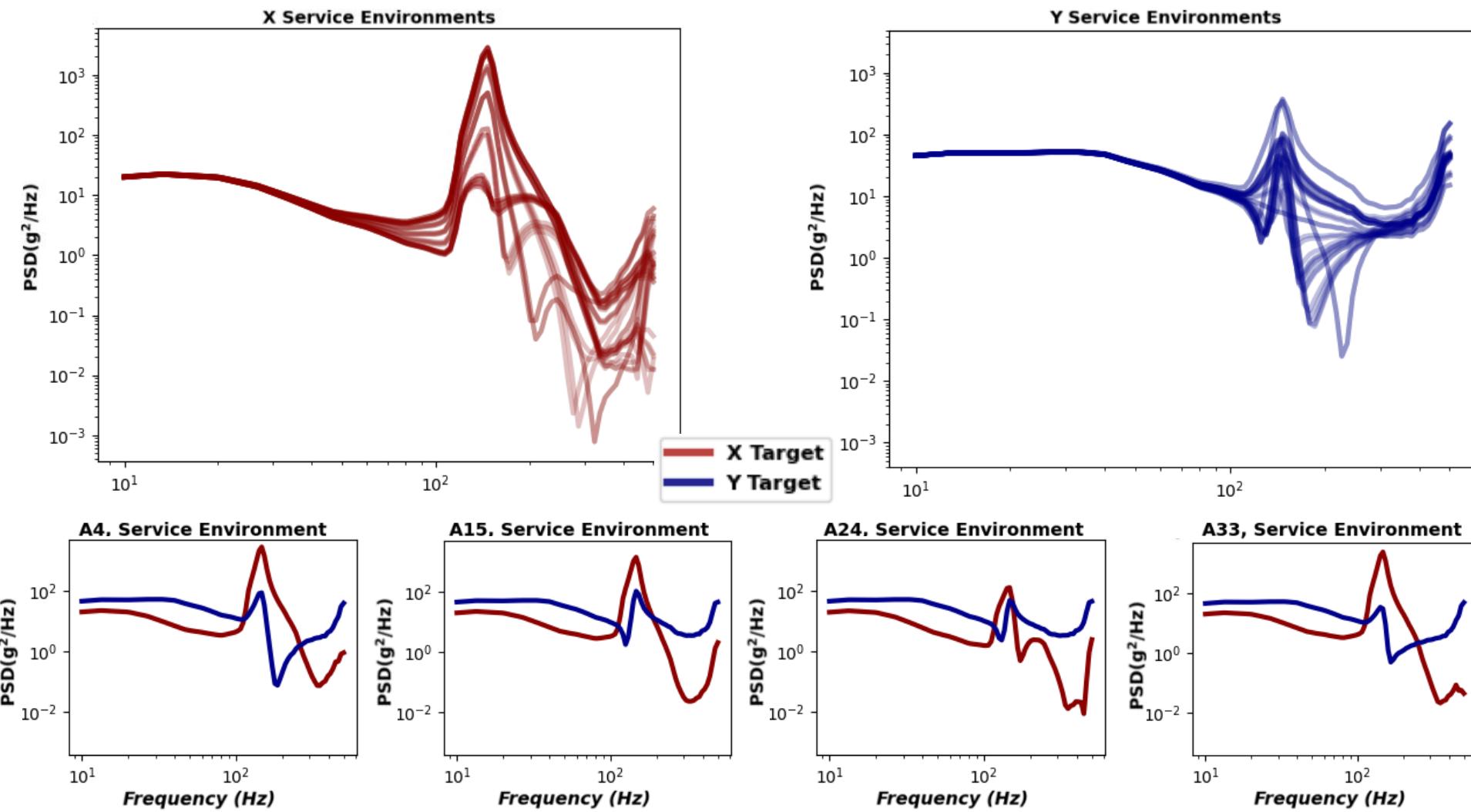
- A **service environment** is the real environment the test article is expected to experience in its lifetime defined as a set of target Power Spectral Densities (PSD) at one or more locations.



Defense Logistics Agency, 2019

- This a common transportation environment base excitation profile from MIL-STD-810H.
- Conducted MIMO simulation to generate a set of response targets from this base excitation...
 1. Apply X and Y excitations simultaneously to the base of the model.
 2. Measure acceleration responses on the BARC.
- Targets were generated using a flexible boundary condition (k_x and $k_y = 10^6 \text{ N/m}$).

Service Environment



Test Quality Metrics

RMS dB Error (RDBE):

RDBE is a scalar metric that represents the average mismatch between a response PSD and a target PSD across all frequencies.

$$RDBE = \sqrt{\frac{\sum_{i=1}^n (10 \log_{10} \frac{\hat{y}_i}{y_i})^2}{n}}$$

Percentage of Frequency Lines within a 3 dB Tolerance (FTOL):

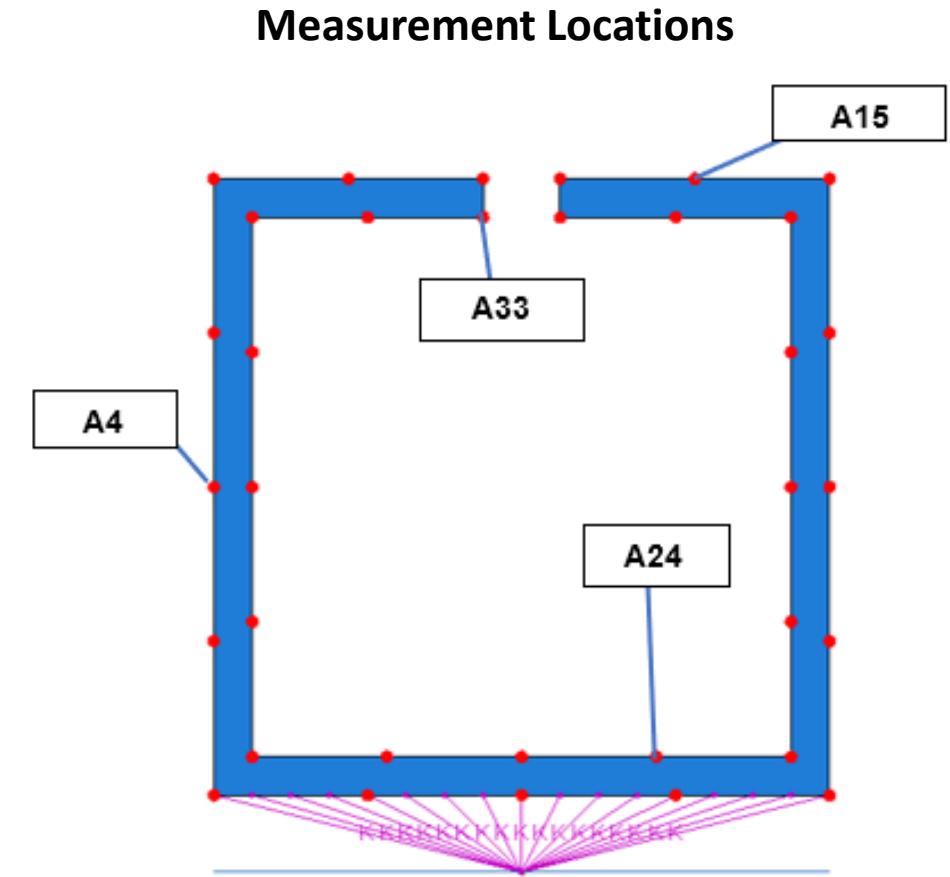
FTOL is a scalar metric that counts the percentage of frequency lines where the response PSD differs from the target PSD by less than 3 dB.

$$\% FL = 100 * \frac{1}{n} \sum_{i=1}^n \begin{cases} 1 & \text{if } \left| 10 \log_{10} \frac{\hat{y}_i}{y_i} \right| \leq 3 \text{ dB} \\ 0 & \text{if } \left| 10 \log_{10} \frac{\hat{y}_i}{y_i} \right| > 3 \text{ dB} \end{cases}$$

In both equations, \hat{y}_i is the value of a response PSD at the i^{th} frequency line, and y_i is the value of the target PSD at the i^{th} frequency line. There are n total frequency lines.

Test Control Simulation

- The locations where measured response is used to derive and control the shaker input PSD are **control locations**.
- The shaker input PSD was derived to achieve the lowest possible RDBE between a target response and measured response at selected control locations.
- **Single Control Location**
 - Common test practice
 - Successfully achieves target response at one location
- **All Control Locations**
 - Better overall test
 - Uncommon in practice due to specification and control software limits



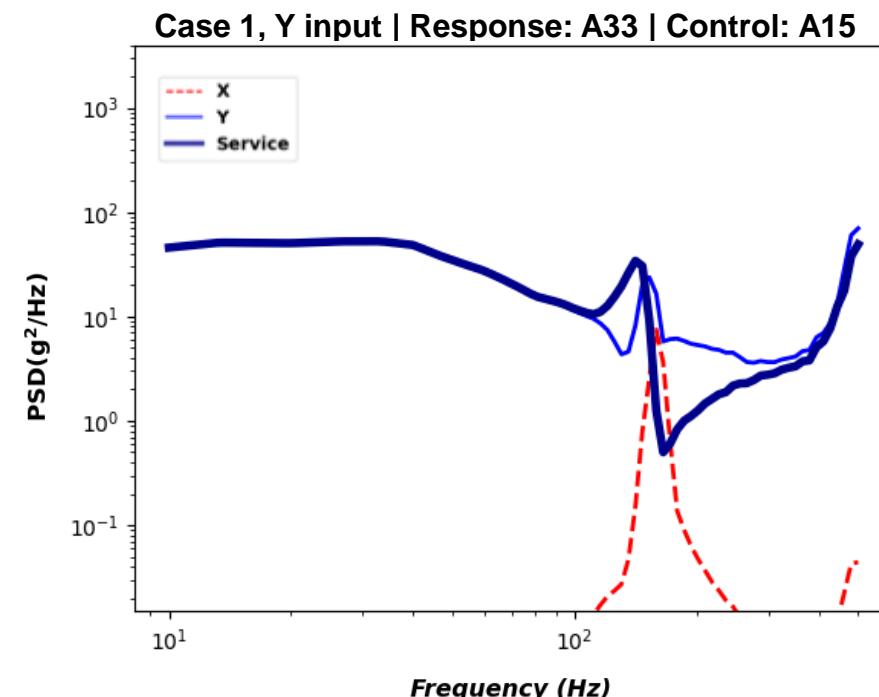
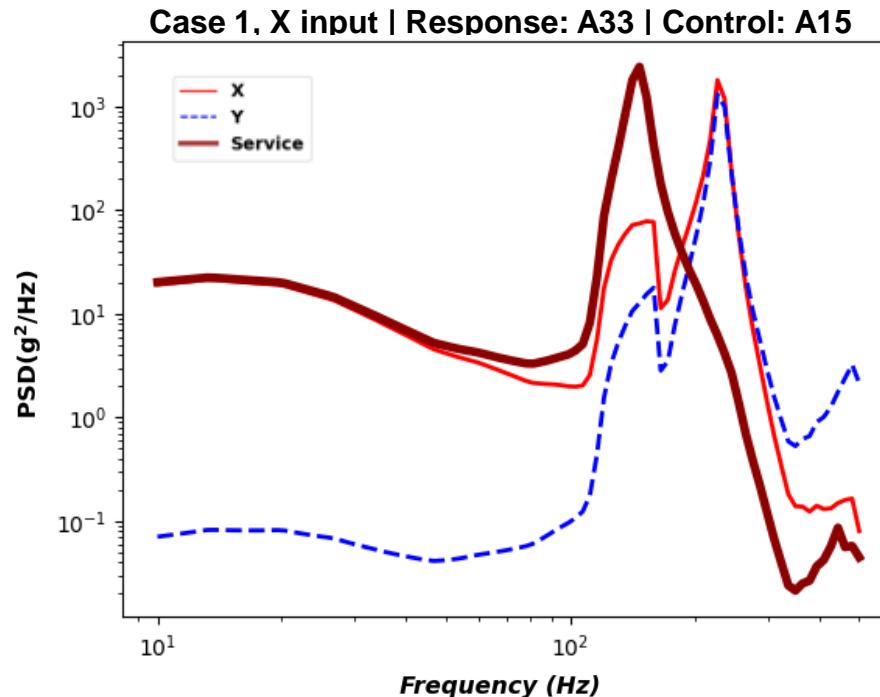
Test Control Simulation

- For single control location sequential tests, choosing the best location improves the RDBE by 2.2 dB and the FTOL by 3.9% over the worst location.
- In all cases, errors are presented as an average across all response locations.

| Sequential Single-Axis (Worst performing control location) | | | | Sequential Single-Axis (Best performing control location) | | | |
|---|------------------|------------------|-----------------------|--|---------------|------------------|-----------------------|
| | X-Error (avg) | Y-Error (avg) | <u>Mean Error</u> | | X-Error (avg) | Y-Error (avg) | <u>Mean Error</u> |
| RDBE | 9.9 dB | 3.2 dB | <u>6.6 dB</u> | RDBE | 5.6 dB | 3.2 dB | <u>4.4 dB</u> |
| FTOL | 64.6% | 83.9% | <u>74.3%</u> | FTOL | 71.6% | 84.7% | <u>78.2%</u> |

Test Control Simulation

With single point control, responses at non-control DOF may be highly undesirable.



Test Control Simulation

When the test is controlled to all locations, the RDBE decreases to **3.8 dB**.

Sequential Single-Axis (All control locations)

| | X-Error (avg) | Y-Error (avg) | <u>Mean Error</u> |
|-------------|------------------|------------------|-----------------------|
| RDBE | 5.1 dB | 2.6 dB | 3.8 dB |
| FTOL | 70.3% | 85.2% | 77.8% |

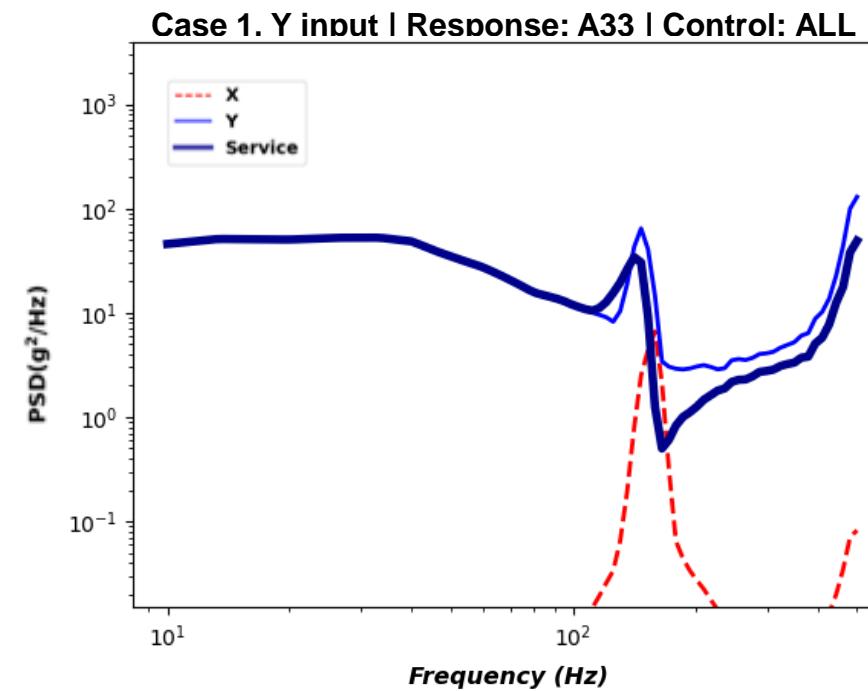
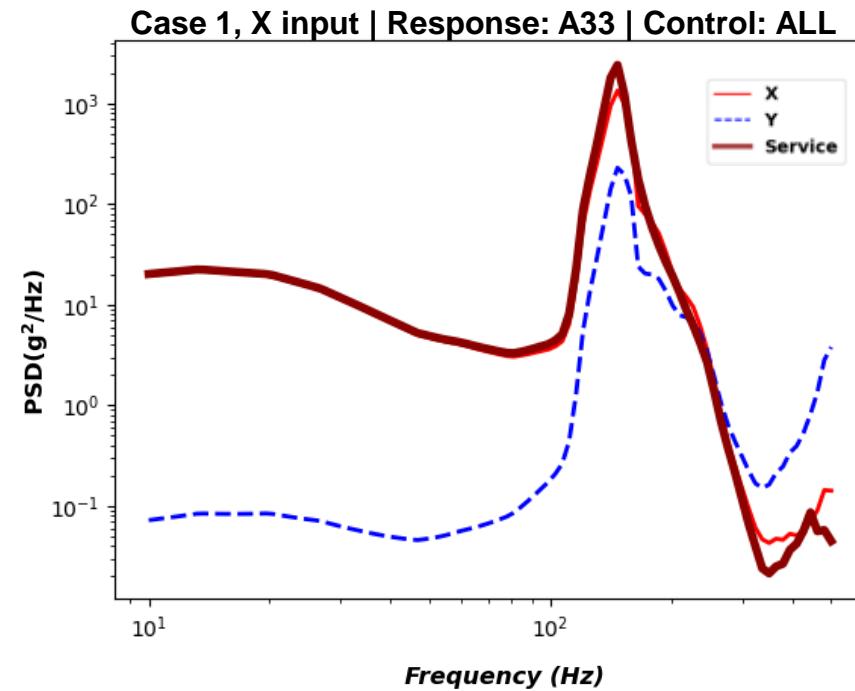
Sequential Single-Axis (Worst performing control location)

Sequential Single-Axis (Best performing control location)

| | X-Error (avg) | Y-Error (avg) | <u>Mean Error</u> | | X-Error (avg) | Y-Error (avg) | <u>Mean Error</u> |
|-------------|------------------|------------------|-----------------------|-------------|---------------|------------------|-----------------------|
| RDBE | 9.9 dB | 3.2 dB | 6.6 dB | RDBE | 5.6 dB | 3.2 dB | 4.4 dB |
| FTOL | 64.6% | 83.9% | 74.3% | FTOL | 71.6% | 84.7% | 78.2% |

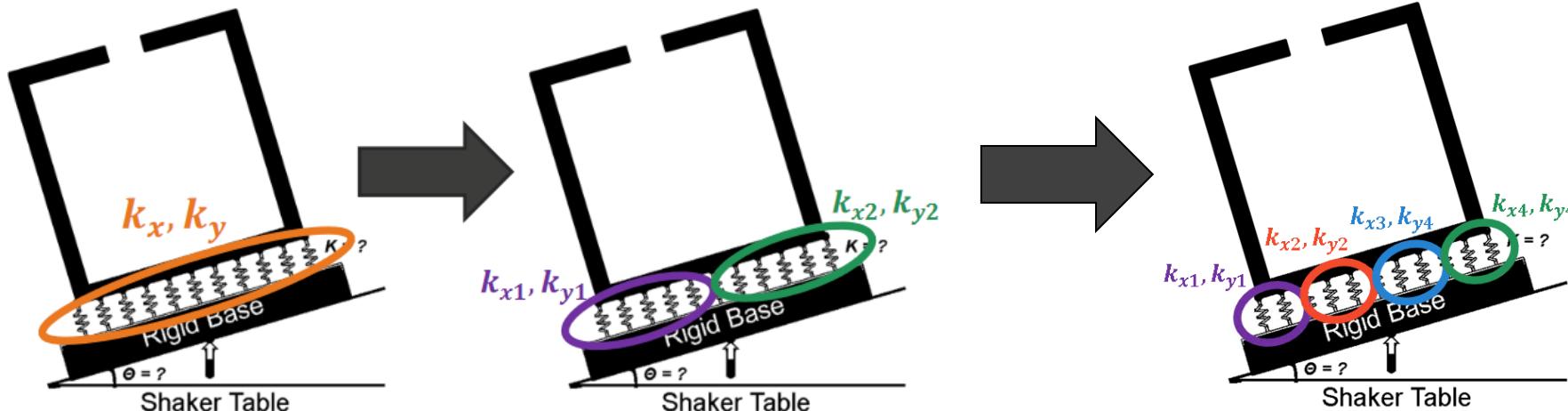
Test Control Simulation

The all control location strategy is used throughout the study to eliminate control location effects.



Case Studies

| Case | Test Fixture Design | Optimization Parameters | |
|------|--------------------------|---|---|
| | | Sequential Single-Axis | SIMO Multi-Axis |
| 1 | Rigid | - | θ |
| 2 | Optimized (2 parameters) | k_x, k_y | k_x, k_y, θ |
| 3 | Optimized (4 parameters) | $k_{x1}, k_{x2}, k_{y1}, k_{y2}$ | $k_{x1}, k_{x2}, k_{y1}, k_{y2}, \theta$ |
| 4 | Optimized (8 parameters) | $k_{x1}, k_{x2}, k_{x3}, k_{x4},$ $k_{y1}, k_{y2}, k_{y3}, k_{y4}$ | $k_{x1}, k_{x2}, k_{x3}, k_{x4},$ $k_{y1}, k_{y2}, k_{y3}, k_{y4}, \theta$ |

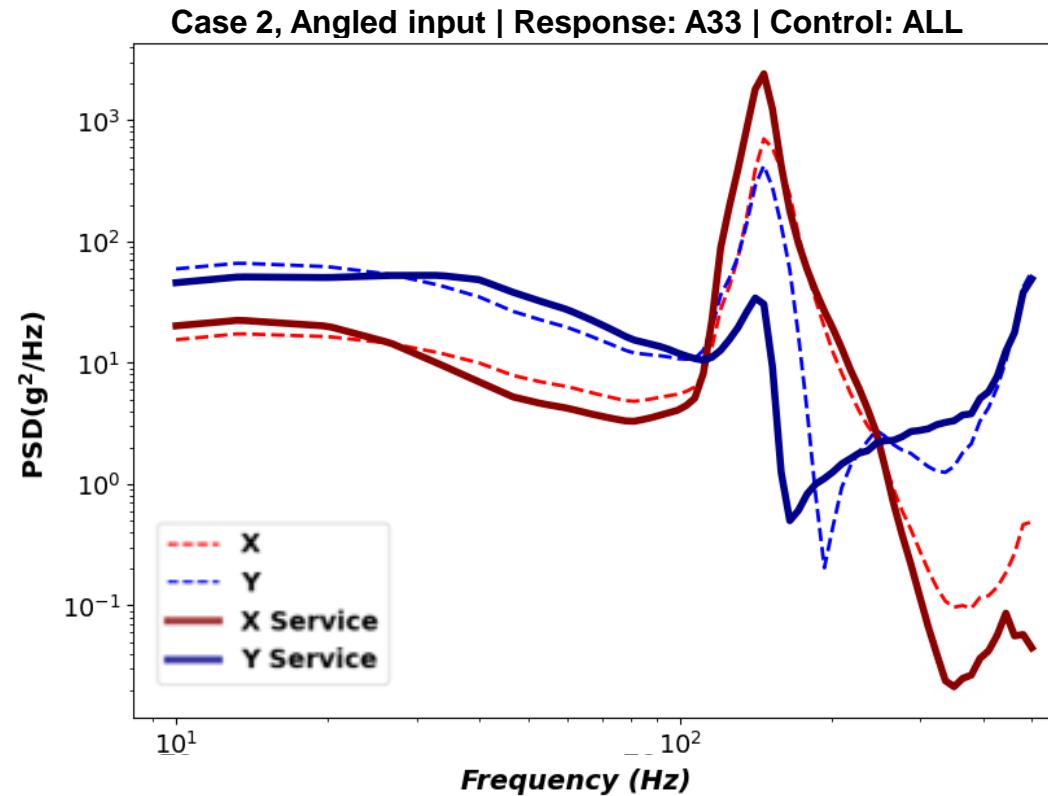


Case Study 1 (rigid)

- The SIMO multi-axis test's **RDBE increases by 1.0 dB** while the **FTOL falls by 3.8%**.

