Version of Record: https://www.sciencedirect.com/science/article/pii/S2352152X22008246
Manuscript_611cdc8f06d10d5eec635443ad69aeee

Economic Dispatch for Electricity Merchant with Energy Storage and Wind Plant: State of Charge Based Decision Making Considering Market Impact and Uncertainties

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Abstract: This paper investigates how the market impact of electricity merchants and uncertainty of wind generation affect their co-optimized scheduling policy, specifically for merchants who have both energy storage and wind plants. In the existing literature, merchants' trading actions are usually assumed not to affect market prices; however, a large-scale energy storage merchant's actions can affect market prices. To this end, we approximate the electricity price by a linear function of the quantity of power traded by the merchant in the reward function to achieve decision-making incorporating the market impact. This paper utilizes the dynamic programming approach to analyze merchants' optimal multi-period decision-making incorporating market impact, uncertain wind generation, and energy storage constraints. First, our results demonstrate that for a merchant with co-located energy storage facilities and wind power plants, the energy storage's feasible state of charge (SOC) range can be segmented into four possible sub-ranges by three analytical SOC reference points. The unique optimal trading decision can be achieved by comparing the current energy inventory and the SOC references in the next period. Second, our results show that market impact and uncertainties substantially change the optimal storage scheduling policy by impacting the values of reference points. To smooth the negative effect of the merchant's market impact on buying and selling actions, the merchant should reduce the amount of electricity generating or pumping each period to maximize profit. Moreover, we identify and investigate the trade-off between increasing the unit power profit and lowering the transaction quantity. Our findings provide co-optimized scheduling guidance for electricity merchants with co-located energy storage and renewable power plants systems.

Keywords: Pumped storage hydro; Wind plants; Market impact, Dynamic programming; State of Charge; Economic dispatch

1. Introduction

Sustainable and renewable energy resources (solar, wind, etc.) have been developing rapidly worldwide in the last two decades owing to no carbon emission, technology-driven cost reduction, and national/state-level regulations and targets. However, these resources are strongly dependent on the weather and so are characterized by intermittency and high levels of uncertainty as well as low forecast reliability (Korpaas et al., 2003; Liu et al., 2022; Cory-Wright & Zakeri, 2020). Moreover, because the electricity supply and demand must be matched in real-time, it is significant for grid operators to deal with electricity insufficiency and surpluses. The energy storage system plays a vital role in dealing with the imbalance (Ahmad et al., 2021; Lai et al., 2021). Energy storage can provide many different types of services for ISO (Independent System Operator), utilities, electricity merchants, and end-users (Bo et al., 2021). From a market participant perspective, energy storage offers an arbitrage opportunity for electricity merchants. Storing electricity for future resale is a typical approach of merchants that sell commodities (Williams and Wright, 1991).

To manage the intermittency of renewable sources and create the flexibility for energy arbitrage, most wind plants owners have embraced collocating electricity generation and grid-connected energy storage facilities such as PSH (pumped storage hydropower) (Al-Masri et al., 2021), compressed air energy storage (Yu et al., 2021), and battery (Ahmad et al., 2021). One example is the Wilmot Energy Center that contains a 30-MW battery energy storage and a 100-MW solar array system (Tucson Electric Power, 2021). Co-optimization of grid-level storage (Garcia-Gonzalez et al., 2008; Zhou et al., 2019) with a wind farm can create value by mitigating the intermittent nature of wind generation by pumping electricity when the wind-generated power output is mismatched power demand (i.e., PSH and storage may benefit the environment also by reducing the wind generation curtailment), by storing wind generation and reselling in future when prices are low, and also by enabling the merchant to buy power for the future. U.S. Department of Energy (DOE) (2018) reported the value of co-located energy storage and wind plants.

When modeling energy storage, research into energy inventory has mainly focused on the optimal scheduling policy or on the optimal bidding decision (McPherson et al., 2020). However, most existent studies (Cheng & Powell, 2018; Ding et al., 2018; Liu et al., 2022; Kim & Powell, 2011; Secomandi, 2010; Zhou et al., 2019) assume that the storage capacity is sufficiently small compared to the wholesale electricity markets, so its operational decisions (i.e., charging and buying or discharging and selling) *do not* affect the electricity prices. Thus, given the price in each period, a merchant (hereafter, she) buys for charging or discharge for selling a certain quantity of energy at a price that is not influenced by her own operational decisions (i.e., in our terminology, *price-taker* merchant). However, the value of large-scale energy storage such as pumped storage hydro (PSH) facilities would be reflected in price arbitrage (Bushnell, 2003; Cruise et al., 2019; Felix et al. 2012; Liu et al., 2021a; Sioshansi et al., 2009); in such case, the electricity merchant's operational trading actions will influence prices in the power market (i.e., *price-maker* merchant). More specifically, the market load will increase when a merchant buys electricity, thus leading to a rise in market prices; conversely, selling power will increase the supply and reduce market prices. Therefore, large-scale electricity storage can reduce its energy arbitrage value by decreasing differences in sale prices on-peak load and purchase cost on off-peak periods (Sioshansi, (2010, 2014)).

Compared to the traditional study for a *price-taker,* Sioshansi et al. (2009) study the arbitrage value of a 1 GW (gigawatts) of an energy storage system in PJM Interconnection from 2002 to 2007 and showed that the price-smoothing *differences in prices on and off-peak* can reduce the arbitrage value over 20% since it results in greater off-peak and less on-peak generation. With decreasing technology costs and boosting renewable deployment, energy storage is poised to be a valuable resource on future power grids. Will et al. (2021) reported that the energy storage would exceed 125 GW by 2050, more than a five-fold increase from the installed storage capacity of 23 GW in 2020. Thus, the market impact of the electricity merchants in trading will significantly affect their own decisions.

In contrast to the previous research, this study analyzes how the market impact affects the co-optimization economic dispatch structure of merchants with co-located energy storage systems and wind plants. The traditional study is mainly based on the optimal scheduling policy that the merchant purchases electricity from the market when prices are low and sells electricity to the market when prices are high (Powell and Meisel, 2016). As a result, considering the market impact, the profit-maximizing merchant's co-optimized scheduling policy depends not only on the traditional operational approach but also on the market impact of the merchant's operational actions on prices and the uncertainty of forecasted wind generation. Therefore, it is valuable to examine the co-optimized economic dispatch policy for electricity merchants who have large-scale energy storage facilities and wind plants and their market impact on energy storage operations. Thus, our study aims to provide new insight into how optimal co-optimized scheduling policies differ for the merchant who has co-located energy storage systems and renewable power plants under these two scenarios (i.e., price impact vs. no price impact).

Prior research in this area commonly supposes that energy/power in storage is worthless in the last period (Liu et al., 2021a; Zhou et al., 2019). This assumption means the merchant should reduce the state of charge (SOC) down to the lower boundary of the energy storage capacity, so the choice is either discharging or remaining idle during the last period of optimization horizon. However, this study incorporates the value of water in the PSH at the terminal period (Liu et al., 2022; Kim & Powell, 2011; Sánchez de la Nieta et al., 2015). In this set-up, the merchant has four options: storing all renewable energy generation and also purchasing power to store; storing partial renewable energy generation and selling the rest of it; idle/offline/do nothing; generating PSH/discharging energy storage and selling all renewable energy generation to the market at the terminal period. In the long term, the residual energy in the storage has potential value for the future then influences the current actions, which is another innovation of this study.

This study was motivated to concentrate on the optimal energy operational decisions scheduling of a merchant who has a co-located storage system and a renewable power plant. In such circumstances, the merchant operates the large-scale energy storage facility to control electricity operation in the wholesale electricity market and incorporate the market impact[1], the forecasted uncertain wind-generated power, the constraints of energy storage (i.e., PSH capacity, pumping/generating limits, and efficiencies), and the residual value of water in the storage when modeling. This paper's analyses are intended to address the following two research questions: (1) How do electricity merchants with co-optimized energy storage and wind farm benefit from considering the market impact of buying and selling power and the uncertain wind generation? (2) What is the difference between the scheduling strategy considering market impact and the traditional scheduling strategy ignoring market impact? We were able to characterize this problem mathematically.

Toward that end, this study relaxes the price-taker assumption and assumes that the impact of the merchant's buy/sell decisions on the market price is approximately linear in the amount of power of buy/sell (Cruise et al., 2019; Liu et al, 2021a; Sioshansi, (2010, 2014)). We formulate this problem as a Markov decision process and explore the electricity merchant's optimal joint operational trading strategies by utilizing the dynamic programming approach to maximize profit. To solve this problem and achieve the closed-form analytical results to support multi-period decision-making, this paper first split the original problem into three sub-optimization problems corresponding to three available activities of the electricity merchant at each period. Then, the optimal solution for each sub-optimization problem will be addressed based on the Bellman equation. Finally, we combine them and achieve the global conclusions of the original problem to obtain the optimal decision rules in the entire optimization horizon. This is the first paper to manage the co-optimized economic dispatch scheduling of the energy storage and wind plants

[1] By market impact, we mean large-scale energy storage merchant's trading actions will influence market prices.

issue, considering the market impact of the merchant's actions and uncertainty of forecasted wind generation through dynamic programming.

The major contributions of this study are as follows: First, this research overcomes the challenges in achieving analytical results when considering market impact because it will change the traditional linear reward functions that overlook the market impact to nonlinear ones. For a storage-and-renewable energy source electricity merchant, we identify analytically three SOC reference points that rely on the currently available energy inventory in the storage, the forecasted prices, the intensity of the market impact of energy storage in trading, and the predicted available renewable energy source. The storage feasible SOC range (i.e., the energy storage capacity space) will be split into four possible sub-ranges by three SOC reference points corresponding to the previously listed four actions. The merchant can choose the optimal action simply by comparing the current energy inventory in the energy storage with the three optimal SOC reference points. Then, the electricity merchant's unique optimal decisions can be achieved through the sub-range within the current energy inventory level falls.

Second, in contrast to the results from existing studies (i.e., those based on price-taker analyses or ignoring the market impact), our results show that market impact and operating cost can raise the cost of pumping/buying and lower the revenue from generating/selling in each period. As a result, a merchant that ignores her impact on electricity prices will overestimate her expected profit when offering the same generating/discharging and pumping/charging maximum limits of the PSH in each period to ISOs as the price-taker merchant. To decrease the negative effect of the market impact of the merchant in operational decisions, the merchant needs to reduce her energy trading amount at each decision period. Our results find that the market impact influence the merchant’s optimal economic dispatch volume by changing the value of optimal SOC reference point. Although the residual value of energy in the storage does not affect the traditional scheduling policy, it influences the value function to affect the SOC and indirectly changes the scheduling quantity of power. Withholding the offered generating/pumping capacity may be needed to offset the market impact. This paper also confirms the corresponding boundary that wind generation benefits merchants' profit if the wind generation cost is low.

Finally, we extend our research to consider how expected profits are affected by the relation between the intensity of market impact and generating/discharging and pumping/charging maximum limits of the PSH offered to ISOs. Our findings suggest that the profit-maximizing merchant should try to make a trade-off between increasing the power transaction quantity directly and limiting the market impact's detrimental effects by reducing the transaction quantity.

This paper is organized into six sections. First, we review the related work in Section 2. Section 3 models an electricity merchant who has co-located energy storage and wind plants; then, we compare our conclusions with the existing literature in which merchants' market impact is not considered. Section 4 demonstrates the proposed results through the synthesis data case study and real data case study of Midcontinent Independent System Operator (MISO), US. Section 5 extends our research by examining cases in which market impact is related to generating/pumping limits that offered to ISOs. Finally, Section 6 summarizes our study and points out the future research directions.

2. Literature review

This section reviews several related works related to energy storage scheduling and co-optimized of renewable energy sources and energy storage. First, Section 2.1 summarizes the previous methods for co-optimizing energy storage and renewable energy sources. Finally, Section 2.2 reviews the works and points out the market impact of energy storage.

2.1. Energy storage and renewable source co-optimization methods

2.1.1 Renewable source with energy storage

Renewable power generation (e.g., wind/solar electricity generation) has high uncertainty levels and is intermittent, and the forecast reliability is low (Liu et al., 2022, Memarzadeh & Keynia, 2021). Energy storage systems (ESS) can solve this problem benefiting renewable energy market participation (Ding et al., 2014; Gomes et al.,2017) and maintaining the stability of the power system (Liu et al., 2015). Li et al. (2022) and Liu & Du (2020) discussed the problem of renewable energy selection, and they proposed a novel PROMETHEE method to rank different types of renewable energies and made a sensitivity analysis for decision results. Various energy storage technologies including battery storage (Cheng & Powell, 2018; Rehman et al., 2022) and PSH (Deane et al., 2010; Wang et al., 2021) were also discussed.

Many scholars target renewable sources with energy storage. Considering the cost of energy storage system installation, Wang et al. (2008) and Dui et al. (2018) determined the optimal energy storage power and energy capacity based on profit maximization using second-order cone programming (SOCP). Liu et al. (2015) used the artificial neural network (ANN) to forest wind generation and LMP (locational marginal pricing), and to study the dispatch of wind farms with hybrid energy storage. Shi et al. (2018) optimized the generating scheduling of wind-storage systems by analyzing the link between wind power fluctuation and ESS based on quantization index (QI) clustering. Orsini et al. (2021) proposed a comprehensive computational framework for the optimal operation for a solar thermal plant with energy storage. Roslan et al. (2021) proposed a day-ahead optimized scheduling controller for the optimal operation of distributed energy resources with energy in the microgrid. Heine et al. (2021) modeled MILP to design and dispatch packaged cool thermal energy storage (CTES) in connected communities to minimize the annual cost. Li et al. (2021) studied the capacity design of an integrated energy system based on the active dispatch mode (ADM). Bafrani et al. (2021) built a stochastic optimization operation model for compressed air energy storage (CAES) considering generator reliability. Savolainen and Lahdelma (2022) solved the optimal dimension and operation of renewable energy with storage in the building based on a 15-minute power balance settlement. Shi et al. (2022) proposed a hierarchical optimization algorithm to optimize renewable energy generation and storage capacity.

In this paper, unlike the research above, both capacity optimization for energy storage and optimal coordination framework are not evaluated. Instead, from the profit-maximizing perspective, we target how to get the analytically optimal co-optimized economic dispatch policy of the electricity merchant with a wind farm and energy storage.

2.1.2 Co-optimization of energy storage and wind plant

For the optimization of co-located energy storage and a wind plant system, Castronuovo and Lopes (2004) proposed a discrete optimization method to maximize daily profits and find the optimal daily operational strategy for a merchant with wind plants and hydroelectric power generation. Garcia-Gonzalez et al. (2008) proposed two decision-making frameworks for a wind energy generator participating in day-ahead, intraday, reserve, and balancing markets. Lee (2008) solved the short-term electricity scheduling problem by applying MIPSO (multi-iteration particle swarm optimization) method on the combined wind farms and PSH system. Zhang et al. (2016) obtained the optimal day-ahead economic dispatch for a smart grid with renewable and storage device by a fully distributed algorithm. Ding et al. (2016), Kim and Powell (2011), Zhou et al. (2019) examined the optimal scheduling policy of a wind plant with a storage system. Levieux et al. (2019) discussed the complementary operation between an existing hydropower plant and a projected wind plant based on heuristic algorithm (HA). Bhoi et al. (2020) studied the optimal scheduling of Photovoltaic (PV) systems with a battery and incorporate the storage health and consuming cost. Taghikhani (2021) studied micro-grid optimal scheduling with renewable resources and storage considering uncertainty. He et al. (2022) proposed a multi-objective evolutionary algorithm with decision-making (MOEA-DM) based on planning-operation co-optimization of renewable energy with storage. However, they all ignore the market impact of energy storage's operating activities on prices because these analyses consider that energy storage activities are small, and merchant’s operational decisions do not influence electricity prices.

Various methods have been used to model price takers; examples include the heuristic approach (Zhang & Wirth, 2010), mixed-integer linear programming (MILP) method (Wang et al., 2021), dynamic programming theory (Liu et al., 2022; Xiao et al., 2021), Lagrangian relaxation technique (Cruise et al., 2019), stochastic optimization scheme (Powell & Meisel, 2016a), and approximate dynamic programming algorithms to co-optimize energy storage for arbitrage (Al-Kanj et al., 2020). Liu et al. (2022) investigate the impact of the PTC (production tax credit) on the optimal scheduling policy of energy storage and ignoring the market impact and the uncertainty of wind generation. Our paper, however, targets the optimal policy of electricity merchants considering both the market impact and the uncertainty of forecasted wind generation, which will definitely affect the economic dispatch policy design.

In conclusion, an independent merchant with co-located energy storage and a wind plant can effectively enhance the stability of power system operation. Meanwhile, by optimizing the energy arbitrage strategy, it can maximize its income in the real-time market. In our work, a series of physical constraints on the energy storage system, the maximum and minimum limits of the generating and pumping, the capacity of the energy storage system, the efficiency, and the residual value of water have been taken into consideration. Our study also considers merchants' operation costs, which may be daily maintenance costs or battery self-discharge loss.

2.2. Market impact of energy storage

For merchants in the electricity market, most studies focus on assuming that the merchant's operational actions (i.e., pumping/charging and generating/charging) do not influence market prices, which we call price-taker. Notably, large-scale energy storage such as PSH will be reflected in energy arbitrage actions on the power market. This is because the merchant's trading actions (i.e., buying and selling) are sufficiently large to affect the electricity prices (Cruise et al., 2019; Felix et al., 2012). Felix et al. offer a pioneering approach to storage valuation that incorporates the effect of a market impact. Along similar lines, Baslis and Bakirtzis (2011) used stochastic MILP to model how a hydropower company’s short-term profit maximization decisions affect its medium-term plans, which adopt an annual stochastic self-dispatching model. Steeger et al. (2018) studied the optimal bidding plan of a single hydropower company whose bidding behavior influences the market price using Stochastic Dual Dynamic Programming (SDDP). Cruise et al. (2019) identified storage trading decisions that affect the market price and addressed decoupling the optimization horizon through the Lagrangian approach. Habibian et al. (2020) employed Lagrangian methods to the optimal power purchase decision making of price-maker enterprises that consume a large amount of power.

Huang et al. (2018) compared the operation of grid-level energy storage under three market mechanisms and proposed a modified mechanism to balance social cost and owner's profit. Huang et al. (2019) analyzed the investment and operation for price-maker storage under the centralized market and deregulated mechanisms and explored the financial incentives for the cooperative operation of multiple grid-level storage devices. Chabok et al. (2019) focused on the influence of the energy storage system as a price-maker on the operation of the power system from the perspective of ISO and proposed a bi-level optimization problem. These works do not investigate the energy storage economic dispatch problem from the perspective of electricity merchants and do not specifically consider wind plants to be operated with energy storage together. Liu et al. (2021a) investigated the optimal operational policy of merchants who only have energy storage and incorporate the market impact. However, they did not address the joint scheduling policy of merchants who have co-located energy storage systems and wind plants, and they did not consider the residual value of energy in the storage. Nevertheless, it is straightforward; the co-optimization policy of electricity merchants is quite different when uncertain renewable energy generation and the residual value of energy in the storage and the market impact are modeled. Nasiri et al. (2021) examined the scheduling strategy for a multi-energy system as a price-maker player in the day-ahead wholesale market based on a hybrid robust-stochastic approach. Later, Nasiri et al.(2022) investigated the tactical response of a wind integrated MES in the wholesale electricity market (WEM) and the natural gas market (NGM) as a price setter via a bi-level optimization model.

Motivated by existing examinations, we designed a co-optimization energy management model for the merchant with a co-located energy storage and renewable power plant that reflects the market impact of merchants' operational decisions in the power market. Compared to the current study (Liu et al., 2022; Jiang & Powell, 2015a; Zhou et al., 2019), it should be noted that the model is non-trivial in achieving analytical results employing a dynamic programming approach when considering the market impact in the problem, as it will transform the traditional study that considers only piecewise linear reward functions to nonlinear ones. Such research problems are addressed in this paper. We think these are original findings that have not been explored before. The difference between this study and the previous work is summarized in Table 1(See appendix).