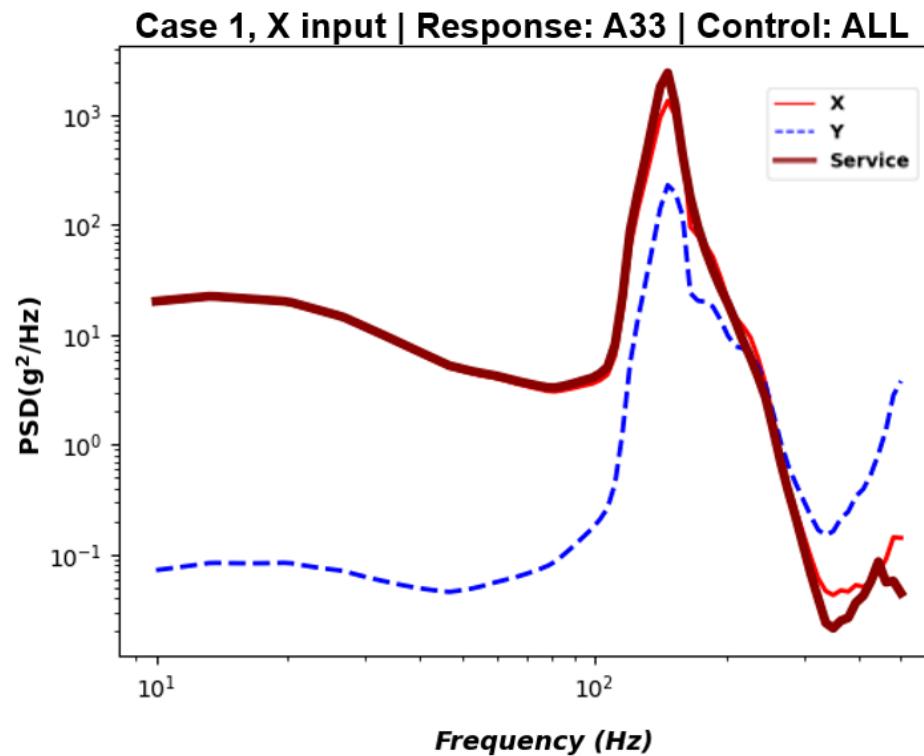
| Sequential Single-Axis (All control locations) | | | | SIMO Multi-Axis (All control locations) | | | |
|---|------------------|------------------|-----------------------|--|---------------|------------------|-----------------------|
| | X-Error (avg) | Y-Error (avg) | <u>Mean Error</u> | | X-Error (avg) | Y-Error (avg) | <u>Mean Error</u> |
| RDBE | 5.1 dB | 2.6 dB | <u>3.8 dB</u> | RDBE | 5.7 dB | 3.9 dB | <u>4.8 dB</u> |
| FTOL | 70.3% | 85.2% | <u>77.8%</u> | FTOL | 69.2% | 78.7% | <u>74.0%</u> |

Case Study 1 (rigid)

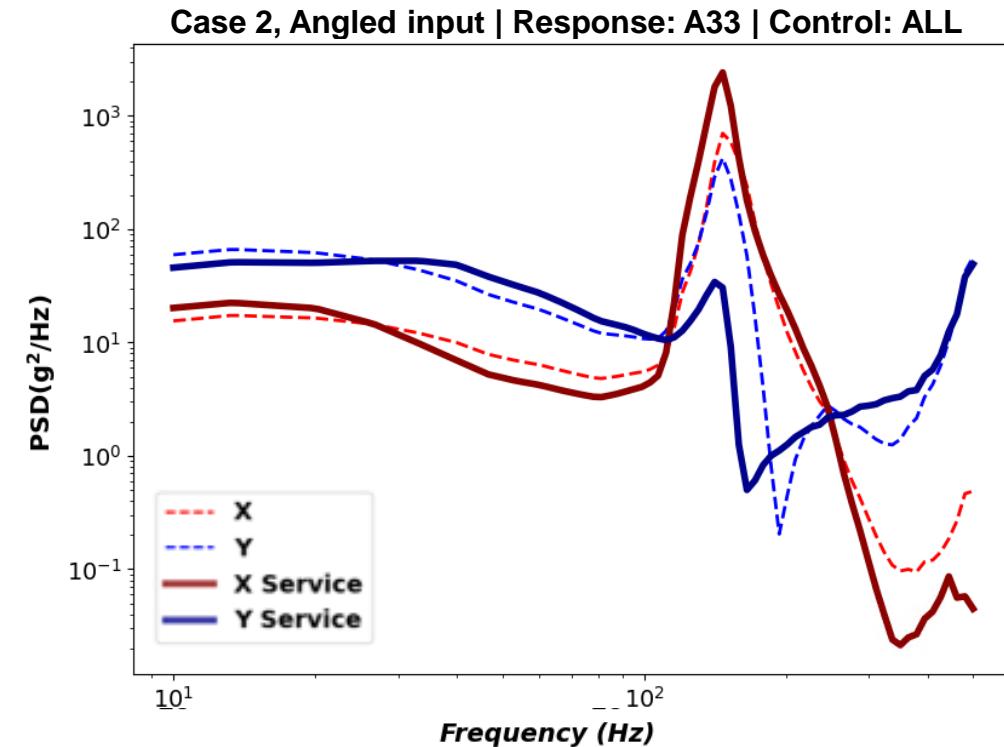


Case Study 1 (rigid)

Sequential X-Axis Test (All control locations)



SIMO Multi-Axis Test (All control locations)



Is a 1.0 dB (26%) increase in error acceptable for the SIMO multi-axis test to eliminate unavoidable cross-axis responses in the sequential test?

Can we do better with an optimized test fixture?

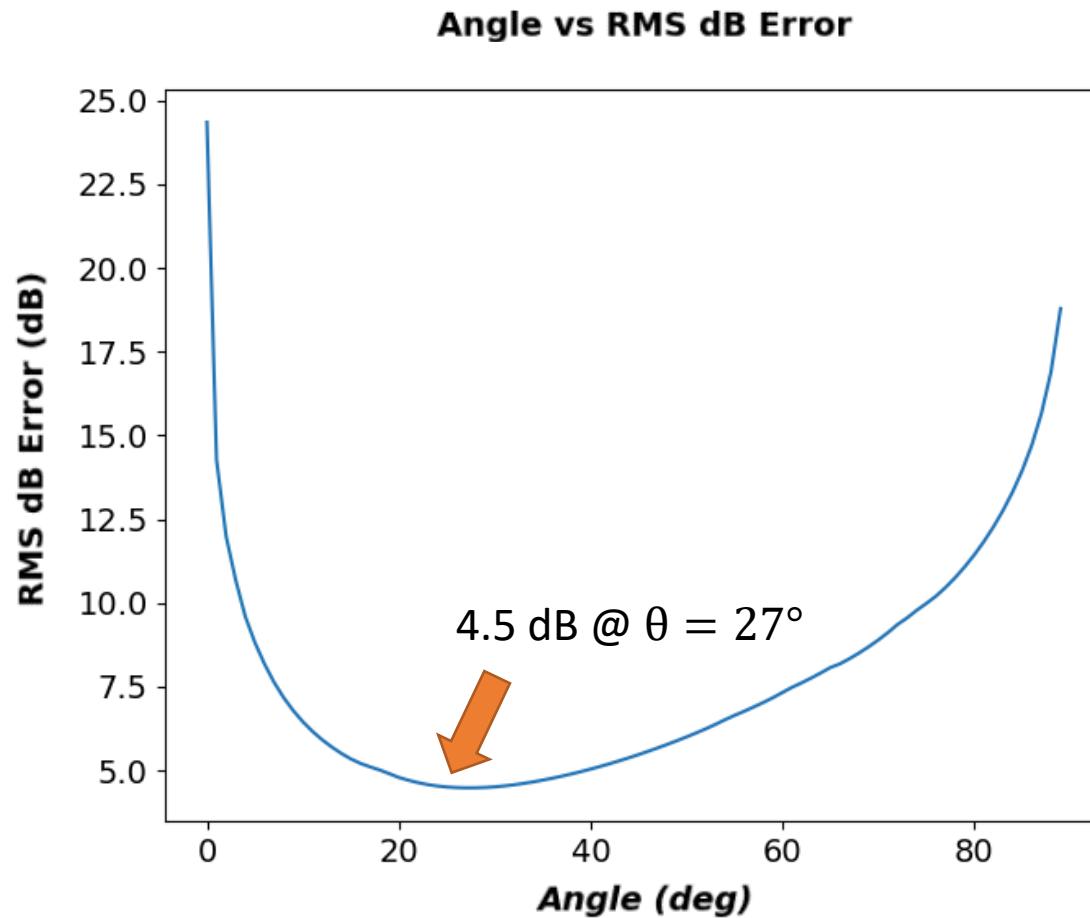
Case Study 2 (2 parameters)

Optimizing the test fixture stiffness with two parameters:

1. Improved the sequential test by 0.1 dB and 2.2%.
2. Improved the SIMO test by 0.3 dB and 1.6%.

| | Mean Error | |
|------|------------------------|-----------------|
| | Sequential Single-Axis | SIMO Multi-Axis |
| RDBE | 3.7 dB | 4.5 dB |
| FTOL | 80.0% | 75.6% |

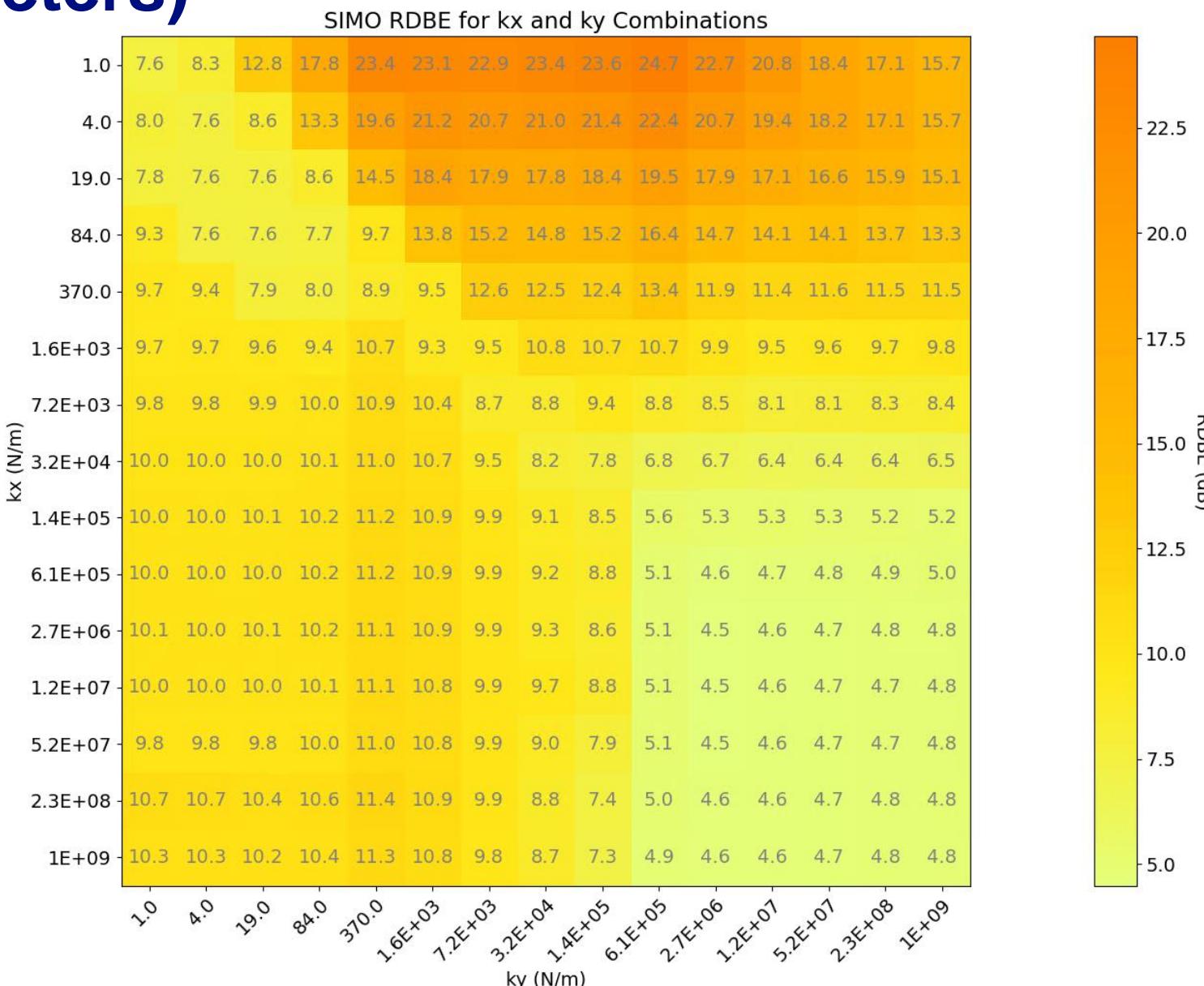
Case Study 2 (2 parameters)



- The SIMO test's angle optimization has a clear minimum.

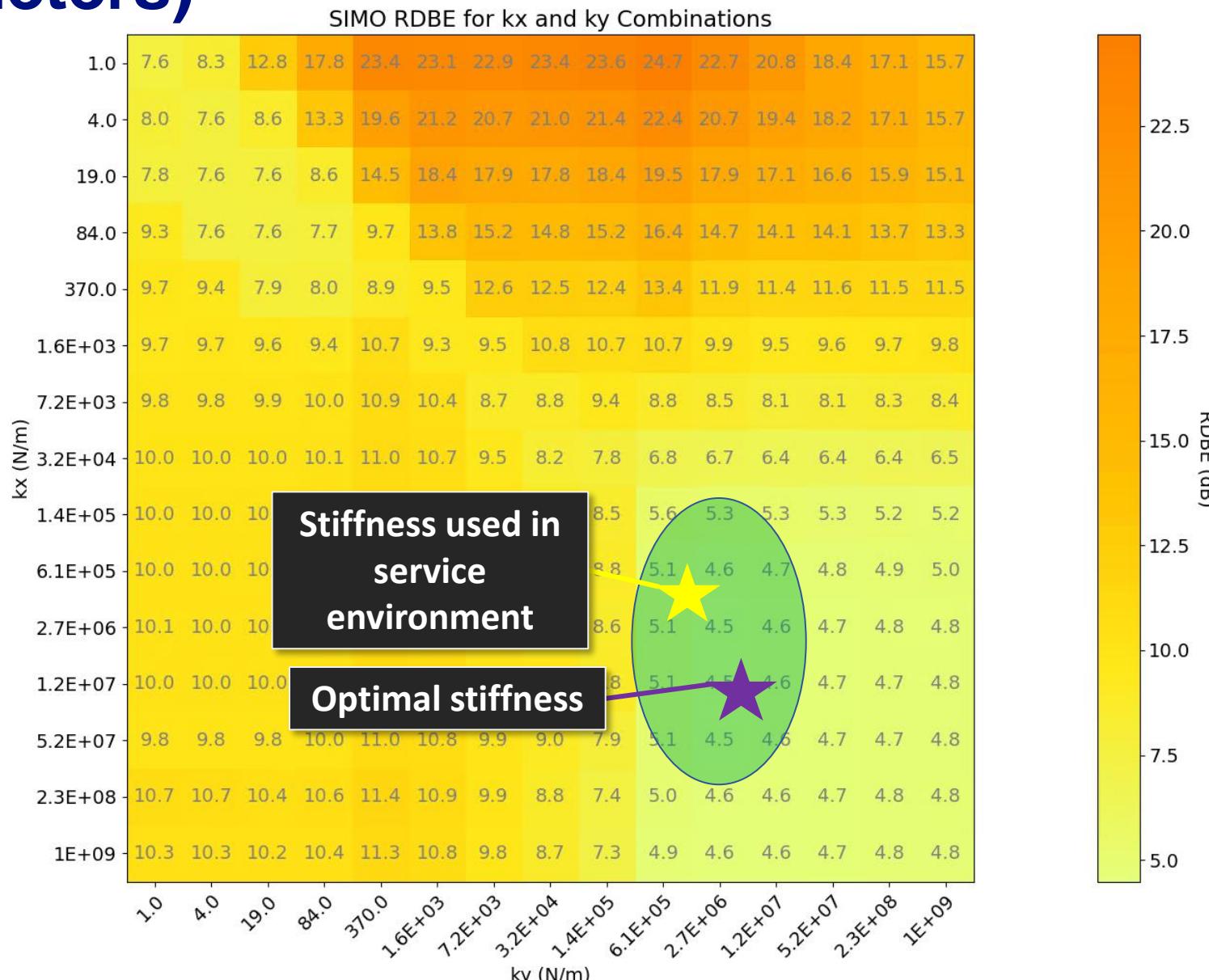
Case Study 2 (2 parameters)

- The SIMO solution space was smooth with a clear minimum region.
- The minimum region provides good values to start a local optimization.

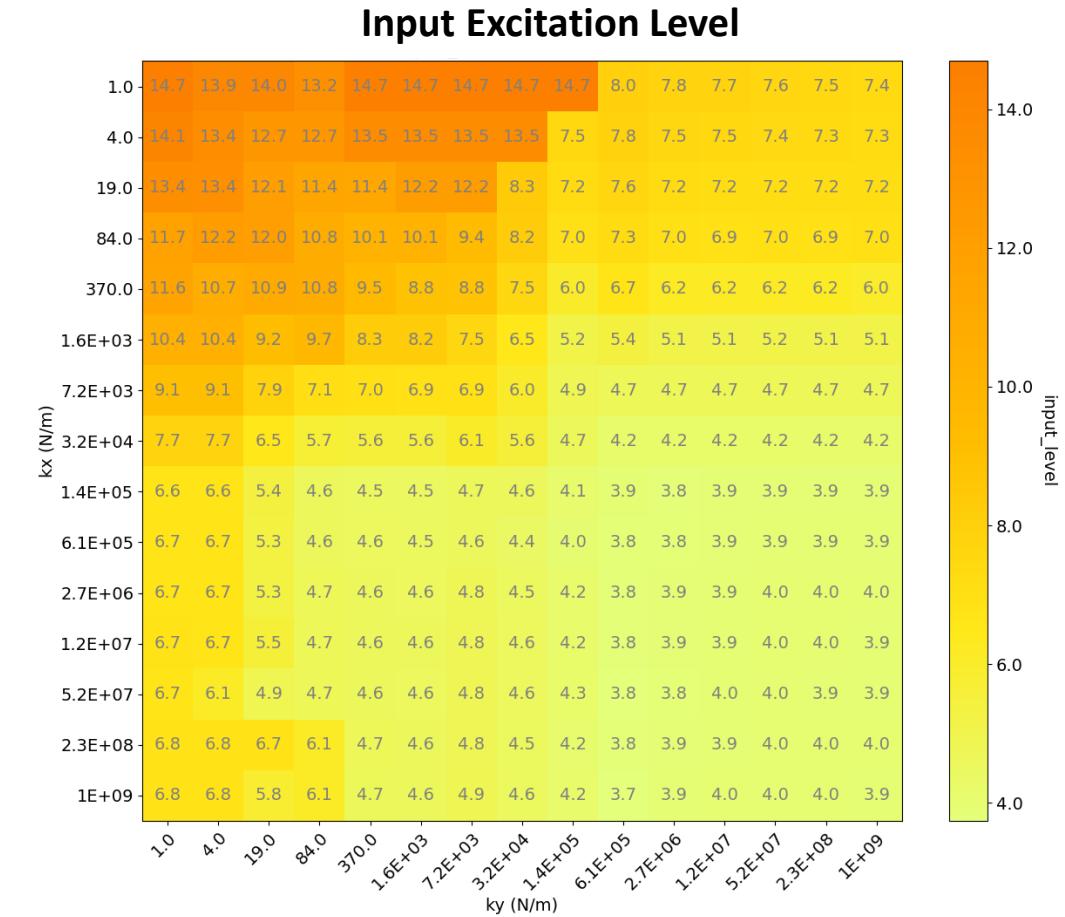
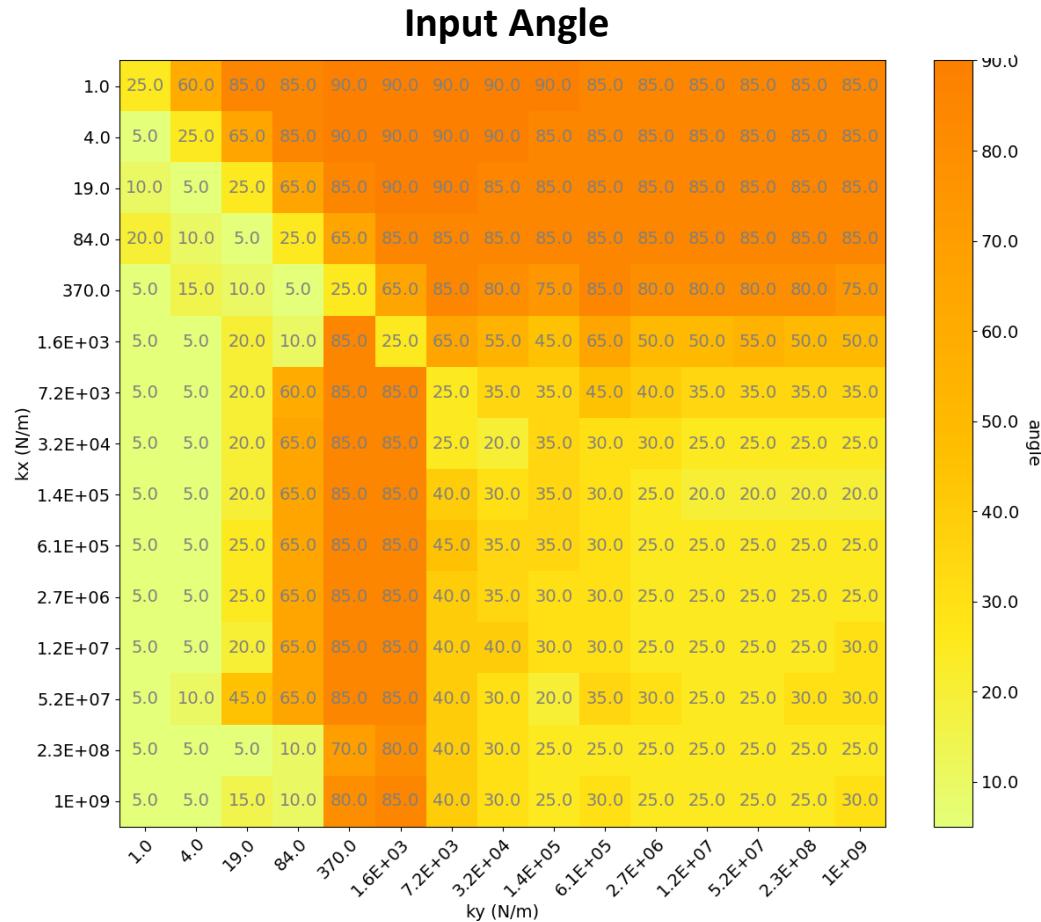


Case Study 2 (2 parameters)

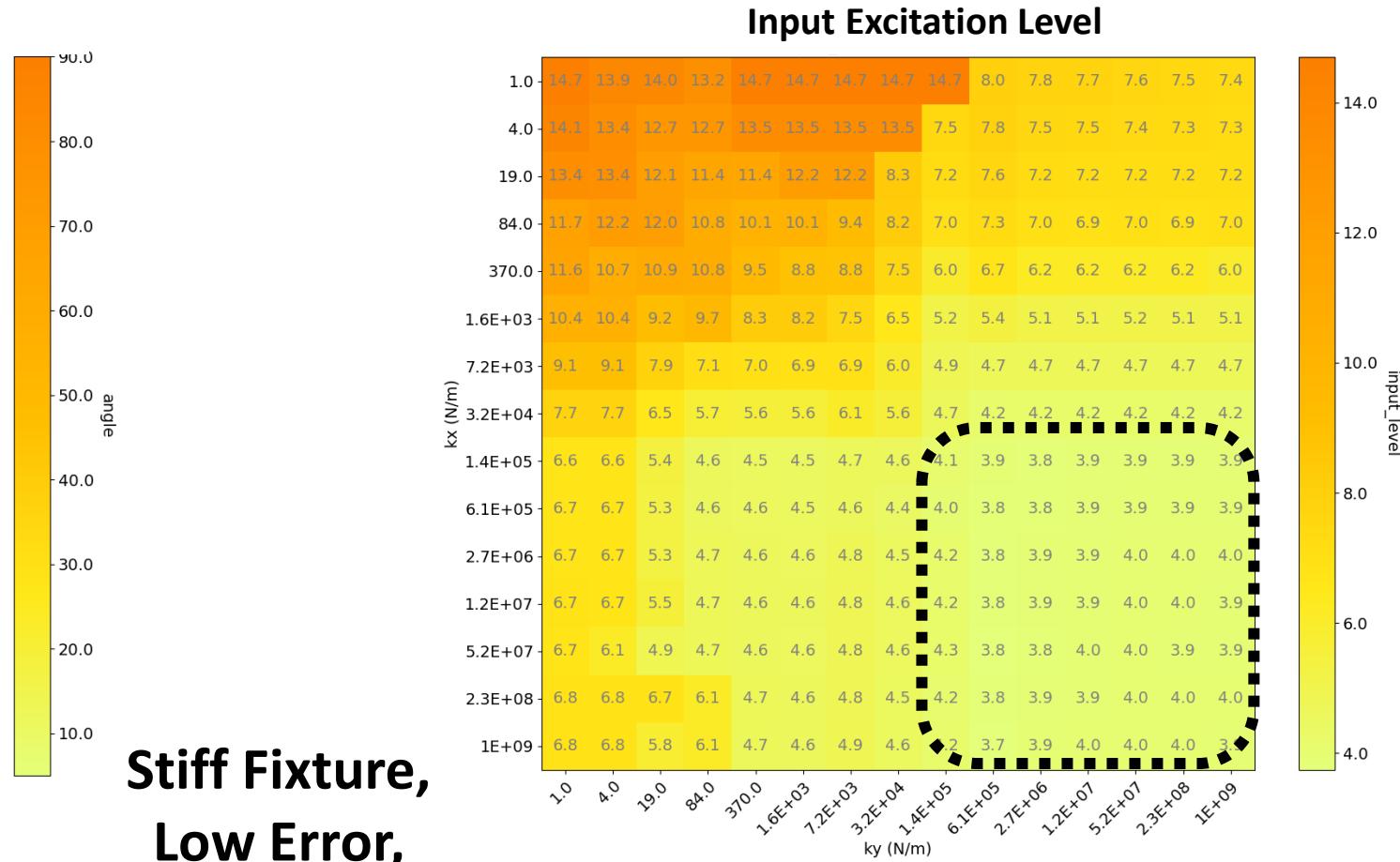
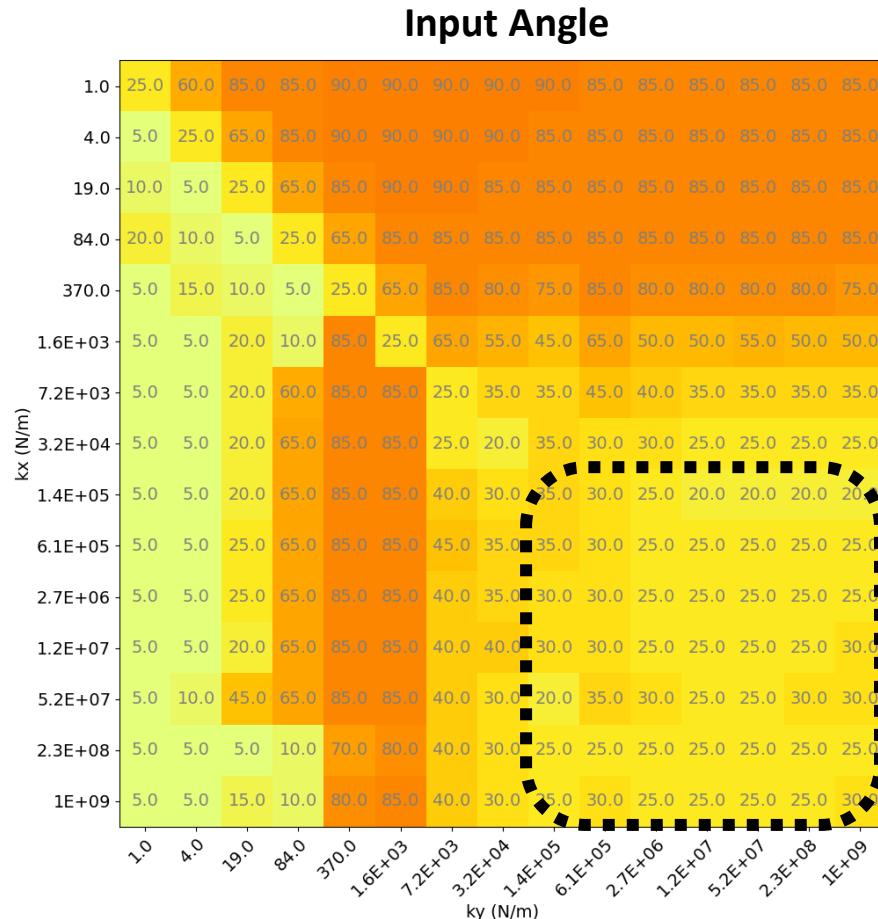
- The SIMO solution space was smooth with a clear minimum region.
- The minimum region provides good values to start a local optimization.



Case Study 2 Input Angle & Excitation Level

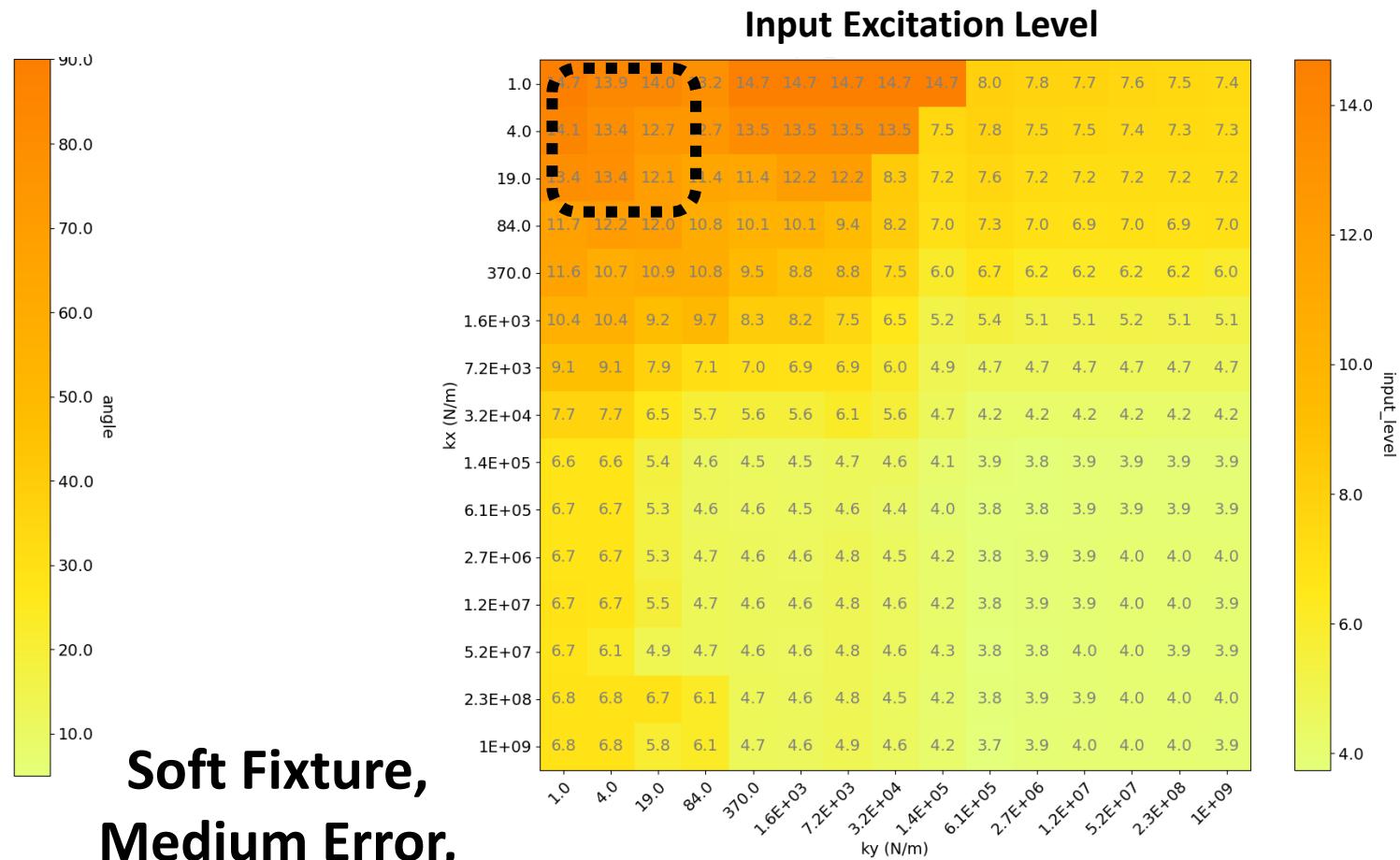
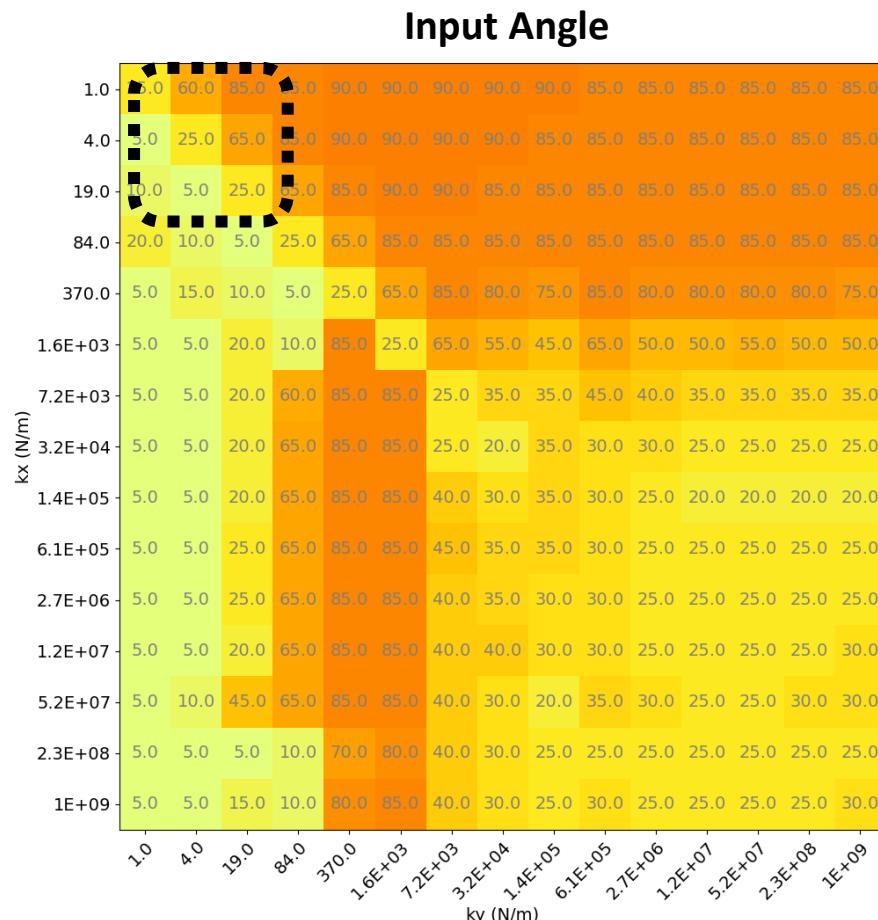


Case Study 2 Input Angle & Excitation Level



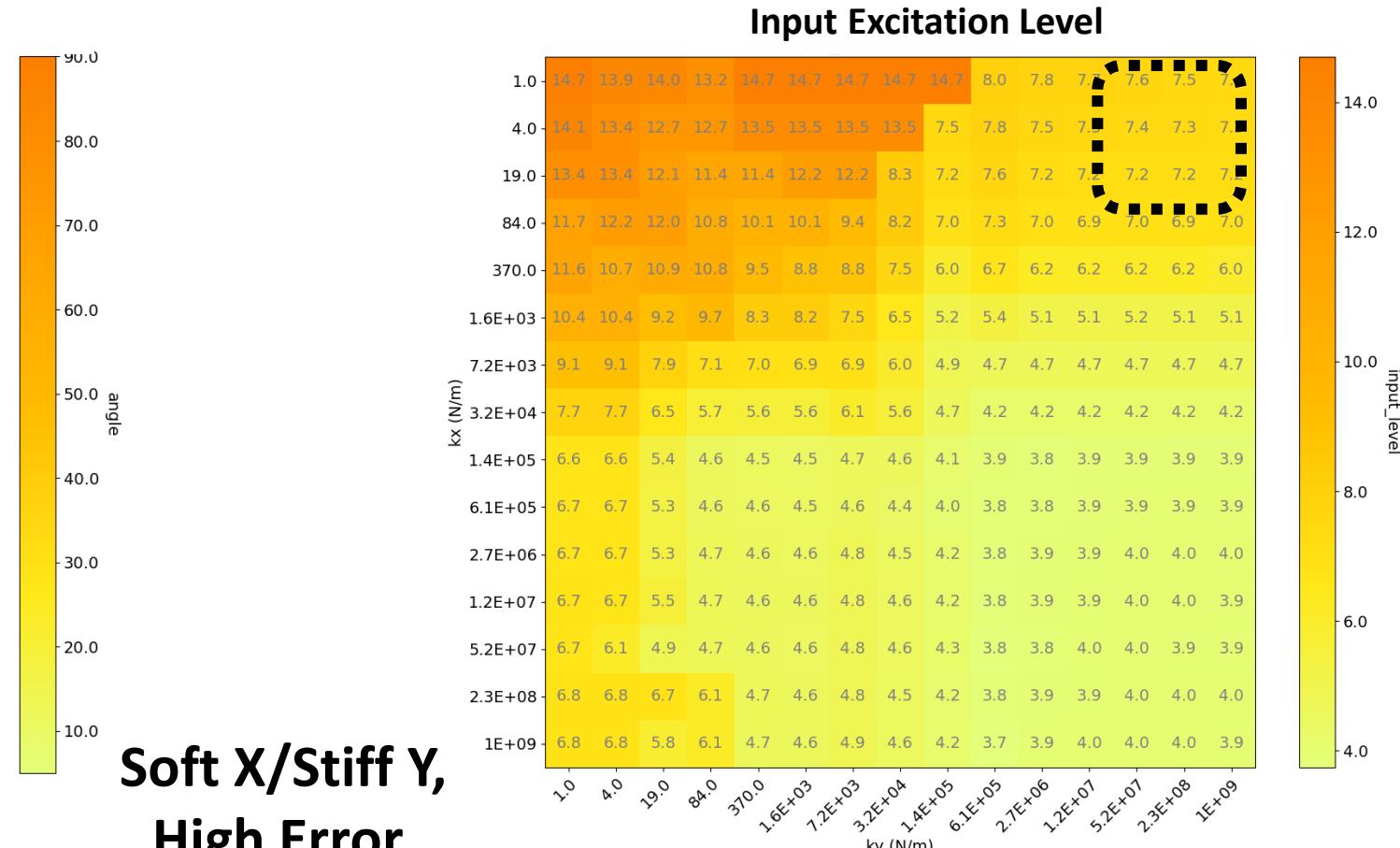
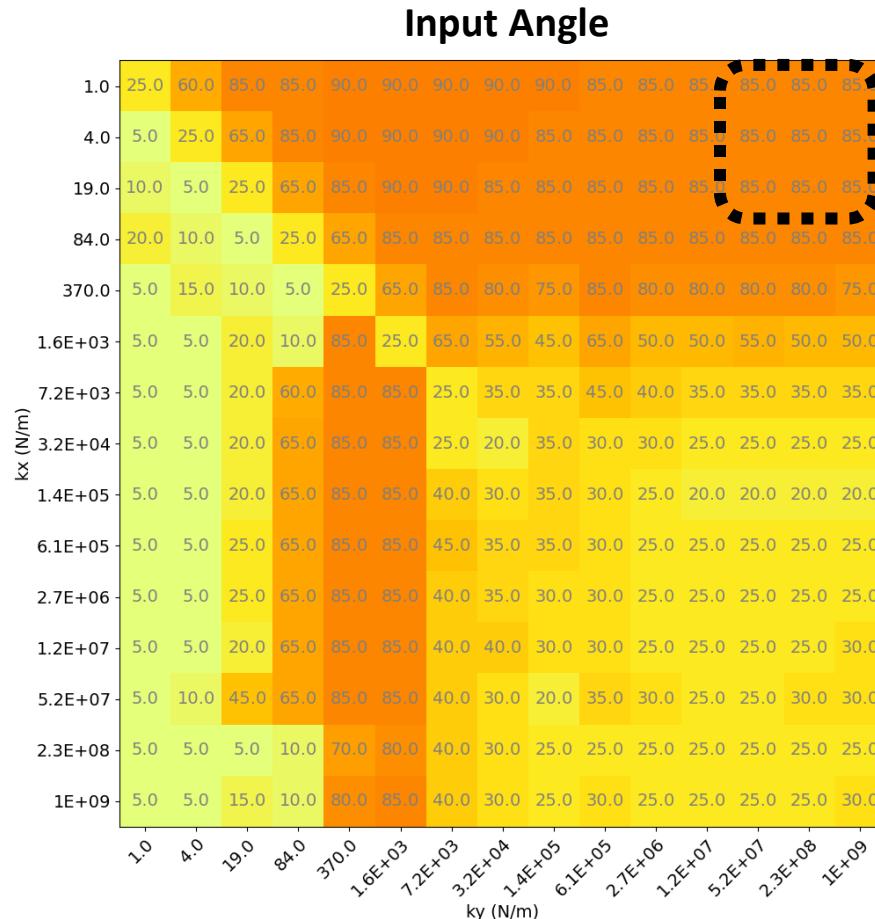
**Stiff Fixture,
Low Error,
25-30 Degree Angle,
Small Excitation**

Case Study 2 Input Angle & Excitation Level



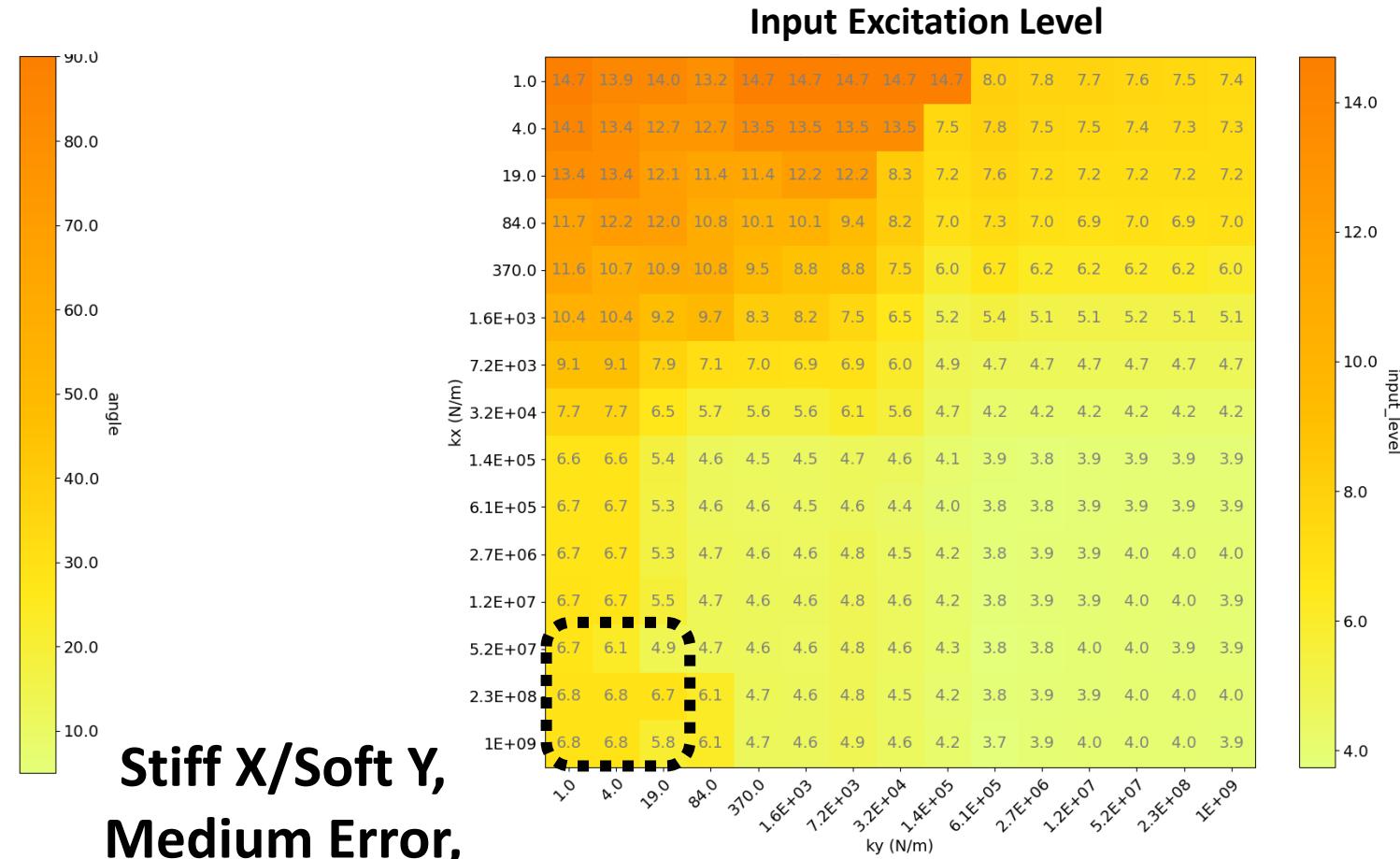
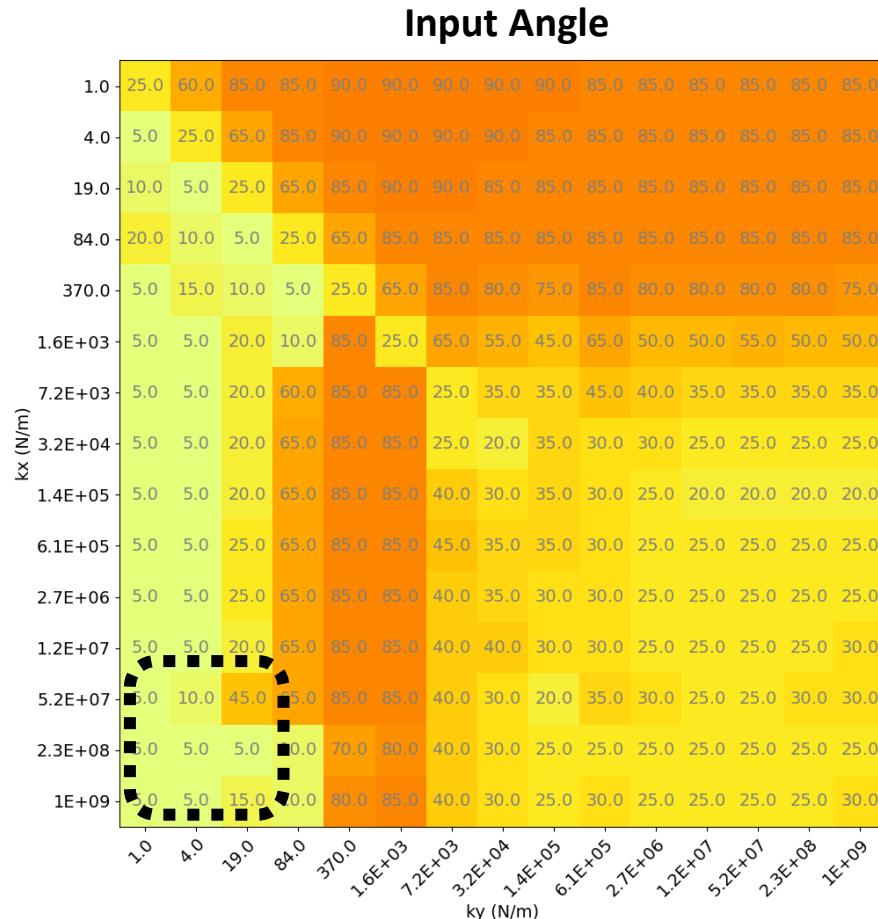
**Soft Fixture,
Medium Error,
Variable Angle,
Very Large Excitation**