3. Modeling and Optimization

In this section, we first model the reward and objective functions for electricity merchants with co-located energy storage and renewable power plants. Then, we study the merchant's optimal joint profit-maximizing strategies and consider the market impact as a function of the forecast price.

3.1. Model Setup

Here, we focus on a merchant with energy storage (here, we use PSH to represent large-scale storage in this paper) and a renewable power plant (for simplicity, henceforth, we use wind plants to refer to renewable power plants), both of which are co-located and connected to the electricity markets via transmission lines. The merchant adopts a co-optimized storage operation strategy and uses her energy storage plant to manage electricity. In this paper, "*we do not study bidding in a forward market, and we assume that any power offered to the wholesale electricity markets is accepted*" (Liu et al., 2022; Sioshansi et al., 2009, Walawalkar et al., 2007; Zhou et al., (2016; 2019). In this paper, we consider discrete time and that the merchant periodically performs operational actions during a finite optimization decision horizon, $t \in \{1, 2, \cdots, T\}$, and assume that the capacity of storage is limited. The PSH has maximum storage capacity $\bar{S}$ (i.e., the total energy/water that could be stored in the upper reservoir) and minimum energy inventory $\underline{S}$, where $\bar{S} > \underline{S} \geq 0$. Following the previous (Harsha & Dahleh, 2015; Jiang & Powell (2015a, 2015 b); Moarefdoost & Snyder, 2015; Zhou et al., (2016, 2019)), we focus on the optimal operating (e.g., charging/pumping, and discharging/generating) policy for a given storage capacity. However, how to optimize the storage capacity, such an approach would be appropriate for solving a different type of problem, thus beyond the scope of this paper. The PSH also has generating and pumping limits. Let $\bar{Q}^p$ and $\underline{Q}^p$ represent (respectively) the maximum and minimum limits of pumping that can be stored into the storage in each period, and let $\bar{Q}^g$ and $\underline{Q}^g$ denote (respectively) the upper and lower limits of released energy from the storage in each period. To ensure that the model will remain analytically tractable, this paper employs the conventional assumption (as in Kim &Powell, 2011; Liu et al., 2022; Zhou et al., (2016; 2019)) that $\underline{Q}^g = \underline{Q}^p = 0$ to build the continuous reward functions. We use W_t to represent the available *wind generation of* the wind plant in period t (in energy units/period). The vector $W = (w_1, w_2, \cdots, w_T)$ represents the sequential levels of available forecasted wind generation. Following the previous work (Jiang & Powell, 2015a; Kim & Powell, 2011; Qi et al., 2015), wind generation is constrained by the maximum generation capacity $\bar{W}$ of the wind plants to show the uncertainty in modeling. Here, the $w_t \in [0, \bar{W}]$ follows a uniform distribution. In reality, the utility would require the transmission capacity to be sufficiently large for the wind plant, so we do not consider the transmission capacity.

Our research involves three types of efficiency with PSH. The first type of efficiency is a portion $\varphi_t \in (0,1]$, a time-independent efficiency of stored energy that dissipates in one optimization period due to the evaporation, spill rate, and leakage of the PSH. The second type efficiency is denoted by θ and ξ , which represent the efficiency of (respectively) the pumping and generating of the PSH; here, $\theta, \xi \in (0,1]$. The other is $\sigma \in (0,1]$, which represents the efficiency of transmission line, that is, the proportion of electricity that flows out of the transmission line to that which flows into this transmission line. Transmission losses will be happened in two directions of the line (Liu et al., 2022; Zhou et al., 2019). It follows that the quantities $\xi\sigma \cdot \bar{Q}^g$ and $\bar{Q}^p/\theta\sigma$ are, respectively, the gross generating power capacity and the net pumping power capacity.

We suppose that the merchant's energy storage is large enough, and her generating and pumping decisions have a market impact on electricity prices. As noted previously, there are four possible actions: storing all renewable energy generation and also purchasing electricity to store; storing partial wind generation and selling the rest of renewable energy generation; remaining idle/offline, and generating PSH storage to sell and also selling all wind electricity to the market. Following previous work (e.g., Cruise et al. 2019, Liu et al. 2021a, Sioshansi 2010, 2014), this paper approximates market impact via a linear function of the quantity of power traded by the merchant. Therefore, we get the following updated prices:

$$\hat{P}_t = \begin{cases} \left(P_t + \lambda P_t \left(q_t^p/\theta - w_t\right)/\sigma\right) = P_t\left(1 + \lambda\left(q_t^p/\theta - w_t\right)/\sigma\right) & (q_t^p > \theta w_t) \\ \left(P_t - \lambda P_t \left(w_t - q_t^p/\theta\right)\sigma\right) = P_t\left(1 - \lambda\left(w_t - q_t^p/\theta\right)\sigma\right) & (0 \le q_t^p \le \theta w_t) \\ \left(P_t - \lambda P_t (q_t^g \xi + w_t)\sigma\right) = P_t\left(1 - \lambda(q_t^g \xi + w_t)\sigma\right) & (q_t^g > 0) \end{cases} \quad (1)$$

Here, $\hat{P}_t$ is the updated price that results from storing all renewable power generation and purchasing power from the market in energy units of $\left(q_t^p/\theta - w_t\right)/\sigma$, storing partial wind-generated power and selling the rest of to the market in units of $\left(w_t - q_t^p/\theta\right)\sigma$, and generating PSH and selling all wind source in units of $(q_t^g \xi + w_t)\sigma$. Here, the parameter $\lambda \ge 0$ reflects *the market impact factor* of the electricity merchant on electricity prices in trading decisions. The special case of $\lambda = 0$ represents the scenario of a price taker merchant for the traditional study. In the electricity market, time-coupling constraints require that the merchant should decide whether to buy or sell electricity in quantities that reflect the optimal policy based on forecasted prices. The electricity price in period t is denoted by P_t (dollars per unit energy). Both buying and selling prices at time t are shown by P_t conveniently for a price taker. The sequential levels of the price by a vector of $P = (P_1, P_2, \cdots, P_T)$. The P_t is the forecast electricity price, and λP_t is a measurement of the market impact of the energy storage on the electricity price at decision time.

From ISO perspective, power transmission network must be considered explicitly in market clearing. From merchant perspective, power transmission network can be considered in two different approaches, explicitly (through building a quasi-ISO clearing model where power transmission network is often treated as constraints of a lower-level optimization problem) and implicitly (through price forecasting model where historical congestion of power transmission network can be included as an input). Due to concerns with the explicit approach (such as data and model availability, uncertainty and computational challenges), this paper uses the latter approach, i.e., implicit consideration of power transmission network which is common in merchant strategy analysis (Li et al. 2007; Radovanovic et al., 2019; Wang et al. 2017).To maximize the profit of the electricity merchant and get the optimal economics dispatch policy of the energy storage, following the previous study (Liu et al., (2021a, 2022); Zhou et al., (2016, 2019)), we assume for the merchant that all forecasted prices are known in advance.

By incorporating the market impact in operational decisions and analyzing the co-optimization policy of a merchant who has both co-located energy storage and wind plants, this method produces the model novel and practical and generalizes the current problem (Liu et al., 2021a; Zhou et al., 2019), as it makes the first contribution of this paper. Thus, the reward function $R(q_t^g, q_t^p, w_t, P_t)$ from making the decision $\left(q_t^g, q_t^p\right)$, which corresponds to the decision time t, the forecast electricity prices P_t, and the forecasted wind power generation W_t, are, when considering the market impact, defined as follows:

$$R(q_t^g, q_t^p, w_t, P_t) = \begin{cases} -P_t\left(1+\lambda\left(q_t^p/\theta - w_t\right)/\sigma\right)\cdot\left(q_t^p/\theta - w_t\right)/\sigma - c^p q_t^p/\theta\sigma - c_w w_t \ (q_t^p > \theta w_t) \\ P_t\left(1-\lambda\left(w_t - q_t^p/\theta\right)\sigma\right)\cdot\left(w_t - q_t^p/\theta\right)\sigma - c^p q_t^p/\theta\sigma - c_w w_t \ (0 \le q_t^p \le \theta w_t) \\ P_t\left(1-\lambda(q_t^g\xi + w_t)\sigma\right)\cdot(q_t^g\xi + w_t)\cdot\sigma - c^g q_t^g\xi\sigma - c_w w_t \ (q_t^g \ge 0) \end{cases} \quad (2)$$

The first line in equation (2) indicates the costs of buying power of electricity merchants from the market. For example, W_t represents available wind generation, $\left(q_t^p/\theta - w_t\right)/\sigma$ indicates the units that the merchant purchases from the market to pump at time t, and q_t^p is the increase in storage inventory. This study lets c^g (resp. c^p) (dollar-unit energy) denote the generating (resp. pumping) operating cost for PSH or the discharging (resp. charging) operating cost of the battery (Huang et al., 2018, 2019; Xu et al., 2017). Following Liu et al. (2022) and Xu et al. (2017), we assume that the generating and pumping operating costs of energy storage are a linear function. The term $c^p \cdot q_t^p/\theta\sigma$ is the pumping operating cost, and $c_w W_t$ is the wind power plant's cost of generation. The second line gives the merchant's rewards from storing part of her wind generation q_t^p while selling the remaining units $\left(w_t - q_t^p/\theta\right)\sigma$ to the market. In the third line, $(q_t^g\xi + w_t)$ represents the electricity merchants generated by the PSH and all available wind sources that are sold to the market. The term $c^g q_t^g \xi\sigma$ denotes the generating operating cost of PSH.

This paper uses SOC_t to denote as the current available energy inventory in the upper reservoir of PSH at the beginning of decision time t. The sequential SOC inventories are represented by

$\hat{S}=(SOC_1, SOC_2, \cdots, SOC_T)$, where $SOC_t \in [\underline{S}, \bar{S}]$ and $\forall t \in \{1,2,\cdots,T\}$. Feasible actions set based on $SOC_t \in \hat{S}$ is defined as follows:

$$\text{Action}(SOC_t) := \{(q_t^g, q_t^p) \in \square : 0 \le q_t^p \le \bar{Q}^p \cdot U_t^p,\ q_t^p \le \bar{S} - SOC_t, 0 \le q_t^g \le \bar{Q}^g \cdot U_t^g,\ q_t^g \le SOC_t - \underline{S}\}. \quad (3)$$

This expression gives the upper limit of the quantity of energy that can be charged/pumped and discharged/generated at each optimization period. The first and second constraints define, respectively, the maximum limit of pumping and the space capacity of the upper reservoir. The third and the fourth constraints represent the maximum limit of generating and available energy in the reservoir. Both binary variables U_t^p and U_t^g denote the unit commitment of pumping and generating in decision period $[t, t+1)$ (respectively). Thus, we have $U_t^p + U_t^g \le 1$; here, $U_t^g \in \{0,1\}$ and $U_t^p \in \{0,1\}$, meaning the PSH cannot generate and pump simultaneously. If the PSH unit is offline/idle, then $U_t^p + U_t^g = 0$.

At decision time $t \in \{1,2,\cdots,T\}$, the merchant will know the storage inventory SOC_t, the wind generation W_t, the price P_t, and the market impact λ. The decision for each time t is denoted by q_t^g or q_t^p, which represents the *SOC change* from time t to time t+1 prior to considering, respectively, the generating loss and the pumping loss. The "storage self-loss" occurs at the end of decision time t, so the energy level at the beginning of decision time t+1 is equal to $\varphi_t(SOC_t + q_t^p - q_t^g)$. Hence, the following equation that summarizes the state transition from decision time t to decision time t+1 for the PSH storage is accurate:

$$SOC_{t+1} = \varphi_t(SOC_t + q_t^p - q_t^g) \quad (4)$$

Following Liu et al. (2022), Secomandi (2010), and Zhou et al. (2019), this study also adopts a single decision (action) variable, and lets q_t (i.e., $q_t = q_t^p - q_t^g$) at as the decision variable of electricity merchant at each decision time $t \in \{1,2,\cdots,T\}$ to substitute for the original two decision (action) variables q_t^g and q_t^p, which represent *the change of energy inventory or of SOC between two optimization periods t and t+1* (i.e., prior to considering accounting for the efficiency loss). Here, $q_t > 0$ denotes the SOC increase due to the pumping action, $q_t < 0$ means the SOC decrease because of generating, and $q_t = 0$ indicates that the SOC does not change or that the storage remains idle or is offline. The state decision variables at each stage t are SOC_t, W_t, and P_t. Thus, the decision state at stage t can be indicated by $S(t) = S_t(SOC_t, w_t, P_t)$. The merchant aims to achieve the optimal decision policy π to maximize her total expected reward functions overall feasible policies.

Her objective function is

$$\max_{\pi} \sum_{t=1}^{T} E\left[R(q_t^g, q_t^p, w_t, P_t) \mid S(1)\right] = \max_{\pi} \sum_{t=1}^{T} E\left[R(q_t, w_t, P_t) \mid S(1)\right] \quad (5)$$

subject to the capacity constraints $\max\{-\bar{Q}^g, \underline{S}-SOC_t\} \le q_t \le \min\{\bar{Q}^p, \bar{S}-SOC_t\}$ and to the storage energy balance constraints $SOC_{t+1} = \varphi_t(SOC_t + q_t)$, as well as $w_t \in [0, \bar{W}]$, where $t \in \{1,2,3,\cdots,T\}$. Both E_1, P_1, and w_1 are the given initial level of the storage and the price in advance. Because the optimization horizon is finite, this paper ignores the discount factor in this paper. This paper uses E to denote the expectations concerning SOC_t, w_t, P_t. In our notation, SOC_1, W_1, and P_1 are, respectively, the given *initial* energy storage inventory, the forecasted available wind generation, and advanced electricity price. Let $V_t(S(t))$ represent the value function of electricity merchant at decision time t and state $S(t) = S_t(SOC_t, w_t, P_t) \in \hat{S} \times W \times P$. This function of $V_t(S(t))$ satisfies the Bellman equation.

Thus, the merchant's value function can be created as

$$V(S(t)) = \max_{Action(SOC_t)} [R(q_t, w_t, P_t) + E\left(V_{t+1}(S(t+1) \mid S(t)\right)] \tag{6}$$

Most on this topic expresses the value of water (VOW) at the last optimization period (residual value of water in the storage) as $V_{T+1}(S(T+1)) = 0$ (e.g., Secomandi 2010; Zhou et al., 2019). In equation (6), however, $V_{T+1}(S(T+1)) = V(SOC_{T+1}, w_{T+1}, VOW_{T+1}) = VOW_{T+1} \cdot SOC_{T+1}$. Here, VOW_{T+1} denotes the VOW in the upper reservoir of PSH at the terminal period (Liu et al. 2022, Kim and Powell 2011), and SOC_{T+1} denotes the energy inventory level at the beginning of decision time T+1, which also represents the SOC at the end of decision time T.

3.2. Model Optimization and Analysis

To obtain the optimal co-optimized decision rules of the electricity merchant, this study first splits the optimization problem in equation (6) into three sub-problems, as in (7), corresponding to the three different actions described in (2) since only one of these actions is allowed at the same period. Then, we obtain the optimal result to each of these three sub-problems. The corresponding value functions of the electricity merchant on three available actions are shown as follows:

$$V(S(t)) = \begin{cases} -P_t\left(1+\lambda\left(q_t/\theta - w_t\right)/\sigma\right)\cdot\left(q_t/\theta - w_t\right)/\sigma - c^p q_t^p/\theta\sigma - c_w w_t + E[V_{t+1}\left(S(t+1) \mid S(t)\right)] \; (q_t > \theta w_t) \\ -P_t\left(1+\lambda\left(q_t/\theta - w_t\right)\sigma\right)\cdot\left(q_t/\theta - w_t\right)\cdot\sigma - c^p q_t^p/\theta\sigma - c_w w_t + E[V_{t+1}\left(S(t+1) \mid S(t)\right)] \; (0 \le q_t \le \theta w_t) \\ -P_t\left(1+\lambda((q_t\xi - w_t)\sigma\right)\cdot(q_t\xi - w_t)\cdot\sigma + c^g q_t \xi\sigma - c_w w_t + E[V_{t+1}\left(S(t+1) \mid S(t)\right)] \; (q_t < 0) \end{cases} \tag{7}$$

Since there is $q_t = SOC_{t+1}/\varphi_t - SOC_t$, to simplify, we use SOC_{t+1} substitute q_t as the decision variable to gain the analytical results, then maximizing Equation (8) enables us to obtain the optimal results by removing the values in the observed current state $S(t)$. The optimal unique action of the electricity merchant at each period will be achieved by comparing the optimal SOC in the next period (i.e.,

SOC_{t+1}) and the current available SOC (i.e., SOC_t) in the storage. Then, the Bellman equation (Liu et al., 2021a; Liu et al., 2022; Zhou et al., 2019) can be used to derive the following results:

$$\begin{cases} V_t^{(1)*}(S(t)) = \max_{\underline{E}\le E_{t+1}\le\overline{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t))] - \frac{\lambda P_t}{\theta^2\sigma^2}\frac{SOC_{t+1}}{\varphi_t^2}^2 + \frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t} SOC_{t+1}\cdot SOC_t + (\frac{2\lambda P_t w_t}{\theta\sigma^2} - \frac{P_t + c^p}{\theta\sigma})^2 \frac{SOC_{t+1}}{\varphi_t} \right) \\ V_t^{(2)*}(S(t)) = \max_{\underline{E}\le E_{t+1}\le\overline{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t))] - \frac{\lambda\sigma^2 P_t}{\theta^2}\frac{SOC_{t+1}}{\varphi_t^2}^2 + \frac{2\lambda\sigma^2 P_t}{\theta^2\varphi_t} SOC_{t+1}\cdot SOC_t + (2\lambda P_t\sigma^2\frac{w_t}{\theta} - \frac{P_t\sigma^2 + c^p}{\theta\sigma})^2 \frac{SOC_{t+1}}{\varphi_t} \right) \\ V_t^{(3)*}(S(t)) = \max_{\underline{E}\le E_{t+1}\le\overline{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t))] - \lambda P_t\xi^2\sigma^2\frac{SOC_{t+1}}{\varphi_t^2}^2 + 2\lambda P_t\xi^2\sigma^2\frac{SOC_{t+1}}{\varphi_t} SOC_t + (2\lambda P_t w_t\xi\sigma^2 - P_t\xi\sigma + c^g\xi\sigma)^2 \frac{SOC_{t+1}}{\varphi_t} \right) \end{cases} \quad (8)$$

This study next investigates the optimal results based on these expressions. Finally, we get the closed-form optimal co-optimization policy structure of merchant in equation (9) by following previous research on this topic (i.e., Kim & Powell, 2011; Liu et al., 2022; Zhou et al. 2019). When incorporating the market impact into the reward function, for the forecast price of electricity P_t, if $P_t<\infty$, then, at each decision state t, the merchant's value function $V_t(S(t))$ and expected total reward $E[V_{t+1}(S(t+1)|S(t))]$ are concave in $SOC_t \in [\underline{S}, \overline{S}]$ for each observed state $S(t) = S_t(SOC_t, w_t, P_t)$. The SOC optimal analytical solution is given by the following lemma (all proofs are provided in Appendix A).

LEMMA 1. When considering an electricity merchant's market impact in trading decisions, let $SOC_{t+1}^{(1)*}$, $SOC_{t+1}^{(2)*}$, and $SOC_{t+1}^{(3)*}$ be the closed-form optimal SOC results (e.g., SOC reference points in next period) in (9). Then, there are

$$\begin{cases} SOC_{t+1}^{(1)*} = \arg\max_{\underline{S}\le SOC_{t+1}\le\overline{S}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t))] - \frac{\lambda P_t}{\theta^2\sigma^2}(\frac{SOC_{t+1}}{\varphi_t} - SOC_t)^2 + (\frac{2\lambda P_t w_t}{\theta\sigma^2} - \frac{P_t + c^p}{\theta\sigma})\frac{SOC_{t+1}}{\varphi_t} \right) \\ SOC_{t+1}^{(2)*} = \arg\max_{\underline{S}\le SOC_{t+1}\le\overline{S}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t))] - \frac{\lambda\sigma^2 P_t}{\theta^2}(\frac{SOC_{t+1}}{\varphi_t} - SOC_t)^2 + (2\lambda P_t\sigma^2\frac{w_t}{\theta} - \frac{P_t\rho^2 + c^p}{\theta\sigma})\frac{SOC_{t+1}}{\varphi_t} \right) \\ SOC_{t+1}^{(3)*} = \arg\max_{\underline{S}\le SOC_{t+1}\le\overline{S}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t))] - \lambda P_t\xi^2\sigma^2(\frac{SOC_{t+1}}{\varphi_t} - SOC_t)^2 + (2\lambda P_t w_t\xi\sigma^2 - P_t\xi\sigma + c^g\xi\sigma)\frac{SOC_{t+1}}{\varphi_t} \right) \end{cases} \quad (9)$$

Based on the scenario of an electricity merchant who has co-located PSH and a wind power plant, this lemma has a critical implication. Because the merchant can choose the optimal action simply by comparing the current SOC level in the storage with the above three optimal SOC reference points separately. It now follows from the preceding discussion that our first proposition gives the corresponding optimal results.

PROPOSITION 1. *For positive forecast electricity prices* $\hat{P}_t \in P$ *(negative forecast electricity prices) at each stage* $t \in \{1,2,3,\cdots,T\}$ *: if* $0 \le \lambda \le \min\{\overline{\lambda}_t^{(1,2)}, \overline{\lambda}_t^{(2,3)}\}$ [2], *then there exist unique optimal storage*

[2]We discuss large-scale storage (such as PSH) with 1–2 gigawatts (GW) capacities. Considering a sizeable competitive wholesale electricity market (such as MISO, which has approximately 50 (off-peak period)–80 GW (on-peak period) demand, roughly 100 GW online capacity) and the limited presence of locational market electricity because of transmission capacity constrained and electricity market monitoring, we only address the case of a

inventories $\underline{S} \le SOC_{t+1}^{(1)*} \le SOC_{t+1}^{(2)*} \le SOC_{t+1}^{(3)*} \le \overline{S}$ (*resp.,* $\overline{S} \ge SOC_{t+1}^{(1)*} \ge SOC_{t+1}^{(2)*} \ge SOC_{t+1}^{(3)*} \ge \underline{S}$) *that depend on the state* $S(t)$, *where*

$$\begin{cases} \overline{\lambda}^{(1,2)} = P_t \Big/ \dfrac{2P_t}{\theta}\left(\dfrac{1+\sigma^2}{\sigma}\right)\left(SOC_t + w_t\theta - \dfrac{\underline{S}}{\varphi_t}\right); \\ \overline{\lambda}^{(2,3)} = \left(\dfrac{P_t\sigma^2 + c^p}{\theta\sigma} - (P_t\xi\sigma - c^g\xi\sigma)\right) \Big/ 2\sigma^2 P_t\left((SOC_t - \dfrac{\underline{S}}{\varphi_t})(\dfrac{1}{\theta^2} - \xi^2) + (\dfrac{w_t}{\theta} - w_t\xi)\right) \end{cases} \tag{10}$$

Therefore, an optimal economic dispatch decision in each state $S(t) = S_t(SOC_t, w_t, P_t) \in \hat{S} \times W \times P$ *can be specified as described in the following two cases.*

CASE 1: *If* $\theta w_t < \min\{SOC_{t+1}^{(1)*}, \overline{Q}^p\}$ (*less forecasted available wind-generated power*), *then the feasible SOC range (i.e., the from the lower boundary to the upper boundary of energy storage capacity) can be split into four sub-ranges (i.e., regions or areas): storing all wind power generation and purchasing electricity to store, storing partial wind power generation and selling the rest of it to market, remaining idle or do nothing*, *and generating PSH and also selling all wind power to the electricity market.*

$$q_t^*(S_t) = \begin{cases} \min\{SOC_{t+1}^{(1)*} - SOC_t, \overline{Q}^p\},\ SOC_t \in [\underline{S}, SOC_{t+1}^{(1)*} - \theta w_t]\ \text{(store renewable and buy electricity, up to } SOC_{t+1}^{(1)*}) \\ \min\{SOC_{t+1}^{(2)*} - SOC_t, \theta w_t\}, SOC_t \in (SOC_{t+1}^{(1)*} - \theta w_t, SOC_{t+1}^{(2)*}]\ \text{(store renewable without buying up to } SOC_{t+1}^{(2)*}) \\ 0,\ SOC_t \in (SOC_{t+1}^{(2)*}, SOC_{t+1}^{(3)*}]\ \text{(keep SOC unchanged)} \\ \max\{SOC_{t+1}^{(3)*} - SOC_t, -\overline{Q}^g\}, SOC_t \in (SOC_{t+1}^{(3)*}, \overline{S}]\ \text{(generate and sell renewable down to } SOC_{t+1}^{(3)*}) \end{cases} \tag{11}$$

CASE 2: *If* $\theta w_t \ge \min\{SOC_{t+1}^{(1)*}, \overline{Q}^p\}$ (*more forecasted available wind-generated power*), *then the feasible SOC range of the storage can be segmented into three possible sub-ranges: storing partial wind power generation and selling the rest of it to market, generating PSH and also selling all wind power generation to the electricity market, and idle*:

$$q_t^*(S_t) = \begin{cases} \min\{SOC_{t+1}^{(2)*} - SOC_t, \overline{Q}^p\}, SOC_t \in [\underline{S},\ SOC_{t+1}^{(2)*}]\ \text{(store renewable without purchasing, up to } SOC_{t+1}^{(2)*}) \\ 0, SOC_t \in [SOC_{t+1}^{(2)*}, SOC_{t+1}^{(3)*}]\ \text{(keep SOC unchanged)} \\ \max\{SOC_{t+1}^{(3)*} - SOC_t, -\overline{Q}^g\}, SOC_t \in (SOC_{t+1}^{(3)*},\ \overline{S}]\ \text{(generate and sell renewable down to } SOC_{t+1}^{(3)*}) \end{cases} \tag{12}$$

Case 1 of Proposition 1 shows analytically that, for an electricity merchant who has both co-located PSH and wind plant and pursues to maximize her expected profit, if there is *less* available forecasted wind power, the SOC of the storage will be segmented into four possible sub-ranges by three analytical SOC reference points ($SOC_{t+1}^{(1)*}, SOC_{t+1}^{(2)*}$, and $SOC_{t+1}^{(3)*}$, which depend on the price forecast P_t, the energy in storage SOC_t, the forecast wing generation W_t, and the market impact λ) that correspond to four possible different operational decisions: (1) storing all renewable generation and also purchasing electricity to store, (2) storing partial wind power and selling the rest of it, (3) remaining idle (i.e., offline/do nothing), and (4)

relatively small market impact.

releasing PSH and also selling all wind power. If the current available energy in the storage is more than reference point $SOC_{t+1}^{(3)*}$, the merchant will release water from the PSH to generate electricity and also sell all wind-generated electricity to the market, then reduce the SOC level down to $SOC_{t+1}^{(3)*}$. If there is less available energy in the PSH than $SOC_{t+1}^{(1)*} - \theta w_t$ and less available wind power (i.e., $\theta w_t < \min\{SOC_{t+1}^{(1)*}, \overline{Q}^p\}$), the merchant should (1) store all the wind power and buy electricity and then (2) increase the SOC inventory up to $SOC_{t+1}^{(1)*}$.

According to Case 2 of the proposition, if there is more available wind generation (i.e., $\theta w_t \geq \min\{SOC_{t+1}^{(1)*}, \overline{Q}^p\}$), then the *feasible storage inventory range* will be divided into three sub-ranges by two analytical SOC reference points ($SOC_{t+1}^{(2)*}$ and $SOC_{t+1}^{(3)*}$) that correspond to operational decisions 2–4. In this case, decision one will not happen since the merchant does not need to purchase power from the market to store when there is more available wind power generation. If there is less water in the PSH than $SOC_{t+1}^{(2)*}$, the merchant does not need to buy electricity to increase the SOC level but she can store partially wind power and increase the SOC so that it is to $SOC_{t+1}^{(2)*}$, and then sell the rest of her wind generation. Likewise, if the current available energy inventory in the storage falls within the boundaries established by two analytical reference points (i.e., $SOC_{t+1}^{(2)*} \leq SOC_t \leq SOC_{t+1}^{(3)*}$), then the merchant should do nothing for the PSH storage; and if there is more water in the upper reservoir than the SOC reference point $SOC_{t+1}^{(3)*}$, then the profit-maximizing merchant should (4) release energy from the PSH for generating and also sell all wind power, thereby decreasing the current inventory to $SOC_{t+1}^{(3)*}$.

Further, this study has three special degenerated cases with fewer thresholds, as seen below.

Special Case A: If $\sigma = 1$ (i.e., ignoring the efficiency loss of transmission line), then our results have $SOC_{t+1}^{(1)*} = SOC_{t+1}^{(2)*}$. This means that storing wind power generation or purchasing electricity from the power market to store will yield the merchant the same profit, that is, without considering the energy loss from the power market to storage via the transmission line, as when the merchant purchases electricity to store. Considering the efficiency loss of transmission line, storing merchant's own generated renewable source to storage is better than purchasing power from the market.

Special Case B: If $\theta = \xi = 1$ (i.e., ignoring the pumping and generating efficiency loss) and if $c^p = c^g = 0$ (i.e., ignoring the generating and pumping operating costs), then $SOC_{t+1}^{(2)*} = SOC_{t+1}^{(3)*}$. Moreover, the SOC range can be split into only three (or two) subranges that depend on the forecasted wind generation. In this case, however, no optimal strategy will include the "idle" state.

Special Case C: If $w_t = 0$ (i.e., the available forecasted wind generation equals zero or no wind source), in this case, there will be no storing or selling of wind generation, and our study has only $SOC_{t+1}^{(1)*}$ and $SOC_{t+1}^{(3)*}$ as optimal reference points (See Appendix). Then this paper obtains the optimal policy for the previous study for a merchant with PSH or storage only (Liu et al. 2021a). In our results, the storage state of charge (SOC) is segmented into four possible subranges by three analytical SOC reference points that correspond to four different decisions for the co-optimization merchant, compared to the three decisions in the previous study (Liu et al. 2021a). Obviously, the scenario that electricity merchant only has storage is a particular case for the merchant has storage and wind plant.

Proposition 1 yields our first insight and application, as follows.

INSIGHT AND APPLICATION 1. *For an electricity merchant with co-located energy storage and a wind plant, the feasible SOC range of the energy storage is segmented into different sub-ranges by the analytical SOC reference points, which depends mainly upon the current SOC, forecasted electricity price, and available forecasted wind source, and the intensity of market impact. As a result, the merchant will achieve the corresponding optimal operational decision for each subrange.*

To maximize the profit, and if less available renewable source, the SOC of storage will be split into four possible sub-ranges by three analytical reference points $SOC_{t+1}^{(1)*}$,$SOC_{t+1}^{(2)*}$ and $SOC_{t+1}^{(3)*}$, which correspond to four possible operational actions: storing all wind-generated power and also purchasing electricity to store, storing and selling partial renewable generation, do nothing/idle/offline, and generating PSH to sell and also selling all wind power. By comparing the current SOC level in the storage with the obtained SOC reference points for next period, the merchant can obtain the related optimal operational decisions. However, if *more* available forecasted wind generation, the storage SOC will be segmented into three sub-ranges by two analytical reference points $SOC_{t+1}^{(2)*}$ and $SOC_{t+1}^{(3)*}$, which correspond to three possible different operational decisions: storing and selling partial wind electricity, doing nothing (idle/offline), and generating electricity by PSH to sell and also selling all wind generation. Obviously, the optimal SOC reference points will be adjusted based on the intensity of market impact to support decision-making.

3.3. Market Impact and Wind Generation Analysis

This research studies the optimal co-optimized scheduling strategy of a merchant with large-scale energy storage and wind plant, whose trading decisions (i.e., buying or selling) are able to affect electricity prices. In traditional treatments, the electricity merchant is a price taker (Kim and Powell, 2011; Liu et al., 2022; Zhou et al., 2019) or only addressed the base problem without considering the wind plant (Cruise et al., 2019; Liu et al., 2021a; Secomandi,2010). In light of our assumptions and the preceding analysis, the optimal results are described in the next proposition.

PROPOSITION 2. *(a) If the electricity merchant has large-scale energy storage and wind plant, optimal expected profit is decreasing in the market impact and operating cost of energy storage.*

(b) For the electricity merchant with energy storage and wind plant, optimal expected profit increases with the forecasted wind generation $w_t \in [0, \bar{W}], \forall t = \{1, 2, \cdots, T\}$.

(c) Suppose the $q^{*(M)}_{t\,(\lambda\geq 0)}$ *(resp.* $q^{*(M)}_{t\,(\lambda=0)}$ *) represents the optimal actions of electricity merchants accounting for the market impact (resp. ignoring the market impact) on power prices, we can draw the following intuitive conclusions for the optimal expected profit of the merchant:*

$$\sum_{t=1}^{T} E\left[R(q^{*(M)}_{t\,(\lambda=0)}, w_t, P_t)_{(\lambda\geq 0)} \mid S(1)\right] \leq \sum_{t=1}^{T} E\left[R(q^{*(M)}_{t\,(\lambda\geq 0)}, w_t, P_t)_{(\lambda\geq 0)} \mid S(1)\right] \quad (13)$$

Proposition 2 is quite intuitive. These conclusions in Part (a) are consistent with the insights stated by Felix et al. (2012) and Liu et al. (2021a). It is straightforward; the merchant will achieve less profit with the increasing of operating cost and market impact. It will increase the cost of buying power from the market and decrease the revenue of selling power to the market by smoothing the difference between the high price at peak hours and low prices at off-peak. Part (b) demonstrates that the electricity merchant should take advantage of renewable wind generation to maximize reward at each period and optimal profit in the optimization horizon. It implies that the merchant with energy storage and wind plant should not curtail wind generation (i.e., generate the wind power based on the max generation capacity of the wind plants installed) to benefit their profit as long as the electricity prices are larger than the generation cost of wind if we do not consider bidding in a forward market. Part (c) shows that if a merchant ignores market impact on the power price and decides from the scenario of price-taker (i.e., the optimal economics dispatch of the storage is optimized on the wrong assumption $\lambda = 0$; however, where the corresponding profit of the merchant is calculated according to the real value of market impact factor λ), she will get less expected profit. Proposition 2 states that the market impact considerably alters the optimal policy structure and optimal expected profit, as detailed by the numerical results presented in Appendix.

INSIGHT AND APPLICATION 2. *The co-optimization merchants will have a lower expected profit with the increase of market impact if price-maker merchant and price-taker merchant submit the same generating and pumping maximum capacity in one optimization period to ISO. However, if the price-maker merchant ignores market impact in trading decisions and follows the price-taker's solutions, she will achieve less optimal expected profit. On the other hand, wind generation benefits merchant's profit if the wind generation cost is low.*

Insight 2 has an important implication for the price-maker merchant. The decisions of a merchant naturally affect the market price, so the merchant will have a lower profit when the cost of buying power is increasing, and the revenue from selling power is decreasing. Therefore, to smooth the negative effect of the market impact on buying and selling actions of the merchant, they should reduce the amount of electricity generating or pumping each period. Thus, a merchant with PSH and wind plants must perfectly balance the power transition quantity and market impact intensity and reduce wind power curtailment to maximize profit.

4. Case Study and Numerical Simulation

Section 4.1 validates the presented approaches and results employing one three-period case to represent the calculation procedure in detail and then compare them with the MILP method through the synthesis data. Additionally, Section 4.2 employs real data from MISO electricity prices and wind power generation to demonstrate the related results and insights.

4.1. Synthesis Data Case Study

For simplicity, this section employs a three-period example to show the detail of the proposed method in Section 3. Here, we suppose there are three optimization decision periods ($T=3$). The forecasted electricity price takes set $P_t=\{P_1,P_2,P_3\}=\{5,2,10\}=\{P^M,P^L,P^H\}$ at each period. This paper also supposes the merchant cannot fill her energy storage fully in one decision period but less than two (i.e., $\underline{S}+\bar{Q}^p\le\bar{S}$, and $\underline{S}+2\bar{Q}^p\ge\bar{S}$). Meanwhile, the full storage can be emptied in one decision period (i.e., resp. $\bar{S}-\underline{S}\le\bar{Q}^g$). In detail, we suppose the energy storage capacity is 10 (i.e., $\underline{S}=0,\bar{S}=10$), and we suppose the pumping capacity in one period is 7, and the generating capacity is 12 in one period. Suppose the pumping and generating operating costs of the storage $c^p=c^g=0.1$, the pumping and generating efficiencies of the energy storage as well as the transmission efficiency of the line are $\theta=\xi=\sigma=0.9$.

In this case, to illustrate the effect of market impact, this section supposes the intensity market impact parameter of the merchant is $\lambda=0.01$. In the case study, we focus on the scenario that the electricity merchant has energy storage and a wind plant and also assume the forecasted wind generation is $w_t=\{3,5,0\}=\{w_1,w_2,w_3\}$. Based on Lemma 1, our results show that both the generation cost of wind and the self-discharging do not affect the optimal solutions, so we assume the generation cost of wind equals zero (i.e., $c_g=0$). Let the operating cost be 0.1(i.e., $c^p=c^g=0.1$), and the pumping and generating efficiencies, and efficiency of transmission line be 0.9 (i.e., $\theta=\xi=\sigma=0.9$). For simplification, in this case study, we assume the residual value of water in storage is zero (i.e., $VOW_4=0$). On this basis, we employ the backward dynamic programming approach to achieve the following optimal outcomes:

In decision State 3:

Action 3: Since the energy in the storage and the end of the third period is valueless, to maximize the profit, the electricity merchant needs to sell power to the electricity market and bring the SOC $\underline{S}=0=E_4^*$ down to the minimum boundary of the storage as long as the electricity prices are positive, thus

$$q_3^*(S_3) = -SOC_3, SOC_3 \in (0,\overline{S}] \tag{14}$$

Thus, the following value function at stage 3 is achieved:

$$\begin{aligned} V_3^* &= \max\{R_3 + V_4^*\} = -P_t[1+\lambda(q_t\xi - w_t)\sigma]\cdot(q_t\xi - g_t)\cdot\sigma + c^g q_t\xi\sigma - c_w w_t (q_t < 0) \\ &= \{-P_3[1+\lambda([0-SOC_3]\xi - w_3)\sigma]\cdot([0-SOC_3]\xi - w_3)\cdot\sigma + c^g[0-SOC_3]\xi\sigma - 0 + 0\} \\ &= 8.019SOC_3 - 0.06561SOC_3^2 \end{aligned}$$

In decision state 2:

By utilizing the functions (9), (10), and (11) in section 3, we obtain the following outcomes for the optimal SOC reference points at initial of third period or the end of second period:

$$\begin{cases} SOC_3^{(1)*} = \underset{SOC_3\in[0,10]}{\arg\max}\left(V_3^* - \frac{\lambda P_2}{\theta^2\sigma^2}[\frac{SOC_3}{\varphi_3} - SOC_2]^2 + (\frac{2\lambda P_2 w_2}{\theta\sigma^2} - \frac{P_2 + c^p}{\theta\sigma})\frac{SOC_3}{\varphi_3} \right) \\ SOC_3^{(2)*} = \underset{SOC_3\in[0,10]}{\arg\max}\left(V_3^* - \frac{\lambda\sigma^2 P_2}{\theta^2}[\frac{SOC_3}{\varphi_3} - SOC_2]^2 + (2\lambda P_2\sigma^2\frac{w_2}{\theta} - \frac{P_2\sigma^2 + c^p}{\alpha\sigma})\frac{SOC_3}{\varphi_3} \right) \\ SOC_3^{(3)*} = \underset{SOC_3\in[0,10]}{\arg\max}\left(V_3^* - \lambda P_2\xi^2\theta^2[\frac{SOC_3}{\varphi_3} - SOC_2]^2 + (2\lambda P_2 w_2\xi\sigma^2 - P_2\xi\sigma + c^g\xi\sigma)\frac{SOC_3}{\varphi_3} \right) \end{cases}$$

$$\Rightarrow \begin{cases} SOC_3^{(1)*} = \underset{SOC_3\in[0,10]}{\arg\max}\left(4.590\cdot SOC_3 - 0.096\cdot SOC_3^2 + 0.061\cdot SOC_3 SOC_2 - 0.030\cdot SOC_2^2\right) = \frac{4.590 + 0.061SOC_2}{0.192} > 10 = \overline{S} \\ SOC_3^{(2)*} = \underset{SOC_3\in[0,10]}{\arg\max}\left(4.964\cdot SOC_3 - 0.086\cdot SOC_3^2 + 0.040\cdot SOC_3 SOC_2 - 0.020\cdot SOC_2^2\right) = \frac{4.964 + 0.04SOC_2}{0.172} > 10 = \overline{S} \\ SOC_3^{(3)*} = \underset{SOC_3\in[0,10]}{\arg\max}\left(7.355\cdot SOC_3 - 0.079\cdot SOC_3^2 + 0.026\cdot SOC_3 SOC_2 - 0.013\cdot SOC_2^2\right) = \frac{7.355 + 0.026SOC_2}{0.158} > 10 = \overline{S} \end{cases} \tag{15}$$