Case Study 2 Input Angle & Excitation Level



**Soft X/Stiff Y,
High Error,
X-Axis Excitation Angle,
Medium Level Excitation**

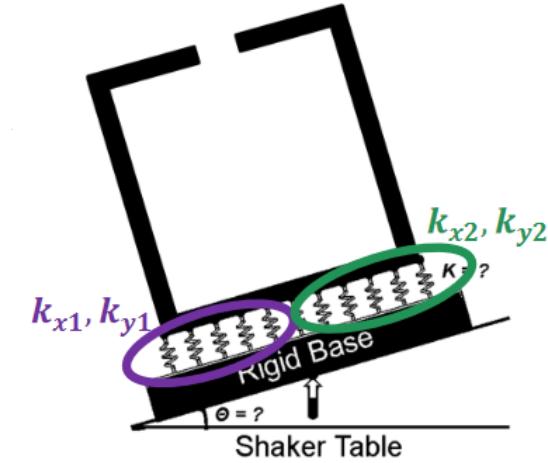
Case Study 2 Input Angle & Excitation Level



**Stiff X/Soft Y,
Medium Error,
Y-Axis Excitation Angle,
Medium Level Excitation**

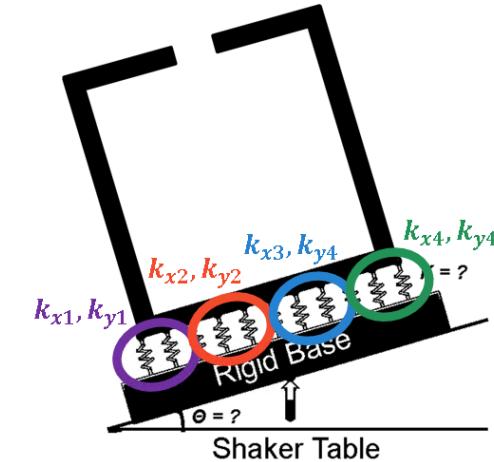
Case Study 3 (4 parameters)

| | Mean Error | |
|------|------------------------|-----------------|
| | Sequential Single-Axis | SIMO Multi-Axis |
| RDBE | 3.7 dB | 4.5 dB |
| FTOL | 80.0% | 75.6% |



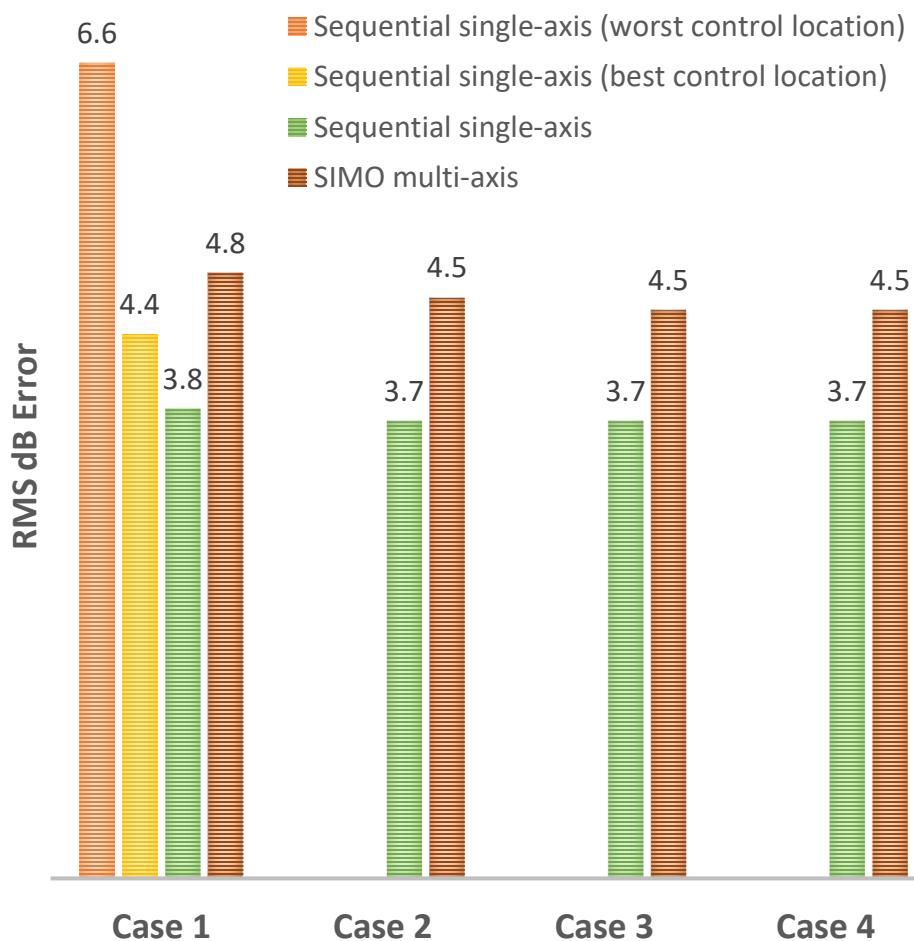
Case Study 4 (8 parameters)

| | Mean Error | |
|------|------------------------|-----------------|
| | Sequential Single-Axis | SIMO Multi-Axis |
| RDBE | 3.7 dB | 4.5 dB |
| FTOL | 79.6% | 75.3% |

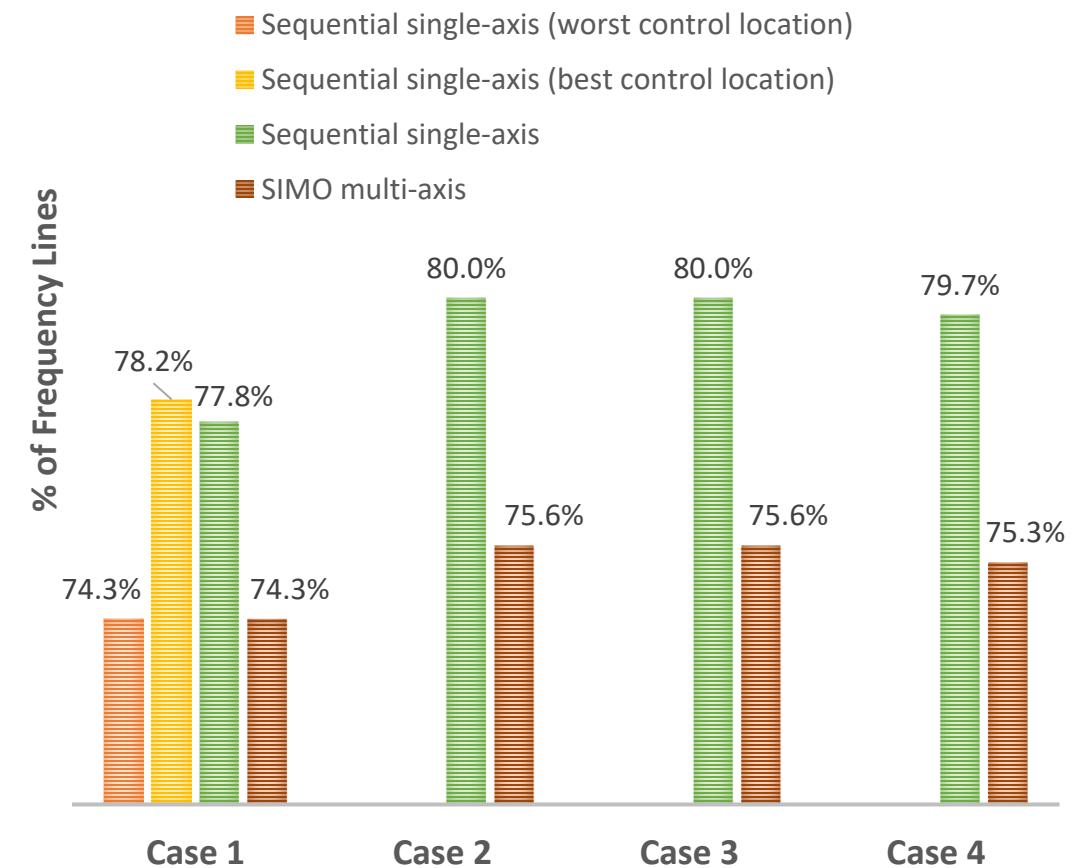


Result Summary

RMS DB ERROR BY CASE



% OF FREQUENCY LINES WITHIN A 3DB TOLERANCE BY CASE



Further Studies

The initial case studies showed the proposed method approximated a multi-axis test with on-axis errors only slightly worse than best-case sequential single-axis testing.

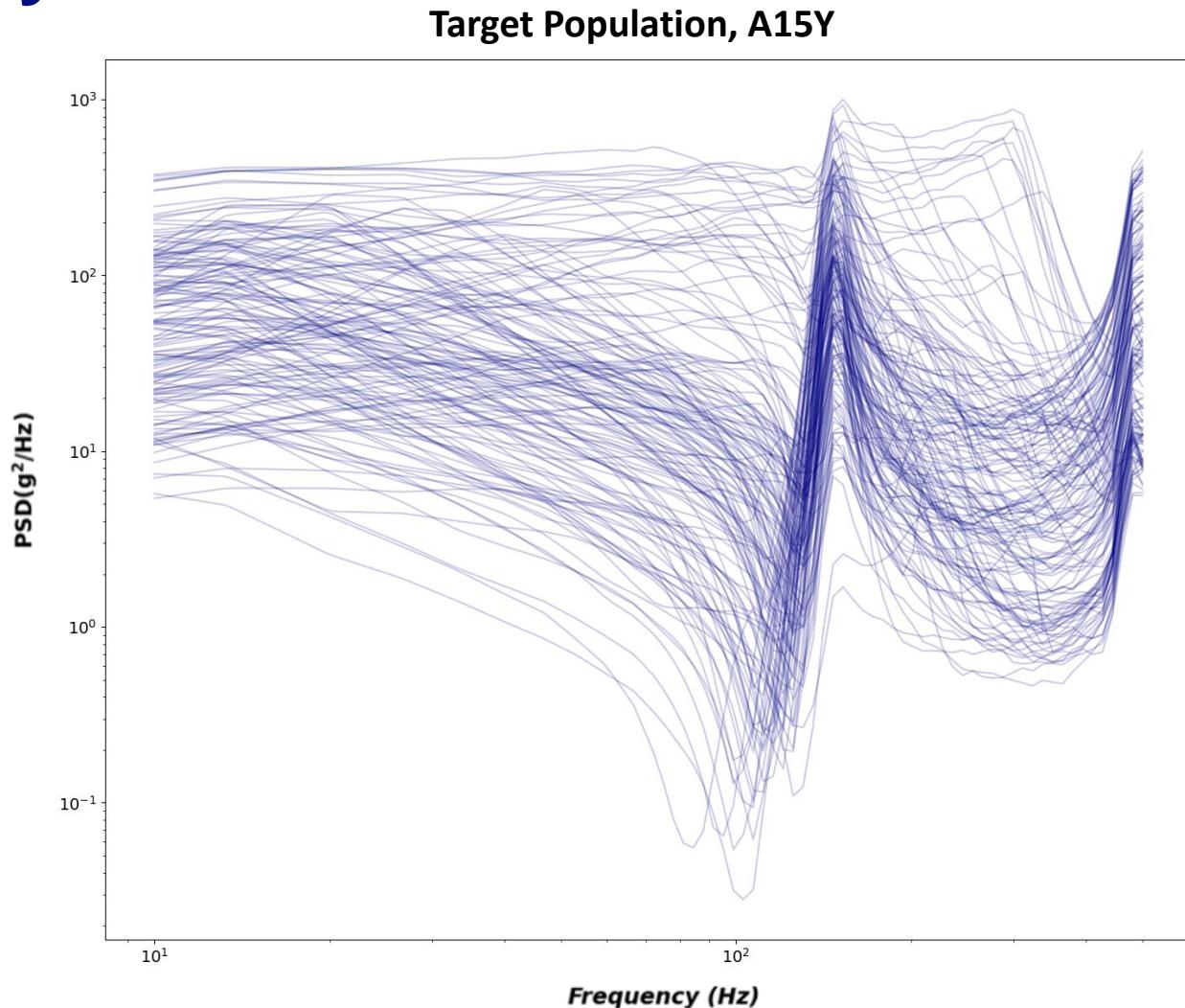
- Would this result hold for different target test environments?
 - Repeated the study for a large population of target test environments to evaluate more general case.

A dynamically optimized test fixture only slightly improved both the sequential test and SIMO test.

- Can we achieve further improvements in SIMO test performance with more exotic test fixture designs?
 - Assessed four additional test fixture configurations with less similarity to the boundary condition imposed to generate the target test environment.

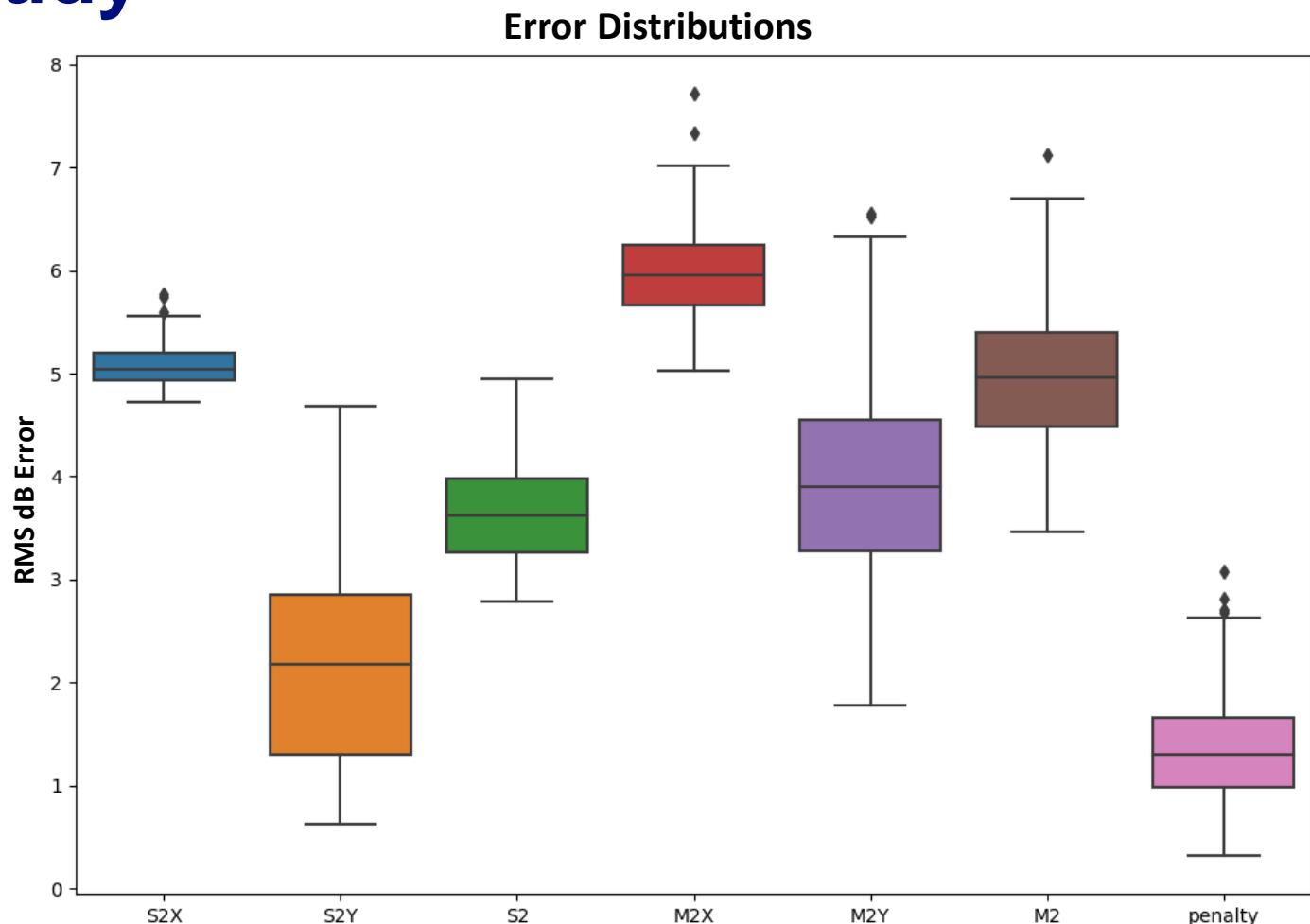
Target Population Study

- 150 target test environment sets were generated by applying significant perturbations to the MIL-STD-810H base excitation environment and repeating the entire process.
- The resulting target test environment variation for one control location is shown.
- For each of the 150 target sets, the entire analysis was repeated for the rigid test fixture case only.



Target Population Study

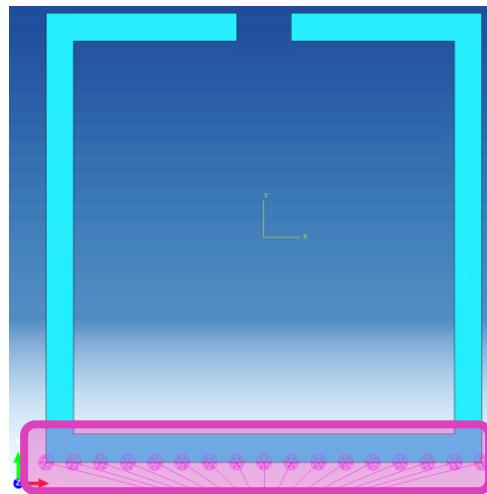
- Error distributions for target population study:
 - S2X: Sequential, X-test error
 - S2Y: Sequential, Y-test error
 - S2: Sequential, mean error
 - M2X: SIMO, X response error
 - M2Y: SIMO, Y response error
 - M2: SIMO, mean error
 - Penalty: Difference between S2 and M2
- The on-axis error penalty for a simultaneous SIMO test remains small for most of the target test environment population.



Is the increase in error acceptable for the SIMO multi-axis test to eliminate unavoidable cross-axis responses in the sequential test?

Can we do better with other test fixtures?

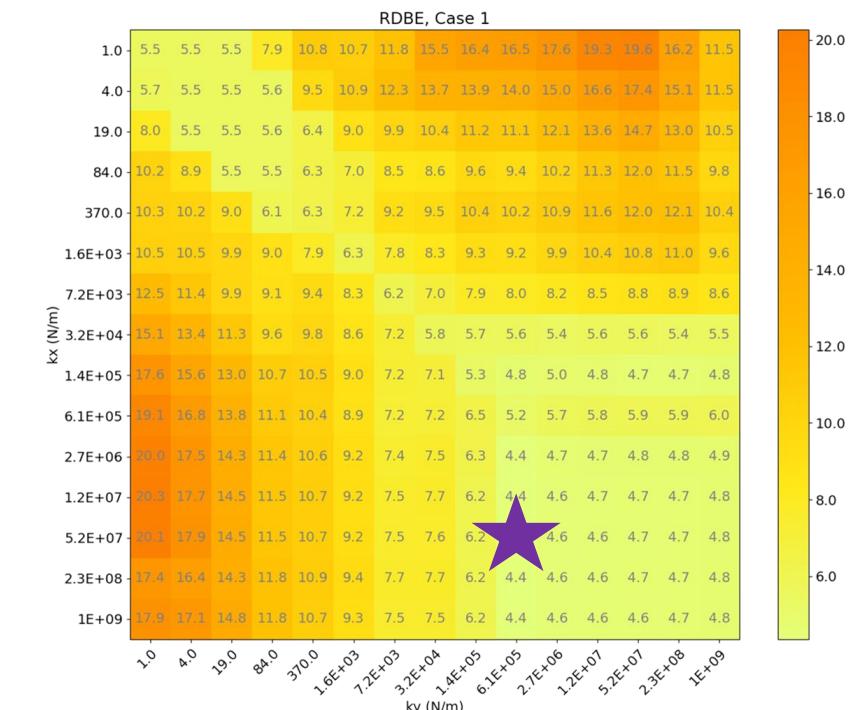
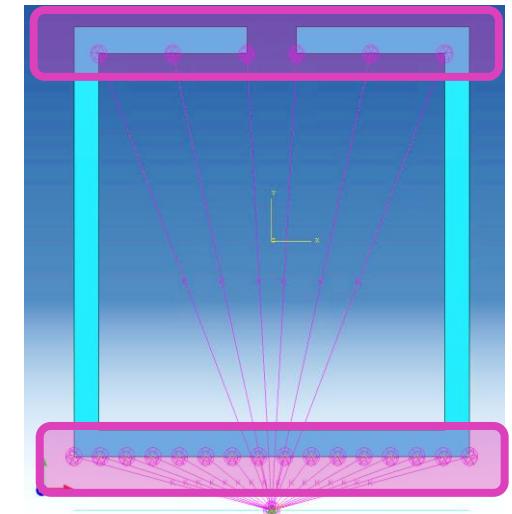
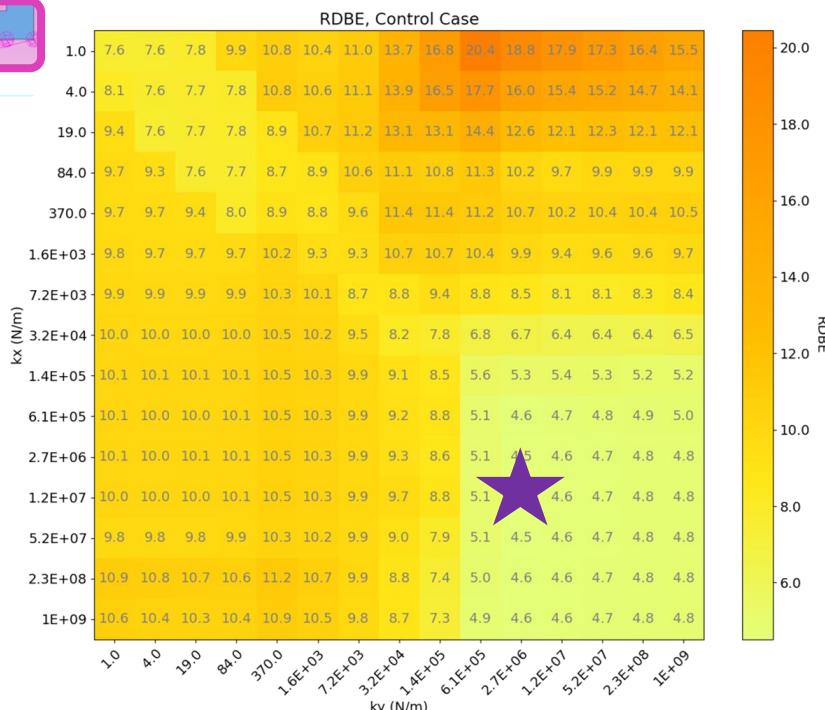
Test Fixture Attachment Points



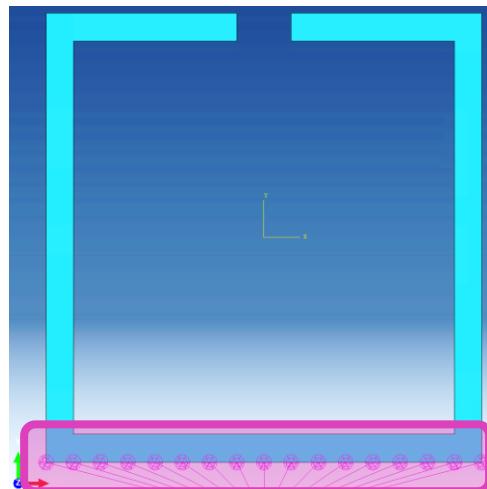
Case Study 2

2 parameters of X and Y
Stiffnesses Along Base

Minimum Error 4.5 dB



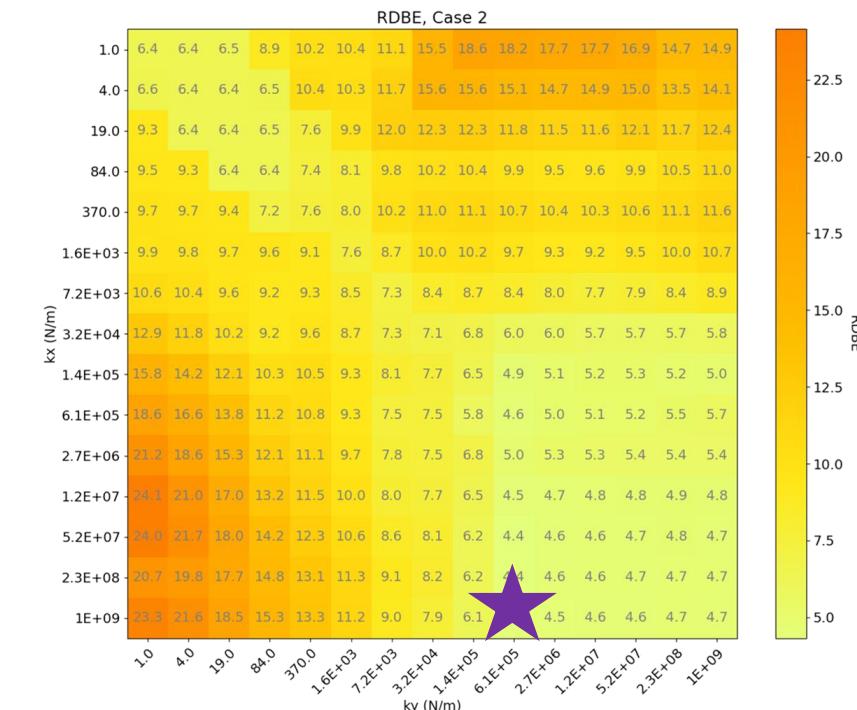
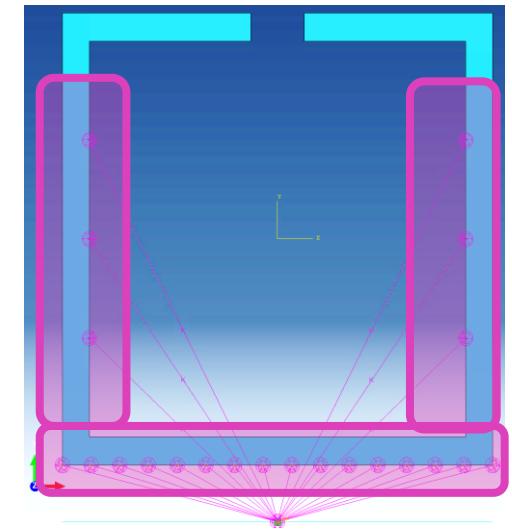
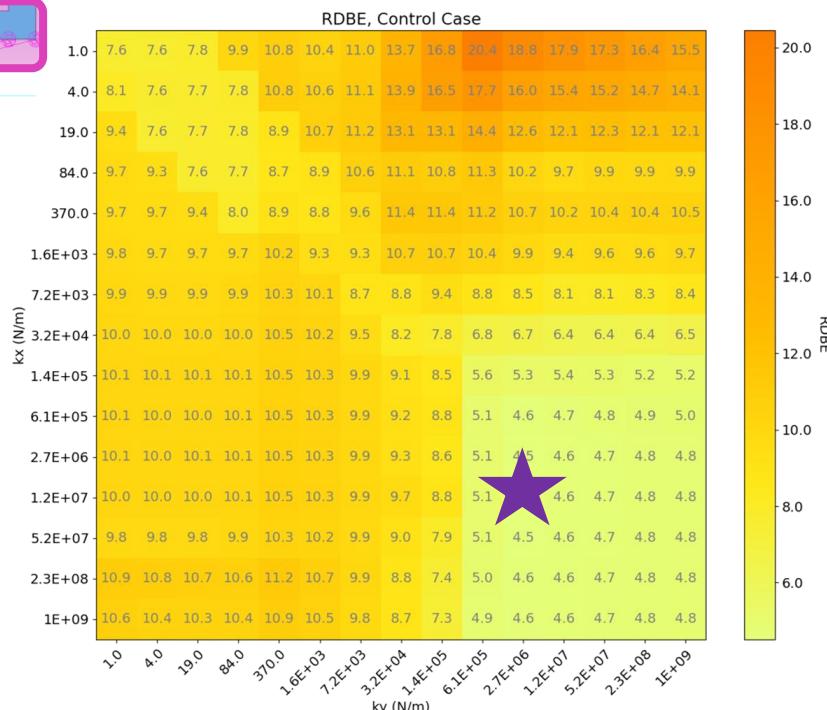
Test Fixture Attachment Points



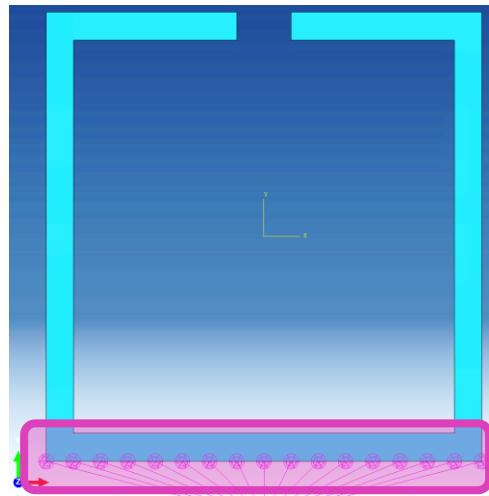
Case Study 2

2 parameters of X and Y
Stiffnesses Along Base

Minimum Error 4.5 dB



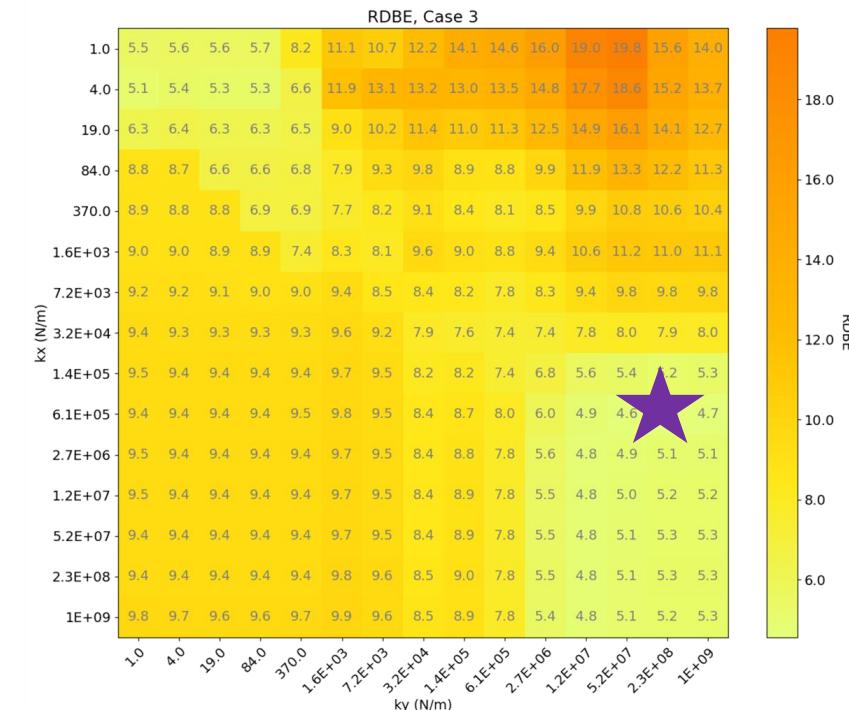
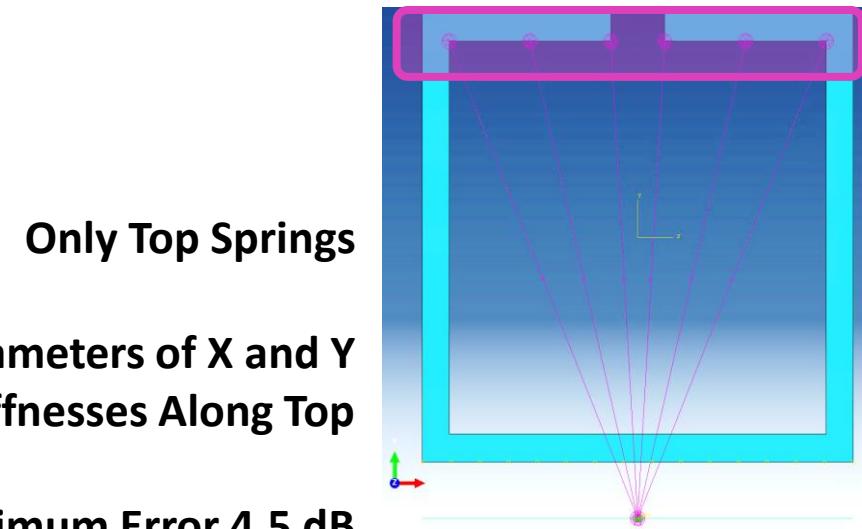
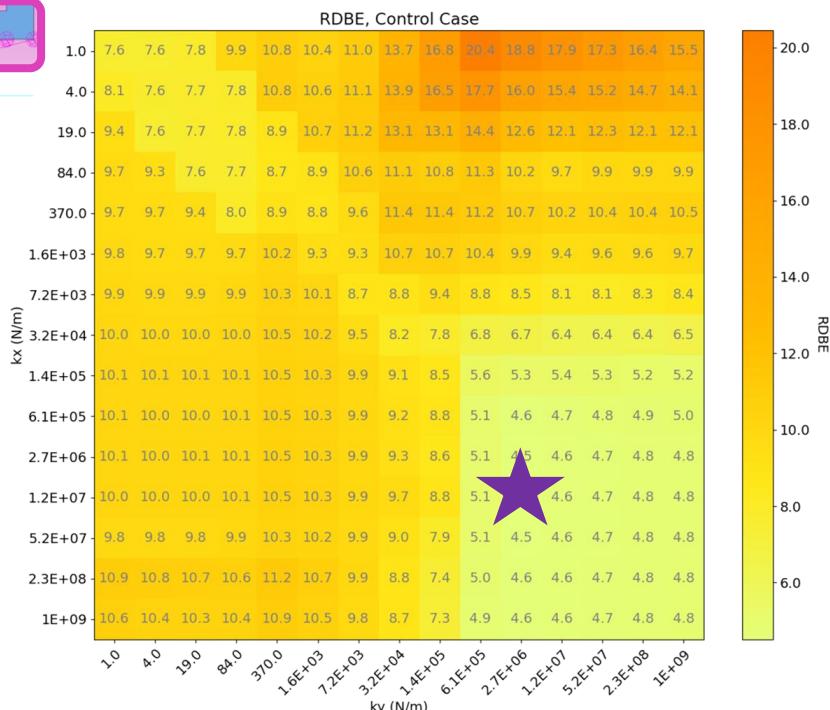
Test Fixture Attachment Points



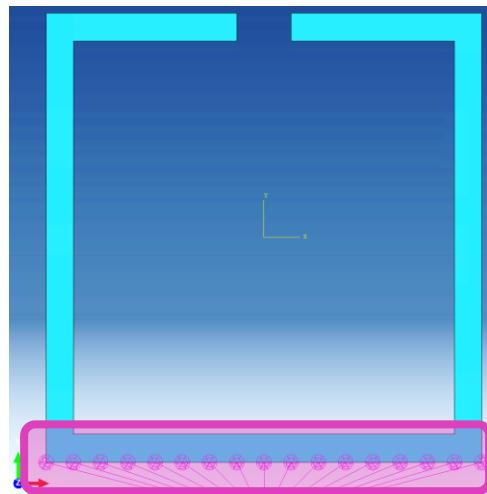
Case Study 2

2 parameters of X and Y
Stiffnesses Along Base

Minimum Error 4.5 dB



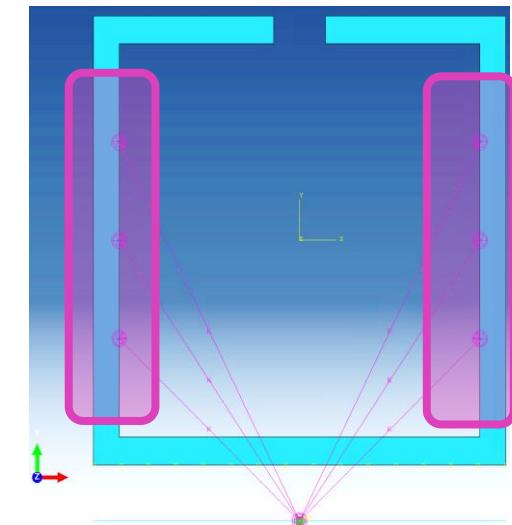
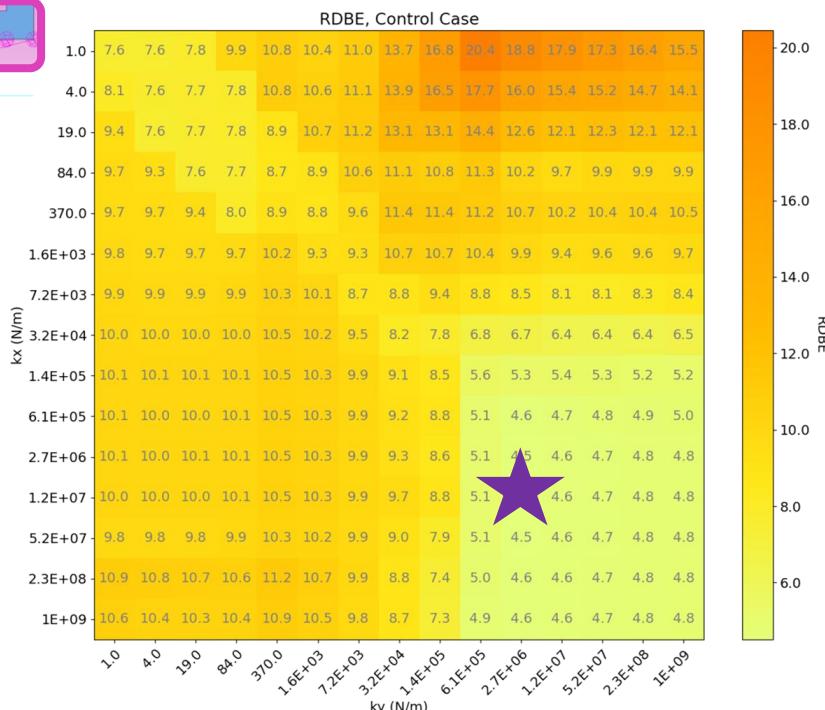
Test Fixture Attachment Points



Case Study 2

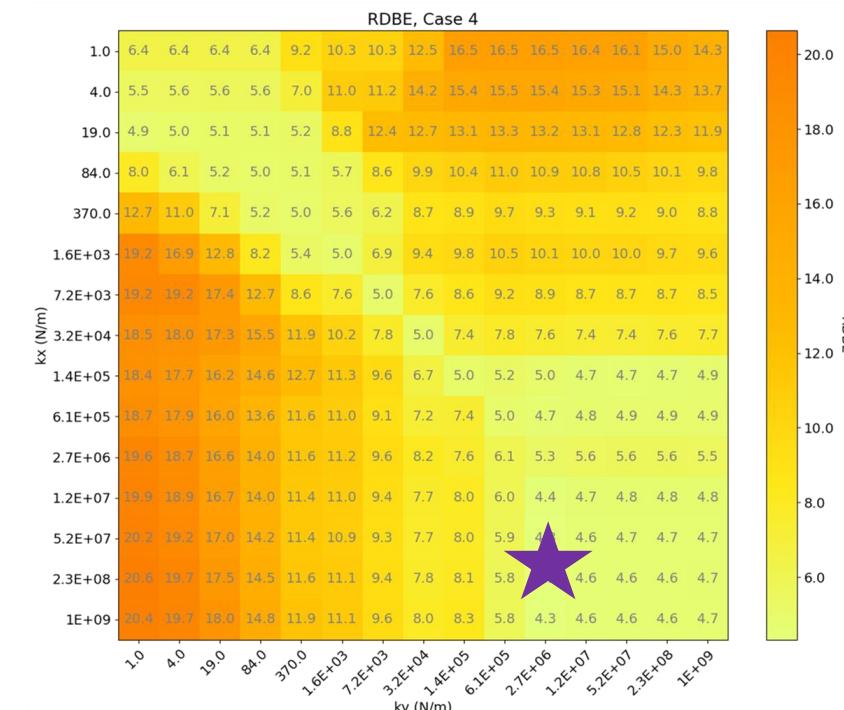
2 parameters of X and Y
Stiffnesses Along Base

Minimum Error 4.5 dB



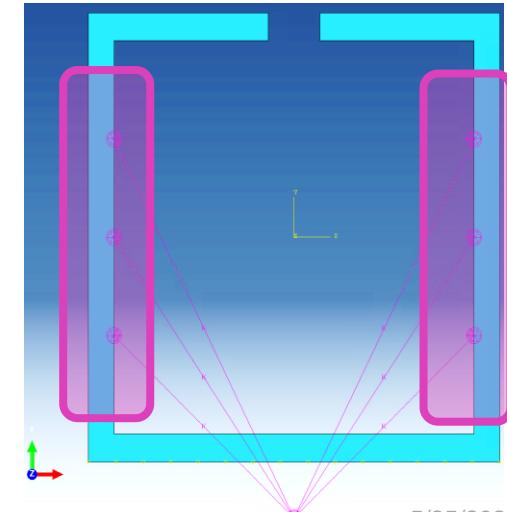
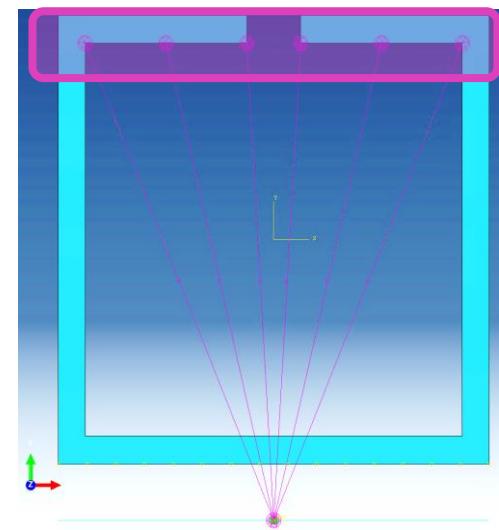
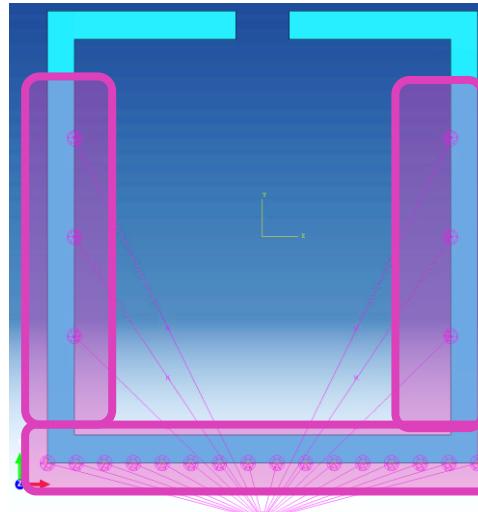
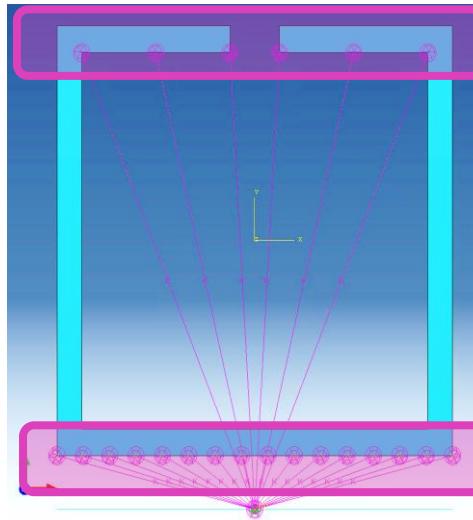
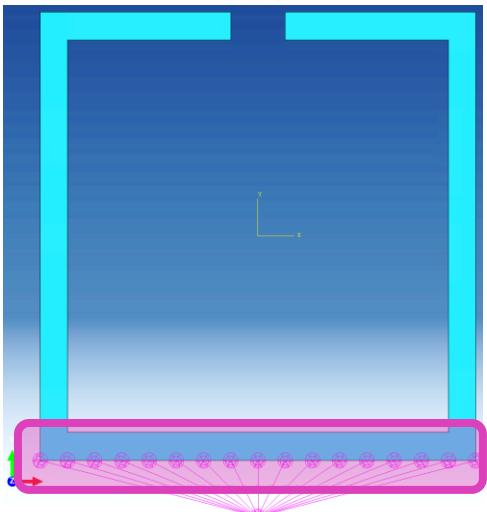
2 parameters of X and Y
Stiffnesses Along Sides

Minimum Error 4.3 dB



Test Fixture Attachment Points

- Unexpectedly, all of these possible fixture designs can result in RDBE of **4.3 - 4.5 dB**.
- With rigid fixtures, all of these fixture designs produce a test with RDBE of **4.7 - 5.3 dB**.
- More exotic, improved fixture designs may exist but are difficult to find and may not be intuitive.



Preliminary Study:

Can single-axis testing techniques provide a sufficient rapid approximation to a multi-axis test?

How much can we improve multi-axis test quality by optimizing passive test fixture hardware instead of adding active excitation sources?

Question:

Is the simplest possible multi-axis test close to single-axis testing in test quality?

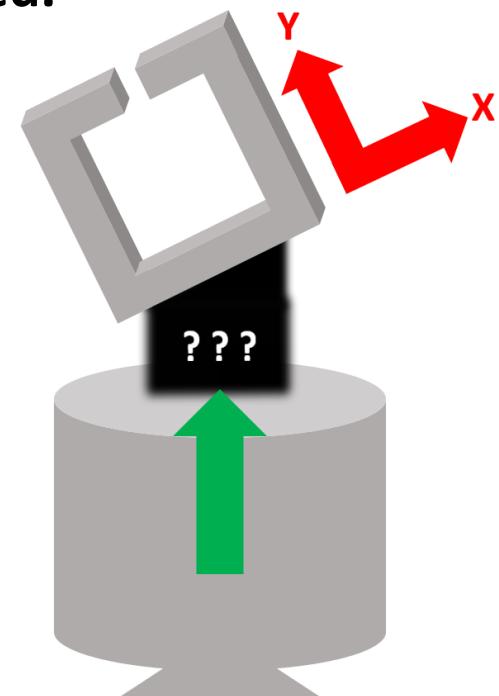
How good do multi-axis tests need to be to replace single-axis testing in practice?

SIMO multi-axis test quality may be in family with traditional test strategies.

Very good fixture designs are challenging while bad designs abound and need to be avoided.

Preliminary study suggests yes.

Unsettled



Questions?