Here, $\theta w_t = 0.9\times 5 = 4.5 < 7$, by comparing the current SOC at the initial of second period and the above-obtained reference points, the merchant will obtain the following optimal decision at period 2:

$$q_2^*(S_2) = \begin{cases} 7,\ SOC_2 \in [0,3] \text{ (store generation and purchase electricity up to } SOC_3^{1*} = \overline{S}) \\ 10 - SOC_2,\ SOC_2 \in (3,5.5] \text{ (store generation and purchase electricity up to } SOC_3^{1*} = \overline{S}) \\ 10 - SOC_2,\ SOC_2 \in (5.5,10] \text{ (store generation without buying up to } SOC_3^{2*} = \overline{S}) \end{cases} \tag{16}$$

Then, the optimal value functions at decision time 3 can be rewritten as

$$V_3^* = \begin{cases} 8.019\cdot SOC_3 - 0.06561\cdot SOC_3^2{}_{|SOC_3 = SOC_2 + 7} = 7.1\cdot SOC_2 + 52.92 - 0.06561\cdot SOC_2^2;\ \text{if } SOC_2 \in [0,3] \\ 8.019\cdot SOC_3 - 0.06561\cdot SOC_3^2{}_{|SOC_3 = \overline{S} = 10} = 80.19 - 6.561 = 73.629;\ \text{if } SOC_2 \in (3,10] \end{cases} \tag{17}$$

By combining the optimal actions and the corresponding price at period 2, we obtain the following reward functions at period 2:

$$R(q_2, w_2, P_2) = \begin{cases} -7.23 & SOC_2 \in [0,3] \\ -15.74 - 0.6361 \cdot SOC_2^{\ 2} + 2.93 \cdot SOC_2 & SOC_2 \in (3,5.5] \\ -12.84 + 2.34 \cdot SOC_2 - 0.01 \cdot SOC_2^{\ 2} & SOC_2 \in (5.5,10] \end{cases}$$

Hence, incorporating the equation (17) and the reward function at period 2, the optimal value functions at the second decision time are obtained:

$$V_2^* = \begin{cases} 7.1 \cdot SOC_2 + 45.69 - 0.06561 \cdot SOC_2^{\ 2} & SOC_2 \in [0,3] \\ -0.6361 \cdot SOC_2^{\ 2} + 2.93 \cdot SOC_2 + 57.89 & SOC_2 \in (3,5.5] \\ 2.34 \cdot SOC_2 - 0.01 \cdot SOC_2^{\ 2} + 60.79 & SOC_2 \in (5.5,10] \end{cases} \tag{18}$$

In decision state 1:

Similarly, by employing the functions (A9), (A10), and (A11), we will reach optimal SOC reference points at the end of the first period or the initial of the second period as the following solutions:

$$\begin{cases} SOC_2^{(1)*} = \underset{SOC_2 \in [0,10]}{\arg\max} \left(V_2^* - \frac{\lambda P_1}{\theta^2 \sigma^2} [\frac{SOC_3}{\varphi_3} - SOC_2]^2 + (\frac{2\lambda P_1 w_2}{\theta \sigma^2} - \frac{P_1 + c^p}{\theta \sigma}) \frac{SOC_3}{\varphi_3} \right) \\ SOC_2^{(2)*} = \underset{SOC_2 \in [0,10]}{\arg\max} \left(V_2^* - \frac{\lambda \sigma^2 P_1}{\theta^2} [\frac{SOC_3}{\varphi_3} - SOC_2]^2 + (2\lambda P_1 \sigma^2 \frac{w_2}{\theta} - \frac{P_1 \sigma^2 + c^p}{\theta \sigma}) \frac{SOC_3}{\varphi_3} \right) \\ SOC_2^{(3)*} = \underset{SOC_2 \in [0,10]}{\arg\max} \left(V_2^* - \lambda P_1 \xi^2 \sigma^2 [\frac{SOC_3}{\varphi_3} - SOC_2]^2 + (2\lambda P_1 w_2 \xi \sigma^2 - P_1 \xi \sigma + c^g \xi \sigma) \frac{SOC_3}{\varphi_3} \right) \end{cases}$$

$$\Rightarrow \begin{cases} SOC_2^{(1)*} = \underset{SOC_2 \in [0,10]}{\arg\max} \left(V_2^* - \frac{0.05}{0.81 \times 0.81} [SOC_2 - SOC_1]^2 - 5.88 \cdot SOC_2 \right) \\ SOC_2^{(2)*} = \underset{SOC_2 \in [0,10]}{\arg\max} \left(V_2^* - 0.05 [SOC_2 - SOC_1]^2 - 4.85 \cdot SOC_2 \right) \\ SOC_2^{(3)*} = \underset{SOC_2 \in [0,10]}{\arg\max} \left(V_2^* - 0.05 \cdot 0.81 \cdot 0.81 [SOC_2 - SOC_1]^2 - 3.75 \cdot SOC_2 \right) \end{cases} \tag{19}$$

Next, we analyze the SOC reference points separately based on equation (18) and energy storage capacity.

(1) scenario: If $SOC_2 \in [0,\ 3]$

$$\begin{cases} SOC_2^{(1)*} = \underset{SOC_2 \in [0,3]}{\arg\max} \left(-0.142 \cdot SOC_2^{\ 2} + (1.22 + 0.15 \cdot SOC_1) SOC_2 + 45.69 - 0.076 \cdot SOC_1^{\ 2} \right) = \frac{1.22 + 0.15 SOC_1}{0.284} > 3 \Rightarrow SOC_2^{(1)*} = 3 \\ SOC_2^{(2)*} = \underset{SOC_2 \in [0,3]}{\arg\max} \left(-0.116 \cdot SOC_2^{\ 2} + (2.25 + 0.1 \cdot SOC_1) SOC_2 + 45.69 - 0.05 \cdot SOC_1^{\ 2} \right) = \frac{2.25 + 0.1 SOC_1}{0.232} > 3 \Rightarrow SOC_2^{(2)*} = 3 \\ SOC_2^{(3)*} = \underset{SOC_2 \in [0,3]}{\arg\max} \left(-0.098 \cdot SOC_2^{\ 2} + (3.35 + 0.066 \cdot SOC_1) SOC_2 + 45.69 - 0.033 \cdot SOC_1^{\ 2} \right) = \frac{3.35 + 0.066 SOC_1}{0.196} > 3 \Rightarrow SOC_2^{(3)*} = 3 \end{cases}$$

(2) Scenario2: If $SOC_2 \in (3,\ 5.5]$

$$\begin{cases} SOC_2^{(1)*} = \underset{SOC_2\in(3,5.5]}{\arg\max}\left(-0.71\cdot SOC_2^{\,2}-2.95\cdot SOC_2+0.152\cdot SOC_1SOC_2\ +57.89-0.076\cdot SOC_1^{\,2}\right)=\dfrac{(-2.95+0.152\cdot SOC_1)}{1.42}<0\Rightarrow SOC_2^{(1)*}=3 \\ SOC_2^{(2)*} = \underset{SOC_2\in(3,5.5]}{\arg\max}\left(-0.686\cdot SOC_2^{\,2}-1.92\cdot SOC_2+0.1\cdot SOC_1SOC_2+57.89-0.05\cdot SOC_1^{\,2}\right)=\dfrac{(-1.92+0.1\cdot SOC_1)}{1.372}<0\Rightarrow SOC_2^{(2)*}=3 \\ SOC_2^{(3)*} = \underset{SOC_2\in(3,5.5]}{\arg\max}\left(-0.669\cdot SOC_2^{\,2}-0.82\cdot SOC_2+0.066\cdot SOC_1SOC_2+57.89-0.033\cdot SOC_1^{\,2}\right)=\dfrac{-0.82+0.066\cdot SOC_1}{1.338}<0\Rightarrow SOC_2^{(3)*}=3 \end{cases}$$

(2) Scenario 3: If $SOC_2 \in (5.5,\ 10]$

$$\begin{cases} SOC_2^{(1)*} = \underset{SOC_2\in[5.5,10]}{\arg\max}\left((-3.54+0.152\cdot SOC_1)SOC_2+60.79-0.086\cdot SOC_2^{\,2}-0.076\cdot SOC_1^{\,2}\right)=\dfrac{(-3.54+0.152\cdot SOC_1)}{0.172}<0\Rightarrow SOC_2^{(1)*}=5.5 \\ SOC_2^{(2)*} = \underset{SOC_2\in[5.5,10]}{\arg\max}\left((-2.51+0.1\cdot SOC_1)SOC_2++60.79-0.06\cdot SOC_2^{\,2}-0.05\cdot SOC_1^{\,2}\right)=\dfrac{(-2.51+0.1\cdot SOC_1)}{0.12}<0\Rightarrow SOC_2^{(2)*}=5.5 \\ SOC_2^{(3)*} = \underset{SOC_2\in[5.5,10]}{\arg\max}\left((-1.41+0.066\cdot SOC_1)SOC_2+60.79-0.043\cdot SOC_2^{\,2}-0.0328\cdot SOC_1^{\,2}\right)=\dfrac{(-1.41+0.066\cdot SOC_1)}{0.086}<0\Rightarrow SOC_2^{(3)*}=5.5 \end{cases}$$

By comparing the max value, we can find the optimal references among scenario 1, scenario 2, and scenario 3. Thus, the merchant obtains the following three optimal SOC reference points: $SOC_2^{(1)*}=SOC_2^{(2)*}=SOC_2^{(3)*}=3$.

Since $\theta w_t < \min\{SOC_{t+1}^{(1)}, \overline{Q}^p\}$ (i.e., $0.9\cdot 3 = 2.7 < \min\{3,7\} = 3$), based on proposition 1 in section 3, the optimal decisions of the merchant at stage 1 are

$$q_1^*(S_1)=\begin{cases} 3-SOC_1,\ \text{ if } SOC_1\in[0,0.3]\text{(store generation and purchase electricity up to 3)} \\ 3-SOC_1,\ \text{ if } SOC_1\in(0.3,3]\text{(store generation without buying up to 3)} \\ 3-SOC_1,\ \text{ if } SOC_1\in(3,10]\text{(sell inventory down to 3)} \end{cases} \tag{20}$$

When incorporating the market impact of the merchant, based on the forecasted price at period 1 and the optimal action in equation (20), the reward functions of electricity merchants at stage 1 are shown:

$$R_1(q_1,w_1,P_1)=\begin{cases} -P_1[1+\lambda((3-SOC_1)/0.9-3)/0.9]\cdot((3-SOC_1)/0.9-3)/0.9-0.1(3-SOC_1)/0.81 & \text{if } SOC_1\in[0,0.3] \\ -P_1[1+\lambda((3-SOC_1)/0.9-3)0.9]\cdot((3-SOC_1)/0.9-3)0.9-0.1(3-SOC_1)/0.81 & \text{if } SOC_1\in(0.3,3] \\ -P_1[1+\lambda((3-SOC_1)0.9-3)0.9]\cdot((3-SOC_1)0.9-3)0.9-0.1(3-SOC_1)0.81 & \text{if } SOC_1\in(3,10] \end{cases}$$

Thus, the optimal value functions of the merchants at first decision state are:

$$V_1^*=\begin{cases} -P_1[1+\lambda(\frac{3-SOC_1}{0.9}-3)/0.9]\cdot(\frac{3-SOC_1}{0.9}-3)/0.9-0.1\frac{3-SOC_1}{0.81}+7.1\cdot SOC_2+45.69-0.066\cdot SOC_2^{\,2} & \text{if } SOC_1\in[0,0.3] \\ -P_1[1+\lambda(\frac{3-SOC_1}{0.9}-3)0.9]\cdot(\frac{3-SOC_1}{0.9}-3)0.9-0.1\frac{3-SOC_1}{0.81}+7.1\cdot SOC_2+45.69-0.066\cdot SOC_2^{\,2} & \text{if } SOC_1\in(0.3,3] \\ -P_1[1+\lambda((3-SOC_1)0.9-3)0.9]\cdot((3-SOC_1)0.9-3)0.9-0.1(3-SOC_1)0.81-0.636\cdot SOC_2^{\,2}+2.93\cdot SOC_2+57.89 & \text{if } SOC_1\in(3,5.5] \\ -P_1[1+\lambda((3-SOC_1)0.9-3)0.9]\cdot((3-SOC_1)0.9-3)0.9-0.1(3-SOC_1)0.81+2.34\cdot SOC_2-0.01\cdot SOC_2^{\,2}+60.79 & \text{if } SOC_1\in(5.5,10] \end{cases} \tag{21}$$

Recall the previous steps, the following optimal trading actions of the merchant at three periods are obtained.

In decision state 1,

$$q_1^*(S_1)=\begin{cases}3-SOC_1, & \text{if } SOC_1\in[0,0.3] \text{(store generation and purchase electricity up to 3)}\\ 3-SOC_1, & \text{if } SOC_1\in(0.3,3] \text{(store generation without buying up to 3)}\\ 3-SOC_1, & \text{if } SOC_1\in(3,10] \text{(sell inventory down to 3)}\end{cases}$$

In decision state 2,

$$q_2^*(S_2)=\begin{cases}7, & SOC_2\in[0,3] \text{ (store generation and purchase electricity up to } SOC_3^{1*}=\overline{S})\\ 10-SOC_2, & SOC_2\in(3,5.5] \text{ (store generation and purchase electricity up to } SOC_3^{1*}=\overline{S})\\ 10-SOC_2, & SOC_2\in(5.5,10] \text{ (store generation without buying up to } SOC_3^{2*}=\overline{S})\end{cases}$$

In decision state 3,

$$q_1^*(S_1)=\begin{cases}3-SOC_1, & \text{if } SOC_1\in[0,0.3] \text{(store generation and purchase electricity up to 3)}\\ 3-SOC_1, & \text{if } SOC_1\in(0.3,3] \text{(store generation without buying up to 3)}\\ 3-SOC_1, & \text{if } SOC_1\in(3,10] \text{(sell inventory down to 3)}\end{cases}$$

1) If $SOC_1=1$ (The SOC in energy storage at the beginning of decision time 1)

State 1: If $SOC_1=1$, (store wind generation 2, and make the SOC up to 3, also sell $2/0.9-3=-7/9$ to the market), then the SOC in the storage will approach to $SOC_2=3$ (i.e., $q_1^*=2$, $R_1=3.23$);

State 2: If $SOC_2=3$, (buying and pumping), then, there is $SOC_3=10$ (i.e., $q_2^*=7, R_2=-7.23$);

State 3: If $SOC_3=10$, (generating and selling), the SOC in the storage will down to $SOC_4=0$ (i.e., $q_3^*=-10, R_3=73.63$).

By using the predicted electricity prices, the total rewards of the merchant during the optimization horizon are shown as $R=R_1+R_2+R_3=69.63=V_1^*$.

2) If $SOC_1=5$ (The SOC in energy storage at the beginning of decision time 1)

State 1: If $SOC_1=5$,(idle), then, there is $SOC_2=3$ (i.e., $q_1^*=-2$, $R_1=20.5$) holding;

State 2: If $SOC_2=3$,(buying and pumping), then there exists $SOC_3=10$ (i.e., $q_2^*=7, R_2=-7.23$);

State 3: If $SOC_3=10$,(generating and selling), since we have $SOC_4=0=\underline{S}$, so the optimal action in the third period $q_3^*=-10$), so there has $R_3=73.63$).

Accordingly, the total rewards of the merchant during the given three optimization periods are

$$R=R_1+R_2+R_3=86.91=V_1^*.$$

Compared to the previous study (Liu et al., 2021a) in which the electricity merchant with energy storage only, or the predicted wind power is zero (i.e., *Special Case C*), considering the market impact and $\lambda=0.01$, the corresponding optimal SOC reference points and profits are shown in Table 2 under different two initial SOC in the storage.

Table 2: Optimal dispatching strategies and profit of the electricity merchant with energy storage only

	Optimal SOC reference points	Optimal economic dispatch	Total rewards

$SOC_1=1$	$SOC_2^{(1)*}=1.5$; $SOC_3^{(1)*}=10$	$q_1^*=0.5$; $q_2^*=7$; $q_3^*=-8.5$	$R=34.1$
$SOC_1=5$	$SOC_2^{(3)*}=3$; $SOC_3^{(1)*}=10$	$q_1^*=-0.16$; $q_2^*=5.16$; $q_3^*=-10$	$R=55.7$

This table displays that two optimal SOC reference points, $SOC_{t+1}^{(1)*}$ and $SOC_{t+1}^{(3)*}$, were created based on the method proposed in section 3 when ignoring wind power generation. For the scenario, the profit-maximizing merchant has energy storage only and only needed to buy power from the electricity market to store and make the current energy level in the storage up to $SOC_{t+1}^{(1)*}$ as close as possible when there is less energy in the storage. If the current available energy level in the storage is larger than reference point $SOC_{t+1}^{(3)*}$, the merchant needs to discharge energy from the storage for selling, then bring the SOC down to $SOC_{t+1}^{(3)*}$ as close as possible.

To verify our research and the proposed method in this paper, we also adopted the classic MILP method (Bo et al., 2021; Liu et al., 2021b; Wang et al., 2021; Wang et al., 2022) to solve the above three-periods case and get the optimal results as well as compare them with the optimal outcomes in Section 4.1. It yielded the same optimal results under both the dynamic programming method (i.e., our method in section 4.1) and the MILP (i.e., traditional approach). The above optimal solutions are verified in AIMMS.

4.2. Real Data Case Study

This section will use hourly optimization period units as the electricity prices and wind generation sequence $P=\{P_1,P_2,\dots,P_T\}$ (\$/M.W.) and $W=\{w_1,w_2,\dots,w_T\}$ (MWH) with 336 decision periods ($T=336$) corresponding to two-weeks optimization horizons from Dec. 3 to Dec. 18, 2020) in MISO as supplied (the prices data is available at: https://www.misoenergy.org/). The maximum and minimum capacity of the PSH upper reservoir $\bar{S}$ and $\underline{S}$ are 20 and 2 (respectively). Here, $\underline{S}>0$ denotes that the merchant cannot empty the upper reservoir of PSH, which is common in the electricity market for a PSH. The pumping and generating capacity are $\bar{Q}^p=2$ and $\bar{Q}^g=2$. The unit of measurement of PSH can be described as GWH. The units of generating and pumping capacity measurement can be represented as a GW.

Following the previous study, we also assume the pumping and the generating efficiencies of the PSH are $\alpha=\beta=0.9$. The optimization period $(\bar{S}-\underline{S})/\bar{Q}^g=9$ hours units for the PSH to empty the storage, while $(\bar{S}-\underline{S})/\bar{Q}^p=9$ hours units for the PSH to fill the storage fully correspond approximately to the

Ludington PSH in Michigan USA (the PSH detail are available at: https://www.consumersenergy.com/). Based on the existing report (Mongird, et al., 2020), we assume the operating cost $c=1$ (\$/MWh). We also ignore the transmission efficiency loss and suppose $\rho=1$ and $\eta=1$. To simplify, we assume that the residual value of the water in the storage is equal to the expected electricity prices during the optimization horizon (i.e., $VOW_{T+1} = \sum_{t=1}^{T} p_t \Big/ T$).

Using the same method proposed by Cruise et al. (2019) to calculate the market impact (e.g., we used the off-peak load and on-peak load and the corresponding prices in the optimization horizon and pumping and generating limits in each period offered to the ISOs to achieve the lambda approximately as a proxy for the market impact[3]). For a merchant who owns a large storage *(such as the Ludington PSH)* and a wind farm, the results are as follows.

The merchants' optimal co-optimized economics dispatch actions are obtained from the value functions (6) when the merchant with co-located energy storage and wind plants is displayed in Figure 1 and Figure 2 under two different initial SOC in the PSH, respectively.

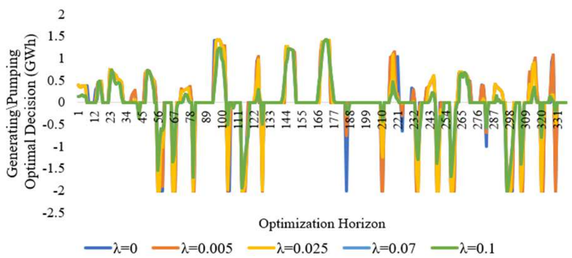


Fig 1: The optimal decisions when $SOC_1 = 2$ GWh

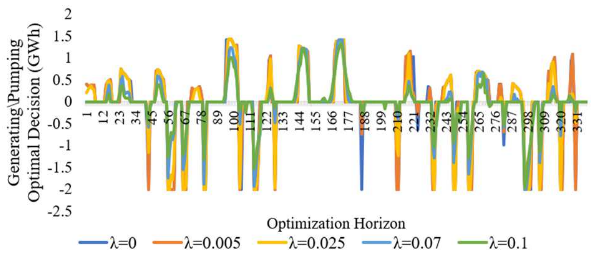


Fig 2: The optimal decisions when $SOC_1 = 10$ GWh

Figures 1 and 2 show that when the market impact factor is small, merchants with a co-located energy storage and wind plant will choose a similar strategy to the traditional strategy (that is, as a price-taker merchant and ignoring the market impact of the energy storage in trading), that is, when the market price of electricity is low, the merchant will buy electricity from market and will resale it later at a high price to maximize the profit. As the intensity of market impact increases (such as $\lambda=0.1$), the transaction quantity of electricity merchant who has energy storage and wind farm (see green and blue curves) in each period decreases. In this situation, the merchant's profit mainly depends on wind

[3]Although the merchant has PSH and wind plants, we will ignore the effect of wind generation when we calculate the market impact due to the high uncertainty of renewable generation.

generation and indirectly reduces the energy storage arbitrage function by decreasing the frequent pumping and generating actions.

The optimal actions are obtained from equation (11) when the merchant with energy storage only (i.e., without wind generation) is shown in Fig3 and Fig 4 under different initial SOC in the storage.

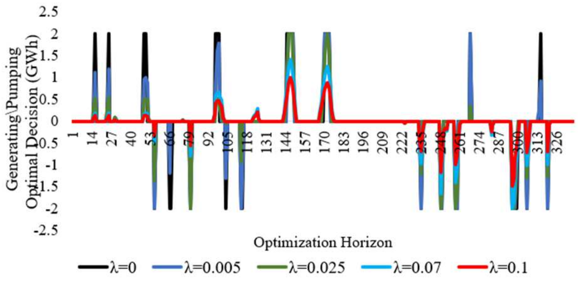


Fig 3: The optimal decisions when $SOC_1 = 2$ GWh

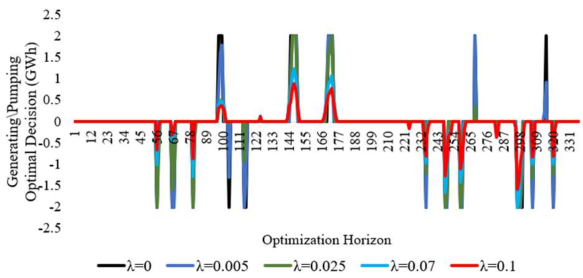
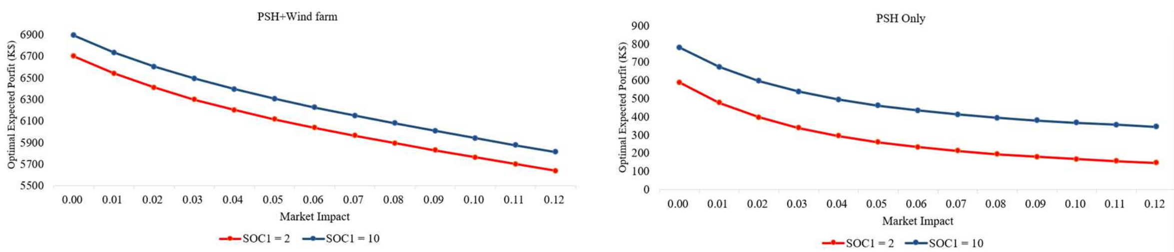


Fig 4: The optimal decisions when $SOC_1 = 10$ GWh

Figures 3 and 4 show that when there is no wind plant, the merchant's dispatching strategy is the same as when there is a wind plant. With the intensity of market influence increasing (such as λ=0.1), each period's transaction quantity decreases (see, red curve). Figures 3 and 4 show the relationship between the optimal action and the intensity of market influence under such a situation, which is the same as that of merchants with only energy storage. With the increasing market impact of the merchant in trading, the cost of purchasing power to pump will rise; however, the revenue will decrease through discharging energy for selling. Therefore, to decrease the negative effect of market impact on operational decisions, the merchant should lower the power transition amount at each decision period to benefit her own profit. Consequently, a profit-maximining merchant with energy storage and wind plants must balance market impact intensity and energy transition quantity.

Figure 5 corresponds to the Ludington case for the relationship between the optimal expected profit and the intensity of market impact with wind and without wind plants, respectively.



Figs 5: The relationship between the optimal expected profit and market impact

Figure 5 indicates that regardless of whether there is wind power generation, for the large-scale energy storage, the operational trading decisions will *influence the market prices.* However, compared with the existing study (i.e., price taker scenario and without considering the market impact), the increases of market impact will lead to decreased maximum expectation profit because purchase costs increase, and sale revenues decrease. It is intuition, considering the market impact of merchants in trading will increase the cost of buying electricity from the market and decrease the revenue of selling electricity to the market. Obviously, if the large-scale energy storage merchant schedules energy in the storage following the scenario of a price taker, she will lose more profit. This part further proves the conclusion of the previous section through numerical simulation. These results are similar to the reported consequences by and Cruise et al. (2019), Felix et al. (2012), and Liu et al. (2021a). To maximize expected profits, merchants should mitigate the market impact and increase profits by reducing the amount of electricity trading each period to offset the negative effect of market impact.

5. Extension Research: Market Impact as a Function of Offered Limits to ISO

The results presented in Section 3.2 and Section 4 show that electricity merchants get less expected profit with growing market impact if both the price-taker electricity merchant and the price-maker electricity merchant offer the exact pumping/generating maximum capacity in one period offered to ISOs. However, in the electricity market, where capacity withholding is allowed, the merchant can adjust their pumping and generating capacity offered to ISOs to change her market impact (Mehdipourpicha and Bo, (2020; 2021)). The implications of electricity merchants' market impact change substantially when considering the relationship between that impact and the generating and pumping capacity in each optimization period offered to ISOs. If the market impact is related to offered maximum pumping and generating limits, when the merchant changed her offered pumping (resp. generating) limit from $\bar{Q}_n^p$ (resp. $\bar{Q}_n^g$) to $\bar{Q}_m^p$ (resp. $\bar{Q}_m^g$), and if $\bar{Q}_m^p \le \bar{Q}_n^p$ and $\bar{Q}_m^g \le \bar{Q}_n^g$ hold, we will get $0 \le \lambda_m \le \lambda_n$. Here, the different subscript values show different generating and pumping limits offered to the ISOs.

For the intensity market impact parameter, in this section, following the previous study (Mehdipourpicha & Bo, (2020; 2021)), we shall use the ratio of electricity merchants' offered limits to the total (MISO-wide) online capacity of generators, where the latter is commonly about 100 GW. Thus, different market impacts correspond to different generating and pumping maximum limits in one period that is offered to the ISOs, which may result in different optimal actions and expected profits. For example, a market impact factor of $\lambda=0.02$ (resp., $\lambda=0.01$) corresponds to a merchant generating/pumping maximum limit of 2 GW (resp.,1 GW) offered to ISOs. Our results are derived simply by increasing the upper limits of generating and pumping that offered to MISO from 0.1 GW to

3 GW (i.e., $0.001 \le \lambda \le 0.03$); here we also suppose all other parameters are the same as in Section 4. Figure 6 illustrates the impact of market impact by adjusting the limits that offered to MISO on the expected profit of the merchant for cases with wind generation (left panel) and without wind generation (right panel).

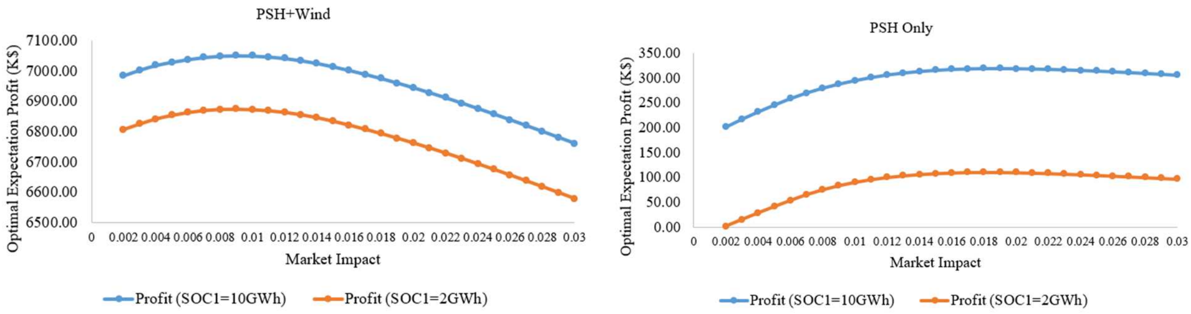


Figure 6: Merchant's optimal expected profit as a function of market impact

Figure 6 indicates that, regardless of whether there is wind power generation, the merchant's optimal expected profits first increase and then decrease with their market impact. It follows that a merchant can maximize expected profits by balancing market impact with offered pumping/generating maximum limits to ISOs. From the perspective of profit maximization, the electricity merchant must decide which is more important: the limits of offered transaction to ISOs or the approximately market impact. There is an inherent trade-off between these two factors, since the merchant can—in each period—increase the unit energy/power profit while lowering transaction quantity.

Suppose the market impact is low; in that case, there is a low revenue (due to the limited power transaction) although the unit power profit is high. Hence the merchant should increase the power transmission quantity to enhance her profit by enlarging the max capacity offered to ISOs. The most intriguing result is that raised market impact would result in a reducing unit power profit by raising the cost of purchasing and lowering sales revenue. In that case, we recommend that the merchant should limit their market impact's detrimental effects by—in each period—reducing her generating/pumping limits offered to ISO and increasing profit from unit power.

We affirm these conclusions by conducting additional analyses, as briefly described following. Accordingly, we change the power prices and wind generation corresponding to one day period with 24 stages and seven days period with 168 stages, respectively, corresponding to one day 12/01/2020 and one week from December 1 to December 7, 2020) in MISO for the year 2020 as provided. Once again, our previous findings are mainly supported.

6. Conclusion and Future Work

The main objective of this paper is to analyze the scenario when the merchants with both co-located large-scale energy storage systems and wind plants and build the co-optimized policy structure of electricity merchants whose actions are sufficiently important to have a market impact on electricity prices. We formulate this problem as a Markov decision process and employ the dynamic programming method to achieve the closed-form analytical results to support multi-period decision-making of merchants. Although there are multiple activities available each period for the electricity merchant, only one of these decisions/actions is allowed at the same time. On this basis, to solve this problem, this paper first split the original problem into three sub-optimization problems corresponding to three different actions. Then, the optimal solution for each sub-optimization problem will be addressed based on the Bellman equation. Finally, we combine them and achieve the global conclusions of the original problem. We demonstrate that the obtained optimal strategy policy in this paper generalizes the traditional results and differs significantly from usual strategies reported to be optimal in the current published work, neglecting the market impact and the residual value of energy in the storage.

To maximize the profit of electricity merchant who has large-scale energy storage and wind power plant, considering the generating and pumping operating costs and three types of efficiency loss, we find the current optimal economic dispatch strategy of the storage relies on the SOC reference points. These SOC reference points depend on the current SOC inventory in the energy storage, the forecasted electricity prices, available forecasted wind generation currently, and the market impact of energy storage in trading. We show analytically that, for a merchant with both PSH and wind plant, there exist three SOC reference points such that the SOC range is divided into four possible sub-ranges, each of which corresponds to one of four distinct options. The merchant will achieve the unique optimal action by comparing the current SOC in the storage and the SOC reference points. However, suppose operating costs and efficiency loss of the energy storage are not modeled. Then, the feasible SOC range of the storage can also be segmented into two sub-ranges by one unique optimal SOC reference point. In this case, storing renewable generation or buying power for pumping will bring the same cost for a merchant. If we ignore the wind generation or not available renewable source, it equals an electricity merchant with only large-scale energy storage. Our study finds the condition that wind generation benefits merchants' profit.

We recognize that the merchant's market impact and the residual value of energy in the storage play essential roles in the optimal strategy design. Although the residual value of energy in the storage affects the value function then influences the optimal decision, this paper finds that it does not change the relationship among three optimal SOC reference points, so the residual value cannot revise the traditional policy. Our results also show that the price-maker merchant will obtain similar strategies as the price-taker merchant scenario when the market impact is small. However, considering the market impact and offering the same generating and pumping capacity as the price-taker, we find the market impact would drive profit-reducing by raising the cost of purchasing and lowering sales revenue. If, besides, the market impact of the merchant is high, then the sales revenue can only somewhat offset the increased cost of purchasing power. In that case, our findings recommend that the merchant needs to mitigate the market impact's negative effect as much as possible by lowering the power transition amount at each decision period to benefit her profit. These new conclusions provide more knowledge of managing differentiated forecasted wind generation, market impact, and co-optimized economic dispatch of energy storage and wind plant.

To the extent that a merchant can influence the market impact (e.g., through adjustment of the pumping and generating maximum limits offered to ISOs), we identify conditions under which the trade-off is either beneficial or detrimental to the merchant. These new findings augment our collective knowledge about managing the intensity of market impact and are an essential contribution to research on this topic.

There are usually two approaches to model market impact---an equilibrium model or a conjecture variation model. This paper's approach is a conjecture variation. Another connected concern for future research is confirming how to model the market impact in an equilibrium model and construct the corresponding reward functions. To establish a reasonable and tractable framework and derive insightful results, we have followed the conventional assumptions about the market impact and generating and pumping minimum limitations to get continuous reward functions. Further research could be undertaken that relaxes these assumptions and extends our research on this problem. Our main consequences and insights are robust for other kinds of relations—when electricity merchants purchase electricity from the power market, and the demand/load will increase, conducting to increasing electricity prices; in contrast, selling power by a merchant will enhance the supply resulting in decreasing the selling prices. Therefore, exploring this topic is a promising avenue for future research. It would also be worthwhile to investigate generating and pumping lower limitations in positive values other than zero. The results and optimal joint optimization scheduling proposed in this paper are developed via dynamic programming based on the static price forecast for the entire horizon. Another related consideration for future work arises: Should the merchant's decision be adjusted to account for this changing price uncertainty? Finally, the effects of transmission constraints, nodes' voltage, and bus voltages on the energy storage planning problem also need to investigate.

Acknowledgments

This research is based upon work supported by the US Department of Energy's Office of Energy Efficiency and Renewable Energy (EERE) under the Water Power Technologies Office (Grant no. DE-EE0008781). The views expressed herein do not necessarily represent the views of the U.S. Department of Energy or the United States Government. We thank Prof. Ross Baldick (The University of Texas at Austin) and Dr. Yong-hong Chen (Midcontinent ISO) for their helpful comments and discussion on previous drafts.

References

Ahmad, F., Khallid, M., Panigrahi, B. K. (2021). Development in energy storage system for electric transportation: A comprehensive review. *Journal of Energy Storage*, 43, 103153. https://doi.org/10.1016/j.est.2021.103153

Al-Kanj L., JulianaNascimento J., Powell, W. B. (2020). Approximate dynamic programming for planning a ride-hailing system using autonomous fleets of electric vehicles. *European Journal of Operational Research*, 284(3): 1088-1106.

Al-Masri, H. M. K., Magableh, S. K., Abuelrub, A., Alzaareer, K. (2021). Realistic coordination and sizing of a solar array combined with pumped hydro storage system. *Journal of Energy Storage*, 41,102915. https://doi.org/10.1016/j.est.2021.102915

Bafrani, H. A., Sedighizadeh, M., Dowlatshahi, M., Hosein Ershadi, M. H., Rezaei, M. M. (2021). Reliability and reserve in day ahead joint energy and reserve market stochastic scheduling in presence of compressed air energy storage. *Journal of Energy Storage*, 43,103194. https://doi.org/10.1016/j.est.2021.103194

Baslis, C. G., Bakirtzis, A. G. (2011). Mid-Term Stochastic Scheduling of a Price-Maker Hydro Producer With Pumped Storage. *IEEE Transactions on Power Systems*, 26(4):1856-1865.

Bhoi, S.K., Nayak, M.R. (2022). Optimal scheduling of battery storage with grid tied PV systems for trade-off between consumer energy cost and storage health. *Microprocessors and Microsystems*,79, 103274.
https://doi.org/10.1016/j.micpro.2020.103274

Bo, R., Chen, Y. H., Wu, L., Baldick, R., Huang, B., Ghesmati, A. et al (2021). Modeling and Optimizing Pumped Storage in a Multi-stage Large Scale Electricity Market under Portfolio Evolution. United

States: N. p., 2021. Web. doi:10.2172/1833111.

Bushnell, J. (2003). A Mixed Complementarity Model of Hydrothermal Electricity Competition in the Western United States. *Operations Research*, 51(1):1-14.

Castronuovo, E. D., Lopes, J. A. P. (2004). On the optimization of the daily operation of a wind-hydro power plant. *IEEE Transactions on Power Systems*, 19(3):1599-1606.

Chabok, H., Roustai, M., Sheikh, M., Kavousi-Fard, A. (2020). On the assessment of the Impact of a Price-Maker Energy Storage Unit on the operation of Power System: the ISO Point of View. *Energy*,190,116224. https://doi.org/10.1016/j.energy.2019.116224.

Cheng, B., Powell, W. B. (2018). Co-Optimizing Battery Storage for the Frequency Regulation and Energy Arbitrage Using Multi-Scale Dynamic Programming. *IEEE Transactions on Smart Grid*, 9(3):1997-2005.

Cruise, J., Flatley, L., Gibbens, R., Zachary, S. (2019). Control of Energy Storage with Market Impact: Lagrangian Approach and Horizons. *Operations Research*, 67(1):1-9.

Deane, J. P., Gallachoir, B. P. O., McKeogh, E. J. (2010). Techno-economic review of existing and new pumped hydro energy storage plant. *Renewable & Sustainable Energy Reviews*, 14(4):1293-1302.

Ding, H., Hu, Z., Song, Y. (2014). Rolling optimization of wind farm and energy storage system in electricity markets. *IEEE Transactions on Power Systems*, 30 (5): 2676-2684.

Ding, H., Pinson, P., Hu, Z., Song, Y. (2016). Optimal Offering and Operating Strategies for Wind-Storage Systems With Linear Decision Rules. *IEEE Transactions on Power Systems*, 31(6): 4755-4764.

DOE (2018). Wind Technologies Market Report. https://www.energy.gov/sites/prod/files/2019/08/f65/2018%20Wind%20Technologies%20Market%20Report%20FINAL.pdf

Dui, X. W., Zhu, G. P., Yao, L. Z. (2018). Two-Stage Optimization of Battery Energy Storage Capacity to Decrease Wind Power Curtailment in Grid-Connected Wind Farms. *IEEE Transaction on Power Systems*, 33(3):3296-3305.

Felix, B., Woll, O., Weber, C. (2012). Gas storage valuation under limited Market liquidity: An application in Germany. *European Journal Finance*, 19(7-8):715-733.