Single-Axis vs. Multi-Axis

- The field and lab environment's mismatch is exacerbated in sequential single-axis testing.

| Compared to a 6DOF test, sequential single-axis testing produces different... | | | | |
|---|-----------------------|--------------------------|--------------------------------------|----------------------|
| Failure times | Failure distributions | Maximum Von Mises stress | Location of maximum Von Mises stress | Modal participations |
| French et al., 2006 | | | Gregory et al., 2009 | |

Impedance Modification Approaches

- Two categories of approaches are trying to rectify the mismatch between field and lab environments:

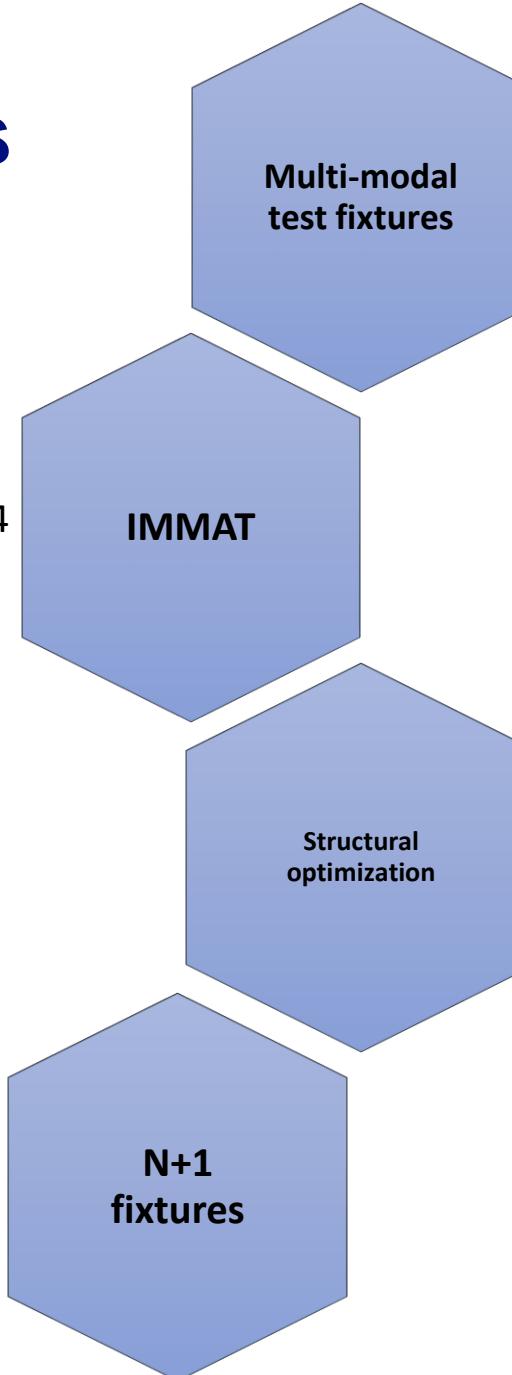
1. **Impedance modification**
2. Input control

Daborn et al., 2014

Hall, 2020

Scharton, 1969

Schoenherr, 2018

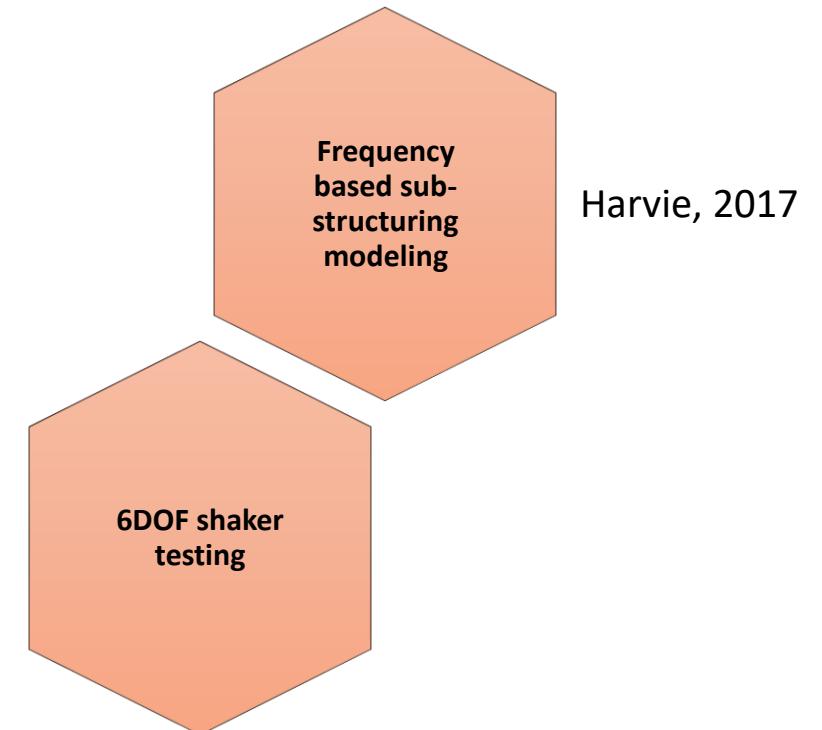


Input Control Approaches

- Two categories of approaches are trying to rectify the mismatch between field and lab environments:

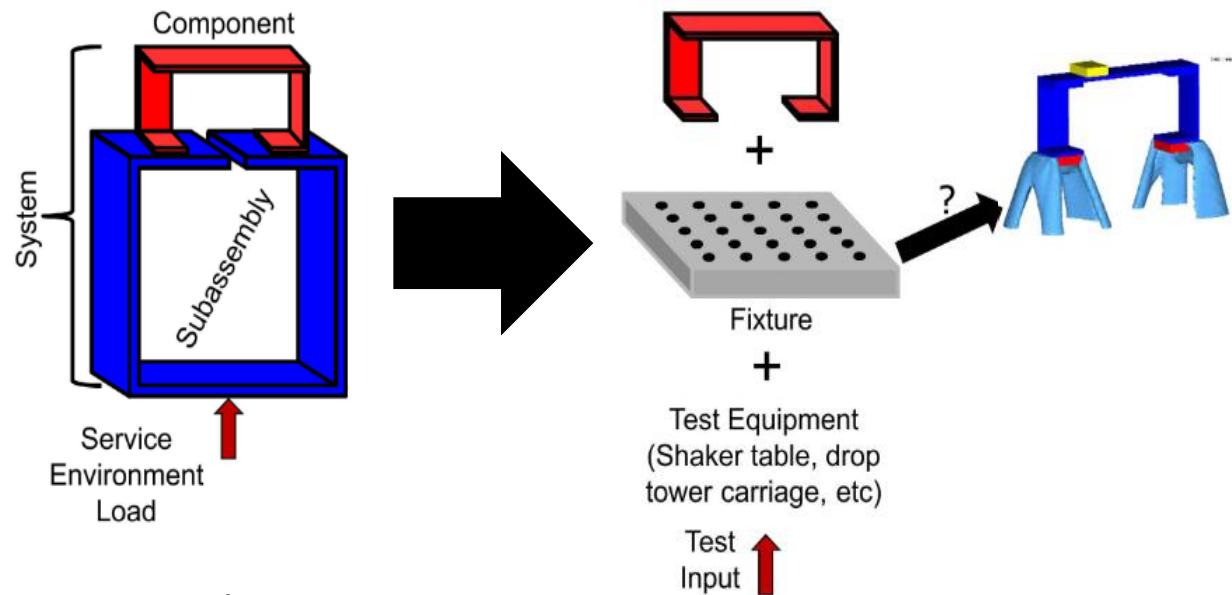
1. Impedance modification
2. **Input control**

Schoenherr et al.,
2019



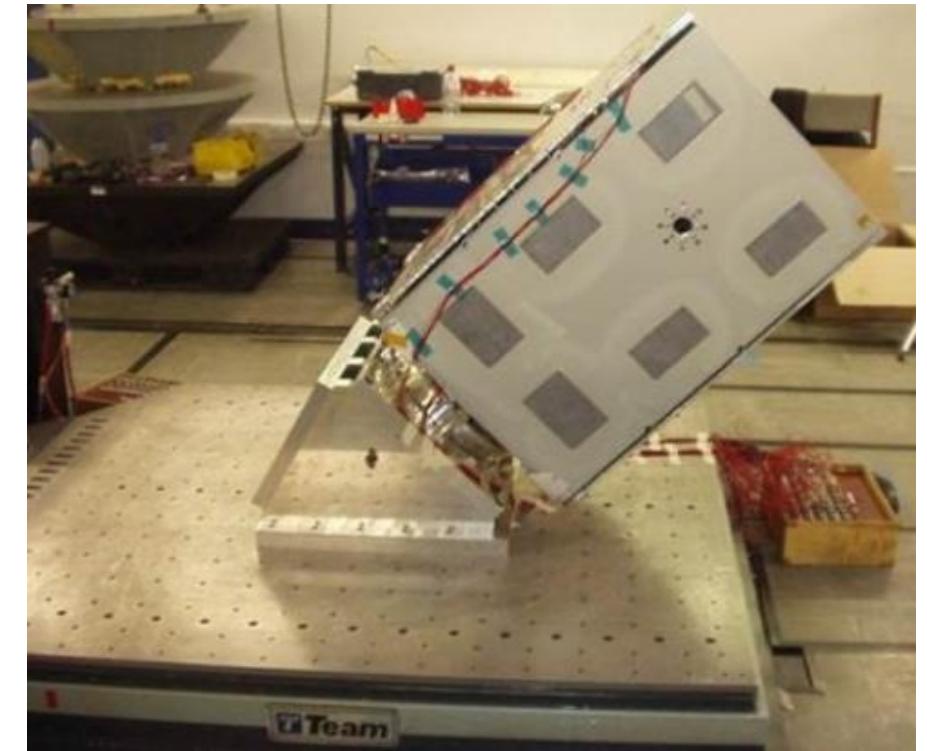
Structural Optimization

Topological Optimization



Jones et al., 2018

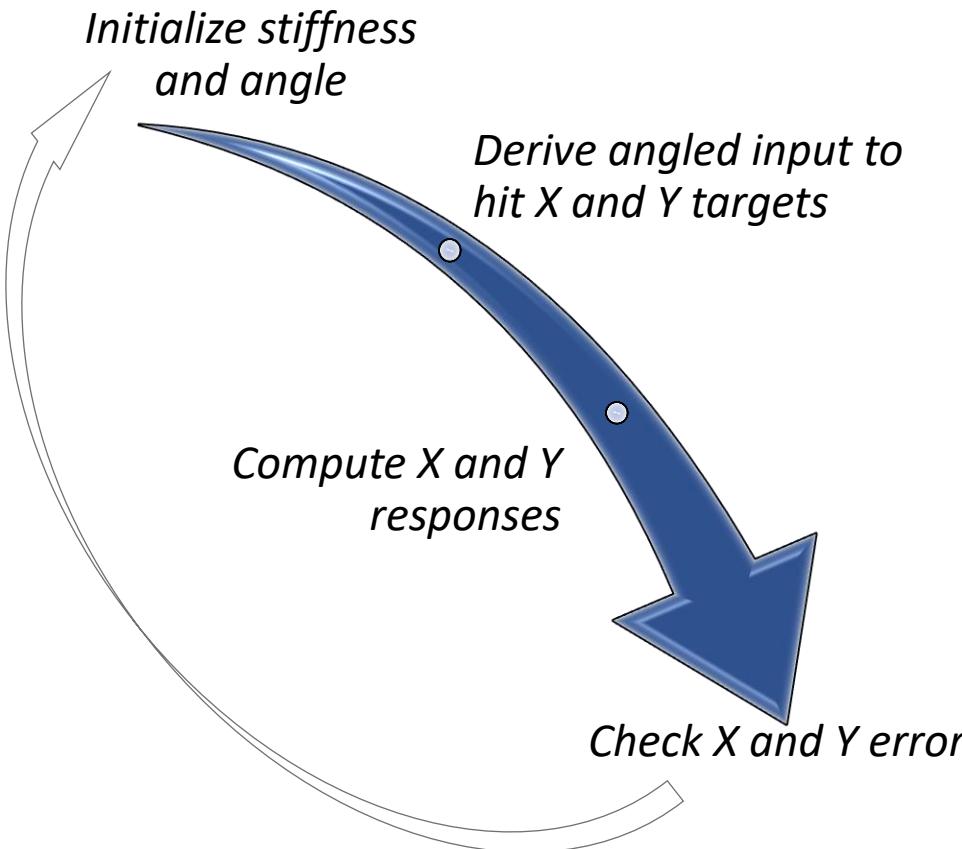
Angle Optimization



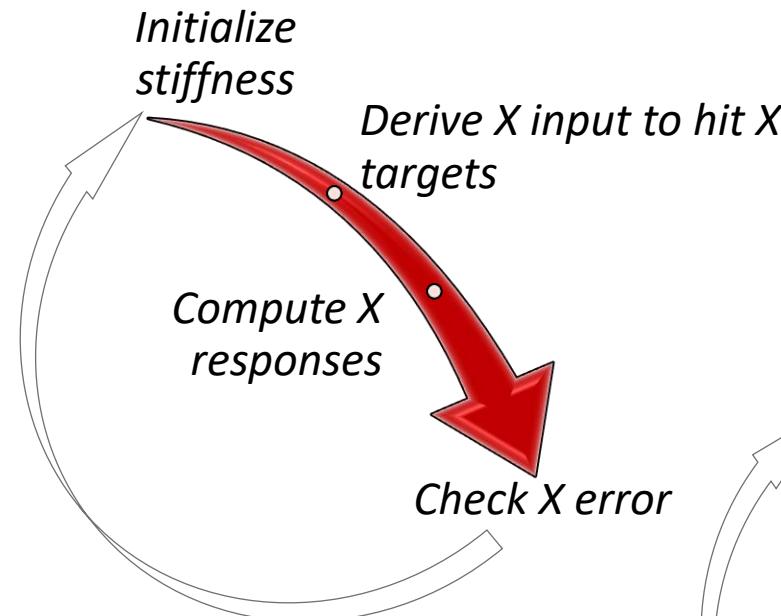
Knight et al., 2018

Simulation Approach

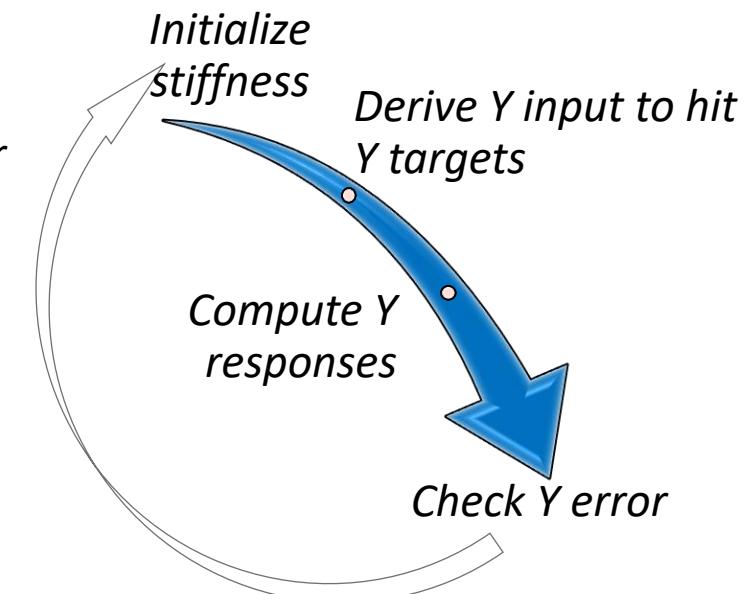
SIMO Multi-Axis Test



X-Axis Sequential Test



Y-Axis Sequential Test



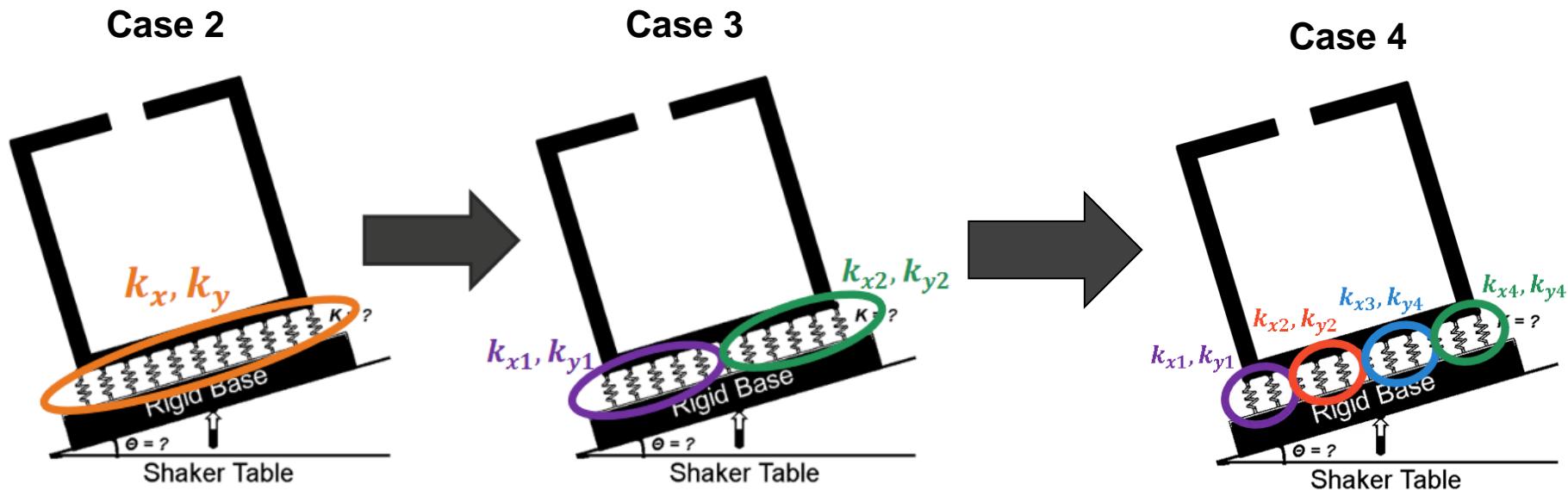
Case Studies

Sequential, single-axis testing and the proposed method are compared with increasing levels of test fixture complexity and design effort.

| Case | Test Fixture Design | Comparisons |
|------|--------------------------|--|
| 1 | Rigid | <ol style="list-style-type: none">1. Sequential single-axis (single control location)2. Sequential single-axis (all control locations)3. SIMO multi-axis (all control locations) |
| 2 | Optimized (2 parameters) | <ol style="list-style-type: none">1. Sequential single-axis2. SIMO multi-axis |
| 3 | Optimized (4 parameters) | <ol style="list-style-type: none">1. Sequential single-axis2. SIMO multi-axis |
| 4 | Optimized (8 parameters) | <ol style="list-style-type: none">1. Sequential single-axis2. SIMO multi-axis |

Case Studies

- In our model, the stiffness of the springs providing the boundary condition for the BARC test article is varied to change the test fixture.
- In Case 1, the boundary condition is rigid.
- In Cases 2, 3, and 4, the boundary condition is optimized using an increasing number of parameters to determine how much test fixture complexity contributes to improving the possible solutions.



Case Study 2 (2 parameters)

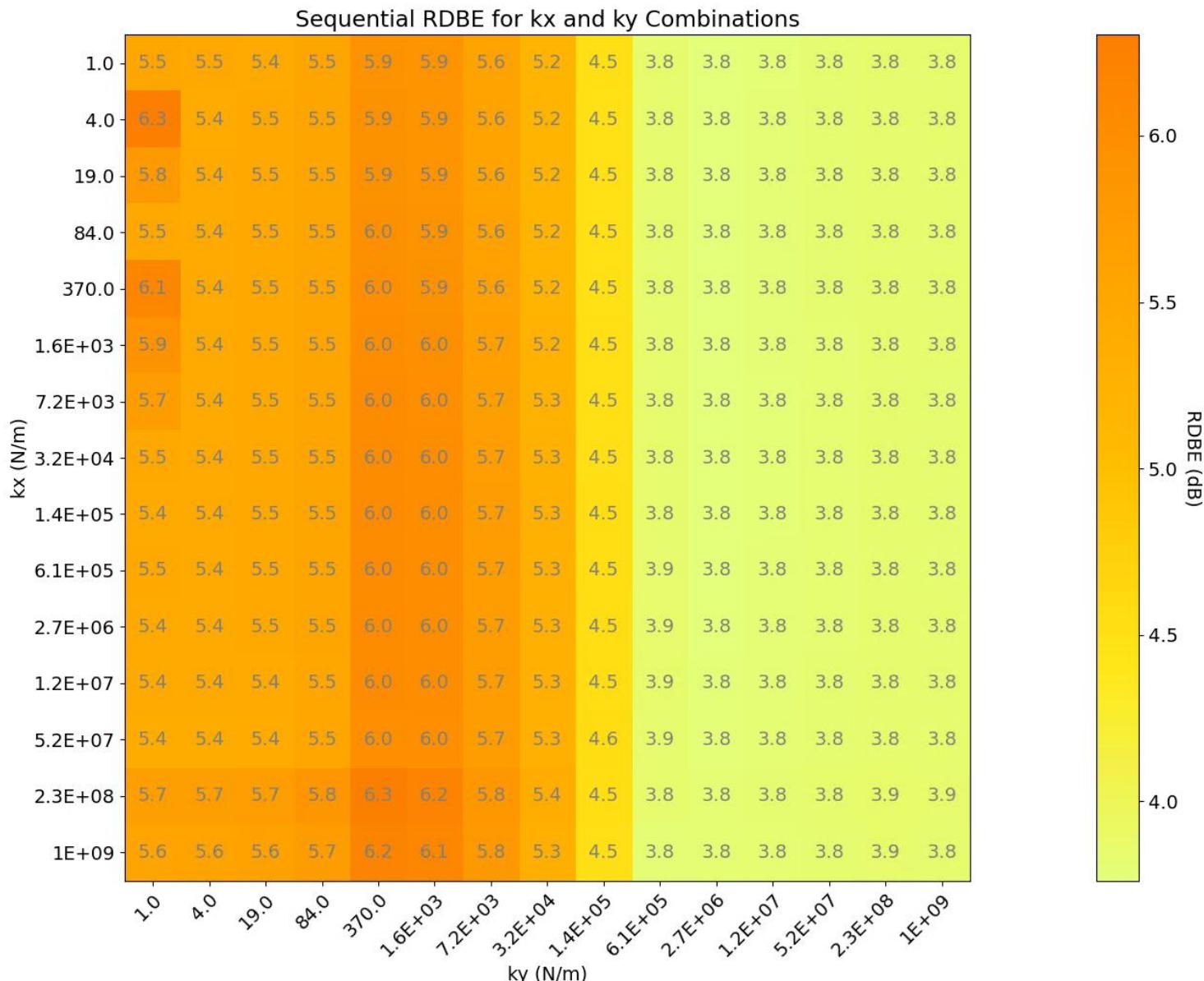
The solution found for each test was:

1. For the sequential test, $k_x = 3.2 * 10^4$ and $k_y = 2.7 * 10^6$ N/m.
2. For the SIMO test, $k_x = 1.2 * 10^7$, $k_y = 2.7 * 10^6$ N/m, and $\theta = 27^\circ$.

| | Mean Error | |
|------|------------------------|-----------------|
| | Sequential Single-Axis | SIMO Multi-Axis |
| RDBE | 3.7 dB | 4.5 dB |
| FTOL | 80.0% | 75.6% |

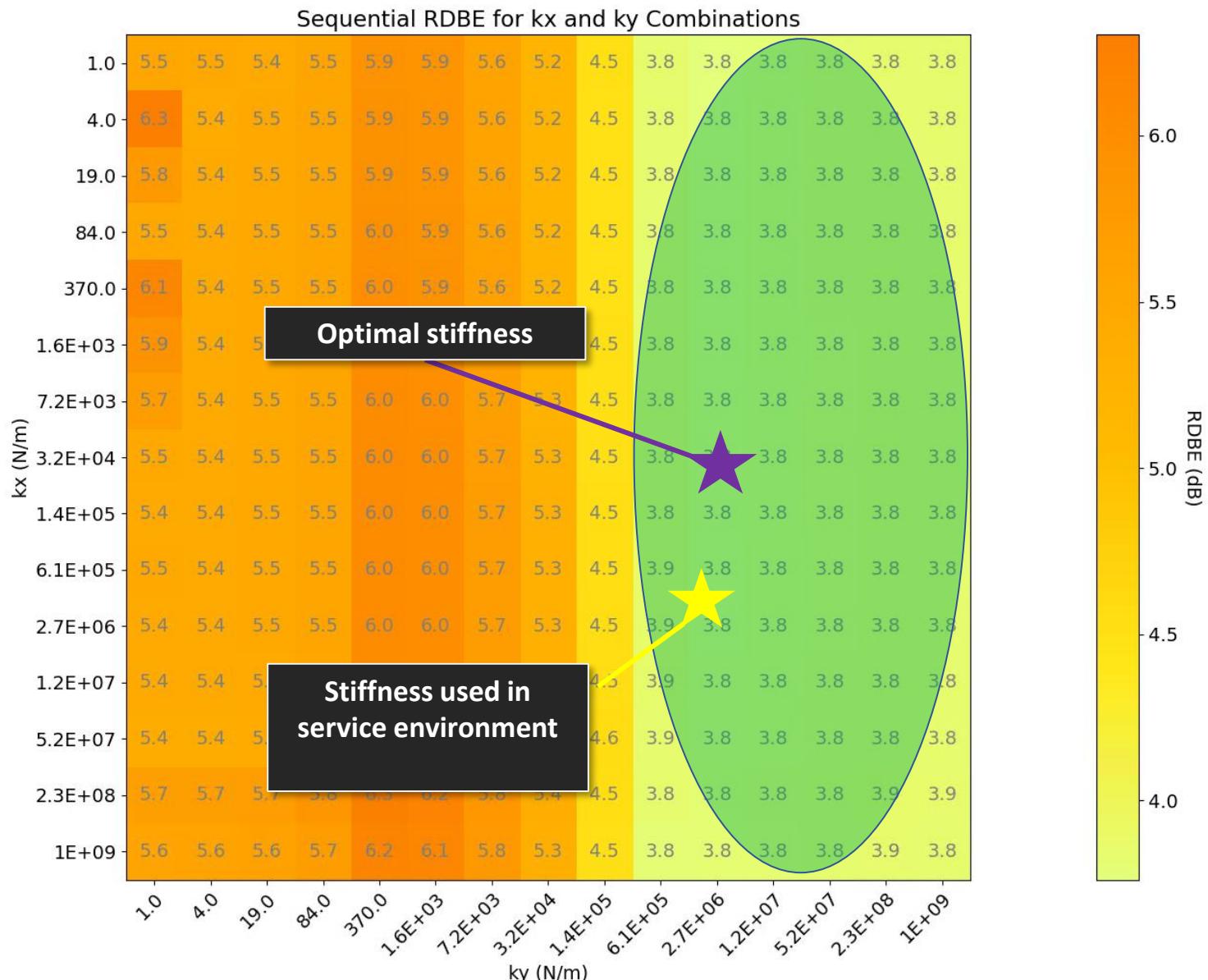
Case Study 2

- The sequential test's stiffness optimization found the X-stiffness had little impact while the Y-stiffness needed to be above $1.4 * 10^5 \text{ N/m}$ to produce good error.