Garcia-Gonzalez, J., Moraga, R., Santos, L. M., Gonzalez, A. M. (2008). Stochastic joint optimization of wind generation and pumped-storage units in an electricity market. *IEEE Transaction on Power System,* 23(2): 460-468.

Gomes, I. L., Pousinho, H. M.I., Melicio, R., Mendes, V. M. F. (2017). Stochastic coordination of joint wind and photovoltaic systems with energy storage in day-ahead market. *Energy*, 124:310-320.

Grillo, S., Pievatolo, A., Tironi, E. (2016). Optimal storage scheduling using Markov Decision Processes. *IEEE Transaction on Sustainable Energy*, 7(2):755-764.

Habibian, M., Downward, A., Zakeri, G. (2019). Multistage stochastic demand-side management for price-making major consumers of electricity in a co-optimized energy and reserve market. *European Journal of Operational Research*, 280(2):671-688.

Harsha, P., Dahleh, M. (2015). Optimal management and sizing of energy storage under dynamic pricing for the efficient integration of renewable energy. *IEEE Transactions on Power systems*, 30(3): 1164-1181.

He, Y., Guo, S., Zhou, J. X., Ye, J. L., Huang, J., Zheng, K., Du, X. R. (2022). Multi-objective planning-operation co-optimization of renewable energy system with hybrid energy storages. *Renewable Energy*,184:776-790.

Heine, K., Tabares-Velasco, P. C., Deru, M. (2021). Design and dispatch optimization of packaged ice storage systems within a connected community. *Applied Energy*, 298,117147. https://doi.org/10.1016/j.apenergy.2021.117147.

Huang, Q. S., Xu, Y. J., Courcoubetis, C. (2019). Financial Incentives for Joint Storage Planning and Operation in Energy and Regulation Markets. *IEEE Transactions on Power Systems*, 34(5):3326-3339.

Huang, Q. S., Xu, Y. J., Wang, T., Courcoubetis, C. A. (2018). Market Mechanisms for Cooperative Operation of Price-Maker Energy Storage in a Power Network. *IEEE Transactions on Power Systems*, 33(3):3013-3028.

Jiang, D. R., Powell, W. (2015a). An approximate dynamic programming algorithm for monotone value functions. *Operations Research*, 63(6): 1489-1511.

Jiang, D. R., Powell, W. (2015b). Optimal hour-ahead bidding in the real-time electricity market with battery storage using approximate dynamic programming. *INFORMS Journal on Computing*, 27(3): 525-543.

Kim, J. H., Powell, W. B. (2011). Optimal energy commitments with storage and intermittent supply. *Operations Research*, 59(6):1526-5463.

Lai, C. S., Locatelli, G., Pimm, A., Wu, X. M. Lai, L. L.(2021). A review on long-term electrical power system modeling with energy storage. *Journal of Cleaner Production*, 280(1):124298.

Lee, T. Y. (2008). Short term hydroelectric power system scheduling with wind turbine generators using the multi-pass iteration particle swarm optimization approach. *Energy Conversion and Management*, 49(4): 751-760.

Levieux, L. L., Inthamoussou, F. A., Battista, H. D. (2019). Power dispatch assessment of a wind farm and a hydropower plant: A case study in Argentina. *Energy Conversion and Management*, 180(15):391-400.

Li, G., Liu C. C., Mattson, C., Lawarrée, J. (2007). Day-ahead electricity price forecasting in a grid environment. *IEEE Transactions on Power Systems*, 22(1):266-274.

Li, P., Xu, Z. W., Wei C. P., Bai, Q. G., Liu J. (2022). A novel PROMETHEE method based on GRA-DEMATEL for PLTSs and its application in selecting renewable energies. *Information Sciences*, 589,142-161. https://doi.org/10.1016/j.ins.2021.12.090.

Lifshitz, D., Weiss, G. (2015). Optimal energy management for grid-connected storage systems. *Optimal Control Applications & Methods*, 36(4):447-462.

Liu, J., Bo, R., Wang, S. Y., Chen, H. T. (2021a). Optimal Scheduling for Profit Maximization of Energy Storage Merchants Considering Market Impact Based on Dynamic Programming. *Computers & Industrial Engineering,* 155,107212. https://doi.org/10.1016/j.cie.2021.107212

Liu, J., Ou, M., Sun, X. Y., Chen, J., Mi, C. M., Bo, R. (2022). Implication of production tax credit on economic dispatch for electricity merchants with storage and wind farms. *Applied Energy*, 308,118318.
https://doi.org/10.1016/j.apenergy.2021.118318

Liu, M., Quilumba, F. L., Lee, W. J. (2015). Dispatch Scheduling for a Wind Farm With Hybrid Energy Storage Based on Wind and LMP Forecasting. *IEEE Transactions on Industry Application*, 51(3):1970-1977.

Liu, Y., Du, J. L. (2020). A multi criteria decision support framework for renewable energy storage technology selection. *Journal of Cleaner Production*, 277(20):122183. https://doi.org/10.1016/j.jclepro.2020.122183

Liu, Y. K., Wu, L., Yang, Y. F., Chen, Y. H., Baldick, R., Bo, R. (2021b). Secured Reserve Scheduling of Pumped-Storage Hydropower Plants in ISO Day-ahead Market. *IEEE Transactions on Power Systems*, 36(6): 5722-5733.

McPherson, M. McBennett, B., Sigler, D., Denholm, P. (2020). Impacts of storage dispatch on revenue in electricity markets. *Journal of Energy Storage*, 31, 101573. DOI: 10.1016/j.est.2020.101573.

Mehdipourpicha, H., Bo, R. (2020). Risk-constrained Bi-level Optimization for Virtual Bidder Bidding Strategy in Day-Ahead Electricity Markets. *2020 IEEE Power & Energy Society General Meeting (PESGM), 2020, pp. 1-5*. doi: 10.1109/PESGM41954.2020.9282117

Mehdipourpicha, H., Bo, R. (2021). Optimal Bidding Strategy for Physical Market Participants With Virtual Bidding Capability in Day-Ahead Electricity Markets. *IEEE Access*, 9, 85392-85402.

Memarzadeh, G., Keynia, F. (2021). A new optimal energy storage system model for wind power producers based on long short-term memory and Coot Bird Search Algorithm. *Journal of Energy Storage*, 44(A),103401. https://doi.org/10.1016/j.est.2021.103401

Mongird, K., Viswanathan,V., Alam, J., Vartanian, C. Sprenkle, V., Baxter, R. (2020). 2020 Grid Energy Storage Technology Cost and Performance Assessment. Pacific Northwest National Laboratory Technical Report, Publication No. DOE/PA-0204 December 2020. https://www.pnnl.gov/sites/default/files/media/file/Final%20-%20ESGC%20Cost%20Performance%20Report%2012-11-2020.pdf

Moarefdoost, M. M., Snyder, L. V. (2015). Generation and storage dispatch in electricity networks with generator disruptions. *Naval Research Logistics*, 62(6): 493-511.

Nasiri, N., Zeynali, S., Ravadanegh, S. N., Marzband, M.(2021). A hybrid robust-stochastic approach for strategic scheduling of a multi-energy system as a price-maker player in day-ahead wholesale market. *Energy*, 235(15): 121398.

Nasiri, N., Zeynali, S., Ravadanegh, S. N., Marzband, M.(2022). A tactical scheduling framework for wind farm-integrated multi-energy systems to take part in natural gas and wholesale electricity markets as a price setter. *IET Generation, Transmission & Distribution*, DOI: 10.1049/gtd2.1242

Orsini, R. M., Brodrick, P. G., Brandt, A. R., Durlofsky, L. J. (2021). Computational optimization of solar thermal generation with energy storage. *Sustainable Energy Technologies and Assessments*, 47(3), 101342. DOI: 10.1016/j.seta.2021.101342

Powell, W. B., Meisel, S. (2016). Tutorial on Stochastic Optimization in Energy---Part II: An Energy Storage Illustration. *IEEE Transactions on Power Systems*, 31(2): 1468-1475.

Qi, W., Liang, Y., Shen, Z. M. (2015). Planning energy storage and transmission for wind energy generation.

Operations Research, 63(6): 1280-1293.

Radovanovic, A., Nesti, T., Chen, B. (2019). A Holistic Approach to Forecasting Wholesale Energy Market Prices. *IEEE Transactions on Power Systems*, 34(6): 4317-4328.

Rehman, W., Bo, R., Mehdipourpicha, H., Kimball, J. W.(2022). Sizing battery energy storage and PV System in an extreme fast charging station considering uncertainties and battery degradation. *Applied Energy*, 313, 118745. https://doi.org/10.1016/j.apenergy.2022.118745

Roslan, M. F., Hannan, M. A., Ker, P.J., Muttaqi, K. M., Mahlia, T. M. I. (2021). Optimization algorithms for energy storage integrated microgrid performance enhancement. *Journal of Energy Storage*, 43, 103182. https://doi.org/10.1016/j.est.2021.103182

Sánchez de la Nieta, A. A., Contreras, J., Catalão, J. P. S. (2015). Impact of the future water value on wind-reversible hydro offering strategies in electricity markets. *Energy Conversion and Management*, 105(15): 313-327.

Savolainen, R., Lahdelma, R. (2022). Optimization of renewable energy for buildings with energy storages and 15-minute power balance. *Energy*, 243(15):123046. https://doi.org/10.1016/j.energy.2021.123046

Secomandi, N. (2010). Optimal commodity trading with a capacitated storage asset. *Management Science*, 56(3): 449-467.

Shi, J., Lee, W. J., Liu, X. F. (2018). Generation Scheduling Optimization of Wind-Energy Storage System Based on Wind Power Output Fluctuation Features. *IEEE Transactions on Industry Applications*, 54(1):10-17.

Shi, Z. D., Wang, W. S., Huang, Y. H., Li, P., Dong, L. (2022). Simultaneous Optimization of Renewable Energy and Energy Storage Capacity with the Hierarchical Control. *CSEE Journal of Power and Energy Systems*, 8(1):95-103.

Sioshansi, R., Denholm, P., Jenkin, T., Weiss, J. (2009). Estimating the value or electricity storage in PJM: Arbitrage and some welfare effects. *Energy Economics,* 31(2):269-277.

Sioshansi, R. (2010). Welfare impacts of electricity storage and the implications of ownership structure. *Energy Journal,* 31(2):173-198.

Steeger, G., Lohmann, T., Rebennack, S. (2018). Strategic bidding for a price-maker hydroelectric producer: Stochastic dual dynamic programming and Lagrangian relaxation. *IISE Transactions*,50(11):929-942.

Su, C., Cheng, C., Wang, P., Shen, J., Wu, X. (2019). Optimization model for long-distance integrated transmission of wind farms and pumped-storage hydropower plants. *Applied Energy*, 242, 285-293.

Taghikhani, M. A. (2021). Renewable resources and storage systems stochastic multi-objective optimal energy scheduling considering load and generation uncertainties. *Journal of Energy Storage*, 43,103293. https://doi.org/10.1016/j.est.2021.103293

Tucson Electric Power. (2021). Building a Cleaner, Greener Grid: TEP's Largest Solar Plus Storage System is Now in Service. https://tucson.com/business/building-a-cleaner-greener-grid-tep-s-largest-solar-plus-storage-system-is-now-in/article_ae5b610b-bea9-548d-adef-02d7c2b4db74.html

Walawalkar, R., Apt, J., Mancini, R. (2007). Economics of electric energy storage for energy arbitrage and regulation in New York. *Energy Policy,* 35(4):2558-2568.

Wang, S. Y., Liu, J., Bo, R., Chen, Y. H. (2022). Approximating Input-Output Curve of Pumped Storage Hydro Plant: A Disjunctive Convex Hull Method, *IEEE Transactions on Power Systems*, DOI: 10.1109/TPWRS.2022.3158629

Wang, S. Y., Liu, J., Chen, H. T., Bo, R., Chen, Y. H. (2021). Modeling state transition and head-dependent efficiency curve for pumped storage hydro in look-ahead dispatch. *IEEE Transactions on power systems*, 36(6): 5396-5407.

Wang, X. Y., Vilathgamuwa, D. M., Choi, S. S. (2008). Determination of Battery Storage Capacity in Energy Buffer for Wind Farm. *IEEE Transaction on Energy Conversion*, 23(3):868-878.

Will, F. A., Cole, W., Denholm, P., Machen, S., Gates, N., Blair, N. (2021). Storage Futures Study: Economic Potential of Diurnal Storage in the U.S. Power Sector. Golden, CO: National Renewable Energy Laboratory. NREL/TP-6A20-77449. https://www.nrel.gov/docs/fy21osti/77449.pdf.

Williams, J. C., Wright, B. D. (1991). Storage and Commodity Markets. Cambridge University Press, Cambridge, UK.

Xiao, Y. Z., Sun, W., Sun L. (2021). Dynamic programming based economic day-ahead scheduling of integrated tri-generation energy system with hybrid energy storage. *Journal of Energy Storage*, 44 (A):103395. https://doi.org/10.1016/j.est.2021.103395

Xu, B. L., Wang, Y. S., Dvorkin, Y., Fern´andez-Blanco, R., Silva-Monroy C. A., Watson, J. P., Kirschen, D. S. (2017). Scalable Planning for Energy Storage in Energy and Reserve Markets. IEEE Transaction on Power Systems, 32(6):4515-4527.

Xu, X., Hu, W., Cao, D., Qi, H., Zhou, L., Wen, L., Zhe, C., Frede, B. (2020). Scheduling of wind-battery hybrid system in the electricity market using distributionally robust optimization. *Renewable Energy*, 156, 47-56.

Yang, W. D., Wang, J.Z., Niu T., Du, P. (2020) A novel system for multi-step electricity price forecasting for electricity market management. *Applied Soft Computing*, 88, 106029.

Yu, H. S., Engelkemier, S., Gencer, E. (2022). Process improvements and multi-objective optimization of compressed air energy storage (CAES) system. *Journal of Cleaner Production*, 335, 130081. https://doi.org/10.1016/j.jclepro.2021.130081

Zhang, L., Wirth, A. (2010). Wind energy management with battery storage. *Journal of Operational Research Society*, 61(10):1510-1522.

Zhang, Y., Rahbari-Asr, N., Duan, J., Chow, M. Y. (2016). Day-Ahead Smart Grid Cooperative Distributed Energy Scheduling With Renewable and Storage Integration. *IEEE Transaction on sustainable energy*,7(4):1739-1748.

Zhou, Y., Scheller-Wolf, A., Secomandi, N., Smith, S. (2016). Electricity trading and negative prices: storage vs. disposal. *Management Science*, 62(3): 880-898.

Zhou, Y., Scheller-Wolf, A., Secomandi, N., Smith, S. (2019). Managing Wind-Based Electricity Generation in the Presence of Storage and Transmission Capacity. *Production and Operations*

Management, 28(4): 970-989.

Appendix: Table 1

Table1: A comparative summary of this study and previous publication

Literature	Objective		Price-maker	Problem	model
	Renewable	ESS			
Al-Kanj et al., 2020	×	✓	×	Storage arbitrage	ADP
Bafrani et al.(2021)	×	✓	×	ISO operation	MINLP
Baslis et al. (2011)	✓	×	✓	Market impact	MILP
Bhoi et al.(2020)	✓	✓	×	Optimal scheduling	DP
Castronuovo et al.(2004)	✓	✓	×	Operation	Discrete optimization
Chabok et al.(2019)	×	✓	✓	Operation of power system	Stochastic optimization
Dui et al.(2018)	×	✓	×	Storage design	SOCP
He et al.(2022)	✓	✓	×	Multi-objective co-optimization	MOEA-DM
Habibian et al. (2020)	×	✓	✓	Power purchase decision	Stochastic programming
Heine et al.(2021)	×	✓	×	Community optimization	MILP
Huang et al.(2019)	×	✓	✓	Market mechanisms design	Bi-level
Huang et al.(2019)	×	✓	×	Storage operation and investment	Bi-level
Kim and Powell (2011)	✓	✓	×	Economic dispatch	MILP
Lee (2008)	✓	✓	×	Short-term scheduling	MIPSO
Levieux et al. (2019)	✓	×	×	RE complementary operation	HA
Liu et al.(2015)	✓	✓	×	Schedule	ANN
Liu et al. (2021a)	×	✓	✓	Economic dispatch	DP
Liu et al. (2022)	✓	✓	×	Economic dispatch	DP
Jiang and Powell（2015a)	✓	✓	×	Economic dispatch	ADP
Secomandi (2010)	×	✓	×	Economic dispatch	DP
Shi et al.(2022)	✓	✓	×	Generation and capacity	MILP
Shi et al.(2018)	✓	✓	×	Design and operation	QI
Steeger et al. (2018)	✓	×	✓	Bidding optimization	SDDP

Taghikhani et al.(2021)	✓	✓	×	Optimal scheduling	MILP
Zhang et al. (2016)	✓	✓	×	Economic dispatch	Fully distributed
Zhou et al. (2016)	×	✓	×	Economic dispatch	DP
Zhou et al. (2019)	✓	✓	×	Economic dispatch	DP
This study	✓	✓	✓	Economic dispatch	DP

Appendix A: Optimal Scheduling for Electricity Merchant

Proof of Lemma 1:

1) The uniqueness of the SOC references points:

Based on the equation (5), by replacing q_t with SOC_{t+1} as the decision variable via $SOC_{t+1}/\varphi_t - SOC_t = q_t$, we get the following rewards function.

$$R(q_t, w_t, P_t) = \begin{cases} -P_t(q_t/\theta - w_t)/\sigma - \lambda P_t/\sigma^2[(q_t/\theta)^2 - 2(q_t/\theta)w_t + w_t{}^2] - c^p(q_t/\theta\sigma) - c_w w_t \ (q_t \geq \theta w_t) \\ -P_t(q_t/\theta - w_t)\cdot\sigma - \lambda P_t\sigma^2[(q_t/\theta)^2 - 2(q_t/\theta)w_t + w_t{}^2] - c^p(q_t/\theta\sigma) - c_w w_t \ (0 \leq q_t \leq \theta w_t) \\ -P_t(q_t\theta - w_t)\cdot\sigma - \lambda P_t\sigma^2[(q_t\xi)^2 - 2(q_t\xi)w_t + w_t{}^2] + c^g q_t\xi\sigma - c_w w_t \ (q_t \leq 0) \end{cases} \tag{A1}$$

In the end of the decision time T (i.e., the beginning of decision time T+1), the value function is shown:

$$V_T(S(T)) = [R(q_T, w_T, p_T) + E[V_{T+1}(S(T+1)|S(T))] = [R(q_T, w_T, p_T) + VOW_{T+1}\cdot SOC_{T+1}]$$

Thus, we get the following three sub-optimization value functions:

$$\begin{cases} V_T^{(1)*}(S(T)) = \max\limits_{\underline{S}\leq SOC_{T+1}\leq\bar{S}} \left\{ -\frac{\lambda P_T}{\theta^2\sigma^2} q_T{}^2 + (\frac{2\lambda P_T w_T}{\theta\sigma^2} - \frac{P_t + c^p}{\theta\sigma})q_T - w_T(\frac{\lambda P_T}{\sigma^2}w_T - \frac{P_T}{\sigma} + c_w) + E[V_{T+1}^*(S(T+1)|S(T))] \right\} \\ V_T^{(2)*}(S(T)) = \max\limits_{\underline{S}\leq SOC_{T+1}\leq\bar{S}} \left\{ -\frac{\lambda\rho^2 P_T}{\sigma^2} q_T{}^2 + (2\lambda P_T\sigma^2\frac{w_T}{\theta} - \frac{P_T\sigma^2 + c^p}{\theta\sigma})q_T - w_T[P_T\sigma(\lambda w_T\sigma - 1) + c_w] + E[V_{T+1}^*(S(T+1)|S(T))] \right\} \\ V_T^{(3)*}(S(T)) = \max\limits_{\underline{S}\leq SOC_{T+1}\leq\bar{S}} \left\{ -\lambda P_T\xi^2\sigma^2 q_T{}^2 + (2\lambda P_T w_T\xi\sigma^2 - P_T\xi\sigma - c^g\xi\sigma)q_T - w_T[P_T\sigma(\lambda w_T\sigma - 1) + c_w] + E[V_{T+1}^*(S(T+1)|S(T))] \right\} \end{cases} \tag{A2}$$

We can get the optimal results to the equation (A3) by removing the given state $S(T)$ (i.e., the given values SOC_T, w_T, and P_T) when maximizing the (A2) . So, we get the following equivalent equations:

$$\begin{cases} V_T^{(1)*}(S(T)) = \max\limits_{\underline{S}\leq SOC_{T+1}\leq\bar{S}} \left\{ -\frac{\lambda P_T}{\theta^2\sigma^2}\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right)^2 + (\frac{2\lambda P_T w_T}{\theta\sigma^2} - \frac{P_t + c^p}{\theta\sigma})\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right) + VOW_{T+1}\cdot SOC_{T+1} \right\} \\ V_T^{(2)*}(S(T)) = \max\limits_{\underline{S}\leq SOC_{T+1}\leq\bar{S}} \left\{ -\frac{\lambda\rho^2 P_T}{\theta^2}\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right)^2 + (2\lambda P_T\sigma^2\frac{w_T}{\theta} - \frac{P_T\sigma^2 + c^p}{\theta\sigma})\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right) + VOW_{T+1}\cdot SOC_{T+1} \right\} \\ V_T^{(3)*}(S(T)) = \max\limits_{\underline{S}\leq SOC_{T+1}\leq\bar{S}} \left\{ -\lambda P_T\xi^2\sigma^2\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right)^2 + (2\lambda P_T w_T\xi\sigma^2 - P_T\xi\sigma - c^g\xi\sigma)\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right) + VOW_{T+1}\cdot SOC_{T+1} \right\} \end{cases} \tag{A3}$$

The first-order derivative of $V_T^*(S(T))$ (i.e., best response functions) on SOC_{T+1} are shown as:

$$\begin{cases} \frac{\partial V_T^{(1)*}(S(T))}{\partial SOC_{T+1}} = -2\frac{\lambda P_T}{\theta^2\sigma^2}\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right)\frac{1}{\varphi_T} + (\frac{2\lambda P_T w_T}{\theta\sigma^2} - \frac{P_t + c^p}{\theta\sigma})\frac{1}{\varphi_T} + VOW_{T+1} \\ \frac{\partial V_T^{(2)*}(S(T))}{\partial SOC_{T+1}} = -2\frac{\lambda\sigma^2 P_T}{\theta^2}\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right)\frac{1}{\varphi_T} + (2\lambda P_T\sigma^2\frac{w_T}{\theta} - \frac{P_T\sigma^2 + c^p}{\theta\sigma})\frac{1}{\varphi_T} + VOW_{T+1} \\ \frac{\partial V_T^{(3)*}(S(T))}{\partial SOC_{T+1}} = -2\lambda P_T\xi^2\sigma^2\left(\frac{SOC_{T+1}}{\varphi_T} - SOC_T\right)\frac{1}{\varphi_T} + (2\lambda P_T w_T\xi\sigma^2 - P_T\xi\sigma - c^g\xi\sigma)\frac{1}{\varphi_T} + VOW_{T+1} \end{cases} \quad (A4)$$

We also obtain the following second-order derivative functions of $V_T^*(S(T))$ on SOC_{T+1}.

$$\frac{\partial^2 V_T^{(1)*}(S(T))}{\partial {SOC_{T+1}}^2} = \left(-2\frac{\lambda P_T}{\theta^2\sigma^2}\frac{1}{{\varphi_T}^2}\right) < 0;\ \frac{\partial^2 V_T^{(2)*}(S(T))}{\partial {SOC_{T+1}}^2} = -2\frac{\lambda\sigma^2 P_T}{\theta^2}\frac{1}{{\varphi_T}^2} < 0;\ \frac{\partial^2 V_T^{(3)*}(S(T))}{\partial {SOC_{T+1}}^2} = -2\lambda P_T\xi^2\sigma^2\frac{1}{{\varphi_T}^2} < 0 \quad (A5)$$

Because the second-order derivative function is negative, we can achieve the unique optimal results by using the first-order function. Therefore, $V_T^{(1)*}(S(t))$, $V_T^{(2)*}(S(t))$,and $V_T^{(3)*}(S(t))$ have a unique optimal solution on $SOC_{T+1} \in [\underline{S}, \bar{S}]$. Then the Bellman equation and Puterman (1994) can be used to derive the following results:

$$\begin{cases} SOC_{T+1}^{(1)*} = \arg\max_{\underline{S} \le SOC_{T+1} \le \bar{S}}\left(E[V_{T+1}^*(S(T+1)\,|\,S(T))] - \frac{\lambda P_t}{\theta^2\sigma^2}(\frac{SOC_{T+1}}{\varphi_T} - SOC_T)^2 + (\frac{2\lambda P_T w_T}{\theta\sigma^2} - \frac{P_T + c^p}{\theta\sigma})\frac{SOC_{T+1}}{\varphi_T}\right) \\ SOC_{T+1}^{(2)*} = \arg\max_{\underline{S} \le SOC_{T+1} \le \bar{S}}\left(E[V_{T+1}^*(S(T+1)\,|\,S(T))] - \frac{\lambda\sigma^2 P_T}{\theta^2}(\frac{SOC_{T+1}}{\varphi_T} - SOC_T)^2 + (2\lambda P_T\rho^2\frac{w_T}{\theta} - \frac{P_T\sigma^2 + c^p}{\theta\sigma})\frac{SOC_{T+1}}{\varphi_T}\right) \\ SOC_{T+1}^{(3)*} = \arg\max_{\underline{S} \le SOC_{T+1} \le \bar{S}}\left(E[V_{T+1}^*(S(T+1)\,|\,S(T))] - \lambda P_T\xi^2\sigma^2(\frac{SOC_{T+1}}{\varphi_T} - SOC_T)^2 + (2\lambda P_T w_T\xi\sigma^2 - P_T\xi\sigma + c^g\xi\sigma)\frac{SOC_{T+1}}{\varphi_T}\right) \end{cases} \quad (A6)$$

Similarly, for the any state at $t \in \{1,2,\cdots,T\}$,by maximizing of the value function $V_t(SOC_t, w_t, P_t)$, subject to $SOC_{t+1} \in [\underline{S}, \bar{S}]$, we will obtain the following optimal functions based on the Bellman equation.

$$
\begin{cases}
V_t^{(1)*}(S(t)) = \max_{\underline{S}\le SOC_{t+1}\le \overline{S}} \left\{ -\frac{\lambda P_t}{\theta^2\sigma^2} q_t^{\ 2} + (\frac{2\lambda P_t w_t}{\theta\sigma^2} - \frac{P_t + c^p}{\theta\sigma}) q_t - w_t(\frac{\lambda P_t}{\sigma^2} w_t - \frac{P_t}{\sigma} + c_w) + E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] \right\} \\
\quad \Leftrightarrow \max_{\underline{S}\le SOC_{t+1}\le \overline{S}} \left(E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] - \frac{\lambda P_t}{\theta^2\sigma^2} \frac{SOC_{t+1}^{\ \ 2}}{\varphi_t^{\ 2}} + \frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t} SOC_{t+1}\cdot SOC_t + (\frac{2\lambda P_t w_t}{\theta\sigma^2} - \frac{P_t + c^p}{\theta\sigma}) \frac{SOC_{t+1}}{\varphi_t} \right) \qquad \text{(A7-1)} \\
V_t^{(2)*}(S(t)) = \max_{\underline{S}\le SOC_{t+1}\le \overline{S}} \left\{ -\frac{\lambda\sigma^2 P_t}{\theta^2} q_t^{\ 2} + (2\lambda P_t\sigma^2 \frac{w_t}{\theta} - \frac{P_t\sigma^2 + c^p}{\theta\sigma}) q_t - w_t[P_t\sigma(\lambda w_t\sigma - 1) + c_w] + E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] \right\} \\
\quad \Leftrightarrow \max_{\underline{S}\le SOC_{t+1}\le \overline{S}} \left(E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] - \frac{\lambda\rho^2 P_t}{\theta^2} \frac{SOC_{t+1}^{\ \ 2}}{\varphi_t^{\ 2}} + \frac{2\lambda\sigma^2 P_t}{\theta^2} \frac{SOC_{t+1}}{\varphi_t} SOC_t + (2\lambda P_t\sigma^2 \frac{w_t}{\theta} - \frac{P_t\sigma^2 + c^p}{\theta\sigma}) \frac{SOC_{t+1}}{\eta_t} \right) \qquad \text{(A7-2)} \\
V_t^{(3)*}(S(t)) = \max_{\underline{S}\le SOC_{t+1}\le \overline{S}} \left\{ -\lambda P_t\xi^2\sigma^2 q_t^{\ 2} + (2\lambda P_t w_t\xi\sigma^2 - P_t\xi\sigma + c^g\xi\sigma) q_t - w_t[P_t\sigma(\lambda w_t\sigma - 1) + c_w] + E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] \right\} \\
\quad \Leftrightarrow \max_{\underline{S}\le SOC_{t+1}\le \overline{S}} \left(E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)] - \lambda P_t\xi^2\sigma^2 \frac{SOC_{t+1}^{\ \ 2}}{\varphi_t^{\ 2}} + 2\lambda P_t\xi^2\sigma^2 \frac{SOC_{t+1}}{\varphi_t} SOC_t + (2\lambda P_t w_t\xi\sigma^2 - P_t\xi\sigma + c^g\xi\sigma) \frac{SOC_{t+1}}{\varphi_t} \right) \text{(A7-3)}
\end{cases}
$$

Based on the proof at last decision period T, we know that for every optimization period $t \in \{1, 2, \cdots, T\}$, and in every state t, both $V_t(S(t))$ and $E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]$ are concave functions on $SOC_t \in [\underline{S},\ \overline{S}]$ for any given decision state $S(t) = S_t(SOC_t, w_t, P_t)$. Clearly, $E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]$ and functions (A7-1)-(A7-3) are concave in $SOC_{t+1} \in [\underline{S},\ \overline{S}]$ for each given state $S(t) = S_t(SOC_t, w_t, P_t)$ by using

$$
\begin{aligned}
\frac{\partial E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]}{\partial SOC_{t+1}^{\ \ 2}} &= \frac{\partial\left(\frac{\partial E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]}{\partial SOC_{t+1}}\right)}{\partial SOC_{t+1}} = \frac{\partial\left(\frac{\partial E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]}{\partial SOC_t}\cdot\frac{\partial SOC_t}{\partial SOC_{t+1}}\right)}{\partial SOC_t}\cdot\frac{\partial SOC_t}{\partial SOC_{t+1}} \\
&= \left(\frac{\partial E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]}{\partial SOC_t^{\ 2}}\cdot\frac{\partial SOC_t}{\partial SOC_{t+1}} + \frac{\partial E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]}{\partial SOC_t}\cdot\frac{\partial SOC_t}{\partial SOC_{t+1}\partial SOC_t}\right)\cdot\frac{\partial SOC_t}{\partial SOC_{t+1}} \\
&= \left(\frac{\partial E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]}{\partial SOC_t^{\ 2}}\cdot\left(\frac{\partial SOC_t}{\partial SOC_{t+1}}\right)^2\right) \le 0
\end{aligned}
$$

(1) When $q_t > \theta w_t$, by maximizing the equation (A7-1), subject to $SOC_{t+1} \in [\underline{S},\ \overline{S}]$, we can also get the following best response function (i.e., first-order derivative):

$$
\frac{\partial V_t^{(1)*}(S(t))}{\partial SOC_{t+1}} = \frac{\partial E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]}{\partial SOC_{t+1}} - \frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t^{\ 2}} SOC_{t+1} + \frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t} SOC_t + \frac{2\lambda P_t w_t}{\theta\sigma^2\varphi_t} - \frac{P_t + c^p}{\theta\sigma\varphi_t}
$$

The second-order derivative function: $\frac{\partial V_t^{(1)*}(S(t))}{\partial SOC_{t+1}^2} = \frac{\partial E[V_{t+1}^{*}\left(S(t+1)\,|\,S(t)\right)]}{\partial SOC_{t+1}^2} - \frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t^{\ 2}} < 0.$

Thus, we can achieve the optimal references points solutions using the first-order function because the second-order derivative is negative. Therefore, we will obtain the subsequent optimal consequences:

$$\begin{cases} SOC_{t+1}^{(1)*} = \arg\max_{\underline{S}\le SOC_{t+1}\le \overline{S}} \left(E[V_{t+1}^{*}(S(t+1)\,|\,S(t))] - \frac{\lambda P_t}{\theta^2\sigma^2}\frac{SOC_{t+1}}{\varphi_t^2}^2 + \frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t} SOC_{t+1}\cdot SOC_t + (\frac{2\lambda P_t w_t}{\theta\sigma^2} - \frac{P_t + c^p}{\theta\sigma})\frac{SOC_{t+1}}{\varphi_t} \right) \\ \text{or } \frac{\partial V_t^{(1)*}(S(t))}{\partial SOC_{t+1}} = \frac{\partial E[V_{t+1}^{*}(S(t+1)\,|\,S(t))]}{\partial SOC_{t+1}} - \frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t^2} SOC_{t+1} + \frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t} SOC_t + \frac{2\lambda P_t w_t}{\theta\sigma^2\varphi_t} - \frac{P_t + c^p}{\theta\sigma\varphi_t}\Big|_{SOC_{t+1}=SOC_{t+1}^{(1)*}} = 0 \end{cases} \quad (A8)$$

(2) Similarly, when $0\le q_t \le \theta w_t$, by optimizing the function (A7-2), we can obtain the unique optimal reference points using the first-order function, and the optimal solutions are:

$$\begin{cases} SOC_{t+1}^{(2)*} = \arg\max_{\underline{S}\le SOC_{t+1}\le \overline{S}} \left(E[V_{t+1}^{*}(S(t+1)\,|\,S(t))] - \frac{\lambda\sigma^2 P_t}{\theta^2}\frac{SOC_{t+1}}{\varphi_t^2}^2 + \frac{2\lambda\sigma^2 P_t}{\theta^2\varphi_t} SOC_{t+1}\cdot SOC_t + (2\lambda P_t\sigma^2\frac{w_t}{\theta} - \frac{P_t\sigma^2 + c^p}{\theta\sigma})\frac{E_{t+1}}{\varphi_t} \right) \\ \text{or } \frac{\partial V_t^{(2)*}(S(t))}{\partial SOC_{t+1}} = \frac{\partial E[V_{t+1}^{*}(S(t+1)\,|\,S(t))]}{\partial SOC_{t+1}} - \frac{2\lambda\sigma^2 P_t}{\theta^2\varphi_t^2} E_{t+1} + \frac{2\lambda\sigma^2 P_t}{\theta^2\varphi_t} SOC_t - (2\lambda P_t\sigma^2\frac{w_t}{\theta} - \frac{P_t\sigma^2 + c^p}{\theta\sigma})\frac{1}{\varphi_t}\Big|_{SOC_{t+1}=SOC_{t+1}^{(2)*}} = 0 \end{cases} \quad (A9)$$

(3) When $q_t < 0$, by optimizing the function (A7-3), we will obtain the optimal SOC results as follows:

$$\begin{cases} SOC_{t+1}^{(3)*} = \arg\max_{\underline{S}\le SOC_{t+1}\le \overline{S}} \left(E[V_{t+1}^{*}(S(t+1)\,|\,S(t))] - \lambda P_t\xi^2\sigma^2\frac{E_{t+1}}{\varphi_t^2}^2 + 2\lambda P_t\xi^2\sigma^2\frac{E_{t+1}}{\varphi_t}E_t + (2\lambda P_t w_t\xi\sigma^2 - P_t\xi\sigma + c^g\xi\sigma)\frac{SOC_{t+1}}{\varphi_t} \right) \\ \text{or } \frac{\partial V_t^{(3)*}(S(t))}{\partial SOC_{t+1}} = \frac{\partial E[V_{t+1}^{*}(S(t+1)\,|\,S(t))]}{\partial SOC_{t+1}} - \frac{2\lambda P_t\xi^2\sigma^2}{\varphi_t^2} SOC_{t+1} + \frac{2\lambda P_t\xi^2\sigma^2}{\varphi_t} SOC_t + \frac{2\lambda P_t w_t\xi\sigma^2 - P_t\xi\sigma + c^g\xi\sigma}{\eta_t}\Big|_{SOC_{t+1}=SOC_{t+1}^{(3)*}} = 0 \end{cases}$$

(A10)

2) **The relations among three SOC optimal results/SOC reference points:**

We define three auxiliary functions based on (A8) - (A10) to simplify illumination.

$$\left\{ F(SOC_{t+1})^{(1)} = \frac{\partial V_t^{(1)*}(S(t))}{\partial SOC_{t+1}};\ F(SOC_{t+1})^{(2)} = \frac{\partial V_t^{(2)*}(S(t))}{\partial SOC_{t+1}};\ F(SOC_{t+1})^{(3)} = \frac{\partial V_t^{(3)*}(S(t))}{\partial SOC_{t+1}} \right. \quad (A11)$$

The related first-order functions of (A11) correspond to the second-order derivative functions of (A7-1), (A7-2), and (A7-3) are shown:

$$\begin{cases} \frac{\partial F(SOC_{t+1})^{(1)}}{\partial SOC_{t+1}} = \frac{\partial V_t^{(1)*}(S(t))}{\partial SOC_{t+1}^2} = \frac{\partial E[V_{t+1}^{*}(S(t+1)\,|\,S(t))]}{\partial SOC_{t+1}^2} - \frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t^2} < 0 \\ \frac{\partial F(SOC_{t+1})^{(2)}}{\partial SOC_{t+1}} = \frac{\partial V_t^{(2)*}(S(t))}{\partial SOC_{t+1}^2} = \frac{\partial E[V_{t+1}^{*}(S(t+1)\,|\,S(t))]}{\partial SOC_{t+1}^2} - \frac{2\lambda\sigma^2 P_t}{\theta^2\varphi_t^2} < 0 \\ \frac{\partial F(SOC_{t+1})^{(3)}}{\partial SOC_{t+1}} = \frac{\partial V_t^{(3)*}(S(t))}{\partial SOC_{t+1}^2} = \frac{\partial E[V_{t+1}^{*}(S(t+1)\,|\,S(t))]}{\partial SOC_{t+1}^2} - \frac{2\lambda P_t\xi^2\sigma^2}{\varphi_t^2} < 0 \end{cases} \quad (A12)$$

Based on (A12), we find that the above three defined auxiliary functions are all decreasing with $SOC_{t+1} \in [\underline{S},\ \overline{S}]$. We also get the following relations among the first-order functions of (A11).

$$\left|\partial F(SOC_{t+1})^{(1)}/\partial SOC_{t+1}\right| \ge \left|\partial F(SOC_{t+1})^{(2)}/\partial SOC_{t+1}\right| \ge \left|\partial F(SOC_{t+1})^{(3)}/\partial SOC_{t+1}\right|.$$

1) For all $SOC_{t+1} \in [\underline{S},\ \overline{S}]$, if $\max F(SOC_{t+1})^{(1)} \le \max F(SOC_{t+1})^{(2)}$, then we will obtain $SOC_{t+1}^{(1)*} \le SOC_{t+1}^{(2)*}$.