Case Study 2

- The sequential test's stiffness optimization found the X-stiffness had little impact while the Y-stiffness needed to be above $1.4 * 10^5 \text{ N/m}$ to produce good error.



Case Study 3

The solution found for each test was:

1. For the sequential test, $k_{x1} = 3.2 * 10^4$, $k_{x2} = 1$, $k_{y1} = 2.7 * 10^6$, $k_{y2} = 2.7 * 10^6$ N/m.
2. For the SIMO test, $k_{x1} = 19$, $k_{x2} = 2.3 * 10^8$, $k_{y1} = 2.7 * 10^6$, $k_{y2} = 2.7 * 10^6$ N/m, $\theta = 27^\circ$.

| | Mean Error | |
|------|------------------------|-----------------|
| | Sequential Single-Axis | SIMO Multi-Axis |
| RDBE | 3.7 dB | 4.5 dB |
| FTOL | 80.0% | 75.6% |

Case Study 4

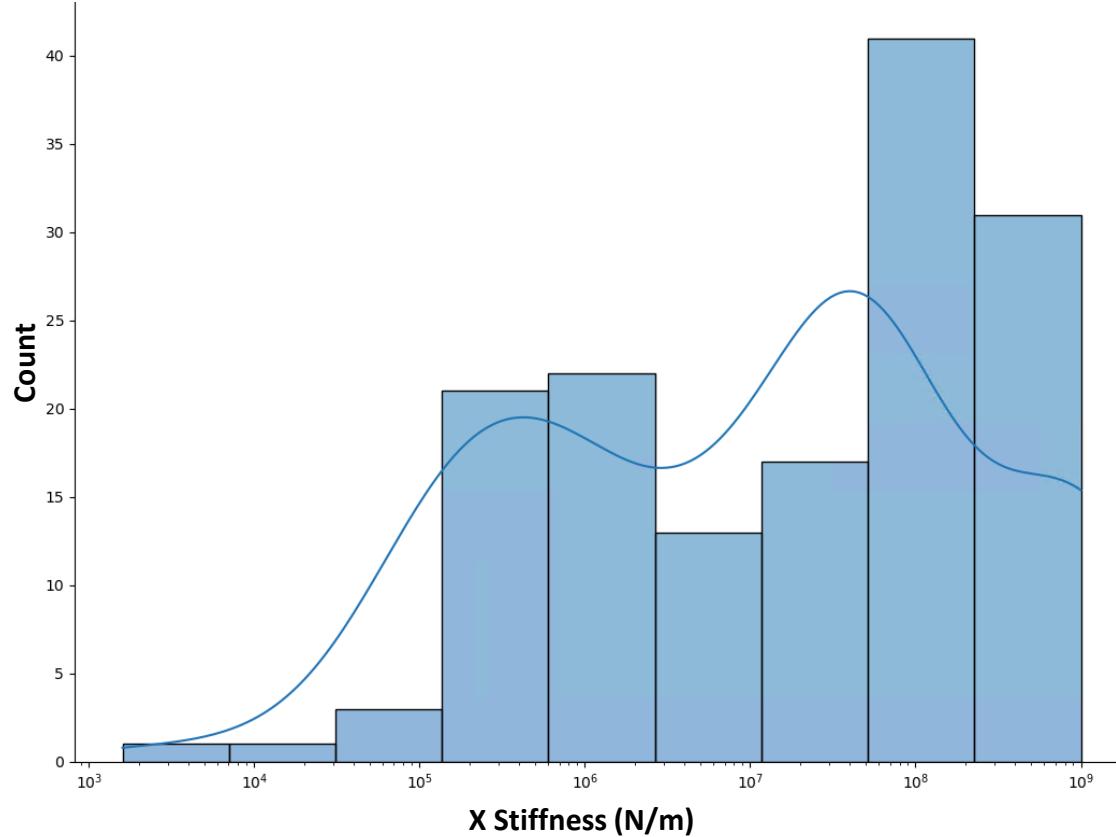
The solution found for each test was:

1. For the sequential test, $k_{x1} = 1, k_{x2} = 1, k_{x3} = 1, k_{x4} = 1, k_{y1} = 1, k_{y2} = 10^9, k_{y3} = 10^9, k_{y4} = 1 \text{ N/m.}$
2. For the SIMO test, $k_{x1} = 10^9, k_{x2} = 3.2 * 10^4, k_{x3} = 2.7 * 10^6, k_{x4} = 2.7 * 10^6, k_{y1} = 2.7 * 10^6, k_{y2} = 2.7 * 10^6, k_{y3} = 10^9, k_{y4} = 2.7 * 10^6 \text{ N/m, } \theta = 27^\circ.$

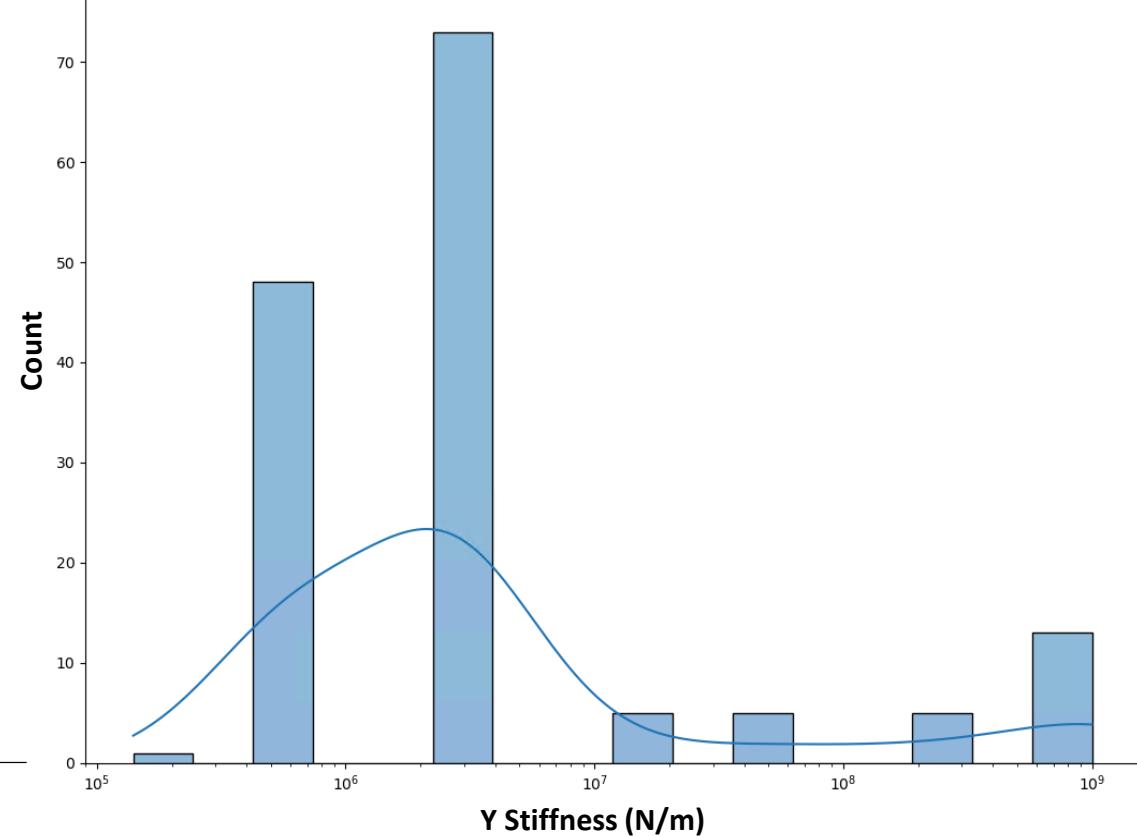
| | Mean Error | |
|------|------------------------|-----------------|
| | Sequential Single-Axis | SIMO Multi-Axis |
| RDBE | 3.7 dB | 4.5 dB |
| FTOL | 79.6% | 75.3% |

Target Population Study

X Stiffness Distribution

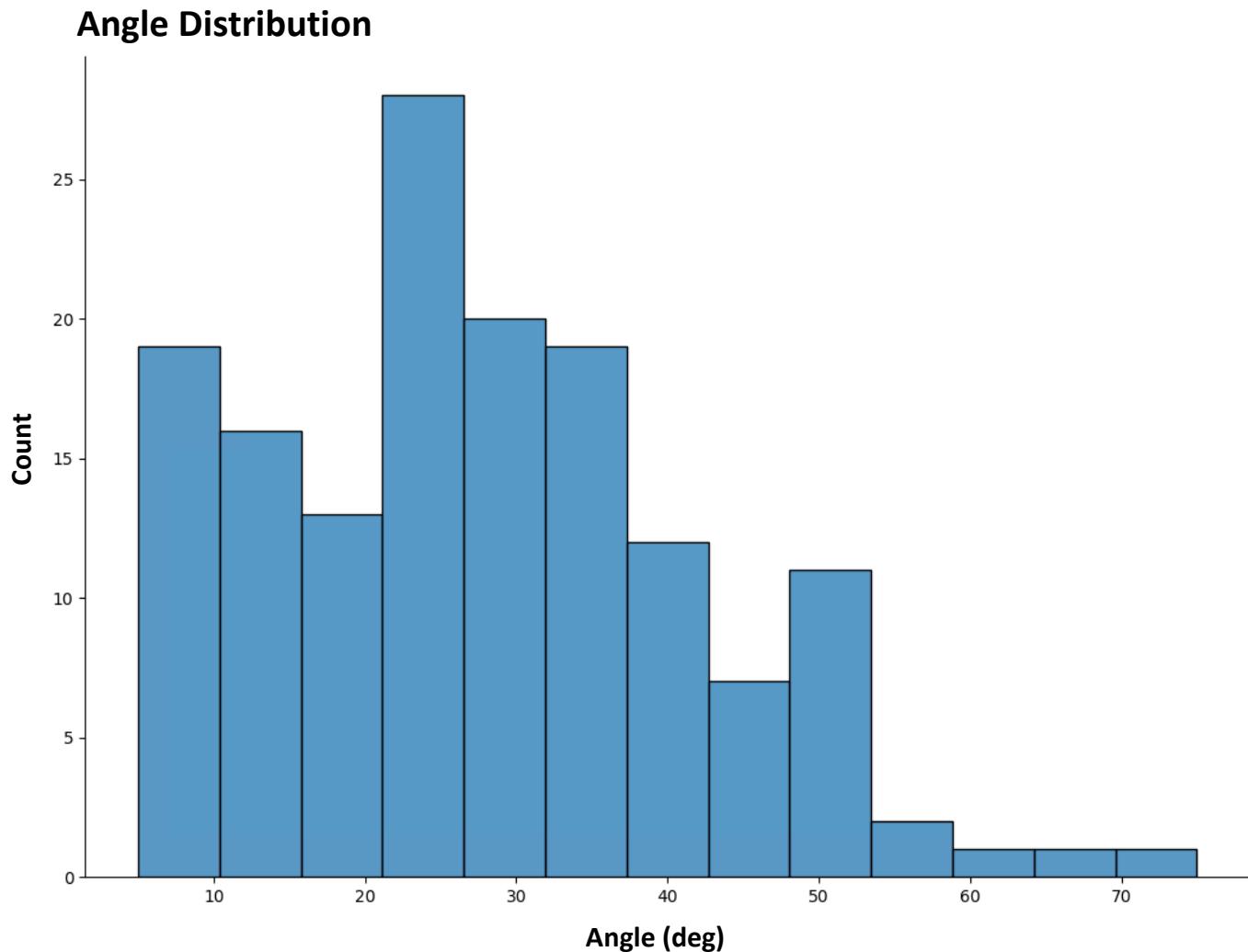


Y Stiffness Distribution



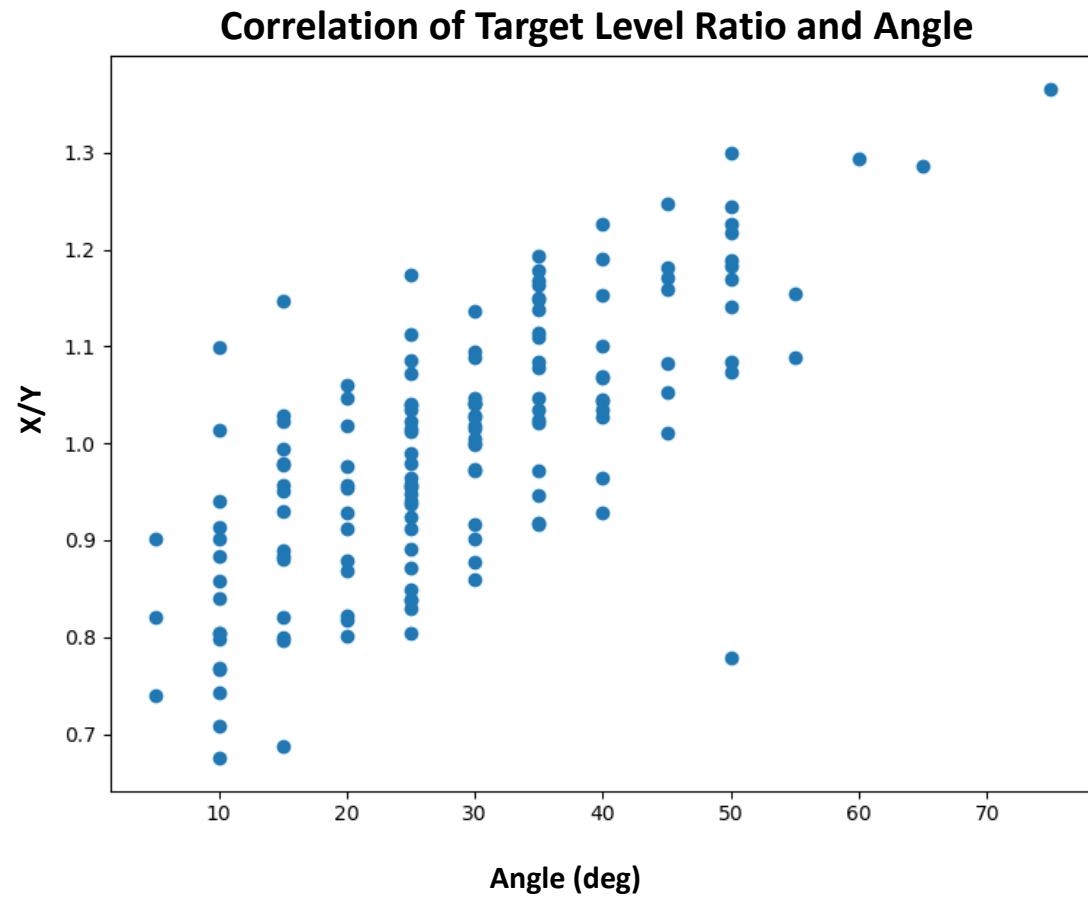
Target Population Study

- For the population of targets, the distribution of optimal excitation SIMO test excitation angles are shown.
- The angle most commonly fell between 20 and 40 degrees

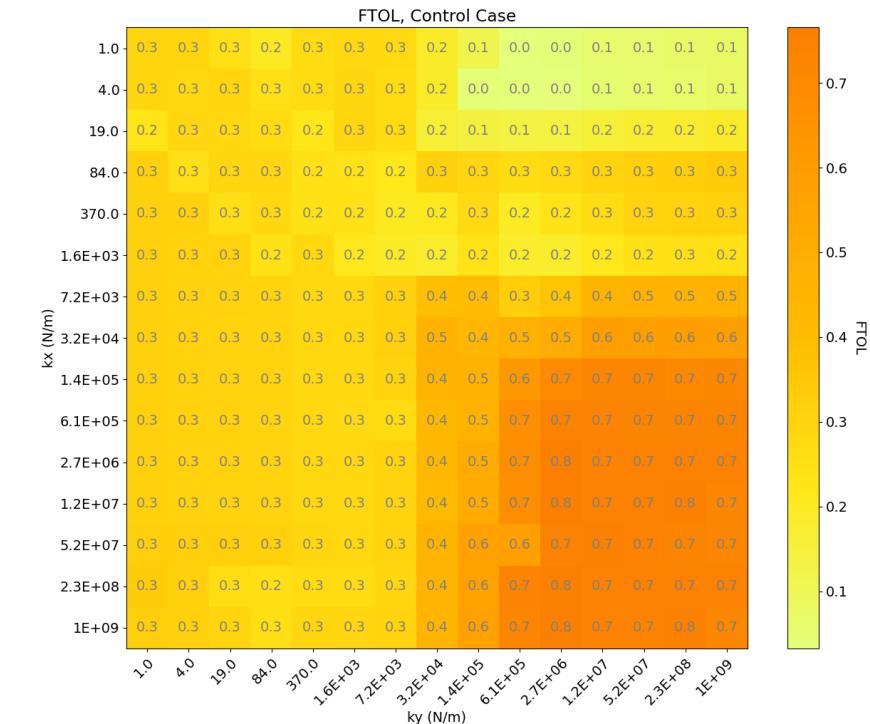
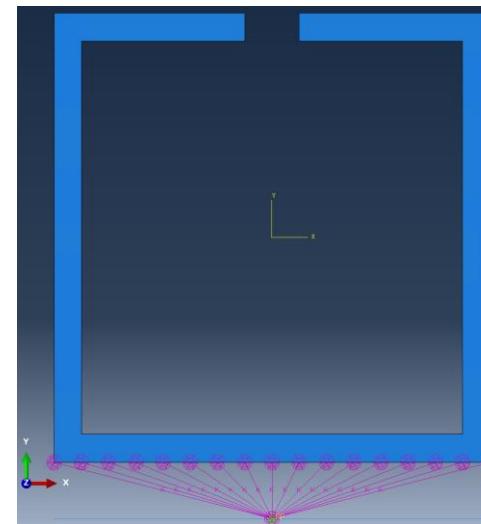
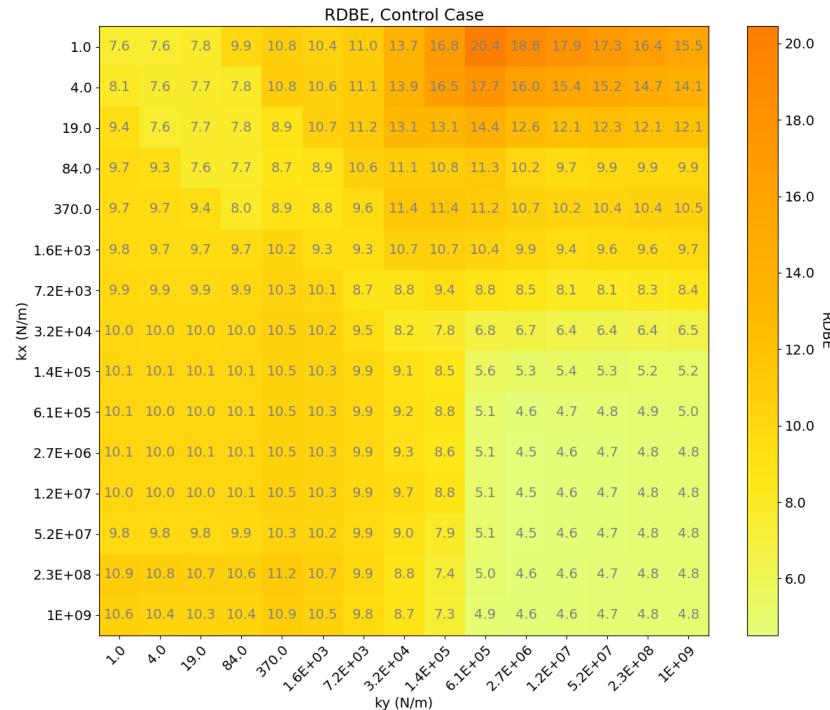


Target Population Study

- Evaluated possible predictive metrics for optimal excitation angle based on target environment alone.
- A ratio of energy between X-axis and Y-axis targets (X/Y) has a correlation coefficient with optimal angle of 0.728.

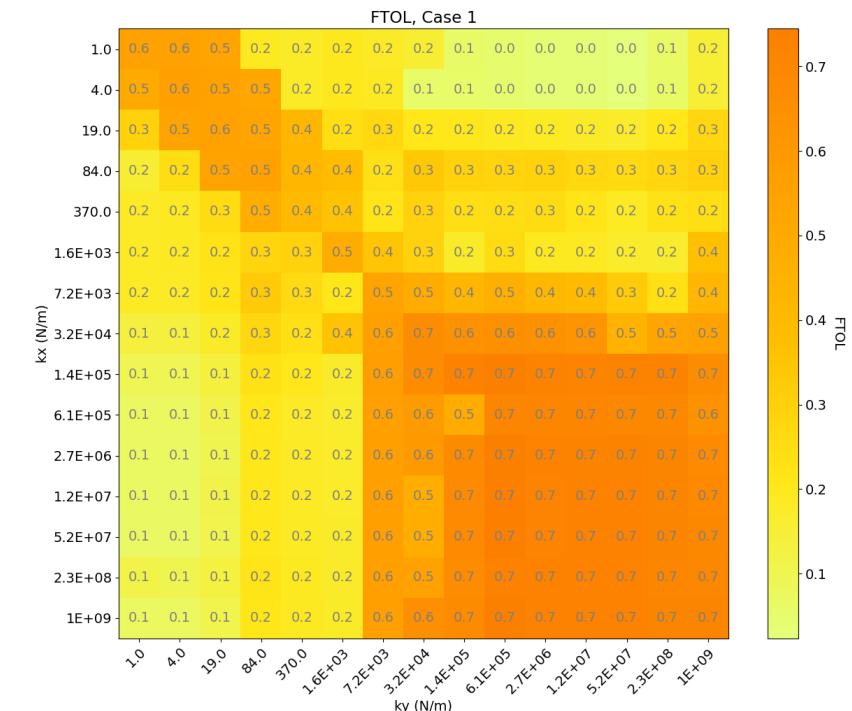
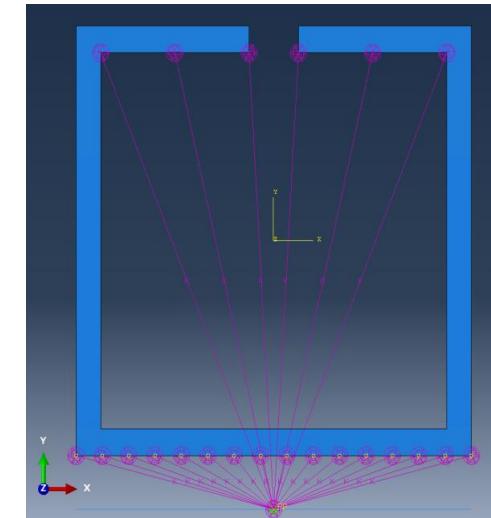
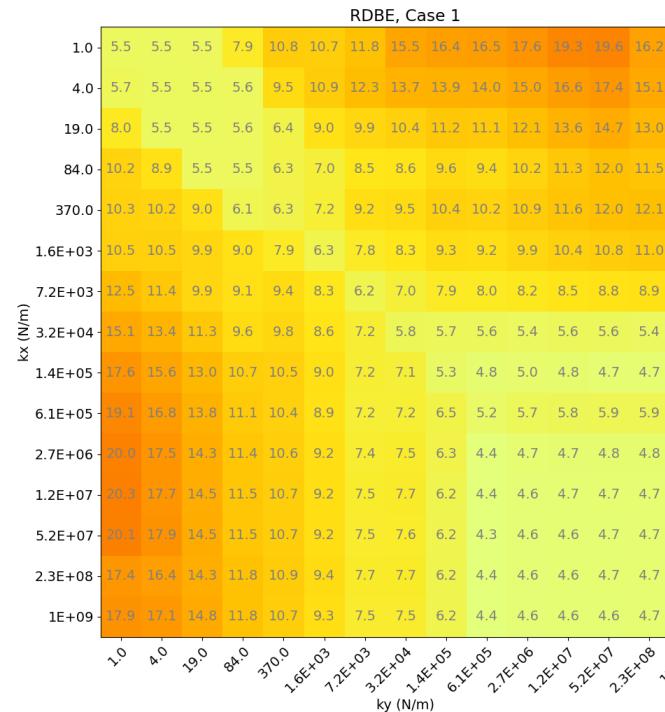


Test Fixture Attachment Points



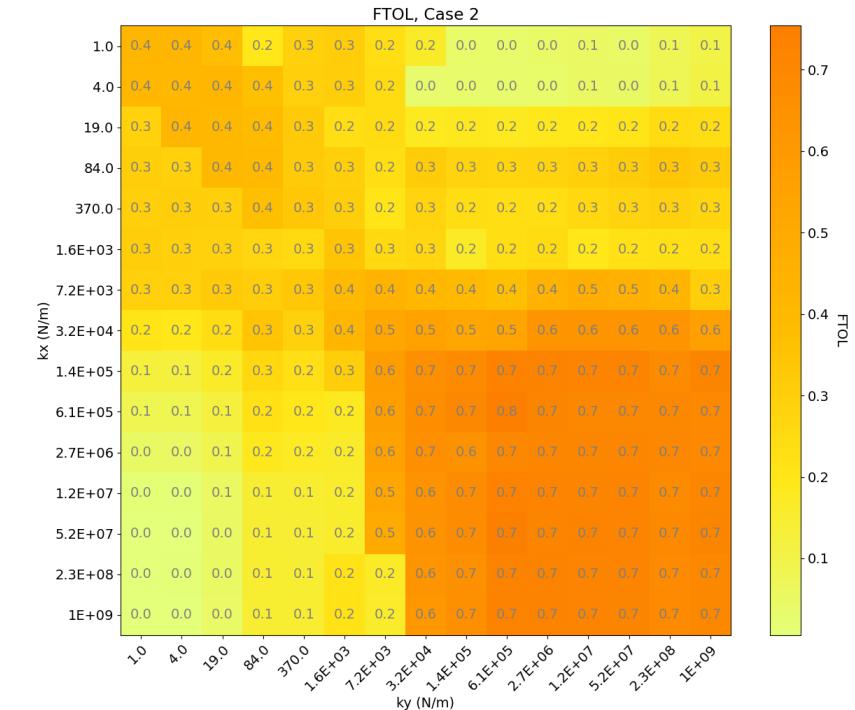
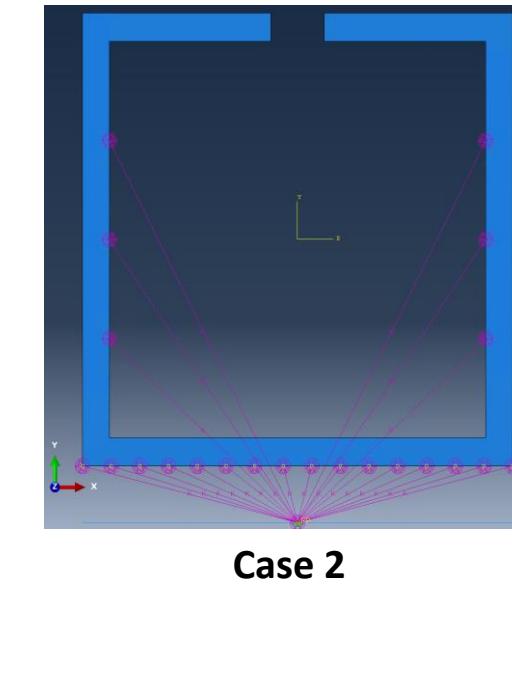
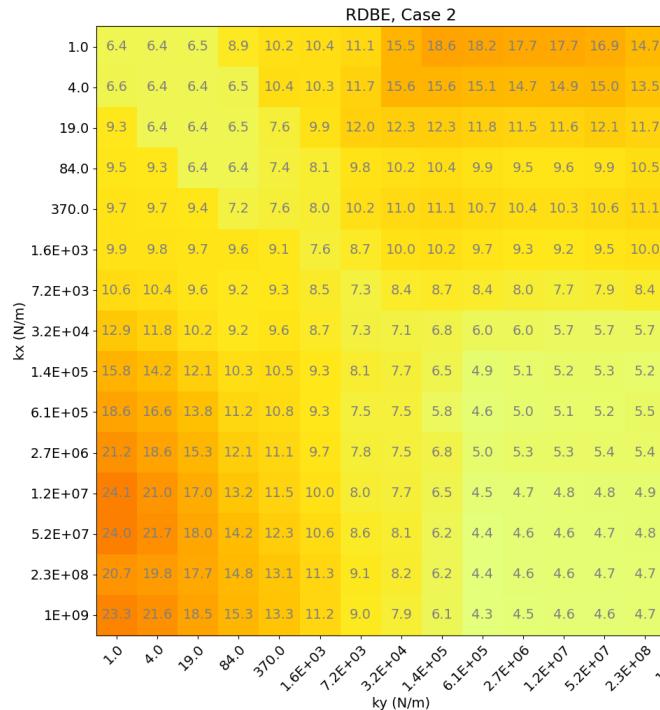
- To assess potential improvements via varied test fixture attachment point, we assign the default target's test fixture optimization as the control case.

Test Fixture Attachment Points



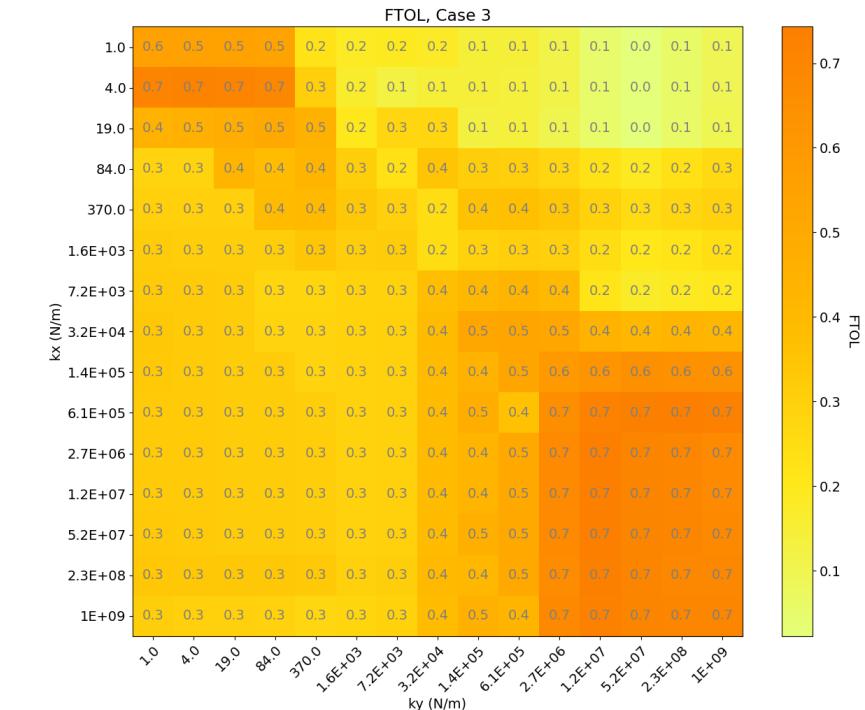
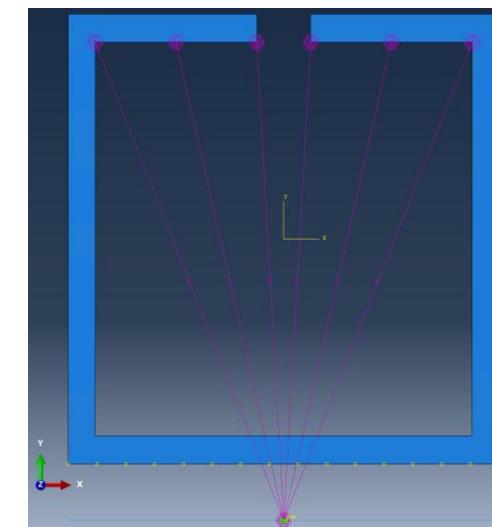
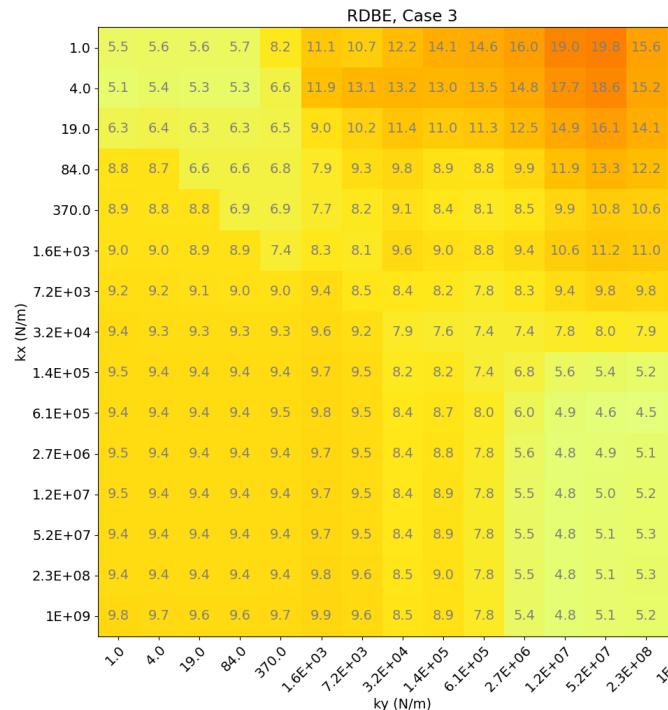
- The first assessed case includes additional springs attached to the top of the BARC. This setup found an improved RMS dB error of 4.3 dB.

Test Fixture Attachment Points



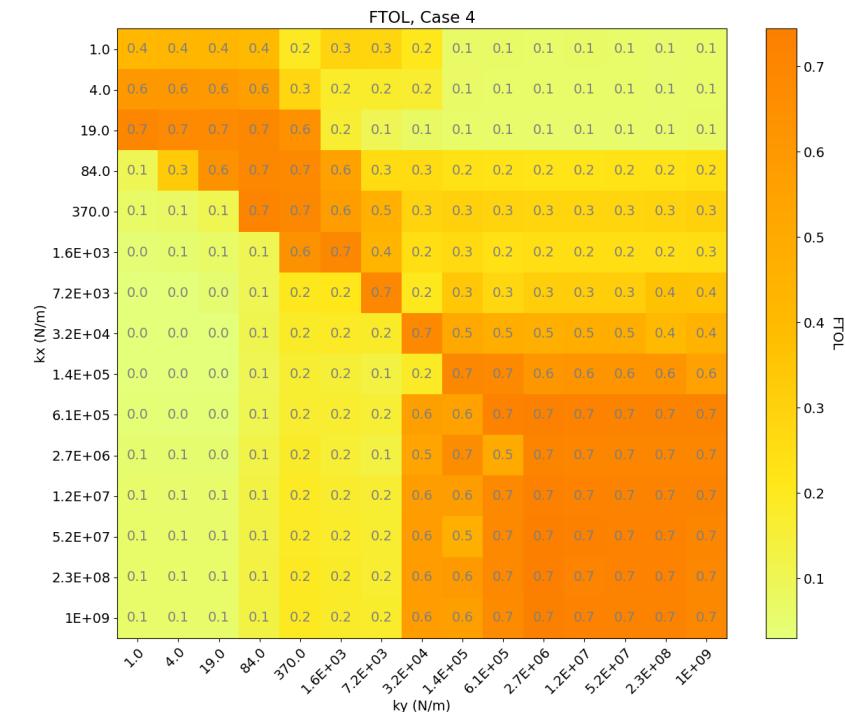
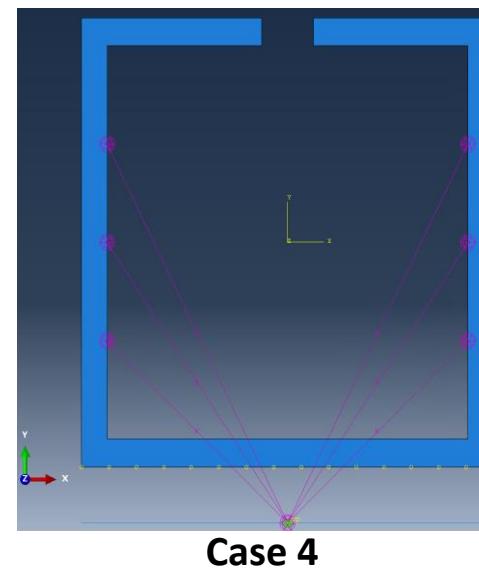
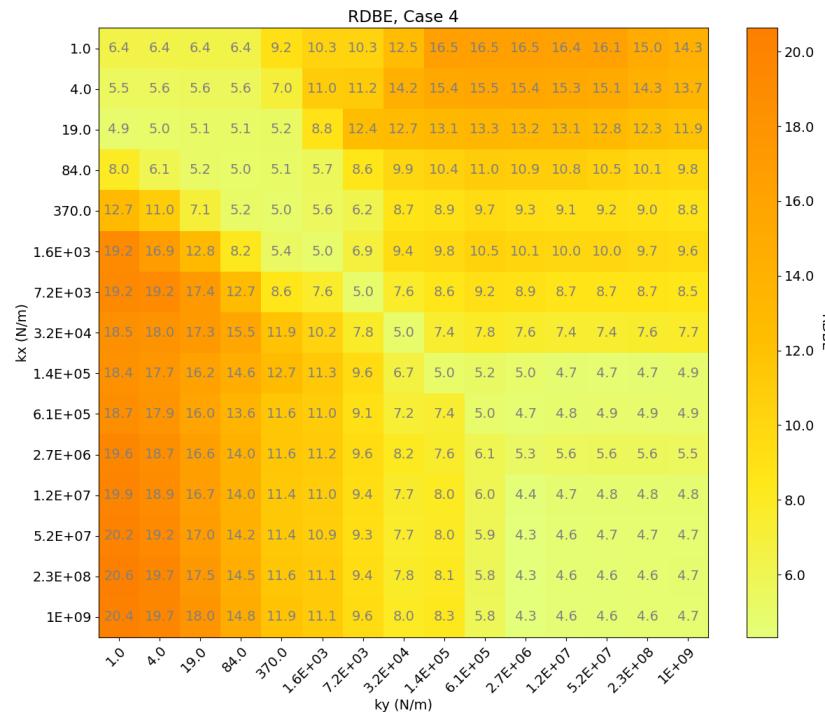
- The second assessed case includes additional springs attached to the side of the BARC. This setup found an improved RMS dB error of 4.3 dB.

Test Fixture Attachment Points



- The third assessed case has springs attached only to the top of the BARC. This case failed to improve upon the control case.

Test Fixture Attachment Points



- The fourth assessed case has springs attached only to the sides of the BARC. This setup found an improved RMS dB error of 4.3 dB.

References

- K. Ahlin, “Comparison of Test Specifications and Measured Field Data,” *SOUND AND VIBRATION*, p. 3, 2006.
- M. P. Bendsøe and O. Sigmund, “Material interpolation schemes in topology optimization,” *Archive of Applied Mechanics (Ingenieur Archive)*, vol. 69, no. 9–10, pp. 635–654, Nov. 1999, doi: 10.1007/s004190050248.
- Bouma, A. Campbell, T. Roberts, S. Taylor, C. Haynes, and D. Harvey, “Accumulated Lifetimes in Single-Axis Vibration Testing,” in *Sensors and Instrumentation, Aircraft/Aerospace, Energy Harvesting & Dynamic Environments Testing*, Volume 7, vol. 7, 2019.
- Chandrasekhar and K. Suresh, “TOuNN: Topology Optimization using Neural Networks,” *Struct Multidisc Optim*, vol. 63, no. 3, pp. 1135–1149, Mar. 2021, doi: 10.1007/s00158-020-02748-4.
- P. M. Daborn, “Replicating Aerodynamic Excitation in the Laboratory,” in *Topics in Modal Analysis II*, Volume 7: Proceedings of the 31st IMAC, A Conference and Exposition on Structural Dynamics, 2014.
- P. M. Daborn, C. Roberts, D. J. Ewins, and P. R. Ind, “Next-Generation Random Vibration Tests,” in *Topics in Modal Analysis II*, Volume 8: Proceedings of the 32nd IMAC, A Conference and Exposition on Structural Dynamics, 2014, pp. 397–410.
- W. de Silva, “Vibration: Fundamentals and Practice Chapter 10,” in *Vibration: Fundamentals and Practice*, 2nd ed., 2006.
- R. M. French, R. Handy, and H. L. Cooper, “A COMPARISON OF SIMULTANEOUS AND SEQUENTIAL SINGLE-AXIS DURABILITY TESTING,” *Exp Techniques*, vol. 30, no. 5, pp. 32–37, Sep. 2006, doi: 10.1111/j.1747-1567.2006.00083.x.
- J. Gatscher, “Comparison of Mechanical Impedance Methods for Vibration Simulation,” 1996.
- F. Gomez and B. F. Spencer, “Topology optimization framework for structures subjected to stationary stochastic dynamic loads,” *Struct Multidisc Optim*, vol. 59, no. 3, pp. 813–833, Mar. 2019, doi: 10.1007/s00158-018-2103-3.

References

- Gregory, A. Nm, and D. O. Smallwood, “Comparison of the Response of a Simple Structure to Single Axis and Multiple Axis Random Vibration Inputs,” p. 9.
- T. M. Hall, “Analytically Investigating Impedance-Matching Test Fixtures,” in Sensors and Instrumentation, Aircraft/Aerospace, Energy Harvesting & Dynamic Environments Testing, Volume 7, vol. 7, Society of Experimental Mechanics, 2020, pp. 21–31.
- J. M. Harvie, “Using Modal Substructuring to Improve Shock & Vibration Qualification,” in Topics in Modal Analysis & Testing, Volume 9, M. Mains and B. J. Dilworth, Eds. Cham: Springer International Publishing, 2017, pp. 227–239. doi: 10.1007/978-3-319-74700-2_24.
- J. M. Harvie and M. van der Seijs, “Application of Transfer Path Analysis Techniques to the Boundary Condition Challenge Problem,” in Sensors and Instrumentation, Aircraft/Aerospace, Energy Harvesting & Dynamic Environments Testing, Volume 7, C. Walber, P. Walter, and S. Seidlitz, Eds. Cham: Springer International Publishing, 2020, pp. 157–166. doi: 10.1007/978-3-030-12676-6_15.
- S. Hoyer, J. Sohl-Dickstein, and S. Greydanus, “Neural reparameterization improves structural optimization,” arXiv:1909.04240 [cs, stat], Sep. 2019, Accessed: Nov. 10, 2021. [Online]. Available: <http://arxiv.org/abs/1909.04240>
- R. Jones, D. Soine, J. Harvie, T. Schoenherr, T. Skousen, and M. Starr, “Boundary Conditions in Environmental Testing Round Robin,” p. 53, 2018.
- A. J. Knight, “More Representative Spacecraft Random Vibration Testing,” p. 158, 2019.
- C. Knight, M. Remedia, G. S. Aglietti, and G. Richardson, “Satellite Vibration Testing: Angle optimisation method to Reduce Overtesting,” *Acta Astronautica*, vol. 147, pp. 205–218, Jun. 2018, doi: 10.1016/j.actaastro.2018.04.004.
- R. Kolaini, W. Tsuha, and J. P. Fernandez, “Spacecraft vibration testing: Benefits and potential issues,” *Advances in aircraft and spacecraft science*, vol. 5, no. 2, pp. 165–175, Mar. 2018, doi: 10.12989/AAS.2018.5.2.165.

References

- Z. Nie, T. Lin, H. Jiang, and L. B. Kara, “TopologyGAN: Topology Optimization Using Generative Adversarial Networks Based on Physical Fields Over the Initial Domain,” p. 13.
- J. On, “Mechanical impedance analysis for lumped parameter multi-degree of freedom/multi-dimensional systems,” p. 41.
- M. G. Ostergaard, A. R. Ibbotson, O. L. Roux, and A. M. Prior, “Virtual testing of aircraft structures,” *CEAS Aeronaut J*, vol. 1, no. 1–4, pp. 83–103, Sep. 2011, doi: 10.1007/s13272-011-0004-x.
- Z. Qiao, Z. Weihong, Z. Jihong, and G. Tong, “Layout optimization of multi-component structures under static loads and random excitations,” *Engineering Structures*, vol. 43, pp. 120–128, Oct. 2012, doi: 10.1016/j.engstruct.2012.05.013.
- S. Rawat, “A Novel Topology Optimization Approach using Conditional Deep Learning,” p. 8.
- M. Remedia, G. S. Aglietti, M. Appolloni, A. Cozzani, and A. Kiley, “An Enhanced Methodology for Spacecraft Correlation Activity Using Virtual Testing Tools,” p. 41, 2017.
- C. Roberts and D. Ewins, “Multi-axis vibration testing of an aerodynamically excited structure,” *Journal of Vibration and Control*, vol. 24, no. 2, pp. 427–437, Jan. 2018, doi: 10.1177/1077546316642064.
- D. P. Rohe. Modal data for the BARC challenge problem Test Report. United States: N. p., 2018. Web. doi:10.2172/1418738.
- D. P. Rohe, R. A. Schultz, T. F. Schoenherr, T. J. Skousen, and R. J. Jones, “Comparison of Multi-Axis Testing of the BARC Structure with Varying Boundary Conditions,” in *Sensors and Instrumentation, Aircraft/Aerospace, Energy Harvesting & Dynamic Environments Testing, Volume 7*, C. Walber, P. Walter, and S. Seidlitz, Eds. Cham: Springer International Publishing, 2020, pp. 179–193. doi: 10.1007/978-3-030-12676-6_17.
- T. D. Scharton, “Vibration and Acoustic Testing of Spacecraft,” p. 5, 2002.

References

- T. D. Scharton, “DEVELOPMENT OF IMPEDANCE SIMULATION FIXTURES FOR SPACECRAFT VIBRATION TESTS,” p. 64.
- T. F. Schoenherr, “Designing an Impedance Matched Test Fixture Using Parameterized Optimization and the Modal Projection Error,” p. 25.
- T. F. Schoenherr, “Derivation of Six Degree of Freedom Shaker Inputs Using Sub-structuring Techniques,” in *Topics in Modal Analysis & Testing, Volume 9*, M. Mains and B. J. Dilworth, Eds. Cham: Springer International Publishing, 2019, pp. 5–14. doi: 10.1007/978-3-319-74700-2_2.
- T. F. Schoenherr, P. Coffin, and B. Clark, “Use of Topology Optimization to Design Shock and Vibration Test Fixtures,” in *Sensors and Instrumentation, Aircraft/Aerospace, Energy Harvesting & Dynamic Environments Testing, Volume 7*, C. Walber, P. Walter, and S. Seidlitz, Eds. Cham: Springer International Publishing, 2020, pp. 77–92. doi: 10.1007/978-3-030-12676-6_8.
- D. O. Smallwood, “The challenges of multiple input vibration testing and analysis,” p. 46, 2013.
- D. E. Soine, R. J. Jones, J. M. Harvie, T. J. Skousen, and T. F. Schoenherr, “Designing Hardware for the Boundary Condition Round Robin Challenge,” p. 4.
- B. H. V. Topping, Ed., *Optimization and Artificial Intelligence in Civil and Structural Engineering*. Dordrecht: Springer Netherlands, 1992. doi: 10.1007/978-94-017-2490-6.
- P. S. Varoto and L. P. R. de Oliveira, “Interaction Between a Vibration Exciter and the Structure Under Test,” *SOUND AND VIBRATION*, p. 6, 2002.
- S. Waimer, S. Manzato, B. Peeters, M. Wagner, and P. Guillaume, “Modelling and simulation of a closed-loop electrodynamic shaker and test structure model for spacecraft vibration testing,” *Advances in aircraft and spacecraft science*, vol. 5, no. 2, pp. 205–223, Mar. 2018, doi: 10.12989/AAS.2018.5.2.205.

References

- Wein, P. D. Dunning, and J. A. Norato, “A review on feature-mapping methods for structural optimization,” *Struct Multidisc Optim*, vol. 62, no. 4, pp. 1597–1638, Oct. 2020, doi: 10.1007/s00158-020-02649-6.
- Y. Yang, M. Zhu, M. D. Shields, and J. K. Guest, “Topology optimization of continuum structures subjected to filtered white noise stochastic excitations,” *Computer Methods in Applied Mechanics and Engineering*, vol. 324, pp. 438–456, Sep. 2017, doi: 10.1016/j.cma.2017.06.015.
- “MIL-STD-810H, Department of Defense Test Method Standard: Environmental Engineering Considerations and Laboratory Tests.” Defense Logistics Agency, United States. 31 Jan 2019.