$$\max F(SOC_{t+1})^{(1)} = F(SOC_{t+1} = \underline{S})^{(1)} \le \max F(SOC_{t+1})^{(2)} = F(SOC_{t+1} = \underline{S})^{(2)}$$
$$\Leftrightarrow \left(-\frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t^{\,2}}\underline{S} + \frac{2\lambda P_t}{\theta^2\sigma^2\varphi_t}SOC_t + \frac{2\lambda P_t w_t}{\theta\sigma^2\varphi_t} - \frac{P_t + c^p}{\theta\sigma\varphi_t} \right) \le \left(-\frac{2\lambda\sigma^2 P_t}{\theta^2\varphi_t^{\,2}}\underline{S} + \frac{2\lambda\sigma^2 P_t}{\theta^2\varphi_t}SOC_t + 2\lambda P_t\sigma^2\frac{w_t}{\theta}\frac{1}{\varphi_t} - \frac{P_t\sigma^2 + c^p}{\theta\sigma}\frac{1}{\varphi_t} \right)$$
$$\Leftrightarrow \frac{P_t + c^p}{\theta\sigma} - \frac{P_t\sigma^2 + c^p}{\theta\sigma} \ge \frac{2\lambda P_t}{\theta^2}SOC_t\left(\frac{1}{\sigma^2} - \sigma^2\right) - \frac{2\lambda P_t}{\theta^2\varphi_t}\left(\frac{1}{\sigma^2} - \sigma^2\right)\underline{S} + 2\lambda P_t\frac{w_t}{\theta}\left(\frac{1}{\sigma^2} - \sigma^2\right)$$
$$\Rightarrow \lambda \le \frac{P_t(1-\sigma^2)}{\theta\sigma} \Big/ \frac{2P_t}{\theta^2}\left(\frac{1}{\sigma^2} - \sigma^2\right)\left(SOC_t + w_t\theta - \frac{\underline{S}}{\varphi_t}\right) = \overline{\lambda}_t^{(1,2)} = P_t \Big/ \frac{2P_t}{\theta}\left(\frac{1+\sigma^2}{\sigma}\right)\left(SOC_t + w_t\theta - \frac{\underline{S}}{\varphi_t}\right)$$

2) For all $SOC_{t+1} \in [\underline{S},\ \overline{S}]$, if $\max F(SOC_{t+1})^{(2)} \le \max F(SOC_{t+1})^{(3)}$, then we will obtain $SOC_{t+1}^{(2)*} \le SOC_{t+1}^{(3)*}$.

$$\max F(SOC_{t+1})^{(2)} = F(SOC_{t+1} = \underline{S})^{(2)} \le \max F(SOC_{t+1})^{(3)} = F(SOC_{t+1} = \underline{S})^{(3)}$$
$$\Leftrightarrow \left(-\frac{2\lambda\sigma^2 P_t}{\theta^2\varphi_t^{\,2}}\underline{S} + \frac{2\lambda\sigma^2 P_t}{\theta^2\varphi_t}E_t + 2\lambda P_t\sigma^2\frac{w_t}{\theta}\frac{1}{\varphi_t} - \frac{P_t\sigma^2 + c^p}{\theta\sigma}\frac{1}{\varphi_t} \right) \le \left(-\frac{2\lambda P_t\xi^2\sigma^2}{\varphi_t^{\,2}}\underline{S} + \frac{2\lambda P_t\xi^2\sigma^2}{\varphi_t}SOC_t + \frac{2\lambda P_t w_t\xi\sigma^2}{\varphi_t} - \frac{(P_t\xi\sigma - c^g\xi\sigma)}{\varphi_t} \right)$$
$$\Leftrightarrow \lambda 2\sigma^2 P_t\left((SOC_t - \frac{\underline{S}}{\varphi_t})(\frac{1}{\theta^2} - \xi^2) + (\frac{w_t}{\theta} - w_t\xi) \right) \le \left(\frac{P_t\sigma^2 + c^p}{\theta\rho} - (P_t\xi\sigma - c^g\xi\sigma) \right)$$
$$\Rightarrow \lambda \le \left(\frac{P_t\sigma^2 + c^p}{\theta\sigma} - (P_t\xi\sigma - c^g\xi\sigma) \right) \Big/ 2\rho^2 P_t\left((SOC_t - \frac{\underline{S}}{\varphi_t})(\frac{1}{\theta^2} - \xi^2) + w_t(\frac{1}{\theta} - \xi) \right) = \overline{\lambda}_t^{(2,3)}$$

If both the available wind generation equals zero (i.e., $w_t = 0$), and the current energy inventory reaches the minimum limit of storage (i.e., $SOC_t - \underline{S}/\varphi_t = 0$) at optimization period t, for any forecasted price $P_t \ge 0$ and market impact of energy storage $\lambda \ge 0$, there exists $SOC_{t+1}^{(1)*} \le SOC_{t+1}^{(2)*} \le SOC_{t+1}^{(3)*}$.

To sum up, for positive prices $P_t \ge 0$, when the market impact of energy storage meets condition $0 \le \lambda \le \min\{\overline{\lambda}_t^{(1,2)}, \overline{\lambda}_t^{(2,3)}\}$, thus, we can get the following relations among three optimal SOC reference points:

$$SOC_{t+1}^{(1)*} \le SOC_{t+1}^{(2)*} \le SOC_{t+1}^{(3)*}$$

Obviously, if $P_t \le 0$, we get $SOC_{t+1}^{(1)*} \ge SOC_{t+1}^{(2)*} \ge SOC_{t+1}^{(3)*}$ when there is $0 \le \lambda \le \min\left\{\overline{\lambda}_t^{(1,2)}, \overline{\lambda}_t^{(2,3)}\right\}$.

Proof of Proposition 1:

(1) Optimal Solutions (without consider the capacity of transmission line):

1) $\theta w_t < \min\{SOC_{t+1}^{(1)*}, \overline{Q}^p\}$

$$q_t^*(S_t) = \begin{cases} \min\{SOC_{t+1}^{(1)*} - SOC_t, \overline{Q}^p\},\ SOC_t \in [\underline{S}, SOC_{t+1}^{(1)*} - \theta w_t] (\text{store generation and buy electricity up to } SOC_{t+1}^{(1)*}) \\ \min\{SOC_{t+1}^{(2)*} - SOC_t, \theta w_t\}, SOC_t \in (SOC_{t+1}^{(1)*} - \theta w_t, SOC_{t+1}^{(2)*}] (\text{store generation without buying up to } SOC_{t+1}^{(2)*}) \\ 0,\ SOC_t \in (SOC_{t+1}^{(2)*}, SOC_{t+1}^{(3)*}]\ (\text{keep inventory unchanged}) \\ \max\{SOC_{t+1}^{(3)*} - SOC_t, -\overline{Q}^g\}, SOC_t \in (SOC_{t+1}^{(3)*}, \overline{S}] (\text{sell inventory down to } SOC_{t+1}^{(3)*}) \end{cases} \quad (A13)$$

2) $\theta w_t \geq \min\{SOC^{(1)*}_{t+1}, \overline{Q}^p\}$

$$q_t^*(S_t) = \begin{cases} \min\{SOC^{(2)*}_{t+1} - SOC_t, \overline{Q}^p, \theta w_t\}, SOC_t \in [\underline{S}, SOC^{(2)*}_{t+1}] (\text{store generation up to } SOC^{(2)*}_{t+1}) \\ 0,\ SOC_t \in [SOC^{(2)*}_{t+1}, SOC^{(3)*}_{t+1}]\ (\text{keep inventory unchanged}) \\ \max\{SOC^{(3)*}_{t+1} - SOC_t, -\overline{Q}^g\}, SOC_t \in (SOC^{(3)*}_{t+1}, \overline{S}] (\text{sell inventory down to } SOC^{(3)*}_{t+1}) \end{cases} \quad (A14)$$

Special case:

a) If $(\theta = \xi = \sigma = 1, c^p = c^g = 0)$**, then we will get** $SOC^{(1)*}_{t+1} = SOC^{(2)*}_{t+1} = SOC^{(3)*}_{t+1} = SOC^*_{t+1}$**.**

1) $\theta w_t < \min\{SOC^*_{t+1}, \overline{Q}^p\}$

$$q_t^*(S_t) = \begin{cases} \min\{SOC^*_{t+1} - SOC_t, \overline{Q}^p\},\ SOC_t \in [\underline{S}, SOC^*_{t+1} - \theta w_t] (\text{store generation and purchased electricity up to } SOC^*_{t+1}) \\ \max\{SOC^*_{t+1} - SOC_t, -\overline{Q}^g\}, SOC_t \in (SOC^*_{t+1}, \overline{S}] (\text{sell inventory down to } SOC^*_{t+1}) \end{cases} \quad (A15)$$

2) $\theta w_t \geq \min\{SOC^*_{t+1}, \overline{Q}^p\}$

$$q_t^*(S_t) = \begin{cases} \min\{SOC^*_{t+1} - SOC_t, \overline{Q}^p, \theta w_t\}, SOC_t \in [\underline{S}, SOC^*_{t+1}] (\text{store generation up to } SOC^*_{t+1}) \\ \max\{SOC^*_{t+1} - SOC_t, -\overline{Q}^g\}, SOC_t \in (SOC^*_{t+1}, \overline{S}] (\text{sell inventory down to } SOC^*_{t+1}) \end{cases} \quad (A16)$$

b) If $\sigma = 1$ **(transmission efficiency), then we will get** $SOC^{(1)*}_{t+1} = SOC^{(2)*}_{t+1}$**.**

1) $\theta w_t < \min\{SOC^{(1)*}_{t+1}, \overline{Q}^p\}$

$$q_t^*(S_t) = \begin{cases} \min\{SOC^{(1)*}_{t+1} - SOC_t, \overline{Q}^p\},\ SOC_t \in [\underline{S}, SOC^{(1)*}_{t+1} - \theta w_t] (\text{store generation and buy power to } SOC^{(1)*}_{t+1}) \\ 0,\ SOC_t \in (SOC^{(2)*}_{t+1}, SOC^{(3)*}_{t+1}]\ (\text{keep inventory unchanged}) \\ \max\{SOC^{(3)*}_{t+1} - SOC_t, -\overline{Q}^g\}, SOC_t \in (SOC^{(3)*}_{t+1}, \overline{S}] (\text{sell power to } SOC^{(3)*}_{t+1}) \end{cases} \quad (A17)$$

2) $\theta w_t \geq \min\{SOC^*_{t+1}, \overline{Q}^p\}$

$$q_t^*(S_t) = \begin{cases} \min\{SOC^{(2)*}_{t+1} - SOC_t, \overline{Q}^p, \theta w_t\}, SOC_t \in [\underline{S}, SOC^{(2)*}_{t+1}] (\text{store generation up to } SOC^{(2)*}_{t+1}) \\ 0, SOC_t \in [SOC^{(2)*}_{t+1}, SOC^{(3)*}_{t+1}] (\text{keep inventory unchanged}) \\ \max\{SOC^{(3)*}_{t+1} - SOC_t, -\overline{Q}^g\}, SOC_t \in (SOC^{(3)*}_{t+1}, \overline{S}] (\text{sell inventory down to } SOC^{(3)*}_{t+1}) \end{cases} \quad (A18)$$

c) If $(\theta = \xi = 1, c^p = c^g = 0)$ **(no pumping and generating loss, no operating cost,** $SOC^{(2)*}_{t+1} = SOC^{(3)*}_{t+1}$**)**

1) $\theta w_t < \min\{SOC^{(1)*}_{t+1}, \overline{Q}^p\}$

$$q_t^*(S_t) = \begin{cases} \min\{SOC^{(1)*}_{t+1} - SOC_t, \overline{Q}^p\},\ SOC_t \in [\underline{S}, SOC^{(1)*}_{t+1} - \theta w_t] (\text{store renewable and buy power up to } SOC^{(1)*}_{t+1}) \\ \min\{SOC^{(2)*}_{t+1} - SOC_t, \theta w_t\}, SOC_t \in (SOC^{(1)*}_{t+1} - \theta w_t, SOC^{(2)*}_{t+1}] (\text{store renewable without buying up to } SOC^{(2)*}_{t+1}) \\ \max\{SOC^{(3)*}_{t+1} - SOC_t, -\overline{Q}^g\}, SOC_t \in (SOC^{(3)*}_{t+1}, \overline{S}] (\text{sell energy down to } SOC^{(3)*}_{t+1}) \end{cases} \quad (A19)$$

2) $\theta w_t \geq \min\{SOC^*_{t+1}, \overline{Q}^p\}$

$$q_t^*(S_t) = \begin{cases} \min\{SOC^{(2)*}_{t+1} - SOC_t, \overline{Q}^p, \theta w_t\}, SOC_t \in [\underline{S}, SOC^{(2)*}_{t+1}] (\text{store renewable generation up to } SOC^{(2)*}_{t+1}) \\ \max\{SOC^{(3)*}_{t+1} - SOC_t, -\overline{Q}^g\}, SOC_t \in (SOC^{(3)*}_{t+1}, \overline{S}] (\text{sell energy down to } SOC^{(3)*}_{t+1}) \end{cases} \quad (A20)$$

Proof of Proposition 2: Market impact and available wind generation analysis

Recall the proof the proposition 1, when the merchant who has PSH and wind plants, for any given state

$S(t)$, we can also obtain the following outcomes:

$$\begin{cases} SOC^{(1)*}_{t+1\ (\lambda>0)} = \arg\max_{\underline{S}\le SOC_{t+1}\le \bar{S}} \left(E[V^*_{t+1}(S(t+1)\,|\,S(t))] - \frac{\lambda P_t}{\theta^2\sigma^2}(\frac{SOC_{t+1}}{\varphi_t} - SOC_t)^2 + (\frac{2\lambda P_t w_t}{\theta\sigma^2} - \frac{P_t + c^p}{\theta\sigma})\frac{SOC_{t+1}}{\varphi_t} \right) \\ SOC^{(2)*}_{t+1\ (\lambda>0)} = \arg\max_{\underline{S}\le SOC_{t+1}\le \bar{S}} \left(E[V^*_{t+1}(S(t+1)\,|\,S(t))] - \frac{\lambda\sigma^2 P_t}{\theta^2}(\frac{SOC_{t+1}}{\varphi_t} - SOC_t)^2 + (2\lambda P_t \rho^2 \frac{w_t}{\theta} - \frac{P_t\sigma^2 + c^p}{\theta\sigma})\frac{SOC_{t+1}}{\varphi_t} \right) \\ SOC^{(3)*}_{t+1\ (\lambda>0)} = \arg\max_{\underline{S}\le SOC_{t+1}\le \bar{S}} \left(E[V^*_{t+1}(S(t+1)\,|\,S(t))] - \lambda P_t \xi^2\sigma^2(\frac{SOC_{t+1}}{\varphi_t} - SOC_t)^2 + (2\lambda P_t w_t \xi\sigma^2 - P_t\xi\sigma + c^g\xi\sigma)\frac{SOC_{t+1}}{\varphi_t} \right) \end{cases}$$

Through the rewards function of (A1), for any positive forecasted prices and decision state $t \in \{1,2,3,\cdots,T\}$, there exist the following relationships:

$$\frac{\partial R(q_t, w_t, P_t)}{\partial\lambda} = \begin{cases} -P_t/\sigma^2 \cdot (q_t/\theta - w_t)^2 \le 0 & (q_t > \theta w_t) \\ -P_t\sigma^2 \cdot (q_t/\theta - w_t)^2 \le 0 & (0 \le q_t < \theta w_t) \\ -P_t\sigma^2 \cdot (q_t\xi - w_t)^2 \le 0 & (q_t < 0) \end{cases} \Rightarrow \frac{\partial R(q_t, w_t, P_t)}{\partial\lambda} \le 0 \tag{A21}$$

Suppose the $q^{*(M)}_{t(\lambda\ge0)}$ (resp. $q^{*(M)}_{t\ (\lambda=0)}$) represents the optimal actions of electricity merchants considering the market impact (resp. without considering market impact) in trading decisions.

Thus, $\sum_{t=1}^{T} R(q^{*(M)}_{t\ (\lambda\ge0)}, w_t, P_t) \ge \sum_{t=1}^{T} R(q^{*(M)}_{t\ (\lambda=0)}, w_t, P_t)$ holds, which means the value function of the merchant $V^*_{t+1\ (\lambda>0)}(S(t+1)\,|\,S(t))$ decreases with the increasing of market impact λ, then there are:

$$\begin{cases} E[V^*_{t+1\ (\lambda\ge0)}(S(t+1)\,|\,S(t))] \le E[V^*_{t+1\ (\lambda=0)}(S(t+1)\,|\,S(t))] \\ \max_{\pi} \sum_{t=1}^{T} E\left[R(q_t, w_t, P_t)_{(\lambda\ge0)}\,|\,S(1)\right] \le \max_{\pi} \sum_{t=1}^{T} E\left[R(q_t, w_t, P_t)_{(\lambda=0)}\,|\,S(1)\right] \end{cases} \tag{A22}$$

Obviously, if a price-maker merchant ignores her market impact in trading decisions and follows the price-taker's optimal economic dispatch, we can draw the following relationship:

$$\max_{\pi} \sum_{t=1}^{T} E\left[R(q_t, w_t, P_t)_{(\lambda\ge0)}\,|\,S(1)\right] = \sum_{t=1}^{T} E\left[R(q^{*(M)}_{t\ (\lambda\ge0)}, w_t, P_t)_{(\lambda\ge0)}\,|\,S(1)\right] \ge \sum_{t=1}^{T} E\left[R(q^{*(M)}_{t\ (\lambda=0)}, w_t, P_t)_{(\lambda\ge0)}\,|\,S(1)\right] \tag{A23}$$

Based on the rewards function of (A1), we will get the following first-order response function:

$$\frac{\partial R(q_t, w_t, P_t)}{\partial w_t} = \begin{cases} P_t/\sigma - \lambda P_t/\sigma^2[-2(q_t/\theta) + 2w_t] - c_w = P_t/\sigma - c_w + 2\lambda P_t/\sigma^2\left(q_t/\theta - w_t\right) & (q_t \ge \theta w_t) \\ P_t\sigma - \lambda P_t\sigma^2[-2(q_t/\theta) + 2w_t] - c_w = P_t\sigma - c_w + 2\lambda P_t\sigma^2\left(q_t/\theta - w_t\right) & (0 \le q_t \le \theta w_t) \\ P_t\sigma - \lambda P_t\sigma^2[-2(q_t\xi) + 2w_t] - c_w = P_t\sigma - c_w + 2\lambda P_t\sigma^2\left(q_t\xi - w_t\right) & (q_t \le 0) \end{cases} \tag{A24}$$

We have the following relationship for $\partial R(q_t, w_t, P_t)/\partial w_t$ based on equation (24).

$$\begin{cases} \partial R_1(q_t, w_t, P_t)/\partial w_t = P_t / \sigma - c_w + 2\lambda P_t/\sigma^2 \left(q_t/\theta - w_t\right) \geq 0 \Rightarrow P_t / \sigma \geq c_w \ \ (q_t \geq \theta w_t) \\ \partial R_2(q_t, w_t, P_t)/\partial w_t = P_t\sigma - c_w + 2\lambda P_t\sigma^2 \left(q_t/\theta - w_t\right) \geq 0 \Rightarrow \lambda \leq (P_t\sigma - c_w)/2P_t\sigma^2 \left(w_t - q_t/\theta\right) < \overline{\lambda}_t^{(1,2)} \ \ (0 \leq q_t \leq \theta w_t) \\ \partial R_3(q_t, w_t, P_t)/\partial w_t = P_t\sigma - c_w + 2\lambda P_t\sigma^2 \left(q_t\xi - 2w_t\right) \geq 0 \Rightarrow \lambda \leq (P_t\sigma - c_w)/2P_t\sigma^2 \left(w_t - q_t\xi\right) < \overline{\lambda}_t^{(2,3)} \ \ (q_t \leq 0) \end{cases}$$

(A25)

It implies that the merchant with PSH and wind plants needs to generate the wind power based on the max generation capacity of the wind turbines installed to benefit her profit.

Next, we will analyze how the operation cost influences the optimal scheduling policy of the energy storage and the revenue of the electricity merchant. Then, based on the rewards function of (A1), we will get the following first-order response function:

$$\frac{\partial R(q_t, w_t, P_t)}{\partial c^p} = \begin{cases} -(q_t/\theta\sigma) \ \ (q_t \geq \theta w_t) \\ -(q_t/\theta\sigma) \ \ (0 \leq q_t \leq \theta w_t) \end{cases}; \frac{\partial R(q_t, w_t, P_t)}{\partial c^g} = q_t\xi\sigma \ \ (q_t \leq 0) \qquad \text{(A26)}$$

Based on the equation (A26), We get the following relationship for the reward functions on the generating and pumping cost.

$$\left\{ \frac{\partial R(q_t, w_t, P_t)}{\partial c^p} \leq 0, \frac{\partial R(q_t, w_t, P_t)}{\partial c^g} \leq 0 \right. \qquad \text{(A27)}$$

It is straightforward; the merchant will achieve less profit from increased operating cost. It plays a similar role as the market impact.